NCERT 10.4.3.6

EE24BTECH11053 - S A Aravind Eswar

Question: The diagonal of a rectangular field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field.

0.1 Theoretical Method:

Let,
longer side = l
shorter side = b
Given,

$$\sqrt{l^2 + b^2} = b + 60 \tag{1}$$

$$l = b + 30 \tag{2}$$

Substituting,

$$b^2 - 60b - 2700 = 0 \tag{3}$$

Solving,

$$b = 90 \tag{4}$$

$$b = -30 \tag{5}$$

But as length cannot be negative,

$$b = 90 \tag{6}$$

From this,

$$l = 120 \tag{7}$$

0.2 Fixed Point iterations:

The most primitive method to calculate roots, Given,

$$x^2 - 60\,x - 2700 = 0 \tag{8}$$

Rearranging, we get,

$$x = \frac{2700}{x} + 60 \tag{9}$$

$$x = g(x) \tag{10}$$

Now applying Fixed point iteration, $x_{n+1} = g(x_n)$, we get,

$$x_{n+1} = \frac{2700}{x_n} + 60 \tag{11}$$

Taking inital guess as 150, we get,

$$x_n = 89.99999999957511 \tag{12}$$

Tolerance = 10e-10
Number of iterations = 23

0.3 Newton's iterations:

The equation for Newton-Raphson Method is given by,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \tag{13}$$

Given,

$$f(x_n) = x_n^2 - 60\,x_n - 2700 \tag{14}$$

$$f'(x_n) = 2\,x_n - 60 \tag{15}$$

We get,

$$x_{n+1} = x_n - \frac{x_n^2 - 60\,x_n - 2700}{2\,x_n - 60} \tag{16}$$

Taking inital guess,

$$x_0 = 150 \tag{17}$$

We get,

$$x_n = 90.00000000000007 \tag{18}$$

Tolerance = 10e-10
Number of iterations = 5

0.4 Secant Method:

Even though Newton's Method will suffice in this case, it has a few drawbacks. To fix them, we will also look at Secant Method.

We will need 2 inital guesses for this method.

The equationn for Secant method is given by,

$$x_{n+1} = x_n - \frac{f(x_n)(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})} \tag{19}$$

Substituting,

$$x_{n+1} = x_n - \frac{\left(x_n^2 - 60\,x_n - 2700\right)(x_n - x_{n-1})}{(x_n^2 - 60\,x_n - 2700) - \left(x_{n-1}^2 - 60\,x_{n-1} - 2700\right)} \tag{20}$$

Taking inital guesses as,

$$x_0 = 150 \tag{21}$$
$$x_1 = 50 \tag{22}$$

We get,

$$x_n = 90.0 \tag{23}$$

Tolerance = 10e-10
Number of iterations = 7

0.5 Matrix Method

We also have a method to solve it without any iterations with the help of companion matrix.

If the given polynomial is,

$$P(x) = c_0 + c_1\,x + c_2\,x^2 + \cdots + c_{n-1}\,x^{n-1} + x^n \tag{24}$$

Then the companion matrix is given as,

$$M = \begin{pmatrix} 0 & 0 & \dots & 0 & -c_0 \\ 1 & 0 & \dots & 0 & -c_1 \\ 0 & 1 & \dots & 0 & -c_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & -c_{n-1} \end{pmatrix} \tag{25}$$

For our equation,

$$P(x) = -2700 - 60\,x + x^2 \tag{26}$$

The companion matrix is given as,

$$M = \begin{pmatrix} 0 & 2700 \\ 1 & 60 \end{pmatrix} \tag{27}$$

We can find the eigenvalues using QR algorithm.

$$M_n = Q_n\,R_n \tag{28}$$
$$M_{n+1} = R_n\,Q_n \tag{29}$$

The diagonal elements of M_n after enough iterations will give you the eigenvalues.

The eigenvalues of the above matrix are,

$$\lambda_1 = 90 \tag{30}$$
$$\lambda_2 = -30 \tag{31}$$

As the root cannot be negative,

$$x = 90 \tag{32}$$

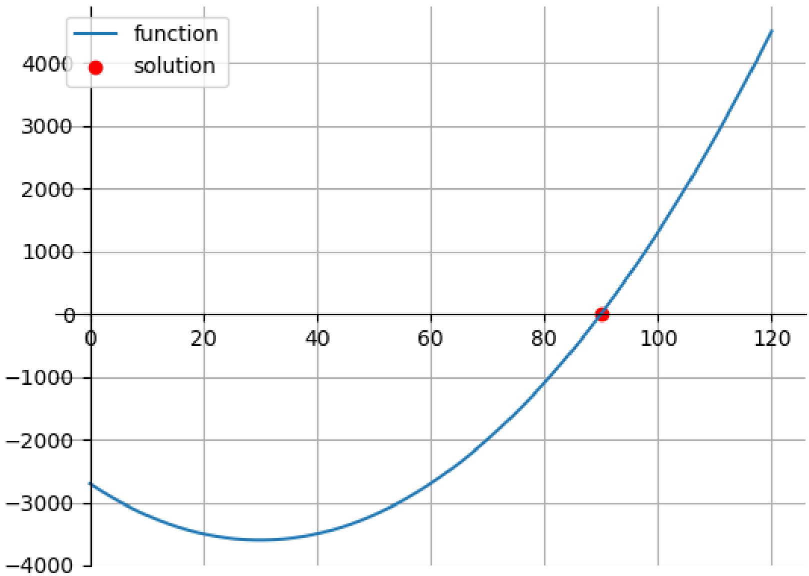

Fig. 0: Verification