NCERT 8.3.19

EE24BTECH11053 - S A Aravind Eswar

Question: The area bound by the y-axis, $y = \cos x$ and $y = \sin x$ when $0 \leq x \leq \frac{\pi}{2}$ is

0.1 Theoretical Solution:

Solving $y = \cos x$ and $y = \sin x$ in the given interval, we can find that they intersect at $x = \frac{\pi}{4}$ Thus, the area can be written as the following integral,

$$A = \int_0^{\pi/2} \min\{\sin x, \cos x\}\, dx \tag{1}$$

$$A = \int_0^{\pi/4} \sin x\, dx + \int_{\pi/4}^{\pi/2} \cos x\, dx \tag{2}$$

Evaluvating the integral, we get,

$$A = 2 - \sqrt{2} \tag{3}$$

0.2 Trapeziodal rule:

The interal can be approximated as,

$$\int_a^b f(x) = \frac{\Delta x}{2} \sum_{k=1}^{N} (f(x_{k-1}) - f(x_k)) \tag{4}$$

where,

$$\Delta x = \frac{b-a}{N} \tag{5}$$

This can be simplified as,

$$\int_a^b f(x) = \Delta x \left(\frac{f(x_0) + f(x_N)}{2} + \sum_{k=1}^{N-1} f(x_k) \right) \tag{6}$$

Thus,

$$\int_0^{\pi/2} \min\{\sin x, \cos x\}\, dx = \Delta x \left(\sum_{k=1}^{N-1} \min\{\sin x_k, \cos x_k\} \right) \tag{7}$$

Taking N as 1000 and solving, we get,

$$Area \approx 0.5852 \tag{8}$$

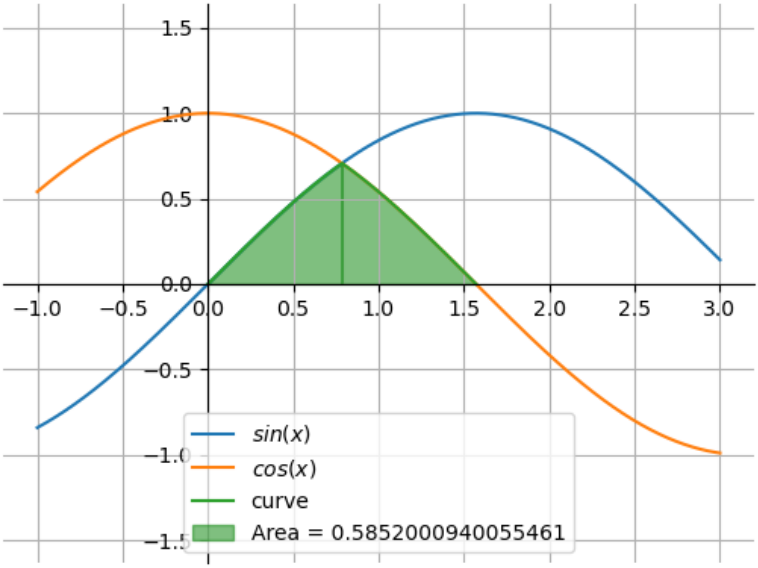

Fig. 0: Verification