NCERT 9.5.1

EE24BTECH11053 - S A Aravind Eswar

Question: Solve the differential equation given below with initial conditions $x = 0$ and $y = 0$.

$$\frac{dy}{dx} + 2y = \sin x \tag{1}$$

1) We can realise that the given equation is a linear differential equation. Then,

$$P = 2 \tag{2}$$

$$Q = \sin x \tag{3}$$

2) Multiplying on both sides with $e^{\int P}$ which is, e^{2x}

$$e^{2x}\frac{dy}{dx} + 2e^{2x}y = \sin x \, e^{2x} \tag{4}$$

3) This can be written as,

$$\frac{d}{dx}\left(y \, e^{2x}\right) = \sin x \, e^{2x} \tag{5}$$

4) Integrating on both sides with respect to dx, we get,

$$y \, e^{2x} = e^{2x}\frac{2\sin x - \cos x}{5} + C \tag{6}$$

5) Diving on both sides with e^{2x} we get,

$$y = \frac{2\sin x - \cos x + C \, e^{-2x}}{5} \tag{7}$$

6) Applying the inital conditions $x = 0$ and $y = 0$, we get,

$$C = 1 \tag{8}$$

7) Thus,

$$y = \frac{2\sin x - \cos x + \, e^{-2x}}{5} \tag{9}$$

is the solution of the given differential equation with given inital conditions

8) **CODING LOGIC:** The solution for the differential equation can be graphically solved using coding by using below logic :
Below is verification:

Algorithm 1 Finite Differnce Algorithm

Inital condition, $x_0 \leftarrow 0$
$y_0 \leftarrow 0$
Number of interations, $interations \leftarrow 20$
Step size, $h = 0.25$
for i in range$(1, interations)$ **do**
$\quad y_i \leftarrow y_{i-1} + \frac{dy}{dx}|_{x_{i-1}, y_{i-1}} h$
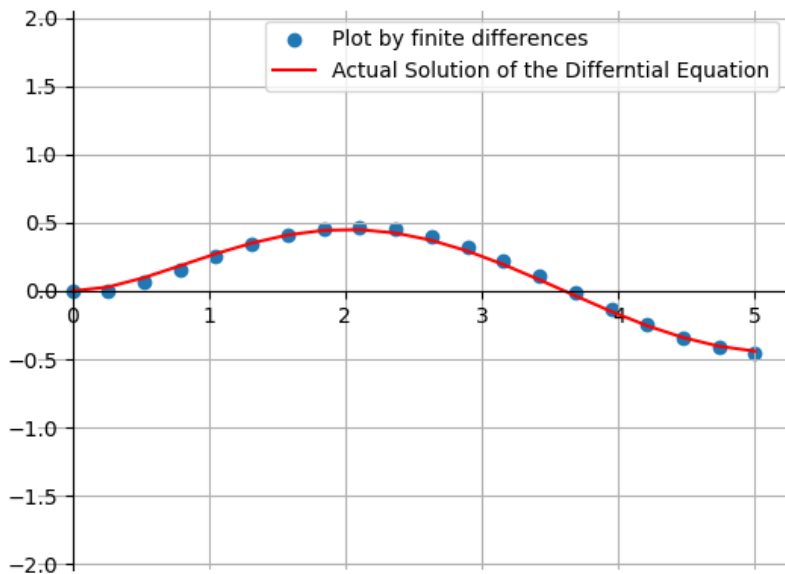$\quad x_i \leftarrow x_{i-1} + h$
end for
plot(x, y)

Fig. 8: Verification