NCERT 9.5.1

EE24BTECH11053 - S A Aravind Eswar

Question: Solve the differential equation given below with initial conditions $x = 0$ and $y = 0$.

$$\frac{dy}{dx} + 2y = \sin x \tag{1}$$

1) We can realise that the given equation is a linear differential equation. Then,

$$P = 2 \tag{2}$$

$$Q = \sin x \tag{3}$$

2) Multiplying on both sides with $e^{\int P}$ which is, e^{2x}

$$e^{2x}\frac{dy}{dx} + 2e^{2x}y = \sin x\, e^{2x} \tag{4}$$

3) This can be written as,

$$\frac{d}{dx}\left(y\, e^{2x}\right) = \sin x\, e^{2x} \tag{5}$$

4) Integrating on both sides with respect to dx, we get,

$$y\, e^{2x} = e^{2x}\,\frac{2\sin x - \cos x}{5} + C \tag{6}$$

5) Diving on both sides with e^{2x} we get,

$$y = \frac{2\sin x - \cos x + C\, e^{-2x}}{5} \tag{7}$$

6) Applying the inital conditions $x = 0$ and $y = 0$, we get,

$$C = 1 \tag{8}$$

7) Thus,

$$y = \frac{2\sin x - \cos x + \ e^{-2x}}{5} \tag{9}$$

is the solution of the given differential equation with given inital conditions

8) **CODING LOGIC:** The solution for the differential equation can be graphically solved using coding by using below logic :

$$\text{Start value of domain, } x_1 = 0 \tag{10}$$

$$\text{End value of domain, } x_2 = 5 \tag{11}$$

$$\text{Number of interations, } resolution = 20 \tag{12}$$

$$\text{Step size, } h = \frac{x_2 - x_1}{resolution} = 0.25 \tag{13}$$

Below is verification:

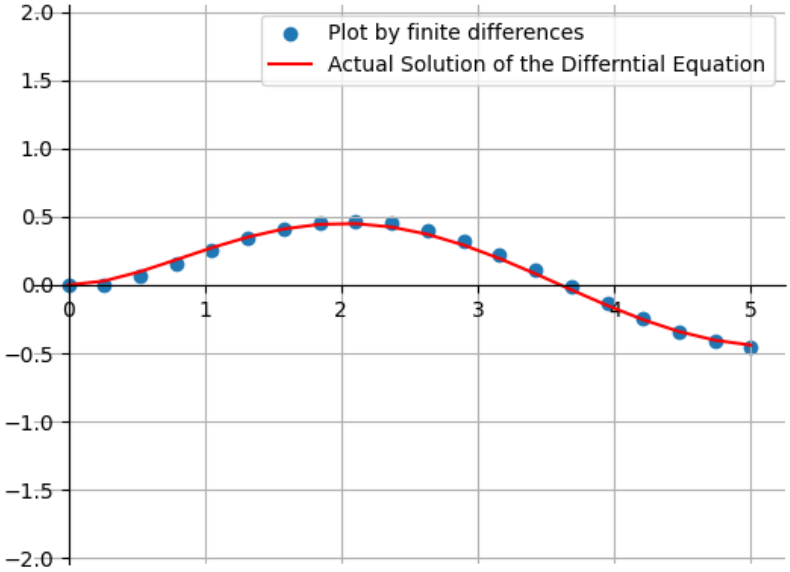

Fig. 8: Verification