NCERT 8.3.19

EE24BTECH11053 - S A Aravind Eswar

Question: The area bound by the y-axis, $y = \cos x$ and $y = \sin x$ when $0 \leq x \leq \frac{\pi}{2}$ is

0.1 *Theoretical Solution:*

Solving $y = \cos x$ and $y = \sin x$ in the given interval, we can find that they intersect at $x = \frac{\pi}{4}$ Thus, the area can be written as the following integral,

$$A = \int_0^{\pi/2} \min\{\sin x, \cos x\}\, dx \tag{1}$$

$$A = \int_0^{\pi/4} \sin x \, dx + \int_{\pi/4}^{\pi/2} \cos x \, dx \tag{2}$$

Evaluvating the integral, we get,

$$A = 2 - \sqrt{2} \tag{3}$$

0.2 *Trapeziodal rule:*

Let,

$$\int_a^b f(x)\, dx = A \tag{4}$$

The interal can be approximated as,

$$A \approx \frac{h}{2} \sum_{k=1}^{N} \left(f(x_{k-1}) - f(x_k)\right) \tag{5}$$

where,

$$h = \frac{b-a}{N} \tag{6}$$

Then the update equation can be written as,

$$J_{n+1} = J_n + h\, \frac{f(x_{n+1} + f_n)}{2} \tag{7}$$

Finding the area from 0 to $\pi/4$,

$$J_{n+1} = J_n + h\, \frac{\sin x_n + \sin x_{n+1}}{2} \tag{8}$$

$$x_{n+1} = x_n + h \tag{9}$$

Giving,

$$A_1 \approx 0.2926 \tag{10}$$

Similarly calculating from $\pi/4$ to $\pi/2$,

$$J_{n+1} = J_n + h\,\frac{\cos x_n + \cos x_{n+1}}{2} \tag{11}$$

$$x_{n+1} = x_n + h \tag{12}$$

Giving,

$$A_2 \approx 0.2926 \tag{13}$$

Total Area,

$$A = A_1 + A_2 \tag{14}$$

$$A \approx 0.5852 \tag{15}$$

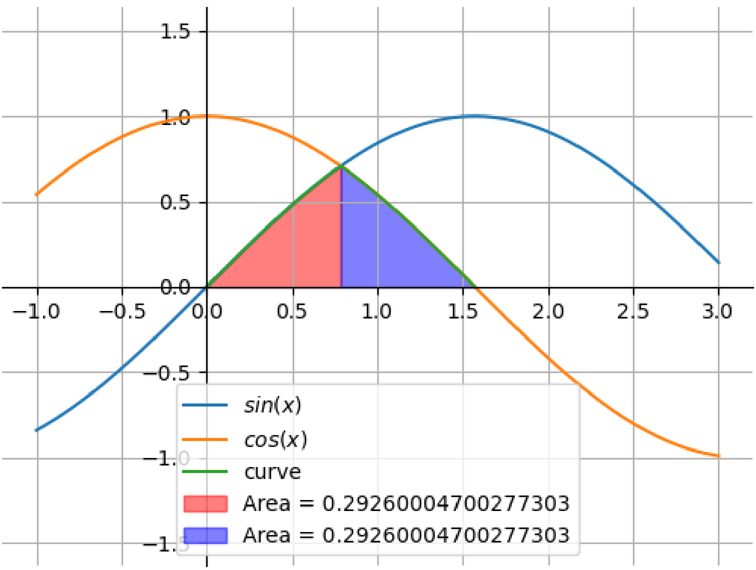

Fig. 0: Verification