

NCERT 8.3.19

EE24BTECH11053 - S A Aravind Eswar

Question: The area bound by the y-axis, $y = \cos x$ and $y = \sin x$ when $0 \leq x \leq \frac{\pi}{2}$ is

0.1 Theoretical Solution:

Solving $y = \cos x$ and $y = \sin x$ in the given interval, we can find that they intersect at $x = \frac{\pi}{4}$. Thus, the area can be written as the following integral,

$$A = \int_0^{\pi/2} \min \{ \sin x, \cos x \} dx \quad (1)$$

Evaluating the integral, we get,

$$A = 2 - \sqrt{2} \quad (2)$$

0.2 Trapezoidal rule:

The interval can be approximated as,

$$\int_a^b f(x) dx = \frac{\Delta x}{2} \sum_{k=1}^N (f(x_{k-1}) + f(x_k)) \quad (3)$$

where,

$$\Delta x = \frac{b - a}{N} \quad (4)$$

This can be simplified as,

$$\int_a^b f(x) dx = \Delta x \left(\frac{f(x_0) + f(x_N)}{2} + \sum_{k=1}^{N-1} f(x_k) \right) \quad (5)$$

Thus,

$$\int_0^{\pi/2} \min \{ \sin x, \cos x \} dx = \Delta x \left(\sum_{k=1}^{N-1} \min \{ \sin x_k, \cos x_k \} \right) \quad (6)$$

Taking N as 1000 and solving, we get,

$$Area \approx 0.5859 \quad (7)$$

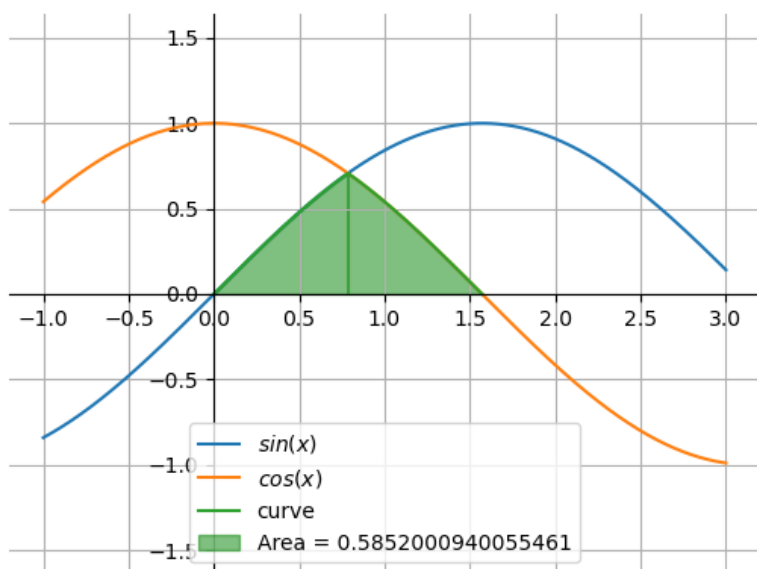


Fig. 0: Verification