# NCERT 6.5.5.4

EE24BTECH11053 - S A Aravind Eswar

**Question:** Find the minima and maxima of the function $f(x) = (x-1)^2 + 3,\ x \in [-3, 1]$

### *0.1 Gradient Decent*

For minima,

$$x_{n+1} = x_n + \mu f'(x_n) \tag{1}$$
$$f\prime(x) = 2(x-1) \tag{2}$$
$$x_{n+1} = x_n + 2\mu(x_n - u[n]) \tag{3}$$

Applying unilateral Z-transform on both sides we get,

$$z\,X(z) - z\,x_0 = X(z) + 2\mu\left(X(z) - \frac{z}{1-z}\right) \tag{4}$$
$$X(z) = \frac{(x_0 - (2\mu+1))}{1-(1+2\mu)z^{-1}} + \frac{1}{1-z^{-1}} \tag{5}$$
$$X(z) = \frac{1}{1-z^{-1}} - \frac{2\mu}{1-(1+2\mu)z^{-1}} \tag{6}$$
$$X(z) = \sum_{k=0}^{\infty}\left(1 - 2\mu\,(1+2\mu)^k\right)z^{-k} \tag{7}$$

from the last equation, we can infer onii-chan,

$$|z| > \max\{2\mu\,|2\mu+1|\,, 1\} \tag{8}$$
$$0 < |2\mu(2\mu+1)| < 1 \tag{9}$$
$$0 < \mu < \frac{\sqrt{5}-1}{4} \tag{10}$$

Now, inital value = -1.5, step size = 0.0001 tolerance = 1.0e-10

we get, $x_{min}$ in the range as,

$x_{min} = 0.999999999500818$

For max, we can do gradient ascent and get the point until we reach the end of the domain This gives,

$$x_{max} = 3 \tag{11}$$

### 0.2 *Quadratic Programming*

We need,

$$\min_x \begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} \tag{12}$$

$$\text{s.t. } \mathbf{x}^\top \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} - \begin{pmatrix} -2 \\ -1 \end{pmatrix}^\top \mathbf{x} + 4 = 0 \tag{13}$$

But the above equation is non-convex and hence we will relax it, given by,

$$\min_x \begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} \tag{14}$$

$$\text{s.t. } \mathbf{x}^\top \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} - \begin{pmatrix} -2 \\ -1 \end{pmatrix}^\top \mathbf{x} \le 0 \tag{15}$$

Using Quadratic Optimization with help of cvxpy, we get,

$$x_{min} = 1 \tag{16}$$

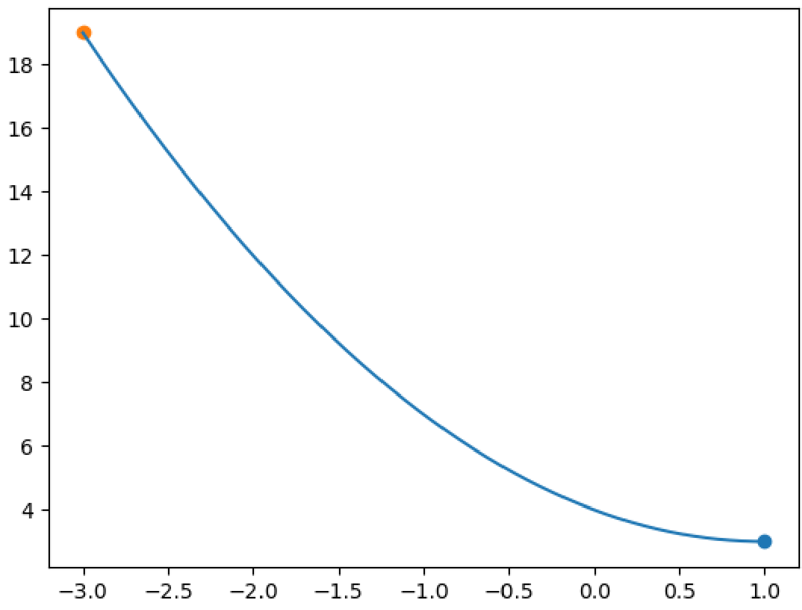


Fig. 0: Verification