

# NCERT 8.3.19

EE24BTECH11053 - S A Aravind Eswar

**Question:** Find the minima and maxima of the function  $f(x) = (x-1)^2 + 3$ ,  $x \in [-3, 1]$   
We shall use the gradient decent method to find the minima and maxima of the given function

For minima,

$$x_{n+1} = x_n + \mu f'(x_n) \quad (1)$$

$$f'(x) = 2(x-1) \quad (2)$$

$$x_{n+1} = x_n + 2\mu(x_n - u[n]) \quad (3)$$

Applying unilateral Z-transform on both sides we get,

$$zX(z) - zx_0 = X(z) + 2\mu \left( X(z) - \frac{z}{1-z} \right) \quad (4)$$

$$X(z) = \frac{(x_0 - (2\mu + 1))}{1 - (1 + 2\mu)z^{-1}} + \frac{1}{1 - z^{-1}} \quad (5)$$

$$X(z) = \frac{1}{1 - z^{-1}} - \frac{2\mu}{1 - (1 + 2\mu)z^{-1}} \quad (6)$$

$$X(z) = \sum_{k=0}^{\infty} \left( 1 - 2\mu(1 + 2\mu)^k \right) z^{-k} \quad (7)$$

from the last equation, we can infer onii-chan,

$$|z| > \max \{ 2\mu |2\mu + 1|, 1 \} \quad (8)$$

$$0 < |2\mu(2\mu + 1)| < 1 \quad (9)$$

$$0 < \mu < \frac{\sqrt{5} - 1}{4} \quad (10)$$

Now, initial value = -1.5, step size = 0.0001 tolerance = 1.0e-10

we get,  $x_{min}$  in the range as,

$$x_{min} = 0.999999999500818$$

For max, we can do gradient ascent and get the point until we reach the end of the domain This gives,

$$x_{max} = 3 \quad (11)$$

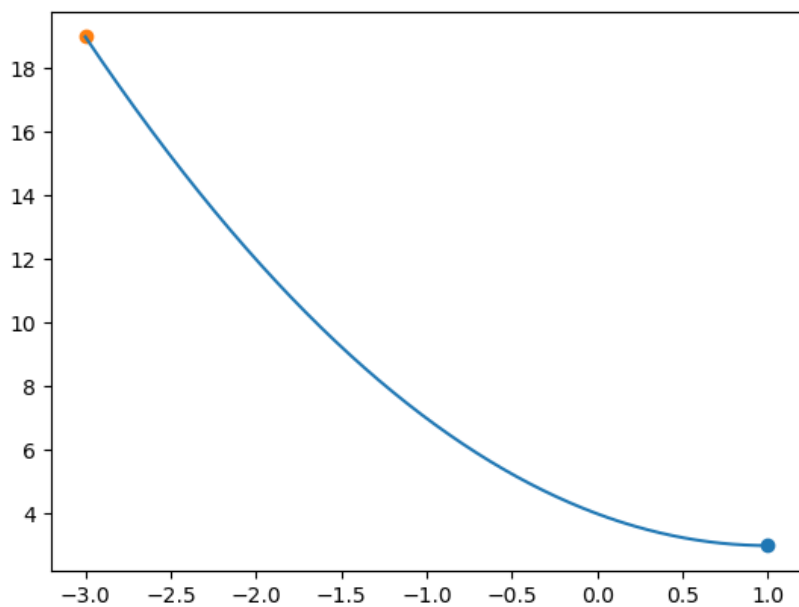


Fig. 0: Verification