NCERT 9.5.1

EE24BTECH11053 - S A Aravind Eswar

Question: Solve the differential equation given below with initial conditions $x = 0$ and $y = 0$.

$$\frac{dy}{dx} + 2y = \sin x \tag{1}$$

1) We can realise that the given equation is a linear differential equation. Then,

$$P = 2 \tag{2}$$

$$Q = \sin x \tag{3}$$

2) Multiplying on both sides with $e^{\int P}$ which is, e^{2x}

$$e^{2x}\frac{dy}{dx} + 2e^{2x}y = \sin x\, e^{2x} \tag{4}$$

3) This can be written as,

$$\frac{d}{dx}\left(y\, e^{2x}\right) = \sin x\, e^{2x} \tag{5}$$

4) Integrating on both sides with respect to dx, we get,

$$y\, e^{2x} = e^{2x}\,\frac{2\sin x - \cos x}{5} + C \tag{6}$$

5) Diving on both sides with e^{2x} we get,

$$y = \frac{2\sin x - \cos x + C\, e^{-2x}}{5} \tag{7}$$

6) Applying the inital conditions $x = 0$ and $y = 0$, we get,

$$C = 1 \tag{8}$$

7) Thus,

$$y = \frac{2\sin x - \cos x + \ e^{-2x}}{5} \tag{9}$$

is the solution of the given differential equation with given inital conditions

8) **CODING LOGIC:** We use the Finite Difference Method to plot the given solution of the differential equation. The Difference equation is given by,

$$y_{i+1} = y_i + \frac{dy}{dx}|_{(x_{i-1}, y_{i-1})}\, h$$

This can be implemented as an algorith as following,

Algorithm 1 Finite Differnce Algorithm

Inital condition, $x_0 \leftarrow 0$
$y_0 \leftarrow 0$
Number of interations, $interations \leftarrow 20$
Step size, $h = 0.25$
for i in range(1, $interations$) **do**
  $y_i \leftarrow y_{i-1} + \frac{dy}{dx}|_{(x_{i-1}, y_{i-1})}\, h$
  $x_i \leftarrow x_{i-1} + h$
end for
plot(x, y)

Below is verification:

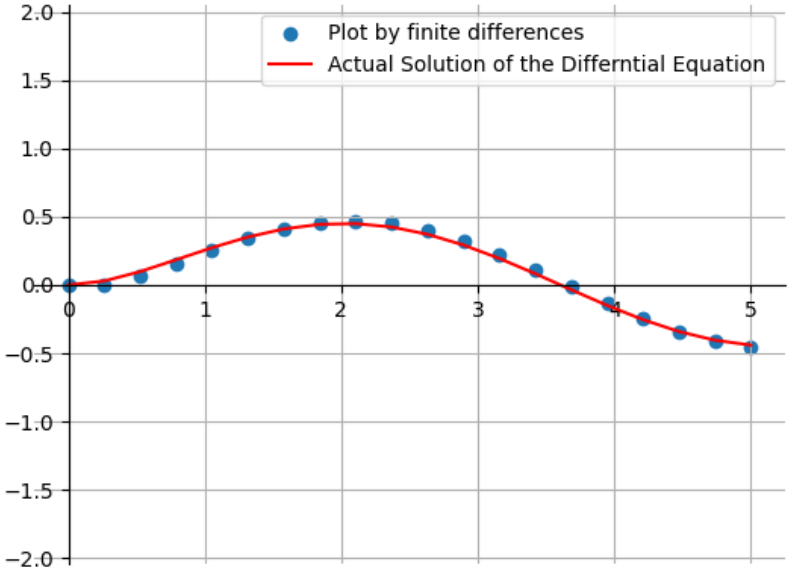

Fig. 8: Verification