NCERT 8.3.19

EE24BTECH11053 - S A Aravind Eswar

Question: The area bound by the y-axis, $y = \cos x$ and $y = \sin x$ when $0 \leq x \leq \frac{\pi}{2}$ is

0.1 Theoretical Solution:

Solving $y = \cos x$ and $y = \sin x$ in the given interval, we can find that they intersect at $x = \frac{\pi}{4}$ Thus, the area can be written as the following integral,

$$A = \int_0^{\pi/2} \min\{\sin x, \cos x\}\, dx \tag{1}$$

$$A = \int_0^{\pi/4} \sin x\, dx + \int_{\pi/4}^{\pi/2} \cos x\, dx \tag{2}$$

Evaluvating the integral, we get,

$$A = 2 - \sqrt{2} \tag{3}$$

0.2 Trapeziodal rule:

Let,

$$\int_a^b f(x)\, dx = A \tag{4}$$

The interal can be approximated as,

$$A \approx \frac{h}{2} \sum_{k=1}^{N} \left(f(x_{k-1}) - f(x_k)\right) \tag{5}$$

where,

$$h = \frac{b-a}{N} \tag{6}$$

Then the update equation can be written as,

$$A_{n+1} = A_n + h\,\frac{f(x_{n+1} + f_n)}{2} \tag{7}$$

Substituting,

$$A_{n+1} = A_n + h\,\frac{\min\{\sin x_n, \cos x_n\} + \min\{\sin x_{n+1}, \cos x_{n+1}\}}{2} \tag{8}$$

$$x_{n+1} = x_n + h \tag{9}$$

Performing the sum interatively,

$$Area \approx 0.5852 \tag{10}$$

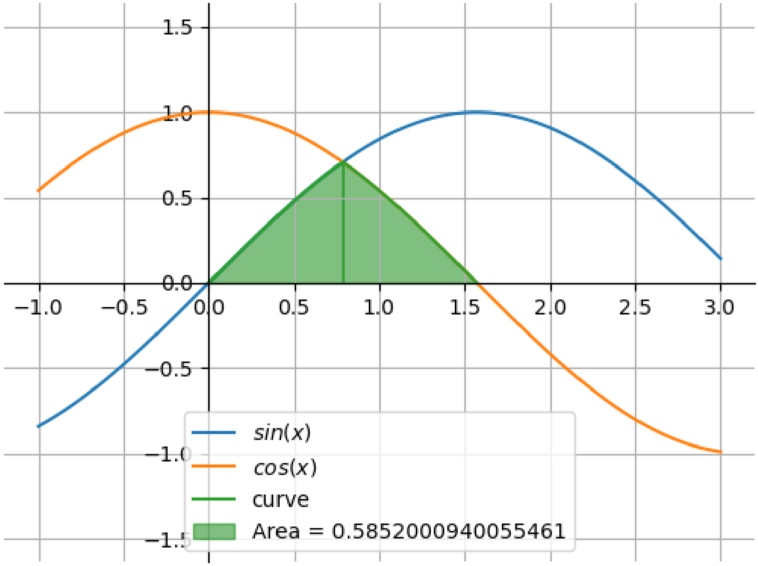

Fig. 0: Verification