NCERT 8.3.19

EE24BTECH11053 - S A Aravind Eswar

Question: Find the minima and maxima of the function $f(x) = (x-1)^2 + 3$, $x \in [-3, 1]$

0.1 Gradient Decent

For minima,

$$x_{n+1} = x_n + \mu f'(x_n) \tag{1}$$

$$f\prime(x) = 2(x-1) \tag{2}$$

$$x_{n+1} = x_n + 2\mu(x_n - u[n]) \tag{3}$$

Applying unilateral Z-transform on both sides we get,

$$z X(z) - z x_0 = X(z) + 2\mu \left(X(z) - \frac{z}{1-z} \right) \tag{4}$$

$$X(z) = \frac{(x_0 - (2\mu + 1))}{1 - (1 + 2\mu)z^{-1}} + \frac{1}{1 - z^{-1}} \tag{5}$$

$$X(z) = \frac{1}{1 - z^{-1}} - \frac{2\mu}{1 - (1 + 2\mu)z^{-1}} \tag{6}$$

$$X(z) = \sum_{k=0}^{\infty} \left(1 - 2\mu (1 + 2\mu)^k \right) z^{-k} \tag{7}$$

from the last equation, we can infer onii-chan,

$$|z| > \max \{2\mu |2\mu + 1|, 1\} \tag{8}$$

$$0 < |2\mu(2\mu + 1)| < 1 \tag{9}$$

$$0 < \mu < \frac{\sqrt{5} - 1}{4} \tag{10}$$

Now, inital value = -1.5, step size = 0.0001 tolerance = 1.0e-10

we get, x_{min} in the range as,

$x_{min} = 0.999999999500818$

For max, we can do gradient ascent and get the point until we reach the end of the domain This gives,

$$x_{max} = 3 \tag{11}$$

0.2 Quadratic Programming

We need,

$$\min_x \begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} \tag{12}$$

$$\text{s.t. } \mathbf{x}^\top \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} - \begin{pmatrix} -2 \\ -1 \end{pmatrix}^\top \mathbf{x} + 4 = 0 \tag{13}$$

But the above equation is non-convex and hence we will relax it, given by,

$$\min_x \begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} \tag{14}$$

$$\text{s.t. } \mathbf{x}^\top \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} - \begin{pmatrix} -2 \\ -1 \end{pmatrix}^\top \mathbf{x} \le 0 \tag{15}$$

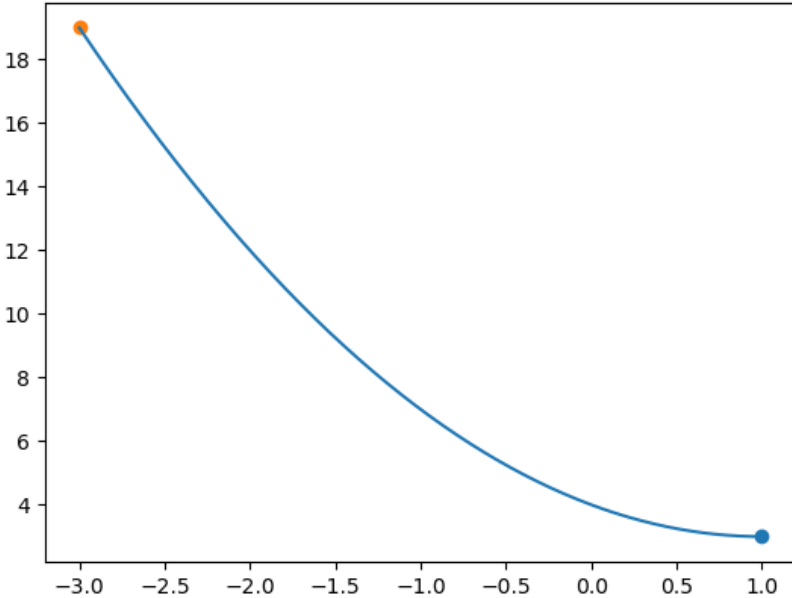Using Quadratic Optimization with help of cvxpy, we get,

$$x_{min} = 1 \tag{16}$$

Fig. 0: Verification