NCERT 9.5.1

EE24BTECH11053 - S A Aravind Eswar

Question: Solve the differential equation given below with initial conditions $x = 0$ and $y = 0$.

$$\frac{dy}{dx} + 2y = \sin x \tag{1}$$

0.1 Theoretical Solution

The Given equation can be written as,

$$y' + 2y = \sin x \tag{2}$$

Applying Laplace Transform on both sides,

$$\mathcal{L}\{y'\} + 2\mathcal{L}\{y\} = \mathcal{L}\{\sin x\} \tag{3}$$

$$\{s\,Y - y(0)\} + 2\{Y\} = \frac{1}{s^2+1} \tag{4}$$

This can be reduced to the following form, and applying the inital condition,

$$Y = \frac{1}{(s+2)(s^2+1)} \tag{5}$$

Decomposing the partial fraction,

$$Y = \left(\frac{2}{s^2+1} - \frac{s}{s^2+1} + \frac{1}{s+2}\right)\frac{1}{5} \tag{6}$$

Now, applying inverse transform, we get the solution,

$$y = \frac{2\sin x - \cos x + e^{-2x}}{5} \tag{7}$$

0.2 Finite Differences

The Difference Equation is given by,

$$\frac{dy}{dx} \approx \frac{y_{i+1} - y_i}{h} \tag{8}$$

This can be written as,

$$y_{i+1} = y_i + \frac{dy}{dx}h \tag{9}$$

Given that,

$$\frac{dy}{dx} = \sin x - 2y \tag{10}$$

The Difference equation can be written as,

$$y_{i+1} = y_i + (\sin x_i - 2y_i)\ h \tag{11}$$

This can be implemented as an algorithm as following,

Algorithm 1 Finite Difference Algorithm

Initial condition, $x_0 \leftarrow 0$
$y_0 \leftarrow 0$
Number of iterations, $iterations \leftarrow 20$
Step size, $h = 0.25$
for i in range(1, $iterations$) **do**
  $y_i = y_{i-1} + (\sin x_{i-1} - 2y_{i-1})\ h$
  $x_i \leftarrow x_{i-1} + h$
end for
plot(x, y)

Below is verification:

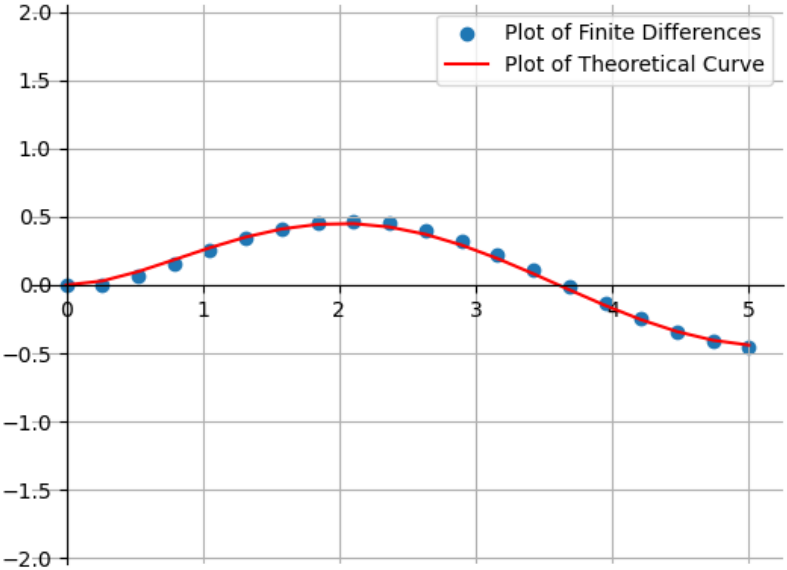

Fig. 0: Verification