

Question: Find the value of x if the distance between points $\mathbf{A} \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, and $\mathbf{B} \begin{pmatrix} x \\ 4 \end{pmatrix}$ is 5 units.

Solution:

Symbol	Value	Description
\mathbf{A}	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	Point \mathbf{A}
\mathbf{B}	$\begin{pmatrix} x \\ -4 \end{pmatrix}$	Point \mathbf{B}
d	5	Distance between points \mathbf{A} and \mathbf{B}
\mathbf{e}_1 and \mathbf{e}_2	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$	Standard basis vectors

TABLE 0: Given Values

Given,

$$\|AB\| = d \quad (0.1)$$

$$\|AB\|^2 = d^2 \quad (0.2)$$

$$AB^T AB = d^2 \quad (0.3)$$

$$(I(\mathbf{B} - \mathbf{A}))^T (I(\mathbf{B} - \mathbf{A})) = d^2 \quad (0.4)$$

$$\left(\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} (\mathbf{B} - \mathbf{A}) + \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} (\mathbf{B} - \mathbf{A}) \right)^T \left(\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} (\mathbf{B} - \mathbf{A}) + \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} (\mathbf{B} - \mathbf{A}) \right) = d^2 \quad (0.5)$$

$$(\mathbf{B} - \mathbf{A})^T \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} (\mathbf{B} - \mathbf{A}) + (\mathbf{B} - \mathbf{A})^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} (\mathbf{B} - \mathbf{A}) = d^2 \quad (0.6)$$

$$(\mathbf{B} - \mathbf{A})^T \mathbf{e}_1 \mathbf{e}_1^T (\mathbf{B} - \mathbf{A}) + (\mathbf{B} - \mathbf{A})^T \mathbf{e}_2 \mathbf{e}_2^T (\mathbf{B} - \mathbf{A}) = d^2 \quad (0.7)$$

$$\|\mathbf{e}_1^T (\mathbf{B} - \mathbf{A})\|^2 + \|\mathbf{e}_2^T (\mathbf{B} - \mathbf{A})\|^2 = d^2 \quad (0.8)$$

$$\mathbf{e}_1^T \mathbf{B} = \mathbf{e}_1^T \mathbf{A} + \sqrt{d^2 - (\mathbf{e}_2^T (\mathbf{B} - \mathbf{A}))} \quad (0.9)$$

Solving,

$$x = 3 \text{ (or) } x = -3 \quad (0.10)$$

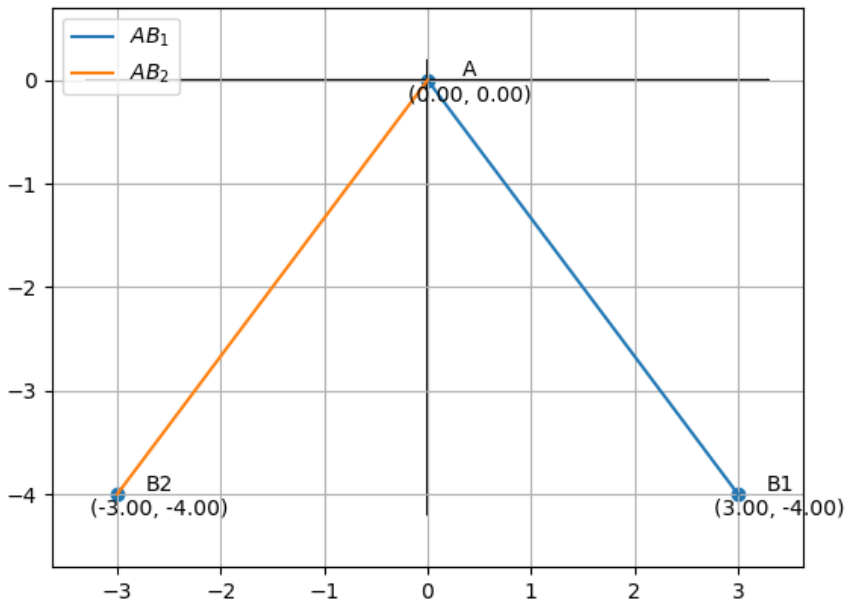


Fig. 0.1: Points **A**, **B**, **C** and **D**