

9.2.12

EE24BTECH11053 - S A Aravind Eswar

Question: Find the area of region bounded by the curve $x^2 = 4y$, $y = 2$, $y = 4$ and the y-axis in the first quadrant.

Solution:

symbol	Value	Description
$\mathbf{V}, \mathbf{u}, f$	$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \end{pmatrix}, 0$	Parameters of the given conic (parabola)
$\mathbf{h}_1, \mathbf{m}_1$	$\begin{pmatrix} 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}$	Parameters of the given line $y = 2$
$\mathbf{h}_2, \mathbf{m}_2$	$\begin{pmatrix} 0 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}$	Parameters of the given line $y = 4$
$\mathbf{a}_1, \mathbf{a}_2$		Points of intersection of given lines to the conic

TABLE 0: Given Values

Using,

$$\kappa_i = \frac{1}{\mathbf{m}^\top \mathbf{V} \mathbf{m}} \left(-\mathbf{m}^\top (\mathbf{V} \mathbf{h} + (\mathbf{u})) \pm \sqrt{[\mathbf{m}^\top (\mathbf{V} \mathbf{h} + \mathbf{u})]^2 - g(\mathbf{h})(\mathbf{m}^\top \mathbf{V} \mathbf{m})} \right) \quad (0.1)$$

and substituting in the line equation,

$$\mathbf{x} = \mathbf{h} + \kappa \mathbf{m} \quad (0.2)$$

we can find the points of intersection of the given lines and the conic in the first quadrant to be,

$$\mathbf{a}_1 = \begin{pmatrix} 2\sqrt{2} \\ 2 \end{pmatrix}, \mathbf{a}_2 = \begin{pmatrix} 4 \\ 4 \end{pmatrix} \quad (0.3)$$

Thus, the area of the parabola between the lines $y = 2$ and $y = 4$ is,

$$\int_0^4 4 - \frac{x^2}{4} dx - \int_0^{2\sqrt{2}} 2 - \frac{x^2}{4} dx = \frac{32 - 8\sqrt{2}}{3} \quad (0.4)$$

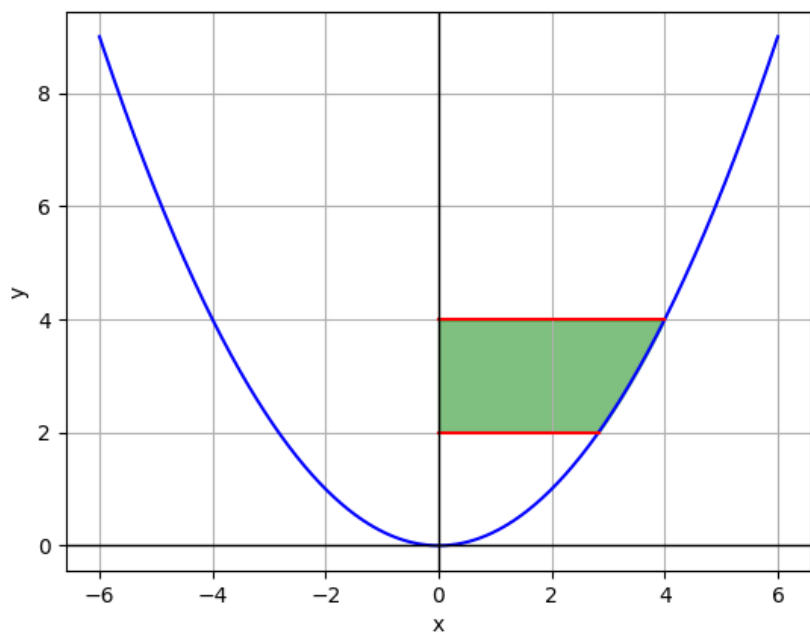


Fig. 0.1: Area Under the graph