EE24BTECH11053 - S A Aravind Eswar

Question: Find the value of x if the distance between points $\mathbf{A}\begin{pmatrix}0\\0\end{pmatrix}$, and $\mathbf{B}\begin{pmatrix}\mathrm{x}\\4\end{pmatrix}$ is 5 units.

Solution:

Symbol	Value	Description
$\mathbf{A}$	$\begin{pmatrix}0\\0\end{pmatrix}$	Point $\mathbf{A}$
$\mathbf{B}$	$\begin{pmatrix}x\\-4\end{pmatrix}$	Point $\mathbf{B}$
d	5	Distance between points $\mathbf{A}$ and $\mathbf{B}$

TABLE 0: Given Values

Given,

$$\|AB\| = d \tag{0.1}$$

$$\|AB\|^2 = d^2 \tag{0.2}$$

$$AB^tAB = d^2 \tag{0.3}$$

$$\left(\begin{pmatrix}1&0\\0&0\end{pmatrix}(\mathbf{B}-\mathbf{A}) + \begin{pmatrix}0&0\\0&1\end{pmatrix}(\mathbf{B}-\mathbf{A})\right)^\top \left(\begin{pmatrix}1&0\\0&0\end{pmatrix}(\mathbf{B}-\mathbf{A}) + \begin{pmatrix}0&0\\0&1\end{pmatrix}(\mathbf{B}-\mathbf{A})\right) = d^2 \tag{0.4}$$

$$(\mathbf{B}-\mathbf{A})^\top \begin{pmatrix}1&0\\0&0\end{pmatrix}(\mathbf{B}-\mathbf{A}) + (\mathbf{B}-\mathbf{A})^\top \begin{pmatrix}0&0\\0&1\end{pmatrix}(\mathbf{B}-\mathbf{A}) = d^2 \tag{0.5}$$

$$(\mathbf{B}-\mathbf{A})^\top \mathbf{e_1}\mathbf{e_1}^\top (\mathbf{B}-\mathbf{A}) + (\mathbf{B}-\mathbf{A})^\top \mathbf{e_2}\mathbf{e_2}^\top (\mathbf{B}-\mathbf{A}) = d^2 \tag{0.6}$$

$$\left\|\mathbf{e_1}^\top (\mathbf{B}-\mathbf{A})\right\|^2 + \left\|\mathbf{e_2}^\top (\mathbf{B}-\mathbf{A})\right\|^2 = d^2 \tag{0.7}$$

$$\mathbf{e_1}^\top \mathbf{B} = \mathbf{e_1}^\top \mathbf{A} + \sqrt{d^2 - (\mathbf{e_2}^\top (\mathbf{B}-\mathbf{A}))} \tag{0.8}$$

Solving,

$$x = 3 \text{ (or) } x = -3 \tag{0.9}$$

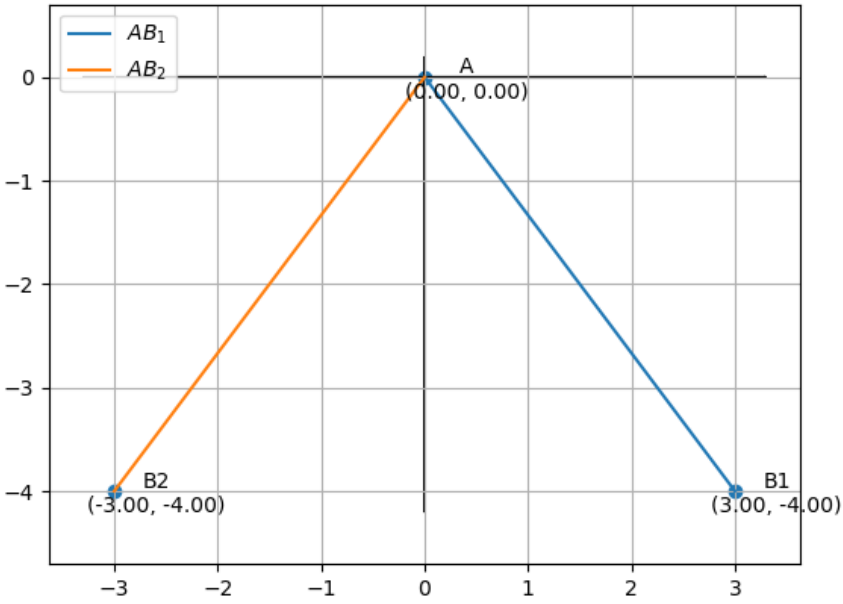

Fig. 0.1: Points **A**,**B**,**C** and **D**