

# Learning L<sup>A</sup>T<sub>E</sub>X

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## I. SECTION B

6. If  $f(1) = 1, f^1(1) = 2$ , then  $\lim_{x \rightarrow 1} \frac{\sqrt{f(x)} - 1}{\sqrt{x} - 1}$  is [2002]
- 1) 2
  - 2) 4
  - 3) 1
  - 4)  $\frac{1}{2}$
7.  $f$  is defined in  $[-5, 5]$  as [2002]  
 $f(x) = x$  if  $x$  is rational  
 $= -x$  if  $x$  is irrational. Then
- 1)  $f(x)$  is continuous at every  $x$ , except  $x = 0$
  - 2)  $f(x)$  is discontinuous at every  $x$ , except  $x = 0$
  - 3)  $f(x)$  is continuous everywhere
  - 4)  $f(x)$  is discontinuous everywhere
8.  $f(x)$  and  $g(x)$  are two differentiable functions on  $[0, 2]$  such that  $f''(x) - g''(x) = 0, f'(1) = 2g'(1) = 4, f(2) = 3g(2) = 9$  then  $f(x) - g(x)$  at  $x = \frac{3}{2}$  is [2002]
- 1) 0
  - 2) 2
  - 3) 10
  - 4) 5
9. If  $f(x+y) = f(x) \cdot f(y) \forall x, y$  and  $f(5) = 2, f'(0) = 3$ , then  $f'(5)$  is [2002]
- 1) 0
  - 2) 1
  - 3) 6
  - 4) 2
10.  $\lim_{x \rightarrow \infty} \frac{1 + 2^4 + 3^4 + \dots n^4}{n^5} - \lim_{x \rightarrow \infty} \frac{1 + 2^3 + 3^3 + \dots n^3}{n^5}$  [2003]
- 1)  $\frac{1}{5}$
  - 2)  $\frac{1}{30}$
  - 3) Zero
  - 4)  $\frac{1}{4}$
11. If  $\lim_{x \rightarrow 0} \frac{\log(3+x) - \log(3-x)}{x} = k$ , then the value of  $k$  is [2003]
- 1)  $-\frac{2}{3}$
  - 2) 0
  - 3)  $-\frac{1}{3}$
  - 4)  $\frac{2}{3}$
12. The value of  $\lim_{x \rightarrow 0} \frac{\int_0^{x^2} \sec^2 t dt}{x \sin x}$  is [2003]
- 1) 0
  - 2) 3
  - 3) 2
  - 4) 1
13. Let  $f(a) = g(a) = k$  and their  $n$ th derivatives  $f^n(a), g^n(a)$  exist and are not equal for some  $n$ . Further if  $\lim_{x \rightarrow a} \frac{f(a)g(x) - f(a) - g(a)f(x) + f(a)}{g(x) - f(x)} = 4$  then the value of  $k$  is [2003]
- 1) 0
  - 2) 4
  - 3) 2
  - 4) 1
14.  $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\left[1 - \tan\left(\frac{x}{2}\right)\right][1 - \sin x]}{\left[1 + \tan\left(\frac{x}{2}\right)\right][\pi - 2x]^3}$  is [2003]
- 1)  $\infty$
  - 2)  $\frac{1}{8}$
  - 3) 0
  - 4)  $\frac{1}{32}$
15. If  $f(x) = \begin{cases} xe^{-(\frac{1}{|x|} + \frac{1}{x})}, & x \neq 0 \\ 0, & x = 0 \end{cases}$  then  $f(x)$  is [2003]
- 1) discontinuous every where
  - 2) continuous as well as differentiable for all  $x$
  - 3) continuous for all  $x$  but not differentiable at  $x = 0$
  - 4) neither differentiable not continuous at  $x = 0$

16. if  $\lim_{x \rightarrow \infty} \left(1 + \frac{a}{x} + \frac{b}{x^2}\right)^{2x} = e^2$ , then the values of  $a$  and  $b$ , are [2004]

- 1)  $a = 1$  and  $b = 2$
- 2)  $a = 1$  and  $b \in \mathbf{R}$
- 3)  $a \in \mathbf{R}, b = 2$
- 4)  $a \in \mathbf{R}, b \in \mathbf{R}$

17.  $f(x) = \frac{1 - \tan x}{4x - \pi}, x \neq \frac{\pi}{4}, x \in \left[0, \frac{\pi}{4}\right]$ . If  $f(x)$  is continuous in  $\left[0, \frac{\pi}{4}\right]$ , then  $f\left(\frac{\pi}{4}\right)$  is

- 1)  $-1$
- 2)  $\frac{1}{2}$
- 3)  $-\frac{1}{2}$
- 4)  $1$

18.  $\lim_{n \rightarrow \infty} \left[ \frac{1}{n^2} \sec^2 \frac{1}{n^2} + \frac{2}{n^2} \sec^2 \frac{4}{n^2} + \dots + \frac{1}{n} \sec^2 1 \right]$   
equals [2005]

- 1)  $\frac{1}{2} \sec 1$
- 2)  $\frac{1}{2} \csc 1$
- 3)  $\tan 1$
- 4)  $\frac{1}{2} \tan 1$

19. Let  $\alpha$  and  $\beta$  be the distinct roots of  $ax^2 + bx + c = 0$ , then,  $\lim_{x \rightarrow \alpha} \frac{1 - \cos(ax^2 + bx + c)}{(x - a)^2}$  is equal to [2005]

- 1)  $\frac{a^2}{2}(\alpha - \beta)^2$
- 2)  $0$
- 3)  $\frac{-a^2}{2}(\alpha - \beta)^2$
- 4)  $\frac{1}{2}(\alpha - \beta)^2$

20. Suppose  $f(x)$  is differentiable at  $x = 1$  and  $\lim_{h \rightarrow 0} \frac{1}{h} f(1 + h) = 5$ , then  $f'(1)$  equals [2005]

- 1)  $3$
- 2)  $4$
- 3)  $5$
- 4)  $6$