Learning LaTeX

EE24BTECH11053 - S A Aravind Eswar*

I. D MCQs with One or More than One Correct

1) Let S be set of all column matrix $\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ such that $b_1, b_2, b_3 \in \mathbb{R}$ and the system of equations (in real variables)

$$\begin{aligned} -x + 2y + 5z &= b_1 \\ 2x - 4y + 3z &= b_2 \\ x - 2y + 2z &= b_3 \end{aligned}$$

has at least one solution. Then, which of the following system(s) (in real variables) has (have) at least one solution for each $\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \in S$ (JEE Adv. 2018)

a) $x+2y+3z = b_1, 4y+5z = b_2$ and $x+2y+6z = b_3$
b) $x + y + 3z = b_1, 5x + 2y + 6z = b_2$ and $-2x - y - 3z = b_3$
c) $-x + 2y - 5z = b_1, 2x - 4y + 10z = b_2$ and $x - 2y + 5z = b_3$
d) $sx+2y+5z = b_1, 2x+3z = b_2, x+4y-5z = b_3$

2) Let $M = \begin{pmatrix} 0 & 1 & a \\ 1 & 2 & 3 \\ 3 & b & 1 \end{pmatrix}$ and $(adj\,M) = \begin{pmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & 1 \end{pmatrix}$ where a and b are real numbers. Which of the following options is/are correct? (JEE Adv. 2019)

a) $a + b = 3$
b) $det(adj\,M^2) = 81$
c) $(adjM)^{-1} + adjM^{-1} = -M$
d) If $M \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ then $\alpha - \beta + \gamma = 3$

3) Let $P_1 = I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, P_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}, P_3 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, P_4 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, P_5 = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, P_6 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$ and $X = \sum_{k=1}^{6} P_k \begin{pmatrix} 2 & 1 & 3 \\ 1 & 0 & 2 \\ 3 & 2 & 1 \end{pmatrix} {P_k}^T$

Where ${P_k}^T$ denotes the transpose of matrix P_k. Then which of the following options is/are correct? (JEE Adv. 2019)

a) X is a symmetric matrix
b) The sum of diagonal elements of X is 18
c) X-30I is an invertible matrix
d) If $X \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \alpha \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, then a is 30

4) Let $x \in R$ and let

$$P = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{pmatrix}, Q = \begin{pmatrix} 2 & x & x \\ 0 & 4 & 0 \\ x & x & 5 \end{pmatrix} \text{ and } R = PQP^{-1}$$

Then which of the following options is/are correct? (JEE Adv. 2019)

a) $\det R = \det \begin{pmatrix} 2 & x & x \\ 0 & 4 & 0 \\ x & x & 5 \end{pmatrix} + 8$, for all $x \in R$
b) For $x = 1$, there exists a unit vector $\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ for which $R \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$
c) There exists a real number x such that $PQ = QP$
d) For $x = 0$, if $R = \begin{pmatrix} 1 \\ a \\ b \end{pmatrix} = 6 \begin{pmatrix} 1 \\ a \\ b \end{pmatrix}$, then a+b=5

II. E Subjective Problems

1) For what value of k do the following system of equations possess a non trivial (i.e., not all zero) solution over the set of rationals Q?

$$x + ky + 3z = 0$$
$$3x + ky - 2z = 0$$
$$2x + 3y - 4z = 0$$

For what value of k, find all the solutions of the system. (1979)

2) Let a, b, c be positive and not all equal.Show that the value of the determinant $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is negative. (1981 - 4 Marks)

3) Without expanding a determinant at any stage, show that $\begin{vmatrix} x^2+x & x+1 & x-2 \\ 2x^2+3x-1 & 3x & 3x-3 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix} = xA + B$, where A and B are determinants of order 3 not involving x. (1982 - 5 Marks)

4) Show that

$$\begin{vmatrix} {}^xC_r & {}^xC_{r+1} & {}^xC_{r+2} \\ {}^yC_r & {}^yC_{r+1} & {}^yC_{r+2} \\ {}^zC_r & {}^zC_{r+1} & {}^zC_{r+2} \end{vmatrix} = \begin{vmatrix} {}^xC_r & {}^{x+1}C_{r+1} & {}^{x+2}C_{r+2} \\ {}^yC_r & {}^{y+1}C_{r+1} & {}^{y+2}C_{r+2} \\ {}^zC_r & {}^{z+1}C_{r+1} & {}^{z+2}C_{r+2} \end{vmatrix}$$

(1985 - 2 Marks)

5) Consider the system of linear equations the system of linear equations in x, y, z:

$$(\sin 3\theta)x - y + z = 0$$
$$(\cos 2\theta)x + 4y + 3z = 0$$
$$2x + 7y + 7z = 0$$

Find the values of θ for which this system has non trivial solutions. (1986 - 5 Marks)

6) Let $\delta a = \begin{vmatrix} a-1 & n & 6 \\ (a-1)^2 & 2n^2 & 4n-2 \\ (a-1)^3 & 3n^3 & 3n^2-3n \end{vmatrix}$

Show that $\sum_{a=1}^{n} \Delta a = c$, a constant (1989-5 Marks)

7) Let the three digit numbers $A28, 3B9$, and $62C$, where A, B, and C are integers between 0 and 9, be divisible by a fixed integer k. Show that the determinant $\begin{vmatrix} A & 3 & 2 \\ 8 & 9 & c \\ 2 & B & 2 \end{vmatrix}$ is divisible by k. (1990 - 4 Marks)

8) If $a \neq p, b \neq q, c \neq r$ and $\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$. Then find the value of $\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c}$ (1991 - 4 Marks)

9) For a fixed positive integer n, if

$$D = \begin{vmatrix} n! & (n+1)! &)(n+2)! \\ (n+1)! & (n+2)! & (n+3)! \\ (n+2)! & (n+3)! & (n+4)! \end{vmatrix}$$

then show that $\left[\frac{D}{(n!)^3} - 4\right]$ is divisible by n.

10) Let λ and α be real. Find the set of all values of λ for which the system of linear equations

$\lambda x + (\sin\alpha)y + (\cos\alpha) = 0, x + (\cos\alpha)y + (\sin\alpha)z = 0, -x + (\text{si}$

has a non trivial solution. For $\lambda = 1$, find all values of α. (1993 - 4 Marks)

11) For all values of A, B, C and P, Q, R show that

$$\begin{vmatrix} \cos(A-P) & \cos(A-Q) & \cos(A-R) \\ \cos(B-P) & \cos(B-Q) & \cos(B-R) \\ \cos(C-P) & \cos(C-Q) & \cos(C-R) \end{vmatrix} = 0$$

(1994 - 4 Marks)