9.2.12

EE24BTECH11053 - S A Aravind Eswar

Question: Find the area of region bounded by the curse $x^2 = 4y$, $y = 2$, $y = 4$ and the y-axis in the first quadrant.
Solution:

symbol	Value	Description
$\mathbf{V}$,$\mathbf{u}$,f	$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \end{pmatrix}, 0$	Parameters of the given conic (parabola)
$\mathbf{h_1}, \mathbf{m_1}$	$\begin{pmatrix} 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}$	Parameters of the given line $y = 2$
$\mathbf{h_2}, \mathbf{m_2}$	$\begin{pmatrix} 0 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}$	Parameters of the given line $y = 4$
$\mathbf{a_1}, \mathbf{a_2}$		Points of intersection of given lines to the conic
κ_i		Parameters of the line equation $\mathbf{x} = \mathbf{h} + \kappa\mathbf{m}$

TABLE 0: Given Values

Using,

$$\kappa_i = \frac{1}{\mathbf{m}^\top \mathbf{V}\mathbf{m}} \left(-\mathbf{m}^\top (\mathbf{V}\mathbf{h} + (u)) \pm \sqrt{[\mathbf{m}^\top (\mathbf{V}\mathbf{h} + \mathbf{u})]^2 - g(\mathbf{h})(\mathbf{m}^\top \mathbf{V}\mathbf{m})} \right) \tag{0.1}$$

and substituting in the line equation,

$$\mathbf{x} = \mathbf{h} + \kappa\mathbf{m} \tag{0.2}$$

we can find the points of intersection of the given lines and the conic in the first quadrant to be,

$$\mathbf{a_1} = \begin{pmatrix} 2\sqrt{2} \\ 2 \end{pmatrix}, \mathbf{a_2} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} \tag{0.3}$$

Thus, the area of the parabola between the lines $y = 2$ and $y = 4$ is,

$$\int_0^4 4 - \frac{x^2}{4} dx - \int_0^{2\sqrt{2}} 2 - \frac{x^2}{4} dx = \frac{32 - 8\sqrt{2}}{3} \tag{0.4}$$

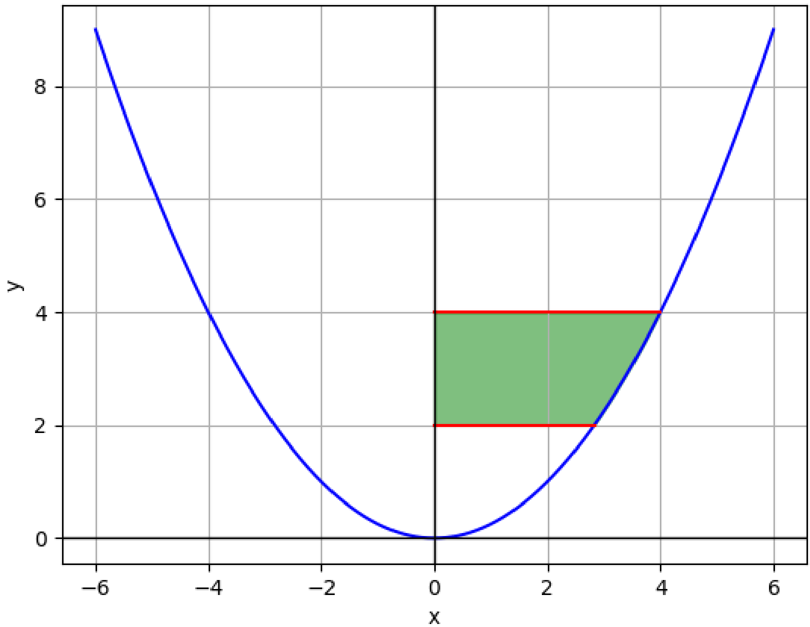

Fig. 0.1: Area Under the graph