

Learning L^AT_EX

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I. SECTION B

- 1) If $f(1) = 1, f'(1) = 2$, then $\lim_{x \rightarrow 1} \frac{\sqrt{f(x)} - 1}{\sqrt{x} - 1}$ is [2002]
- (a) 2 (b) 4
(c) 1 (d) $\frac{1}{2}$
- 2) f is defined in $[-5, 5]$ as [2002]
 $f(x) = x$ if x is rational
 $= -x$ if x is irrational. Then
 a) $f(x)$ is continuous at every x , except $x = 0$
 b) $f(x)$ is discontinuous at every x , except $x = 0$
 c) $f(x)$ is continuous everywhere
 d) $f(x)$ is discontinuous everywhere
- 3) $f(x)$ and $g(x)$ are two differentiable functions on $[0, 2]$ such that $f''(x) - g''(x) = 0, f'(1) = 2g'(1) = 4, f(2) = 3g(2) = 9$ then $f(x) - g(x)$ at $x = \frac{3}{2}$ is [2002]
- (a) 0 (b) 2
(c) 10 (d) 5
- 4) If $f(x + y) = f(x) \cdot f(y) \forall x, y$ and $f(5) = 2, f'(0) = 3$, then $f'(5)$ is [2002]
- (a) 0 (b) 1
(c) 6 (d) 2
- 5) $\lim_{x \rightarrow \infty} \frac{1 + 2^4 + 3^4 + \dots n^4}{n^5} - \lim_{x \rightarrow \infty} \frac{1 + 2^3 + 3^3 + \dots n^3}{n^5}$ [2003]
- (a) $\frac{1}{5}$ (b) $\frac{1}{30}$
(c) Zero (d) $\frac{1}{4}$
- 6) If $\lim_{x \rightarrow 0} \frac{\log(3+x) - \log(3-x)}{x} = k$, then the value of k is [2003]
- (a) $-\frac{2}{3}$ (b) 0
(c) $-\frac{1}{3}$ (d) $\frac{2}{3}$
- 7) The value of $\lim_{x \rightarrow 0} \frac{\int_0^{x^2} \sec^2 t dt}{x \sin x}$ is [2003]
- (a) 0 (b) 3
(c) 2 (d) 1
- 8) Let $f(a) = g(a) = k$ and their n th derivatives $f^n(a), g^n(a)$ exist and are not equal for some n . Further if $\lim_{x \rightarrow a} \frac{f(a)g(x) - f(a) - g(a)f(x) + f(a)}{g(x) - f(x)} = 4$ then the value of k is [2003]
- (a) 0 (b) 4
(c) 2 (d) 1
- 9) $\lim_{x \rightarrow \frac{\pi}{2}} \frac{[1 - \tan(\frac{x}{2})][1 - \sin x]}{[1 + \tan(\frac{x}{2})][\pi - 2x]^3}$ is [2003]
- (a) ∞ (b) $\frac{1}{8}$
(c) 0 (d) $\frac{1}{32}$
- 10) If $f(x) = \begin{cases} xe^{-(\frac{1}{|x|} + \frac{1}{x})}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ then $f(x)$ is [2003]
- a) discontinuous every where
 b) continuous as well as differentiable for all x
 c) continuous for all x but not differentiable at $x = 0$
 d) neither differentiable not continuous at $x = 0$
- 11) If $\lim_{x \rightarrow \infty} \left(1 + \frac{a}{x} + \frac{b}{x^2}\right)^{2x} = e^2$, then the values of a and b , are [2004]
- (a) $a = 1$ and $b = 2$ (b) $a = 1$ and $b \in \mathbf{R}$
(c) $a \in \mathbf{R}, b = 2$ (d) $a \in \mathbf{R}, b \in \mathbf{R}$

12) $f(x) = \frac{1 - \tan x}{4x - \pi}$, $x \neq \frac{\pi}{4}$, $x \in \left[0, \frac{\pi}{4}\right]$. If $f(x)$ is continuous in $\left[0, \frac{\pi}{2}\right]$, then $f\left(\frac{\pi}{4}\right)$ is

- (a) -1 (b) $\frac{1}{2}$
 (c) $-\frac{1}{2}$ (d) 1

13) $\lim_{n \rightarrow \infty} \left[\frac{1}{n^2} \sec^2 \frac{1}{n^2} + \frac{2}{n^2} \sec^2 \frac{4}{n^2} + \dots + \frac{1}{n} \sec^2 1 \right]$
 equals [2005]

- (a) $\frac{1}{2} \sec 1$ (b) $\frac{1}{2} \operatorname{cosec} 1$
 (c) $\tan 1$ (d) $\frac{1}{2} \tan 1$

[2pt]

14) Let α and β be the distinct roots of $ax^2 + bx + c = 0$, then, $\lim_{x \rightarrow \alpha} \frac{1 - \cos(ax^2 + bx + c)}{(x - \alpha)^2}$ is equal to [2005]

- (a) $\frac{a^2}{2}(\alpha - \beta)^2$ (b) 0
 (c) $\frac{-a^2}{2}(\alpha - \beta)^2$ (d) $\frac{1}{2}(\alpha - \beta)^2$

15) Suppose $f(x)$ is a differentiable at $x = 1$ and $\lim_{h \rightarrow 0} \frac{1}{h} f(1 + h) = 5$, then $f'(1)$ equals [2005]

- (a) 3 (b) 4
 (c) 5 (d) 6