9.2.12

EE24BTECH11053 - S A Aravind Eswar

Question: Find the area of region bounded by the curse $x^2 = 4y$, $y = 2$, $y = 4$ and the y-axis in the first quadrant.

Solution:

symbol	Value	Description
$\mathbf{V}$,$\mathbf{u}$,f	$\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}, \begin{pmatrix}0\\2\end{pmatrix}, 0$	Parameters of the given conic (parabola)
$\mathbf{h_1}, \mathbf{m_1}$	$\begin{pmatrix}0\\2\end{pmatrix}, \begin{pmatrix}1\\0\end{pmatrix}$	Parameters of the given line $y = 2$
$\mathbf{h_2}, \mathbf{m_2}$	$\begin{pmatrix}0\\4\end{pmatrix}, \begin{pmatrix}1\\0\end{pmatrix}$	Parameters of the given line $y = 4$
$\mathbf{a_1}, \mathbf{a_2}$		Points of intersection of given lines to the conic
κ_i		Parameters of the line equation $\mathbf{x} = \mathbf{h} + \kappa\mathbf{m}$
A_1		Area under the parabola from $y = 0$ to $y = 4$
A_2		Area under the parabola from $y = 0$ to $y = 2$

TABLE 0: Given Values

Using,

$$\kappa_i = \frac{1}{\mathbf{m}^\top \mathbf{V}\mathbf{m}} \left(-\mathbf{m}^\top \left(\mathbf{V}\mathbf{h} + (u) \right) \pm \sqrt{\left[\mathbf{m}^\top \left(\mathbf{V}\mathbf{h} + \mathbf{u} \right) \right]^2 - g(\mathbf{h})(\mathbf{m}^\top \mathbf{V}\mathbf{m})} \right) \tag{0.1}$$

and substituting in the line equation,

$$\mathbf{x} = \mathbf{h} + \kappa\mathbf{m} \tag{0.2}$$

we can find the points of intersection of the given lines and the conic in the first quadrant to be,

$$\mathbf{a_1} = \begin{pmatrix}2\sqrt{2}\\2\end{pmatrix}, \mathbf{a_2} = \begin{pmatrix}4\\4\end{pmatrix} \tag{0.3}$$

Calculating A_1 and A_2,

$$A_1 = \int_0^4 \frac{x^2}{4} dx = \frac{16}{3} \tag{0.4}$$

$$A_2 = \int_0^{2\sqrt{2}} \frac{x^2}{4} dx = \frac{4\sqrt{2}}{3} \tag{0.5}$$

Thus, the area of the parabola between the lines $y = 2$ and $y = 4$ is,

$$\left(\int_0^4 4dx - A_1\right) - \left(\int_0^{2\sqrt{2}} 2dx - A_2\right) = \frac{32 - 8\sqrt{2}}{3} \tag{0.6}$$

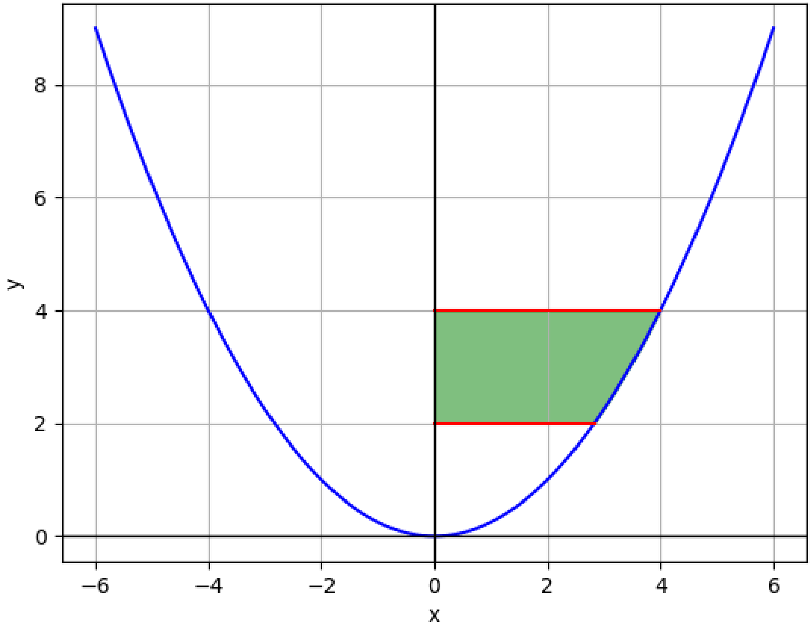

Fig. 0.1: Area Under the graph