Experiment 4

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1 Aim

To study and analyze the transient response of an LC circuit, determine the natural frequency (Ω_n), and calculate the damping ratio (ξ) using theoretical and experimental methods.

2 Materials and Apparatus Required

1) 100 μF Capacitor
2) Largest available inductor in the lab (denoted as L)
3) Resistor (small value for practical considerations)
4) DC Power Supply
5) Oscilloscope

3 Theory

An LC circuit consists of an inductor (L) and a capacitor (C) connected in parallel. When a charged capacitor is connected to an inductor, energy oscillates between the capacitor's electric field and the inductor's magnetic field. This oscillatory behavior is governed by the second-order differential equation:

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = 0,$$

where $q(t)$ is the charge on the capacitor as a function of time.

The natural frequency of oscillation is given by:

$$\omega_n = \frac{1}{\sqrt{LC}},$$

where: - L is the inductance in henries (H), - C is the capacitance in farads (F).

For an ideal LC circuit (no resistance), the damping ratio ($\xi = 0$) indicates purely oscillatory behavior. However, in practical circuits, resistance (R) introduces damping, and the damping ratio becomes:

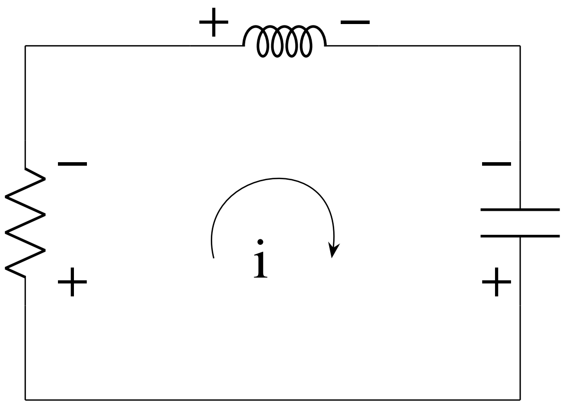
$$\xi = \frac{R}{2}\sqrt{\frac{C}{L}}.$$

From this, we get,

$$\omega = \omega_n\sqrt{1-\xi^2}$$

where, ω is the oscillating frequency

For experimental analysis, we monitor voltage waveforms using an oscilloscope. The observed oscillation frequency can be compared with theoretical calculations to validate the model.



4 Procedure

1) Precharge the Capacitor:
   - Connect the capacitor to a 5V DC power supply.
   - Once charged, disconnect it carefully without discharging.
2) Construct the LC Circuit:
   - Connect the charged capacitor in parallel with the largest available inductor.
   - Ensure minimal resistance in wiring.
3) Capture Transient Response:
   - Use an oscilloscope to monitor voltage across the capacitor.
   - Observe natural oscillations.
4) Calculate Theoretical Values:
   - Compute natural frequency ($\Omega_n = 1/\sqrt{LC}$).
   - Estimate damping ratio ($\xi = R/2\sqrt{\frac{C}{L}}$) if resistance is non-negligible.
5) Compare with Experimental Results:
   - Extract oscillation frequency from oscilloscope data.
   - Compare with theoretical calculations.

5 Observations

We use $1nF$ capacitor with a $2.2mH$ inductor. This gives,

$$\omega_n = \frac{1}{\sqrt{LC}} = 9.94053 \times 10^5 rad/s$$

Given,10% error in L, and 20% error in C,

Tolerance is given by,

$$\frac{\Delta\omega_n}{\omega_n} = \frac{1}{2}\left(\frac{\Delta L}{L} + \frac{\Delta C}{C}\right)$$
$$\frac{\Delta\omega_n}{\omega_n} = 0.15$$

Giving us a tolerance of 15%

$$\omega_n = (9.94 \pm 1.49) \times 10^5 rad/s$$

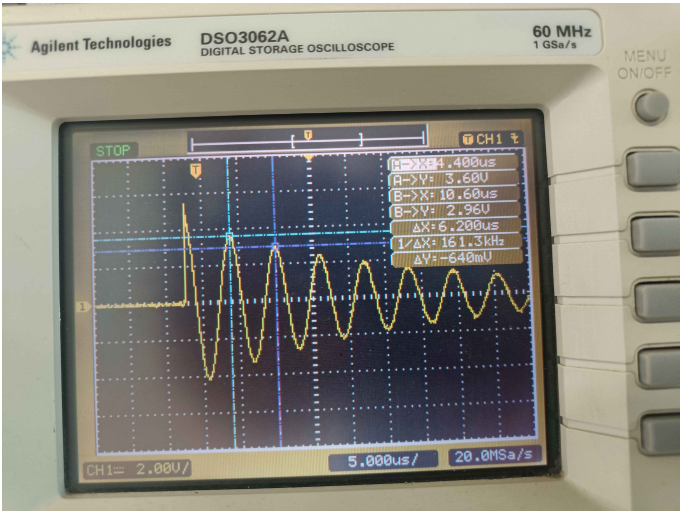


Fig. 1: Transient Response of LC[$L = 2.2mH, C = 560pF$]

Above reading is across the inductor. Taking reading across the capacitor was impractical as it discharded quick before the oscillation could begin. From the observation,

$$T = 6.2\mu s$$
$$\omega = \frac{2\pi}{T} = 1.013 \times 10^6 rad/s$$

We know that,

$$\omega < \omega_n$$

Thus, we assume ω_n takes values from 1.013×10^6 to $1.143 \times 10^6 rads/s$

Damping ratio is given by,

$$\xi = \sqrt{1 - \left(\frac{\omega}{\omega_n}\right)^2}$$
$$\xi_{max} = 0.4631$$

We can observe that damping ratio is less than 1, implying it is underdamped
The internal resistance in the circuit is given by,

$$R_{max} = 2\xi_{max}\sqrt{\frac{L}{C}}$$
$$R_{max} = 2.038k\Omega$$

To find the exact value of R, we can use the peak value. From the observation,

$$V(t = T) = V_0 e^{-\alpha T}$$
$$V_0 e^{-\alpha T} = 680mV$$
$$\alpha = 5.2984 \times 10^4 rad/s$$
$$R = 233\Omega$$

6 Precautions

1) Make sure not to accidentally discharge the capacitor
2) Make sure the DC generator is connected to ground properly
3) Use a push button for shorting the inductor and capacitor
4) Repeat the experiment multiple times to be sure of the reading as it is a sensitive measurement