Data-Driven State Awareness for Fly-by-Feel Aerial Vehicles via Adaptive Time Series and Gaussian Process Regression Models

Shabbir Ahmed, Ahmad Amer, Carlos Varela, and Fotis Kopsaftopoulos

Rensselaer Polytechnic Institute, Troy, NY 12180, USA
{ahmeds6,amera2,cvarela,kopsaf}@rpi.edu

Abstract. This work presents the investigation and critical assessment, within the framework of Dynamic Data Driven Applications Systems (DDDAS), of two probabilistic state awareness approaches for fly-by-feel aerial vehicles based on (i) stochastic adaptive time-dependent time series models and (ii) Bayesian learning via homoscedastic and heteroscedastic Gaussian process regression models (GPRMs). Stochastic time-dependent autoregressive (TAR) time series models with adaptive parameters are estimated via a recursive maximum likelihood (RML) scheme and used to represent the dynamic response of a self-sensing composite wing under varying flight states. Bayesian learning based on homoscedastic and heteroscedastic versions of GPRM is assessed via the ability to represent the nonlinear mapping between the flight state and the vibration signal energy of the wing. The experimental assessment is based on a prototype self-sensing UAV wing that is subjected to a series of wind tunnel experiments under multiple flight states.

1 Introduction

Future intelligent aerial vehicles will be able to "feel," "think," and "react" in real time based on high-resolution ubiquitous sensing leading to autonomous operation based on unprecedented self-awareness and self-diagnostic capabilities. But flight in complex dynamic environments requires unprecedented levels of sensing, awareness and diagnostic capabilities. Such capabilities can be enabled via the concept of "fly-by-feel" aerial vehicles, i.e., vehicles that can "feel," "think," and "react" inspired by avian flight. Such systems fall within the core of *Dynamic Data-Driven Application Systems (DDDAS)* concept as they have to dynamically incorporate real-time data into the modeling, learning and decision making application phases, and in reverse, steer the data measurement process based on the system's dynamic data integration and interpretation [5, 3, 8, 4].

Towards the "fly-by-feel" concept, in this study two dynamic data-driven state awareness approaches based on stochastic time series models and Bayesian Gaussian process regression models (GPRMs) are presented and experimentally assessed on a prototype self-sensing composite wing subjected to a series of wind tunnel experiments under multiple flight states –defined by a pair of angle of attack (AoA) and airspeed [9, 8]. Adaptive parametric time-dependent autoregressive (TAR) models are used to represent the time-varying dynamics of the wing as it undergoes different flight states. Model parameter estimation is based on a recursive maximum likelihood (RML) statistical scheme that allows the AR parameters to adapt with time in order to capture the non-stationary dynamic response of the wing [12, 14]. In addition, non-parametric Bayesian learning via GPRMs [13, 10] is used to "learn" the nonlinear relationship between sensor signal energy and the flight state, as defined by the AoA and airspeed (GPRM covariates). Both homoscedastic [13, 1], i.e. model observations' noise is assumed constant throughout the input space, and heteroscedastic [10], i.e. considering input-dependent variance, GPRM versions are presented and critically assessed.

This study is a continuation of recent DDDAS work by the authors and coworkers [8, 7, 4, 3], with the main novel contributions related to addressing the DDDAS fly-by-feel state awareness concept within (i) a non-stationary framework via adaptive time series models with unstructured time-dependent parameter evolution, and (ii) a Bayesian learning framework that represents the relationship between several flight-state inputs (covariates) and data-driven flight-state-sensitive features accounting for potential input-dependent noise variance.

2 Bayesian learning via Gaussian process regression

Being kernel-based linear regression models, GPRMs allow for the modeling of complex, nonlinear relationships between observations (targets) and covariates (inputs), and the extraction of prediction confidence intervals (CIs) at a relatively small computational cost [13]. As a result, they have been widely used in many applications in the machine learning community [13] and recently in Structural Health Monitoring (SHM) applications [2, 1]. However, the inherent and oftentimes unrealistic assumption of a fixed noise variance across the input space [13, Chapter 2, pp. 16] that governs standard (homoscedastic) GPRMs, makes them inappropriate in modeling many real-life processes. As such, heteroscedastic GPRMs have been proposed [10, 6] that allow for input-dependent variance with the cost that the predictive density and marginal likelihood are no longer analytically tractable [10].

2.1 Homoscedastic Gaussian process regression

In this section, a concise overview of homoscedastic GPRMs will be provided. For a full treatment, the reader is directed to [13]. Given a training data set $\mathcal{D}$ containing n inputs-observation pairs $\{(\mathbf{x}_i \in \mathbb{R}^D, y_i \in \mathbb{R},\ i = 1, 2, 3, \ldots, n\}$, a standard GPRM can be formulated as follows:

$$y = f(\mathbf{x}) + \epsilon, \quad f(\mathbf{x}) \sim \mathcal{GP}(m(\mathbf{x}), k(\mathbf{x}, \mathbf{x}')), \quad \epsilon \sim iid\, \mathcal{N}(0, \sigma_n^2) \tag{1}$$

where, in a Bayesian setting, a GP prior with mean $m(\mathbf{x})$ and covariance $k(\mathbf{x}, \mathbf{x}')$ is placed on the latent function $f(\mathbf{x})$, and an independent, identically-distributed (iid), zero-mean Gaussian prior with variance σ_n^2 is placed on the noise term ϵ. $\mathcal{N}(\cdot, \cdot)$ indicates normal distribution with the indicated mean and variance. The mean $m(\mathbf{x})$ may be set to zero and the squared exponential covariance function (kernel) is used for the latent function GP $k(\mathbf{x}, \mathbf{x}') = \sigma_0^2 \exp(-\frac{1}{2}(\mathbf{x} - \mathbf{x}')^T \Lambda^{-1} (\mathbf{x} -$

$\mathbf{x}')$). σ_0^2 is the output variance and Λ^{-1} designates the inverse of a diagonal matrix of the characteristic input length scales corresponding to each dimension (D, i.e each covariate) in the input data.

Training involves optimizing the hyperparameters ($\theta \equiv \sigma_0^2, \Lambda, \sigma_n^2$), which is typically done *via* Type II Maximum Likelihood [13, Chapter 5, pp. 109], whereas the marginal likelihood (evidence) of the training observations is maximized, or its negative log is minimized with respect to θ:

$$-\log p(\mathbf{y}|X,\theta) = -\frac{1}{2}\mathbf{y}^T (K_{XX} + \sigma_n^2\mathbb{I})^{-1}\mathbf{y} - \frac{1}{2}\log|K_{XX} + \sigma_n^2\mathbb{I}| - \frac{n}{2}\log 2\pi \qquad (2)$$

Prediction can be achieved by assuming joint Gaussian distribution between the training observations $\mathbf{y}$, and a test observation (to be predicted) at the set of test inputs ($\mathbf{x}_*$) [13].

2.2 Heteroscedastic Gaussian process regression

One of the inherent drawbacks of homoscedastic GPRMs is the assumption of a fixed noise variance throughout the input space, which, in many real-life applications, is impractical. Thus, a number of extensions have been proposed to allow for the noise variance to vary with the input within a heteroscedastic GP (HGP) framework. In this work, we have implemented the variational inference that is based on variational Bayes and Gaussian approximation [10]:

$$y = f(\mathbf{x}) + \epsilon(\mathbf{x}), \quad \epsilon \sim \mathcal{N}(0, r(\mathbf{x})) \qquad (3)$$

The added complexity of the heteroscedastic formulation results in not analytically tractable marginal likelihood and predictive distribution. One of the proposed approaches for their approximation was put forward by Lázaro-Gredilla and Titsias and is based on variational inference [10].

For training, the number of free variational heteroscedastic GPRM (VHGPRM) parameters to be determined becomes $n + n(n+1)/2$, which makes the training process computationally exhaustive. Thus, Lázaro-Gredilla and Titsias [10] proposed a reparametrization of μ and Σ at the maxima of the marginal variational bound. The predictive distribution for a new point in terms of the first two moments can be calculated analytically [10].

2.3 GPRM-based flight awareness results

The demonstration and assessment of the methods presented is based on wind-tunnel experiments for a self-sensing composite UAV wing under varying AoA

Table 1. Performance of standard GPRMs and VHGPRMs based on validation data.

GPRM Input	Standard GPRMs	VHGPRMs
AoA – MSE[a]	5.7337	5.7624
AoA – NMSE[b]	0.0947	0.0952
AS – MSE	0.0822	0.0816
AS – NMSE	0.1124	0.1115

[a]Mean Square Error; [b]Normalized Mean Square Error [10].

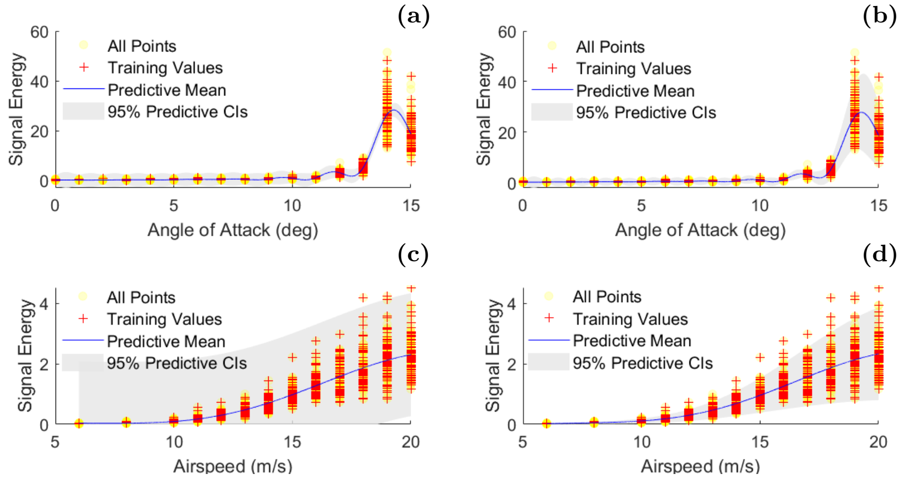

Fig. 1. Indicative GPRM results: (a) standard and (b) variational heteroscedastic (right) GPRMs representing the evolution of signal energy vs AoA (set airspeed of 15 m/s; top row); and (c) (d) airspeed (set AoA of 10 deg; bottom row).

(from 0 to 17 degrees) and airspeed (0 m/s to 20 m/s); for details see [9, 8]. Embedded piezoelectric sensors recorded stochastic vibration 90,000-sample-long (90 s) signals (sampling frequency $f_s = 100$ Hz) for which the signal energy for varying time-windows was calculated (indicative results are currently presented for one-second-long windows). Model inputs (covariates) are represented via a flight state vector consisting of the AoA and airspeed values, and the signal energy is the output. For training, after an initial investigation in terms of model effectiveness versus computational cost, 1000 signal energy points were randomly selected under the considered flight states, and 486 and 183 test points were used for the AoA and airspeed, respectively. In the said format, the trained GPRMs are capable of predicting signal energy for a given flight state; however, the flight state can be identified via the trained GPRMs based on the predictive confidence intervals (CIs) at the test signal energies and the calculation of the probability that a point sampled from the predictive distribution of each set of flight states falls within the calculated CIs. The flight state that has the highest probability is determined as the actual state corresponding to the observed test signal energy. Figure 1 presents indicative GPRM results for the standard and VHGPRM cases for varying AoA (top row; Figure 1a and b) and airspeed (bottom row; Figure 1c and d). It can be readily observed that the VHGPRM predictive mean and variance can accurately represent the evolution of the signal energy along with the corresponding variance that varies with the input state. On the other hand, the standard GPRM, as expected, fails to capture the predictive variance, as evident by either too narrow or too broad CIs.

Figure 2 presents indicative results of the flight state prediction based on the standard and VHGPRM models. It can be observed that the VHGPRM provides more accurate predictions especially in the case of the airspeed for which the standard GPRM fails to capture the variance (see Figure 1). Table 1 presents the comparison of the standard GPRM and VHGPRM performance.

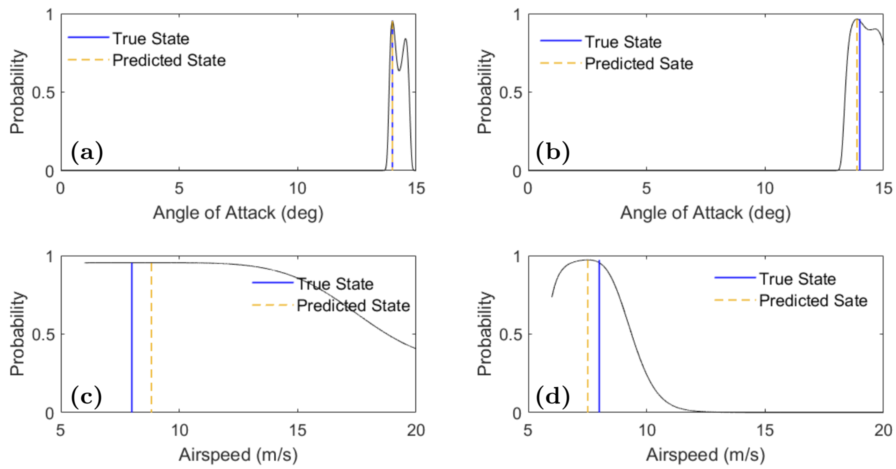

Fig. 2. Indicative flight state probabilities: (top) AoA predictions at an airspeed of 15 m/s for (a) standard and (b) VHGPRM models; (bottom) airspeed predictions at an AoA of 10 degrees for (c) standard and (d) VHGPRM models.

3 Adaptive modeling via time-dependent AR models

The dynamic response of aerial vehicles is governed by non-stationary stochastic vibrations under varying operating and environmental states characterized by time-dependent (evolutionary) characteristics. From a physical standpoint, non-stationary behavior is due to time-dependent and/or inherently non-linear dynamics. Non-stationary models can be based on non-parametric or parametric representations [12, 14, 11]; for a review of non-stationary random vibration modeling and analysis see [12]. In this study, TAR models are used to represent stochastic time-varying vibration signals recorded from piezoelectric sensors embedded within the composite layup of the wing under the aforementioned flight states (for details see [9, 7]). TAR models resemble their stationary AR counterparts allowing their parameters depend upon time and can *adapt* based on the time-dependent dynamics of the system [12]. A TAR(na) model, with na designating its AR order, is thus of the form:

$$y[t] + \sum_{i=1}^{na} a_i[t] \cdot y[t-i] = e[t] \quad \text{with} \quad e[t] \sim \text{iid}\,\mathcal{N}\big(0, \sigma_e^2[t]\big) \tag{4}$$

with t designating discrete time, $y[t]$ the signal to be modeled, $e[t]$ an (unobservable) uncorrelated innovations sequence with zero mean and time-dependent variance $\sigma_e^2[t]$, and $a_i[t]$, the time-dependent AR model parameters. The TAR representation imposes no "structure" on the evolution of its parameters, which are thus "free" to change with time, and is thus directly parameterized in terms of time-dependent parameters $a_i[t]$ and innovations variance $\sigma_e^2[t]$.

Given a single, N-sample-long, non-stationary signal record $\{y[1], \ldots, y[N]\}$, TAR model identification involves selecting the corresponding model structure, and estimating the model parameters $a_i[t]$ and the innovations variance $\sigma_e^2[t]$

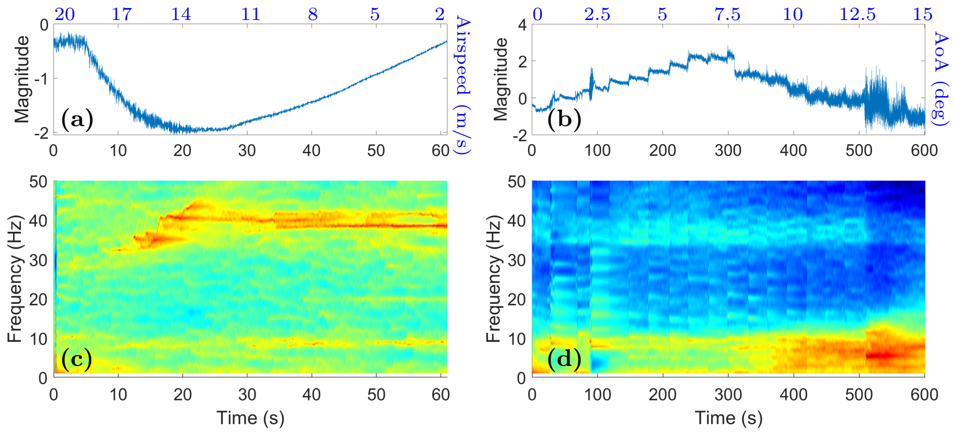

Fig. 3. Top: indicative non-stationary signals under continuously varying (a) airspeed and (b) AoA. Bottom: RML-TAR$(40)_{0.998}$-based time-dependent power spectral density estimates under continuously varying (a) airspeed and (b) AoA.

that "best" fit the available data. The TAR model is parameterized via the parameter vector $\boldsymbol{\theta}[t] = [a_1[t] \ \ldots \ a_{na}[t]]$ to be estimated based on the recorded non-stationary signal. For a detailed review see [12]. In this work, parameter estimation is based on an exponentially weighted prediction error criterion and a recursive estimation scheme accomplished via the recursive maximum likelihood (RML) method [12, 11].

3.1 Adaptive TAR-based flight awareness results

The parametric identification via TAR models is based on 60, 100 (601 s) and 6, 100 (61 s) sample-long response signals (sampling frequency $f_s = 100$ Hz) under *continuously* varying AoA (from 0 to 15 degrees) and airspeed (decreasing from 20 m/s to 0 m/s), respectively, recorded via embedded piezoelectric sensors (see Figure 3a and b). The model structure selection problem, i.e. determination of the model order and forgetting factor [12], is based on the successive estimation of TAR(na) models for orders $na = 2, \ldots, 50$ and forgetting factors $0.900, \ldots, 0.999$, with the best model selected based on the combined consideration of the Bayesian Information Criterion [12] and the comparison with the corresponding non-parametric power spectral density (PSD) estimates. This process resulted in RML-TAR$(40)_{0.998}$ models for representing the non-stationary dynamics due to time-dependent evolution of the AoA and airspeed of the wing.

Figure 3c and d presents indicative RML-TAR$(40)_{0.998}$-based time-dependent PSD estimates for continuously varying airspeed and AoA, respectively. Observe the time-dependent nature of the wing dynamics; in the case of varying airspeed (Figure 3c) observe the separation of the 9 Hz natural frequency at 20 s, as the two vibrational modes are decoupled as the airspeed decreases and the aeroelastic flutter diminishes. Figure 4(a-c) presents the first three RML-TAR$(40)_{0.998}$-based time-dependent AR parameters along with their estimated 95% CIs for a close-up time window of one second. Again, the time-dependent nature of the parameters is evident with the evolution of the flight state dynamics. In addition, observe the narrow confidence intervals of the model parameters that are

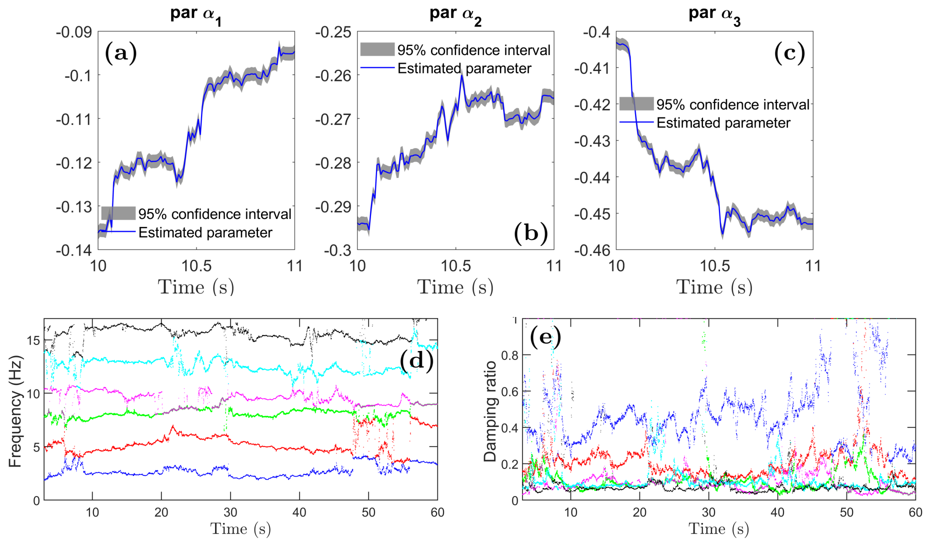

Fig. 4. Top (a-c): RML-TAR$(40)_{0.998}$-based time-dependent evolution of three indicative model parameters. Bottom (d-e): identified RML-TAR$(40)_{0.998}$-based time-dependent (d) natural frequencies and (e) damping rations in the $[0-15]$ Hz range.

based on the recursive estimation of the parameter covariance matrix. Figure 4d and e depicts RML-TAR$(40)_{0.998}$-based the time-dependent natural frequencies of the wing and their identified damping ratios within the frequency bandwidth of $[0-15]$ Hz. Again, observe the time-dependent nature of the identified modes based on the RML-TAR model and compare with Figure 3.

4 Conclusions

The investigation and assessment of non-parametric Bayesian Gaussian process regression homoscedastic and variational heteroscedastic models, and adaptive time-dependent models for flight awareness, were presented based on experimental data collected from a UAV wing during wind tunnel experiments under varying flight states. The VHGPRMs outperformed their homoscedastic counterparts in terms of the predictive input-dependent variance estimation accuracy. Stochastic adaptive RML-TAR models were shown to be capable of identifying the time-dependent stochastic wing vibration dynamics under continuously varying AoA and airspeed by imposing no structure on the time evolution of their parameters. Ongoing work addresses the investigation of time-dependent stochastic models that impose structured stochastic (parameters are random variables allowed to change with time) and deterministic (parameters are projected on time-dependent functional subspaces) evolution on their parameters.

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