DSP- Digital Signal Processing:

DSP is the processing of signals by digital means. In other words DSP is a field of study that deals with the manipulation, analysis and synthesis of digital signals.

DSP includes

Signal:

A signal is defined as a function of one or more variables which conveys information. A signal is a physical quantity that varies with time in general, or any other independent variable. It can be dependent one or more independent variables.

Noise:

Any variables which does not convey information is called Noise. Noise is a random phenomenon in which physical parameters are time-variant. It usually does not carry useful information and is almost always considered undesirable.

→ Dimension of a signal may be defined based on the number of independent variables.

There are three types of signal based on Dimension:

1. One-Dimension
2. Two-Dimension
3. Multi-dimension

1. One-dimension -

When a function depends on a single independent variable to represent the signal, it is said to be a one-dimensional signal.

The ECG and speech signal are examples of one-dimensional signals where the independent variable is time. The magnitude of the signals is dependent variable.

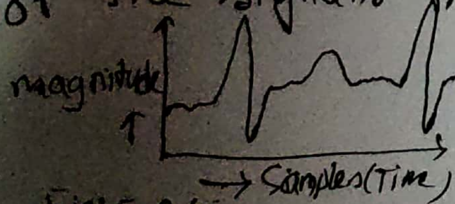


Fig: ECG

2. Two-dimensional Signal:

When a function depends on two independent variables to represent the signal, it is said to be a two-dimensional signal. For example a photograph is a two-dimensional

3. Multi-dimensional Signal:

When a function depends on two or more than two independent variables to represent the signal, it is said to be a multi-dimensional signal.

System:

A system takes input signal and gives output signal.

Input signal:

A signal that enters a system from an external source is refferred to as an input signal. For example the voltage from a function generator, electrocardiogram from heart, temperature from the human body etc.

Output Signal:

A signal produced by the system in response to the input signal is called the output signal. Example: output voltage from an amplifier.

Ⅲ Why signal should be analyzed? / Importance/Application

Signal can be two types: Analog and Digital.

1. **Analog Signal:**

2. **Digital Signal:**

1.4 - Convert Analog to Digital and Digital to analog.

* Analog to Digital: There are three steps

1. Sampling
2. Quantization
3. Coding

ⓒ We gets from (b), $y(n) = 13\cos\frac{2\pi}{5}n - 5\sin\frac{4\pi}{5}n$, so, the analog will be

$y(t) = 13\cos 2000\pi t - 5\sin 4000\pi t$ [$\because F_s = 5000\,Hz$ multiply with F_s] $f = \frac{F}{F_s}$

Proakis

Ex-1.4.2: ⓐ The frequency of the signal $x(t) = 3\cos 100\pi t$ is $50\,Hz$.

So, the minimum sampling rate required to avoid aliasing is

$F_s = 2f = 2 \times 50 = 100\,Hz$.

ⓑ Given, $F_s = 200$. As we know, $f = \frac{F}{F_s}$, So descrete time-signal is $x(n) = 3\cos\frac{100\pi}{200}n = 3\cos\frac{\pi}{2}n$

ⓒ Given, $F_s = 75\,Hz$, $f = \frac{F}{F_s}$, so, descrete time-signal is

$x(n) = 3\cos\frac{\cancel{100}^{4}\pi}{\cancel{75}_{3}}n = 3\cos\frac{4\pi}{3}n = 3\cos\left(2\pi - \frac{2\pi}{3}\right)n$

$\therefore x(n) = 3\cos\frac{2\pi}{3}n$.

ⓓ Given, $F_s = 75\,Hz$, we have, $F = fF_s = 75f = \cancel{75}^{25} \times \frac{1}{\cancel{3}}$ [$\because f = \frac{1}{3}$ from (c)]

$\therefore F = 25\,Hz$

So, the sinusoidal signal $y_a(t) = 3\cos 2\pi Ft = 3\cos 50\pi t$

Ex-1.4.3: $x(t) = 3\cos 50\pi t + 10\sin 300\pi t - \cos 100\pi t$

$F_1 = 25\,Hz$, $F_2 = 150\,Hz$, $F_3 = 50\,Hz$

$\therefore$ Nyquest Rate, $F_N = 2F_{max} = 2F_2 = 2 \times 150 = 300\,Hz$

Ex-1.4.4: ⓐ $F_1 = 1000\,Hz$, $F_2 = 3000$, $F_3 = 6000$, $F_N = 2F_{max} = 2 \times 6000 = 12000 = 12\,KHz$

ⓑ $x(n) = x(nT) = x\left(\frac{n}{F_s}\right) = 3\cos 2\pi\frac{1}{5}n + 5\sin 2\pi\frac{3}{5}n + 10\cos 2\pi\frac{6}{5}n = 3\cos 2\pi\frac{1}{5}n + 5\sin 2\pi\left(1 - \frac{2}{5}\right)n + 10\cos 2\pi\left(1 + \frac{1}{5}\right)n = 3\cos 2\pi\frac{1}{5}n + 5\sin\left(-\frac{2}{5}\right)n + 10\cos 2\pi\left(\frac{1}{5}\right)n = 13\cos\frac{2\pi}{5}n - 5\sin\frac{4\pi}{5}n$

Poornachandra

2.2 - Classification of Signals

1. Continuous-time signal and Discrete-Time signal (Analog, Digital)
2. Periodic and Aperiodic signal
3. Even signal and odd signal
4. Deterministic and Random signal
5. Energy and Power signal.

1. Continuous and Discrete:

→ **Continuous:** (natural signals are continuous)

A signal $x(t)$ is said to be a continuous-time signal if it is defined for all time t. The amplitude of the signal varies continuously with time.

→ **Discrete:**

Most of the signals that are obtained from their sources are continuous in time. A signal $x(n)$ is said to be discrete-time signal if it can be defined for a discrete instant of time.

Problem-2.2: given, $x(t) = e^{-2t}$ for $2 \geq t \geq -2$, $T = 0.1s$.

Solve:

t	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
$x(t)$	54.6	20	7.4	2.7	1	0.37	0.13	0.05	0.02

This will plot the continuos signal. Now, the descrete-time signal will be,

$$x(t) = x(nT)\Big|_{t=nT}$$

$$= x(0.1n) \qquad [T = 0.1]$$

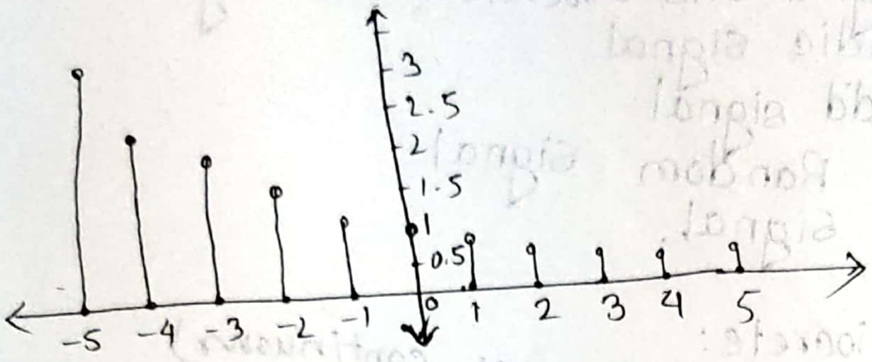
$$= e^{-2 \times 0.1n}$$

$$\therefore x(n) = e^{-0.2n}$$

n	-5	-4	-3	-2	-1	0	1	2	3	4	5
$x(n)$	2.7	2.23	1.82	1.5	1.2	1	0.8	0.67	0.5	0.45	0.37

2. Periodic and Aperiodic:

Discrete - time signal



$\boxed{2.2.2}$ - Periodic and Aperiodic continuous - time signal

A continuous - time signal $x(t)$ is said to be periodic if $x(t) = x(t+T)$, $T > 0$, T is a period of cycle.

$x(t) = x(t+1T) = x(t+2T) = x(t+3T) = \dots = x(t+nT)$

So, a periodic signal with period $T > 0$ is also periodic with period nT.

Problem 2.8: Test periodic or not

(i) $x(t) = e^{\sin(t)}$

$t = t+T$, substitute $t = t+T$

$$x(t+T) = e^{\sin(t+T)}$$

$$= e^{\sin(t+2\pi)} \quad [\because T = 2\pi]$$

$$= e^{\sin(2\pi+t)}$$

$$x(t+T) = e^{\sin t}$$

$\therefore x(t+T) = x(t)$

Hence, the signal $x(t) = e^{\sin(t)}$ is periodic.

(ii) $x(t) = te^{\sin t}$, From periodicity definition, $t = t+T$

$x(t+T) = (t+T)e^{\sin(t+T)}$

$= (t+2\pi)e^{\sin(t+2\pi)}$

$= (t+T)e^{\sin(t)}$

$\therefore x(t+T) \neq x(t)$

So, this is aperiodic.

Problem-2.6

Given, $x(t) = \sin(\frac{3\pi}{5})t$

$\omega = \frac{2\pi}{5}$, $T = \frac{2\pi}{\omega} = \frac{2\pi}{\frac{2\pi}{5}} = 2\pi \times \frac{5}{2\pi} = 5$

$\therefore$ Periodicity is 5.

Problem-2.9: Given, $x(t) = je^{j10t}$ $[\because e^{j\omega t}$

Solve: The frequency, $\omega_0 = 10$

$T = \frac{2\pi}{\omega} = \frac{2\pi}{10} = \frac{\pi}{5}$

The signal is periodic with fundamental period $\frac{\pi}{5}$.

Problem-2.10:

$x(n) = \sin \frac{2\pi}{3}n$

frequency, $\Omega = \frac{2\pi}{3}$ is in radians in the discrete domain

Fundamental period,

$$N = \frac{2\pi m}{\Omega} = \frac{2\pi m}{2\pi/3} = 3 \quad [m = 1]$$

$$\boxed{N = \frac{2\pi}{\Omega}}$$

2.2.3 - Even and odd continuous-time Signal.

→ A continuos-time signal $x(t)$ is said to be even, if it satisfies the cond.

$$x(t) = x(-t)$$

Even signals are symmetric about the vertical axis

→ A C-T signal $x(t)$ is said to be odd if it satisfy the cond

$$x(t) = -x(-t)$$

odd are antisymetric.

Problem-2.18: Find the odd and even components of $x(t) = e^{j2t}$

Solve: we know that,

$$x(t) = x_e(t) + x_o(t) = e^{j2t}$$

The even signal is given by,

$$x_e(t) = \frac{x(t) + x(-t)}{2} = \frac{e^{j2t} + e^{-j2t}}{2}$$

$$= \cos 2t$$

$$x_o(t) = \frac{x(t) - x(-t)}{2}$$

$$\left[\begin{array}{l} e^{j2t} = \cos 2t + j\sin 2t \\ e^{-j2t} = \cos 2t - j\sin 2t \end{array}\right]$$

$$= \frac{e^{j2t} - e^{-j2t}}{2}$$

$$= j\left[\frac{e^{j2t} - e^{-j2t}}{2j}\right]$$

$$= j \sin 2t$$

$$= \frac{\cos 2t + j\sin 2t - (\cos 2t - j\sin 2t)}{2}$$

$$= \frac{\cancel{\cos 2t} + j\sin 2t - \cancel{\cos 2t} + j\sin 2t}{2}$$

$$\therefore x_o(t) = \frac{\cancel{2} j\sin 2t}{\cancel{2}} = j\sin 2t$$

$$\therefore x(t) = x_e(t) + x_o(t) = \cos 2t + j\sin 2t$$

Problem-2.19: Show that

Solve: Let's consider two even signals $x_1(t)$ and $x_2(t)$

$$x(t) = x_1(t) \times x_2(t)$$

Replace $t = -t$ then.

$$x(-t) = x_1(-t) \times x_2(-t)$$
$$= x(t)$$

Hence those are even signal

let, two odd signal $x_3(t)$ and $x_4(t)$

$$x(t) = x_3(t) \times x_4(t)$$

Replace, $t = -t$

$$x(-t) = x_3(-t) \times x_4(-t)$$

$$x(-t) = -x_3(t) \times -x_4(t)$$

$$= x_3(t) \times x_4(t)$$

$$\therefore x(-t) = x(t)$$

so, it is an even signal.

$$x(t) = x_1(t) \times x_3(t)$$

$$x(-t) = x_1(-t) \times x_3(-t)$$

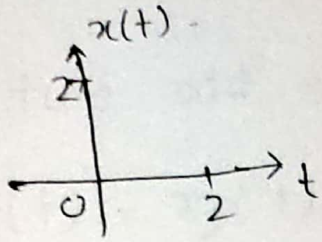
$$= x_1(t) \times -x_3(t)$$

$$= -x_1(t) \times x_3(t)$$

$\therefore x(-t) = -x(t)$ which is odd.

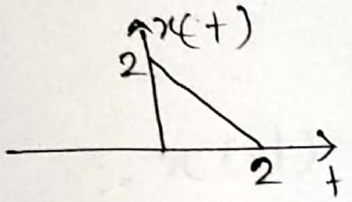
Problem-2.20 :

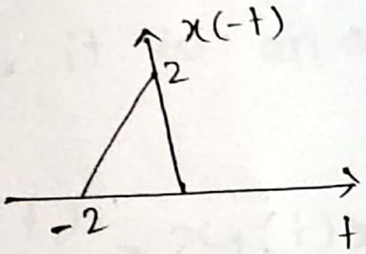
Pro 2.22 :



solve: For even

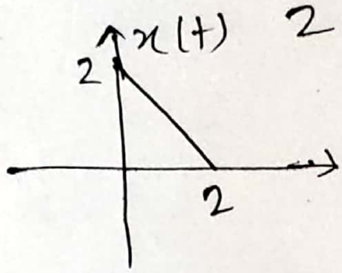
$$x_e(t) = \frac{x(t) + x(-t)}{2}$$

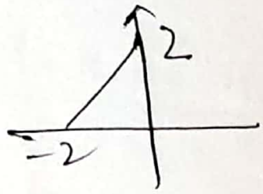


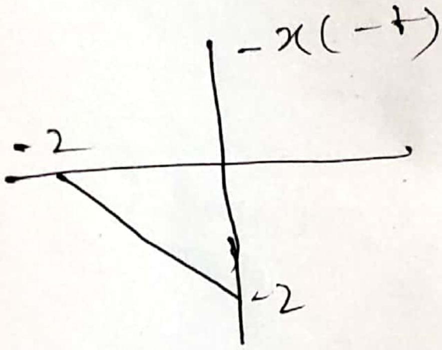


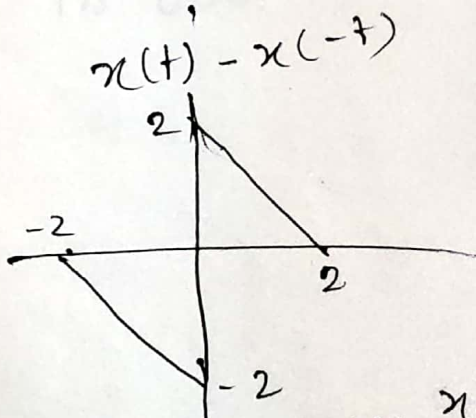
For odd

$$x_o(t) = \frac{x(t) - x(-t)}{2}$$

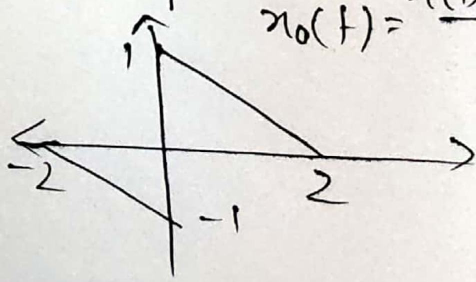








$$x_o(t) = \frac{x(t) - x(-t)}{2}$$



2.2.4 - Energy and power Signal

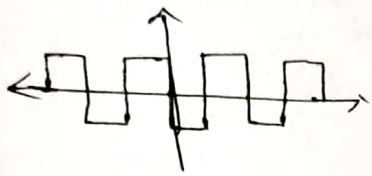
If integration is finite then it is power signal.

2.2.5 - Deterministic Signal and Random Signal.

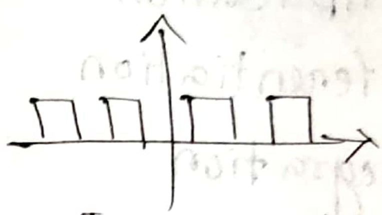
(i) Deter-

A DS signal is a signal about which there is certainty with respect to its values at any time. In a deterministic signal, the future values of the signal are predictable.

Ex:



Square wave

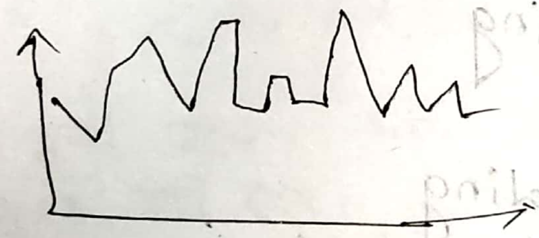


Train pulse

(ii) Random

A RS is a signal about which there is uncertainty with respect to its values at any time. In RS, the future values of the signal are ~~p~~ unpredictable.

Ex:



Speech signal

2.3 - Basic operations on Signals

1. Operations performed on dependent variables
   - (i) Amplitude Scaling
   - (ii) Addition
   - (iii) Multiplication
   - (iv) Differentiation
   - (v) Integration

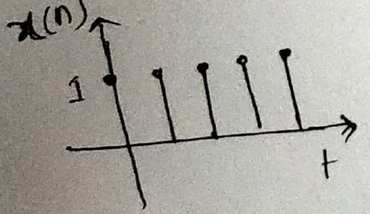
2. Operations performed on independent variables-
   - (i) Time scaling of signals
   - (ii) Reflection
   - (iii) Time shifting

2.3.1 - Amplitude Scaling

Amplitude scaling factor = α

→ if $\alpha < 1$, the signal attenuates, if $\alpha > 1$, the signal aplifies.

$$y(t) = \alpha x(t)$$

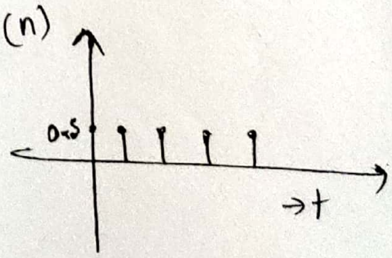


original

let,

$\alpha = 0.5$

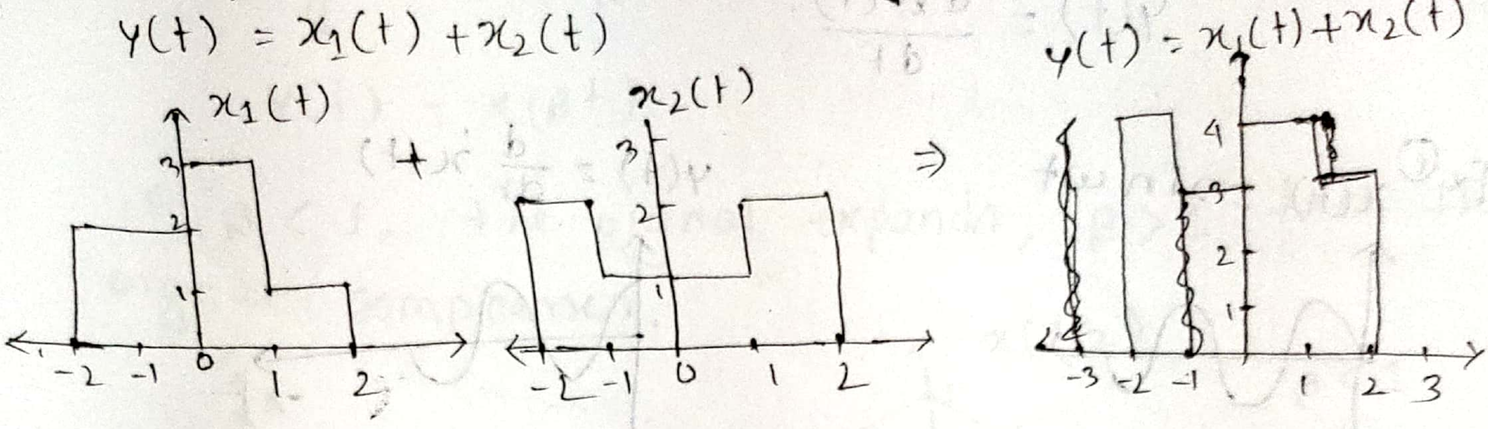
$y(t) = 0.5\,x(t)$



2.3.2 - Addition of signals

The period of the output signal is unaltered.

$y(t) = x_1(t) + x_2(t)$



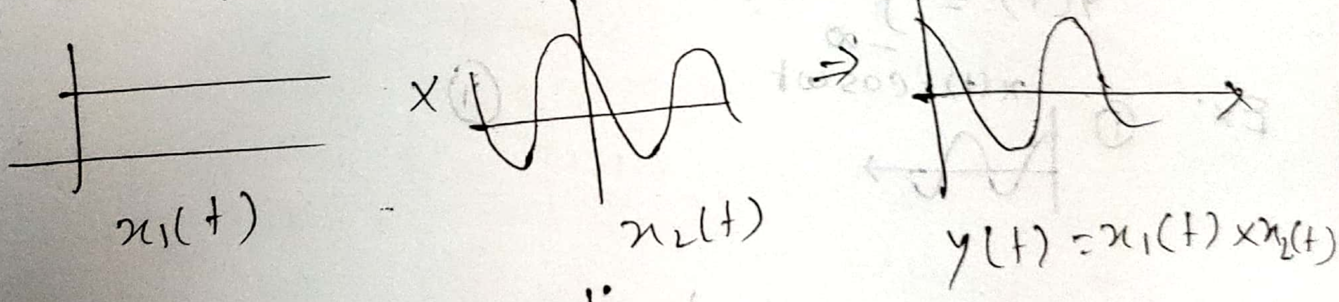
2.3.3 - Multiplication:

Multiplication of message signal $x_1(t)$ over the carrier signal $x_2(t)$ results in modulated signal, which is used to transmit over communication media.

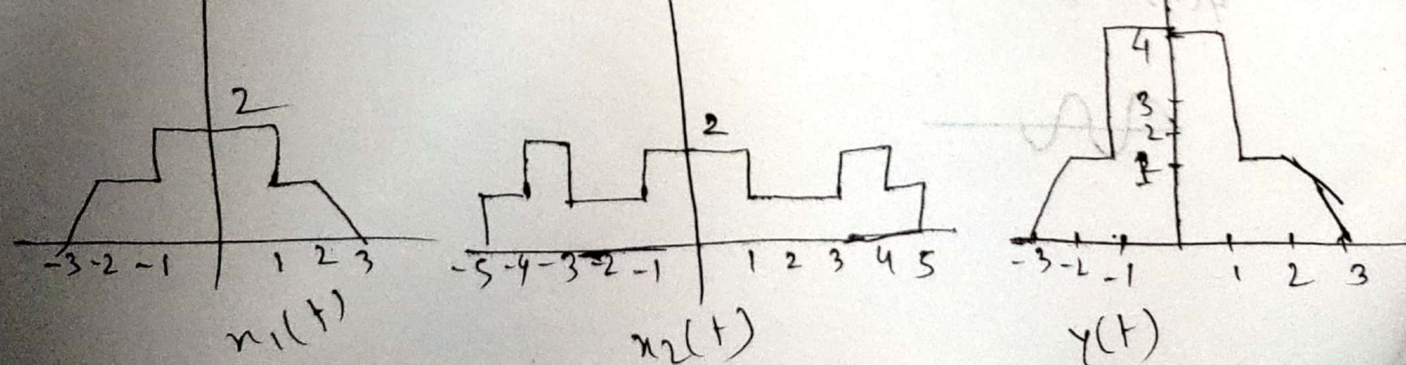
$$y(t) = x_1(t) \times x_2(t)$$

⊛ সর্বনিম্ন মূল্য হবে। যদি যেকোনো টার একদিকে থাকে তাহলে তাই হবে।

Ex: ①



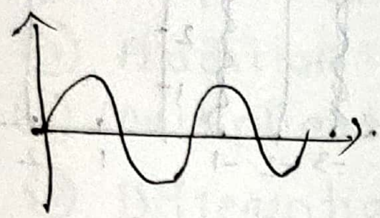
(ii)

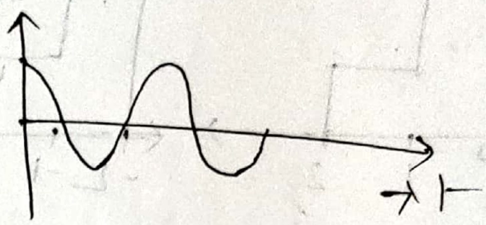


2.3.4 - Differentiation

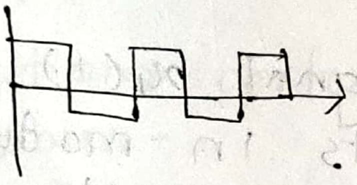
$$y(t) = \frac{d x(t)}{dt}$$

Ex ① $x(t) = \sin \omega t$ $\qquad y(t) = \frac{d}{dt} x(t)$

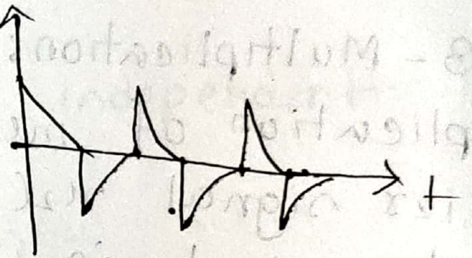




Ex - ⓘⓘ



$x(t)$

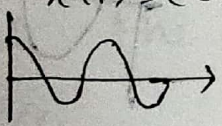


$y = \frac{d}{dt} x(t)$

2.3.5 - Integration

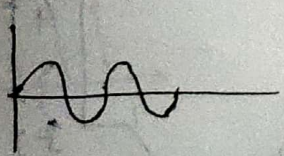
$$y(t) = \int_{-\infty}^{t} x(t)\, dt$$

Ex: ① $x(t) = \cos \omega t$



ⓘⓘ

$y(t) = \int x(t) dt = \sin \omega t$

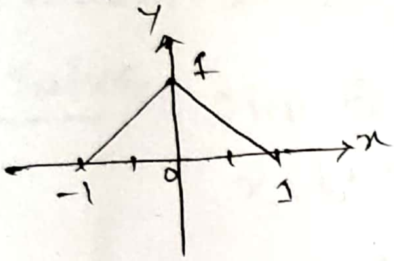


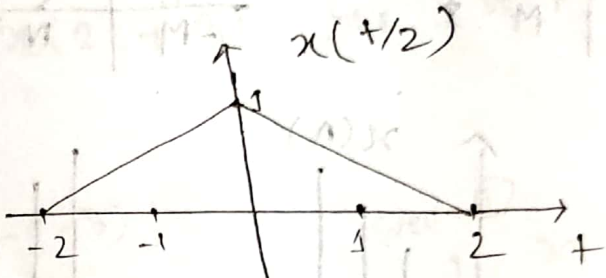
2.3.6 - Time scaling

scaling factor β.

$$y(t) = x(\beta t)$$

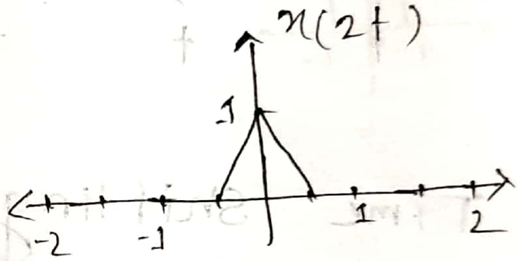
if $\beta < 1$, the signal expands, $\beta > 1$ then the signal compresses.





$\beta < 1$

(*) Discrete এর time scaling বুঝিনি নাই।



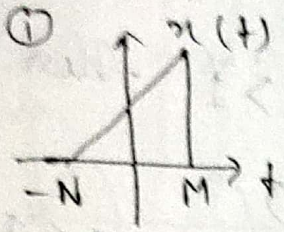
2.3.7 – Reflection / Folding

$$y(t) = x(-t)$$

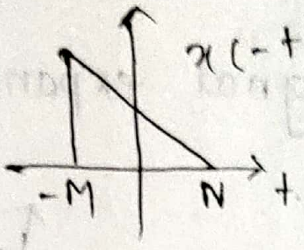
$$y(n) = x(-n)$$

Ex: (i)

$x(t)$

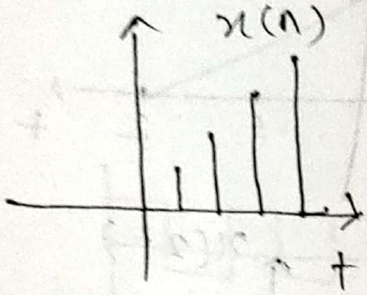


$x(-t)$

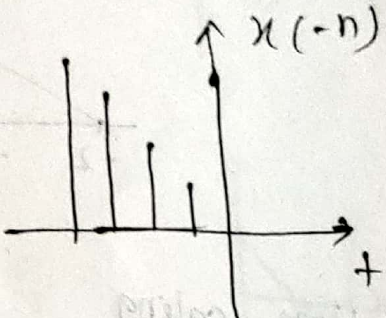


(ii)

$x(n)$



$x(-n)$

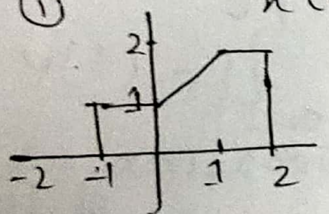


2.3.8 – Time Shifting

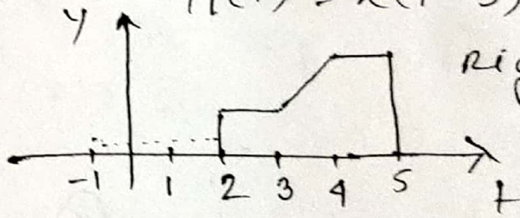
$$y(t) = x(t - t_0)$$

Ex: (i)

$x(t)$



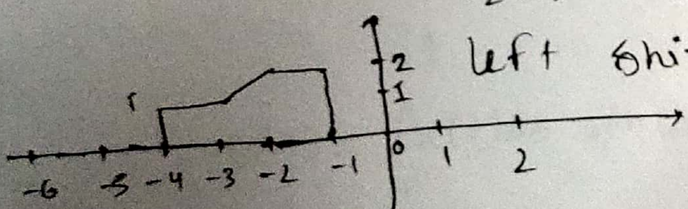
$y_1(t) = x(t-3) = x(t-(+3$



Right shift

$y_2(t) = x(t+4) = x(t-(-4))$

left shift



2.3.9 - Time shifting and Time scaling

$y(t) = x(t-a) \longrightarrow$ shifting

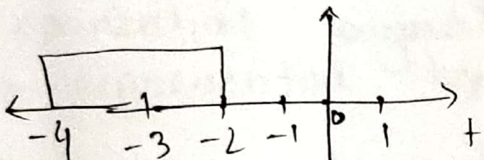
scaling, $y(t) = x(\beta t)$

Problem - 2.33(i) - Find $x(2t+3)$ for $x(t)$.

<u>Solve:</u>

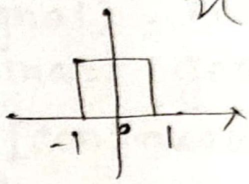
Step - (ii)

$x(t+3) = x[t-(-3)]$



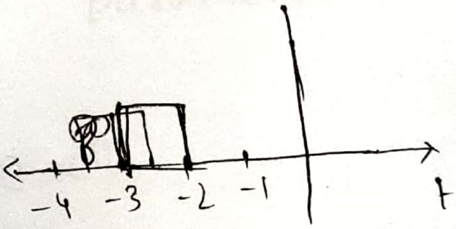
Step - (i)

$x(t)$



Step - (ii)

$x(2t)$



Lower Bound	Upper Bound
$2t+3 = -1$	$2t+3 = 1$
$2t = -4$	$2t = -2$
$t = -2$	$t = -1$

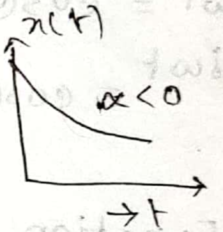
2.4 - Types of Signals

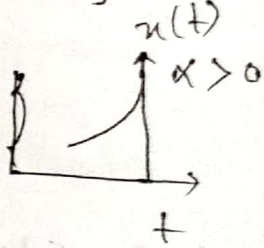
1. Exponential
   (i) Real
   (ii) Complex
2. Sinusoidal
3. Step
4. Impulse
5. Ramp.

2.4.1 – Exponential:

(i) Real Exponential (continuous-time signal) - A real exponential signal $x(t)$ in its most general form is represented by $x(t) = Be^{\alpha t}$ [continuous]

B is scaling factor,
α is real parameter





Continuous-time Exponential signal

For discrete,
$x(n) = B\,\alpha^n$

(ii) Complex exponential signal (continuos-time signal)

$x(t) = e^{j\omega t}$

periodicity, $x(t) = e^{j\omega(t+T)}$, If $\omega = 0$, then $x(t) = 1$, periodic for any value of T.

$e^{j\omega T} = \cos\omega T + j\sin\omega T \stackrel{?}{=} 1$

CES (Discrete-signal)

$x(n) = C\alpha^n$

C = scaling factor

$\alpha = e^{\beta}$, complex parameter

2.4.2 - Sinusoidal signal (Continuos - time)

It's general form,

$$x(t) = A\cos(\omega t + \phi)$$

A = amplitude
ω = frequency (radians)
ϕ = phase (radians)

$$\boxed{\omega = \frac{2\pi}{T}}$$

For Discrete-time signal,

$$x(n) = A\cos(\Omega n + \phi) \qquad \boxed{N = \frac{2\pi}{\Omega}}$$

$$x(n+N) = A\cos(\Omega n + \phi) = x(n)$$

complex sinusoidal

$$e^{j\omega t} = \cos(\omega t) + j\sin\omega t$$

$$e^{-j\omega t} = \cos(\omega t) - j\sin\omega t$$

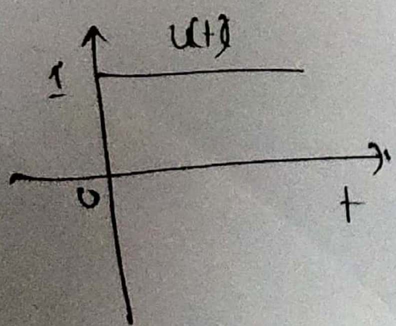
2.4.3 - Step Function

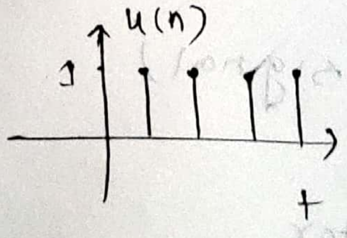
The continuous-time step function is commonly denoted by $u(t)$ and is defined as,

$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

Discrete,

$$u(n) = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$





2.4.4 - Impulse Function

The impulse function is a derivative of the step function $u(t)$ with respect to time. It is denoted by $\delta(t)$

$$\delta(t) = 0, \quad t \neq 0$$

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

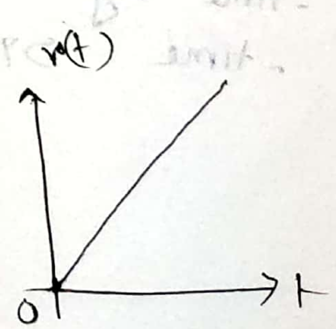
2.4.5 - Ramp Function -

The integral of the step function $u(t)$ is a ramp function of unit slope. It is denoted by $r(t)$

$$r(t) = \begin{cases} t, & t \geq 0 \\ 0 & t < 0 \end{cases} \quad [\text{continuous}]$$

Discrete,

$$r(n) = \begin{cases} n, & n \geq 0 \\ 0 & n < 0 \end{cases}$$



2.5 - System:

A system is an entity that manipulates one or more input signals to perform a function which resuls a new output function.

Input → $\boxed{\text{System}}$ → output

1. Continuos and Discrete time System
2. Stable and Unstable system
3. Memory and Memorylem System
4. Invertible and Noninvertible system
5. Time-variant and time-invariant system
6. Linear and Non-linear System
7. Causal and Non-causal System

1. Continuos and Discrete-time System:

→ If the input and output of the system are continuos-time signals, then the system is called continuous-time system.

Input (cont. time) → $\boxed{\text{System}}$ → output (continuous-time)

→ If the input and output of the system are discrete-time signals, then the system is called discrete-time system.

2.6.2 - Stable and Unstable System

→ A given system is said to be stable if and only if every bounded input produces a bounded output. It is also known as 'BIBO'

$$|x(t)| \leq M_x < \infty \text{ for all } t / n.$$

Ex - 2.39: (i) Given $h_1(n) = 2^n u(n-3)$, So, general condition for this system will be, $h_1(n) = \sum_{n=3}^{\infty} |2^n| < \infty$

$$\therefore \sum_{n=3}^{\infty} |2^n| = 2^3 + 2^4 + 2^5 + \dots + 2^{\infty} = \infty$$

Hence, the system is stable

2.6.3 - Memory and Memoryless

→ The given system is said to posses memory if the output of the system depends on past and future values. Also known as 'Dynamic System'

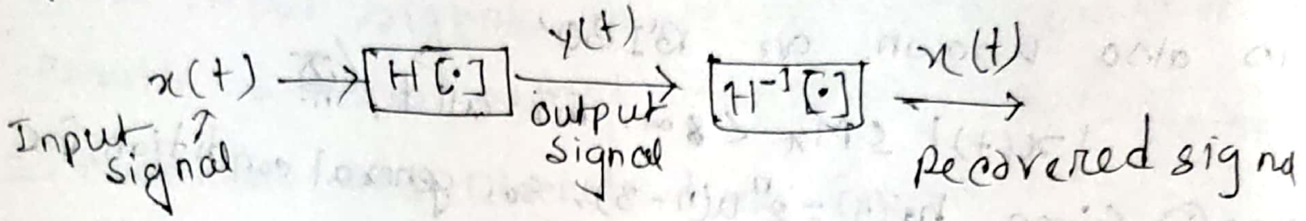
Ex - $y(t) = x(t) + x(t-1) + x(t+1)$

→ The given system is said to be memoryless if the output of the system depends only on the present values.

Ex $y(t) = x(t)$

2.6.4 - Invertible and Noninvertible

A system is said to be an invertible system if the input signal given to the system can be recovered.

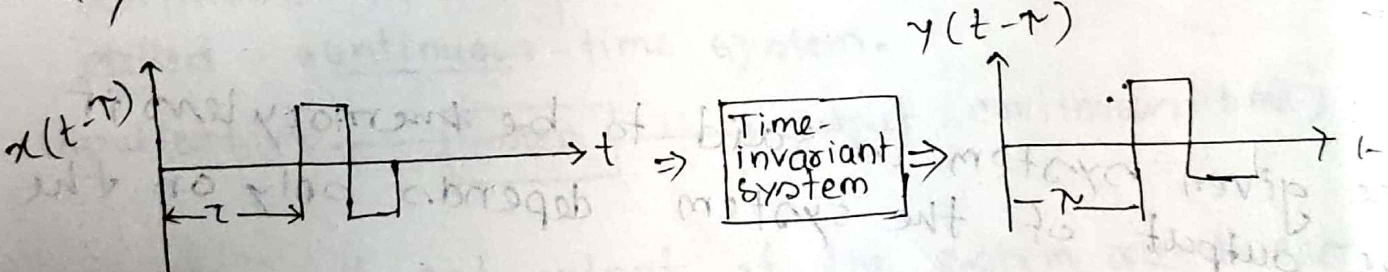


Noninvertible:

A system is said to be noninvertible if the input signal given to the system cannot be recovered from the output signal of the system.

2.6.5 - Time-invariant and Time variant

A system is said to be time-invariant if the input input signal is delayed or advanced by any factor that leads to some delay or advancement in the time scale by the same factor, the system responds to an input which is given at any instant of time and results in an output.



Problem-2.41: The input-output relation is given by $y(t) = \sin[x(t)]$. Determine whether the system is time invariant or not.

Ans:

$$y(t) = \sin[x(t)]$$

Let us assume the signal of the form

Let us introduce time delay t_0 in the input signal in equation (1), then

$$x_2(t) = x_1(t-t_0)$$

The delay input therefore results in the output

$$y_2(t) = \sin[x_2(t)] = \sin[x_1(t-t_0)]$$

Let us introduce the same time delay t_0 in the output of the equation

$$y_1(t-t_0) = \sin[x_1(t-t_0)]$$

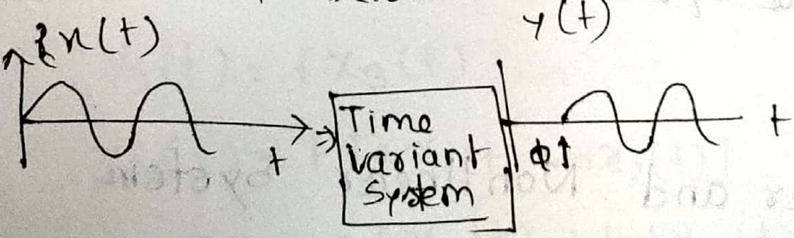
On computing equations (2) and (3),

$$y_2(t) = y_1(t-t_0)$$

Hence system is time-invariant.

Time variant system-

A system is said to be time-variant if the output signal is delayed or advanced with respect to input signal as shown below:



Problem 2.42 -

The input-output relation is given by $y(t) = t\,x(t)$. Determine whether the system is time-variant or not.

Solution:

$y(t) = t\,x(t)$

Let us assume the signal of the form

$$y_1(t) = t\,x_1(t) \quad \text{—— (i)}$$

Let us introduce time delay t_0 in the input signal in equation 1

$$x_2(t) = x_1(t-t_0)$$

The delay in input results in a output

$$y_2(t) = t\,x_2(t) = t\,x_1(t-t_0) \quad \text{—— (ii)}$$

Let us introduce the same delay t_0 in the output of the system

$$y(t-t_0) = (t-t_0)\,x_1(t-t_0) \quad \text{—— (iii)}$$

On comparing equation (2) and (3),

$$y_2(t) \neq y_1(t-t_0)$$

Hence, the system is time-variant.

2.6.6 Linear and Nonlinear System

Input output

$x_1(t) = y_1(t)$

$x_2(t) = y_2(t)$ Then the system is said to be linear if

1. The response to $x_1(t)+x_2(t)$ is $y_1(t)+y_2(t)$

2. The response to $ax_1(t)+bx_2(t)$ is $ay_1(t)+by_2(t)$, where a and b are complex constants (scaling property)

$$T[ax_1(t)+bx_2(t)] = aT[x_1(t)] + bT[x_2(t)]$$

$$T[ax_1(t)+bx_2(t)] = ay_1(t)+by_2(t)$$

Problem:

2.47 - Determine whether the given continuous-time system is linear or not. $y(t) = tx(t)$

Solve: Given

$$y(t) = tx(t)$$

$$y_1(t) = tx_1(t)$$

$x_2(t)$ is response $y_2(t)$

$$y_2(t) = t\,x_2(t)$$

The two defined signals are related by

$$x_3(t) = ax_1(t)+bx_2(t)$$

Then the output $y_3(t)$ is defined as

$$y_3(t) = tx_3(t)$$

$$y_3(t) = t[ax_1(t)+bx_2(t)]$$

$$y_3(t) = atx_1(t) + btx_2(t)$$

$$= ay_1(t) + by_2(t)$$

Therefore the system is linear.

2.6.7 — Causal and Noncausal

Causal system-

In a causal system, the output response of the system at any time depends only on the present input or on the past input, but not on the future inputs.

Ex:

$$y(n) = x(n) - x(n-1)$$

$$y(t) = t\,x(t)$$

In noncausal system, the output response of the system depends on the future input values also.

Ex:

$$y(n) = n^2 x(n)$$

$$y(t) = x(n+1) - x(n)$$

Problem - 2.54:

Check causal or not.

(i) $y(t) = t\,x(t)$ (ii) $y(t) = x(t^2)$ (iii) $y(t) = x^2(t)$

(iv) $y(t) = x(\sin(t))$ (v) $y(t) = \int_{-\infty}^{4i} x(\tau)\,d\tau$

(vi) $y(t) = \frac{dx(t)}{dt}$

Solution:

(i) $y(t) = t\,x(t)$

Let $t=0$, then $y(0)=0$

$t=1$, then $y(1) = x(1)$

$t=-1$, then $y(-1) = -x(-1)$

For all values of t, the output depends on present and past values of the input.

So, this is causal.

(ii) $y(t) = x(t^2)$

Let, $t=0$ then $y(0)=0$

$t=1$, then $y(1) = x(1)$

$t=-1$ then $y(-1) = x(1)$

$t=2$ then $y(2) = x(4)$

In the last two cases, the output value depends on the future value of the input.

So, the system is noncausal.

(iii) $y(t) = x^2(t)$

git add .

git commit -m

git push

Let, $t=0$, then $y(0) = x^2(0)$

$t=-1$, " $y(-1) = x^2(-1)$

$t=1$, then $y(1) = x^2(1)$

The output value does not depend on the future value of the input. Hence, the system is causal

(iv) $y(t) = x(\sin t)$

Let, $t=0$, then $y(0) = x[\sin(0)]$

$t=-1$, then $y(-1) = x[\sin(-1)]$

$t=1$, then $y(1) = x[\sin(1)]$

The output value does not depend on the future value of the input. Hence, the system is causal.

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Chapter - 3 (LTI Systems)

112 - page

Problem

3.1 - Express the given signal sequence as a time-shifted impulse.

$$x(n) = \{1, -2, 8, \underset{\uparrow}{4}, 5, -3, 7\}$$

Solve:

↑ shows the value of the data for n = 0.

n	-3	-2	-1	0	1	2	3
x(n)	1	-2	8	4	5	-3	7

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)$$

$$x(n) = \sum_{k=-3}^{3} x(k)\,\delta(n-k)$$

$$x(n) = x(-3)\,\delta(n+3) + x(-2)\,\delta(n+2) + x(-1)\,\delta(n+1) + x(0)\,\delta(n)$$
$$+ x(1)\,\delta(n-1) + x(2)\,\delta(n-2) + x(3)\,\delta(n-3)$$

$$x(n) = \delta(n+3) - 2\,\delta(n+2) + 8\,\delta(n+1) + 4\,\delta(n) + 5\,\delta(n-1)$$
$$- 3\,\delta(n-2) + 7\,\delta(n-3)$$

Proble - 3.2 - express as time-shifted impulse.

$$x(n) = \{2, 3, \underset{\uparrow}{0}, 7, 8, -15, 18, 20\}$$

Solve:

n	-2	-1	0	1	2	3	4	5
x(n)	2	3	0	7	8	-15	18	20

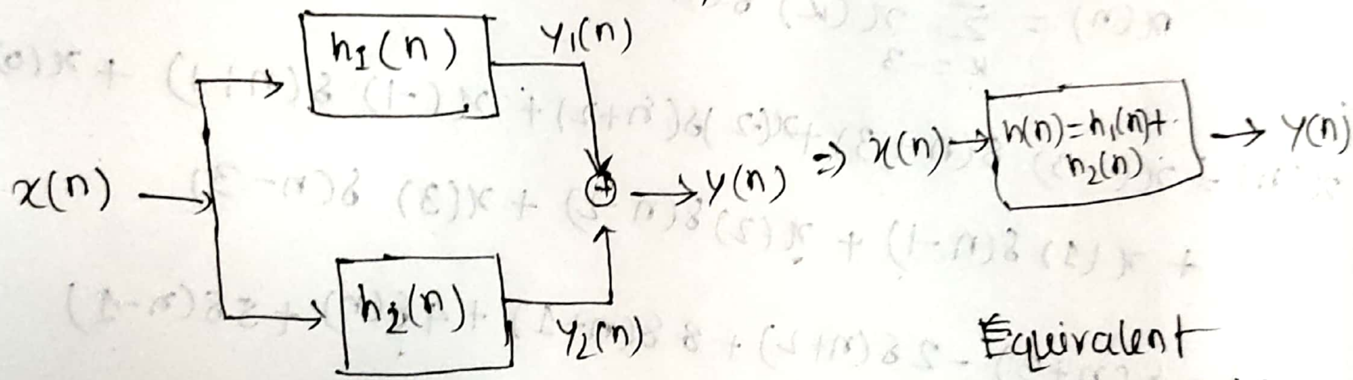
$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)$$

$$x(n) = \sum_{k=-2}^{5} x(k)\,\delta(n-k)$$

$$x(n) = x(-2)\,\delta(n+2) + x(-1)\,\delta(n+1) + x(0)\,\delta(n) + x(1)\,\delta(n-1) + x(2)\,\delta(n-2) + x(3)\,\delta(n-3) + x(4)\,\delta(n-4) + x(5)\,\delta(n-5)$$

$$x(n) = 2\delta(n+2) + 3\delta(n+1) + 7\delta(-1) + 8\delta(n-2) - 15\delta(n-3) + 18\delta(n-4) + 20\delta(n-5)$$

3.2.1- Distributive Property (DD) Divison



Equivalent Representation

Systems connected in parallel

By def of distributive property,

$$x(n) * [h_1(n) + h_2(n)] = x(n) * h_1(n) + x(n) * h_2(n)$$

In short, ভাগ করে নেওয়া;

3.2.2 – Associative property (A B) Bracket

impulse responses $h_1(n)$, $h_2(n)$

$$x(n) \rightarrow \boxed{h_1(n)} \xrightarrow{y_1(n)} \boxed{h_2(n)} \xrightarrow{y(n)} \;\equiv\; x(n) \rightarrow \boxed{h(n) = h_1(n) * h_2(n)} \rightarrow y(n)$$

By definition of the associative property

$$[x(n) * h_1(n)] * h_2(n) = x(n) * [h_1(n) * h_2(n)]$$

Proof: The output of the first system

$$y_1(n) = x(n) * h_1(n) \quad \text{——— (1)}$$

The output of the second system

$$y(n) = y_1(n) * h_2(n)$$

$$\therefore y(n) = x(n) * h_1(n) * h_2(n) \quad [\text{From (1)}]$$

3.2.3 – Commutative property (CP) position

$$x(n) \rightarrow \boxed{h_1(n)} \rightarrow \boxed{h_2(n)} \rightarrow y(n)$$

$$\equiv$$

$$x(n) \rightarrow \boxed{h_2(n)} \rightarrow \boxed{h_1(n)} \rightarrow y(n)$$

By definition,

$$h_1(n) * h_2(n) = h_2(n) * h_1(n)$$

$$h_1(n) * h_2(n) = \sum_{k=-\infty}^{\infty} h_1(k)\, h_2(n-k)$$

3.3

3.3.1 – Memory with and without

A system is memoryless if the output at any time depends only on the present input.

$$y(n) = \sum_{-\infty}^{\infty} x(k)\, k\, \delta(n-k) \quad \left[\begin{array}{l} \delta(n) = 1 \\ \delta(n-k) = 1 \end{array} \quad \begin{array}{l} k = h(0) \\ \hookrightarrow \text{constant} \end{array}\right]$$

$$y(n) = k\, x(k) \rightarrow h(n) = 0$$

$y(n) = k\,x(n)$ is a memoryless LTI system.

If $h(n) \neq 0$ and $n \neq 0$ then the LTI system is called a memory system.

3.3.2 - Invertibility of LTI system

A system is invertible only if an inverse system exists. Similarly, an LTI system is invertible only if an inverse LTI system exists.

$$x(n) \xrightarrow{h(n)} \boxed{h(n)} \xrightarrow{y(n)} \boxed{h_1(n)} \longrightarrow w(n) = x(n)$$

3.3.3 — Stability

A system is said to be stable if every bounded input produces a bounded output. The statement can be extended to LTI system.

Pro - 3.3 - Find whether the system with impulse response $h(n) = 2e^{-2|n|}$ is stable or not.

Ans :

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty \text{ , Stability condition,}$$

$$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=-\infty}^{\infty} 2e^{-2|n|}$$

$$= 2\sum_{n=-\infty}^{\infty} e^{-2|n|} = 2\left[\sum_{n=-\infty}^{-1} e^{-2(-n)} + \sum_{n=0}^{\infty} e^{-2(n)}\right]$$

$$= 2\left[\sum_{n=1}^{\infty} e^{-2n} + \sum_{n=0}^{\infty} e^{-2n}\right]$$

$$= 2\left[\frac{e^{-2a}}{1-e^{-2a}} + \frac{1}{1-e^{-2a}}\right]$$

$$= 2\left(\frac{1+e^{-2}}{1-e^{-2}}\right) = 2\left(\frac{1+\frac{1}{e^2}}{1-\frac{1}{e^2}}\right) = 2\left(\frac{\frac{e^2+1}{e^2}}{\frac{e^2-1}{e^2}}\right) = 2\left(\frac{e^2+1}{e^2-1}\right)$$

$$= 2\left(\frac{1+e^{-2}}{1-e^{-2}}\right) < \infty$$ So the system is stable.

Problem - 3.4 - $h(n) = e^{2n}$, stable or not.

Ans: $\sum_{n=\infty}^{\infty} |h(n)| = \sum_{n=-\infty}^{\infty} e^{2n} = 1 + e^2 + e^4 + e^6 + \dots = \infty$

So, this is unstable.

3.3.4: Causal System-

For a discrete-time LTI system, $h(n) = 0$ for $n<0$.
The output of the system must be expressed as

$$y(n) = \sum_{k=0}^{\infty} x(k)\, h(n-k)$$

3.4 – Linear Convolution.

The convolution equation can be given as an algorithm

1. Plot both $x(k)$ and $h(k)$
2. Reflect $h(k)$ about $k=0$ to obtain
3. Shift $h(-k)$ by n (towards left for $-n$ and towards right for $+n$)
4. Let the initial value of n be negative
5. Multiply each element of $x(k)$ with $h(n-k)$ and add all the product terms to obtain $y(n)$
6. Shift $h(n-k)$ by incrementing the value of n by one and do step 5.
7. Do step 6 until the product of $x(k)$ and $h(n-k)$ reduces to zero.

P-3.5:

Convolution: It is a mathematical way of combining two signals to produce a new signal which is having the characteristics of both signals.

Ex: 3.5 – Perform the convolution of the two sequences

$$x(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}; \quad h(n) = \{1, 1, 1, 1\}$$

Solution:

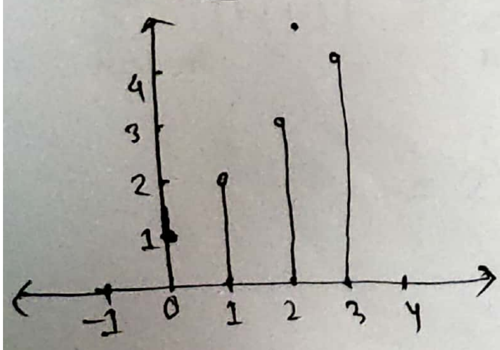
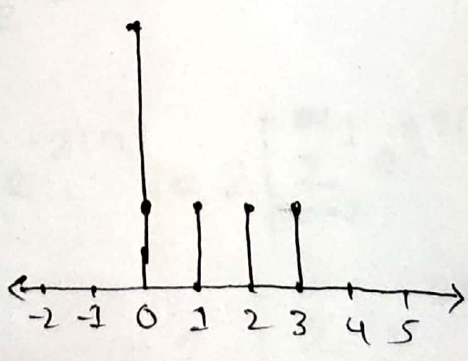
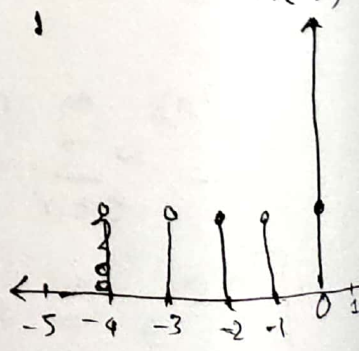


Fig: (a)

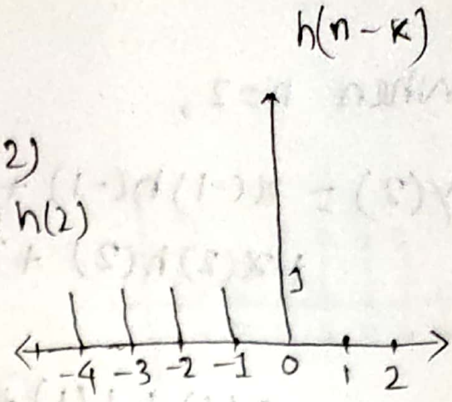


(b)



(c)

When $n = -1$



$$y(-1) = x(-4)h(-4) + x(-3)h(-3) + x(-2)h(-2) + x(-1)h(-1) + x(0)h(0) + x(1)h(1) + x(2)h(2) + x(3)h(3)$$

$$= 0(1) + 0(1) + 0(1) + 0(1) + 1(0) + 2(0) + 3(0) + 4(0)$$

$$= 0$$

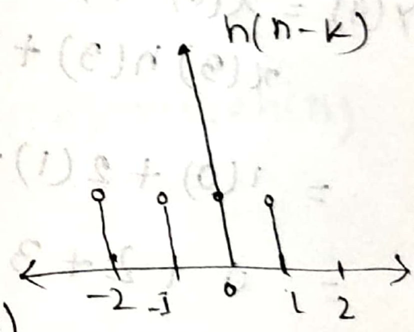
$n = 0$

$$y(0) = x(-3)h(-3) + x(-2)h(-2) + x(-1)h(-1) + x(0)h(0) + x(1)h(1) + x(2)h(2) + x(3)h(3)$$

$$= 0(1) + 0(1) + 0(1) + 1(1) + 2(0) + 3(0) + 4(0)$$

$$= 1$$

When $n = 1$,



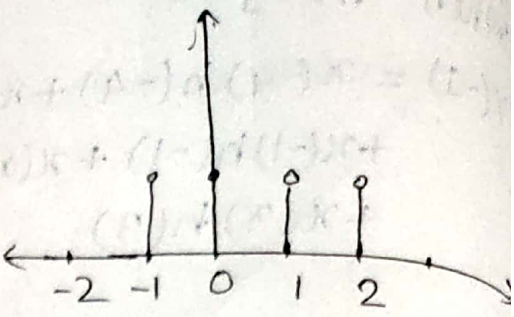
$$y(1) = x(-2)h(-2) + x(-1)h(-1) + x(0)h(0)$$
$$x(1)h(1) + x(2)h(2) + x(3)h(3)$$

$$= 0(1) + 0(1) + 1(1) + 2(1) + 3(0) + 4(0)$$

$$= 1 + 2 = 3$$

When $n=2$,

$y(2) = x(-1)h(-1) + x(0)h(0) + x(1)h(1)$
$+ x(2)h(2) + x(3)h(3)$



$= 0(1) + 1(1) + 2(1) + 3(1) + 4(0)$

$= 1 + 2 + 3 + 0$

$= 6$

When $n=3$,

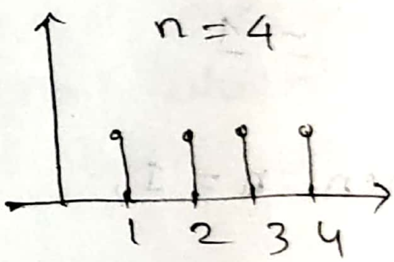
$y(3) = x(0)h(0) + x(1)h(1) + x(2)h(2) + x(3)h(3)$

$= 1(1) + 2(1) + 3(1) + 4(1)$

$= 1 + 2 + 3 + 4$

$= 10$

When $n = 4$,



$y(4) = x(0)h(0) + x(1)h(1) + x(2)h(2) +$
$x(3)h(3) + x(4)h(4)$

$= 1(0) + 2(1) + 3(1) + 4(1) + 0(1)$

$= 0 + 2 + 3 + 4 + 0$

$= 9$

When $n=5$

$y(5) = x(0)h(0) + x(1)h(1) + x(2)h(2) +$
$x(3)h(3) + x(4)h(4) + x(5)h(5)$

$= 1(0) + 2(0) + 3(1) + 4(1) + 0(1) + 0(1)$

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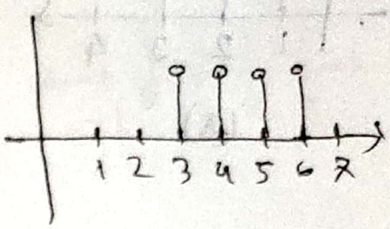
$= 3+4 = 7$

when $n = 6$



$$y(6) = x(0)h(0) + x(1)h(1) + x(2)h(2) + x(3)h(3) + x(4)h(4) + x(5)h(5) + x(6)h(6)$$

$$= 1(0) + 2(0) + 3(0) + 4(1) + 0(1) + 0(1) + 0(1)$$

$$= 4$$

when $n = 7$

$$y(7) = x(0)h(0) + x(1)h(1) + x(2)h(2) + x(3)h(3) + x(4)h(4) + x(5)h(5) + x(6)h(6) + x(7)h(7)$$

$$= 1(0) + 2(0) + 3(0) + 4(0) + 0(1) + 0(1) + 0(1) + 0(1)$$

$$= 0$$

$\therefore$ The result of the convolution is $y(n) = x(n) * h(n)$

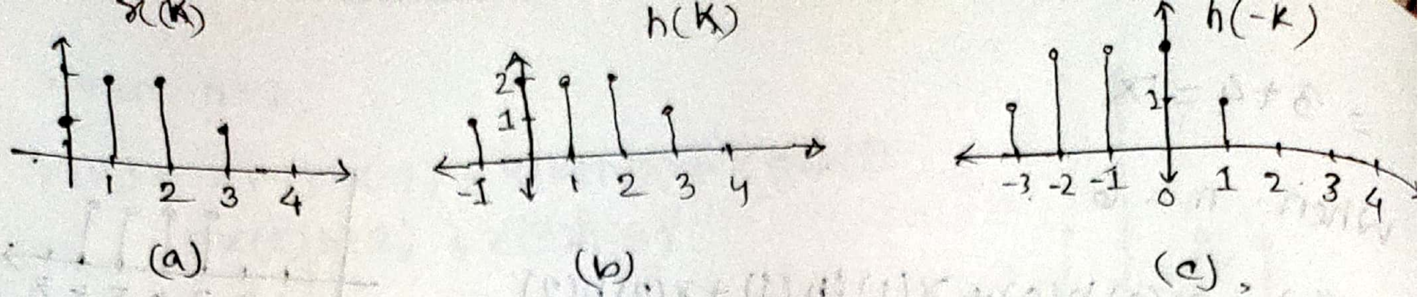
$$= \{1, 3, 6, 10, 9, 7, 4\}$$

Problem-3.6: Perform convolution

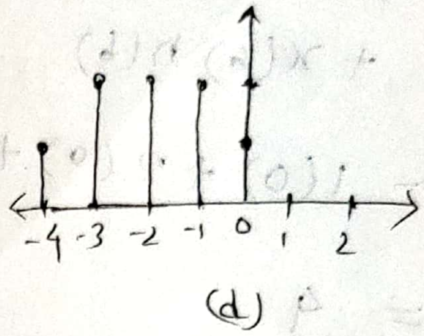
$$x(n) = \{\underset{\uparrow}{1}, 2, 2, 1\}; \quad h(n) = \{1, \underset{\uparrow}{2}, 2, 2, 1\}$$

Solution -

Plot the $x(n)$ and $h(n)$



when $n=-1$, multiply (a) x (d)



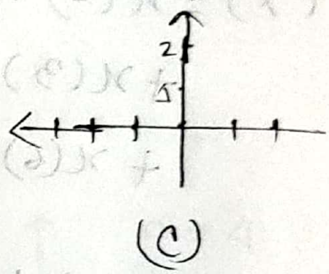
$$y(-1) = x(-4)h(-4) + x(-3)h(-3) + x(-2)h(-2) + x(-1)h(-1) + x(0)h(0) + x(1)h(1) + x(2)h(2) + x(3)h(3)$$

$$= 0(1) + 0(2) + 0(2) + 0(2) + 1(1) + 2(0) + 2(0) + 1(0)$$

$$= 1$$

when, $n=0$

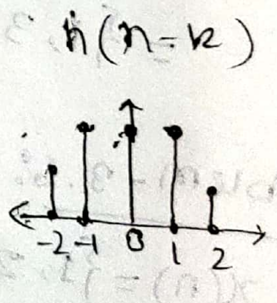
$$y(0) = x(-3)h(-3) + x(-2)h(-2) + x(-1)h(-1) + x(0)h(0) + x(1)h(1) + x(2)h(2) + x(3)h(3)$$



$$= 0(1) + 0(2) + 0(2) + 1(2) + 2(1) + 2(0) + 2(0) + 1(0)$$

$$= 2+2 = 4$$

when, $n=1$,



$$y(1) = x(-2)h(-2) + x(-1)h(-1) + x(0)h(0) + x(1)h(1) + x(2)h(2) + x(3)h(3)$$

$$= 0(1) + 0(2) + 1(2) + 2(2) + 2(1) + 1(0)$$

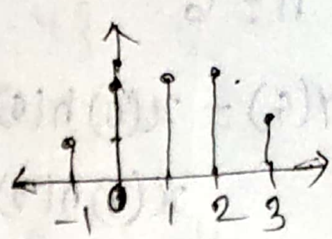
$$= 2+4+2 = 8$$

when, $n=2$

$$y(2) = x(-1)h(-1) + x(0)h(0) + x(1)h(1) + x(2)h(2) + x(3)h(3)$$

$$= 0(1) + 1(2) + 2(2) + 2(2) + 1(1)$$

$$= 2 + 4 + 4 + 1 = 11$$



when, $n = 3$

$$y(3) = x(0)h(0) + x(1)h(1) + x(2)h(2) + x(3)h(3) + x(4)h(4)$$

$$= 1(1) + 2(2) + 2(2) + 1(2) + 0(1)$$

$$= 1 + 4 + 4 + 2$$

$$= 11$$

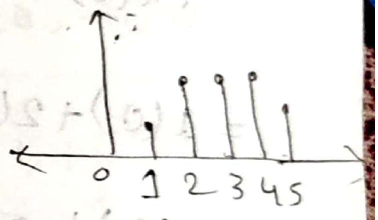
when, $n = 4$

$$y(4) = x(0)h(0) + x(1)h(1) + x(2)h(2) + x(3)h(3) + x(4)h(4) + x(5)h(5)$$

$$= 1(0) + 2(1) + 2(2) + 1(2) + 0(2) + 0(1)$$

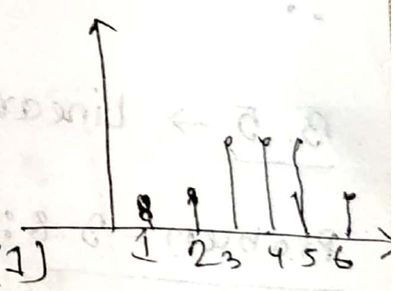
$$= 2 + 4 + 2$$

$$= 8$$



when, $n = 5$

$$y(5) = x(0)h(0) + x(1)h(1) + x(2)h(2) + x(3)h(3) + x(4)h(4) + x(5)h(5) + x(6)h(6)$$



$$= 1(0) + 2(0) + 2(1) + 1(2) + 0(2) + 0(2) + 0(1)$$

$$= 2 + 2$$

$$= 4$$

When

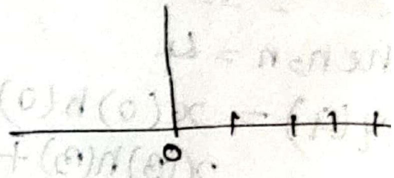
$n = 6$

$$y(6) = x(0)h(6) + x(1)h(1) + x(2)h(2) + x(3)h(3) + x(4)h(4) + x(5)h(5) + x(6)h(6) + x(7)h(7)$$

$$= 1(0) + 2(0) + 2(0) + 1(1) + 0(2) + 0(2) + 0(2) + 0(1) = 1$$

When

$n = 7$



$$y(7) = x(0)h(0) + x(1)h(1) + x(2)h(2) + x(3)h(3) + x(4)h(4) + x(5)h(5) + x(6)h(6) + x(7)h(7) + x(8)h(8)$$

$$= 1(0) + 2(0) + 2(0) + 1(0) + 0(1) + 0(2) + 0(2) + 0(2) + 0(1)$$

$$= 0$$

$\therefore$ The convolution $= x(n) * h(n) = \{1, 4, 8, 11, 11, 8, 4, 1\}$

<u>3.5</u> $\rightarrow$ Linear Convolution cross-table method

<u>Problem - 3.8:</u>

Perform the convolution method matrix] of $x(n)$ and $h(n)$ where,

$$x(n) = \{1, 2, 3, 4\}$$
$$h(n) = \{1, 1, 1, 1\}$$

$$\begin{array}{c|cccc} & \multicolumn{4}{c}{\rightarrow x(n)} \\ & 1 & 2 & 3 & 4 \\ \hline \downarrow h(n)\ 1 & 1 & 2 & 3 & 4 \\ 1 & 1 & 2 & 3 & 4 \\ 1 & 1 & 2 & 3 & 4 \\ 1 & 1 & 2 & 3 & 4 \end{array}$$

$\therefore y(n) = x(n) * h(n)$

$= \{1, (2+1), (3+2+1), (4+3+2+1), (4+3+2), (3+4), 4\}$

$= \{1, 3, 6, 10, 9, 7, 4\}$

Problem-3.9: Convolution

$x(n) = \{1, -2, 3, -4\}$

$h(n) = \{4, -3, 2, -1\}$

Solution:

$$\begin{array}{c|cccc} & \multicolumn{4}{c}{\rightarrow x(n)} \\ & 1 & -2 & 3 & -4 \\ \hline 4 & 4 & -8 & 12 & -16 \\ -3 & -3 & 6 & -9 & 12 \\ 2 & 2 & -4 & 6 & -8 \\ -1 & -1 & 2 & -3 & 4 \end{array}$$

$y(n) = x(n) * h(n) = \{4, (-8-3), (2+6+12), (-1-4-9-16), (2+6+12), (-3-8), 4\}$

$= \{4, -11, 20, 30, 20, -11, 4\}$

3.6 - Matrix Method: convolution

P-3.10: $x(n) = \{1, 2, 3, 4\}$, $h(n) = \{1, 1, 1, 1\}$

Solve:

$$X = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 3 & 2 & 1 & 0 \\ 4 & 3 & 2 & 1 \\ 0 & 4 & 3 & 2 \\ 0 & 0 & 4 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix} \qquad H = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$Y = XH = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 3 & 2 & 1 & 0 \\ 4 & 3 & 2 & 1 \\ 0 & 4 & 3 & 2 \\ 0 & 0 & 4 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2+1 \\ 3+2+1 \\ 4+3+2+1 \\ 4+3+2 \\ 4+3 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 6 \\ 10 \\ 9 \\ 7 \\ 4 \end{bmatrix}$$

3.11 $x(n)=\{1,-2,3,-4\}$, $h(n)=\{4,-3,2,-1\}$

Solve

$$X=\begin{bmatrix}1&0&0&0\\-2&1&0&0\\3&-2&1&0\\-4&3&-2&1\\0&-4&3&-2\\0&0&-4&3\\0&0&0&-4\end{bmatrix}\quad H=\begin{bmatrix}4\\-3\\2\\-1\end{bmatrix}$$

$$XH=\begin{bmatrix}1&0&0&0\\-2&1&0&0\\3&-2&1&0\\-4&3&-2&1\\0&-4&3&-2\\0&0&-4&3\\0&0&0&-4\end{bmatrix}\times\begin{bmatrix}4\\-3\\2\\-1\end{bmatrix}=\begin{bmatrix}4\\-8-3\\12+6+2\\-16-9-4-1\\12+6+2\\-8-3\\4\end{bmatrix}=\begin{bmatrix}4\\-11\\20\\-30\\20\\-11\\4\end{bmatrix}$$

3.8 - Deconvolution.

Deconvolution is 'undo' procedure of convolution.

P-3.21:

Class -(12.07.23)

Fourier analysis is a method to analysis every type of signal.

1. Fourier Series for continuous time periodic

1.1 → In trigonometric form :

calculation to find out the value of a_0,

we know, $f(\theta) = a_0 + \sum_{n=1}^{\infty}(a_n \cos n\theta + b_n \sin n\theta)$

$$\int_0^{2\pi} f(\theta)\, d\theta = \int_0^{2\pi} a_0\, d\theta + \sum_{n=1}^{\infty}\left(\int_0^{2\pi} a_n \cos n\theta\, d\theta + \int_0^{2\pi} b_n \sin n\theta\, d\theta\right)$$

$$\text{or, } \int_0^{2\pi} f(\theta)\, d\theta = \int_0^{2\pi} a_0\, d\theta + a_n \sum_{n=1}^{\infty}\int_0^{2\pi} \cos n\theta\, d\theta + \sum_{n=1}^{\infty} b_n \int_0^{2\pi} \sin n\theta\, d\theta$$

$$= \int_0^{2\pi} a_0\, d\theta + 0 + 0 \quad \left[\because \cos 2\pi = 1,\ \cos\pi = -1 \,\middle|\, \sin 2\pi = 0\right]$$

$$= a_0 \int_0^{2\pi} d\theta$$

$$\text{or, } \int_0^{2\pi} f(\theta)\, d\theta = a_0 [\theta]_0^{2\pi} = a_0(2\pi - 0) = 2\pi a_0$$

$$\text{or, } 2\pi a_0 = \int_0^{2\pi} f(\theta)\, d\theta$$

$$\text{or, } a_0 = \frac{1}{2\pi}\int_0^{2\pi} f(\theta)\, d\theta$$

Calculation to find out the value of a_n:

We know that,

$$f(\theta) = a_0 + \sum_{n=1}^{\infty}(a_n \cos n\theta + b_n \sin n\theta)$$

$$\Rightarrow f(\theta) = a_0 + a_1 \cos\theta + a_2 \cos 2\theta + \dots + a_n \cos n\theta + b_1 \sin\theta + b_2 \sin 2\theta + \dots + b_n \sin n\theta$$

$$\Rightarrow \int_0^{2\pi} f(\theta) \cos n\theta\, d\theta = \int_0^{2\pi} a_0 \cos n\theta\, d\theta + \int_0^{2\pi} a_1 \cos\theta \cos n\theta + \int_0^{2\pi} a_2 \cos\theta$$

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$\cos 2\theta \, d\theta + \dots + \int_0^{2\pi} a_n \cos n\theta \cos n\theta \, d\theta + \int_0^{2\pi} b_1 \sin\theta \cos n\theta \, d\theta +$

$\int_0^{2\pi} b_2 \sin 2\theta \cos n\theta \, d\theta + \dots + \int_0^{2\pi} b_n \sin n\theta \cos n\theta \, d\theta$

$$\Rightarrow \int_0^{2\pi} f(\theta) \cos n\theta \, d\theta = a_0 \overset{0}{\cancel{\int_0^{2\pi} \cos n\theta \, d\theta}} + 0 + 0 + \dots + a_n \pi + 0 + 0 + \dots + 0$$

$$\Rightarrow \int_0^{2\pi} f(\theta) \cos n\theta \, d\theta = a_n \pi$$

$$\Rightarrow a_n \pi = \int_0^{2\pi} f(\theta) \cos n\theta \, d\theta$$

$$\Rightarrow a_n = \frac{1}{\pi} \int_0^{2\pi} f(\theta) \cos n\theta \, d\theta$$

cos-এর ক্ষেত্রে
যদি same হয় → π $\left(\int_0^{2\pi} \cos n\theta \times \cos n\theta\right)$
" " না " → 0

sin-এর ক্ষেত্রে,
যদি same হয় → π
" " না

$\sin \times \cos = 0$ always

Calculation to find out the value of b_n:

$\int_0^{2\pi} \cos n\theta \cos n\theta \, d\theta = \pi$

$$f(\theta) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\theta + b_n \sin n\theta)$$

$$\Rightarrow \int_0^{2\pi} \sin n\theta \, f(\theta) \, d\theta = \int_0^{2\pi} a_0 \sin n\theta \, d\theta + \sum_{n=1}^{\infty} \int_0^{2\pi} a_n \cos n\theta \sin n\theta \, d\theta + \sum_{n=1}^{\infty} \int_0^{2\pi} b_n \sin n\theta \sin n\theta \, d\theta$$

$$= 0 + 0 + \int_0^{2\pi} b_1 \sin\theta \sin\theta \, d\theta + \dots + \int_0^{2\pi} b_n \sin n\theta \sin n\theta \, d\theta$$

$$= 0 + \dots + b_n \pi$$

$$\Rightarrow \int_0^{2\pi} f(\theta) \sin n\theta \, d\theta = b_n \pi$$

$$\Rightarrow b_n = \frac{1}{\pi}\int_0^{2\pi} f(\theta)\sin n\theta \, d\theta$$

<u>Note</u>:

$$f(\theta) = a_0 + \sum_{n=1}^{\infty}\left(a_n\cos n\theta + b_n \sin n\theta\right)$$

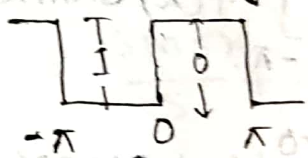
$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} f(\theta)\, d\theta$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(\theta)\cos n\theta \, d\theta$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(\theta)\sin n\theta \, d\theta$$

Example-1: Find the mathematical equation for given signal using Fourcier series as trigonometric form.

$$f(x) = \begin{cases} 0 & -\pi \le x \le 0 \\ 1 & 0 \le x \le \pi \end{cases}$$



<u>Solution</u>: We know that,

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$= \frac{1}{2\pi}\int_{-\pi}^{0} f(x)\,dx + \frac{1}{2\pi}\int_0^{\pi} f(x)\,dx$$

$$= 0 + \frac{1}{2\pi}\int_0^{\pi} dx$$

$$= \frac{1}{2\pi}\left[x\right]_0^{\pi}$$

$$= \frac{1}{2\pi}(\pi - 0)$$

$$= \frac{1}{2\pi} \times \pi = 1/2$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx \, dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} f(x)\cos nx\, dx + \frac{1}{\pi}\int_{0}^{\pi} f(x)\,\cancel{\sin nx}\ \cos nx\, dx$$

$$\cancel{= 0 + \frac{1}{\pi}\left[\int_0^{\pi} \sin nx\, dx\right]} = 0 + 0$$

$$\cancel{= \frac{1}{\pi}\left[-\frac{\cos nx}{n}\right]_0^{\pi}} = 0$$

$$\cancel{= \frac{1}{\pi}\left(-\frac{\cos n\pi}{n} - \frac{\cos n0}{n}\right)} \quad \therefore a_n = 0$$

$$\cancel{= \frac{1}{\pi}}$$

$\cos n\pi = 1$ [$n = 2, 4, 6 ..$]
$\cos n\pi = -1$ [$n = 1, 3, 5 ...$]

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\, dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} f(x)\sin nx\, dx + \frac{1}{\pi}\int_{0}^{\pi} f(x)\sin nx\, dx$$

$$= 0 + \frac{1}{\pi}\int_0^{\pi} \sin nx\, dx$$

$$= \frac{1}{\pi}\left[-\frac{\cos nx}{n}\right]_0^{\pi}$$

$$= \frac{1}{\pi}\left[-\frac{\cos n\pi}{n} + \frac{\cos n0}{n}\right]$$

$$= \frac{1}{\pi}\left(-\frac{\cos n\pi}{n} + \frac{1}{n}\right)$$

$$= \frac{1}{n\pi}(1 - \cos n\pi)$$

$$= \begin{cases} 0 & \text{for even } n \\ \frac{1}{n\pi}(1+1) & \text{for odd } n \end{cases}$$

$$= \begin{cases} 0 & \text{for even } n \\ \frac{2}{n\pi} & \text{for odd } n \end{cases}$$

So, The equation.

$$f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx) \quad \text{--- (i)}$$

putting the values of a_0, a_n and b_n into (i)

$$f(x) = \frac{1}{2} + \sum_{n=1}^{\infty} 0 \cos nx + \sum_{\substack{n=1 \\ (\text{odd})}}^{\infty} \frac{2}{n\pi} \sin nx$$

$$= \frac{1}{2} + \frac{2}{\pi} \sin x + \frac{2}{3\pi} \sin 3x + \frac{2}{5\pi} \sin 5x + \cdots + \infty$$

$$= \frac{1}{2} + \frac{2}{\pi}\left(\frac{\sin x}{1} + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \cdots + \infty \right)$$

1.2- Conversion of Trigonometric form fourier series into complex form Fourier series for continuous Time Periodic Signal

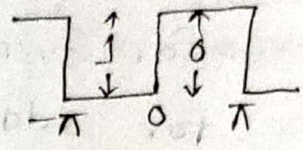
Complex form not complicated form but also simple form compare to others form such as trigonometric

$$f(\theta) = \sum_{n=\infty}^{\infty} c_n e^{jn\theta} \quad \text{and} \quad c_n = \frac{1}{2\pi} \int_0^{2\pi} f(\theta)\, e^{-jn\theta}\, d\theta$$

Or $$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn2\pi F_0 t} \quad \text{and} \quad c_n = \frac{1}{T} \int_0^{T} f(t)\, e^{-jn2\pi F_0 t}\, dt$$

Ex-2: Find the mathematical eqn for given signal using Fourier series on complex form.

$$f(x) = \begin{cases} 0 & -\pi \le x \le 0 \\ 1 & 0 \le x \le \pi \end{cases}$$



Solution: We know that,

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{inx} \quad \text{(i)} \qquad \text{where, } c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\, e^{-inx}\, dx$$

$$c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\, e^{-inx}\, dx$$

$$= \frac{1}{2\pi}\int_{-\pi}^{0} f(x)\, e^{-inx}\, dx + \frac{1}{2\pi}\int_{0}^{\pi} f(x)\, e^{-inx}\, dx$$

$$= \frac{1}{2\pi}\int_{-\pi}^{0} 0\, e^{inx}\, dx + \frac{1}{2\pi}\int_{0}^{\pi} 1.\, e^{-inx}\, dx$$

$$= 0 + \frac{1}{2\pi}\int_{0}^{\pi} e^{-inx}\, dx$$

$$= \frac{1}{2\pi}\left[\frac{e^{-inx}}{-in}\right]_0^{\pi}$$

$$= -\frac{1}{2in\pi}\left(e^{-in\pi} - e^{-in0}\right)$$

$$= -\frac{1}{2in\pi}\left(e^{-in\pi} - e^{0}\right)$$

$$c_n = -\frac{1}{2in\pi}\left(e^{-in\pi} - 1\right) \quad \text{(ii)}$$

When n is odd then,

$$c_1 = -\frac{1}{2i.1.\pi}\left(e^{-i.1.\pi} - 1\right)$$

$$= -\frac{1}{2i\pi}\left(e^{-i\pi}-1\right)$$

$$= -\frac{1}{2i\pi}\left(\cos\pi - i\sin\pi - 1\right)$$

$$= -\frac{1}{2i\pi}\left(-1-0-1\right)$$

$$= -\frac{1}{2i\pi}\left(-1-1\right)$$

$$= -\frac{1}{2i\pi} \times -2$$

$$c_1 = \frac{1}{i\pi}$$

$$c_3 = \frac{1}{i3\pi}$$

$$\therefore c_n = \frac{1}{in\pi}$$

If n = even, but not 0.

$$c_2 = -\frac{1}{2i.2.\pi}\left(e^{-i.2.\pi}-1\right)$$

$$= -\frac{1}{4i\pi}\left(e^{-i2\pi}-1\right)$$

$$= -\frac{1}{4i\pi}\left(\cos 2\pi - i\sin 2\pi - 1\right)$$

$$= -\frac{1}{4i\pi}\left(\cancel{1}-0-\cancel{1}\right)$$

$$= -\frac{1}{4i\pi}(0)$$

If $n=0$ then

$$c_0 = \frac{1}{2\pi}\int_0^{\pi} 1\, dx = \frac{1}{2\pi}[x]_0^{\pi} = \frac{1}{2\pi}\times\pi = \frac{1}{2}$$

$$= 0$$

$$c_4 = 0$$

$$\therefore c_n = 0$$

Putting the value of c_n in eqn (i)

$$f(x) = c_0 + \sum_{n=(odd)(-,+)}^{\infty} c_n e^{inx} + \sum_{n=even}^{\infty} c_n e^{inx}$$

$$= \frac{1}{2} + c_1 e^{inx} + c_3 e^{i3x} + c_5 e^{i5x} + \dots + c_2 e^{i2x}$$

$$+ c_4 e^{i4x} + \dots + c_{-1} e^{-i.1.x} + c_{-3} e^{-i3x} + c_{-5} e^{-i5x} + \dots$$

$$= \frac{1}{2} + \frac{1}{i\pi} e^{ix} + \frac{1}{i3\pi} e^{i3x} + \frac{1}{i5\pi} e^{i5x} + \dots + 0 + 0 + 0 + \dots 0$$

$\frac{1}{in\pi}$

$$+ \frac{1}{-i\pi} e^{-ix} + \frac{1}{-i3\pi} e^{-i3x} + \frac{1}{-i5\pi} e^{-i5x} + \dots$$

$$= \frac{1}{2} + \frac{1}{i\pi}\left(\frac{e^{ix}}{1} + \frac{e^{i3x}}{3} + \frac{e^{i5x}}{5} + \dots\right) + \frac{1}{i\pi}\left(\frac{e^{-ix}}{-1} + \frac{e^{-i3x}}{-3} + \frac{e^{-i5x}}{-5} + \dots\right)$$

$$= \frac{1}{2} + \frac{2}{\pi}\left(\frac{1}{1}\frac{e^{ix} - e^{-ix}}{2i} + \frac{1}{3}\cdot\frac{e^{i3x} - e^{-i3x}}{2i} + \frac{1}{5}\cdot\frac{e^{i5x} - e^{-i5x}}{2i} + \dots\right)$$

$$= \frac{1}{2} + \frac{2}{\pi}\left(\frac{1}{1}\cdot\frac{2i\sin x}{2i} + \frac{1}{3}\cdot\frac{2i\sin 3x}{2i} + \frac{1}{5}\cdot\frac{2i\sin 5x}{2i} + \dots\right)$$

$$= \frac{1}{2} + \frac{2}{\pi}\left(\frac{\sin x}{1} + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \dots\right).$$ Or

Sasikala

Thus, a linear combination of harmonically related complex exponential is of the form

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

$$x(0) = a_0$$

For $k=0$, which is a constant.

For $k = \pm 1$,

$$x(t) = a_{-1} e^{-j\omega t} + a_0 + a_1 e^{j\omega t}$$

P-4.1 Determine the periodic signal whose fundamental frequency is 2π and Fourier coefficients are $a_0 = 1$; $a_2 = a_{-2} = \frac{1}{2}$; $a_4 = a_{-4} = \frac{1}{4}$, $a_6 = a_{-6} = \frac{1}{6}$

Solution: A periodic signal can be represented exponentially as

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

For $\omega_0 = 2\pi$,

$$x(t) = \sum_{k=-6}^{6} a_k e^{jk2\pi t}$$

$$x(t) = a_{-6} e^{j(-6)2\pi t} + a_{-4} e^{j(-4)2\pi t} + a_{-2} e^{j(-2)2\pi t} + a_0 + a_2 e^{j2.2\pi t} + a_4 e^{j.4.2\pi t} + a_6 e^{j.6.2\pi t}$$

$$= \frac{1}{6} e^{-j12\pi t} + \frac{1}{4} e^{-j8\pi t} + \frac{1}{2} e^{-j4\pi t} + 1 + \frac{1}{2} e^{j4\pi t} + \frac{1}{4} e^{j8\pi t} + \frac{1}{6} e^{j12\pi t}$$

$$= 1 + \frac{1}{6} e^{j12\pi t} + \frac{1}{6} e^{-j12\pi t} + \frac{1}{4} e^{j8\pi t} + \frac{1}{4} e^{-j8\pi t} + \frac{1}{2} e^{j4\pi t} + \frac{1}{2} e^{-j4\pi t}$$

$$= 1 + \frac{1}{3}\left(\frac{e^{j12\pi t} + e^{-j12\pi t}}{2}\right) + \frac{1}{2}\left(\frac{e^{j8\pi t} + e^{-j8\pi t}}{2}\right)$$

$$+ \left(\frac{e^{j4\pi t} + e^{-j4\pi t}}{2}\right)$$

$$= 1 + \frac{1}{3}\left(\frac{\cancel{2}\cos 12\pi t}{\cancel{2}}\right) + \frac{1}{2}\left(\frac{\cancel{2}\cos 8\pi t}{\cancel{2}}\right) + \frac{\cancel{2}\cos 4\pi t}{\cancel{2}}$$

$$= 1 + \frac{1}{3}\cos 12\pi t + \frac{1}{2}\cos 8\pi t + \cos 4\pi t \quad \text{Ans}$$

Summery – continuous-time Fourier series pair.

Inverse CTFS $\quad x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$

CTFS $\quad a_k = \frac{1}{T}\int_{t=0}^{T} x(t)\, e^{-jk\omega_0 t}\, dt$

P-4.2: Find the Fourier coefficients of the given signal,

$$x(t) = 1 + \sin 2\omega_0 t + 2\cos 2\omega_0 t + \cos\left(3\omega_0 t + \frac{\pi}{3}\right)$$

Solve: Given,

$$x(t) = 1 + \sin 2\omega_0 t + 2\cos 2\omega_0 t + \cos\left(3\omega_0 t + \frac{\pi}{3}\right)$$

Rewriting the given eqation

$$x(t) = 1 + \frac{e^{j2\omega_0 t} - e^{-j2\omega_0 t}}{2j} + \left(e^{j2\omega_0 t} + e^{-j2\omega_0 t}\right)$$

$$+ \left\{\frac{e^{j(3\omega_0 t + \frac{\pi}{3})} + e^{-j(3\omega_0 t + \frac{\pi}{3})}}{2}\right\}$$

$$x(t) = 1 + \frac{1}{2j} e^{j2\omega_0 t} - \frac{1}{2j} e^{-j2\omega_0 t} + e^{j2\omega_0 t} + e^{-j2\omega_0 t} + \frac{1}{2} e^{j3\omega_0 t} \cdot e^{j\pi/3} + \frac{1}{2} e^{-j3\omega_0 t} \cdot e^{-j\pi/3}$$

$$= 1 + \left(\frac{1}{2j} + 1\right) e^{j2\omega_0 t} + \left(1 - \frac{1}{2j}\right) e^{-j2\omega_0 t} + \frac{e^{j\pi/3}}{2} e^{j3\omega_0 t} + \frac{e^{-j\pi/3}}{2} e^{-j3\omega_0 t}$$

$$a_0 = 1$$

$$a_2 = \left(1 + \frac{1}{2j}\right)$$

$$a_{-2} = \left(1 - \frac{1}{2j}\right)$$

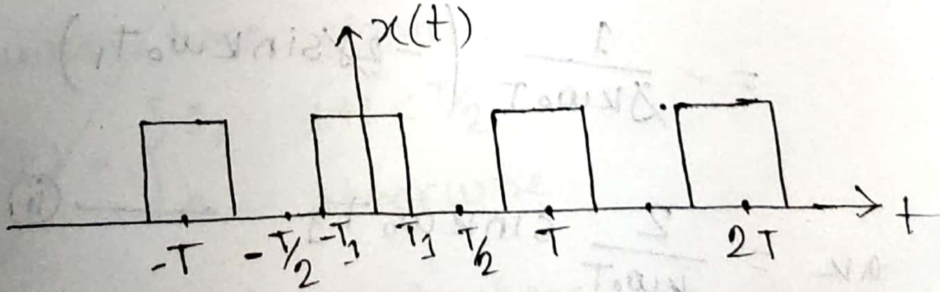
$$a_3 = \frac{e^{j\pi/3}}{2}$$

$$a_{-3} = \frac{e^{-j\pi/3}}{2}$$

P-4.3: Determine the Fourier series coefficient of exponential representation of,

$$x(t) = \begin{cases} 1, & |t| < T_1 \\ 0 & T_1 < |t| < \frac{T}{2} \end{cases}$$

Solution:



The fourier series coefficient is given by

$$a_k = \frac{1}{T} \int_T x(t) e^{-jk\omega_0 t} dt$$

Since the pulse exists between $-T_1$ to T_1 equation (1) becomes

$$a_k = \frac{1}{T}\int_{-T_1}^{T_1} 1 . e^{-jk\omega_0 t}\,dt$$

For $k=0$,

$$a_0 = \frac{1}{T}\int_{-T_1}^{T_1} 1\,dt$$

$$a_0 = \frac{1}{T}(t)_{-T_1}^{T_1} = \frac{2T_1}{T}, \; k=0 \quad \text{(i)}$$

where a_0 is the average or DC value of the signal $x(t)$, $k \neq 0$

$$a_k = \frac{1}{T}\int_{-T_1}^{T_1} 1 . e^{-jk\omega_0 t}\,dt$$

$$a_k = \frac{1}{T}\left[\frac{-1}{jk\omega_0} . e^{-jk\omega_0 t}\right]_{-T_1}^{T_1}$$

$$= -\frac{1}{jk\omega_0 T}\left(e^{-jk\omega_0 T_1} - e^{+jk\omega_0 T_1}\right)$$

$$= -\frac{1}{jk\omega_0 T}\left\{\cancel{\cos k\omega_0 T_1} - j\sin k\omega_0 T_1 - \cancel{\cos k\omega_0 T_1} - j\sin k\omega_0 T_1\right\}$$

$$= -\frac{1}{\cancel{j}k\omega_0 T}\left(-2\cancel{j}\sin k\omega_0 T_1\right)$$

$$a_k = \frac{2}{k\omega_0 T}\sin k\omega_0 T_1 \quad \text{(ii)}$$

The frequency of the pulse train is $\omega_0 = \frac{2\pi}{T}$

$$\omega_0 T = 2\pi$$

Eqn (ii) becomes

$$a_k = \frac{\sin k\omega_0 T_1}{k\pi}$$

For $T = 4T_1$,

$$a_k = \frac{\sin(k\frac{\pi}{2})}{k\pi} \quad , k \neq 0$$

Eqn (i) becomes

$$a_0 = \frac{2T_1}{4T_1} = \frac{1}{2} \quad ; k = 0$$

Fourier Transform for continuous time Aperiodic signal

3.1- Fourier Transform for Continuous Time A-periodic Signal

Let's consider an a-periodic signal $x(t)$ with finite dura

skip - This is not for the CT-2 No skip idiot

এখনো আলোচনা page থেকে শুরু আছে From এখানে

$\boxed{\text{Fourier Transform}}$: Math Example (Fourier series sanikal)

ওখানে পরে দেখে নিও বেশি

Problem - 5.1: Determine the Fourier transform of the continuous time signal

$$x(t) = \begin{cases} 1, & |t| < T/2 \\ 0, & \text{otherwise} \end{cases}$$

Plot the magnitude and phase spectrum.

Solution: ~~By definition of FT,~~

We can create a periodic signal $x_p(t)$ with period T_p (F3.2) that is

Clearly $x_p(t) = x(t)$ in the limit as $T_p \to \infty$

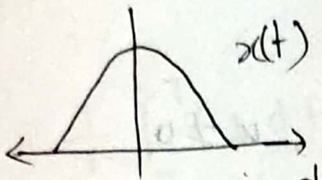
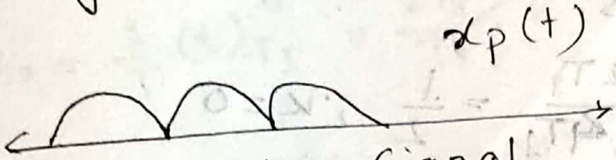


Fig: Aperiodic signal



periodic Signal

$$x(t) = \lim_{T_p \to \infty} x_p(t)$$

$$x_p(t) = \sum_{K=-\infty}^{\infty} c_K e^{j2\pi K F_0 t}$$

$$c_K = \frac{1}{T_p} \int_{-\infty}^{\infty} x(t) e^{-j2\pi K F_0 t} dt$$

$$\therefore X(F) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi F t} dF$$ [X(F) is the Fourier Transform of $x(t)$]

$$x(t) = \int_{-\infty}^{\infty} X(F) e^{j2\pi F t} dF$$

3.2 - Energy Density and Power Density Spectrum for continuous Time Aperiodic signal.

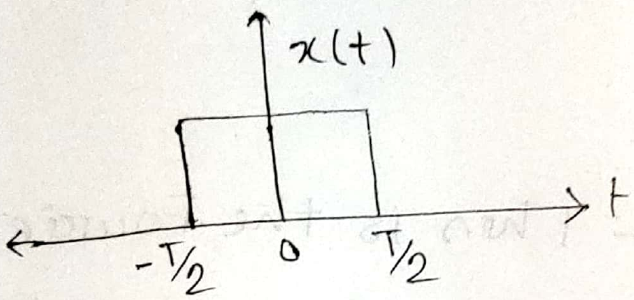
A continuous time aperiodic signal has infinite energy which is given as.

$$E_x = \int_{-\infty}^{\infty} [x(t)]^2 dt = \int_{-\infty}^{\infty} [X(F)]^2 dF$$

The $[X(F)]^2$ is represented the energy density spectrum of CTAS. In this case, we can't calculate power density spectrum of aperiodic signal due to unknown period.

Example-5: Determine the Fourier transform and energy density spectrum of rectangular pulse define as

$$x(t) = \begin{cases} 0 & \frac{T}{2} < |t| \\ 1 & \frac{T}{2} \geq |t| \end{cases}$$



$-t < \frac{T}{2} < t$

Solution: We know that,

$$X(F) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi Ft}\, dt$$

$$X(F) = \int_{-\infty}^{-T/2} x(t)\, e^{-j2\pi Ft}\, dt + \int_{-T/2}^{T/2} x(t)\, e^{-j2\pi Ft}\, dt$$

$$+ \int_{T/2}^{\infty} x(t)\, e^{-j2\pi Ft}\, dt$$

$$X(F) = \int_{-\infty}^{-T/2} 0\, e^{-j2\pi Ft}\, dt + \int_{-T/2}^{T/2} 1\, e^{-j2\pi Ft}\, dt + \int_{T/2}^{\infty} 0\, e^{-j2\pi Ft}\, dt$$

$$X(F) = \int_{-T/2}^{T/2} e^{-j2\pi Ft}\, dt = \int_{-T/2}^{T/2} (\cos 2\pi Ft - j\sin 2\pi Ft)\, dt$$

$$= \int_{-T/2}^{T/2} \cos 2\pi Ft\, dt \quad [\sin 2\pi = 0]$$

$$X(F) = \frac{\sin(\pi Ft)}{}$$

$$= \left[\frac{\sin 2\pi F t}{2\pi F}\right]_{-T/2}^{T/2}$$

$$= \frac{1}{2\pi F}\left\{\sin \cancel{2}\pi F\left(\frac{T}{\cancel{2}}\right) - \sin \cancel{2}\pi F\left(-\frac{T}{\cancel{2}}\right)\right\}$$

$$= \frac{1}{2\pi F}\left(\sin\pi F T + \sin \pi F T\right)$$

$$= \frac{1}{\cancel{2}\pi F} \times \cancel{2}\sin\pi F T$$

$$= \frac{\sin \pi F T}{\pi F}$$

$$= T \cdot \frac{\sin \pi F T}{\pi F T}$$

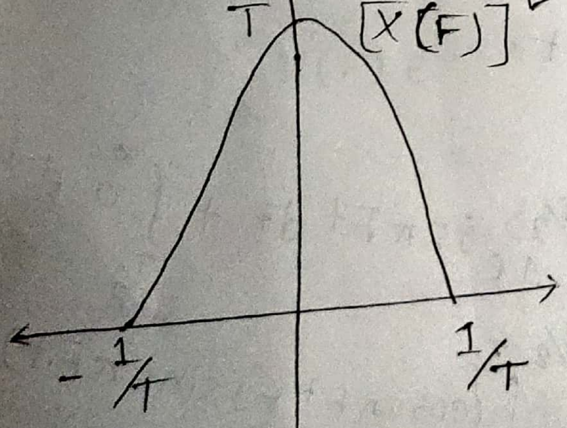
$\therefore X(F) = T\,\mathrm{sinc}(\pi F T)$ — This is the Fourier Transform.

So, energy density spectrum for rectangular pulse

is $X(F)^2 = \left[T\,\frac{\sin(\pi F T)}{\pi F T}\right]^2$.

$$= \frac{T^2 \sin^2(\pi F T)}{\pi^2 F^2 T^2}$$

The graphical representation of energy density spectrum for given



1-8-23 DSP

এই ক্লাস টা কোন pdf থেকে নেওয়া তা না বুঝতে পারা আমার অবুঝ মন।

(i) How FT form?

(ii) Trapezoidal integration

(iii) composite signal plot.

4.08.23 DSP ([illegible])

Propertien of the Fourier Transform

✓ **Linearity:**

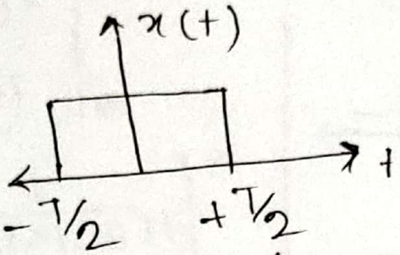
Sasikala - Fourier Transform

P-5.1: Determine the Fourier transform of the continuous-time signal

$$x(t) = \begin{cases} 1, & |t| < T/2 \\ 0, & \text{otherwise} \end{cases}$$

plot the magnitude and phase spectrum.

Solution: By definition of FT,



$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt = \int_{-T/2}^{T/2} 1 \cdot e^{-j\omega t}\, dt$$

$$X(j\omega) = \left[\frac{e^{-j\omega t}}{-j\omega} \right]_{-T/2}^{T/2}$$

$$= -\frac{1}{j\omega}\left(e^{-j\omega \frac{T}{2}} - e^{+j\omega \frac{T}{2}} \right)$$

$$= -\frac{1}{j\omega}\left(e^{-j\cdot\frac{2\pi}{T}\cdot\frac{T}{2}} - e^{j\cdot\frac{2\pi}{T}\cdot\frac{T}{2}} \right)$$

$$= -\frac{1}{j\omega}\left(e^{-j\pi} - e^{j\pi} \right)$$

$$= -\frac{1}{j\omega}\left(\cos\pi - j\sin\pi - \cos\pi - j\sin\pi \right)$$

$$= -\frac{1}{j\omega} \times -2j\sin\pi$$

$$= \frac{2\sin\pi}{\omega} =$$

$|x| < -3$

$+3 \neq x \neq 3$

$|t| < T/2$

$-T/2 < x < T/2$

Intermediate

$|x| < 3$

$-3 < x < 3$

$$= \frac{2}{\omega}\left(\frac{e^{j\omega T/2} - e^{-j\omega T/2}}{2j}\right)$$

$$X(j\omega) = \frac{2}{\omega} \sin\left(\frac{\omega T}{2}\right)$$

$$= \frac{T \sin\left(\frac{\omega T}{2}\right)}{\frac{\omega T}{2}}$$

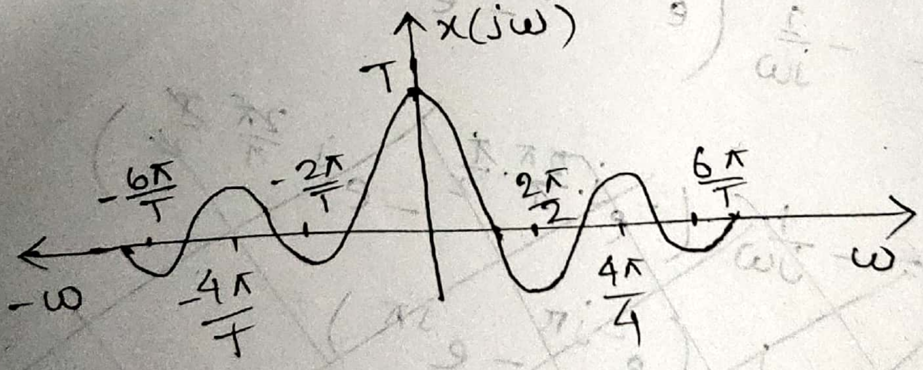
$$= T \operatorname{sinc}\left(\frac{\omega T}{2}\right)$$

The magnitude spectrum,

$$|X(j\omega)| = T \left|\operatorname{sinc}\left(\frac{\omega T}{2}\right)\right|$$

The phase spectrum.

$$\phi(j\omega) = \begin{cases} 0, & \operatorname{sinc}\left(\frac{\omega T}{2}\right) > 0 \\ \pi, & \operatorname{sinc}\left(\frac{\omega T}{2}\right) < 0 \end{cases}$$



<u>P-5.2</u>: Determine the Fourier transform of the signal $x(t) = e^{-\omega t} u(t)$, $a > 0$. Plot the magnitude and phase spectrum,

<u>Solution</u>: By definition of F.T,

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$X(j\omega) = \int_{-\infty}^{\infty} e^{-\omega t}\, u(t)\, e^{-j\omega t}\, dt$$

$$= \int_{0}^{\infty} e^{-at}\, e^{-j\omega t}\, dt$$

$$= \int_{0}^{\infty} e^{-(a+j\omega)t}\, dt$$

$$X(j\omega) = \left. \frac{e^{-(a+j\omega)t}}{-(a+j\omega)} \right|_{0}^{\infty}$$

$$= -\frac{1}{(a+j\omega)} \cdot \left. \frac{1}{e^{(a+j\omega)t}} \right|_{0}^{\infty}$$

$$= -\frac{1}{(a+j\omega)} \left(\frac{1}{e^{\infty}} - \frac{1}{e^{0}} \right)$$

$$= -\frac{1}{(a+j\omega)} \left(0 - \frac{1}{1} \right)$$

$$= -\frac{1}{a+j\omega} (-1)$$

$$X(j\omega) = \frac{1}{a+j\omega}$$

The magnitude spectrum

$$|X(j\omega)| = \left| \frac{1}{a+j\omega} \right|$$

$$= \frac{1}{\sqrt{a^2+\omega^2}}$$

phase spectrum

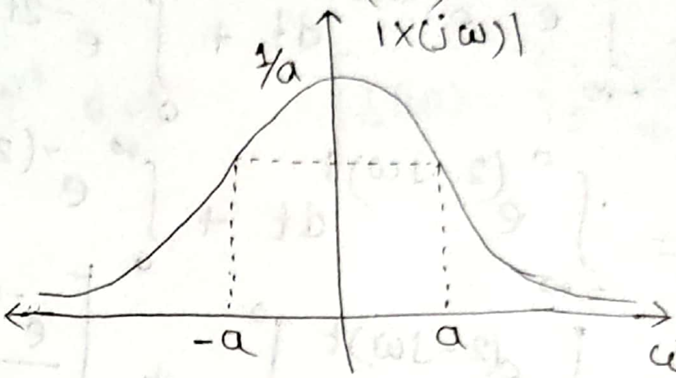
$$\angle X(j\omega) = \frac{1}{a+j\omega} \times \frac{a-j\omega}{a-j\omega}$$

$$= \frac{a-j\omega}{a^2+\omega^2}$$

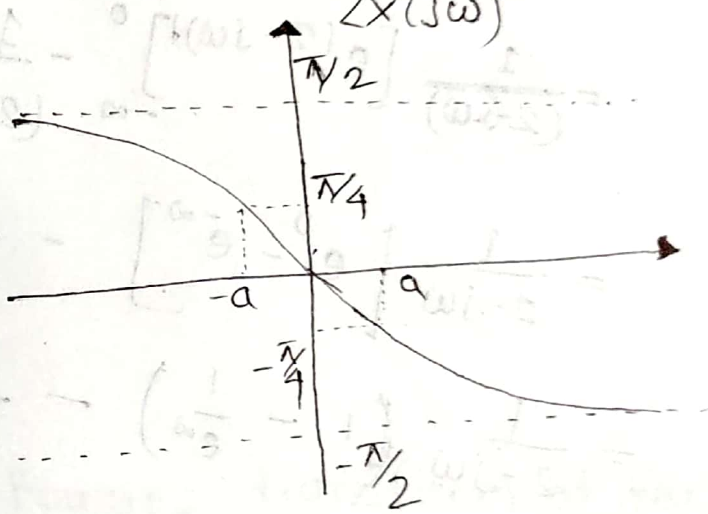
$$= \frac{a}{a^2+\omega^2} - j\frac{\omega}{a^2+\omega^2}$$

$$\therefore \angle X(j\omega) = \tan^{-1}\left(\frac{\frac{-\omega}{a^2+\omega^2}}{\frac{a}{a^2+\omega^2}} \right)$$

$$= -\tan^{-1}\left(\frac{\omega}{a}\right)$$



(a) Magnitude Spectrum



(b) phase spectrum

Problem - 5.3: Determine the Fourier transform of the signal $x(t) = e^{-2|t|}$

Solution: By definition of FT,

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} e^{-2|t|} e^{-j\omega t} dt$$

$$= \int_{-\infty}^{0} e^{-2(-t)} e^{-j\omega t} dt + \int_{0}^{\infty} e^{-2t} e^{-j\omega t} dt$$

$$= \int_{-\infty}^{0} e^{2t} e^{-j\omega t} dt + \int_{0}^{\infty} e^{-2t} e^{-j\omega t} dt$$

$$= \int_{-\infty}^{0} e^{(2-j\omega)t} dt + \int_{0}^{\infty} e^{-(2+j\omega)t} dt$$

$$= \left[\frac{e^{(2-j\omega)t}}{2-j\omega}\right]_{-\infty}^{0} + \left[\frac{e^{-(2+j\omega)t}}{-(2+j\omega)}\right]_{0}^{\infty}$$

$$= \frac{1}{(2-j\omega)}\left[e^{(2-j\omega)t}\right]_{-\infty}^{0} - \frac{1}{(2+j\omega)}\left[e^{-(2+j\omega)t}\right]_{0}^{\infty}$$

$$= \frac{1}{2-j\omega}\left[e^{0} - e^{-\infty}\right] - \frac{1}{2+j\omega}\left[e^{-\infty} - e^{0}\right]$$

$$= \frac{1}{2-j\omega}\left(1 - \frac{1}{e^{\infty}}\right) - \frac{1}{2+j\omega}\left(\frac{1}{e^{\infty}} - (1)\right)$$

$$= \frac{1}{2-j\omega} - \frac{1 \cdot -1}{2+j\omega}$$

$$= \frac{1}{2-j\omega} + \frac{1}{2+j\omega} = \frac{2+\cancel{j\omega} + 2-\cancel{j\omega}}{2^2 - (j\omega)^2}$$

$$= \frac{4}{4-j^2\omega^2} = \frac{4}{4+\omega^2} \text{ Ans}$$

Problem-5.4: Determine the inverse Fourier Transform of $X(j\omega) = \delta(\omega)$

Solution: By definition of inverse Fourier Transform

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \delta(\omega)\, e^{j\omega t}\, d\omega \qquad \left[\delta(\omega) = \begin{cases} 1, & \omega = \\ 0, & \omega = \end{cases}\right.$$

$$= \frac{1}{2\pi}(1)$$

$$\therefore x(t) = \frac{1}{2\pi}$$

$$\Rightarrow IFT[\delta(\omega)] = \frac{1}{2\pi}$$

$$\Rightarrow IFT[2\pi\delta(\omega)] = 1$$

or, $FT(1) = 2\pi\delta(\omega)$

Problem-5.5: Determine the Fourier transform of the unit impulse $x(t) = \delta(t)$.

Solution: By definition of FT,

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$X(j\omega) = \int_{-\infty}^{\infty} \delta(t)\, e^{-j\omega t}\, dt$$

[unit impulse ∵]

$$= 1$$

Problem 5.6 - Determine the FT of the signal

$x(t) = \sin \omega_0 t$

Solution: By def of FT, we know

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$= \int_{-\infty}^{\infty} \sin\omega_0 t \, e^{-j\omega t} dt$$

$$= \int_{-\infty}^{\infty} \left[\frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j}\right] e^{-j\omega t} dt$$

$$\frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j} = \sin\omega_0 t$$

$\cos\omega_0 t + j\sin\omega_0 t - (\cos\omega_0 t - j\sin\omega_0 t)$

$= 2j\sin\omega_0 t$

$\therefore \sin\omega_0 t = \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j}$

$$FT\left\{\frac{e^{-j\omega_0 t}}{2\pi}\right\} = \delta(\omega_0)$$

$$= \frac{1}{2j}\int_{-\infty}^{\infty} e^{j\omega_0 t} e^{-j\omega t} dt - \frac{1}{2j}\int_{-\infty}^{\infty} e^{-j\omega_0 t} \cdot e^{-j\omega t} dt$$

$$= \frac{1}{2j}\int_{-\infty}^{\infty} e^{-j(\omega_0 - \omega)t} dt - \frac{1}{2j}\int_{-\infty}^{\infty} e^{-j(\omega_0 + \omega)t} dt$$

$$\Rightarrow \cancel{\frac{1}{2j}\left[\frac{e^{-j(\omega_0 - \omega_0)t}}{-j(\omega_0 - \omega_0)}\right]_{-\infty}^{\infty} - \frac{1}{2j}\left[\frac{e^{-j(\omega_0 + \omega)t}}{-j(\omega_0 + \omega)}\right]_{-\infty}^{\infty}}$$

$$= \frac{1}{2j}\left[2\pi\delta(\omega - \omega_0)\right] - \frac{1}{2j}\left[2\pi\delta(\omega + \omega_0)\right]$$

$$= \frac{2\pi}{2j}\left[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)\right]$$

$$= \frac{\pi}{j}\left[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)\right] \quad \text{Ans}$$

CamScanner

Problem 5.9 - Determine the Fourier tramform of the signal

$$x(t) = e^{-at} \sin \omega_0 t \, u(t)$$

Solution : By def of FT,

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$= \int_{-\infty}^{\infty} e^{-at} \sin \omega_0 t \, u(t) e^{-j\omega t} dt$$

$$= \int_{0}^{\infty} e^{-at} \sin \omega_0 t . 1 . e^{-j\omega t} dt$$

$$= \int_{0}^{\infty} e^{-(a+j\omega)t} \left[\frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j} \right] dt$$

$$= \int_{0}^{\infty} \frac{e^{-(a+j\omega)t} e^{j\omega_0 t}}{2j} dt - \int_{0}^{\infty} \frac{e^{-(a+j\omega)} . e^{-j\omega_0 t}}{2j} dt$$

$$= \int_{0}^{\infty} \frac{e^{-(a+j\omega - j\omega_0)t}}{2j} dt - \int_{0}^{\infty} \frac{e^{-(a+j\omega + j\omega_0)t}}{2j} dt$$

$$= \frac{1}{2j} \left[\frac{e^{-(a+j\omega - j\omega_0)t}}{-(a+j\omega - j\omega_0)} \right]_0^{\infty} - \frac{1}{2j} \left[\frac{e^{-(a+j\omega + j\omega_0)t}}{-(a+j\omega + j\omega_0)} \right]_0^{\infty}$$

$$= \frac{-1}{2j(a+j\omega - j\omega_0)} \left[e^{-\infty} - e^{0} \right] - \frac{-1}{2j(a+j\omega + j\omega_0)} \left(e^{-\infty} - e^{0} \right)$$

$$= \frac{-1 . -1}{2j(a+j\omega - j\omega_0)} - \frac{-1 . -1}{2j(a+j\omega + j\omega_0)}$$

$$= \frac{1}{2j}\left[\frac{a+j\omega+j\omega_0 - (a+j\omega-j\omega_0)}{(a+j\omega-j\omega_0)(a+j\omega+j\omega_0)}\right]$$

$$= \frac{1}{2j}\left[\frac{\cancel{a}+\cancel{j\omega}+j\omega_0-\cancel{a}-\cancel{j\omega}+j\omega_0}{(a+j\omega)^2-(j\omega_0)^2}\right]$$

$$= \frac{1}{\cancel{2j}}\left[\frac{\cancel{2j}\omega_0}{(a+j\omega)^2+\omega_0^2}\right]$$

$$= \frac{\omega_0}{(a+j\omega)^2+\omega_0^2}.$$ Ans

6.08.23 - DSP

Fourier Transforms properties:

Linearity

The Fourier transform is linear: if $x_1(t) \leftrightarrow X_1(\omega)$ and $x_2(t) \leftrightarrow X_2(\omega)$ then $c_1x_1(t)+c_2x_2(t) \leftrightarrow c_1X_1(\omega)+c_2X_2(\omega)$ for any two numbers c_1 and c_2.

proof: obvious -

$$F[c_1x_1(t)+c_2x_2(t)] = \int_{-\infty}^{\infty}[c_1x_1(t)+c_2x_2(t)]e^{-j\omega t}\,dt$$

$$= c_1\int_{-\infty}^{\infty}x_1(t)e^{-j\omega t}\,dt + c_2\int_{-\infty}^{\infty}x_2(t)e^{-j\omega t}\,dt$$

$$= c_1X_1(\omega) + c_2X_2(\omega)$$

Time Shifting:

If $x(t) \leftrightarrow X(\omega)$, then $x(t-c) \leftrightarrow X(\omega) e^{-j\omega c}$ for any constant c.

Proof: $F[x(t-c)] = \int_{-\infty}^{\infty} x(t-c)\, e^{-j\omega t}\, dt$

$$= \int_{-\infty}^{\infty} x(t)\, e^{-j\omega (t+c)}\, dt$$

$$= e^{-j\omega c} \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= X(\omega)\, e^{-j\omega c}$$

Frequency Shifting:

If $x(t) \leftrightarrow X(\omega)$, then $x(t)\, e^{j\omega_0 t} \leftrightarrow X(\omega-\omega_0)$ for any real ω_0.

Proof: $F[x(t)\, e^{j\omega t}] = \int_{-\infty}^{\infty} x(t)\, e^{j\omega_0 t}\, e^{-j\omega t}\, dt$

$$= \int_{-\infty}^{\infty} x(t)\, e^{-j(\omega-\omega_0)}\, dt$$

$$= X(\omega - \omega_0)$$

Time Reversal:

If $x(t) \leftrightarrow X(\omega)$, then $f(-t) \longleftrightarrow F(-jW)$

Proof:

Time Reversal

Proof:

$$F[f(-t)] = \int_{-\infty}^{\infty} f(-t)\, e^{-j\omega t}\, dt$$

$$= \int_{t=-\infty}^{t=\infty} f(-t)\, e^{-j\omega t}\, dt$$

$$= \int_{-t=-\infty}^{-t=\infty} f(t)\, e^{j\omega t}\, d(-t)$$

$$= \int$$

$$= \int_{t=\infty}^{t=-\infty} f(t)\, e^{j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} f(t)\, e^{j\omega t}\, dt$$

$$= F(-j\omega)$$

The Modulation theorem:

If $x(t) \leftrightarrow X(\omega)$, then $x(t)\cos(\omega_0 t) \leftrightarrow \frac{1}{2}[X(\omega+\omega_0) + X(\omega-\omega_0)]$.

Proof: Use linearity and the last property to get.

$$F[x(t)\cos(\omega_0 t)] = F\left[\frac{1}{2}x(t)\left(e^{j\omega_0 t} + e^{-j\omega_0 t}\right)\right]$$

$$= \frac{1}{2}F[x(t)e^{j\omega_0 t}] + \frac{1}{2}F[x(t)e^{-j\omega_0 t}]$$

$$= \frac{1}{2}[X(\omega-\omega_0) + X(\omega+\omega_0)]$$

PDF Name: Combine DFT

DFT

In mathematics, the DFT converts a finite list of equally-spaced samples of a function into the list of coefficients of a finite combination of complex sinusoids, ordered by their frequencies, that has those same sample values. It can be said that to convert the sampled function from its original domain to the frequency domain.

DTFT → Discrete - Time Fourier Transform

The DFT differs from the DTFT in that its input and output sequences are both finite.

Definition

The DFT can be regarded as a special case of the DTFT. The DTFT of a causal signal $x(k)$ is defined as follows.

$$X(f) = \sum_{k=0}^{\infty} x(k)\, e^{-jk2\pi fT}, \quad -\frac{f_s}{2} < f \leq \frac{f_s}{2}$$

It has two drawbacks.

DFT and its inverse

DFT

It is a transformation that maps an N-point Discrete-time (DT) signal $x(n)$ into a function of the N complex discrete harmonies. That is given $x[n]$; $n=0, 1, 2, \ldots N-1$, an N-point Discrete-time signal $x[n]$ then DFT is given by :

$$X(k) = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}nk} \quad \text{for } k=0, 1, 2, \ldots, N-1$$

and the inverse DFT (IDFT) is given by

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)\, e^{j\frac{2\pi}{N}nk} \quad \text{for } n=0,1,2,\ldots N-1$$

Example: compute the DFT of the following two sequences:

$h[n] = \{1, 3, -1, -2\}$ and $x[n] = \{1, 2, 0, -1\}$

Solve:

where $N=4$ $\quad e^{j\frac{2\pi}{N}} = e^{j\frac{2\pi}{4}} = e^{\frac{j\pi}{2}} = j$

lets use this info (6.1) to compute DFT values.

$$H(k) = \sum_{n=0}^{3} h[n] e^{-j\frac{\pi}{2}nk} \quad \text{for } k=0, 1, 2, 3$$

$H(0) = h[0] + h[1] + h[2] + h[3] = 1$

$H(1) = h[0] + h[1] e^{-j\frac{\pi}{2}} + h[2] \cdot e^{-j\pi} + h[3] \cdot e^{-j\frac{3\pi}{2}}$

$= 2 - j5$

$H(2) = h(0) + h(1) e^{j\pi} + h(2)\, e^{-j2\pi}$