Problem Set 4 Solutions

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ME 257/357: Propulsion System and Gas-Turbine Analysis

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1 Problem 1: Velocity Triangle (30 pts)

(a) (10 pts) $r_{\text{hub}}/r_{\text{tip}} = 0.8 \implies U_{\text{hub}}/U_{\text{tip}} = 0.8$ since $U = \Omega r$, where Ω is the rotational speed. Given the approximate nature of the analysis, use of the mean blade radius is sufficient for high-level design purposes.

(b) (10 pts) Figure 1 shows the velocity triangles at the four stations.

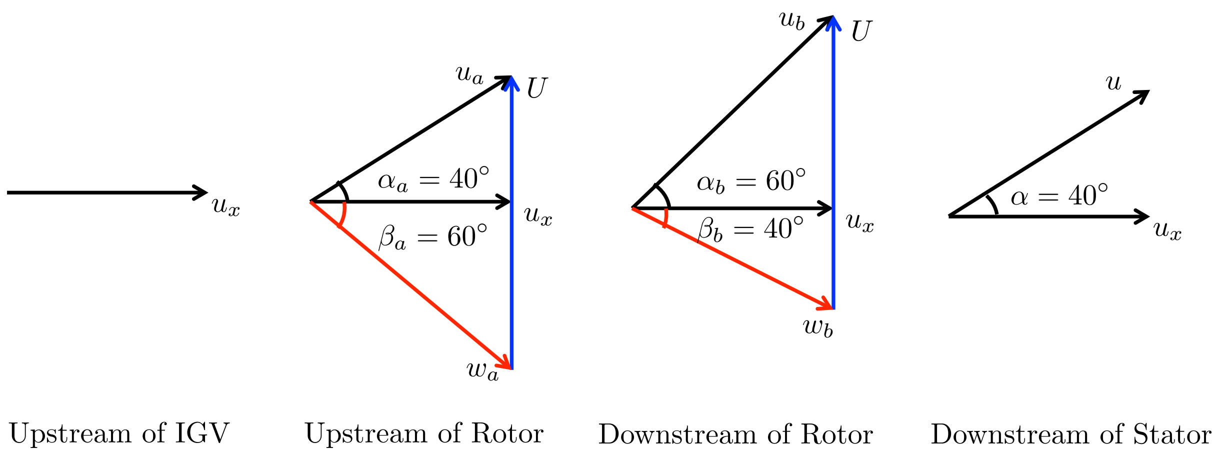

Figure 1: Velocity triangles at four stations.

(c) (10 pts) The power required is given by

$$\begin{aligned} P_c &= \dot{m}_a(h_{0b} - h_{0a}) \\ &= \dot{m}_a U^2 \left[1 - \frac{u_x}{U}(\tan(\alpha_a) + \tan(\beta_b))\right] . \end{aligned} \tag{1}$$

Furthermore, $\dot{m}_a = \rho u_x A$ and

$$\begin{aligned} A &= \pi(r_{\text{tip}}^2 - r_{\text{hub}}^2) \\ &= \pi r_{\text{tip}}^2 \left[1 - \left(\frac{r_{\text{hub}}}{r_{\text{tip}}}\right)^2\right] \\ &= \pi r_m^2 \left(\frac{2}{1 + \frac{r_{\text{hub}}}{r_{\text{tip}}}}\right)^2 \left[1 - \left(\frac{r_{\text{hub}}}{r_{\text{tip}}}\right)^2\right] \\ &= 0.125 \text{ m}^2 . \end{aligned} \tag{2}$$

Since $\rho = 1.2$ kg/m^3 for air at 20° and 1 atm and $u_x = 125$ m/s, $\dot{m}_a = 18.8$ kg/s.

Additionally, $U = ux(\tan(\alpha_a) + \tan(\beta_a)) = 321$ m/s. Therefore, $\boxed{P = 680 \text{ kW}}$.

The rotational speed is $N = U/(2\pi r_m) = \boxed{170 \text{ RPS}}$ or $\boxed{10,200 \text{ RPM}}$

2 Problem 2: Compressor Map (40 pts)

(a) (10 pts) Firstly, the turbine inlet temperature is given to be $\boxed{T_{04} = 1600 \text{ K}}$. The mass flow of air through the compressor is given by

$$\dot{m}_c = \frac{T}{(U_e - U_0) + \beta(U_{1e} - U_0)} \,, \tag{3}$$

where the thrust is known to be $T = 3400$ N and the bypass ratio is $\beta = 2.9$. Additionally, $U_0 = M_0 a_0 =$ m/s at 30 kft.

U_e and U_{e1} are found in a manner similar to the real turbofan case in the first problem set (details are given in the solution code). The primary difference is the use of the stage efficiency, η_{st}. For Point A, the reference condition is taken to be the design condition (i.e., engine operation at 30 kft.). Hence, $\boxed{\eta_{st} = 0.87}$.

Furthermore, the stagnation pressure leaving the compressor is $\boxed{p_{03} = 4.3 \text{ bar}}$ and the mass flow rate is $\boxed{\dot{m}_c = 2.9 \text{ kg/s}}$.

Finally, by inverting Eq. 2, one can find $\bar{r}$ once A is known since $r_{\text{hub}}/r_{\text{tip}} = 0.6$. A is found using relations for isentropic compressible flow (see code) with $M_{2.5} = u_{2.5}/a_{2.5}$ where $u_{2.5} = u_{x,\text{ref}} = 75$ m/s. Hence, $\boxed{\bar{r} = 0.12 \text{ m}}$. It should be noted that in actuality the mean radius may change to preserve the axial velocity through the compressor. Hence, $\bar{r}$ should be thought of as an effective mean radius.

(b) (10 pts) The mass flow is slightly less than that used in the previous analysis (about 3 kg/s at 30 kft); however, is within a somewhat reasonable range. The maximum diameter of the Honda Jet engine is 21.2 in. The tip radius was found to be 5.7 in. Hence, the compressor in this analysis is somewhat smaller than expected. Also, it should be noted that the compressor pressure ratio found is significantly less than the design condition ($\pi_c = 5.2$ rather than $\pi_c = 12$).

(c) (10 pts) The compressor efficiency, η_c is given by

$$\eta_c = \frac{\pi_c^{\frac{\gamma-1}{\gamma}} - 1}{\tau_c - 1} \tag{4}$$

Now, the temperature ratio is found by the update relation

$$\begin{aligned} T_{0,i+1} &= T_{0,i} + \frac{U \Delta u_\theta}{c_p} \\ \implies T_{03} &= T_{02.5} + n_{\text{stages}} \frac{U \Delta u_\theta}{c_p} \\ \implies \tau_c &= 1 + n_{\text{stages}} \frac{U \Delta u_\theta}{c_p T_{02.5}} \,. \end{aligned} \tag{5}$$

For a single stage, the update relation for pressure is given by

$$\begin{aligned}
p_{0,i+1} &= p_{0,i}\left(1+\eta_{st}\frac{U\Delta u_\theta}{c_p T_{0,i}}\right)^{\frac{\gamma}{\gamma-1}} \\
&= p_{0,i}\left[1+\eta_{st}\frac{U\Delta u_\theta}{c_p\left(T_{02.5}+i\frac{U\Delta u_\theta}{c_p}\right)}\right]^{\frac{\gamma}{\gamma-1}} \\
&= p_{0,i}\left[1+\eta_{st}\left(\frac{c_p T_{02.5}}{U\Delta u_\theta}+i\right)^{-1}\right]^{\frac{\gamma}{\gamma-1}} \\
\implies \pi_c &= \left[\prod_{i=1}^{n_{\text{stages}}} 1+\eta_{st}\left(\frac{c_p T_{02.5}}{U\Delta u_\theta}+i\right)^{-1}\right]^{\frac{\gamma}{\gamma-1}}
\end{aligned} \tag{6}$$

Substitution into Eq. 4 yields

$$\eta_c = \boxed{\left[\prod_{i=1}^{n_{\text{stages}}} 1+\eta_{st}\left(\frac{c_p T_{02.5}}{U\Delta u_\theta}+i\right)^{-1}\right] - \left(n_{\text{stages}}\frac{U\Delta u_\theta}{c_p T_{02.5}}\right)^{-1}} \tag{7}$$

(d) (10 pts) The compressor map is shown in Fig. 2. Your map may differ given your assumptions.

3 Problem 3: Different Operating Conditions (30 pts)

(a) (10 pts) The CRUISE point is marked in Fig. 2 with a red diamond.

(b) (10 pts) A drag calculation for this point is not required since it is negligible; thrust is primarily used in this stage to accelerate the aircraft and to climb.

The sea-level static thrust is marked on the graph with a magenta square. It is shown that this is not an operable condition in our analysis.

(c) (10 pts) This analysis shows that the current compressor design is not operable during takeoff. This differs from the cruise condition, which is simply inefficient.

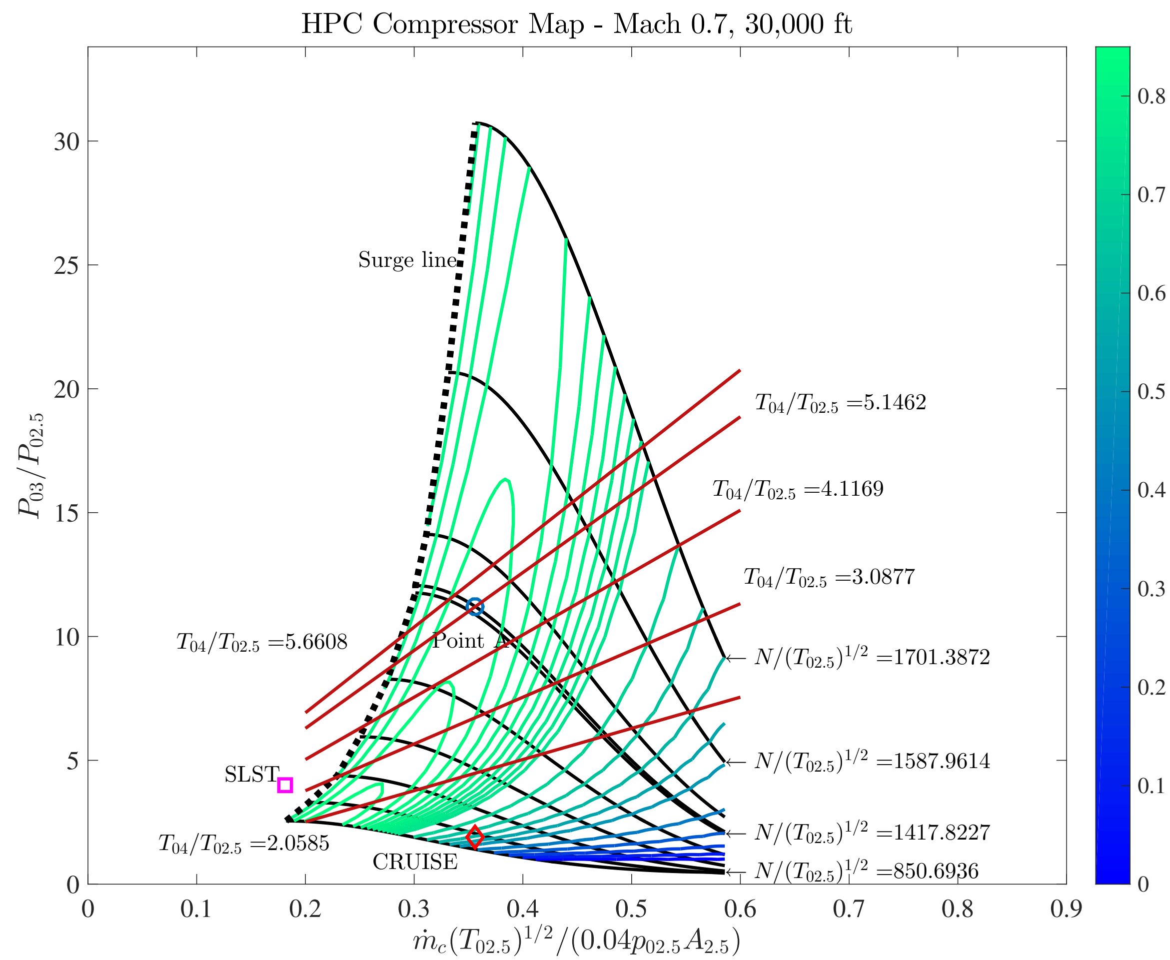

Figure 2: Compressor map. Red lines are of constant $T_{04}/T_{02.5}$, contour lines are of constant η_c, and black lines are of constant $N/T_{02.5}^{1/2}$.