ME 257/357: Propulsion Analysis
Midterm Exam – Part 1
May 28, 2013, 9:00 – 10:15 am
Instructor: M. Ihme

Write your name on this handout and on the notebook(s) and sign the honor code.

Name: ....

Honor Code: *I have neither given nor received unauthorized aid on this examination, nor have I concealed any violations of the Honor Code:* ....

This exam consists of two parts. Part 1 is a *closed-book, closed-notes quiz*, which tests your conceptual understanding of the course-material. Once you finished this part, you are not allowed to return to this part. Provide a brief but clear explanation or sketch to all questions. Write or sketch all answers in the open space under each question. Part 1 of the exam amounts to about 30 % to the exam. Manage your time carefully, and allocate not more than **20 minutes** time to answer part 1. At the end of the exam, return part 1 and part 2 together with all your notes!

Part 1: Quiz – Closed Book, Closed Notes (15 pts)

1. Write down the first law of thermodynamics for a closed system and name all terms in this equation. (2 pts)
 From the equation above derive the corresponding first law of thermodynamics for

$$\underbrace{de}_{\text{Change in internal energy}} = \underbrace{dq}_{\text{HEAT TO SYSTEM}} - \underbrace{dw}_{\text{WORK EXTRACTED FROM SYSTEM}}$$

 an open system, relating enthalpy to heat, pressure, and specific volume. (1 pts)

$h = e + pv; \quad dw = pdv$

$d(h - pv) = dq - pdv$

$dh = dq - pdv + d(pv) = dq - pdv + [pdv + vdp]$

$dh = dq + vdp$

2. Compute the mean molecular weight for a gas composition which corresponds to the Martian atmosphere, consisting of 95 % CO_2 and 5 % N_2 by volume. (2 pts)

$W = \sum X_i W_i$

$W = 0{,}05 \cdot 28 + 0{,}95 \cdot 44$

$= \frac{1}{20}[28 + 19 \cdot 44]$

$\boxed{W = 43.2 \frac{g}{mol}}$

	N_2	CO_2
X_i	0.05	0,95
W_i	28	44 [g/mol]

3. How many translational, rotational, and vibrational degrees of freedom (DOF) has methane (CH_4) at 5,000 K? (2 pts)

$DOF_{\text{trans}} = 3$

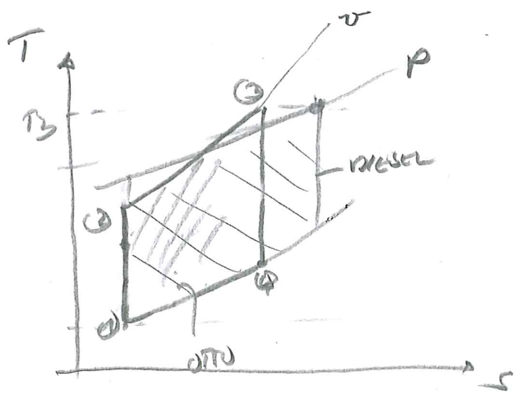
$DOF_{\text{rot}} = 3$

$DOF_{\text{vib}} = 2(3N-6) = 18$

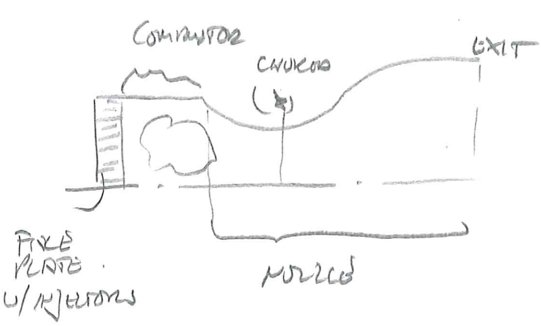
From this compute the ratio of specific heats, γ. (1 pt)

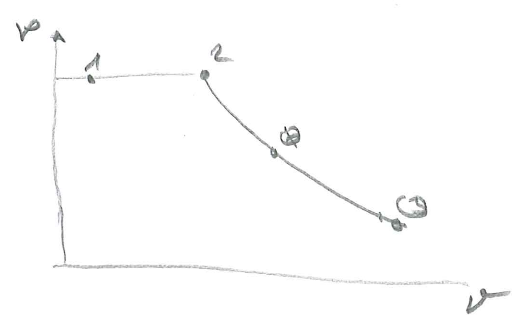
$$\gamma = 1 + \frac{2}{DOF} = 1 + \frac{2}{3+3+18} = 1 + \frac{2}{24} = \boxed{1.0833}$$

4. Sketch the thermodynamic cycle for an Otto and a Diesel engine into *one* T-s diagram, and label all thermodynamic states and processes. Both engines operate in a two-stroke cycle, having the same V_{BDC} and the same combustion temperature T_3; consider appropriate scaling for the compression ratio. (3 pts)



5. Sketch a rocket thrust chamber assembly (TCA) and label all components. Plot the corresponding thermodynamic cycle in a $p - v$ diagram and label state 1: injector; state 2: end of combustion; state $*$: chocked point; state e: exit TCA. (2 pts)





6. Name three ways of how to increase the thrust in a rocket thrust chamber. (1 pts)

- low molecular weight of product
- high heating value
- expand to as low exit pressure as possible
- burn at high chamber pressures as possible.

7. What is the definition and meaning of thermal efficiency? (1 pts)

$$\eta_{th} = \frac{\text{net work from system}}{\text{heat added to system}}$$

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Part 2 of the exam is an open-book, open-notes section. Show all your work for all problems to obtain partial credit wherever possible. At the end of this exam, return this handout and all your work to the open-book, open-notes section together with the closed-book part.

Part 2: Open Book, Open Notes

1 Space Shuttle Orbital Maneuvering System (10 pts.)

The Space Shuttle Orbital Maneuvering System (SS-OMS) is a small rocket system on the Space Shuttle orbiter. It is used for orbital injection and for performing orbital trajectory corrections. The OMS is operated with a hypergolic propellant mixture, consisting of monomethyl hydrazine (CH_3NHNH_2) as fuel and nitrogen tetroxide (N_2O_4) as oxidizer. The SS-OMS is operated at stoichiometric condition, so that the reaction products consist only of carbon dioxide (CO_2), water (H_2O), and nitrogen (N_2). Answer the following questions:

6 (a) Write down the stoichiometric reaction for the monomethyl hydrazine/nitrogen tetroxide reaction.

4 (b) The space shuttle carries 100 kilograms of monomethyl hydrazine during a typical mission. Compute the mass of nitrogen tetroxide that is needed to consume all fuel. What is the total mass of H_2O that is formed during the mission?

2 Isentropic State Change (12 pts.)

Consider the Peng-Robinson state equation:

$$p = \frac{RT}{v} - \frac{aT}{v^2}, \tag{1}$$

where a is a constant which is related to the critical condition of the mixture.

2 (a) From Eq. (1) derive an expression for the compressibility coefficient Z.

8 (b) Consider an isentropic state change between states 1 and 2, use Eq. (1) to derive an relation that relates the temperature ratio T_2/T_1 to the ratio of specific volume v_2/v_1. Assume that the specific heats (c_p and c_v) and the gas constant (R) obey the calorically perfect gas relation. Show all your work in this derivation!

2 (c) Under which condition is the isentropic relation recovered:

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} . \tag{2}$$

3 The Emhi Cycle (23 pts.)

Recently, a so-called Emhi-cycle has been proposed as a new concept for an internal combustion engine. Unlike conventional Otto/Diesel engines, in this cycle the pressure and specific volume are controlled so that they obey the following relation:

$$(\widehat{v} - a)^2 + (\widehat{p} - b)^2 = c^2 , \tag{3}$$

In this cycle, $\widehat{v}$ and $\widehat{p}$ are the normalized specific volume and the normalized pressure, respectively:

$$\widehat{v} = \frac{v}{v_{\text{ref}}} \quad \text{and} \quad \widehat{p} = \frac{p}{p_{\text{ref}}} . \tag{4}$$

The corresponding reference quantities and coefficients for Eqs. (3) and (4) are summarized in Tab. 1. For simplicity, assume that the gas-mixture is calorically perfect with $\gamma = 1.4$ and $R = 287$ J/(kg-K).

2 (a) Sketch the $\widehat{p} - \widehat{v}$ diagram for this cycle.

3 (b) Compute the minimum and maximum specific volume for this cycle. Evaluate the compression ratio which is defined as the ratio between maximum and minimum specific volume.

3 (c) Compute the minimum and maximum pressure of this cycle. Evaluate the pressure ratio which is defined as the ratio between maximum and minimum pressure.

3 (d) Compute the net specific mechanical work in units of MJ/kg.

6 (e) In the $\widehat{p} - \widehat{v}$ diagram of part (a), identify the locations of maximum and minimum temperature of the cycle. Derive an equation for $\widehat{v}$ that allows you to determine these points. Do not attempt to solve this equation; I am only interested in the analytic expression.

3 (f) Qualitatively sketch the T–s diagram for this cycle.

3 (g) Comment on the thermal efficiency of this cycle.

Quantity	v_{ref}	p_{ref}	a	b	c
Value	0.861 m^3/kg	1 bar	2.5	3	2

Table 1: Reference quantities and coefficients for Eqs. (3) and (4).

1 SPACE SHUTTLE OMS

a) STOICHIOMETRIC REACTION:

$$\underbrace{CH_3NHNH_2}_{\text{FUEL}} + \underbrace{a\,N_2O_4}_{\text{OXIDIZER}} \rightarrow b\,CO_2 + c\,H_2O + d\,N_2$$

FROM ATOMIC CONSERVATION:

C	1	b	→	$b = 1$
H	6	2c	→	$c = 3$
O	4a	2b+c		$a = 5/4$
N	2+2a	d		$d = 4.5$

2

→ STOICHIOMETRIC EQUATION

$$\boxed{\begin{array}{c} CH_3NHNH_2 + 1.25\,N_2O_4 \rightarrow CO_2 + 3\,H_2O + 4.5\,N_2 \\ a = 1.25;\ b = 1;\ c = 3;\ d = 4.5 \end{array}}$$

4

b) MASS OF FUEL

WITH $m_i = \nu_i M_i$

	CH_3NHNH_2	N_2O_4	CO_2	H_2O	N_2
$M_i\,[\frac{g}{mol}]$	46	92	44	18	28
ν_i [mol]	1	1,25	1	3	4,5

2 FOR MW'S

FROM STATEMENT: $\nu_{MMH} = \dfrac{m_{MMH}}{M_{MMH}} = \dfrac{100\ g\ mol}{46\ g}$

$$\boxed{\nu_{MMH} = 2{,}174\ mol}$$

1

$$m_{NTO} = \nu_{MMH} \cdot \nu_{NTO} \cdot M_{NTO}$$

$$= 2{,}174 \cdot 1{,}25 \cdot 92$$

$$\boxed{m_{NTO} = 250\ g}$$

1/2

$$m_{H_2O} = \nu_{MAH}\, \nu_{H_2O}\, M_{H_2O}$$

$$= 2{,}714 \cdot 3 \cdot 18$$

$$\boxed{m_{H_2O} = 117.4\ kg}$$

1/2

2 ISENTROPIC STATE CHANGE

$$p = \frac{RT}{v} - \frac{aT}{v^2}$$

2 a) FROM LECTURE

$$pv = ZRT \quad ; \quad Z \text{ IS COMPRESSIBILITY}$$

$$Z = \frac{pv}{RT} = 1 - \frac{a}{Rv}$$

$$\boxed{Z = 1 - \frac{a}{Rv}} \quad 2$$

8 b) FIRST LAW FOR CLOSED SYSTEM

$$de = dq - pdv \; 1 \quad ; \quad de = c_v\, dT \; 1$$

SECOND LAW

$$ds = \frac{dq}{T} \Rightarrow dq = Tds \quad 1$$

SOLVE FOR ds

$$ds = c_v \frac{dT}{T} + \frac{p}{T} dv \quad 2$$

FROM PROBLEM STATEMENT
- c_v, c_p, R = CONST
- a = CONST
- ISENTROPIC : $ds = 0$

INSERT PR-EOS

$$ds = c_v \frac{dT}{T} + \left(\frac{R}{v} - \frac{a}{v^2}\right) dv \quad 1$$

∴ AND INTEGRATE BTW STATE 1 & STATE 2

$$\Delta s = c_v \int \frac{dT}{T} + R \int \frac{dv}{v} - a \int \frac{dv}{v^2}$$

FOR ISENTROPIC PROCESS: $\Delta S = 0$

$$c_v \ln\left(\frac{T_2}{T_1}\right) = -a\left(\frac{1}{v_2} - \frac{1}{v_1}\right) - R \ln\left(\frac{v_2}{v_1}\right)$$

$$\ln\left(\frac{T_2}{T_1}\right) = -\frac{a}{v_2 c_v}\left(1 - \frac{v_2}{v_1}\right) - (\gamma-1)\ln\left(\frac{v_2}{v_1}\right)$$

$$\frac{T_2}{T_1} = \exp\left\{\frac{a}{c_v v_2}\left(\frac{v_2}{v_1} - 1\right) + \ln\left(\frac{v_1}{v_2}\right)^{\gamma-1}\right\}$$

$$\boxed{\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} \exp\left\{\frac{a}{c_v v_2}\left(\frac{v_2}{v_1} - 1\right)\right\}}$$ 2

2 c) FOR $a = 0$, WE RECOVER ($\Leftrightarrow Z = 1$) 1
IDEAL GAS LAW,

$$pv = RT$$ 1

AND

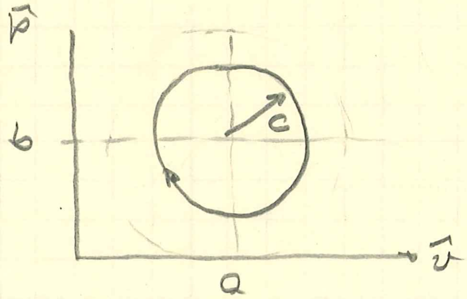
$$\boxed{\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1}}$$

3) EMHI - CYCLE

$$(\hat{v} - a)^2 + (\hat{p} - b)^2 = c^2$$

$$\hat{v} = \frac{v}{v_{ry}} \quad ; \quad \hat{p} = \frac{p}{p_{ry}}$$

a) $\hat{p}$ - $\hat{v}$ - DIAGRAM



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b) MIN/MAX SPECIFIC VOLUME

$$\hat{v}^{\pm} = a \pm c$$

$$\sim v^{\pm} = v_{ry}(a \pm c)$$

$$\hat{v} = \{0{,}5 \, ; \, 4.5\}$$ 2

$$v = \{0{,}4305 ; \, 3{,}8745\}$$

COMPRESSION RATIO $r = \frac{v^+}{v^-} = \frac{a+c}{a-c} = 9$ 1

c) MIN/MAX SPECIFIC PRESSURE

$$p^{\pm} = b \pm c$$ 2

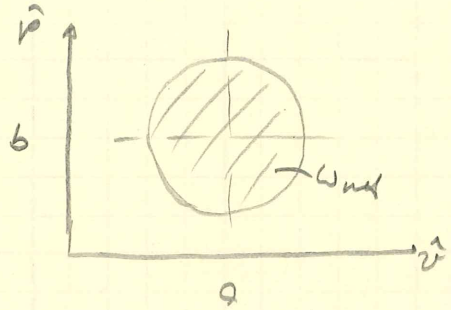
$$p^{\pm} = p_{ry}(b \pm c)$$

$$\hat{p}^{\pm} = \{1, 5\}$$

$$p^{\pm} = \{1, 5\} \text{ bar}$$

$$\tilde{p} = \frac{p^+}{p^-} = 5$$ 1

e) NET-SPECIFIC MECHANICAL WORK:



- NET WORK IS AREA ENCLOSED IN CYCLE.
- IN PRINCIPLE, WE COULD INTEGRATE

$$w_{NET} = p_{k} v_{k} \oint \hat{p}\, d\hat{v}$$ 1

WITH $\hat{p} = \sqrt{c^2 - (\hat{v} - a)^2} + b$; THIS HOWEVER RESULTS IN A TRANSCENDENTAL FCT:

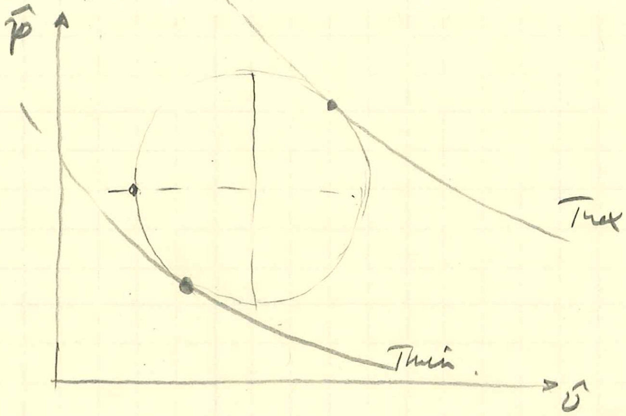
- THE SIMPLER SOL^{N} IS TO DETERMINE w_{N} FROM THE GEOMETRIC PROPERTIES, AS

$$w_{net} = p_{k} v_{k} A \quad ; \quad A = \pi c^2$$

$$= 1 \cdot 10^5 \cdot 0{,}0861 \frac{J}{kg} \pi c^2$$ 1

$$\boxed{w_{N} = 1.082 \text{ kJ/kg}}$$ 1

e) MAXIMUM & MINIMUM TEMPERATURE



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- THE LOCATION OF MAXIMUM / MINIMUM TEMPERATURE IS WHERE ISOTHERMS ARE TANGENTIAL TO CYCLE

- FROM LECTURE, WE KNOW

$$\left.\frac{\partial \hat{p}}{\partial \hat{v}}\right|_T = -\frac{\hat{p}}{\hat{v}} \quad 1$$

DIFFERENTIATE WRT $\hat{v}$ $\quad (\hat{p}-b)^2 + (\hat{v}-a)^2 = c$

$$\left[\ \hat{p} = \pm\sqrt{c^2-(\hat{v}-a)^2} + b\ \right]$$

$$\frac{\partial \hat{p}}{\partial \hat{v}} = -\frac{(\hat{v}-a)}{\sqrt{c^2-(\hat{v}-a)^2}} \quad 1$$

AND EQUATE WITH $-\hat{p}/\hat{v}$

$$\frac{(\hat{v}-a)}{\sqrt{c^2-(\hat{v}-a)^2}} = \frac{\hat{p}}{\hat{v}} = \frac{\sqrt{c^2-(\hat{v}-a)^2}+b}{\hat{v}}$$

$$\boxed{\hat{v}(\hat{v}-a) - \left[c^2-(\hat{v}-a)^2\right] = b\left[c^2-(\hat{v}-a)^2\right]^{1/2}} \quad 2$$

$\Rightarrow$ QUARTIC EQUATION.

WITH SOLUTION (ALTHOUGH NOT REQUIRED)

$$(\hat{v}, \hat{p}, T)_1 = (0{,}542 /\ 2{,}59 /\ 421\,K)$$

$$(\hat{v}, \hat{p}, T)_2 = (3{,}88 /\ 4{,}35 /\ 5200\,K)$$

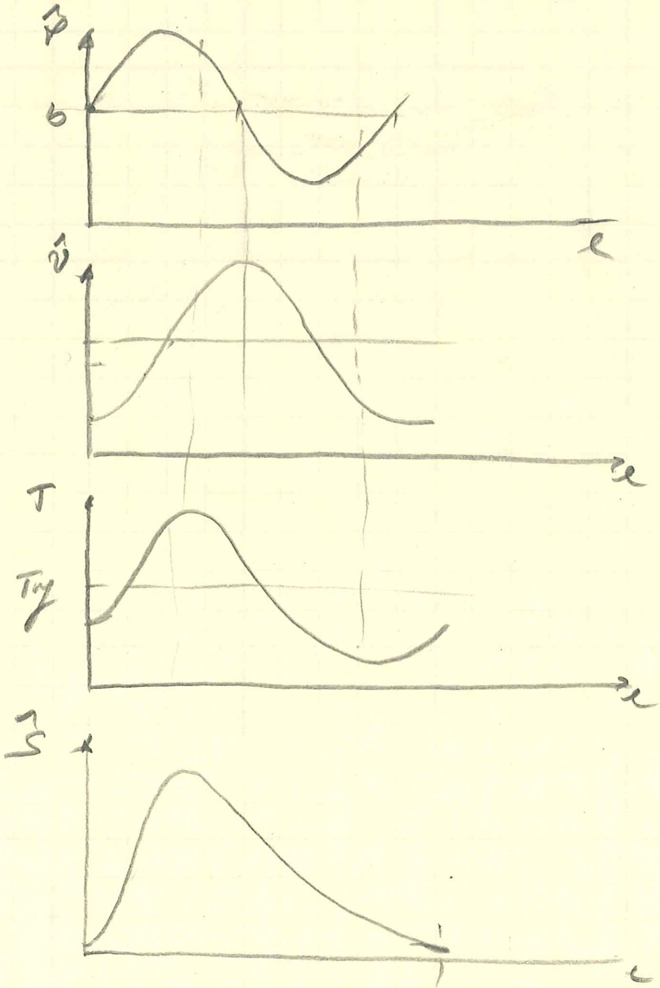
f) T-S-DIAGRAM

- SINCE THIS QUESTION ASKS FOR A <u>QUALITATIVE</u> SKETCH, A POSSIBLE WAY FOR THIS IS TO DERIVE THIS DIRECTLY FROM THE P-V-PLOT.

FOR THIS, WE FIRST INTRODUCE A PATH-COORDINATE,

$$d\ell = c \, d\phi$$

WITH THIS DEFINITION OF ℓ, WE CAN PLOT $\hat{p}, \hat{v}, \hat{T}$, AND s AS FCT OF ℓ



WHERE:

$$\hat{T} = \hat{p}\hat{v}$$

AND $\hat{T} = \frac{T}{T_{cr}}$

WITH $T_{cr} = \frac{p_{cr} v_{cr}}{R}$

- ANOTHER, MORE RIGOROUS, APPROACH IS TO USE THE GIBBS EQUATION.

- FOR THIS, CONSIDER THE NON-DIMENSIONAL QUANTITIES:

$$\hat{p} = \frac{p}{p_{cr}} \; ; \; \hat{v} = \frac{v}{v_{cr}} \; ; \; \hat{T} = \frac{TR}{p_{cr} v_{cr}} \; ; \; \hat{s} = \frac{s}{R}$$

— WITH THIS, WE CAN WRITE GIBBS' EQUATIONS AS:

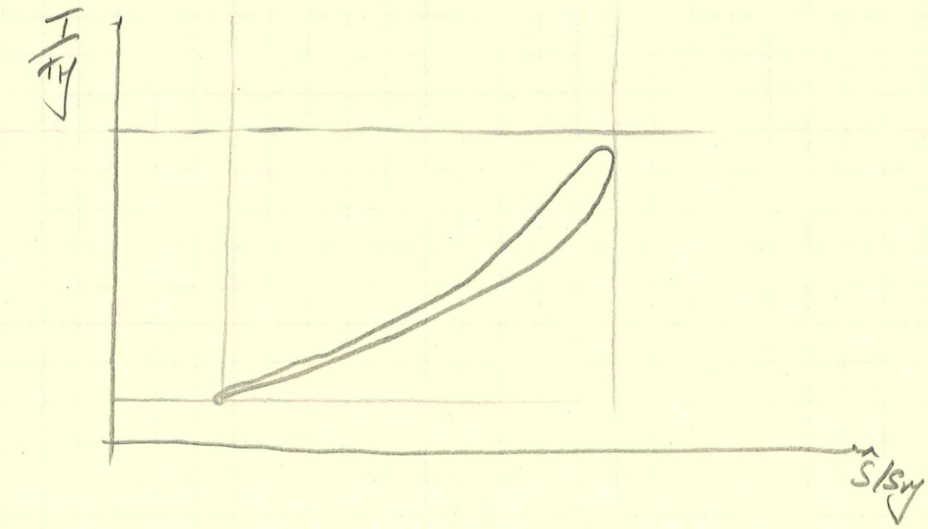
$$T ds = c_p dT - v dp \rightarrow \hat{T} d\hat{s} = \frac{\gamma}{\gamma-1} d\hat{T} - \hat{v} d\hat{p}$$

$$T ds = c_v dT + p dv \rightarrow \hat{T} d\hat{s} = \frac{1}{\gamma-1} d\hat{T} + \hat{p} d\hat{v}$$

AND BY ELIMINATING THE TEMPERATURE-DIFFERENTIAL, WE HAVE:

$$d\hat{s} = \left(\frac{\gamma}{\gamma-1}\right) \frac{d\hat{v}}{\hat{v}} + \frac{1}{\gamma-1} \frac{d\hat{p}}{\hat{p}}$$

$$\Rightarrow \Delta\hat{s} = \left(\frac{\gamma}{\gamma-1}\right) \ln\hat{v} + \left(\frac{1}{\gamma-1}\right) \ln\hat{p} + C$$



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g) TO ASSESS THE THERMAL EFFICIENCY, WE CAN CONSIDER THE T-S -DIAGRAM,

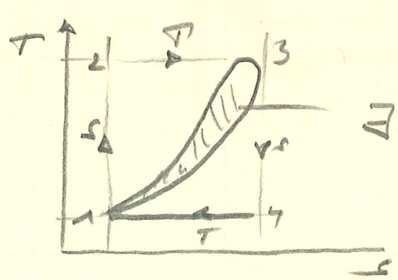
FROM LECTURE, WE KNOW THAT

$\eta_{th} \rightarrow$ MAX IF (*) HEAT ADDITION AT MAX TEMP
(*) HEAT REJECTION AT MIN TEMP.

$$\eta_{th} = 1 - \frac{q_{out}}{q_{in}}$$

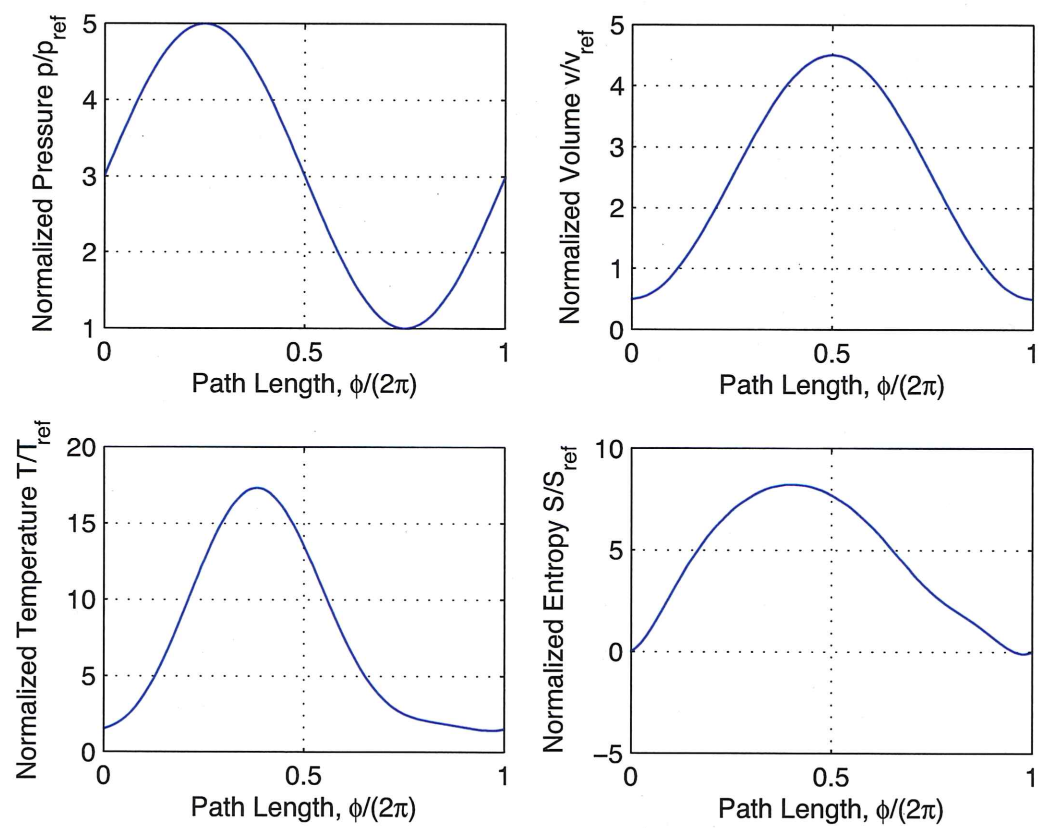
THE OPTIMUM CYCLE WOULD CORRESPOND TO A CARNOT PROCESS (S-T-S-T)

3 for discussion

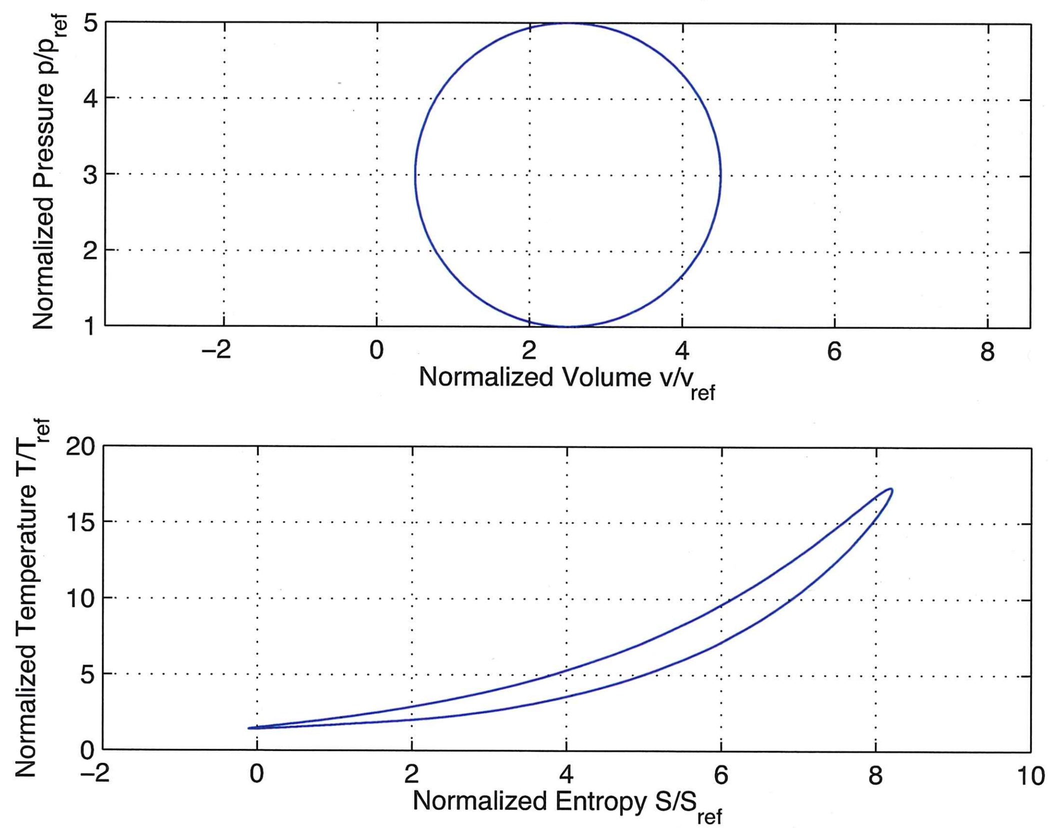


ZHNI-CYCLE

IF WE COMPARE THIS WITH THE ZHNI CYCLE, WE SEE THAT THE LATER PROCESS DOESN'T HAVE THIS PROPERTY, AND THEREFORE HAS A LOWER EFFICIENCY.



Normalized Pressure p/p_ref
Path Length, φ/(2π)
Normalized Volume v/v_ref
Path Length, φ/(2π)
Normalized Temperature T/T_ref
Path Length, φ/(2π)
Normalized Entropy S/S_ref
Path Length, φ/(2π)



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