Spring 2015

ME 257/357 – Midterm Examination

Write your name on this handout and on the notebook(s), and sign the honor code.

Name: . . . .

Honor Code: I have neither given nor received unauthorized aid on this examination, nor have I concealed any violations of the Honor Code.

Signature

OPEN-BOOK PART – Part 2 of the exam is an open-book, open-notes section. However, no laptops or phones with internet connections are allowed. Show all your work on every problem to obtain partial credit wherever possible. After the exam, return this handout and all other work to the open-book, open-notes section.

Problem 1 [20 points]: GE 90 Engine Analysis

We will analyze a GE 90 turbofan (shown in Fig. 1) with specifications listed in Table 1. Assume all components operate ideally EXCEPT the turbines – in your analysis, include the losses in the turbine stages with the given adiabatic efficiencies.

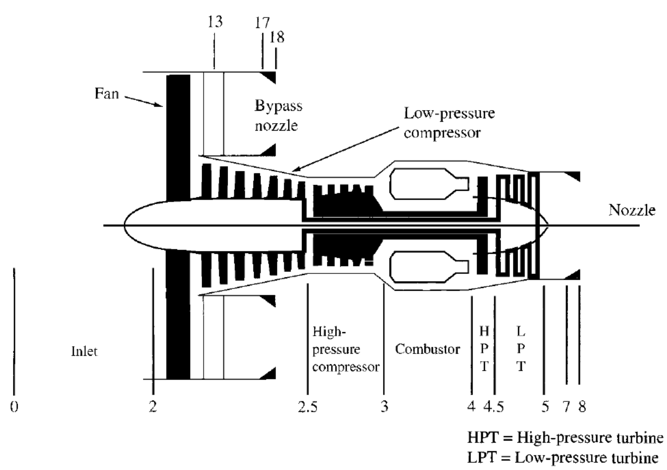

Figure 1. GE90 engine (left) and schematic with station numbering (right).

Type	Axial flow, twin-shaft, bypass turbofan engine
Length	287 in (7,290 mm)
Diameter	Overall: 134 in (3,404 mm); fan 123 in (3,124 mm)
Dry weight	16,644 lb (7,550 kg)
Compressor	Axial: 1 wide chord fan; HPC: 3 stages; LPC: 10 stages
Turbine	Axial: LPT: 6 stages; HPT: 2 stages
Bypass ratio, β	8.1
Fan pressure ratio, Π_F	1.7
LPC pressure ratio, Π_{LPC}	1.14
HPC pressure ratio, Π_{HPC}	21.0
HPT isentropic efficiency, $\eta_{T,HPT}$	0.97
LPT isentropic efficiency, $\eta_{T,LPT}$	0.97
Heating value of fuel, q_{HV}	48 MJ/kg_{Fuel}
Mach number, M	0.85
Altitude	10,600 m
Ambient temperature, T_a	220 K
Ambient pressure, p_a	24 kPa
Specific heat ratio, γ	1.3 (calorically perfect gas)
Gas constant, R	280 J/(kg-K)
Turbine inlet temperature, T_{04}	1380 K

Table 1. Specification of GE90-engine and flight operating conditions (H/LPC/T: High/low-pressure compressor/turbine).

a) Work through the engine station by station to calculate the fuel/air ratio f (simple combustor with the given T_{04} and fuel heating value) as well as the core and fan stream exit velocities. Please put boxes around temperature and pressure at each station.

b) Calculate the thrust specific fuel consumption. For $TSFC = \frac{\dot{m}_{fuel}\,g}{Thrust}$ do we actually need the mass flow of air through the (core of the) engine? (Hint: simplify the trust equation; assume perfectly expanded nozzle flow).

Problem 2 [40 points]: Boeing 777 Aerodynamic Analysis

Now, you will analyze drag and thrust of a Boeing 777, which uses the GE 90 engine. You will first find the velocity, and then calculate drag and thrust.

a) Begin with the drag equation

$$D = \frac{1}{2}\rho_\infty U_\infty^2 S_{ref} C_{D0} + \frac{W^2}{\frac{1}{2}\rho_\infty U_\infty^2 S_{ref}\, \pi\, AR\, e}$$

and derive a symbolic expression for the speed U^* that results in minimum drag. Assume other parameters (aircraft weight and size, as well as altitude) are constant.

b) Find an expression for the minimum drag required, D_{min}. Your answer should be as simplified as possible. Does this minimum drag depend on operating conditions (speed, altitude), or on aircraft parameters, or both? Comment on the relative contributions at speed U^* of profile drag,

$$D_p(U^*) = \frac{1}{2}\rho_\infty (U^*)^2 S_{ref} C_{D0},$$

and induced drag,

$$D_i(U^*) = \frac{W^2}{\frac{1}{2}\rho_\infty (U^*)^2 S_{ref}\, \pi\, AR\, e}\ .$$

Answers up to this point should be symbolic expressions, but now we will proceed with values for analyzing a Boeing 777 at the altitude and airspeed from Problem 1 (Table 1). Use the maximum takeoff weight $MTOW = 2.425\times10^6$ N, wing area $S_{ref} = 427.8\ \mathrm{m}^2$, wing span $b = 60.9$ m, profile drag coefficient $C_{D0} = 0.02$, Oswald efficiency $e = 0.7$.

c) Calculate the actual values of D_{min} and U^* for this aircraft at this altitude (10,600 m).

d) Does the given cruise velocity ($U_0 \approx 240\ m/s$) seem reasonable? Comment on the speed being above or below the velocity for minimum drag, and the practical reason(s) for flying at that speed.

Problem 3 [20 points]: Range and Passenger-Miles-Per-Gallon

Finally, we will calculate the range (and "millage") and compare the Boeing 777 to a Honda Civic! The Boeing 777 carries a fuel volume $V_{fuel} = 117350\,L$ and weight $W_{fuel} = 945120\,N$.

a) Use the Breguet range equation and calculate the range of the Boeing 777 (Using engine analysis from Problem 1 and the aircraft aerodynamic analysis from Problem 2; use cruise velocity of 240 m/s if needed).

b) What is the fuel efficiency, in miles per gallon, of the 777? (Unit conversions: $1\,mile = 1.609\,km,\ 1\,gal = 3.875\,L$)

c) A better measure of efficiency is Passenger-Miles-Per-Gallon (PMPG), which would account for how many people are traveling. A Boeing 777 carries approximately 400 passengers – compare the PMPG of the Boeing 777 to a Honda Civic, which gets around 40 mpg on the highway and carries 4 passengers (comfortably). Is this result surprising in terms of efficiency? Performance? Practicality?

Problem 4 [10 points]: Compressor Stage

Consider the compressor stage shown in Figure 2 to the right, where all flow vectors are indicated at their correct relative lengths. Use the following values for the parameters shown:

V_B = 180 m/s

U_a = 45 m/s; α_a = 15°

U_b = 197 m/s; α_b = 77°

W_a = 197 m/s; β_a = 77°

W_b = 45 m/s; β_b = 15°

Take the stagnation temperature and pressure entering the stage to be T_0 = 600 K and $p_0 = 1.5 \times 10^6$ N/m^2, take the ratio of specific heats to be $\gamma = 1.4$; and take the mass-specific heat of the air to be c_p = 1040 J/(kg-K) at this temperature.

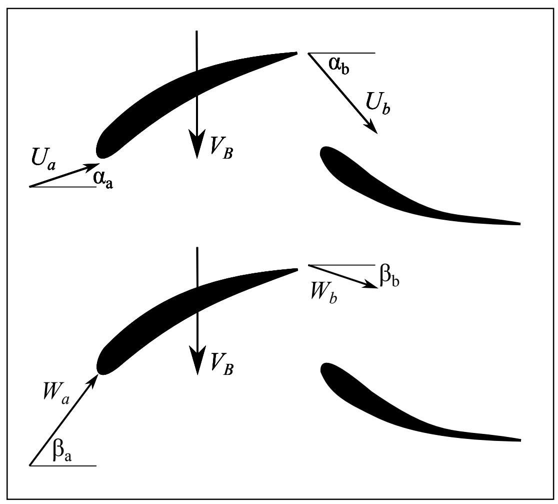

Figure 2. Compressor stage, showing velocity components in absolute (top) and relative (bottom) frame.

a) Carefully draw the velocity triangle for the rotor, showing all vectors at their correct length and orientation, and indicate the change ΔU_θ in the azimuthal component of the flow velocity across the rotor.

b) Compute the work done by the rotor per unit mass of air flowing through this compressor stage, in J/kg.