Spring 2014

ME 257/357 – Midterm Examination

Write your name on this handout and on the notebook(s), and sign the honor code.

Name: ....

– REFERENCE SOLUTION –

Honor Code: I have neither given nor received unauthorized aid on this examination, nor have I concealed any violations of the Honor Code:

Signature

CLOSED-BOOK PART – This part tests your conceptual understanding of gas turbine propulsion systems and your ability to perform elementary thermodynamic calculations. Show brief answers in the spaces provided or circle your answer below. At the end of the exam, return this part together with the open-book part of the examination. You are allowed **not more than fifteen (15) minutes** to complete this part of the exam; after finishing you are not allowed to return to this section.

1. [1 points] Define the *sectional* lift coefficient c_l.

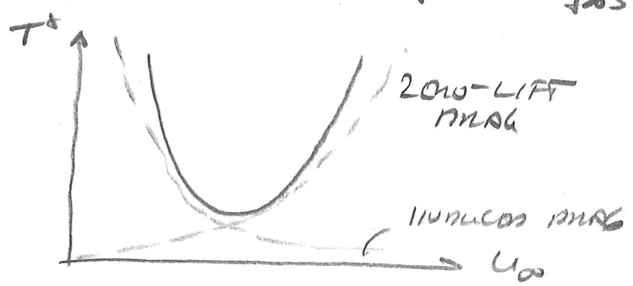
$c_l = \frac{l}{q_\infty c}$ $\quad q_\infty = \frac{1}{2}\rho_\infty U_\infty^2$ IS THE DYNAMIC PRESSURE

l IS LIFT PER UNIT SPAN ($l = L/b$)

c IS CORD

2. [3 point] Write down an equation for the required thrust, and sketch required thrust T^* as a function of aircraft speed U_∞.

$$T^* = \frac{W}{L} D = C_{D0}\, q_\infty S + \frac{W^2}{q_\infty S \pi AR\, e}$$



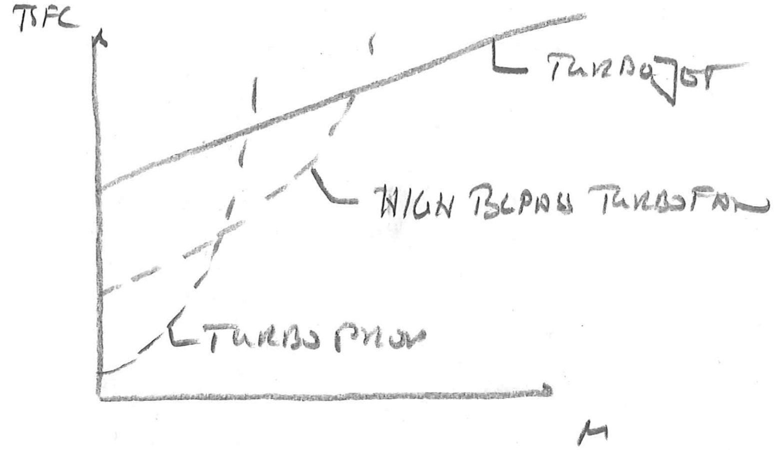
3. [2 points] Write down the thrust equation. Explain the terms in this equation.

$$T = \underbrace{\dot{m}_a\left([1+f]u_e - u_\infty\right)}_{\text{JET THRUST}} + \underbrace{A_e\left(p_e - p_\infty\right)}_{\text{PRESSURE THRUST}}$$

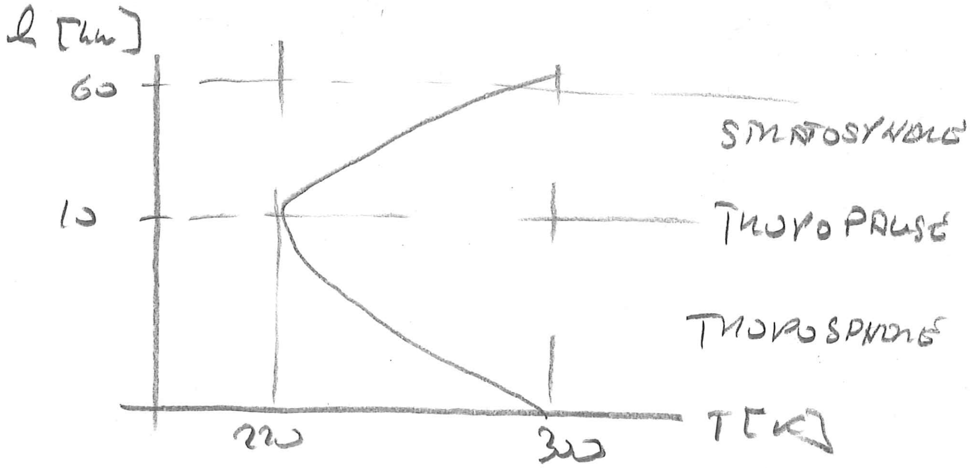
... turn over ...

4. [3 point] What is the definition of thrust-specific fuel consumption. Plot a graph showing TSFC as a function of Mach number for (a) turbo-prop, (b) high-bypass-ratio turbofan, and (c) turbojet.

$$TSFC = \frac{\dot{m}_f}{T} = \frac{\dot{m} f}{T}$$



5. [2 point] Sketch temperature as function of altitude, include troposphere and stratosphere and approximately label the location of the tropopause.



6. [3 point] Name three different alternative fuels and briefly summarize advantages and issues as replacement fuels for conventional aviation fuels.

HYDROGEN, ETHANOL, SYNTHETIC FUEL / FISCHER-TROPSCH

ADVANTAGES: - RENEWABLE
- REDUCE GHG EMISSIONS
- REDUCE POLLUTANT EMISSIONS

DISADVANTAGES: - COST
- THERMAL & MECHANICAL STABILITY
- REGULATIONS
- LARGE RANGE PRODUCTION

... turn over ...

7. [1 point] Write down the first law of thermodynamics.

$du = dq - dw$

$du = Tds - pdv$

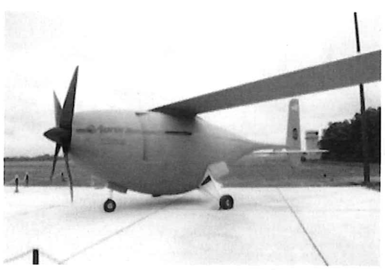
8. [3 point] With a critical Weber number of $We_{crit} = 1$, estimate the maximum droplet diameter for kerosene-fuel that is injected into a gas turbine combustor (the air velocity is 100 m/s, the density is 0.1 kg/m^3, and surface tension of kerosene is 0.02 N/m).

$$We_{crit} = \frac{\rho u^2 D}{\sigma} \Rightarrow D = \frac{We_{crit}\,\sigma}{\rho u^2}$$

$$D = \frac{1 \cdot 0.02}{0.1 \cdot 100^2} \quad \frac{N\,m^3\,s^2}{m\,kg\,m^2} \quad [m]$$

$$\boxed{D = 20\ \mu m \quad (\text{REASONABLE})}$$

9. [2 point] For the three aircraft, labeled a, b, c in the figure below, which do you think would use ethanol, hydrogen, or kerosene as fuel? Write your answers next to each aircraft along with a brief explanation.



a) HYDROGEN
LARGE FUSELAGE FOR STORAGE

b) KEROSENE,
THRUST / PERFORMANCE REQUIREMENT

c) ETHANOL
LARGE WING FOR STORAGE, & INCREASED LIFT TO OVERCOME PENALTY IN HEATING VALUE

Spring 2014

ME 257/357 – Midterm Examination

41 + 14 (Bonus)

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Signature

OPEN-BOOK PART – Part 2 of the exam is an open-book, open-notes section. However, no laptops or phones are allowed. Show all your work on every problem to obtain partial credit wherever possible. After the exam, return this handout and all other work to the open-book, open-notes section.

Background for Problems 1 & 2: The Pratt & Whitney J57 turbojet engine (see Figure 2) was one of the first turbojets that was widely used and helped revolutionize aviation from 1951-1965. The J57 had great performance and versatility, powering early aircraft (see Figure 1) from the B-52 (8 J57 engines!) to the U-2 spy plane to the F-8 supersonic fighter.

In this first problem, you are asked to analyze the B-52 and J57 engine at a single operating condition. The second problem will look at diagrams of the engine operation and the effects of adding a generator to provide electrical power.

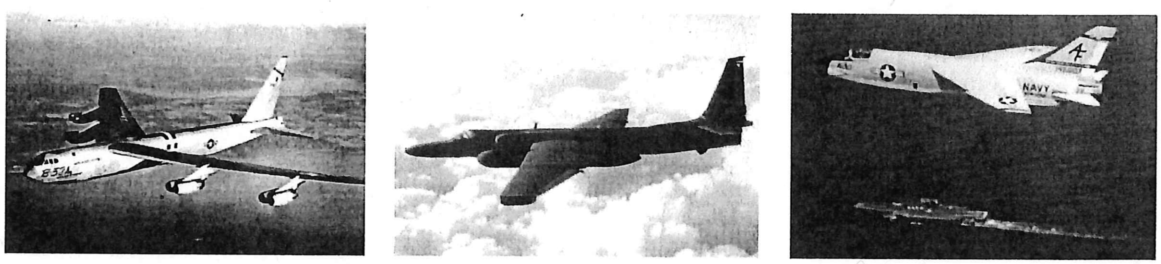


Figure 1: Aircraft powered by Pratt & Whitney J57 turbojet engine, showing B-52 (left), U-2 high altitude spy plane (middle), and F-8 supersonic fighter (right).

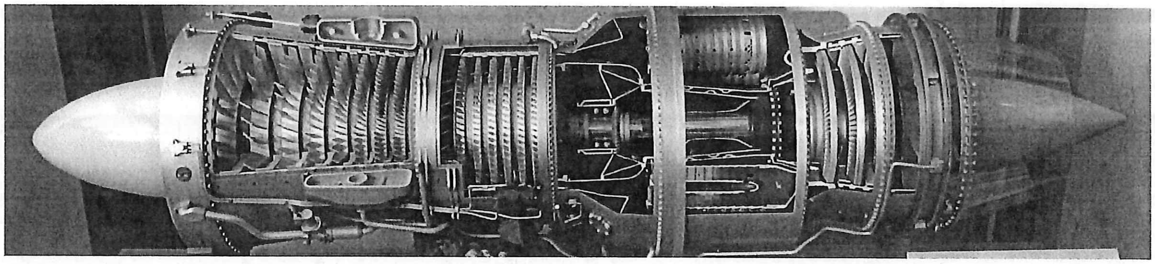


Figure 2: Cut-away of Pratt & Whiteny J57 engine.

Problem 1: B-52 Analysis 27

In this problem, you will analyze drag and thrust of a B-52, which uses the J57 engine. The aircraft will operate at sea level, but NOT at static conditions.

Aircraft Analysis: You will first find the velocity, and then calculate drag and thrust.

2 i. Begin with the drag equation

$$D = \frac{1}{2}\rho_\infty U_\infty^2 S_{ref} C_{D0} + \frac{W^2}{\frac{1}{2}\rho_\infty U_\infty^2 S_{ref}\ \pi\ AR\ e}$$

and derive a symbolic expression for the speed U^* that results in minimum drag. Assume other parameters (aircraft weight and size, as well as altitude) are constant.

3 ii. Find an expression for the minimum drag required, D_{min}. Your answer should be as simplified as possible. Does this minimum drag depend on operating conditions (speed, altitude), or on aircraft parameters, or both? Comment on the relative contributions at speed U^* of profile drag,

$$D_p(U^*) = \frac{1}{2}\rho_\infty (U^*)^2 S_{ref} C_{D0}$$

and induced drag,

$$D_i(U^*) = \frac{W^2}{\frac{1}{2}\rho_\infty (U^*)^2 S_{ref}\ \pi\ AR\ e}\ .$$

Answers up to this point should be symbolic expressions, but now we will proceed with values for analyzing a B-52 at sea level and flying at the speed to achieve minimum drag. (This is representative of an aircraft just after takeoff and during climb.) Use density $\rho_{SL} = 1.23\ kg/m^3$, weight $W = 2 \times 10^6$ N, wing area $S_{ref} = 370\ \text{m}^2$, wing span $b = 56.4$ m, profile drag coefficient $C_{D0} = 0.01$, Oswald efficiency $e = 0.8$.

2 iii. Calculate the actual values of D_{min} and U^* for this aircraft.

Engine Analysis: We will approximate the P&W J57 as an ideal turbojet, and employ the calorically perfect gas approximation with constant specific heat of $c_p = 1005$ J/kg K, $\gamma = 1.4$. Use the engine properties for the compressor pressure ratio, $\pi_c = 12.5$, $\dot{m}_{air} = 80$ kg/s, and $T_4 = 1200$ K. The engine operates with dodecane ($C_{12}H_{26}$) and the heat of formation and molecular weights for all reaction products are given in Table 1.

Table 1. Heats of formation and molecular masses for dodecane combustion reaction

Substance	Formation enthalpy [kJ/mol]	Molecular mass [g/mol]
Dodecane: $C_{12}H_{26}$ (g),	-350.6	170.0
Water: H_2O(g)	-241.8	18.0
Carbon Dioxide: CO_2(g)	-393.3	44.0
Oxygen: O_2(g)	0	32.0
Nitrogen: N_2(g)	0	28.0

For a single J57 engine at sea-level and flying at U^* (found in part iii) conditions ($U_0 = U^*$, $T_0 = 288$ K, $p_0 = 101325$ Pa), answer the following questions:

4 iv. Compute the pressure and temperature at the combustor inlet, p_3 and T_3 (don't forget temperature/pressure rise due to diffuser stagnating flow).

7 v. Set up a reaction equation for the fuel-lean combustion of dodecane with air. Write an equation for the adiabatic flame-temperature. Set $T_{ad} = T_4$ (with T_4 given above), evaluate the corresponding equivalence ratio and find the fuel-air ratio f.

6 vi. Calculate the thrust and TSFC at this condition.

Engine Installation on B-52: Note that the B-52 has eight J57 engines (seen in Figure 1, mounted as four podded pairs). Address the following:

2 vii. Calculate the total thrust available, using the result from part vi for a single engine. Does TSFC change once all eight engines are installed?

1 viii. One advantage of many engines is that if an engine fails, the aircraft loses a smaller percentage of thrust than if it has only two engines. How many of the B-52's engines could fail while still maintaining steady level flight?

Problem 2: J57 Engine Diagrams and Generator 14

Using your analysis from parts iv – vi in Problem 1:

4 i. Sketch a *T-s* diagram for this engine. Clearly label the engine stations, and draw the diagram reasonably to scale with values marked on the axes.

Additional generator: We will now analyze the effect of adding a generator, which is needed to provide power to the airplane's electronics, air conditioning, controls, hydraulic pumps, etc. A reasonable estimate for the single engine is a generator producing 1.5 MW. Consider the same conditions as in Problem 1 ($U_0 = U^*$, $T_0 = 288$ K, $p_0 = 101325$ Pa).

3 ii. Sketch the engine block diagram (inlet, compressor, combustor, etc), with the addition of the generator – clearly show where the generator connects to extract power.

7 iii. What is the new value of thrust, and the percent of thrust lost? Do you think this reduction in thrust is a severe penalty, or relatively acceptable?

Problem 3: Wind Turbine — Bonus — 14

A wind turbine (see Figure 3) is operated in the so-called generator mode to convert kinetic energy into electricity. For the present consideration, the wind blows in horizontal direction (without any crosswind component) at speed $u = 10$ m/s. The rotor-blade speed is $U_b = 2$ m/s at the mid-radius of $r_m = 1$ m. The turbine blades itself are operating under ideal conditions without any losses. The blade angle is $\alpha = 10°$: The specific work that is effectively extracted from each blade is 0.42 J/kg.

8 i. Evaluate the specific work that is lost due to friction or other mechanical losses.

3 ii. Compute the maximum blade-radius so that the relative blade-tip Mach number is $M_{tip} = 0.8$.

3 iii. Assume that the work remains constant along the blade-radius, compute the blade angle at the tip.

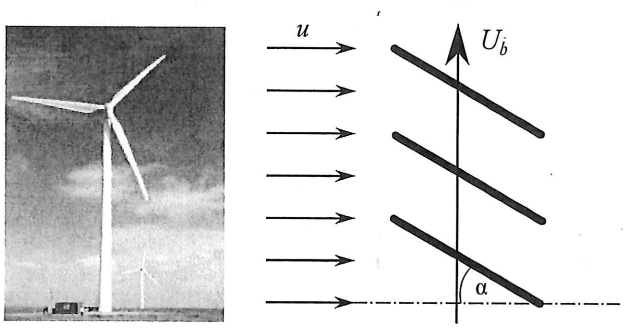


Figure 3: Wind turbine (left) and schematic of problem formulation.

– REFERENCE SOLUTION –

PROBLEM #1 : B52 – ANALYSIS

(2) i) $$D = \frac{1}{2}\rho_\infty U_\infty^2 S_W C_{D0} + \frac{2W^2}{\rho_\infty U_\infty^2 S_W \pi AR e}$$

* TAKE $\frac{\partial D}{\partial U_\infty} \to 0$ TO FIND U^*

$$\frac{dD}{dU_\infty} = \rho_\infty U_\infty S_W C_{D0} - \frac{4W^2}{\rho_\infty U_\infty^3 S_W \pi AR e} = 0$$

AND SOLVE FOR U

$$\boxed{U^* = \left(\frac{4W^2}{\rho_\infty^2 S_W^2 \pi AR e C_{D0}}\right)^{1/4}} \; 2$$

(3) ii) $D_{min} = D(U^*)$ (OMIT SHOWING LOCAL MIN–PROPERTY)

$$D_{min} = \frac{1}{2}\rho_\infty U^{*2} S_W C_{D0} + \frac{2W^2}{\rho_\infty U^{*2} S_W \pi AR e}$$

WITH $U^{*2} = \frac{2W}{\rho_\infty S_W}\sqrt{\frac{1}{\pi AR e C_{D0}}}$

$$D_{min} = \cancel{\tfrac{1}{2}}\cancel{2} W\sqrt{\frac{C_{D0}}{\pi AR e}} + W\sqrt{\frac{C_{D0}}{\pi AR e}}$$

$$\boxed{D_{min} = 2W\sqrt{\frac{C_{D0}}{\pi AR e}}} \; 2$$

– MINIMUM DRAG IS ONLY A FUNCTION OF DRAG – COEFFICIENT & AIRCRAFT GEOMETRY, BUT NOT OF FLIGHT CONDITION

– AT D_{min} BOTH CONTRIBUTIONS DUE TO PROFILE DRAG & INDUCED DRAG ARE EQUAL!

$\rho_{SL} = 1.23 \ kg/m^3$

$W = 2 \cdot 10^6 \ N$

$S_y = 370 \ m^2$

$b = 56.4 \ m$

$C_{D0} = 0.01$

$e = 0.8$

(2) iii) $D_{min} = 2W \sqrt{\dfrac{C_{D0}}{\pi AR \, e}}$; $AR = \dfrac{b^2}{S_y}$

$= 2 \cdot 2 \cdot 10^6 \sqrt{\dfrac{0.01}{\pi \cdot \frac{56.4^2}{370} \cdot 0.8}}$

$\boxed{D_{min} = 86{,}052 \ N \qquad (86.05 \ kN)}$ 1

$u^* = \left(\dfrac{4W^2}{\rho_\infty^2 \ S_y^2 \ \pi \ AR \, e \, C_{D0}} \right)^{1/4}$

$= \left(\dfrac{4 \cdot (2 \cdot 10^6)^2}{1.23^2 \cdot 370^2 \cdot \pi \cdot \frac{56.4^2}{370} \cdot 0.8 \cdot 0.01} \right)^{1/4}$

$\boxed{u^* = 137.5 \ m/s}$ 1

ENGINE ANALYSIS

$c_p = 1005\ J/kgK$, $\gamma = 1.4$; $\pi_c = \frac{p_{03}}{p_{02}} = 12.5$

$\dot{m}_A = 80\ kg/s$; $T_4 = 1200\ K$

$T_0 = 288\ K$
$p_0 = 101325\ Pa$

(4) ii) * INLET

$$T_{02} = T_0 + \frac{1}{2}\frac{u^2}{c_p}$$

$$= 288 + \frac{1}{2}\frac{137.5^2}{1005}$$

$$\boxed{T_{02} = 297.4\ K}\ 1$$

$$p_{02} = p_0\left(\frac{T_{02}}{T_0}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\boxed{p_{02} = 113{,}388\ Pa}\ 1$$

* COMPRESSOR

$$p_{03} = \pi_c\, p_{02}$$

$$= 12.5\,(113{,}388)$$

$$\boxed{p_{03} = 1417.35\ kPa}\ 1$$

$$T_{03} = T_{02}\left(\frac{p_{03}}{p_{02}}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_{03} = 297.4\cdot(12.5)^{\frac{0.4}{1.4}}$$

$$\boxed{T_{03} = 612\ K}\ 1$$

* COMBUSTOR: $\boxed{T_{04} = 1200\ K}$

$$\boxed{p_{04} = p_{03} = 1417.35\ kPa}$$

(7) 2) COMBUSTION PROCESS

* FUEL-LEAN COMBUSTION

$$\phi\, C_{12}H_{26} + 18.5(O_2 + 3.76 N_2) = 12\phi\, CO_2 + 13\phi\, H_2O + 69.56\, N_2 + 18.5(1-\phi)\, O_2 \quad 2$$

* FROM FIRST LAW ENERGY CONSERVATION

$$m c_p (T_0 - T_3) + \Delta H_R^0 + m c_p (T_{04} - T_0) = 0$$

$$m c_p (T_4 - T_3) + \Delta H_R^0 = 0 \quad 1$$

* WITH • $m = m_R = m_P = \sum \nu_i' M_i$

$$= \phi M_{C_{12}H_{26}} + 18.5(M_{O_2} + 3.76 M_{N_2})$$

$$\bullet\, \Delta H_R^0 = \sum_i \nu_i'' h_i^0 - \sum_i \nu_i' h_i^0 \quad 1$$

$$= \phi(12 h_{CO_2}^0 + 13 h_{H_2O}^0 - h_{C_{12}H_{26}}^0) = \phi \mathcal{H}$$

$$\left[\phi M_{C_{12}H_{26}} + 18.5(M_{O_2} + 3.76 M_{N_2})\right] c_p (T_4 - T_3) + \phi \mathcal{H} = 0$$

$$\phi = -\frac{18.5(M_{O_2} + 3.76 M_{N_2})\, c_p (T_4 - T_3)}{\mathcal{H} + c_p M_{C_{12}H_{26}} (T_4 - T_3)}$$

$$\phi = -\frac{18.5(M_{O_2} + 3.76 M_{N_2})}{\frac{\mathcal{H}}{c_p(T_4 - T_3)} + M_{C_{12}H_{26}}} \quad 1$$

WITH $\mathcal{H} = -7512.4$ kJ

$$\phi = -\frac{18.5(0.032 + 3.76\ 0.028)}{-\frac{7512.4}{1.005(1200-612)} + 0.17}$$

$$\boxed{\phi = 0.2025} \quad 1$$

$$\phi = \frac{f}{f_{st}} \Rightarrow f = \phi\, f_{st} = \phi \left.\frac{m_F}{m_{air}}\right)_{st}$$

$$f = 0.2025 \frac{170}{18.5(32 + 3.76 \cdot 28)}$$

$$\boxed{f = 0.01355} \quad 1 \qquad \left(m_F \approx 1.084 \frac{kg}{s} \right)$$

(6) vi) THRUST-SPECIFIC FUEL CONSUMPTION

$$W_T = -W_C$$

$$m_A c_p (T_3 - T_2) = m_A (1+f) c_p (T_{05} - T_{04})$$

$$T_5 = T_4 - \frac{T_3 - T_2}{(1+f)}$$

$$= 1200 - \frac{612 - 257.4}{1.01355}$$

$$\boxed{T_5 = 885.6\,K} \quad 1$$

$$p_{05} = p_{04} \left(\frac{T_{05}}{T_{04}} \right)^{\frac{\gamma}{\gamma-1}}$$

$$= 1417.35 \left(\frac{885.6}{1200} \right)^{\frac{1.4}{0.4}}$$

$$\boxed{p_{05} = 483.85\,kPa} \quad 1$$

CONSIDER PERFECTLY EXPANDED NOZZLE

$$p_e = p_0$$

$$T_e = T_{05} \left(\frac{p_e}{p_{05}} \right)^{\frac{\gamma-1}{\gamma}}$$

$$T_e = 885.8\,K \left(\frac{101325}{483850} \right)^{\frac{0.4}{1.4}}$$

$$\boxed{T_e = 564.7\,K} \quad 1$$

FROM ENTHALPY

$$u_e = \sqrt{2 c_p (T_{0T} - T_e)}$$

$$= \sqrt{2 \cdot 1005 \cdot (885.8 - 564.7)}$$

$$\boxed{u_e = 803 \ \frac{m}{s}} \ 1$$

THRUST

$$T = \dot{m}_a \left([1+f] u_e - u_0 \right)$$

$$TSFC = f \frac{\dot{m}_a}{T} = f \frac{1}{([1+f] u_e - u_0)}$$

$$TSFC = \frac{0.01355}{(1.01355 \cdot 803 - 137.5)}$$

$$\boxed{TSFC = 0.02 \ \frac{g}{kN} = 72 \ \frac{g}{kN\,hr}} \ 2$$

(2) a ii) THRUST AVAILABLE

$$T_{avail} = n_{engine} \cdot T$$
$$= 8 \cdot \dot{m}_a \left((1-\beta) u_e - u_0 \right)$$
$$= 8 \cdot 80 \left(\left[(1+\beta) \; 803 - 137.5 \right) \right.$$

$$\boxed{T_{avail} = 432{,}883 \text{ kN}} \; 2$$

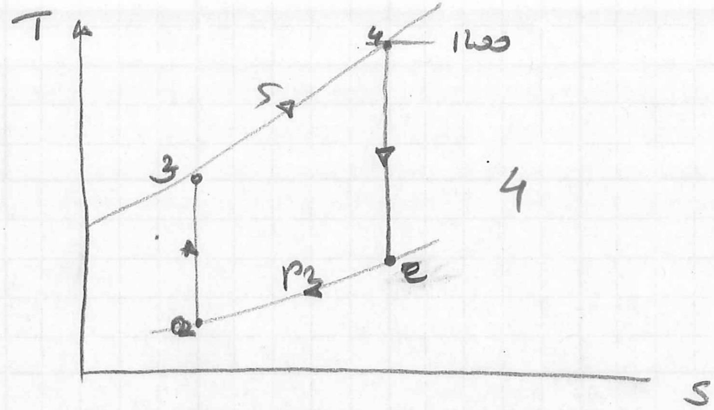
(1) a iii) $n_{fail} = n_{engine} - \left\lceil \frac{T_{avail}}{D_m} \right\rceil$

$$\boxed{n_{fail} = 6} \; 1$$

6 ENGINES ARE ALLOWED TO FAIL

PROBLEM #2] 5? ENGINE DIAGRAM

(4) i) T-S -DIAGRAM



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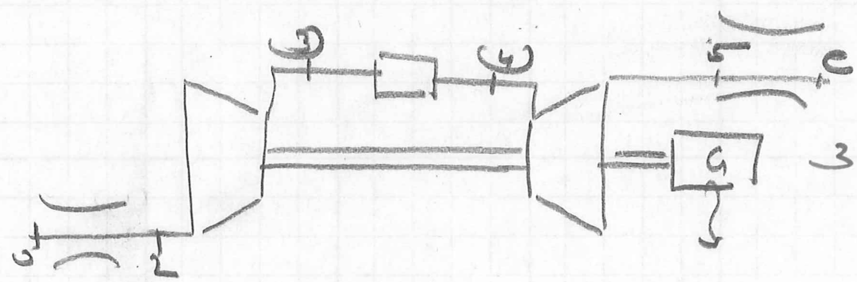
ADDITIONAL GENERATOR

(3) ii) ENGINE BLOCK DIAGRAM



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(7) iii) CONSIDERATION OF GENERATOR REQUIRES REEVALUATION OF ALL CONDITIONS UPSTREAM OF TURBINE INLET

$$-\dot m_A c_{pa}(T_{03}-T_{02}) - P_G = \dot m_A(1+f)\,c_p(T_{05}-T_{04})$$ 1

$$T_{05} = T_{04} - \frac{T_{03}-T_{02}}{1+f} - \frac{P_G}{\dot m_A(1+f)\,c_p}$$

$$= 1200 - \frac{612-287.4}{1.01355} - \frac{1.5\cdot 10^6}{80\,(1.01355)\,1225}$$

$\boxed{T_{05}^* = 871.2\,K}$ (TEMP. -REDUCTION BY 18K)

$\boxed{p_{05}^* = 462.1\ kPa}$

$\boxed{T_e^* = 564.7\ K}$ (SAME AS BEFORE) 2

$$u_e^* = \sqrt{2 c_p \left(T_{05}^* - T_e^*\right)}$$

$$u_e^* = \sqrt{2 \cdot 1005 \cdot \left(871.2 - 564.7\right)}$$

$$\boxed{u_e^* = 784.9 \text{ m/s}}$$ 1

$$T^* = \dot{m}_a \left[(1+f)\, u_e^* - u_0 \right]$$

$$\boxed{T^* = 52\,641.5 \text{ N}}$$

$$\boxed{T = 54\,110 \text{ N}}$$ 3

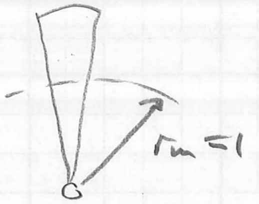
⟹ DROP IN THRUST BY 2.7%

THIS IS ACCEPTABLE

PROBLEM #3 WIND TURBINE

(i)

8



$u = 10\ m/s$

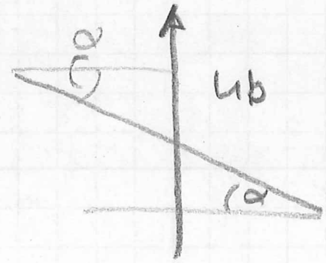
$u_b = 2\ m/s$

$r_m = 1\ m$

$\alpha = 10^\circ$

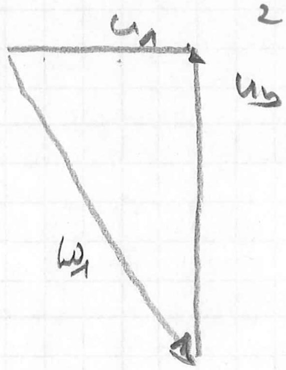
WORK EXTRACTED FROM TURBINE

$w = u_b \Delta u_\theta$

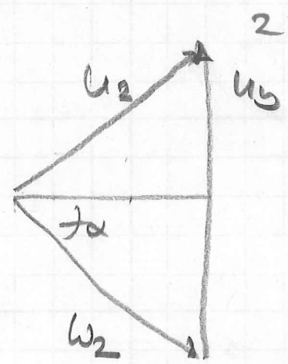


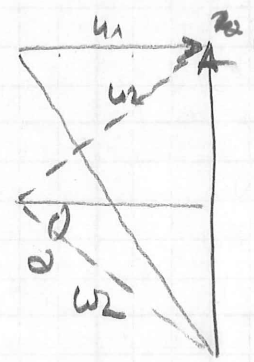
VELOCITY TRIANGLE

LE



TE





$$u_{\theta 2} = u_b - u \tan\alpha$$
$$= 2 - 10 \tan 10^\circ$$

$$\boxed{u_{\theta 2} = 0{,}2367 \text{ m/s}}\ 1$$

$$w = u_b \Delta u_\theta \qquad \Delta u_\theta = u_{\theta 2} - u_{\theta 1}$$
$$\boxed{w = 0{,}473 \frac{m^2}{s^2}}\ 1 \qquad = u_{\theta 2}$$

→ THIS IS THE THEORETICAL RESULT

$$w_{loss} = w - w_{act}$$
$$= 0{,}473 - 0{,}42$$

$$\boxed{w_{loss} = 0{,}053 \frac{J}{kg} \quad (\approx 12\%)}\ 1$$

(3) ii) MAXIMUM BLADE RADIUS

$$M_{tip} = \frac{u}{a} = \frac{\frac{u_b}{r_m} r}{\sqrt{\gamma R T}}\ 1 \qquad u_{tip} = \Omega r = \frac{u_b}{r_m} r$$

$$r = M_{tip} \sqrt{\gamma R T} \frac{r_m}{u_b}$$

$$= 0{,}8 \sqrt{1{,}4 \cdot 287 \cdot 288} \frac{1 m}{2 \frac{m}{s}}\ 1$$

$$\boxed{r_{tip} = 136 \text{ m}}\ 1$$

(2) iii) $u_{b,tip} = u_b \frac{r_{tip}}{r_m} = 2 \cdot 136$

$$\boxed{u_{b,tip} = 272 \frac{m}{s}}\ 2$$

$$u_{\theta 2} = u_{b\,tip} - u \tan\alpha$$

$$\alpha = \arctan\left(\frac{u_{b\,tip} - u_{\theta 2}}{u}\right)$$

$$= \arctan\left(\frac{272 - 0{,}2367}{10}\right)$$

$$\boxed{\alpha = 87.9^\circ}$$ //

THIS IS REASONABLE, AND EXPLAINS THE BLADE TWISTING !