Relevant equations for compressor map

Conservation of energy:

$$H_1 = H_2 \ , \quad \text{or} \ \dot{m}_1\left(h_1 + \frac{1}{2}u_1^2\right) = \dot{m}_2\left(h_2 + \frac{1}{2}u_2^2\right)$$

$$\boxed{\dot{m}_1\left(c_pT_1 + \frac{1}{2}u_1^2\right) = \dot{m}_2\left(c_pT_2 + \frac{1}{2}u_2^2\right)}$$

Isentropic State Relations:

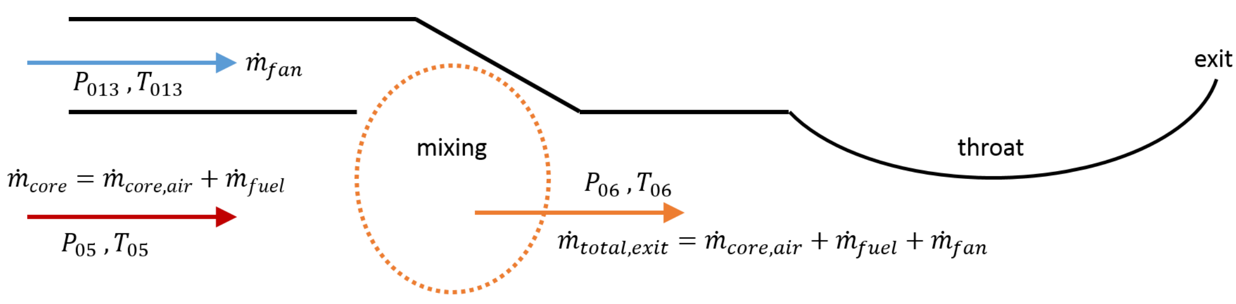
$$\boxed{\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{\rho_2}{\rho_1}\right)^{\frac{1}{\gamma-1}}}$$

Adiabatic efficiencies:
(with our turbojet engine station numbering)

$$\eta_d = \frac{T_{02s} - T_{01}}{T_{02} - T_{01}} \ , \quad \eta_c = \frac{T_{03s} - T_{02}}{T_{03} - T_{02}} \ , \quad \eta_t = \frac{T_{05} - T_{04}}{T_{05s} - T_{04}} \ , \quad \eta_t = \frac{T_{0e} - T_{06}}{T_{0es} - T_{06}}$$

Stream Mixing:

Consider fan and core streams mixing downstream of turbine.
This is modeled by constant volume mixing of two fluids.



$$T_{06} = \frac{\dot{m}_{core}T_{05} + \dot{m}_{fan}T_{013}}{\dot{m}_{core} + \dot{m}_{fan}}$$

And, if $\dot{m}_{core} = \dot{m}_{core,air} + \dot{m}_{fuel}$, $f = \dot{m}_{fuel}/\dot{m}_{core,air}$, and $\beta = \dot{m}_{fan}/\dot{m}_{core,air}$

$$\boxed{T_{06} = \frac{(1+f)T_{05} + \beta T_{013}}{(1+f) + \beta}}$$

Similarly, from masses and ideal gas equation of state,

$$\boxed{P_{06} = \frac{(1+f)T_{05} + \beta T_{013}}{(1+f)\dfrac{T_{05}}{P_{05}} + \beta\dfrac{T_{013}}{P_{013}}}}$$

Normalized Measures of Thrust:

Mass-specific Thrust

$$MST = \frac{Thrust}{\dot{m}_{core,air}} = (1 + f + \beta)u_{e,mixed} - (1 + \beta)U_0$$

Non-dimensional Specific Thrust

$$T_S = \frac{Thrust}{\dot{m}_{core,air} * U_0} = (1 + f + \beta)\frac{u_{e,mixed}}{U_0} - (1 + \beta)$$

Nozzle Flow Equations:

references: https://www.grc.nasa.gov/www/k-12/airplane/astar.html
http://www.stanford.edu/~cantwell/AA283_Course_Material/AA283_Course_Notes/Ch_03_Ramjet_Cycle.pdf

Mass flow rate

$$\boxed{\dot{m} = \rho U A}$$

which is true at any location in the nozzle. For choked flow (for instance, the throat of the exhaust nozzle), we know the Mach number is 1, and denote these sonic conditions with the " * " superscript, and stagnation or total properties with " 0 " subscript.

$$\dot{m} = \rho^* U^* A^*$$

$$\frac{T^*}{T_0} = \frac{2}{\gamma + 1}\ ,\ \frac{\rho^*}{\rho_0} = \left(\frac{2}{\gamma + 1}\right)^{\frac{1}{\gamma - 1}}\ ,\ \frac{P^*}{P_0} = \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma}{\gamma - 1}}$$

NOTE: you can find the density at any engine station using the ideal gas law, BUT we are already keeping track of total temperature and pressure at each station, so we can use those quantities.

After substituting $\rho^* = {P^*}/{RT^*}$, and at sonic conditions $M = 1 = {U^*}/{\sqrt{\gamma R T^*}}$, and some algebra to simplify the expression (APPLICABLE ONLY AT SONIC CONDITIONS),

$$\boxed{\dot{m} = \frac{P_0}{\sqrt{T_o}} A^* \sqrt{\frac{\gamma}{R}} \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma + 1}{2(\gamma - 1)}}}$$

For a general location where $M \neq 1$,

$$\boxed{\dot{m} = \frac{P_0}{\sqrt{T_o}} A \sqrt{\frac{\gamma}{R}} M \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-\left(\frac{\gamma + 1}{2(\gamma - 1)}\right)}}$$

These equations give rise to the Area-Mach # relation,

$$\boxed{f(M) = \frac{A}{A^*} = \frac{1}{M}\left(\frac{\gamma+1}{2}\right)^{-\left(\frac{\gamma+1}{2(\gamma-1)}\right)}\left(1+\frac{\gamma-1}{2}M^2\right)^{\left(\frac{\gamma+1}{2(\gamma-1)}\right)}}$$

Compressor Stage:

Rotor Work

$$W_{rotor} = U \cdot \Delta u_\theta$$

With generic numbering for the rotor-stator combination (NOT our engine station numbering), conservation of energy across a stage leads to $h_{02} - h_{01} = U \cdot \Delta u_\theta$, or

$$\boxed{\frac{T_{02} - T_{01}}{T_{01}} = \frac{U \cdot \Delta u_\theta}{c_p T_{01}}}$$

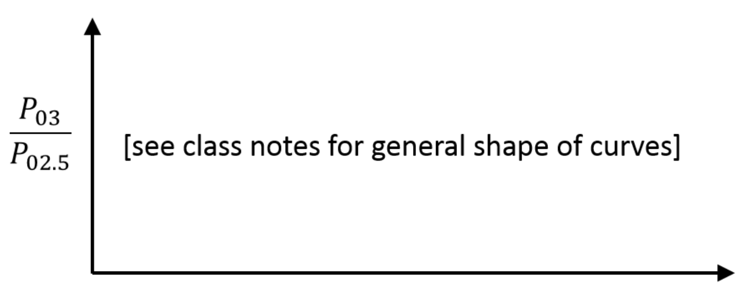
Then, from isentropic relations,

$$\boxed{\frac{T_{02s}}{T_{01}} = 1 + \eta_{stage}\frac{U \cdot \Delta u_\theta}{c_p T_{01}}}$$

$$\boxed{\frac{P_{02s}}{P_{01}} = \left(1 + \eta_{stage}\frac{U \cdot \Delta u_\theta}{c_p T_{01}}\right)^{\frac{\gamma}{\gamma-1}}}$$

Compressor Map:

The x-axis is a measure of mass flow or velocity, typically in some normalized quantity; y-axis is pressure ratio. See Hill & Peterson, Chapter 8 for more details.



$$\tilde{m} = \frac{\dot{m}\sqrt{T_{02.5}}}{P_{02.5}A_{2.5}} \quad \text{or} \quad \frac{A^*}{A_{2.5}} = \frac{\dot{m}\sqrt{T_{02.5}}}{P_{02.5}A_{2.5}}\sqrt{\frac{R}{\gamma}}\left(\frac{2}{\gamma+1}\right)^{-\left(\frac{\gamma+1}{2(\gamma-1)}\right)}$$