ME 257/357 Final Project
Due Thursday, June 12, 2018, 10 AM
(Prof. Ihme's Office or electronic submission, *NO LATE EXTENSION*)

This project fuses all problems that we covered in the previous homework sets with the object to develop a fully coupled gas-turbine combustor engine model. The focus of this project is on the coupling of the compressor and the turbine in order to evaluate the engine performance. The format of this project is similar to previous homework sets. Expected deliverables are documentation of relevant derivations and analysis, results, and non-trivial source code. For further reader, copies of Chapter 8 by Hill and Peterson and Sec. 9.5 by Mattingly are provided on canvas.

Project Policy

- Clearly outline and explain your approach to solving each problem.
- Make ***and state*** any assumptions that you think are necessary in solving problems.
- Provide all solution steps, and clearly mark/box your solutions.
- Start with the general formulation, and simplify all expressions as much as possible.
- Work out symbolic expressions, and plug in numbers only at the end when asked for specific numerical values.
- Show all your results and derivations explicitly worked out by hand. Partial credit is given for intermediate steps of derivations.
- Computer symbolic toolboxes (Maple, Mathematica, Matlab, etc.) should not be used for derivations, algebraic steps, ODE's, etc.
- Attach all non-trivial source code. Code should be ***clearly written and commented*** to explain units, notation, function inputs & outputs, etc.
- You are encouraged to discuss the general approach at the conceptual level in groups; however, you must submit your individual work, including write-up, plots, and code.

Problem formulation

In this assignment, we will design and analyze a turbine that is representative of the high-pressure turbine found in the GE Honda HF-120 engine (shown in Figure 1 below). Note that we will design the high-pressure turbine as *one stage* to match the high-pressure compressor we analyzed in Homework 4.

Figure 1. GE Honda HF-120 Engine Cutaway Drawing (from janes.ihs.com)

Part 1. Turbine Stator-Rotor Analysis

Consider a single turbine stage shown schematically in Figure 2 below. At the mean radius, a cross-sectional cut shows the blade configuration of the stator/rotor cascade.

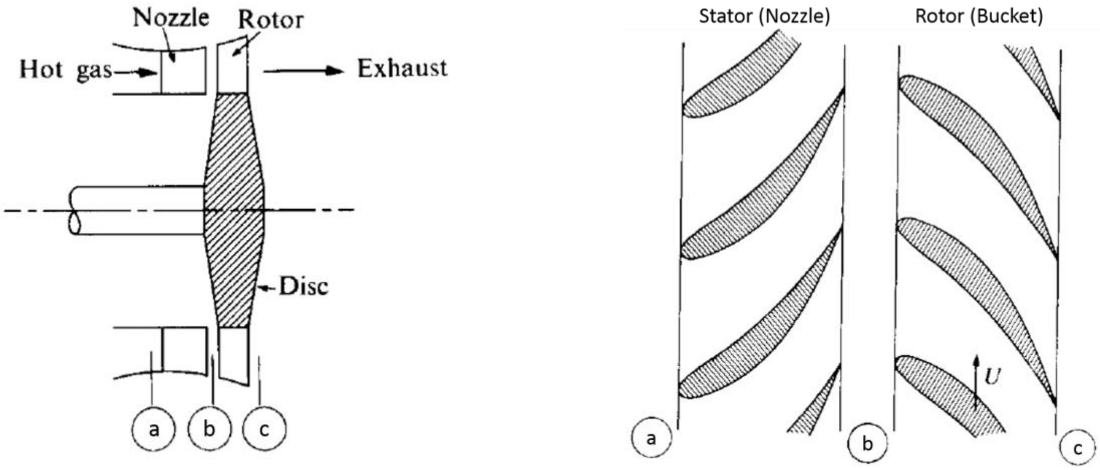

Figure 2. Turbine stage stator and rotor combination, side view and cascade

a) **Background Theory:** Sketch velocity triangles at the turbine stage stations b and c. Label engine-frame and rotor-frame velocity vectors, $\vec{u}$ and $\vec{w}$, and flow angles, α and β.

b) Show that the work extracted for a single stage is

$$\Delta h_0 = h_{0a} - h_{0c} = U^2 \left[\frac{u_x}{U}(\tan\alpha_b + \tan\beta_c) - 1\right]$$

and that this can also be written as

$$1 - \frac{T_{0c}}{T_{0a}} = (\gamma - 1)\frac{U}{\sqrt{\gamma R T_{0a}}}\left[\frac{u_x}{\sqrt{\gamma R T_{0a}}}(\tan\alpha_b + \tan\beta_c) - \frac{U}{\sqrt{\gamma R T_{0a}}}\right].$$

Consult Chapter 8 of Hill & Peterson for a reference (copy of this chapter is available on canvas), but note that we are deriving this for general flow angles leaving the stator and rotor stages – be careful with the derivations in the textbook that are specifically for impulse stages or 50% reaction stages, as these have certain assumptions about the relation between some angles.

c) Write the adiabatic efficiency and pressure ratio for a single turbine stage. How would these extend to a multi-stage turbine?

d) **Code:** Write a function for a generic, multi-stage turbine with n identical stages:

```
[p_ratio, eta_t, T_out, Work_out] = turbfn(cp, gamma, T, eta_st, u_x, U, n, alpha_b, beta_c)
```

To be consistent with our station numbering notation, let a, b, c refer to the single-stage analysis of a stator-rotor pair, and subscripts 4 and 5 refer to upstream and downstream of the entire turbine. The single-stage efficiency is η_{st} and the entire turbine efficiency is η_t. With inputs for air $c_p = 1005\ J/(kg\ K), \gamma = 1.4, T = 1600\ K, \eta_{st} = 0.9, u_x =$

$300\frac{m}{s}, U_{blade} = 200\ m/s, n = 3, \alpha_b = 70°, \beta_c = 60°$, as validation check that the outputs are $p_{05}/p_{04} = 0.1120$, $\eta_t = 0.918$, $T_{05} = 917K$, and $w_T = 6.863 \times 10^5\ J/kg$

Part 2. Turbine Design

The turbine design is usually driven by the compressor design (even though the mechanical components are the other way around!). To summarize HW 4 results and start from a consistent compressor design, use the following values from our design condition, altitude of 30,000 feet and Mach 0.7:

HPC inlet, station 2.5	$T_{0\,2.5} = 311\ K, P_{0\,2.5} = 82.4\ kPa$
Compressor operating	$u_x = 75\ m/s, U = 310\ m/s$
Compressor sizing	$\dot{m}_{core} = 2.825\ kg/s, \bar{r} = 0.115\ m, r_{hub}/r_{tip} = 0.6$

For the turbine, we will take the mean radius as $\bar{r} = 0.08\ m$ and work out a design for the stator and rotor angles. We will be dealing with fluid properties at engine station 4, leaving the combustor and entering the HPT, where $T_{04} = 1600\ K$. Address the following:

e) What mechanical constraint is there between the compressor and turbine? What is the blade velocity U of the turbine?

f) We will design for the inlet of the turbine to be choked. Using the full expression for mass flow in terms of stagnation quantities and the Area-Mach relation $f(M) = {A^*}/{A}$,

$$\dot{m} = \gamma\left(\frac{\gamma+1}{2}\right)^{\frac{-(\gamma+1)}{2(\gamma-1)}} \frac{P_0 A f(M)}{\sqrt{\gamma R T_0}}$$

determine the area A_4 of the turbine inlet. (We have seen that the fuel-air ratio is small; so, you can ignore it when using mass and work balance between the HPC and HPT.) What is the hub-to-tip radius ratio? At this point, do we have any unknowns other than the angles α_b and β_c?

g) Recall compressible flow: what is the relation between velocity and Mach number if we know the **stagnation** temperature rather than **static** temperature? Write an expression for u in terms of M and T_0, and an expression for M in terms of u and T_0.

h) In order to minimize the turbine weight and mechanical complexity, we will try to achieve compressor-turbine work matching with as few stages as possible. With ***one stage*** and $\eta_{st} = 0.95$, construct a contour plot of work extracted with *x*-axis α_b and *y*-axis β_c. What is our design/target work from the HPC? Show this contour on the plot.

i) We see that there is a locus of (α_b, β_c) that would produce the desired amount of work. Is the range of angles reasonable? Why can we achieve higher turning angles in turbines compared to compressors? Is one stage enough for the HPT?

j) Verify that your one-stage turbine with $\alpha_b = 60°, \beta_c = 45°$ is a satisfactory design, and use these blade angles for the remainder of the project.

Part 3. Turbine Map

We have designed our HPT at a certain operating condition and will now construct a turbine map containing lines of constant $N/\sqrt{T_{04}}$ and overall turbine adiabatic efficiency η_t. ***Note that we are using station 4 as a reference since this analysis is for the HPT section between stations 4 and 4.5.***

We will represent the HF-120 HPT as ***one axial stage***. Use the stator and rotor angles $\boldsymbol{\alpha_b = 60}$° and $\boldsymbol{\beta_c = 45}$°. We will use a slightly different expression for stage efficiency, since the turbine efficiency depends on stage loading and less on stage aerodynamics and stall (see Figures 8.23 and 8.24 in Hill & Peterson for a couple sketches). Use the function:

$$\eta_{st}(u_x, U) = 0.95 - 0.03\left(\frac{u_x}{U} - \frac{u_{x,ref}}{U_{ref}}\right)^2 - 0.2\frac{|U - U_{ref}|}{U_{ref}}$$

From our turbine design in Part 1, $u_{x,ref} = 732$ m/s and blade velocity $U_{ref} = 216$ m/s. We will keep the combustion temperature $T_{04} = 1600$ K.

a) First, write your pseudo-code approach to calculate axial flow quantity $f(M_4)$, turbine pressure ratio, and efficiency over a range of N and u_x.
b) Implement your code to construct a turbine map plotting $p_{04}/p_{0\,4.5} = 1/\pi_t$ vs $f(M_4)$, varying N from 15,000 to 30,000 RPM and u_x from 200 to 750 m/s.
c) Since all the lines collapse to a narrow band, it is hard to visualize efficiency contours. See Figures 9.86 and 9.87 in Mattingly – one way to show efficiency contours is in the form of a "consolidated turbine map", with the *x*-axis adjusted to space out the curves. In this map, plot $(p_{04}/p_{0\,4.5} = 1/\pi_t)$ vs $\left(N \times f(M_4)\right)$, and show contours of η_t. (Again, since we are not accounting for supersonic effects near choked flow limits, our contours will not quite be circular "islands", but we will see some efficiency contours that are physically meaningful.)

Part 4. Compressor-Turbine Matching and Operating Line

We will now incorporate all of our compressor-turbine analysis by matching the performance and constructing the operating line! For this analysis, we will consider the turbine inlet choked, which will be an assumption related to the mass flow calculation. We have fixed engine geometry (rotor/stator angles, radii, areas), but the engine operation and flow can change with variables $N, u_{x,comp}, T_{04}$, and other parameters that can be determined from engine analysis $(\pi_c, f(M_{2.5}), W_{comp}, M_4 = 1\ (choked), u_{x,turb}, U_{comp}, U_{turb}, \pi_t, f(M_4), W_{turb})$.

a) Write out (in words) your approach to a scheme for compressor-turbine matching if we specify the rotation speed N. (Mention the variable(s) you will iterate over, or guess and converge, and those that you can calculate.) What is your objective function?
b) Write pseudo-code for your approach, and then implement in Matlab.
c) Vary N from 15,000 to 30,000 RPM and use your iterative scheme to solve for compressor and turbine map variables $(\pi_c, f(M_{2.5}), \pi_t, f(M_4), T_{04})$, making sure you get the root to the right of the surge line.
d) Plot the operating line on the compressor map and the turbine map. With the turbine choked, where is the operating line? Plot it over the consolidated turbine map.