16 pages
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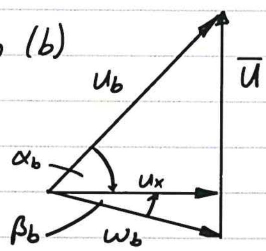
ME 257/357 – Final Project

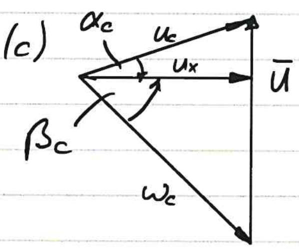
Part 1) Turbine Stator-Rotor Analysis

a) Background theory: sketch velocity triangles at stations (b) and (c)

station (b)



station (c)



($\bar{U}$ = circumferential velocity)

b) Work extracted for a single stage:

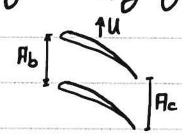
We compute the specific work of a stage from its power as $W_{total} = \frac{P_{total}}{\dot m}$:

We can derive the power as follows:

- Azimuthal force per stage: $F_a = \delta m \left[\frac{\partial u_\theta}{\partial t} + u_r \frac{\partial \theta}{\partial t}\right]$

→ Torque per stage: $T = r \cdot F_a \rightarrow \delta T = \delta m \frac{\partial}{\partial t}[r u_\theta]$

→ integrate & apply Reynolds transport theorem: $T = \underbrace{\frac{\partial}{\partial t}\int_{cv} \rho r u_\theta \, dV}_{\approx 0 \text{ (steady state)}} + \oint_{cs} \rho r u_\theta (\underline{u} \cdot \underline{\hat{n}}) dA$, $A_b = A_c$



→ $T = A \cdot \rho \cdot u \cdot [(r u_\theta)_c - (r u_\theta)_b] = \dot m r \cdot \Delta u_\theta$

→ Compute power: $P_{total} = T_{total} \cdot \Omega = T_{total} \cdot \frac{U}{r} = \dot m \, \Delta u_\theta \cdot U$

→ Compute specific work: $W_{total} = \frac{P_{total}}{\dot m} = \Delta u_\theta \cdot U$

The work extracted for a single stage is therefore: $\boxed{U \cdot \Delta u_\theta = \Delta h_{bc} = \Delta h_{o,total}}$

(There is no work coming from the stator; $\Delta h_{ab} = 0$)

With $u_{\theta,b} = u_x \cdot \tan\alpha_b$ and $u_{\theta,c} = u_x \cdot \tan\alpha_c = U - u_x \cdot \tan\beta_c$ we can substitude Δu_θ by $u_{\theta,b} - u_{\theta,c}$:

$$\Delta h_{ac} = U \cdot \Delta u_\theta = U(u_x \cdot \tan\alpha_b - U + u_x \cdot \tan\beta_c) = U^2\left(\frac{u_x}{U}(\tan\alpha_b + \tan\beta_c) - 1\right)$$

$$\Rightarrow \boxed{\Delta h_o = U^2 \left[\frac{u_x}{U}(\tan\alpha_b + \tan\beta_c) - 1\right]} \qquad (I)$$

We can rewrite this equation by assuming ideal gas:

$$\left.\begin{array}{l} h_{oa} = c_p \cdot (T_{oa} + T_{ref}) \\ h_{oc} = c_p \cdot (T_{oc} + T_{ref}) \end{array}\right\} \; h_{oa} - h_{oc} = c_p\,(T_{oa} - T_{oc})$$

Substituting this in Eq. I we can derive:

$$c_p \cdot (T_{oa} - T_{oc}) = U^2 \left(\frac{u_x}{U}(\tan\alpha_b + \tan\beta_c) - 1\right)$$

with $c_p = R \cdot \frac{\gamma}{\gamma - 1}$ and multiplying by $\frac{1}{c_p T_{oa}}$: $1 - \frac{T_{oc}}{T_{oa}} = \frac{U^2}{c_p T_{oa}} \left(\frac{u_x}{U}(\tan\alpha_b + \tan\beta_c) - 1\right)$

$$\rightarrow 1 - \frac{T_{oc}}{T_{oa}} = \frac{U^2(\gamma-1)}{\gamma R T_{oa}} \left(\frac{u_x}{U}(\tan\alpha_b + \tan\beta_c) - 1\right)$$

$$\rightarrow \boxed{1 - \frac{T_{oc}}{T_{oa}} = \frac{U(\gamma-1)}{\sqrt{\gamma R T_{oa}}} \left(\frac{u_x}{\sqrt{\gamma R T_{oa}}}(\tan\alpha_b + \tan\beta_c) - \frac{U}{\sqrt{\gamma R T_{oa}}}\right)}$$

c)

$$\boxed{\eta_{ad,st} = \frac{h_{oc} - h_{oa}}{h_{ocs} - h_{oa}} = \frac{T_{oc} - T_{oa}}{T_{ocs} - T_{oa}} = \frac{T_{oc}/T_{oa} - 1}{T_{ocs}/T_{oa} - 1}} \rightarrow \frac{T_{ocs}}{T_{oa}} = 1 + \frac{1}{\eta_{ad,st}}\left(\frac{T_{oc}}{T_{oa}} - 1\right)$$

$$\boxed{\Pi_{c,st} = \frac{P_{oc}}{P_{oa}} = \left(\frac{T_{ocs}}{T_{oa}}\right)^{\gamma/\gamma-1} = \left[1 + \frac{1}{\eta_{ad,st}}\left(\frac{T_{oc}}{T_{oa}} - 1\right)\right]^{\gamma/\gamma-1}}$$

For a multistage turbine, the overall adiabatic efficiency and pressure ratio becomes:

$$\eta_{ad} = \frac{\frac{T_{os}}{T_{o4}} - 1}{\frac{T_{oss}}{T_{o4}} - 1} \approx \frac{\left(U\,\Delta u_\theta / c_p \cdot T_{o4} + 1\right)^n - 1}{\left(\frac{U\,\Delta u_\theta}{c_p T_{o4}} \cdot \frac{1}{\eta_{st}} + 1\right)^n - 1} \qquad \Pi_c = \frac{P_{os}}{P_{o4}} = \left(\frac{T_{oss}}{T_{o4}}\right)^{\gamma/\gamma-1} = \left[1 + \frac{1}{\eta_t}\left(\frac{T_{os}}{T_{o4}} - 1\right)\right]^{\gamma/\gamma-1}$$

(for n stages)

d) Generic, multi-stage turbine with n identical stages:

$[\Pi_c, \eta_t, T_{out}, W_{out}] = \text{turbfn}(c_p, \gamma, T, \eta_{st}, u_x, U, n, \alpha_b, \beta_c)$

$T(1) = T$; $\Pi_c(1) = 1$;

for $i = 1:n$

$$T(i+1) = T(i) \cdot \left[1 - \frac{U^2}{c_p \cdot T(i)}\left[\frac{u_x}{U}(\tan\alpha_b + \tan\beta_c) - 1\right]\right];$$

$$\Pi_c(i+1) = \Pi_c(i) \cdot \left[1 + 1/\eta_{st}\left(\frac{T(i+1)}{T(i)} - 1\right)\right]^{\gamma/\gamma-1};$$

end

$\Pi_c = \Pi_c(n+1)$; $T_{out} = T(n+1)$;

$\eta_t = \left(T(n+1)/T(1) - 1\right) / \left(\Pi_c^{\gamma-1/\gamma} - 1\right)$;

$W_{out} = [U \cdot (u_x(\tan\alpha_b + \tan\beta_c) - U)] \cdot n$;

+ see matlab function "turbfn.m" → results were validated.

Part2) Turbine Design

Turbine parameters: $r_{mean} = 0.08m$
$T_{04} = 1600K$

a) The mechanical constraints between the compressor and the turbine are the rotational speed: $\boxed{n_{comp} \overset{!}{=} n_{turb}}$ (if no gear is involved) and the work balance: $\boxed{c_p \cdot (T_{05} - T_{04.5}) = c_p \cdot (T_{03} - T_{02.5})}$
(ignoring mass of fuel $\dot{m}_f \ll \dot{m}_{ac}$)

Additionally, also the mass flow $\boxed{\dot{m} \text{ has to be constant}}$ for the compressor and the turbine if we neglect the mass flow of the fuel ($\dot{m}_f \ll \dot{m}_{ac}$).

$\rightarrow U = n \cdot \frac{2\pi r_m}{60} \left[\frac{m}{s}\right]$ with $n = U_c \cdot \frac{60}{2\pi r_{c,m}}$ [rpm]

Thus, from the compressor we obtain $n = 25742$ rpm, $\dot{m}_{ac} = 2.825\ {kg}/s$

For the turbine this leads to a blade velocity of $\underline{U_t = n \cdot \frac{2\pi r_{m,t}}{60} = 215.65 \frac{m}{s}}$.

b) Choke condition at turbine inlet:

given: $f(M) = \frac{A^*}{A}$ (I) $\quad \dot{m} = \gamma \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \cdot \frac{p_0 A f(M)}{\sqrt{\gamma \cdot R \cdot T_0}}$ (II) with $p_0, T_0 \hat{=}$ total values

$\rightarrow$ For a choked turbine inlet we have: $A_{inlet} = A_4 = A^*$

Thus, we can determine A^* by substituting (I) in (II) and transforming it for A^*:

$$\boxed{A_4 = A^* = \frac{\dot{m}}{\gamma} \left(\frac{\gamma+1}{2}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \cdot \frac{\sqrt{\gamma R T_0}}{p_0}}$$

$\rightarrow$ For $T_{04} = 1600K$ and $p_{04} = p_{03} = 4.3115 \times 10^5$ Pa (from compressor with 4 stages as in HW4):
$R = 8314/28.85 \frac{J}{kg \cdot K} = 288.18 \frac{J}{kg \cdot K}$, $\dot{m}_{ac} = 2.825\ {kg}/s$ (from compressor sizing)

$$A_4 = \frac{2.825\ {kg}/s}{1.4} \left(\frac{2.4}{2}\right)^{2.4/2(0.4)} \cdot \frac{\sqrt{1.4 \cdot 288.18\ {J}/{kgK} \cdot 1600K}}{4.3115 \times 10^5 Pa} = \underline{0.006498 m^2}$$

$\rightarrow$ We can now compute the hub-to-tip ratio as follows:

$$A_4 = \pi \cdot (r_{tip}^2 - r_{hub}^2) = \pi \left(\cancel{r_m^2} + 2r_m \Delta r + \cancel{\Delta r^2} - \cancel{r_m^2} + 2r_m \Delta r - \cancel{\Delta r^2}\right) = \pi \cdot 4 r_m \Delta r$$

with $r_m = r_{tip} - \Delta r = r_{hub} + \Delta r$

$\rightarrow \Delta r = \frac{A_4}{4\pi r_m} = 0.0065 m \qquad \rightarrow \underline{\frac{r_{hub}}{r_{tip}} = \frac{r_m - \Delta r}{r_m + \Delta r} = 0.85}$

The other unknowns (other than α_b and β_c) are the turbine stage efficiency η_{st} and (only not computed yet) the blade tip Mach number.

c) The relation between stagnation temperature T_t, Mach number and static temperature T is:

$$\frac{T_t}{T} = \left(1+\frac{\gamma-1}{2}M^2\right) \quad \text{for compressible flow} \quad \rightarrow T = \frac{T_t}{\left(1+\frac{\gamma-1}{2}M^2\right)}$$

Now, the relation between velocity, Mach number and static temperature is with speed of sound $a = \sqrt{\gamma R T}$:

$$M = \frac{u}{a} = \frac{u}{\sqrt{\gamma R T}} = \frac{u}{\sqrt{\gamma R T_t}} \cdot \left(1+\frac{\gamma-1}{2}M^2\right)^{1/2} \quad \text{with } T_t = T_0$$

$$\rightarrow M^2 = \frac{u^2}{\gamma R T_t}\left(1+\frac{\gamma-1}{2}M^2\right) \quad \rightarrow \left(1-\frac{\gamma-1}{2}\frac{u^2}{\gamma R T_t}\right)M^2 = \frac{u^2}{\gamma R T_t}$$

$$\boxed{\begin{aligned} &\Rightarrow u = M\sqrt{\gamma R T_0}\left(1+\frac{\gamma-1}{2}M^2\right)^{-1/2} = u(M, T_0) \\ &\Rightarrow M = \left(\frac{u^2}{\gamma R T_0}\right)^{1/2} \cdot \left(1-\frac{\gamma-1}{2}\frac{u^2}{\gamma R T_0}\right)^{-1/2} = M(u, T_0) \end{aligned}}$$

d) given:
- one stage $\rightarrow n = 1$
- efficiency of stage $\rightarrow \eta_{st} = 0.95$

Solution: extracted work from turbine $W_T = c_p(T_{05} - T_{04}) = u\Delta u_\theta$
with $\Delta u_\theta = (u_x(\tan\alpha_b + \tan\beta_c) - u)$

[from inlet A_4 we can derive $u_x = \dot{m}_{ac}/(A_4 \cdot \rho_{04}) = \dot{m}_{ac}/\left(A_4 \cdot \frac{P_{04}}{R T_{04}}\right) = 464.96 \frac{m}{s}$
(Not true because ideal gas assumption (and thus ρ_{04}) not valid in transsonic region)]

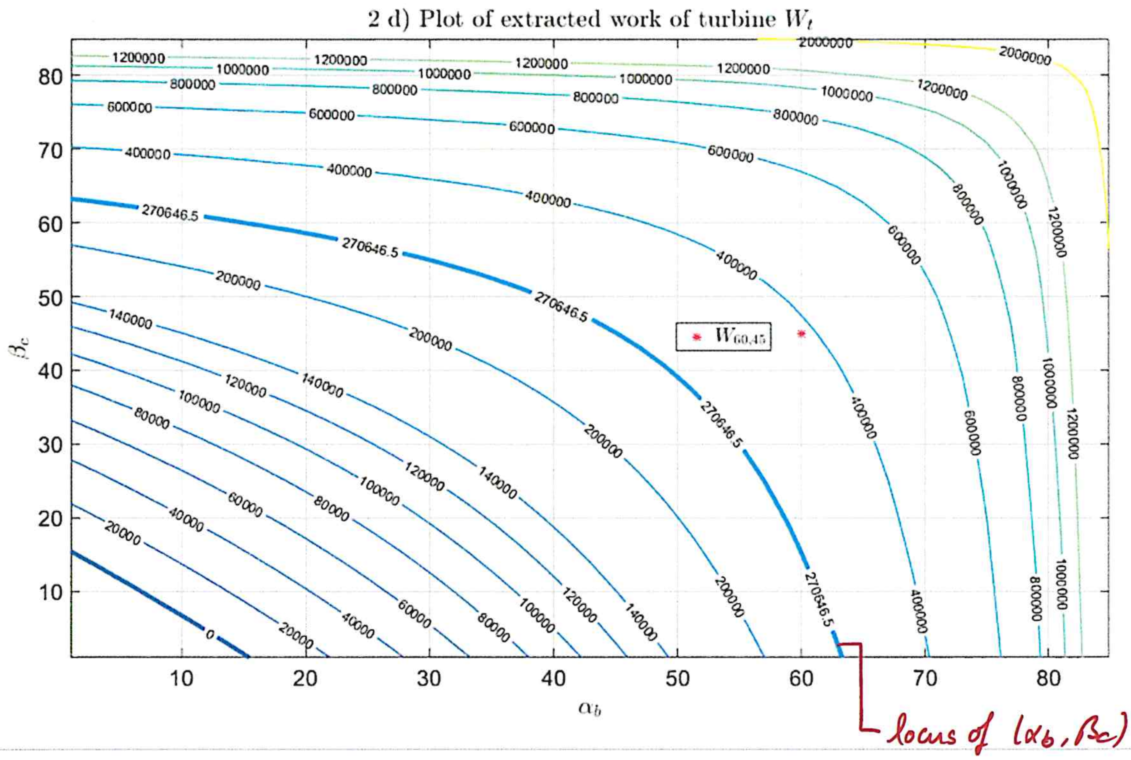
$\rightarrow u_x = \sqrt{\gamma R T_0}\left(1+\frac{\gamma-1}{2}\right)^{-1/2} = 733\ m/s$ (using velocity relation from part 2c)

$\rightarrow W_T = u^2\left(\frac{u_x}{u}(\tan\alpha_b + \tan\beta_c) - 1\right)$

$\rightarrow$ design / target work: $W_c = c_p \cdot (T_{03} - T_{02.5}) = 2.7 \times 10^5\ J/kg$

with $T_{03} = 580.21\ K$ (from compressor)

$\rightarrow$ Plot on next page.



The bold blue line in the plot shows the design/target work from the HPC.

e) The range of angles that would produce the desired amount of work is the bold blue line itself.

For ranges of angles from $\alpha_b \in [30°, 80°]$ and $\beta_c \in [30°, 80°]$ we can find optimal blade angles for the required work from the compressor, so one stage seems reasonable for the HPT.

We can achieve higher turning angles in turbines compared to compressors because of the higher temperature and thus lower density of the fluid at the turbine inlet.

As we can see from the plot above, one stage seems to be enough for the HPC as the angles are still in a reasonable range.

f) As we can verify with the plot, the turbine work for the given mass flow and angles of $\alpha_b = 60°$, $\beta_c = 45°$ (red star) is larger than the required compressor work (blue bold line). Thus, this choice of angles seems to be reasonable and we don't even need to operate at/close to the turbine design point to achieve compressor-turbine matching.

Part 3) Turbine Map

given: station 4 ≙ reference, look for HPT between stations 4 and 4.5

1 axial stage

$\alpha_b = 60°$, $\beta_c = 45°$

$$\eta_{st} = 0.95 - 0.03\left(\frac{u_x}{U} - \frac{u_{x,ref}}{U_{ref}}\right)^2 - 0.2\left(\frac{U - U_{ref}}{U_{ref}}\right)$$

with $U_{ref} = 216\ m/s$, $u_{x,ref} = 732\ \frac{m}{s}$, $T_{04} = 1600K$

a) Pseudo-code:

$T_0 = T_{04}$; $p_0 = p_{04}$; $\gamma, R = \text{const.}$; $r_m = r_{m,t}$; U_{ref}, u_{xref} given;

for $iN = 1 : \text{length}(N)$

for $iU_x = 1 : \text{length}(U_x)$

$$M_4(iN, iU_x) = \left(\frac{U_x(iU_x)^2}{\gamma R T_0}\right)^{1/2} \cdot \left(1 - \frac{\gamma-1}{2}\frac{U_x(iU_x)^2}{\gamma R T_0}\right)^{-1/2}$$

$$\boxed{f_{M_4}(iN, iU_x) = \frac{A^*}{A} = M_4\left(\frac{\gamma+1}{2}\right)^{\left(\frac{\gamma+1}{2(\gamma-1)}\right)}\left(1 + \frac{\gamma-1}{2}M_4^2\right)^{-\left(\frac{\gamma+1}{2(\gamma-1)}\right)}}$$

axial flow quantity $f(M_4)$

pressure ratio

$$U = N(iN) \cdot \frac{2\pi r_m}{60}$$

$$\eta_{st}(iN, iU_x) = 0.95 - 0.03\left(\frac{u_x}{U} - \frac{u_{x,ref}}{U_{ref}}\right)^2 - 0.2\left(\frac{U - U_{ref}}{U_{ref}}\right)$$

$$\Delta U_\theta = u_x(\tan\alpha_b + \tan\beta_c) - U$$

$$T_{04.5} = T_{04} - \frac{U\Delta U_\theta}{c_p} \qquad \left[c_p\left(T_{04.5} - T_{04}\right) = -U\Delta U_\theta\right]$$

$$\boxed{\frac{p_{04}}{p_{04.5}} = \left(1 - \frac{U\Delta U_\theta}{c_p \cdot T_{04}}\frac{1}{\eta_{st}}\right)^{\frac{-\gamma}{\gamma-1}}}$$

$$\left[-U\Delta U_\theta = c_p\left(T_{04.5s} - T_{04}\right)\cdot\eta_{st} \qquad \frac{T_{04.5s}}{T_{04}} = 1 - \frac{U\Delta U_\theta}{c_p T_{04}}\frac{1}{\eta_{st}} \right.$$
$$\left.\left(\frac{p_{04.5s}}{p_{04}}\right) = \frac{p_{04.5}}{p_{04}} = \left(\frac{T_{04.5s}}{T_{04}}\right)^{\gamma/\gamma-1}\right]$$

$\eta(iN, iU_x) = \eta_{st}(iN, iU_x)$ → only one stage

end

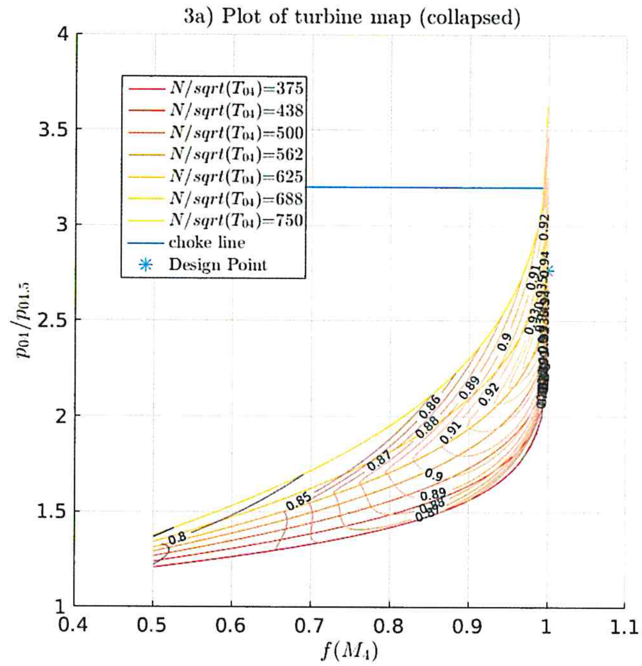
end

plot $\left(f_{M_4}(iN,:), \frac{p_{04}}{p_{04.5}}(iN,:)\right)$ → label $\frac{N(iN)}{\sqrt{T_{04}}}$ (constant $\frac{N}{\sqrt{T_{04}}}$)

contour $\left(f_{M_4}, \frac{p_{04}}{p_{04.5}}, \eta\right)$ (efficiency)

b) plot of turbine map: next page

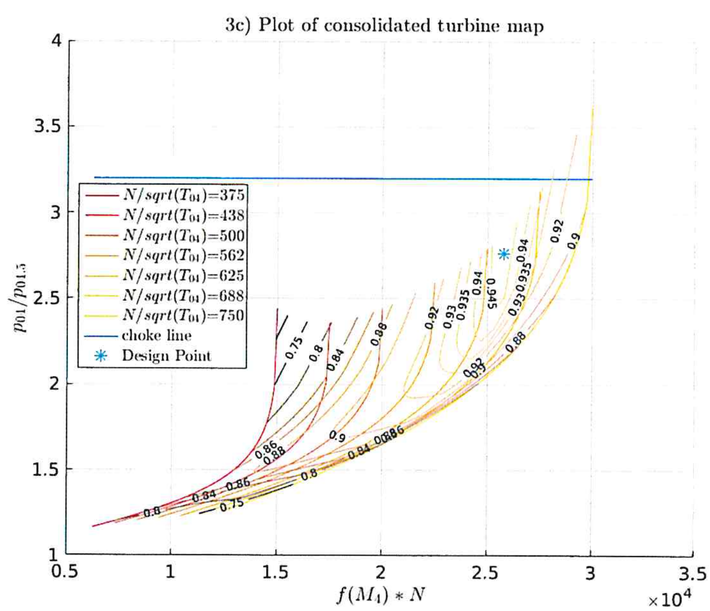
b) turbine map:



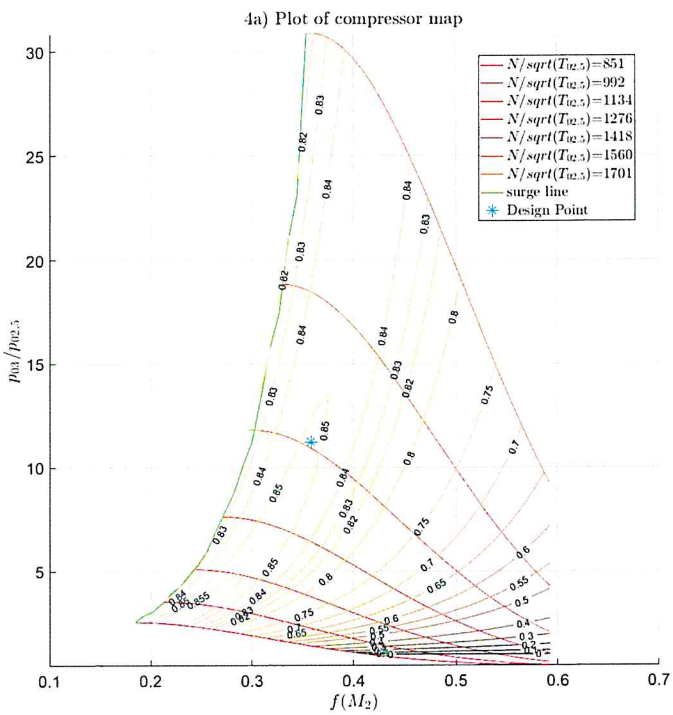
We can observe that all lines of constant rotational speed merge at the right side for a choked flow ($M_4 = 1$).

c) Thus, we can plot a consolidated turbine map with an x-axis of $f(M_4) \cdot N$:

Here, we can distinguish between different efficiencies and observe that the efficiency of the design point is about 0.95.



Before we started with part 4, we adjusted our solution for the compressor map from homework 4 to match the given solution from homework 4:



For this map, we had to assume that for every compressor stage we compute the increase in pressure only depending on $T_{02.5}$:

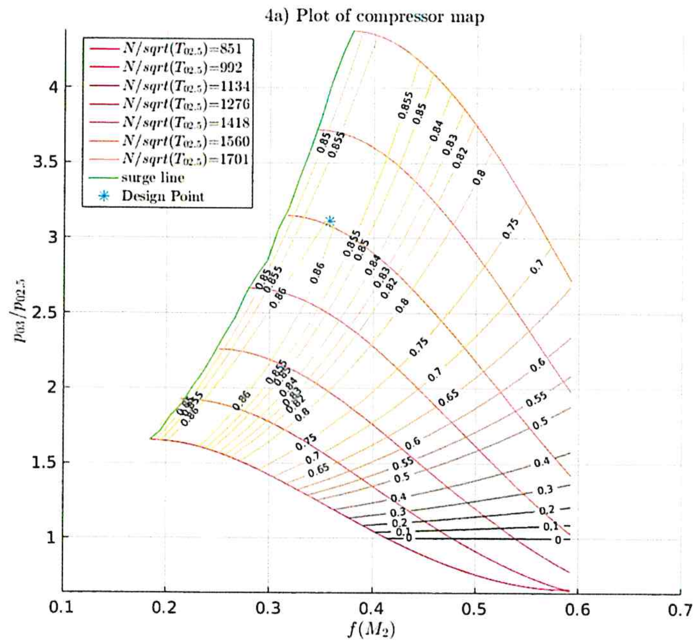
$$p_{03}(i+1) = p_{03}(i) \cdot \left(1 + \eta_{st} \cdot \left(\frac{u \Delta u_\theta}{c_p \cdot T}\right)_{02.5}^{\gamma/\gamma-1}\right)$$

This is a strong assumption as u_x and thus Δu_θ as well as T change for every stage.
The corrected version would be:

$$p_{03}(i+1) = p_{03}(i) \cdot \left(1 + \eta_{st} \left(\frac{u \Delta u_\theta(i)}{c_p \cdot T(i)}\right)^{\gamma/\gamma-1}\right)$$

Thus, for the following analysis, we recomputed $T(i)$ for every stage and adapted $\frac{u \Delta u_\theta(i)}{c_p}$ by multiplying it with a factor 0.8 (assumption) as for a constant area and mass flow but an increasing pressure and density u_{xc} and thus Δu_θ will be reduced.

The new compressor map is shown on the next page: →



We can see, that the pressure ratio per stage is also more reasonnable (smaller) in the adjusted compressor map.

We will keep this adjusted compressor map for the following analysis.

Part 4: Approach to compressor - turbine matching:

a) For the compressor-turbine matching, we will compute the required $u_{x,comp}$, T_{04} and the other parameters for every fixed N.

We match T_{04} and $u_{x,comb}$ by requiring I) $\dot m_t \approx \dot m_c$ (to compute $u_{x,comb}$ from $u_{x,turb}$ at $M_4 = 1$)

II) $W_t \approx W_c$

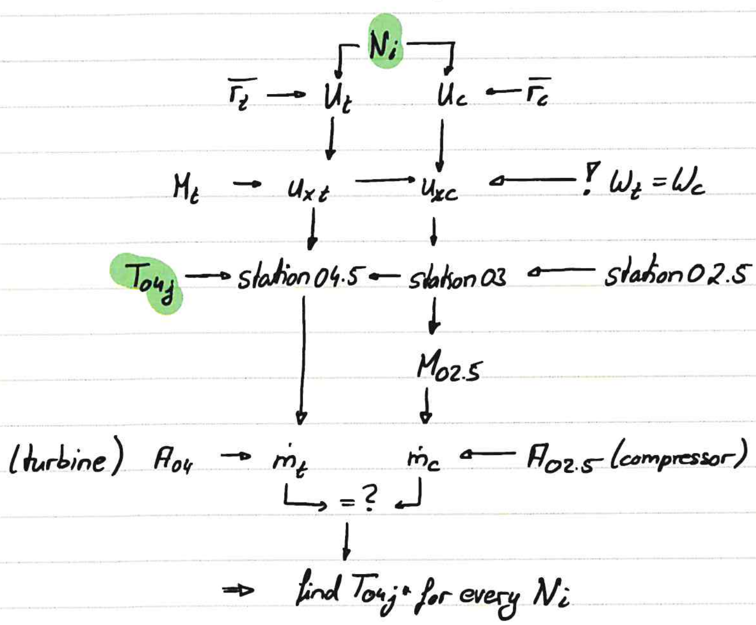
and III) $N_t = N_c$

Thus, for every rotational speed N_i we will iterate over a range of temperatures $T_{04,j} \in [1000, 2500]$ and compute the corresponding $\dot m_{t,ij}$ and $\dot m_{c,ij}$.

Our objective function will then be: $\boxed{\min_j |\dot m_{t,ij} - \dot m_{c,ij}|.} \rightarrow j^*$

This will give us $\dot m_{t i,j^*}$ and $\dot m_{c,ij^*}$ and the corresponding values of the compressor and turbine map: π_{c,ij^*}, $f(M_{2.5})_{ij^*}$, $\pi_{t\,ij^*}$, $f(M_4)_{ij^*}$ and T_{04ij^*}

Thus, we will find the operating conditions for all N_i as j^*.



b) We computed $\dot m_t$ and $\dot m_c$ for every N_i and T_{04j} as follows: (given values are underlined)

N_i

T_i

1) circumpherential speed: $U_t = \frac{N}{60} \cdot 2\pi \underline{r_t}$; $U_c = \frac{N}{60} \, 2\pi \underline{r_c}$

2) for turbine: $u_{xt} = \underline{M_t}\sqrt{\gamma R \underline{T_{04}}}\left(1 + \frac{\gamma - 1}{2}\underline{M_t^2}\right)^{-1/2}$ with $M_t = 1$

3) match compressor - turbine work: $W_t = U_t \Delta u_{\theta,t} \stackrel{!}{=} -W_c = -U_c \Delta u_{\theta,c} \cdot n_{stages,c}$

$\rightarrow U_t\,(u_{x,t}(\tan\alpha_b + \tan\beta_c) - U_t) = U_c\,(U_c - u_{xc}(\tan\alpha_a + \tan\beta_b))\,n_{stages,c}$

$\rightarrow u_{x,c} = \left[\frac{U_t}{U_c \cdot n_{stc}}\left(u_{x,t}\,(\tan\alpha_b + \tan\beta_c) - U_t\right) - U_c\right] \cdot \frac{1}{\tan\alpha_a + \tan\beta_b}$

4) Compute pressure p_{03}: $p_{03} = f(u_{x,c}, U_c, T_{02.5}, p_{02.5}, n_{stages}, \eta_{stc}) \rightarrow \pi_c = \frac{p_{03}}{p_{025}}$

5) Compute pressure $p_{04.5}$: $p_{04.5} = f(p_{04} \approx p_{03}, u_{x,t}, U_t, T_{04}, n_{stages}, \eta_{stt}) \rightarrow \frac{1}{\pi_t} = \frac{p_{04}}{p_{045}}$

6) Compute Mach number $M_{02.5} = \left(\frac{u_{xc}^2}{\gamma R T_{02.5}}\right)^{1/2} \cdot \left(1 - \frac{\gamma-1}{2}\frac{u_{xc}^2}{\gamma R T_{025}}\right)^{-1/2}$

7) Compute mass flow for both stations from $A_{2.5}$ and A_4 and compare:

$$\dot m_t = \frac{A_t\, p_{04}}{\sqrt{T_{04}}}\sqrt{\frac{\gamma}{R}}\left(\frac{\gamma+1}{2}\right)^{-\frac{\gamma+1}{2(\gamma-1)}} \qquad \text{with } A_t = \pi \cdot r_{tip,t}^2\left(1 - \left(\frac{r_{hub}}{r_{tip}}\right)^2\right)$$

$$\dot m_c = \frac{A_c\, p_{02.5}}{\sqrt{T_{02.5}}}\sqrt{\frac{\gamma}{R}} \cdot M_c \cdot \left(1 + \frac{\gamma-1}{2}M_c^2\right)^{-\frac{\gamma+1}{2(\gamma-1)}} \qquad r_{tip,t} = 2\bar r_t \,/\left(1 + \frac{r_{hub}}{r_{tip}}\right)$$

end T_i

8) compute absolut error as $e(j) = |\dot m_t - \dot m_c|$ (for every N_i)

9) For j^* with $e(j^*) = \min(e(j))$ we can compute the x-values for the operating line as
$$f(M_{02.5}^*) = \frac{1}{M_{025}^*}\cdot\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}\left(1 + \frac{\gamma-1}{2}M_{025}^{*2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$
$$f(M_4) = 1$$

end N_i

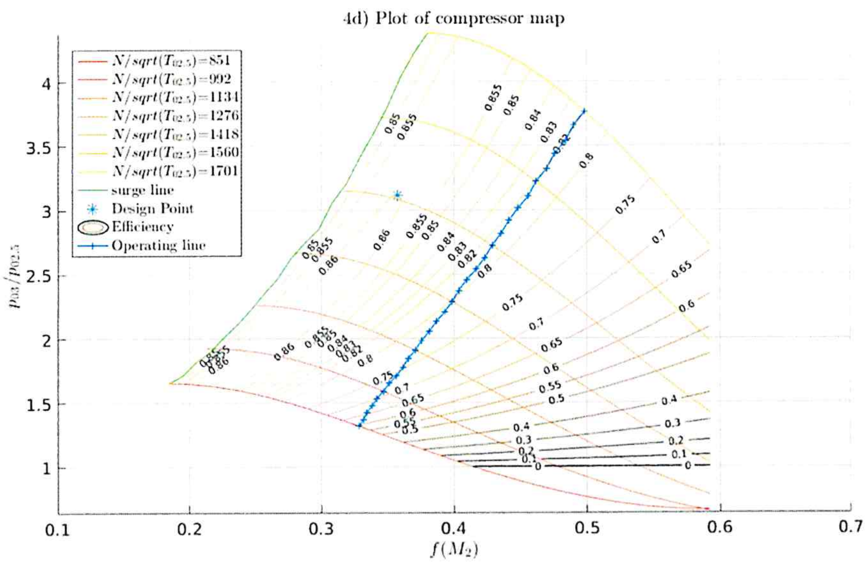
c)

	$N(i)$	$\frac{p_{03}}{p_{02.5}} = \pi_c$	$f(M_{2.5})$	$\frac{p_{04.5}}{p_{04}} = \pi_t$	$f(M_4)$	$T_{04}(j^*)$	
(1)	15'000 rpm	1.3182	0.3281	0.4238	1	1'095	
(11)	20'000 rpm	1.9107	0.3709	0.4350	1	1790	
(21)	25'000 rpm	2.7248	0.4288	0.4493	1	2'705	not feasible. (too hot for turbine)
(31)	30'000 rpm	3.7654	0.4982	0.4361	1	3'860	

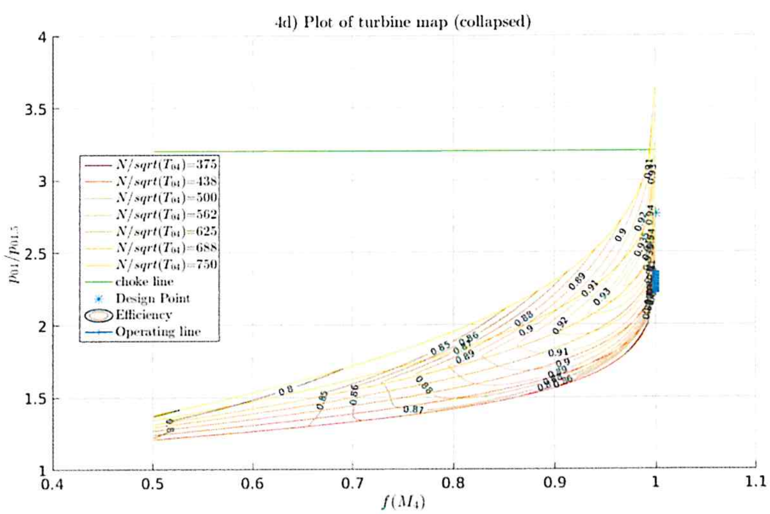
⇒ We can observe that $f(M_{2.5})$ is larger than the surge value of every rotational velocity N (~ 0.32).

Thus, our root will be on the right side of the surge line. (as shown in the plots)

Plotting the operating line on the compressor map, we can achieve the following result: (blue +-line $\hat{=}$ operating line)

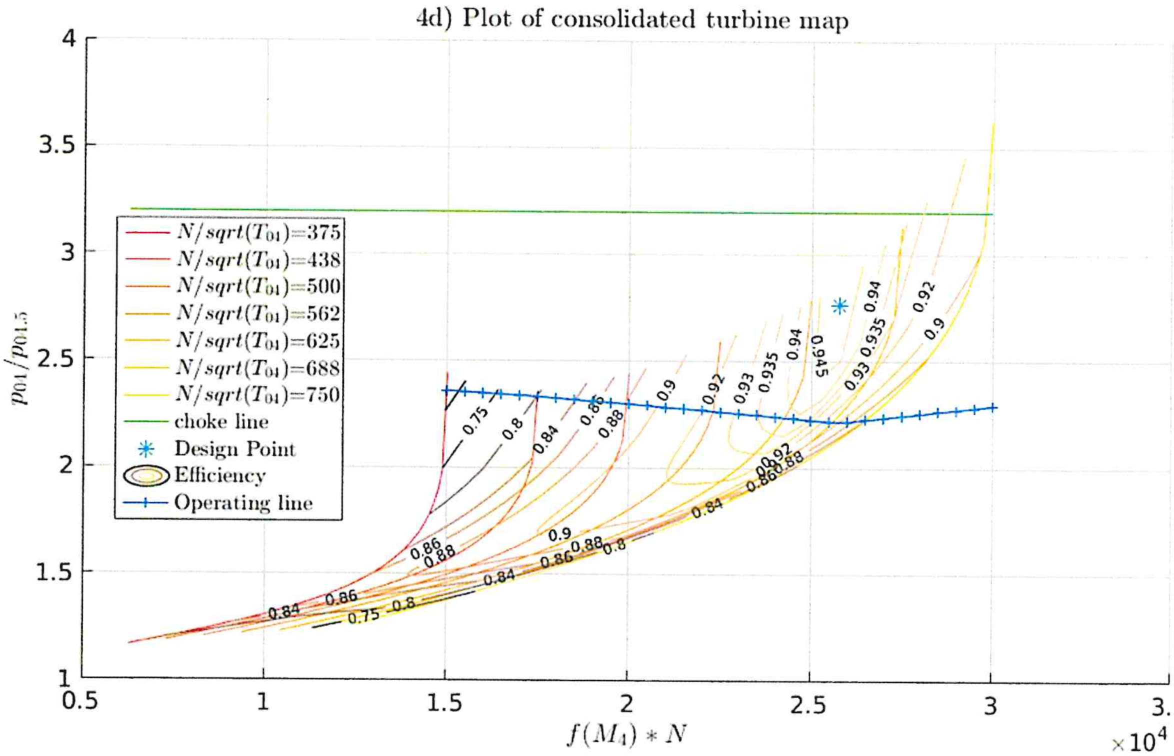


For the turbine, we observe:



We can see in the turbine map that with the turbine choked, the operating line is a vertical line on the right side of the plot at $f(M_4) = 1$.

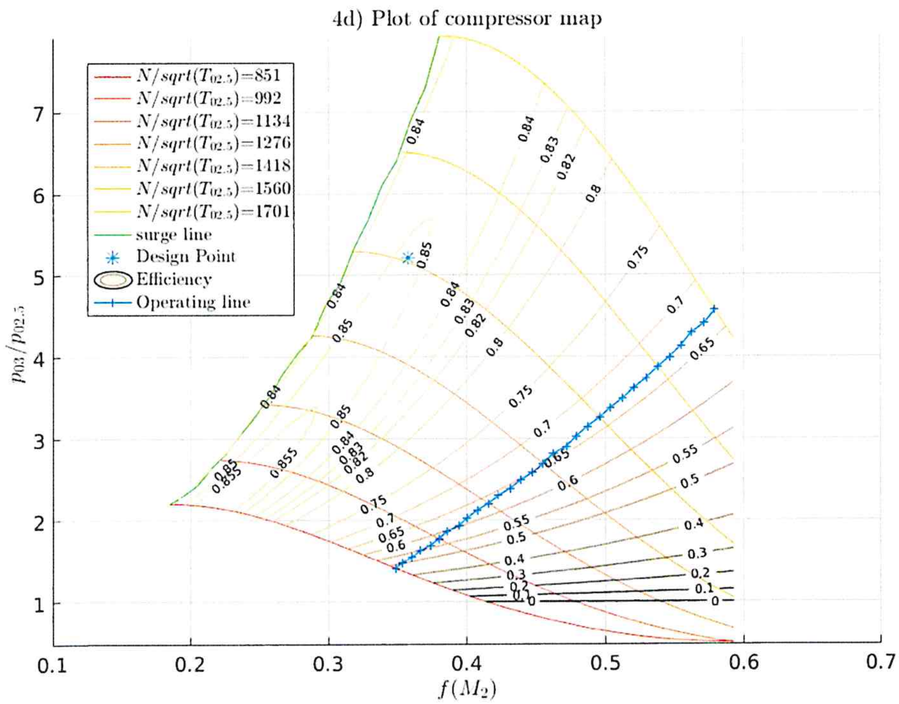
Thus, to evaluate the efficiencies for the operating line, we need to plot it over the consolidated turbine map:



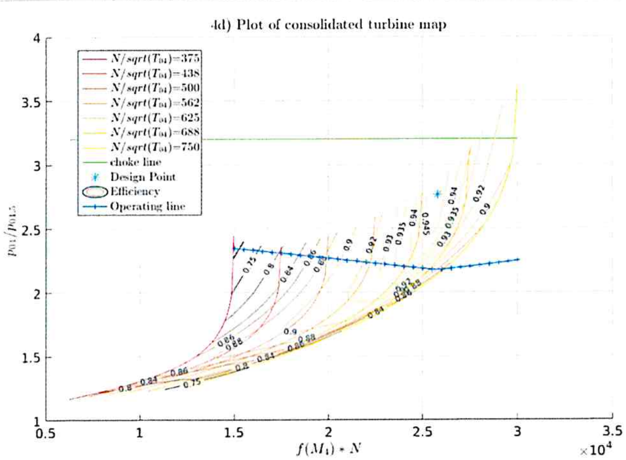
We can see, that only for smaller mass flows, the operating line is within a reasonable range for $T_{04} = 1600K$. As we mentionned before, above $N = 20'000$, the temperature T_{04} to match compressor and turbine becomes too high.

We can also observe, that the operating line leads to efficiencies much lower than the design point and thus, the construction (area and blade angles) can still be optimized.

Finally, we also tested our model without the adjustement of u_x at the compressor stage (assuming $\frac{u \Delta u \theta}{c_p}$ = const. ξ) and got the following compressor map:



We can see that the operating line moves towards decreasing efficiencies.
The turbine map stays similar:



part 1 d)

```
function [ratio_c, e_t, T_out, W_out] =
 turbfn(cp,gamma,T_in,e_st,u_x,U,n,alpha_b,beta_c)
T1(1)=T_in;
ratio_c1(1)=1;

for it=1:n
    T1(it+1) = T1(it)* (1-U^2 /(cp*T1(it)) *(u_x/U
 *(tan(alpha_b)+tan(beta_c)) -1) );
    ratio_c1(it+1) = ratio_c1(it) *(1+1/e_st *(T1(it+1)/
T1(it)-1) )^(gamma/(gamma-1));
end

ratio_c=ratio_c1(end);
T_out=T1(end);
e_t=(T_out/T_in -1) / (ratio_c^((gamma-1)/gamma) -1);
W_out=(U *(u_x*(tan(alpha_b)+tan(beta_c))-U) )*n;

end
```

Not enough input arguments.

Error in turbfn (line 2)
T1(1)=T_in;

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subfunction
of driver

```
function [x_plot, y_plot, e_t, T_out, W_out, s4] =
 TurbinePoint2(u_x,U,fluid, s4, turb, engine)
cp = fluid.cp;
gamma = fluid.g;
T_in = engine.s4_T;
n = turb.nStages;
alpha_b = turb.b_alpha;
beta_c = turb.c_beta;
e_st = 0.95 -0.03 *(u_x/U -turb.ux_ref/turb.U_ref)^2 -0.2 *(abs(U-
turb.U_ref) /turb.U_ref);

T1(1)=T_in;
ratio_pIt(1)=1;

for it=1:n
    T1(it+1) = T1(it)* (1-U^2 /(cp*T1(it)) *(u_x/U
 *(tan(alpha_b)+tan(beta_c)) -1) );
    ratio_pIt(it+1) = ratio_pIt(it) *(1+1/e_st *(T1(it+1)/
T1(it)-1) )^(gamma/(gamma-1));
end

ratio_p=ratio_pIt(end);
T_out=T1(end);
e_t=(T_out/T_in -1) / (ratio_p^((gamma-1)/gamma) -1);
%e_t=e_st;
W_out=(U *(u_x*(tan(alpha_b)+tan(beta_c))-U) )*n;

% Compute mass flow, flow is choked (somewhere ~station 2.5)
%!!! :-/ assume all mass goes through compressor (neglect bypass)
s4.T_static = s4.T -u_x^2/(2*fluid.cp); % static temperature
s4.M = u_x /( sqrt(fluid.g*fluid.R*s4.T_static) ); % Mach number
ratio_A = 1/s4.M *(2 /(fluid.g+1) *(1 +(fluid.g-1)/2
 *s4.M^2) )^((fluid.g+1)/(2*(fluid.g-1))); % A/A_star
s4.m_choked = s4.p/sqrt(s4.T) *sqrt(fluid.g/fluid.R) *((fluid.g
+1)/2)^(-(fluid.g+1)/(2*(fluid.g-1)));
s4.m_a = s4.A *s4.m_choked /ratio_A;

x_plot=1 /ratio_A;
y_plot=1/ratio_p;
end
```

Not enough input arguments.

Error in TurbinePoint2 (line 2)
cp = fluid.cp;

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Subfunction of driver

```
function [legendCell,Fig_Turb] = Plot_TurbineMap(fluid, e, s4, turb,
 engine)

% This function is intended to generate and then plot the compressor
 map
ux=turb.ux;
N=turb.N;

% Number of points for interpolation
np_x=75;
np_surge = 20;

for iN=1:length(N)
    U=N(iN) /60 *(2*pi*s4.r_mean);
    % Compute points on map
    for iUx=1:length(ux)
        % From compressor
        %
 [x_all(iUx),y_all(iUx),e.compressor(iN,iUx)]=TurbinePoint(ux(iUx), U,
 fluid, e, s25, comp);
        % From prokect part 1
        [x_all(iUx),y_all(iUx),e.turbine(iN,iUx), ~,
 ~]=TurbinePoint2(ux(iUx), U, fluid, s4, turb, engine);
    end

    % Get parameters for surge line (= maximum pressure ratio)
    % [~, index_maxy] = max(y_all);
    %[~,index_maxy] = min(abs(e.turbine(iN,:)-0.85));
    [~,index_maxy] = min(abs(x_all-0.5));
    surge_x(iN) = x_all(index_maxy(1));
    surge_y(iN) = y_all(index_maxy(1));

    % Cut solutions at surge line
    c_map_x=x_all(index_maxy(1): end);
    c_map_y=y_all(index_maxy(1): end);
    c_map_eta=e.turbine(iN,index_maxy(1): end);

    % Interpolate lines of constant N and eta
    if length(c_map_x) > 1
        x_plot(iN,:) = linspace(c_map_x(1), c_map_x(end), np_x); %
 x_vector for plot
        y_plot(iN,:) = interp1( c_map_x, c_map_y, x_plot(iN,:)); %
 plot of x-matrix
        eta_matrix(iN,:) = interp1( c_map_x, c_map_eta,
 x_plot(iN,:) );
    else
        x_plot(iN,:) = repmat(c_map_x, 1, np_x);
        y_plot(iN,:) = repmat(c_map_y, 1, np_x);
        eta_matrix(iN,:) = repmat(c_map_eta, 1, np_x);
    end
end
```



```
% % Interpolate surge line
% surge_x_ip = linspace(surge_x(1), surge_x(end), np_surge); %
 x_vector for plot
%
% surge_x_plot(1)=surge_x(1);
% surge_y_plot(1)=surge_y(1);
% iSurge=1;
% for i=2:length(surge_x)-1
%     if surge_x_plot(iSurge)~=surge_x(i)
%         iSurge=iSurge+1;
%         surge_x_plot(iSurge)=surge_x(i);
%         surge_y_plot(iSurge)=surge_y(i);
%
%     end
% end
% surge_x_plot(end)=surge_x(end);
% surge_y_plot(end)=surge_y(end);
% surge_y_ip = interp1(surge_x_plot, surge_y_plot, surge_x_ip); % plot
 of x-matrix
surge_x_ip = surge_x;
surge_y_ip = surge_y;

% Get values for choke line
CL_x=x_plot(:,end);
CL_y=y_plot(:,end);
```

Not enough input arguments.

Error in Plot_TurbineMap (line 3)
ux=turb.ux;

Plot solution

```
Fig_Turb=figure(21);
set(Fig_Turb,'units','normalized','position',[.5 .1 .4 .7])

% Plot lines of constant rotational speed N
colormap autumn
cmap = colormap;
hold on;
n_plot=1;
for iN=1:length(N)
    if floor(mod(iN,5))==1
        h1=plot(x_plot(iN,:), y_plot(iN,:),'color',cmap(n_plot*7,:));
        legendentry(n_plot)=N(iN)/sqrt(s4.T);
        n_plot=n_plot+1;
    end
end
% Get legend entries
legendCell = cellstr(num2str(legendentry', '$N/sqrt(T_{04})$=%.0f'));
```

```
% Plot specific points and choke and surge line
plot([x_plot(1,1) x_plot(end,end)],[3.2 3.2],'g'); % choke line
%plot(surge_x_ip,surge_y_ip,'g'); % surge line
[comp.design_x,comp.design_y,~, ~, ~]=TurbinePoint2(turb.ux_ref,
 turb.U_ref, fluid, s4, turb, engine);
plot(comp.design_x,comp.design_y,'*', 'MarkerSize', 10); % Design
 point A
% Get legend entries
legendCell(end+1) = cellstr('choke line');
%legendCell(end+1) = cellstr('surge line');
legendCell(end+1) = cellstr('Design Point');
legendCell(end+1) = cellstr('Efficiency');
legendCell(end+1) = cellstr('Operating line');


% Plot efficiency
colormap('copper')
eta_levels = [ (0:0.1:0.5) , (0.5:0.05:0.8) , (0.85:0.01:0.92) ,
 (0.93:0.005:0.95) ];
hc=contour(x_plot,y_plot,eta_matrix,eta_levels,'ShowText','on','LineWidth',1);


% Set plot parameters
grid('on');
set(gca, 'FontName', 'Arial')
set(gca, 'FontSize', 16)
title('4d) Plot of turbine map (collapsed)','interpreter','latex');
xlabel('$f(M_4)$','interpreter','latex');
ylabel('$p_{04}/p_{04.5}$','interpreter','latex');
xlim([0.4 1.1])
ylim([1 4])
hold off;
```

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DRIVER

This script intends to generate a compressor map and a turbine map and then computes the operating line by matching the turbine (HPT) work and the compressor (HPC) work for different temperatures.

```
clc
clear all
```

General input parameters Fluid

```
fluid.g = 1.4;
fluid.cp = 1005; %assumed [kJ/(K kg)]
fluid.R = 8314/28.85; %assumed [kJ/(K kg)]

% Efficiencies
e.diffusor = 0.97;
e.fan = 0.92;
e.combustor = 1;
e.LPT = 0.91;
e.nozzle_core = 0.98;
e.nozzle_bypass = 0.98; %assumed

% Engine Design
engine.ratio.pi_fan = 2.0; % fan
engine.ratio.comp = 12; % HPC
engine.ratio.m_bypass = 2.9; % bypass ratio
engine.s4_T = 1600; % [K] turbine inlet temperature
engine.Thrust_req = 3400; %[N]

% Operating condition
h = 9144; %[m] (30000 ft)

s0.M = 0.7; % Flight Mach number
[s0.T, s0.a, s0.p, ~] = atmosisa(h); % station parameters at altitude
s0.u = s0.a *s0.M; % flight speed
```

Compute stations until compressor inlet: station 2

```
s2.T=s0.T *( 1 +(fluid.g-1)/2 *s0.M.^2 ); % stagnation temperature of station 0 by assumption
s2.p=s0.p *( 1 +e.diffusor *(s2.T/s0.T-1) ) ^(fluid.g/(fluid.g-1));

% station 13
s13.p=engine.ratio.pi_fan *s2.p;
s13.T=s2.T *(1 +1/e.fan *(engine.ratio.pi_fan ^((fluid.g-1)/fluid.g) -1) );

% station 2.5
s25.p=s13.p;
s25.T=s13.T;
s25.rho=s25.p /s25.T /fluid.R;
```

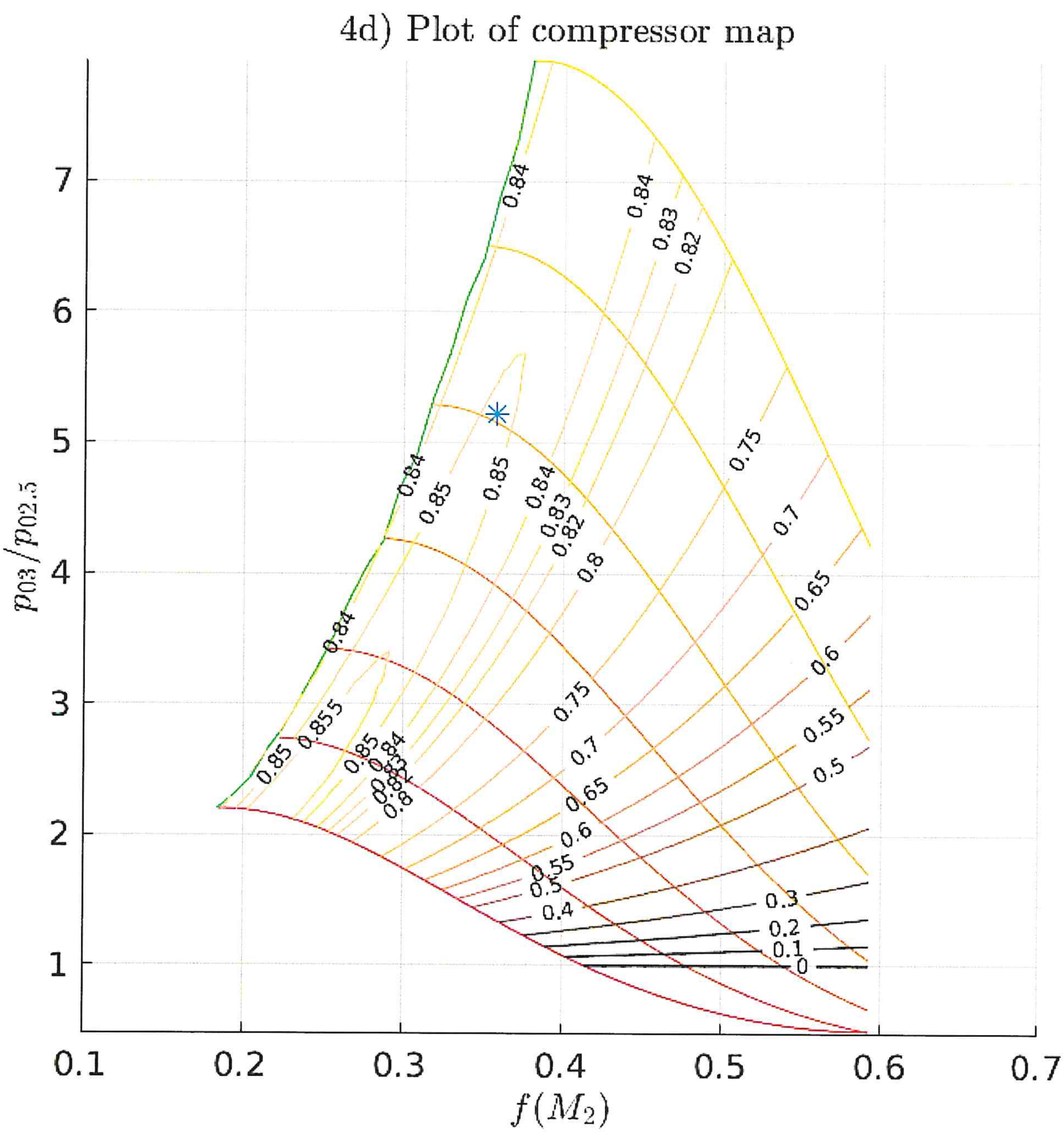
Compute compressor operating condition Fixed parameters

```
comp.a_alpha = 30/360*2*pi(); % [rad]
comp.a_beta = 60/360*2*pi(); % [rad]
comp.b_alpha = 60/360*2*pi(); % [rad]
comp.b_beta = 30/360*2*pi(); % [rad]
comp.ratio_r = 0.6; % radii ratio at stage 1
comp.ux_ref = 75;
```

```
comp.U_ref = 307;
comp.nStages = 4;
comp.rHubTipRatio = 0.6;

% Compute compressor geometry from operating condition
% station 3
comp.U = comp.U_ref;
comp.ux = comp.ux_ref;
e.st = 0.87 -16 *(comp.ux/comp.U -comp.ux_ref/comp.U_ref)^2;
comp.delta_uth = comp.U-comp.ux*(tan(comp.b_beta)+tan(comp.a_alpha));
C = comp.U *comp.delta_uth /fluid.cp; % constant for all stages

s3.T = s25.T;
s3.p = s25.p;
for iStage = 1:comp.nStages
    s3.T(iStage+1) = s3.T(iStage)+C;
    s3.p(iStage+1) = s3.p(iStage) *(1+ e.st*C/s3.T(iStage+1))^(fluid.g/(fluid.g-1));
end

s3.p = s3.p(end);
s3.T = s3.T(end);
e.compressor = ( (s3.p/s25.p)^((fluid.g-1)/fluid.g) -1) /(s3.T/s25.T -1);

% station 4 (neglect mass flow of fuel)
s4.p = s3.p;
s4.T = engine.s4_T;

% station 5 (turbine exit, solving for hpt and lpt in one)
s5.T = s4.T -(s3.T -s2.T +engine.ratio.m_bypass *(s13.T -s2.T) );
%!!! :/ should be 1/e.LPT
s5.p = s4.p *(1 +e.LPT *(s5.T/s4.T -1) )^(fluid.g/(fluid.g-1)) ;

% station 6 (solve mixing)
s6.T = (s5.T +engine.ratio.m_bypass *s13.T)/(1 +engine.ratio.m_bypass);
s6.p = (s5.T +engine.ratio.m_bypass *s13.T)/(s5.T/s5.p +engine.ratio.m_bypass *s13.T/s13.p);

% Get final thrust and velocity

% % Include mixing
% se.u = sqrt(2 *fluid.cp *e.nozzle_core *s6.T *(1 -(s0.p/s6.p) ^((fluid.g-1)/fluid.g) ) );
% engine.m_a = engine.Thrust_req /2 /(se.u -s0.u);

%!!! :-/ Ignore mixing, assume only one engine
se.u5 = sqrt(2 *fluid.cp *e.nozzle_core *s5.T *(1 -(s0.p/s5.p) ^((fluid.g-1)/fluid.g) ) );
se.u13 = sqrt(2 *fluid.cp *e.nozzle_core *s13.T *(1 -(s0.p/s13.p) ^((fluid.g-1)/fluid.g) ) );
engine.m_a =  engine.Thrust_req /(se.u5 -s0.u +engine.ratio.m_bypass *(se.u13 -s0.u));
engine.m_ac=engine.m_a /(1+engine.ratio.m_bypass) ;

% Get area from mass flow, flow is choked (somewhere ~station 2.5)
%!!! :-/ assume all mass goes through compressor (neglect bypass)
s25.T_static = s25.T -comp.ux^2/(2*fluid.cp); % static temperature
s25.M = comp.ux /( sqrt(fluid.g*fluid.R*s25.T_static) ); % Mach number
ratio_A = 1/s25.M *(2 /(fluid.g+1) *(1 +(fluid.g-1)/2 *s25.M^2) )^((fluid.g+1)/(2*(fluid.g-1))); % A/A_star
s25.m_choked = s25.p/sqrt(s25.T) *sqrt(fluid.g/fluid.R) *((fluid.g+1)/2)^(-(fluid.g+1)/(2*(fluid.g-1)));
s25.A = engine.m_a /s25.m_choked * ratio_A;

% Compute the radii from area:
s25.r_tip = sqrt(s25.A /(pi() *(1-comp.ratio_r^2)) );
s25.r_hub = comp.ratio_r *s25.r_tip;
s25.r_mean = 0.5 *(s25.r_tip+s25.r_hub);
comp.design_N = comp.U /(2*pi()*s25.r_mean) *60;
```

Plot compressor map

```
comp.ux=30:2:130;
comp.N=15000:500:30000;
[legendCell1,Fig_comp] = Plot_CompressorMap(fluid, e, s25, comp); % figure 11
```

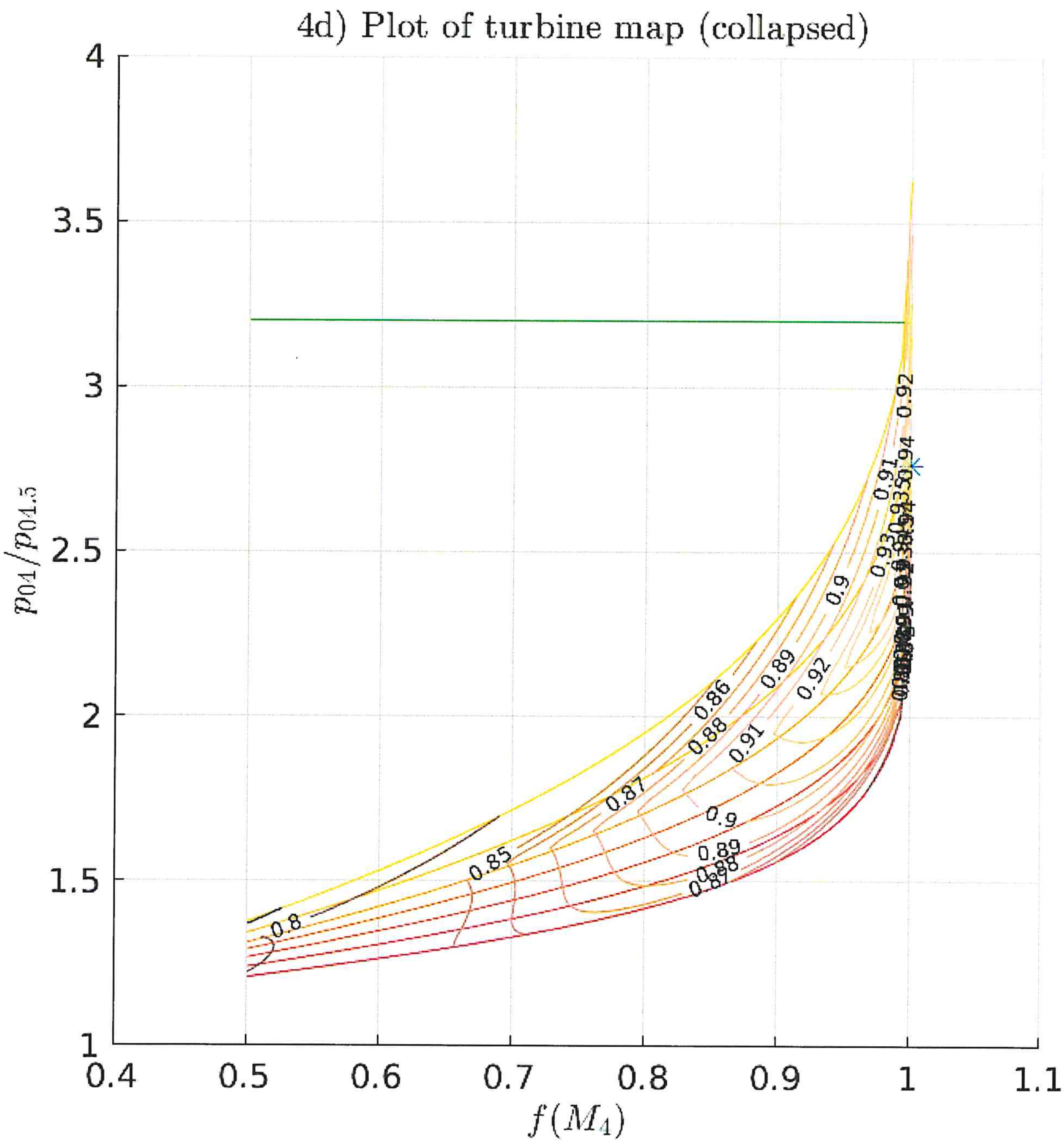


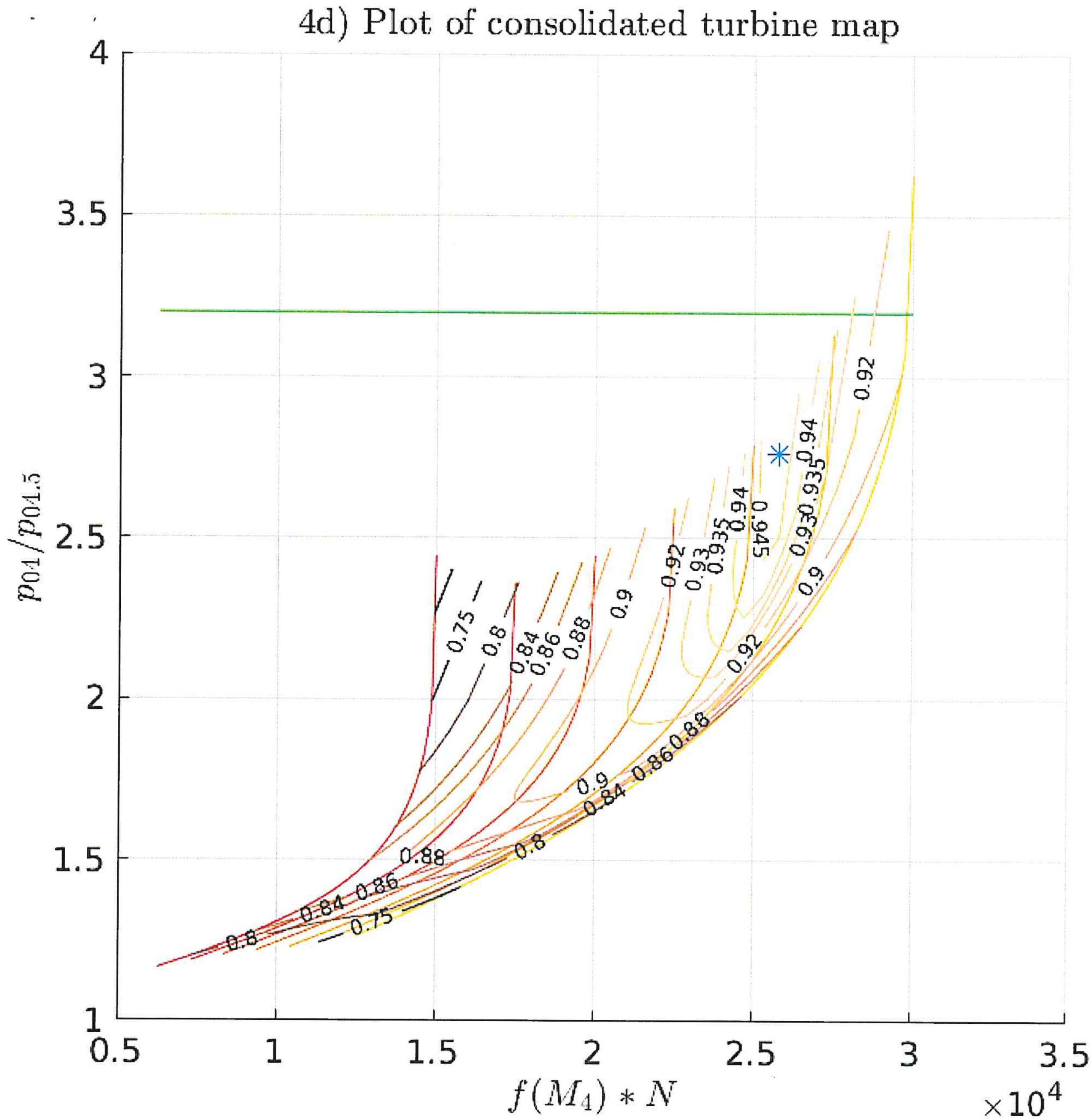
Compute turbine operating condition Fixed parameters

```
turb.b_alpha = 60/360*2*pi(); % [rad]
turb.c_beta = 45/360*2*pi(); % [rad]
turb.ratio_r = 0.6; % radii ratio at stage 1
turb.ux_ref = 732;
turb.U_ref = 216;
turb.nStages = 1;

% Given mean radius:
```

```
s4.r_mean = 0.08; % [m]
s4.r_tip=2*s4.r_mean  /(1+turb.ratio_r);
s4.A=pi()*s4.r_tip^2 *(1 -turb.ratio_r^2);
```

```
% Plot turbine map
turb.ux=200:2:750;
turb.N=15000:500:30000;
[legendCell2,Fig_Turb] = Plot_TurbineMap(fluid, e, s4, turb, engine); % figure 21
[legendCell3,Fig_Turb1] = Plot_TurbineMap1(fluid, e, s4, turb, engine); % figure 31
```



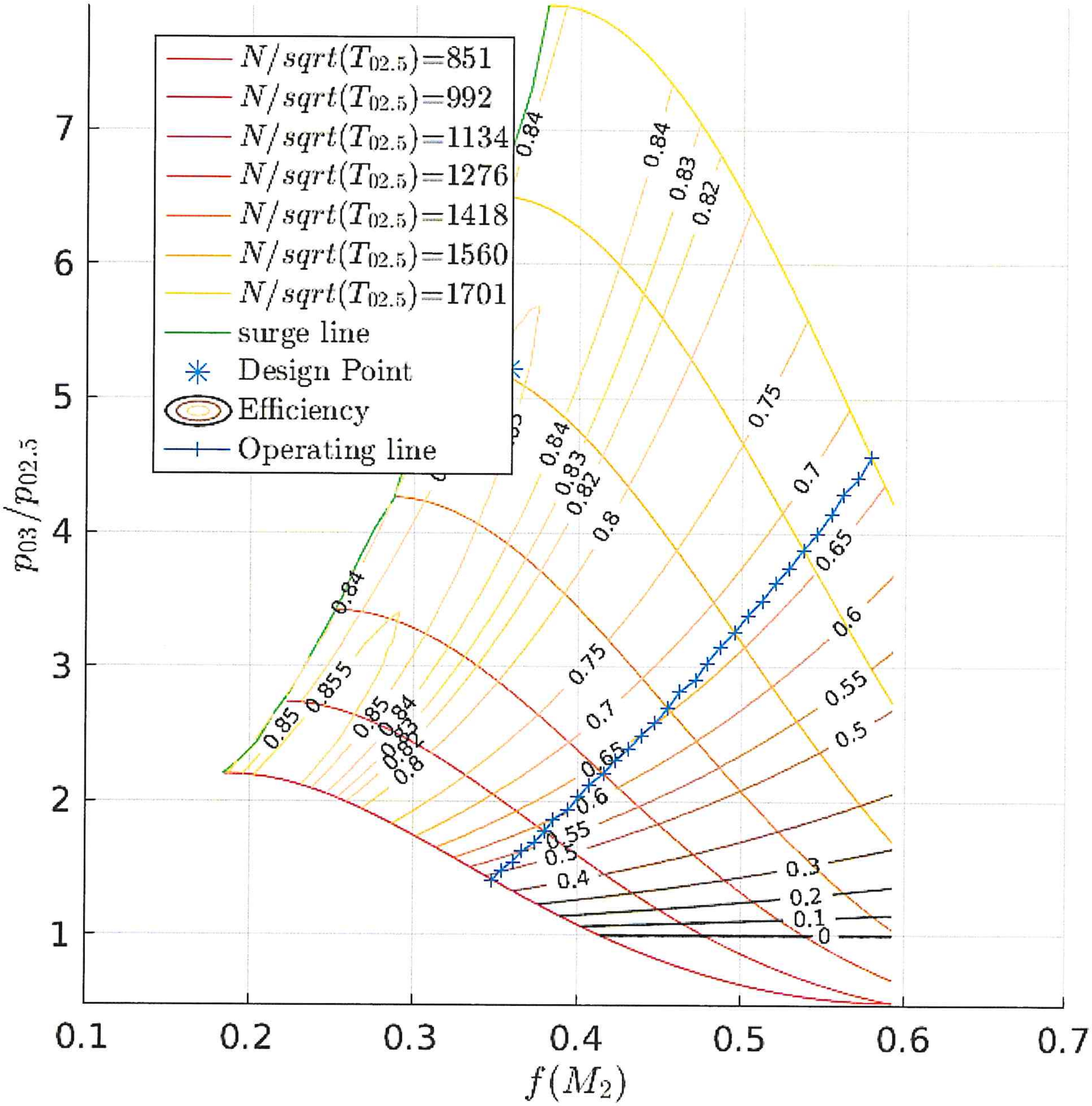


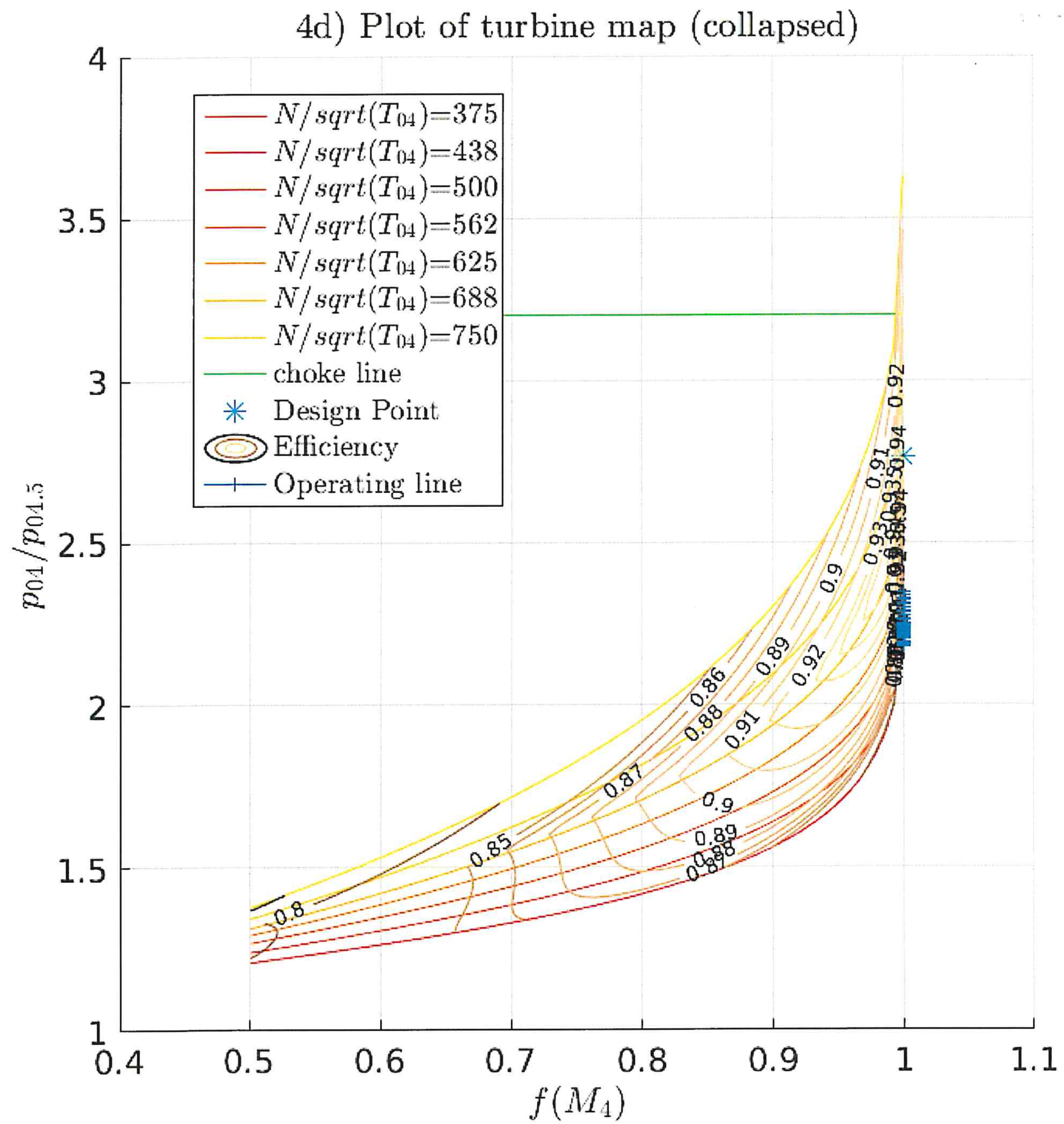
Operating line

```
N=15000:500:30000;
T=1000:5:5000;
for iN=1:length(N)
    U_t=N(iN) /60 *(2*pi*s4.r_mean);
    U_c=N(iN) /60 *(2*pi*s25.r_mean);
    test=0;
    for iT=1:length(T)

        s4.M = 1;
        engine.s4_T = T(iT);
        s4.T=engine.s4_T;
        % 2) axiel speed of turbine
        u_xt = s4.M*sqrt(fluid.g*fluid.R*s4.T) *(1+(fluid.g-1)/2)^-0.5;
```

```matlab
        % 3) match work
        u_xc(iT)=(U_t/U_c/comp.nStages *(u_xt *(tan(turb.b_alpha)+tan(turb.c_beta)) -U_t) -U_c)...
            /(tan(comp.a_alpha)+tan(comp.b_beta));
        % 4) + 6) + 7) Solve HPC
        [x_cm(iT), y_cm(iT),~,s25]=CompressorPoint(u_xc(iT), U_c, fluid, e, s25, comp);
        s4.p = y_cm(iT)*s25.p;
        m_c(iT)=s25.m_a;
        % 5) +7) Solve HPT
        [x_tm(iT), y_tm(iT), e_t, ~, ~,s4] = TurbinePoint2(u_xt,U_t,fluid, s4, turb, engine);
        m_t(iT)=s4.m_a;
        if test==1
            m_c(iT)=0;
            m_t(iT)=100;
        end
        if abs(s25.m_a-s4.m_a)<0.05
            test=1;
        end
        p5(iT)=s4.p/y_tm(iT);

    end
    [err_m(iN), iT_star]=min(abs(m_c-m_t));
    x_c_plot(iN)=x_cm(iT_star);
    y_c_plot(iN)=y_cm(iT_star);
    x_t_plot(iN)=x_tm(iT_star);
    y_t_plot(iN)=y_tm(iT_star);
    T04(iN)=T(iT_star);
    p05(iN)=p5(iT_star);
end


figure (Fig_comp)
hold on;
plot(x_c_plot,y_c_plot,'-+b');
h=legend(legendCell1,'location','best');
set(h,'Interpreter','latex');
hold off;

figure (Fig_Turb)
hold on;
plot(x_t_plot,y_t_plot,'-+b');
h=legend(legendCell2,'location','best');
set(h,'Interpreter','latex');
hold off;

figure (Fig_Turb1)
hold on;
plot(x_t_plot.*N,y_t_plot,'-+b');
h=legend(legendCell3,'location','best');
set(h,'Interpreter','latex');
hold off;
```

4d) Plot of compressor map




4d) Plot of turbine map (collapsed)
N/sqrt(T04)=375
N/sqrt(T04)=438
N/sqrt(T04)=500
N/sqrt(T04)=562
N/sqrt(T04)=625
N/sqrt(T04)=688
N/sqrt(T04)=750
choke line
Design Point
Efficiency
Operating line
p04/p04.5
f(M4)
4
3.5
3
2.5
2
1.5
1
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1
0.92
0.94
0.91
0.93
0.9
0.89
0.92
0.86
0.88
0.91
0.87
0.9
0.89
0.88
0.87
0.85
0.8