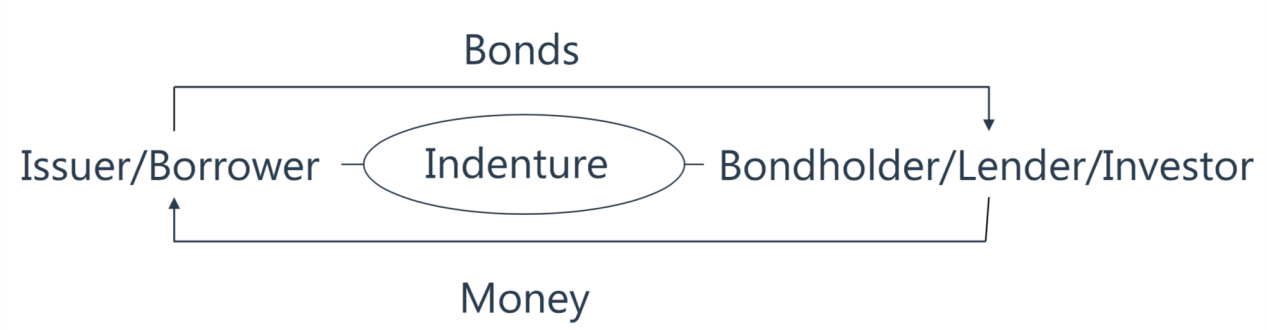
Bond

Section 1: Overview



1.1 Characteristics - 前三个在签约合约时已确定:

- Coupon Rate: 承诺给对方的利息，fixed coupon, floating rate coupon, zero-coupon(折价发行)
- Face Value: 所借的本金 ≠ **price** (债券这一张契约在市场上值多少钱)
- Relationship between Market Price and Face Value
  - **price** > face value, there is premium, then c > y
  - price = face value, par 按面值发行, c = y
  - price < face value, discount 折价发行, c < y
- Maturity

- Yield to Maturity (YTM): 平均收益率，反映即期利率的平均水平。补：一个债券的价值估计是未来现金流的折现求和，会用到一个维度是折现利率（市场利率，常用spot rate），yield 假设未来所有利率都是恒定不变的，用平均利率反应折现利率，倒推出来的，可看作即期利率的平均水平。
  - YTM 可反映债券投资 return 的条件: 1> 持有至到期; 2>再投资仍然按照原利率

Section 2: Interest Rate

2.1 Types - 5种最常见的收益率

- Risk-free rate: at which derivatives are priced is determined from Overnignt Interbank rates using overnight indexed swap.
- Treasury rates: the rates earns on Treasury bills and Treasury bonds, risk-free rates
- LIBOR: interest rate at which the bank is prepared to make a large wholesale deposit with other banks 银行间相互拆放资金时计息用的一种利率
- SOFR: 担保隔夜融资利率是银行同业拆借利率的替代选择，是一种担保银行间隔夜利率和参考利率。
- Repo rares: the difference between selling price and the repurchased price 抵押贷款利息

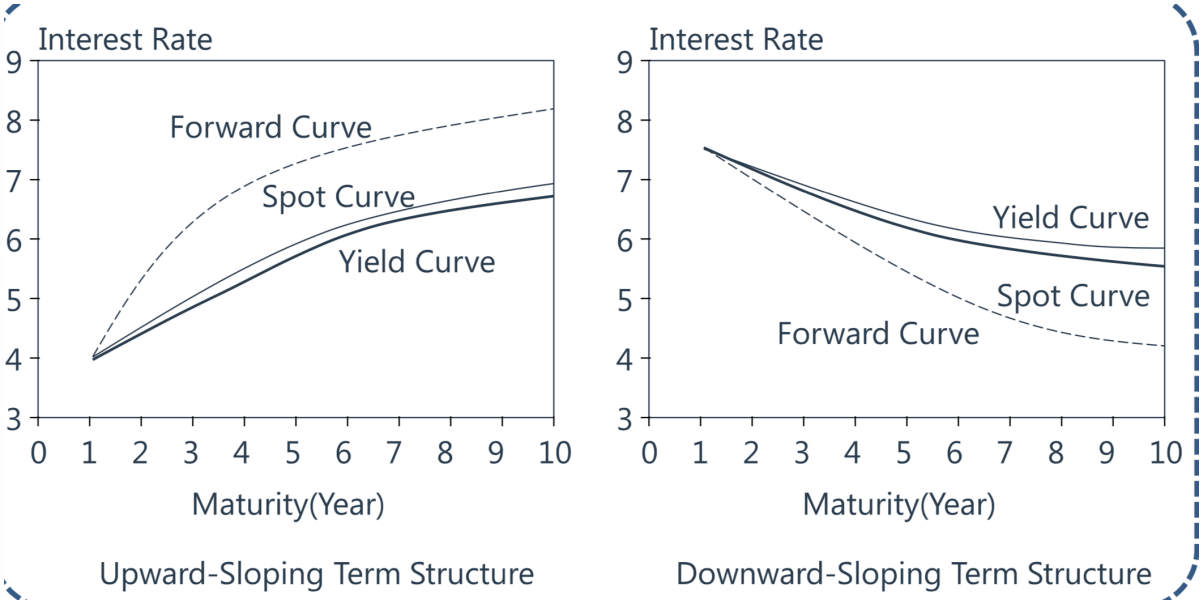
2.2 Payment Methods

- Simple Interest: $FV = PV * (1 + T * R_s)$, e.g. $100 * 10\% * 2 + 100$ -> two-year
- compounding interest: $FV = PV * (1 + \frac{R_m}{m})^{mn}$, $PV = \frac{FV}{(1+R)^n}$, R market rate, 一年m次，共n年
- Continuous compounding interest: $FV = PVe^{R*n}$, $PV = FVe^{-R*n}$, m->∞

2.3 Present Valus & Future Value

- Discount factor 未来的1元，现在的价值是多少
  - $PV = \frac{5}{1+z_1} + \frac{105}{(1+z_2)^2} = 5 * d(1) + 105 * d(2)$, z_1, z_2 is spot rate
- Spot rate:
  - rate on a spot loan (zero-coupon bond rate), 没有coupon, 只有一笔现金流, 折现发行, $80 = \frac{100}{(1+z_3)^3}$
- Forward rate:
  - $P = \frac{5}{1+z_1} + \frac{105}{(1+z_2)^2} = \frac{5}{1+z_1} + \frac{105}{(1+z_1)(1+F_{1,2})}$
  - 复利: $(1 + z_1)^{T_2} = (1 + z_1)^{T_1}(1 + F_{1,2})^{T_2 - T_1}$
  - 连续复利: $e^{z_2 T_2} = e^{z_1 T_1} * e^{F_{1,2}(T_2 - T_1)}$ -> $F_{1,2} = \frac{z_2 T_2 - z_1 T_1}{T_2 - T_1}$
- Par rate (Coupon rate)
  - 可以反映 yield， 因为 price = par 时，c = y, 按面值发行, 定的 coupon rate 叫 par rate
- 利率的期限结构 term-structure, 即期利率曲线和远期利率曲线: z_2 可看成z_1与$F_{1,2}$之间的平均水平, YTM is a kind of average of all spot rates
  - Upward-sloping: $YTM < z_1 < z_2 < F_{1,2}$
  - Flat: 利率是恒定不变的，一般作为假设

- Downward-sloping: $YTM > z_1 > z_2 > F_{1,2}$
- 补充：根据forward rate公式，可以得出z_1、z_2 和 $F_{1,2}$ 的关系。z_1、z_2 和 YTM 的关系 (可以参照下面 **return 那里的 YTM 介绍**)： YTM is a kind of average of all sote rates. $P = \frac{CF_1}{1+YTM} + \frac{CF_2}{(1+YTM)^2} + \ldots + \frac{CF_n}{(1+YTM)^n} = \frac{CF_1}{1+z_1} + \frac{CF_2}{(1+z_2)^2} + \ldots + \frac{CF_n}{(1+z_n)^n}$

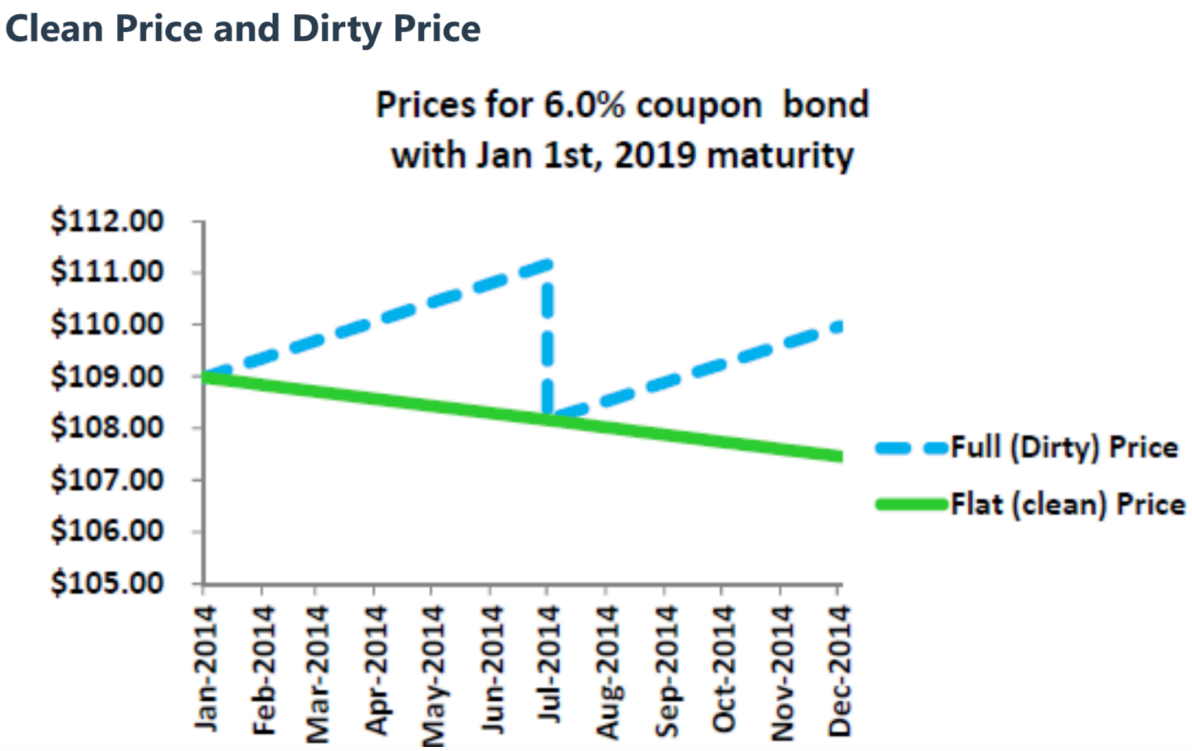


- Three Major theories of the Term Structure of Interest Rates
  - Expectation Theory: 远期利率就是人们对市场的期望，but不能解释why利率曲线总是upward-sloping.
  - Market Segmentation Theory: 债券市场可分为短中长期，供需决定bond price和interest rate.
  - Liquidity Preference Theory: 持有时间越长，流动性越差，利率越高，所以越大多数利率曲线是upward-sloping, 流动性补偿

2.4 Treasury Market & Corporate Bond

- **Treasury Market** Types (Maturity):
  - Treasury Bills (T≤1): zero-coupon, 货币市场, 折价发行，售价低于面值，到期以面值赎回，报价特别
    - cash price = 100 [1 - discount rate * (n / 360)]
  - Treasury Notes (2≤T≤10) 付息债且以半年付息
  - Treasury Bonds (10≤T) 付息债且以半年付息
    - 1> both notes and bonds make interest payments semi-annually;
    - 2> quoted price 按照100块面值，以三十二进制报价 (96-13+) = 96 + 13.5/32, 报价为clean price, 实际交易价是dirty price (参照下面**Price报价体系**)
  - Treasury STRIPS: 以上三种没有中长期的**0息债** (只有一笔现金流，不存在再投资风险，利率风险最高【因为0息债的MacD等于它的到期日】，所以想要有中长期的0息债，便于对冲和风险管理)
    - 假设10年T-bonds, 半年付息, 其中有20笔现金流 (20 coupon, 1 principal), 根据coupon拆分叫C-strips, 根据principal拆分叫 P-strips.
- **Price 报价体系:**
  - dirty price = clean price 报价 + AI (accrues interest) 上家持有应获利息，按比例且不考虑时间价值不折现

- 为什么不以 dirty price 报价？因为这样会使1个债券因为付息日不同而变成2个完全不同的债券，且会出现价格不连续
- AI day count conventions:
  - Treasury Bond: autual / actual
  - Corporate and Municipal Bond: 30 / 360
  - Money Market Instrument (Treasury Bills): actual / 360



- **Corporate Bond**
  - Trading Characteristic:
    - Bonds issued via private placement 私募: held by the original purchasers until maturity, are often not traded.
    - Bond issued via public offering 公募: typically traded in the over-the-counter market.
  - Bond indenture: corporate bond issuer promises and investor's rights.
  - Corporate trustee: represents bondholder's interests, is third party to the contract, ensure that the bond issuer is in compliance with the covenants of the indenture at all times.
  - Main types of interest payment classification:
    - Straight-coupon
    - Zero-coupon corporate bonds
      - interest rate is determined by the original issue discount (OID). OID is the difference between the face amount and the offering price when first issued.
    - Floating-rate bonds
  - Different types of corporate bonds:
    - Mortage Bonds 房屋贷款, 与普通债券(最后一笔现金流还本金)不一样, 须每个付息日本金利息掺在一起还;
    - Collateral Trust Bonds;
    - Equipment Trust Certificates;
    - Debenture Bonds 无抵押, 1> 发行债券公司信用极好; 2> 有保险

    - Guaranteed Bonds.
  - High-yield bond (junk bond): rated below investment grade
    - High-yield bond issuer: 1> original issuers 发行人性质 2> Fallen angles 好公司堕落变成坏公司 3> Restructuring and Leverage Buyouts 重组

2.5 Bond Valuation

- Price of 有限期普通债券: $PV = \sum_{t=1}^{T} \frac{CF_t}{(1+y)^t}$, 所有现金流折现到0时刻的总和. y是市场收益率,千变万化
- Price of Annuity 年金: 每次付息日支付等额金额，有到期日, but never makes a final principal payment 即 FV=0，$P = \frac{c}{y}[1 - (\frac{1}{1+\frac{y}{2}})^{2T}]$
- Price of Perpetuity 永续年金: pay coupons forever, 没有到期日 $P = \frac{c}{y}$, T -> ∞
- 上面的公式可知 <u>收益率和价格成反比</u>，当收益率为coupon rate，债券价格为par。债券收益率低的时候价格加速上升，债券收益率高的时候价格减速下降，曲线是convex. (参照下面的price-yield relationship图)

2.6 Returns, Spreads, Yields

- <u>Gross Realized Returns</u>: $R_{t,t+1} = \frac{P_{t+1}+C-P_t}{P_t}$ ，P_t 为期初的价格，P_{t+1}为期末的价格，C为coupon
  - 每单位期数价格带来的收益，但如果借钱来买，用这个衡量收益就不合适，没有考虑融资成本
- <u>Net Realized Returns</u>: $R = \frac{P_{t+1}+C-B_{founded\_price}}{P_t}$，$B_{founded\_price}$ 为实际融资成本
- <u>Yield to Maturity</u>
  - Internal rate of return found by equating the present value of the cash flows to the current price of the security. 未来有一定现金流入的情况下，我的这笔投资的**内部收益率**。
  - YTM 可反映债券投资 return 的条件: 1> 持有至到期; 2>再投资仍然按照原利率. 如果1>不满足，不可能达到3年期的收益, 如果2>不满足，y不等于r，不能用YTM反映收益。
  - $40 * (1+y)^7 + 40 * (1+y)^6 + \ldots + 1040 = 850 * (1+r)^8 = YTM$, YTM is a kind of average of all spot rates. $P = \frac{CF_1}{1+YTM} + \frac{CF_2}{(1+YTM)^2} + \ldots + \frac{CF_n}{(1+YTM)^n} = \frac{CF_1}{1+z_1} + \frac{CF_2}{(1+z_2)^2} + \ldots + \frac{CF_n}{(1+z_n)^n}$
  - Relationship between spot rate and YTM, (**见上面的term-structure中的图**)
- <u>Spread of Bond</u> 风险议价 risk premium，相对价值分析
  - 国债：risk premium低，价格高；公司债：risk premium高，价格低
  - 风险议价一般是通过market price倒推出来的，如下公式
  - $PV = \frac{CF_1}{1+f(1)+s(1)} + \frac{CF_2}{(1+f(1)+s(1))+(1+f(2)+s(2))} + \ldots$, f()是无风险利率，s()是风险premium
- <u>Coupon Effect</u>: correctly priced bonds with the same maturity but different coupons have different yields to maturity.
- <u>Decomposition of Profit & Loss</u>
  - Price appreciation (or depreciation): carry-roll-down 所有其他因素不变, 只有t变化 -> rate change t发生变化之后, 无风险利率变化 -> spread change t,r 变化之后, 风险议价变化
  - Cash-carry: cash flow such as coupon payments and financing costs

2.7 Bond Replication

- Law of One Price: 不考虑 liquidity, tax …情况下, cash flow一样, 则price一样. 不满足law of one price，就会有套利空间。考点1> 复制现金流 2>违背law of one price, 会存在套利, 低买高卖

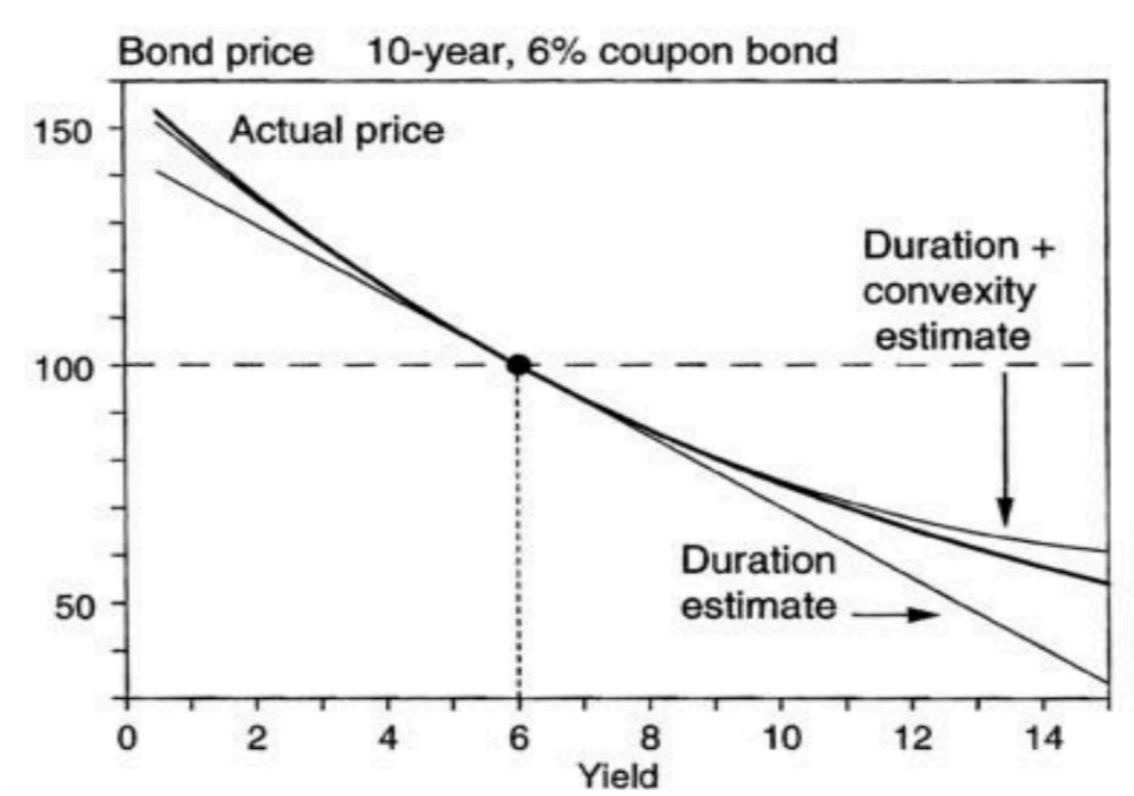
  > coupon & price, 假设债券期限一样都是一年，求？：
  >
  > 2 7/8 $98.40;
  >
  > 4 1/2 ?;
  >
  > 6 1/4 $101.30
  >
  > 根据law of one price, 两笔现金流复制一笔，$(100+2+7/8)w_1+(100+6+1/4)w_2=100+4+1/2$, $w_1+w_2=1$ => ？= $99.80
  >
  > 如果现在这个债券价格为102 > 99.80，说明价格被高估了，策略是买原来两个债券，卖这个债券。
  >
  > 两个债券构造另一个债券，每笔现金流乘以权重。

- 如果mispricing发生，套利可能不存在，由于
  - transaction costs 交易成本存在
  - Bid-ask spread 买卖价差. 若流动性好, 则买卖价差低, 反之高.
  - 债券其他特性...
- 债券复制例题（有coupon）

  > Bond1: 10-year, 8% coupon bond currently sells for $90
  >
  > Bond2: 10-year, 4% coupon bond currently sells for $80
  >
  > Question: 10-year zero rate
  >
  > Answer: $8w_1+4w_2=0$; $108w_1+104w_2=100$

2.8 Risk Metrics 评估风险，市场风险度量维度，主要是利率变动导致

- **interest rate risk** $P=\sum\frac{CF_i}{(1+y)^i}$
  - **单因素 One-factor risk metrics**



- **Duration**: 久期 现金流的平均到期时长。Duration是债券的内在性质，与你什么时候卖掉债券无关。注意⚠️：不可以分析具有一定含权特征的产品，如MBS - 可能会提前偿付, callable - 可提前赎回，因为MD是通过求导得来的，而现金流或定价公式是确定的才能求导。

  > 假设2个债券，par都是100，1个1年到期，1个两年到期, 求duration。
  >
  > 折线到0时刻才可以比大小。
  >
  > $PV_1 = \frac{100}{1+10} = 90.9, PV_2 = \frac{100}{(1+10\%)^2} = 82.6$
  >
  > $D = 1 * \frac{90.9}{90.9+82.6} + 2 * \frac{82.6}{90.9+82.6} = 1.48$ 年，当y变化1%时，p反向变化1.48%

  - **Macaulay Duration:**

$$MacD = \sum_{t=1}^{T}[\frac{PV(CF_t)}{P} * t]$$

$PV(CF_t)$ 每笔现金流折现价值，P 所有现金流折现总价值

  - zero-coupon bond: MacD == Maturity，利率风险高;
  - Plain bond: MacD <接近 Maturity. 因为之前的现金流分散了权重，虽然最后一笔现金流占比最大。
  - Consol 永续债券: MacD=$1 + \frac{1}{y}$

    > $p = \frac{c}{y} = cy^{-1}$ -> DD=$\frac{\partial p}{\partial y} = cy^{-2} = -\frac{c}{y^2}$ -> MD = $\frac{-c/y^2}{c/y} = \frac{1}{y}$ -> MacD=$1 + \frac{1}{y}$

  - **Modified Duration:** 指利率发生变化，债券价格变动的百分比. 考试中无特殊说明, 都是MD
    - 符号：指变化方向，y和p的变化方向一定是反的。

$$MD = -\frac{\Delta P}{\Delta y} * \frac{1}{P} = \frac{MacD}{1+y}$$

y 为每期利率

    - MD 推导：dollar duration $DD = \frac{\Delta P}{\Delta y}$ -> 对y求导，推导过程假设利率是按年的利率

    - MD 与 MacD 变动是一致的，与 T 同向变动，与 c, y 反向变动 (coupon 越小, 越接近与 zero-coupon duration)
- **DV01:** 利率发生特殊变化 (one basic point) 对应的价格变动

$$DV01 = MD * Bond\ Value * 0.0001$$

- **补充MD应用:**
    - Risk Management: 复杂债券产品，直接估算组合风险很难，可以先算利率(风险因子)的变化，再去估算组合价格变化

> $\Delta p = -MD * p * \Delta y$, 线形近似并不准确
>
> $\Delta p = -MD * p_0 * \Delta y + \frac{1}{2} * C * p_0 * \Delta y^2$, 用到convexity，非线性更加准确

    - Hedge: 对冲工具带来的价格变动要能完全cover被对冲组合的价格变动：

> $N = \frac{MD_s * P_s}{MD_h * P_h}$, s是现货 h是期货
>
> 对于欧洲美元期货, DV01 = 25, 特性, 无需计算, 则
>
> $N = \frac{DV01_s}{DV01_F}$, $F_B = \frac{F_A * DV01_A}{DV01_B}$

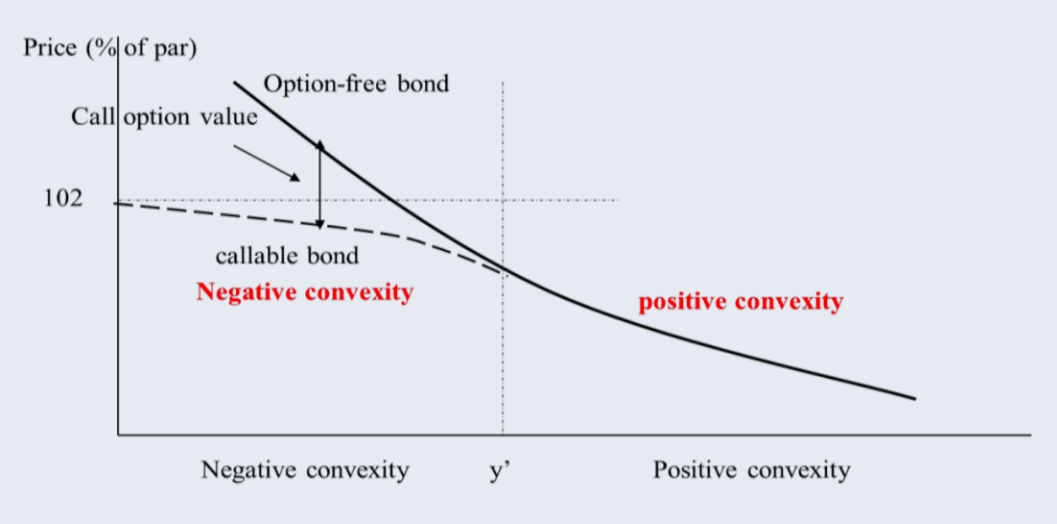
    - 持有债券: 利率上升, p下降, 买D小的; 利率下降, p上升, 买D大的；
- **补充bond分裂 (利息等式加权重) :** a bond can split into a floater (LIBOR) and an inverse floater (6%-LIBOR)

$$V_{bond} = V_F + V_{IF}$$
$$D_{bond} = w_1 D_F + w_2 D_{IF}$$
$$D_F = 0$$
$$C_{bond} = C_F + C_{IF}, coupon$$

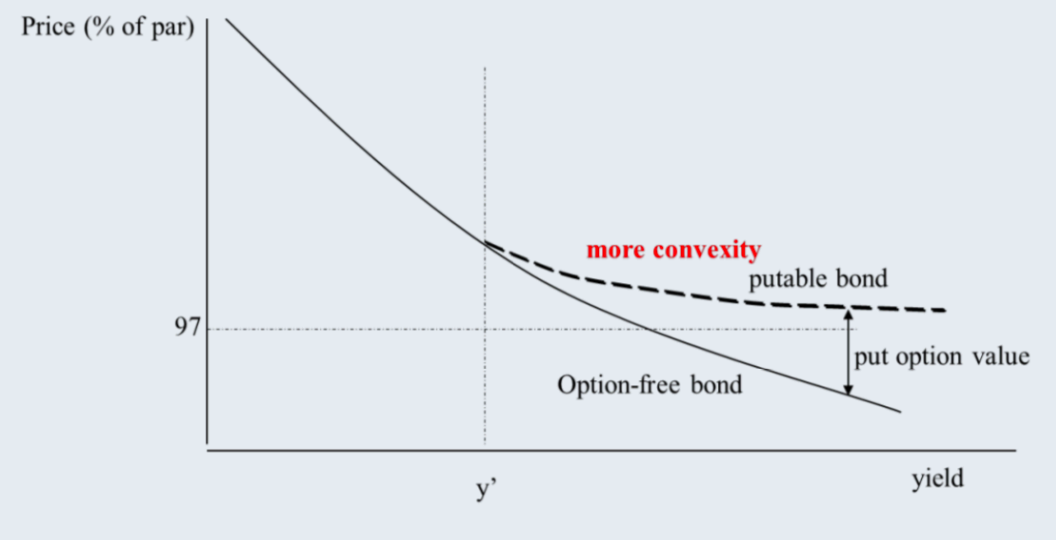
- **<u>Convexity 曲率</u>**:

$$\Delta p = -D * p_0 * \Delta y + \frac{1}{2} * Convexity\ * p_0 * \Delta y^2,\ C = \frac{d^2 p}{dy^2}$$

    - the p-y curve [见上图] is convex (convex是笑凸, concave是哭凹), 斜率在变化, 债券多投来说convex是好的
    - 久期相等，现金流越分散 δ^2，convexity越大。$C = \frac{MacD^2 + MacD + \delta^2}{(1+y)^2}$
    - 一般债券的convexity大于0，特殊债券如含权不一定
        - 一般债券：convexity 和 duration 的变化方向一致, 与 T 同向变动 (与 duration 关系很大, 期限长的债券的 convexity 远超期限短的债券的 convexity), 与 c, y 反向变动
        - 含权债券：negative convexity, 一般指callable bond and mortage bond。当利率下降，债券价格上升，比较容易出现提前赎回，所以债券价格不会无休止上升。（如下图）
            - Callable Bond: issuer has the right to buy back the bond in the future at a set price; as yields fall, bond is likely to be called; prices will rise at a decreasing rate-negative convexity. $V_{pure} = V_{callable} - V_{call}$



- Putable bond: bondholder has the right to sell bond back to the issuer at a set price.



- 对于含权债券 (发行人可提前赎回), 当然下面两个式子也可以分析不含权
  - effective duration 计算逻辑类似MD

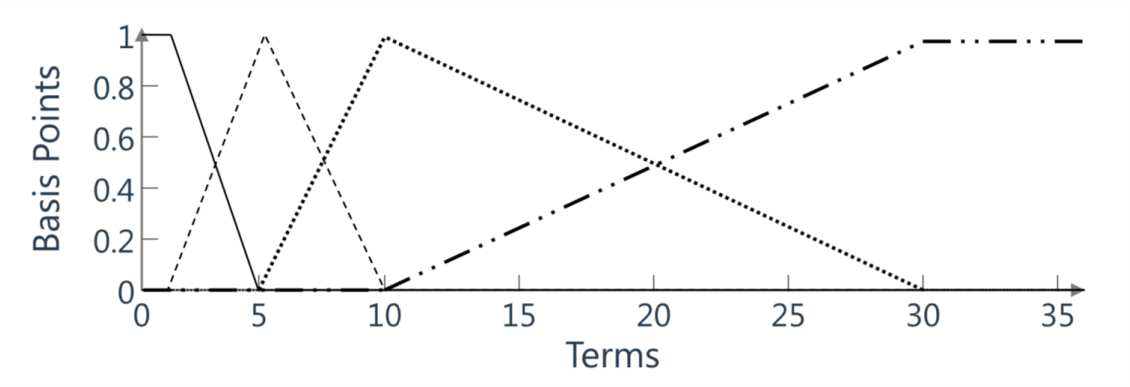
$$ED = \frac{\Delta p/p}{\Delta y} = \frac{p_- - p_+}{p_0 * 2\Delta y},\ [(p_-, y - \Delta y), (p_+, y + \Delta y)]$$

  - effective convexity 计算逻辑类似C

$$EC = \frac{D_- - D_+}{\Delta y} = \frac{p_- + p_+ - 2p_0}{p_0 * \Delta y^2}$$

- **Portfolio duration and convexity:**
  - weighted sum of individual = 单个债券价值(market value) / 总价值
  - Bullet: 组合构造用的都是中间期限的债券
    - Barbell: 短期和长期, benefits more from interest rate volatility
- 多因素 **Multi-factor risk metrics - Nonparallel**
  - Key rate exposure 都是债券, 拟合国债市场利率
    - Key rate shifts:
      - assum that all rates can be determined as a function of a relatively small number of key rates. A few rates along the term-structure are picked which are representative of the

curve.
    - Shifts in the key-rates are decline linearly
        - the rate of a given maturity is affected solely by its closest key-rate



        - key rate 01: 类似DV01, 一个关键利率点变动一个基点对应价格变动
    - key rate duration: 一单位关键利率变动带来的价格变动百分比
- Partial 01s & Partial PV01 对于不仅有债券, 还有swap, 拟合swap rate
    - 不能简单的用国债市场利率变动进行分析，常用是用**swap rate**进行分析。when swaps are taken as the benchmark for interest rates, risk is usually measured with Partial 01s or Partial PV01.
    - Partial 01: 对于那些particular fitted rate发生一个基点变化组合价值的变化
    - PV 01: all swap rates 都发生一基点的变化
    - Forward Bucket 01: 将一组远期利率一起发生同样的变化，再将另一组...

> 0 - 2 年, 2 - 5 年, ...
>
> computed by shifting the forward rates in that bucket by one basis point

Section 3: reinvestment risk

- 未来的coupon未必能以当前的利率进行投资
- 未来的投资利率未知，时间长，coupon大，reinvestment risk大
- reinvestment risk和利率风险此消彼长

Section 4: credit risk

- credit default risk 最严重: unable to meet its financial obligations
- credit spread risk: the credit spread is the difference between a corporate bond's yield and the yield on a comparable-maturity benchmark Treasury security.
- issuer default rate vs. dollar default rate
- recovery rate: 不同行业决定