Fanta Addicts - HW4

Lindsey Crowell, Danny Eckerberg, Ashleigh Mauler

October 13, 2025

1 Background

After leaving our case of Fanta in the car on a hot summer day, we would like to find out how long it will take to cool a can of Fanta to a desirable drinking temperature. Our smart friend is taking the approach of spinning the can in an ice bath. He follows a pattern of spinning the can for 30 seconds, then letting it rest in the water for 5 minutes. Three periods of 30 second spinning are modeled, as described by our smart friend. Here, we will model this scenario using a transient heat conduction equation and employing some simplifying assumptions. Our model will result in an estimate of the time it will take for the can to reach an enjoyable temperature, including our uncertainty about this estimate.

2 Assumptions

First, simplifying assumptions are made about the can and the environment. We assume the can to be similar to a typical 12 oz. soda can: 12 cm in height and 6 cm in diameter. Its starting temperature, uniform throughout the fluid, is 95°F. The can is then submerged in an ice bath assumed to be a fluid of uniform temperature 32°F. Contents of the can are assumed to have water-like heat transfer material properties. Heat transfer through the thin aluminum can wall is ignored. Thus, this becomes a case of a hot cylinder of water submerged in a cold body of water. For the periods of can spinning, the can is assumed to rotate one revolution per second.

Some present—but likely negligible—physical effects are ignored. The pressure in the can and buoyancy forces are deemed negligible. Small changes in water density during the cooling process are ignored. Any fluid mixing in the can and bath are assumed to have no significant effect on cooling time. The problem considers only conduction when the can is still, and additionally forced convection when the can is spun. A temperature-dependent polynomial model is fit to tabular data for thermal conductivity k and Specific heat capacity c_p. Heat transfer coefficient h is calculated at each temperature step in the simulation for both free and natural convection.

3 Approach

Initially, a lumped capacitance model was employed. This model uses average temperature throughout the cylinder, assuming that the temperature gradient across the can is roughly uniform during the cooling process. This proved to be an unrealistic assumption, confirmed by the high Biot number, relative to the 0.1 rule-of-thumb value for this method.

To model the temperature gradient in the can, conduction across cells in the can is simulated in 2-dimensions, radius r and height z. The boundaries of the cylinder are fixed at the temperature of the bath, $T_\infty = 32$°F. Using Python, we estimate the time at which the origin reaches the target temperature, 45°F. Within the simulation, the material properties are taken from "Fundamentals of Heat and Mass Transfer" (Bergman et. all, 2011) and then modeled to a polynomial fit with uncertainties calculated. The material properties are then recalculated at each time step simulated.

The heat transfer coefficient, h is calculated for both natural (sitting can) and forced (spinning can) convection. For natural convection, the Rayleigh number (Ra) is used. For forced convection, the rotational

Figure 1: Frustrated man just wants an ice cold Fanta. Generated by Google Gemini 2.5 Pro

speed and the Reynolds number (Re) are used. Both cases use the Nusselt number (Nu) and the conductivity (k) to find the heat transfer coefficient, which is difference for the forced and natural convection.

A 95% confidence interval around the estimated time to target temperature is developed using uncertainties of parameter values and model assumptions. Uncertainties for conductivity k and specific heat capacity c_p are based on the residual error in fitting a polynomial model to tabulated values. An additional 10% relative error is added to the value of heat transfer coefficient h, accounting for any error in utilized tabulated values. This is conservative, as relative uncertainties for h would typically reach this level only in complex scenarios. After propagating these uncertainties to the final time estimate, an additional 10% relative uncertainty is added in quadrature to the estimate to account for error resulting from model assumptions.

In addition to the 3-period can spinning method estimate, time estimates are calculated for scenarios in which the can is spun continuously and not at all.

4 Unit Testing

The goal of this experiment is to cool a can with time. Therefore, for the unit test, we need to prove that the temperature was in fact decreasing as time passes. For the unit test, the coefficients for c_p, h and k need to be pulled from the main code, as well as the coordinates of N_z and N_r. Using the values in a loop, a new temperature is found and appended as the loop resets. Then, an assertLessEqual statement was created to compare the temperature at time t and the following period of time $t + \Delta t$. If the temperature at $t + \Delta t$ is less than the temperature at t, then the loop is running correctly.

5 Results and Conclusion

The final estimate for the time it takes to cool the can to 45°F is 41.03 minutes (95% CI: 36.89; 45.17). An animation of this cooling process can be seen when running the python file "HW4cancooling.py." If no can spinning is performed, the time estimate is 41.2 minutes (95% CI: 37.04; 45.36). If the can is spun continuously, the estimate is 38.5 minutes (95% CI: 34.61; 42.39). An additional sensitivity analysis showed that with increasing the can-spinning velocity, the cooling time converges to around 36.5 minutes. So, we conclude that "smart friend" may have wasted his time doing any can spinning. Even if you were to spin very fast with the industrial setup shown in Figure 2, you would only save about five minutes. So, your best bet is just letting the can sit in the ice bath for around 41 minutes.

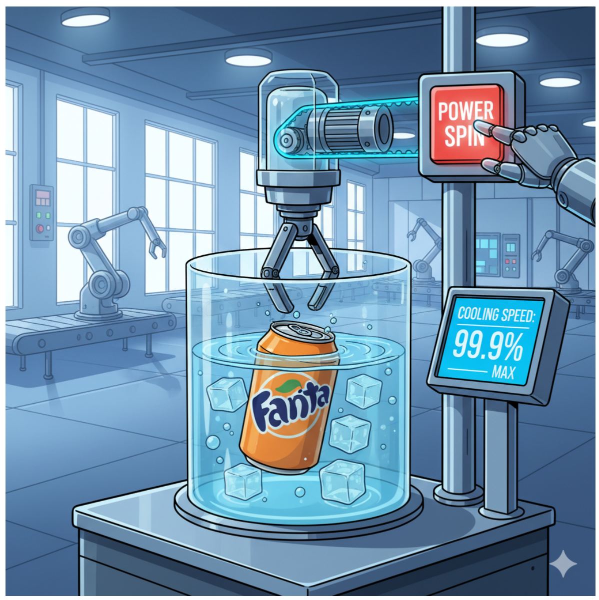

Figure 2: Industrial can spinning setup. Generated by Google Gemini 2.5 Pro