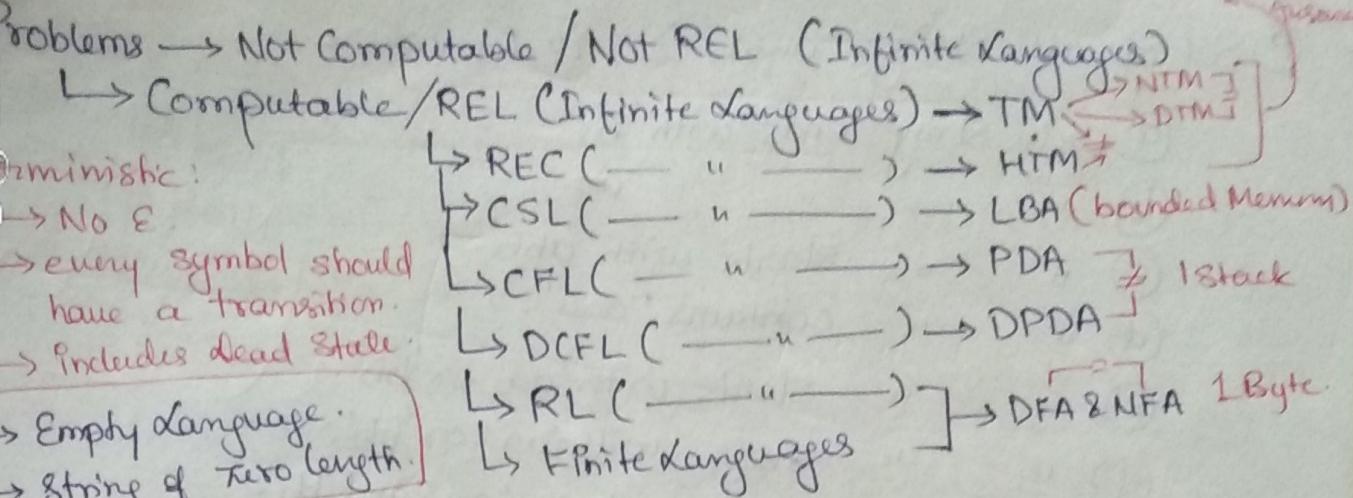


## THEORY OF COMPUTATION: (SHORT NOTES)



Infinite Memory  
2<sup>nd</sup> Generation

\* Regular Grammar generates Regular Languages, that is accepted by Finite Automata.

\* Regular Expression helps in representation of Regular Languages.

↪ Uses (only) 4 operators: + (OR), . (concat), \* (K.C), \* (P.C).

Q. Understand the following Regular Expressions:

- (1)  $\epsilon + \epsilon$
- (2)  $\epsilon + \emptyset$
- (3)  $\emptyset + \emptyset$
- (4)  $\epsilon \cdot \epsilon$
- (5)  $\epsilon \cdot \emptyset$
- (6)  $\emptyset \cdot \emptyset$
- (7)  $\emptyset^*$
- (8)  $\emptyset^+$
- (9)  $\epsilon^*$
- (10)  $\epsilon^+$
- (11)  $a + \emptyset$
- (12)  $a + \epsilon$
- (13)  $a \cdot \epsilon$
- (14)  $a \cdot \emptyset$
- (15)  $a.a$
- (16)  $a.a^*$
- (17)  $a^* \cdot a$
- (18)  $a + a^*$
- (19)  $\epsilon + a^*$
- (20)  $a + a^+$
- (21)  $a + a^+$
- (22)  $a^* + a^*$
- (23)  $a^* + a^+$
- (24)  $a^* \cdot a^*$
- (25)  $a^* \cdot a^+$
- (26)  $a^*(aa)^*$
- (27)  $(a + \epsilon)^*$
- (28)  $(a + \epsilon)^+$
- (29)  $(a^*)^*$
- (30)  $(a^*)^+$
- (31)  $(a^+)^*$
- (32)  $(a^+)^+$
- (33)  $a^+ \cdot a^*$
- (34)  $a^* a^+$
- (35)  $a^+ \cdot a^+$
- (36)  $(a^* b^*)^*$

Answers:

- 1)  $\epsilon$
- 2)  $\epsilon$
- 3)  $\emptyset$
- 4)  $\epsilon$
- 5)  $\emptyset$
- 6)  $\emptyset$
- 7)  $\epsilon$
- 8)  $\emptyset$
- 9)  $\epsilon$
- 10)  $\epsilon$
- 11)  $a$
- 12)  $\epsilon + a$
- 13)  $a$
- 14)  $\emptyset$
- 15)  $a^2$
- 16)  $a^+$
- 17)  $a^+$
- 18)  $a^*$
- 19)  $a^*$
- 20)  $a^*$
- 21)  $a^+$
- 22)  $a^*$
- 23)  $a^*$
- 24)  $a^*$
- 25)  $a^+$
- 26)  $a^*$
- 27)  $a^*$
- 28)  $a^*$
- 29)  $a^*$
- 30)  $a^*$
- 31)  $a^*$
- 32)  $a^+$
- 33)  $a^+$
- 34)  $a^+$
- 35)  $a^+ / a^+ a^+ a^+$
- 36)  $(b^* a^*)^*$
- $(b^* a^*)^+$
- $(b^+ a^+)^*$

Q. Write Regular Expressions for the following

- ①  $L = \{x \mid x \in (a, b)^*\}$
- ②  $L = \{x \mid x \in (a, b)^*, |x| \neq 0\}$
- ③  $L = \{\}$
- ④  $L = \{\epsilon\}$
- ⑤  $L = \{xw \mid x=a, w \in (a, b)^*\}$

# FINITE AUTOMATA

↓  
with Output  
↓

Moore M/c  
O/p with states:  
 $\lambda: Q \rightarrow \Delta$

Mealy M/c  
O/p with transitions:  
 $\lambda: Q \times \Sigma \rightarrow \Delta$

★ For an 'n' length input, the output length of

Moore Machine:  $n+1$   
1st char of output is ignored.

Mealy Machine:  $n$   
→ output alphabet

$\{Q, \Sigma, S, q_0, \Delta, \lambda\}$   
↳ output function.

• No final state

without Output → NFA  $\cong$  DFA  $\cong$  Min DFA.  
 { $Q, \Sigma, S, q_0, F$ }.

$$\begin{aligned} \# \text{states in } \epsilon\text{-NFA} \\ = \\ \# \text{states in NFA} \end{aligned}$$

DFA:  $S: Q \times \Sigma \rightarrow Q$ .

NFA:  $S: Q \times \Sigma \rightarrow 2^Q$ .

\* If every <sup>state</sup> is non-final, both DFA & NFA accepts  $\emptyset$ , but when every state is final, DFA accepts  $\Sigma^*$  but NFA may not.

★ Every Non Regular language has Infinite Equivalence classes. But for Regular languages,

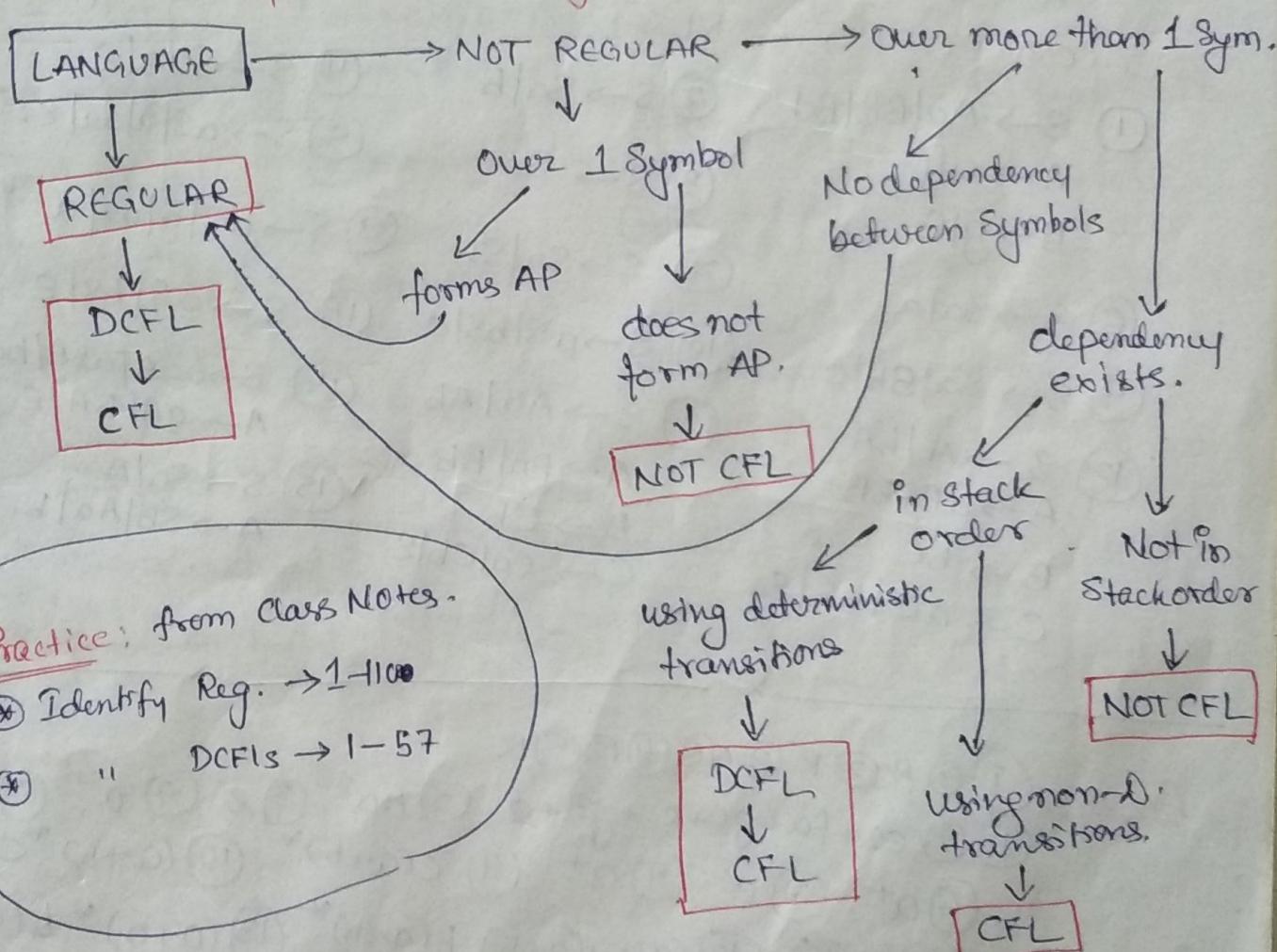
\*\* #Equivalence Classes = #States in min. DFA.

\* If a given NFA has  $Q$  states, then the equivalent min. DFA has  $\leq 2^Q$  states.

NOTES: (1) If  $x_2$  <sub>1<sup>st</sup> last symbol</sub> = 2<sup>n</sup> states (For Q. 34, 36)

- ✓ (1) Starts with contain dead state, but ends with or contains some string does not have a dead state, in the DFA.
- ✓ (2) L and  $\bar{L}$ , for RL, has same no. of states in DFA (Exchange F & NFs)
- ✓ (3) mod 'n' questions  $\rightarrow n$  states in DFA.
- ✓ (4)  $n_a(w) = k$  or  $\leq k \rightarrow (k+2)$  states  
 $n_a(w) \geq k \rightarrow (k+1)$  states.
- ✓ (5)  $a^{xn+y} \rightarrow y+1$  states.
- ✓ (6)  $\{ \text{fixed way } z | x \in (a,b)^* \text{ and } y, z, w, x \in (a,b) \} \rightarrow 2^y$  states.
- ✓ (7)  $n_a$  is divisible by  $2^y$ ,  $n_b$  is divisible by  $y \rightarrow z^*y$  states.  
 ↳ and, or, but not, either or but not both. whose remainders are odd.

### How to Identify REGULAR Languages, DCFLs & CFLs:



**GRAMMAR:** It generates a language using set of rules.  
 $G = \{ V, T, P, S \}$

### ① REGULAR GRAMMAR

↓  
left linear      Right linear

$$V \rightarrow VT^* \mid T^*$$

$$V \rightarrow T^* V \mid T^*$$

# LMD

=  $\epsilon$

# RMD

=

# PT

=

②

### ② CONTEXT FREE GRAMMAR

$$V \rightarrow (V + T)^*$$

③

" SENSITIVE "

$$\begin{aligned} \alpha &\rightarrow \beta \\ |\alpha| &\leq |\beta| \end{aligned}$$

④

Unrestricted Grammar

$$\alpha \rightarrow \beta$$

\* AMBIGUOUS CFGs: String having more than 1 parse tree.

\* UNAMBIGUOUS CFGs: String having exactly 1 parse tree.

\* Checking a CFG is AMBIGUOUS — SEMI DECIDABLE.  
UNAMBIGUOUS — NOT "

Q. Which of the following is Not Ambiguous?

- ①  $S \rightarrow SSS|a$  ②  $S \rightarrow SAB|B|C$  ③  $S \rightarrow SAS|B$  ④  $S \rightarrow a|b$ .

NOTE: If same NT participates in left as well as Right Recursion in a production, then it is surely Ambiguous.  
but vice versa is not true.

e.g.  $\begin{array}{l} S \rightarrow AB \\ A \rightarrow aA|\epsilon|Ab \\ B \rightarrow Bb|\epsilon \end{array}$

Ambiguous

$$\begin{cases} S \rightarrow AB \\ A \rightarrow aA|\epsilon \\ B \rightarrow aB|\epsilon \end{cases}$$

PUSH DOWN AUTOMATA: FA + 1 stack = PDA

$PDA = (Q, \Sigma, \delta, q_0, F, \tau_0, \tau)$

$\tau \xrightarrow{\text{top of stack symbol}}$   
 $\tau \xrightarrow{\text{Bottom element of stack}}$

$\lceil S(DPDA) \Rightarrow Q \times \Sigma \times \tau \rightarrow Q \times \tau^*$   
 $\lfloor S(PDA) \Rightarrow Q \times \Sigma \cup \{\epsilon\} \times \tau \rightarrow Q \times \tau^*$  operation to be performed  
on TOS.

\* Any CFL can be accepted using PDA with only 1 state.

### STACK OPERATIONS:

① PUSH —  $\delta(q_i, x, y) \rightarrow (q_j, xy)$  — DPDA

$\delta(q_i, x, \epsilon) \rightarrow (q_j, x)$

$\delta(q_i, x, X) \rightarrow (q_j, xX)$  — PDA

$\delta(q_i, x, yy) \rightarrow (q_j, xyy)$

② POP —  $\delta(q_i, x, y) = (q_j, \epsilon)$  — DPDA

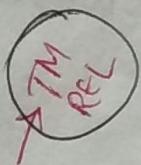
$\delta(q_i, x, yX) = (q_j, X)$  — PDA

$\delta(q_i, x, yy) = (q_j, y)$

③ NO OPERATION —  $\delta(q_i, x, y) = (q_j, y)$  — DPDA

$\delta(q, x, \epsilon) = (q, \epsilon)$  — PDA

$\delta(q, x, X) = (q, X)$  — PDA



\* Inherently Ambiguous : Not even 1 unambiguous grammar exists.

\* Not Inher. Ambiguous :  $\exists$  at least 1 " "

\* For language over 1 symbol,  $[FA \cong PDA]$

NOTE: ①  $\{ww^R \mid w \in (a,b)^*\}$  = CFL

②  $\{ww \mid w \in (a,b)^*\}$  = CFL

③  $\{a^n b^n c^n \mid n \geq 0\}$  = CFL

**PREFIX PROPERTY!** L satisfies Prefix Property, iff every string is not a prefix to every other string.

Q. Check whether the following satisfies Prefix property.

- ①  $L = a^*$
- ②  $L = a+b$
- ③  $L = ab^*$
- ④  $L = (a+b)^*$
- ⑤  $L = \{a^n b^n \mid n \geq 1\}$
- ⑥  $L = \{a, ab, \dots\}$
- ⑦  $L = \{ab, abb\}$

Ans: ②, ⑤.  $\times$  PDA by empty stack never accepts NULL.

Note: DPDA by empty stack **Accepts** set of languages that satisfies Prefix property.

### CLOSURE PROPERTIES:

✓ If Closed:  $h^{-1}$  and  $\subseteq$  finite.

$\times$  If Not Closed:  $\subseteq$  and Infinite <Any>.

✓ RL  $\Rightarrow$  All Closed

✓ DCFL  $\Rightarrow$  L and prefix(L)  $\Rightarrow$  Closed.

$\times$  CFL  $\Rightarrow$  L,  $\cap$ ,  $\cup$ ,  $-$ ,  $\subseteq$  finite; -finite  $\Rightarrow$  Not Closed.

$\times$  REC | CSL  $\Rightarrow$  f, h, f finite  $\Rightarrow$  Not Closed.

$\times$  RE  $\Rightarrow$  L,  $-$ ,  $\subseteq$  finite  $\Rightarrow$  Not Closed.

$$\begin{aligned} \emptyset \cap S &= \emptyset \\ \Sigma^* \cup S &= \Sigma^* \\ \Sigma^* \cap S &= S \\ \Sigma^* - L &= \bar{L} \\ L_1 - L_2 &= L_1 \cap \bar{L}_2 \end{aligned}$$

Q) Write F for Finite, and R for Regular:

- ①  $F, \cup F_2$
- ②  $F \cap F_2$
- ③  $F_1 - F_2$
- ④  $\bar{F}$
- ⑤  $R, \cup R_2$
- ⑥  $R, \cap R_2$
- ⑦  $R_1 - R_2$
- ⑧  $\bar{R}$
- ⑨  $F \cup R$
- ⑩  $F \cap R$
- ⑪  $F - R$
- ⑫  $R - F$

Answer: ① F ② F ③ F ④ Not Finite ⑤ R ⑥ R ⑦ R ⑧ R ⑨ R ⑩ F = ⑪ • ⑫ = R

$n+1 \leq$  No. of Substrings  $\leq \frac{\Sigma n + 1}{2}$

$\downarrow$   
if all characters  
are same in string.  
(over 1 symbol)

Subsequence:  $2^n - 1$

over  $> 1$   
symbol.

??

$$\frac{n(n+1)}{2}$$

Q.  $L_1 = a^*b$ ,  $L_2 = ab^*$

Find  $L_1 / L_2$  and

$$L_2 / L_1 ?$$

Ans!  $a^*$   
 $a+ab^*$

- (\*) NOT CFL = CFL (\*) CFL = may or may not be CFL  
 (\*) CFL  $\cup$  DCFL = CFL (\*) REG  $\cap$  CFL = CFL  
 (\*) CFL  $\cap$  DCFL = Need not be CFL  
 (\*) CFL  $\cap$  CFL = "  
 (\*) DCFL  $\cap$  DCFL = "

dab<sup>n</sup> only  $\cap \{a^nb^nc^n\}$  is not  
 DCFL  
 (CFL)

TURING MACHINE:  $\rightarrow$  It accepts/enumerates/produces REL.

(Unlimited resources and memory)

TM  $\cong$  COMPUTER  
 (Limited Resources & Memory)

DTM  $\cong$  NTM

$S(DTM) : QXT \rightarrow Q \times \Sigma^*, R^*$ .  
 $S(NTM) : QXT \rightarrow 2^{Q \times \Sigma^* \times \{L, R\}}$ .

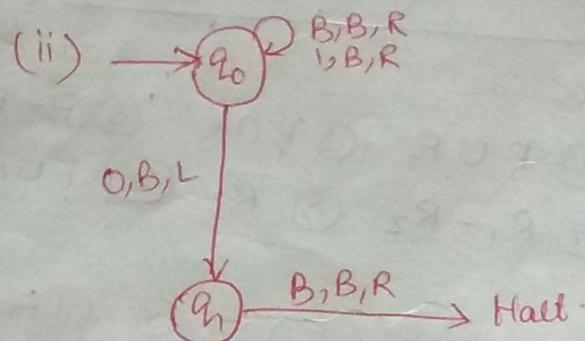
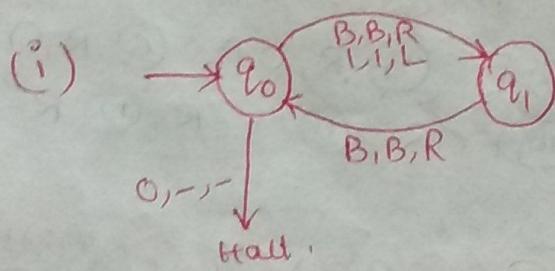
(\*) FA and PDA halts only after reading the entire input,  
 but TM can halt anytime.

Enumerates effectively. { LBA  $\rightarrow$  TM with bounded tape — Always halts.  
 HTM — Unbounded tape — Always halts.  
 Enumerates Not Effectively. { TM — " " — halts for logic only.

$$P(FA) \leq P(PDA) \leq P(PDA) \leq P(LBA) \leq P(HTM) \leq P(TM)$$

(\*) No. of moves in DTM  $\cong$  Time Complexity.  
 (\*) No. of tape cells in DTM  $\cong$  Space Complexity.

Q) UNDERSTAND THE FOLLOWING TMS over  $\Sigma = \{0, 1\}$ .



(Answers)

- (i) String starting with '0' always halts.  
 (ii) String containing '0' as substring always halts.

- ✗  $F = \text{Regular}$  but not Finite      ✗  $\overline{R} = R$   
 ✗  $\overline{\text{DCFL}} = \text{DCFL}$       ✗  $\overline{\text{CFL}} = \text{may or may not be CFL (CSL)}$ .  
 ✗  $\overline{\text{CSL}} = \text{CSL}$       ✗  $\overline{\text{REC}} = \text{REC}$       ✗  $\overline{\text{REL}} = \text{need not be REL}$ .  
 ✗ Decidable = Decidable      ✗  $\overline{SD}$  = Not REL  
 ✗ Reg. But not Finite = Regular (need not be Finite)  
 ✗ DCFL but not Regular = DCFL but not Regular.  
 ✗ CFL but not DCFL = CSL (need not be CFL) (not also DCFL)  
 ✗ NOT REL = RE but not REC, may be Not Rel or Rel but not REC.  
 ① Finite language over  $\Sigma \rightarrow D$       ⑦ REL but not REC  $\rightarrow SD$ .  
 ② Regular "      " "  $\rightarrow D$       ⑧ REL  $\rightarrow SD$   
 ③ DCFL "      " "  $\rightarrow D$       ⑨ Set of ① to ⑧  $\rightarrow SD$ .  
 ④ CFL "      " "  $\rightarrow D$       ⑩  $\{ \text{CFG} \mid \text{CFG is Ambiguous} \} \rightarrow SD$ .  
 ⑤ CSL "      " "  $\rightarrow D$   
 ⑥ REC "      " "  $\rightarrow D$

Decision Properties	FA(Regs)	DPDA	PDA	LBA/ HTM	TM
① Halting Problem   NH	D	D	D	D	SD/NR
② Membership "   NM	D	D	D	D	SD/NR
③ Emptiness "   NE <sub>m</sub>	D	D	D	NR/SD	NR/SD
④ Finiteness "   NF	D	D	D	NR/NR	NR/NR
⑤ Totality "   NT	D	D	NR/SD	NR/SD	NR/NR
⑥ Equivalence "   NE <sub>q</sub>	D	D	NR/SD	NR/SD	NR/NR
⑦ Disjoint "   ND	D	UD	UD	UD	UD
⑧ Set Containment "   NS <sub>c</sub>	D	UD	UD	UD	UD

- (\*) HALTING PROBLEM : A given machine halts or Not.
- (\*) MEMBERSHIP " : Is string 'w' accepted by a Machine 'M'?
- (\*) Emptiness " : Does a machine accept empty language ( $\emptyset$ )?
- (\*) Finiteness " : Does a machine has only finite no. of strings that halts at final?
- (\*) Totality " : Every string  $\in \Sigma^*$  should halt at final.  
( $L = \text{some } " " " \text{ non-final}$ ).
- (\*) Equivalence " : For every string, both machines must halt at either final or non-final, but not both.  
 $L = \text{atleast 1 string, such that } M_1 \text{ halts at final and } M_2 \text{ halts at non-final.}$
- (\*) DISJOINT " : For every string, one machine should accept but other should not accept.  
 $L = \text{atleast 1 string is accepted/rejected by both.}$
- (8) SET CONTAINMENT : Is  $L_1 \subseteq L_2$ ? or  $L_1 \cap \bar{L}_2 = \emptyset$ ?

Some Examples:

- ① Does TM accepts given 'w'?  $\rightarrow$  Membership  $\rightarrow$  SD
- ② Does TM accept some string?  $\rightarrow$  non-emptiness  $\rightarrow$  SD
- ③ Does TM accept only 'w'?  $\rightarrow$  Finiteness  $\rightarrow$  NR
- ④ Can u design a TM?  $\rightarrow$  Decidable.
- ⑤ Can u verify a TM?  $\rightarrow$  Not REL

Some Trivial Questions :

- |  |  |
|--|--|
| <ol style="list-style-type: none"> <li>① <math>\{ \text{DFA}   \text{DFA accepts RL} \}</math></li> <li>② <math>\{ \text{DPDA}   \text{DPDA accepts DCFL} \}</math></li> <li>③ <math>\{ \text{PDA}   \text{PDA accepts CFL} \}</math></li> <li>④ <math>\{ \text{TM}   \text{TM accepts REL} \}</math></li> <li>⑤ <math>\{ \text{HTM}   \text{HTM accepts REC} \}</math></li> </ol> | <div style="border-left: 1px solid black; padding-left: 10px; margin-top: 100px;">           Decidable.         </div> |
|--|--|

Q. Identify D, SD and NR.

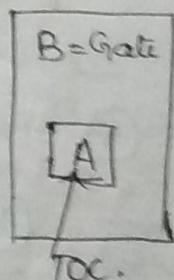
- ① Does given TM accepts Finite Language?
- ② Does " " " Regular " ?
- ③ " " " only string w ?
- ④ {TM | TM reaches a state 'q' with n moves} ?
- ⑤ {TM | TM reaches state 'q'} ?
- ⑥ {TM | " " " " in more than n moves} ?
- ⑦ Does a TM has N states? or  $|T| = 10$ ?
- ⑧  $|L(TM)| = 10$ ?
- ⑨ {CFG | CFG is unambiguous} ?

Answers:

- ① Finiteness - NR    ② Finiteness - NR    ③ Finiteness - NR
- ④ L = Go  $\leq$  moves and check if TM reaches state q — logic  $\geq$  D.  
L = " " " " " " does not " " " — logic  $>$  D.
- ⑤ L = keep checking for state q (if it has to occur, it will occur  
finite moves) — logic  $\rightarrow$  SD
- L = As the string is not present, myc never halts — No logic  $\uparrow$
- ⑥ Same as 5  $\rightarrow$  SD    ⑦ Membership — SD    ⑧ Finiteness — NR
- ⑨ L = Keep checking infinitely for Ambiguity — No logic  $\geq$  NR.  
L = " " " infinitely " " — logic

REDUCIBILITY: \* A  $\leq$  B means A is reducible to B.

\* SWAP & SORT, \* A  $\propto$  B or A  $\leq_m$  B ; A is many to one  
reducible to B.



\* A  $\propto_p$  B means A is polynomially reducible to B.

\* If A  $\leq$  B and B  $\leq$  A, then A = B.

\* If A  $\leq$  B and B  $\leq$  C, and if C is REC and B is REC, then A is REC.

\* If A  $\leq$  B and B  $\leq$  C, C is REC and B is REC, then A is REC.

↓  
Tricky Question.

Assess yourself  
on HOTS

- Assume A ⊂ B, then Answer the following.
- ① If B is decidable, then A is (D/UD)?
  - ② If A " ", then B is
  - ③ If A is undecidable, then B is
  - ④ If B is finite, then A is
  - ⑤ If A is infinite, then B is
  - ⑥ If B takes 100 days, then A takes
  - ⑦ If A " " " ", " B "
  - ⑧ If  $B = \Theta(n^3)$ , then A =
  - ⑨ If  $A = \Theta(n^3)$ , then B =
  - ⑩ If A is NR, then B is
  - ⑪ If A is REL, then B is
  - ⑫ If B is REL, then A is

STUDY Theory  
of Complexity  
from Class Notes

Answers: ① D ② may or may not be D ③ UD ④ finite ⑤ Infinite  
⑥  $\leq 100$  days ⑦  $\geq 100$  days ⑧  $\Theta(n^3)$  ⑨  $\Omega(n^3)$  ⑩ NR  
⑪ may or may not be REL ⑫ REL

### SHORTCUT FOR REGULAR CLOSURE:

$$X \left| \begin{matrix} \cap \\ U \\ \bar{\cdot} \end{matrix} \right| R.L = X$$

Note: REG - CFL  
 $= \text{REG} \cap \overline{\text{CFL}} = \text{REG} \cap \text{CSL} = \text{CSL}$

### \* SPECIAL PROBLEMS ON REGULARS and Non-REGULARS:

- Identify Regulars:
- ①  $\{w | w \in \{0,1\}^*, n_{01}(w) = n_{10}(w)\}$  — Not Reg. <sup>Binary exp. not possible</sup>
  - ②  $\{ \dots \dots \dots , n_{00}(w) = n_{11}(w) \}$  — <sup>Decimal eq. (10) is div. by 1024?</sup>
  - ③  $\{ \dots \dots \dots , \dots \dots \dots \}$  — <sup>27?</sup>
  - ④  $\{ \dots \dots \dots , \dots \dots \dots \}$  —
  - ⑤  $\{w | w \in (a,b)^*, \text{every prefix } s \text{ of } w \text{ satisfies } |n_a(s) - n_b(s)| \leq K\}$

Answers: ① Regular ③ Regular — 11 states ⑤ Regular  
② Non-regular ④ Regular — 4 states

## PUMPING LEMMA (For RLs)

- It is a proof for RLs (or It satisfies Regular Languages).
- It is used to prove Non-regular Languages by Contradiction.
- It can only be applied to Infinite languages, as finite languages are already Regular.
- It cannot be used to check given language is regular or not.
- It can only be used to check why a language is non-regular.

Algorithm:

- ① Choose pumping Constant ( $\epsilon_{\min. \text{ no. of states in DFA}}$ )
- ② Choose a string  $w$ ,  $|w| \geq \text{Pumping Constant}$ .
- ③ Divide  $w$  into 3 parts,  $w = xyz$ , such that,  
 $y \neq \epsilon$ , and  $|xy| \leq \text{Pumping Constant}$ .
- ④  $\forall i \geq 0, xy^i z \in L \text{ iff } L \text{ is Regular.}$

E.g.:  $L = (a+b)^*aaa$

STEP 1: Pumping Constant  $\geq 4$

STEP 2: Consider  $w = baaa$ , such that  $|w| \geq 4$ .

STEP 3: Divide  $w$  into 3 parts, such that  $x = \epsilon, y = b, z = aaa$ .

STEP 4:  $\forall i \geq 0, b^i aaa \in L$

$\therefore L$  is Regular.

E.g.:  $L = a^n b^n$

STEP 1: Pumping Constant  $\geq 2$

STEP 2: Consider  $w = a^n b^n$  such that  $|w| \geq 2$

STEP 3: Divide  $w$  into 3 parts,  $x = a^{n-1}, y = ab, z = b^{n-1}$

STEP 4:  $\forall i \geq 0, a^{n-1} (ab)^i b^{n-1} \in L$ , when  $i=0, a^{n-1} b^{n-1} \in L$   
 $i=1, a^{n-1} abb^{n-1} \in L$   
 $i=2, a^{n-1} abab b^{n-1} \notin L$

$\therefore L$  is not RL by Contradiction.

**ARDEN'S THEOREM:** If  $R = RP + Q$ , then  $R = QP^*$  where P does not contain  $\epsilon$ .

**Note:** If P contains  $\epsilon$ , then R has infinite no. of solutions.

**NOTE!** FA Cannot do - Multiplication of 2 arbitrary Numbers.

→ FA can do - Multiplication of 1 " Number with a constant (by finding the no. of multiplications).

## SIMPLIFICATION OF CFG: (For Parsers to clearly understand CFG)

- ① DELETE NULL PRODUCTIONS
- ② " UNIT "
- ③ " USELESS "

Given CFG.  
 $S \rightarrow Aa|\epsilon$   
 $A \rightarrow aB|AB|\epsilon$   
 $B \rightarrow Sa|Ab|a$

### Elimination of NULL Productions:

- (a) Delete  $S \rightarrow \epsilon$
- |                                |                           |
|--------------------------------|---------------------------|
| $S \rightarrow Aa$             | $S \rightarrow Aa a$      |
| $A \rightarrow aB AB \epsilon$ | $A \rightarrow aB AB B$   |
| $B \rightarrow Sa Ab a a$      | $B \rightarrow Sa Ab a b$ |
- (b) Delete  $A \rightarrow \epsilon$ .

### \* Elimination of UNIT PRODUCTION ( $A \rightarrow B$ )

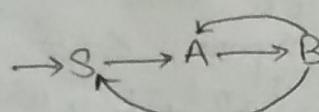
- (a)  $S \rightarrow Aa|a$
- $$A \rightarrow aB|AB|Sa|Ab|a|b$$
- $$B \rightarrow Sa|Ab|a|b.$$

### \* Elimination of USELESS PRODUCTION

STEP 1: Find NT that do not derive any string and delete them.

$$\begin{matrix} NT = \{S, A, B\} \\ \downarrow \quad \downarrow \quad \downarrow \\ a \quad ab \quad aab \end{matrix} \quad \therefore \text{No deletion.}$$

STEP 2: Draw an edge (directed) from A to B if  $A \rightarrow \dots B \dots$



$\therefore \begin{cases} S \rightarrow Aa|a \\ A \rightarrow aB|AB|Sa|Ab|a|b \\ B \rightarrow Sa|Ab|a|b. \end{cases} \rightarrow \text{Simplified CFG.}$

Consider:  $\begin{cases} S \rightarrow Aa|Bb \\ A \rightarrow e|Db \\ D \rightarrow a|Eb \\ E \rightarrow Ea \\ C \rightarrow g \\ F \rightarrow h|e \end{cases}$

STEP 1:  $NT = \{S, A, B, C, F, B\}$

Delete all productions containing E & B

$$\begin{matrix} S \rightarrow Aa \\ A \rightarrow e|Db \\ D \rightarrow a \\ C \rightarrow g \\ F \rightarrow h|e \end{matrix}$$

STEP 2:

$$\begin{matrix} \rightarrow S \rightarrow A \rightarrow D \\ \times C \quad F \times \end{matrix}$$

$\therefore \text{Simplified CFG} =$

$\begin{matrix} S \rightarrow Aa \\ A \rightarrow e|Db \\ D \rightarrow a \end{matrix}$

## Points to Be NOTED:

\* If a DFA has 'x' states and 'y' ifp symbols (alphabets), with designated Initial State, #DFAs =  $x^{(x,y)}$ .

\* No. of Functions for  $F: A \rightarrow B = B^A$ .

\* In CNF, For generating  $|w|=n$ , #Steps =  $(2^{n-1})$ .

$S \rightarrow \epsilon$  { \* In CNF,  $A \rightarrow VV$  or  $A \rightarrow T$  (VENT, T ∈ Terminals).  
only S.      \* In GNF,  $A \rightarrow TVVV\dots$  or  $A \rightarrow T$  (— " —).  
not A,B,etc..

\* If a TM has read only input, but has > 2 Counters, then it is equivalent to basic model of TM.

\* TM that accepts <sup>only</sup> CSLs, cannot leave their input.

\* The language of extended Regular Expressions whose complement is empty is REC but not CSL.

\* Universal TM  $\cong$  TM.

\* Set of all 'N' that are powers of 47 in unary representation is CSL but not CFL. BUP with dynamic programming.

\* Time Complexity of CYK Algorithm is  $O(n^3)$ .

\* Graph Coloring Problem is always decidable (interactable — NP Complete).

\* Register Optimization can be solved using Graph Coloring.

\*  $(A \cap B \cap C)^c = A^c \cup B^c \cup C^c$  [where A,B,C are conditions]

\*  $(RL)^*$  is not always infinite, e.g.,  $\emptyset^* = \emptyset$ .

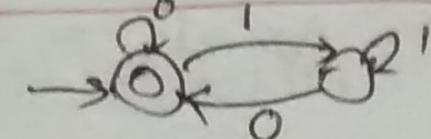
\*  $(RL) \cdot (\text{Non-RL})$  may be RL or not, e.g. -  $\emptyset \cdot a^n b^n = \emptyset$ .

\* Checking if a grammar is RG (CFG/CSG/REC/RE) or not is always decidable.

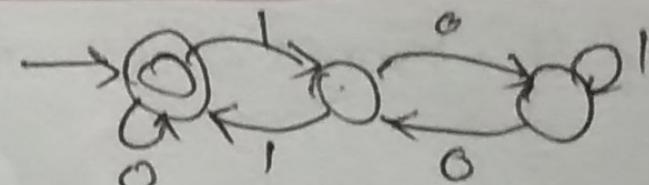
- \* TM accepts REC  $\rightarrow$  not decidable. [Not REC]  
only TM accepts RE  $\rightarrow$  decidable. [REC]

- \* Binary numbers that are divisible/not by 'n' has n states.

(No dead state)



divisible by 2



divisible by 3.

- \* Every TM can be assigned to a unique Binary Number.