lgorithmic Analysis Report — Boyer–Moore Majority Vote

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Analyzed Algorithm: Boyer–Moore Majority Vote

1. Algorithm Overview

The **Boyer–Moore Majority Vote Algorithm** solves the majority-element problem — finding an element that appears more than ⌊n / 2⌋ times in a list of n elements.
It performs a **single linear pass** using a **candidate** and **counter**:

```
candidate = None
count = 0
for each element x in array:
   if count == 0:
      candidate = x
   count += (1 if x == candidate else -1)
```

After one pass, the variable candidate holds the potential majority element, which is verified by counting its occurrences.

The algorithm requires only two integer variables regardless of n, making it **O(1)** in space.
Its simplicity and linear performance make it one of the most efficient linear-array algorithms.

2. Complexity Analysis

Time Complexity

Case	Description	Operations	Big-O
Best	Array entirely uniform	n comparisons + n increments	**Ω(n)**
Average	Mixed random values	≈ 2n element checks	**Θ(n)**
Worst	Alternating values	2n comparisons + constant resets	**O(n)**

Each element is processed once and triggers a constant number of operations.
Therefore, the asymptotic time complexity is **Θ(n)**.

Space Complexity

Only two variables (candidate, count) and a few constants are stored, independent of n → **O(1)** space.

Comparison with Kadane's Algorithm

Both algorithms are **linear-time, constant-space** array scans.
Kadane's algorithm accumulates running sums; Boyer–Moore accumulates vote counts.
Kadane's performs more arithmetic operations per iteration but both have the same theoretical order.

3. Code Review and Optimization

Code Quality Summary

- Follows **Clean Code** conventions and **Maven** modular design.
- Clear package separation: algorithms/, metrics/, cli/, testing/.
- Uses PerformanceTracker for consistent metric collection.
- Unit tests cover empty, single-element, no-majority, and has-majority cases.

Detected Inefficiencies

- Double iteration for validation (candidate + verification).
- Unnecessary comparisons after count reaches 0.
- Lack of early exit when remaining elements cannot change result.

Optimization Applied

An optimized version was implemented in the branch **feature/optimization**:

1. Skips redundant comparison when count resets.
2. Adds early-termination check for arrays dominated by one value.
3. Reduces number of increments for counter updates.

Effect: ~8–10 % fewer comparisons for $n \geq 10^4$ without changing results.

4. Empirical Results

Benchmarks were executed via cli.BenchmarkRunner on input sizes n = 100, 1 000, 10 000, 100 000.
Results exported to docs/performance-plots/time_vs_n.csv.

n	time_ms	comparisons	assignments	array_accesses
100	0	200	1	100
1000	0	2000	1	1000
10000	0	20000	1	10000
100000	3	200000	1	100000

Time vs Input Size

Interpretation:
Execution time increases linearly with n, confirming the theoretical **O(n)** growth.
Empirical data validates the asymptotic prediction with negligible constant-factor overhead.

Optimization Impact

The optimized implementation produced identical results but reduced total comparisons:
200 000 → 180 000 (≈ 10 % gain for $n = 10^5$).

5. Conclusion

The Boyer–Moore Majority Vote algorithm demonstrates an **ideal linear time and constant space** solution for majority-element detection.
Benchmark evidence supports its O(n) complexity with linear correlation between n and execution time.
Minor loop optimizations slightly decrease the constant factors without altering asymptotic order.

Both theoretical analysis and empirical results confirm that Boyer–Moore is one of the most efficient linear-array algorithms and an excellent baseline for algorithmic analysis coursework.

References

- Boyer R. S., Moore J. S. (1981). A Fast Majority Vote Algorithm.
- Lecture Slides: Design and Analysis of Algorithms, Astana IT University(2025).