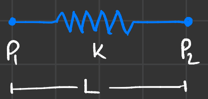
SPRINGS & DAMPER

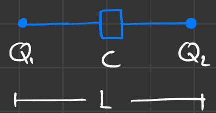
A spring is an element that exerts a restoring force on masses in a dynamic system.



$$F_K = K\left(r_{P_1P_2} - L\right)$$

$$\theta_K = \tan^{-1}\left(\frac{\bar{r}_{P_1P_2}\cdot\hat{j}}{\bar{r}_{P_1P_2}\cdot\hat{i}}\right)$$

A damper is an element that exerts a resistive force on masses in a dynamic system.



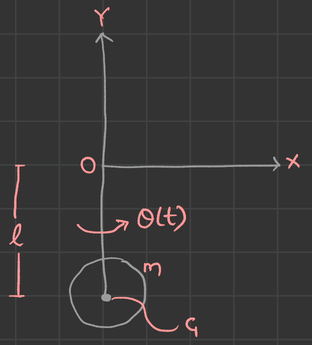
$$F_C = C\,\dot{\bar{r}}_{Q_1Q_2}\cdot\hat{e}_{Q_1Q_2}$$

$$\theta_C(t) = \tan^{-1}\left(\frac{\bar{r}_{Q_1Q_2}\cdot\hat{j}}{\bar{r}_{Q_1Q_2}\cdot\hat{i}}\right)$$

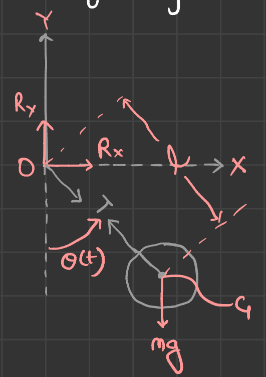
$$\hat{e}_{Q_1Q_2} = \frac{\bar{r}_{Q_1Q_2}}{r_{Q_1Q_2}}$$

Q. Derive the EOM for a simple pendulum.

A.



Free body diagram :



$$\bar{r}_{OG} = l\sin\theta\,\hat{i} - l\cos\theta\,\hat{j}$$

$$\dot{\bar{r}}_{OG} = l\cos\theta\,\dot{\theta}\,\hat{i} + l\sin\theta\,\dot{\theta}\,\hat{j}$$

$$\ddot{\bar{r}}_{OG} = (l\cos\theta\,\ddot{\theta} - l\sin\theta\,\dot{\theta})\,\hat{i} + (l\sin\theta\,\ddot{\theta} + l\cos\theta\,\dot{\theta})\,\hat{j}$$

$$a_{Gx} = l\cos\theta\,\ddot{\theta} - l\sin\theta\,\dot{\theta}$$
$$a_{Gy} = l\sin\theta\,\ddot{\theta} + l\cos\theta\,\dot{\theta}$$

$$\Sigma F_{Gx} = m a_{Gx}$$
$$-T\sin\theta = m a_{Gx}$$
$$T = \frac{-m a_{Gx}}{\sin\theta}$$

$$\Sigma F_{Gy} = m a_{Gy}$$
$$T\cos\theta - mg = m a_{Gy}$$
$$T = \frac{m a_{Gy} + mg}{\cos\theta}$$

$$\therefore \quad \frac{-\cancel{m} a_{Gx}}{\sin\theta} = \frac{\cancel{m} a_{Gy} + \cancel{m} g}{\cos\theta}$$

$$(l\sin\theta\,\dot{\theta} - l\cos\theta\,\ddot{\theta})\cos\theta = \sin\theta\,(l\sin\theta\,\ddot{\theta} + l\cos\theta\,\dot{\theta} + g)$$

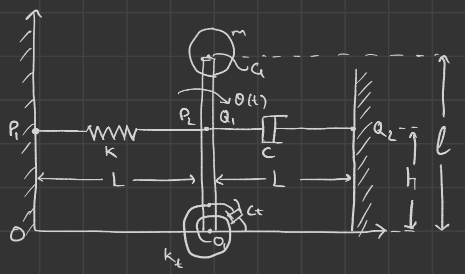
$$\cancel{\sin\theta\cos\theta\,\dot{\theta}} - \cos^2\theta\,\ddot{\theta} = \sin^2\theta\,\ddot{\theta} + \cancel{\sin\theta\cos\theta\,\dot{\theta}} + \frac{g\sin\theta}{l}$$

$$\ddot{\theta}(\sin^2\theta + \cos^2\theta) + g/l \sin\theta = 0$$

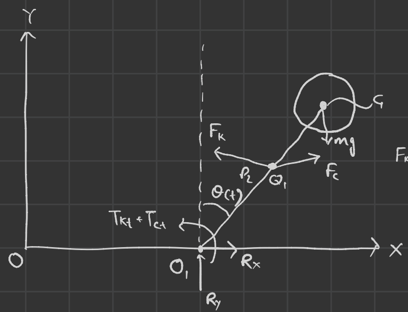
$$\boxed{\ddot{\theta} + \frac{g}{l}\sin\theta = 0}$$

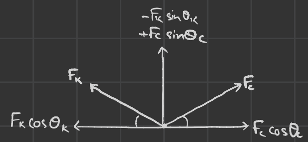
* INVERTED PENDULUM

Q. Derive the EOM for the DOF $\theta(t)$.



A. FBD :-





$$T_{kt} = k_t \theta \qquad T_{ct} = c_t \dot{\theta}$$

Forces :-

$$F_k = (r_{P_1P_2} - L)\, k$$

$$r_{P_1P_2} = |\bar{r}_{P_1P_2}| = \sqrt{(\bar{r}_{P_1P_2} \cdot \hat{i})^2 + (\bar{r}_{P_1P_2} \cdot \hat{j})^2}$$

$$\bar{r}_{P_1P_2} = \bar{r}_{OP_2} - \bar{r}_{OP_1}$$

$$\bar{r}_{OP_2} = (L + h\sin\theta)\,\hat{i} + (h\cos\theta)\,\hat{j}$$

$$\bar{r}_{OP_1} = h\,\hat{j}$$

$$\bar{r}_{P_1P_2} = (L + h\sin\theta)\,\hat{i} + (h\cos\theta - h)\,\hat{j}$$

$$r_{P_1P_2} = \left[(L + h\sin\theta)^2\,\hat{i} + (h\cos\theta - h)^2\,\hat{j} \right]^{1/2}$$

$$F_k = \left[\left[(L + h\sin\theta)^2\,\hat{i} + (h\cos\theta - h)^2\,\hat{j} \right]^{1/2} - L \right] k$$

$$F_c = c\,(\dot{\bar{r}}_{Q_1Q_2} \cdot \hat{e}_{Q_1Q_2})$$

$$\bar{r}_{Q_1Q_2} = \bar{r}_{OQ_2} - \bar{r}_{OQ_1} \qquad \hat{e}_{Q_1Q_2} = \frac{\bar{r}_{Q_1Q_2}}{r_{Q_1Q_2}}$$

$$r_{Q_1Q_2} = \sqrt{(\bar{r}_{Q_1Q_2}\hat{i})^2 + (\bar{r}_{Q_1Q_2}\hat{j})^2}$$

$$\bar{r}_{Q_1Q_2} = (2L\hat{i} + h\hat{j}) - ((L+\sin\theta)\hat{i} + h\cos\theta\hat{j})$$

$$\bar{r}_{Q_1Q_2} = (L+\sin\theta)\hat{i} + (h - h\cos\theta)\hat{j}$$

$$r_{Q_1Q_2} = \left((L+\sin\theta)^2\hat{i} + (h-h\cos\theta)^2\hat{j}\right)^{1/2}$$

$$\hat{e}_{Q_1Q_2} = \frac{(L+\sin\theta)\hat{i} + (h-h\cos\theta)\hat{j}}{\left((L+\sin\theta)^2\hat{i} + (h-h\cos\theta)^2\hat{j}\right)^{1/2}}$$

$$\dot{\bar{r}}_{Q_1Q_2} = \dot{\theta}\cos\theta\,\hat{i} + h\dot{\theta}\sin\theta\,\hat{j}$$

$$F_c = c\left[(\dot{\theta}\cos\theta\,\hat{i} + h\dot{\theta}\sin\theta\,\hat{j}) \cdot \left[\frac{(L+\sin\theta)\hat{i} + (h-\cos\theta)\hat{j}}{((L+\sin\theta)^2\hat{i} + (h-h\cos\theta)^2\hat{j})^{1/2}}\right]\right]$$

Newton's law :-

$$\Sigma F_x = m a_x \qquad \Sigma F_y = m a_y$$

$$\bar{r}_{O4} = (L + h\sin\theta)\,\hat{i} + h\cos\theta\,\hat{j}$$

$$\dot{\bar{r}}_{O4} = h\dot{\theta}\cos\theta\,\hat{i} - h\dot{\theta}\sin\theta\,\hat{j}$$

$$\ddot{\bar{r}}_{O4} = (h\ddot{\theta}\cos\theta - h\dot{\theta}^2\sin\theta)\hat{i} - (h\ddot{\theta}\sin\theta + h\dot{\theta}^2\cos\theta)\hat{j}$$

$$\ddot{\bar{r}}_{O4} = \underbrace{(h\ddot{\theta}\cos\theta - h\dot{\theta}^2\sin\theta)}_{a_{4x}}\hat{i} + \underbrace{(-h\ddot{\theta}\sin\theta - h\dot{\theta}^2\cos\theta)}_{a_{4y}}\hat{j}$$

$$\sum F_x = m a_{G x}$$

$$-F_K \cos\theta_K + F_C \cos\theta_C + R_x = m\left(h\ddot{\theta}\cos\theta - h\dot{\theta}^2 \sin\theta\right)$$

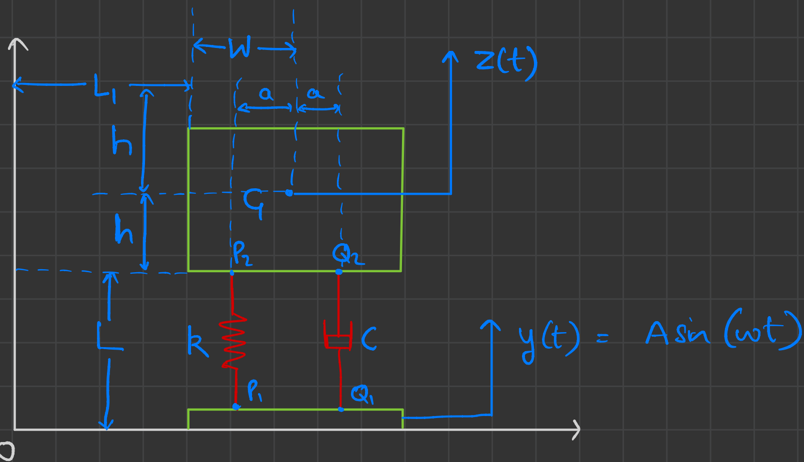
$$\sum F_y = m a_{G y}$$

$$-F_K \sin\theta_K + F_C \sin\theta_C + R_y - mg = m\left(-h\ddot{\theta}\sin\theta - h\dot{\theta}^2 \cos\theta\right)$$

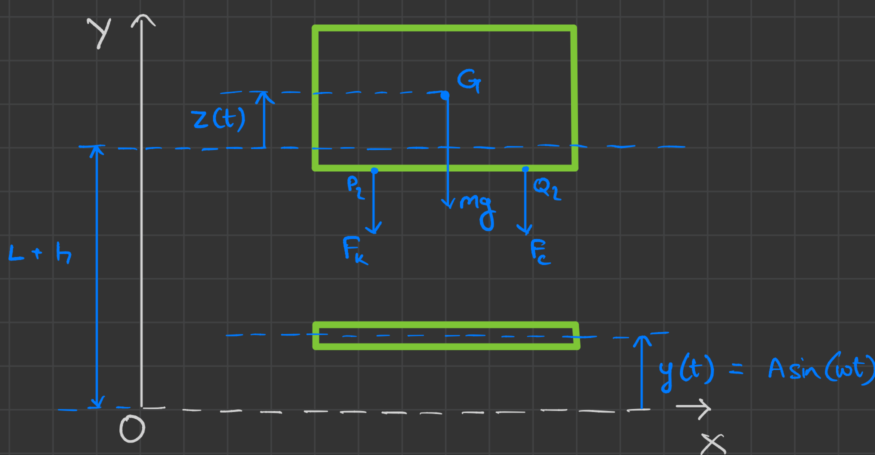
$$\sum M_{O_1} = I_{O_1}(-\ddot{\theta})$$

$$\begin{aligned}\sum M_{O_1} &= -ml^2\ddot{\theta} \\ &= (-F_K \sin\theta_K + F_C \sin\theta_C) h \sin\theta + F_K \cos\theta_K h \cos\theta \\ &\quad - F_C \cos\theta_C h \cos\theta - mgl\sin\theta + k_t\theta + G\dot{\theta}\end{aligned}$$

Base Excitation



FBD :-



$$\bar{r}_{OG} = (L+h)\,\hat{j} + (L_1+w)\,\hat{i} + z(t)\,\hat{j}$$

$$\dot{\bar{r}}_{OG} = \dot{z}\,\hat{j} + 0\,\hat{i}$$

$$\ddot{\bar{r}}_{OG} = \ddot{z}\,\hat{j} + 0\,\hat{i}$$ (Acceleration)

Forces

$$F_k = k\left(\bar{r}_{P_1P_2} - L\right)$$

$$\bar{r}_{P_1P_2} = \bar{r}_{OP_2} - \bar{r}_{OP_1}$$

$$\bar{r}_{OP_1} = (L_1 + w - a)\hat{\imath} + y(t)\hat{\jmath}$$

$$\bar{r}_{OP_2} = (L_1 + w - a)\hat{\imath} + (L + z(t))\hat{\jmath}$$

$$\bar{r}_{P_1P_2} = (z(t) - y(t) + L)\hat{\jmath}$$

$$r_{P_1P_2} = z(t) - y(t) + L$$

So, $F_k = k\left(z(t) - y(t)\right)$

$$F_c = c\left(\dot{z}(t) - \dot{y}(t)\right)$$

$$= c\,\dot{\bar{r}}_{Q_1Q_2} \cdot \hat{e}_{Q_1Q_2}$$

$$\hat{e}_{Q_1Q_2} = \hat{\jmath} = \frac{\bar{r}_{Q_1Q_2}}{r_{Q_1Q_2}}$$

Newton's Laws

$$\sum F_y = m\,a_{ay}$$

$$-mg - k(z-y) - c(\dot{z}-\dot{y}) = m\ddot{z}$$

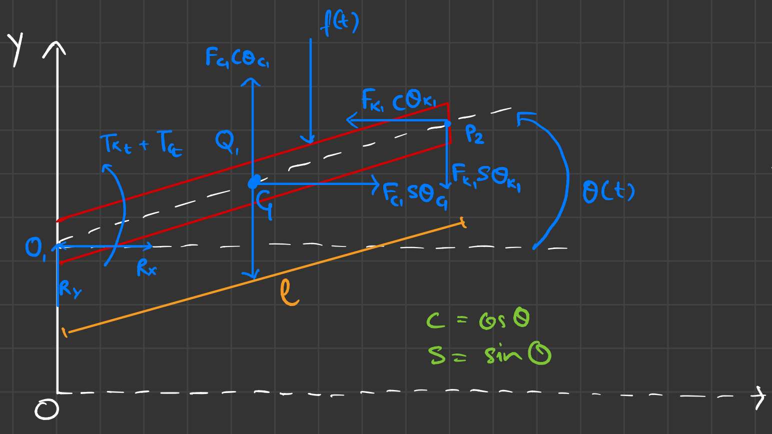
$$m\ddot{z} + c\dot{z} + kz = -mg + c\dot{y} + ky$$

$$z(t) = z_1(t) + \Delta_{st}$$

$$m\ddot{z}_1 + c\dot{z}_1 + k(\cancel{\Delta_{st}} + z_1) = \cancel{-mg} + c\dot{y} + ky$$

HORIZONTAL BAR

.... Continuation



f(t) stays vertical

$$T_{kt} = K_t \theta \qquad T_{ct} = C_t \dot{\theta}$$

$\theta(t)$ is small $\qquad F_{k_1} = K_1 \ell \theta \qquad F_{c_1} = -C_1 \frac{\ell}{2} \dot{\theta}$

Acceleration

$$\bar{r}_{oG} = L_1 \hat{j} + \frac{\ell}{2}\cos\theta\,\hat{i} + \frac{\ell}{2}\sin\theta\,\hat{j}$$

$$\dot{\bar{r}}_{oG} = -\frac{\ell}{2}\dot{\theta}(\sin\theta)\,\hat{i} + \frac{\ell}{2}\dot{\theta}(\cos\theta)\,\hat{j}$$

$$\ddot{\bar{r}}_{oG} = \left(-\frac{\ell}{2}\ddot{\theta}\sin\theta - \frac{\ell}{2}\dot{\theta}^2\sin\theta\right)\hat{i} + \left(\frac{\ell}{2}\ddot{\theta}\cos\theta - \frac{\ell}{2}\dot{\theta}^2\sin\theta\right)\hat{j}$$

$$a_{Gx} = \ddot{\bar{r}}_{oG} \cdot \hat{i} \qquad a_{Gy} = \ddot{\bar{r}}_{oG} \cdot \hat{j}$$

Newton's Laws

$$\sum F_x = m a_{a_x} \qquad \sum M_{G} = +I_G \ddot{\theta} \qquad I_G = \tfrac{1}{12} m l^2$$

$$\sum F_y = m a_{G_y} \qquad \sum M_{O_1} = +I_{O_1} \ddot{\theta} \qquad I_{O_1} = \frac{1}{3} m l^2$$

$\sum F_x$:-

$$R_x + F_{C_1} \cos\theta_{C_1} - F_{K_1} \cos\theta_{K_1} = -m\left(\frac{l}{2}\ddot{\theta}\sin\theta + \frac{l}{2}\dot{\theta}^2\sin\theta\right)$$

$\sum F_y$:-

$$+R_y - mg + F_{G}\sin\theta_{G} - f(t) - F_{K_1}\sin\theta_{K_1} = m\left(\frac{l}{2}\ddot{\theta}\cos\theta - \frac{l}{2}\dot{\theta}^2\sin\theta\right)$$

$\sum M_G$:-

$$\sum M_G = I_G \ddot{\theta}$$

$$\frac{1}{12} m l^2 \ddot{\theta} = -T_{Kt} - T_{ct} - R_y \frac{l}{2}\cos\theta + R_x \frac{l}{2}\sin\theta - F_{K_1}\sin\theta_{K_1}\frac{l}{2}\cos\theta + F_{K_1}\cos\theta_{K_1}\frac{l}{2}\sin\theta - f(t)\,a\cos\theta$$

$\sum M_{O_1}$:-

$$\sum M_{O_1} = I_{O_1}\ddot{\theta}$$

$$\frac{1}{3} m l^2 \ddot{\theta} = -T_{Kt} - T_{ct} - mg\frac{l}{2}\cos\theta + F_{C_1}\sin\theta_{C_1}\, l\cos\theta - F_{G}\cos\theta_{C_1}\frac{l}{2}\sin\theta - f(t)\left[\frac{l}{2} + a\right]\cos\theta$$

$$- F_{K_1} \sin \theta_{K_1} \, l \cos \theta \quad + \quad F_{K_1} \cos \theta_{K_1} \, l \sin \theta$$

When θ is small,

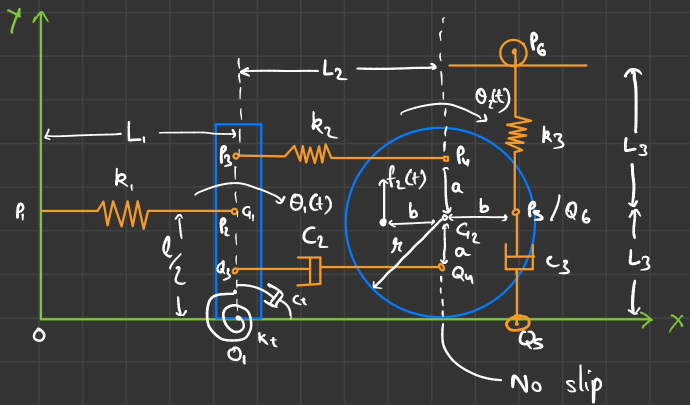
$$\cos \theta = 1$$
$$\sin \theta = 0$$
$$\sin \theta_K = 1$$
$$\cos \theta_K = 0$$
$$\sin \theta_c = 1$$
$$\cos \theta_c = 0$$

Equation's of Motions :

$$\frac{1}{3} m l^2 \ddot{\theta} = -T_{Kt} - T_{ct} - mg \frac{l}{2} + F_{c_1} \frac{l}{2} - f(t) \left(\frac{l}{2} + a \right) - F_{K_1} l$$

Differential Equation governing the motion of the bar

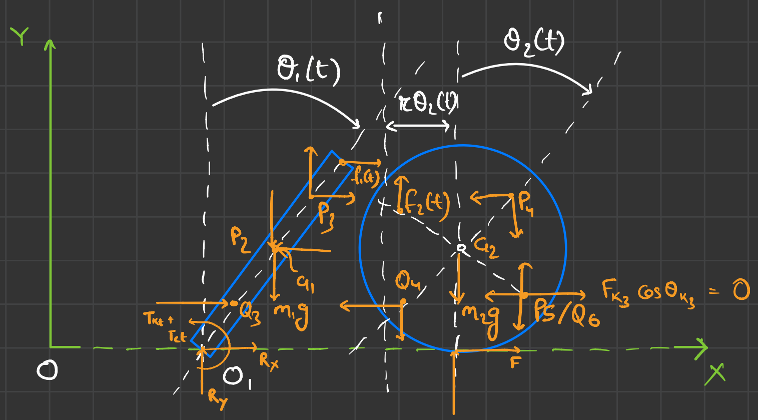
Induced Vibrations



$f_1(t)$ stays horizontal
$f_2(t)$ stays vertical

$\theta_1(t) = A\cos(\omega t)$

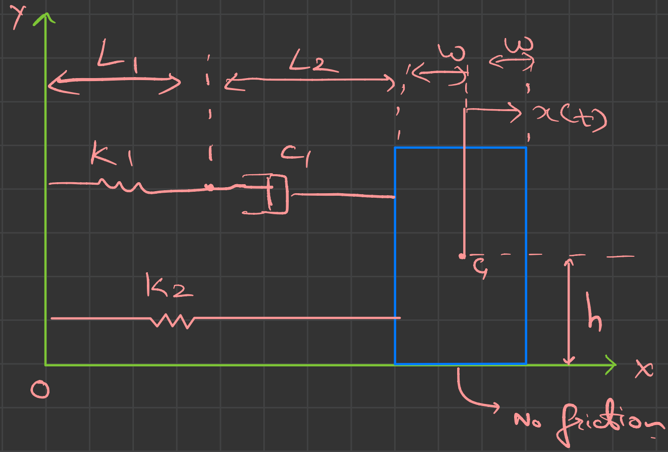
Free Body Diagram



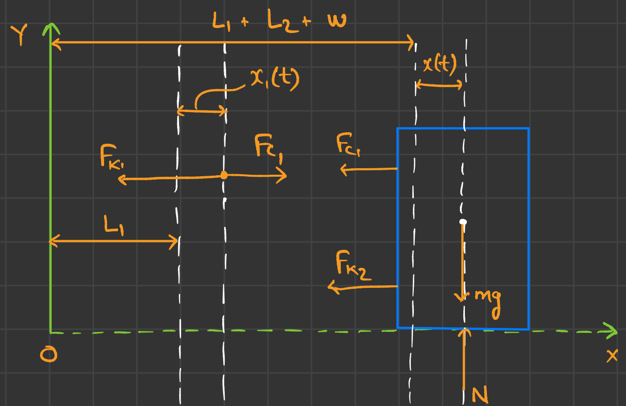
$I_{O_1} = \frac{1}{3} m_1 \ell^2$

$I_2 = \frac{3}{2} m_2 r^2$

1.5 DoF



Free Body Diagram



$$F_{k_1} = k_1 x_1$$

$$F_{k_2} = k_2 x$$

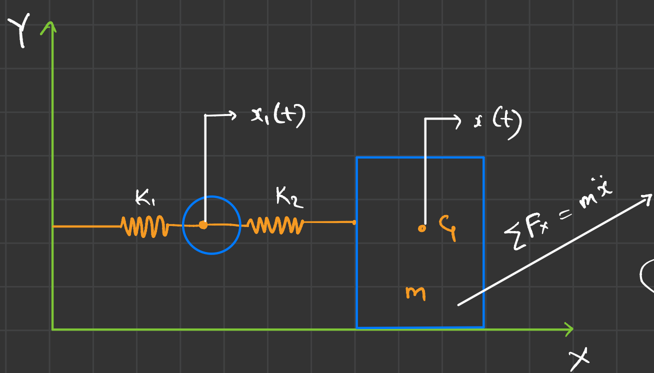
$$F_{c_1} = c_1 (\dot{x} - \dot{x}_1)$$

$$c_1(\dot{x}-\dot{x}_1) = k_1 x_1$$

For mass m,

$$\sum F_x = m\ddot{x}$$
$$m\ddot{x} = -F_{k_2} - F_{c_1}$$
$$m\ddot{x} = -k_2 x - c_1(\dot{x}-\dot{x}_1)$$

$$m\ddot{x} + c_1\dot{x} + k_2 x - c_1\dot{x}_1 = 0$$



$$m\ddot{x} = -k_2(x-x_1)$$
$$k_1 x_1 = k_2(x-x_1)$$
$$(k_1+k_2)x_1 = k_2 x$$

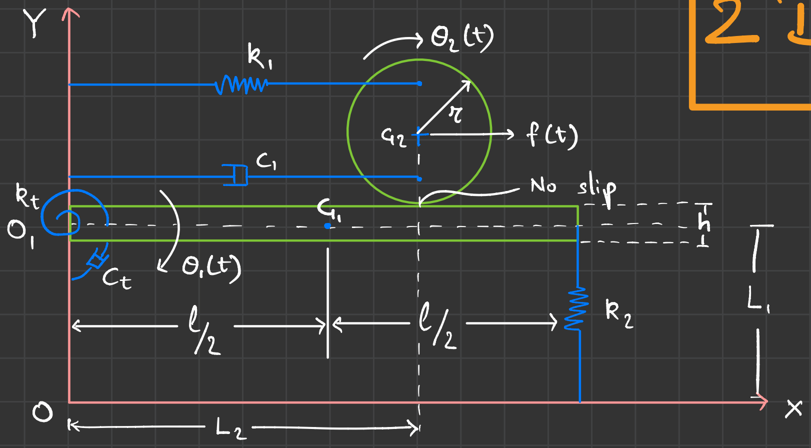
$$x_1 = \left(\frac{k_2}{k_1+k_2}\right)x$$

$$m\ddot{x} + k_2\left(x - \frac{k_2}{k_1+k_2}x\right) = 0$$

$$m\ddot{x} + \left(\frac{k_1 k_2}{k_1+k_2}\right)x = 0$$

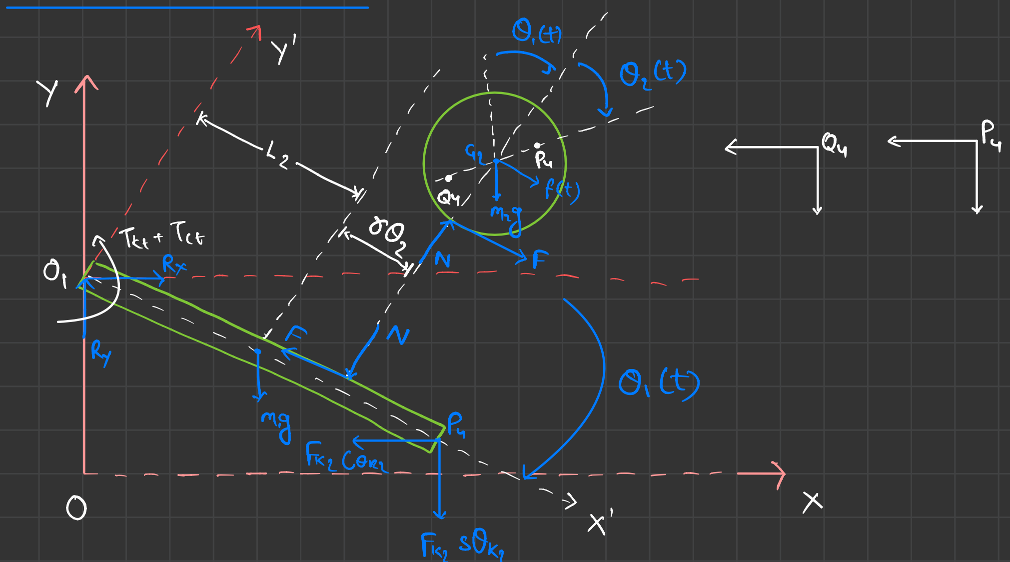
$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$$

2 DoF



f(t) stays parallel to the bar

Free Body Diagram :-



Rotation of Cylinder :- $-(\theta_1(t)+\theta_2(t))\hat{k}$

$\hat{i}' = \cos\theta_1\,\hat{i} - \sin\theta_1\,\hat{j}$ $\quad\quad$ $\dot{\hat{i}}' = -\dot{\theta}_1\,\hat{j}'$

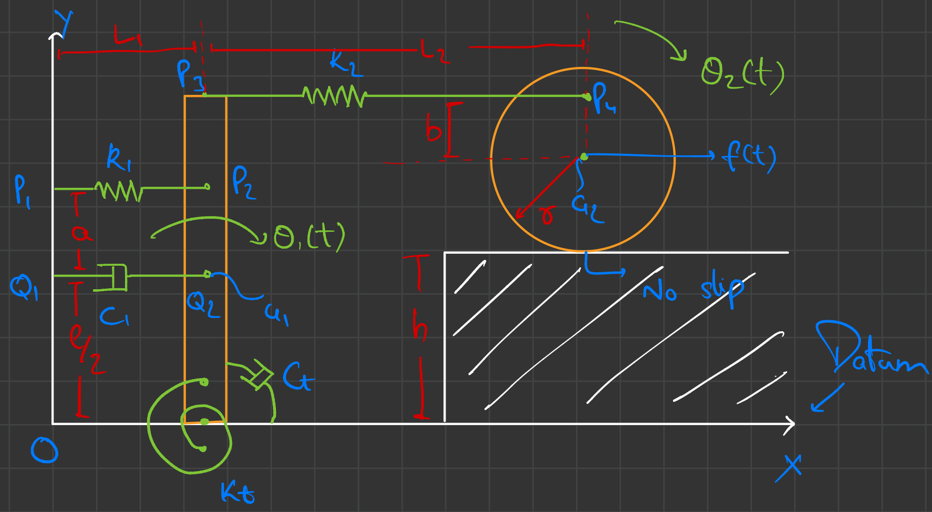
$\hat{j}' = \sin\theta_1\,\hat{i} + \cos\theta_1\,\hat{j}$ $\quad\quad$ $\dot{\hat{j}}' = \dot{\theta}_1\,\hat{i}'$

$$\bar{r}_{OP_4} = (L_2 + r\theta_2)\hat{i}' + L_1\hat{j} + \frac{h}{2}\hat{j} + r\hat{j}' + a\cos\theta_2\hat{j}' + a\sin\theta_2\,\hat{i}'$$

Power & Energy

Kinetic Energy , Potential Energy , Power dissipated

$$T = \frac{1}{2} m_1 \overline{\dot{r}_{OG_1}} \cdot \overline{\dot{r}_{OG_1}}$$



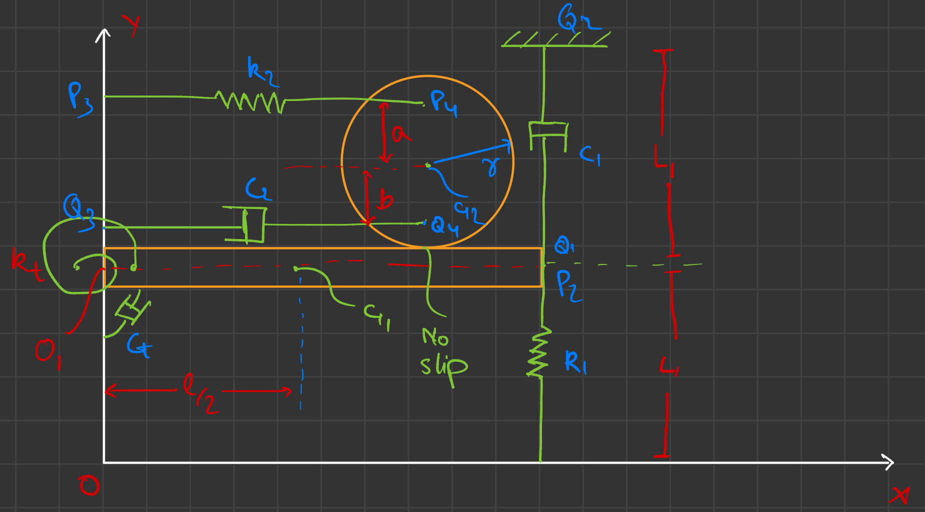
$$KE = \underbrace{\frac{1}{2} m_1 \overline{\dot{r}_{OG_1}} \cdot \overline{\dot{r}_{OG_1}} + \frac{1}{2} I_{G_1} \dot{\theta}_1^2}_{\frac{1}{2} I_{O_1} \dot{\theta}_1^2} + \frac{1}{2} m_2 \overline{\dot{r}_{OG_2}} \cdot \overline{\dot{r}_{OG_2}} + \frac{1}{2} I_{G_2} \dot{\theta}_2^2$$

$$I_{O_1} = I_{G_1} + m\left(\frac{\ell}{2}\right)^2$$

$$PE = \frac{1}{2} k_1 \left(r_{P_1P_2} - L_1\right)^2 + \frac{1}{2} k_2 \left(r_{P_3P_4} - L_2\right)^2 + \frac{1}{2} k_t \theta_1^2 + m_1 g\, \overline{r_{OG_1}} \cdot \hat{j}$$

$$+ m_2 g\, \overline{r_{OG_2}} \cdot \hat{j}$$

$$P_{diss} = c_1 \left(\dot{\bar{r}}_{Q_1Q_2} \cdot \hat{e}_{Q_1Q_2} \right)^2 + c_t \dot{\theta}_1^2$$



$$(KE)_{cyl} = \frac{1}{2} m_2 \dot{\bar{r}}_{OG_2} \cdot \dot{\bar{r}}_{OG_2} + \frac{1}{2} I_{G_2} (\dot{\theta}_1 + \dot{\theta}_2)^2$$

$$(KE)_{BAR} = \frac{1}{2} m_1 \dot{\bar{r}}_{OG_1} \cdot \dot{\bar{r}}_{OG_1} + \frac{1}{2} I_{G_1} \dot{\theta}_1^2 = \frac{1}{2} I_{O_1} \dot{\theta}_1^2$$

TEE

$$\hat{i}' = \cos\theta_1\,\hat{i} - \sin\theta_1\,\hat{j}$$
$$\hat{j}' = \sin\theta_1\,\hat{i} + \cos\theta_1\,\hat{j}$$

$$\hat{i}' = \cos\theta_1\,\hat{i} - \sin\theta_1\,\hat{j}$$
$$\hat{j}' = \sin\theta_1\,\hat{i} + \cos\theta_1\,\hat{j}$$

$$F_{k_1} = k\left(r_{P_1P_2} - L_1\right)$$

$$\bar{r}_{OP_1}\;,\quad \bar{r}_{OP_2}\;,\quad \bar{r}_{P_1P_2} = \bar{r}_{OP_2} - \bar{r}_{P_1}$$

$$r_{P_1P_2} = \left[\left(\bar{r}_{P_1P_2}\cdot\hat{i}\right)^2 + \left(\bar{r}_{P_1P_2}\cdot\hat{j}\right)^2\right]^{1/2}$$

$$\theta_{k_1}(t) = \tan^{-1}\left(\frac{\bar{r}_{P_1P_2}\cdot\hat{j}}{\bar{r}_{P_1P_2}\cdot\hat{i}}\right)$$

$$t_{kt} = K_t\,\theta$$
$$t_{ct} = C_t\,\dot{\theta}$$

$$F_{c_1} = c_1\,\dot{\bar{r}}_{Q_1Q_2}\cdot\hat{e}_{Q_1Q_2}$$

$$\hat{e}_{Q_1Q_2} = \frac{\bar{r}_{Q_1Q_2}}{r_{Q_1Q_2}}$$

$$KE = \frac{1}{2}m_1\,\dot{\bar{r}}_{OG_1}\cdot\dot{\bar{r}}_{OG_1} + \frac{1}{2}m_2\,\dot{\bar{r}}_{OG_2}\cdot\dot{\bar{r}}_{OG_2} + \frac{1}{2}I_{G_1}\dot{\theta}_1^2 + \frac{1}{2}I_{G_2}\dot{\theta}_2^2$$

$$PE = \frac{1}{2}k_1\left(r_{P_1P_2} - L_1\right)^2 + \frac{1}{2}k_2\left(r_{P_3P_4} - L_2\right)^2 + \frac{1}{2}k_t\theta_1^2 + m_1 g\,\bar{r}_{OG_1}\cdot\hat{j}$$
$$+\, m_2 g\,\bar{r}_{OG_2}\cdot\hat{j}$$

$$P_{in} = f_2(t)\,\hat{i}\cdot\dot{\bar{r}}_{OG_2} + M(t)\,\hat{k}\cdot\dot{\theta}_1\,\hat{k}$$

$$P_{diss} = \left(C_t\dot{\theta}_1\right)\dot{\theta}_1 + C_1\left(\dot{\bar{r}}_{Q_1Q_2}\cdot\hat{e}_{Q_1Q_2}\right)^2 + C_2\left(\dot{\bar{r}}_{Q_3Q_4}\cdot\hat{e}_{Q_3Q_4}\right)^2$$

$$KE = \frac{1}{2} m_1 \dot{\bar{r}}_{OG_1} \cdot \dot{\bar{r}}_{OG_1} + \frac{1}{2} m_2 \dot{\bar{r}}_{OG_2} \cdot \dot{\bar{r}}_{OG_2} + \frac{1}{2} I_{G_1} \dot{\theta}_1^2 + \frac{1}{2} I_{G_2} \dot{\theta}_2^2$$

$$PE = \frac{1}{2} k_1 (r_{P_1P_2} - L_1)^2 + \frac{1}{2} k_2 (r_{P_3P_4} - L_2)^2 + \frac{1}{2} k_t \theta_1^2 + m_1 g \bar{r}_{OG_1} \cdot \hat{j} + m_2 g \bar{r}_{OG_2} \cdot \hat{j}$$

$$KE = \frac{1}{2} m_1 \dot{\bar{r}}_{OG_1} \cdot \dot{\bar{r}}_{OG_1} + \frac{1}{2} m_2 \dot{\bar{r}}_{OG_2} \cdot \dot{\bar{r}}_{OG_2} + \frac{1}{2} I_{G_1} \dot{\theta}_1^2 + \frac{1}{2} I_{G_2} \dot{\theta}_2^2$$

$$PE = \frac{1}{2} k_1 (r_{P_1P_2} - L_1)^2 + \frac{1}{2} k_2 (r_{P_3P_4} - L_2)^2 + \frac{1}{2} k_t \theta_1^2 + m_1 g \bar{r}_{OG_1} \cdot \hat{j} + m_2 g \bar{r}_{OG_2} \cdot \hat{j}$$

$$TE = \frac{1}{2} m_1 \dot{\bar{r}}_{OG_1} \cdot \dot{\bar{r}}_{OG_1} + \frac{1}{2} m_2 \dot{\bar{r}}_{OG_2} \cdot \dot{\bar{r}}_{OG_2} + \frac{1}{2} I_{G_1} \dot{\theta}_1^2 + \frac{1}{2} I_{G_2} \dot{\theta}_2^2$$

$$PE = \frac{1}{2} k_1 (r_{P_1P_2} - L_1)^2 + \frac{1}{2} k_2 (r_{P_3P_4} - L_2)^2 + \frac{1}{2} k_t \theta_1^2 + m_1 g \bar{r}_{OG_1} \cdot \hat{j} + m_2 g \bar{r}_{OG_2} \cdot \hat{j}$$

$$P_{in} = f_2(t)\, \hat{i} \cdot \dot{\bar{r}}_{OG_2} + M_1(t)\, \hat{k} \cdot \dot{\theta}_1 \hat{k}$$

$$P_{diss} = c_t \dot{\theta}_1^2 + c_1 (\dot{\bar{r}}_{Q_1Q_2} \cdot \hat{e}_{Q_1Q_2})^2 + c_2 (\dot{\bar{r}}_{Q_3Q_4} \cdot \hat{e}_{Q_3Q_4})^2$$