(Book :- Fundamental of Robotics - Robert)

Categories of Robots :-

1. Hard Automation (no. of units more)

Robot does only one application through its lifecycle.

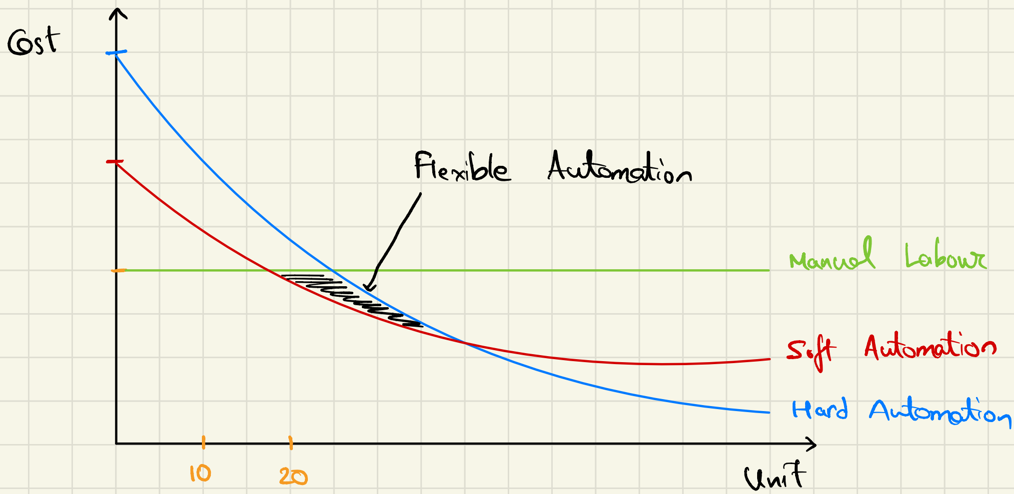
Ex :- In car assembly, robot picks door & places. Here if the car model changes, it wouldn't matter to the robot.

2. Soft Automation

Robot that can be used for multiple applications.

Ex :- Paint job for cars. Different models of car require different programming of the robot

3. Manual Labour



Classification of Robots

1. Drives used

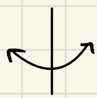
Motors — AC Servo motors
DC Brushless motors (low - medium load carrying capacity)
Stepper motors
— Efficiency, accuracy, low maintainance

Hydraulic Drives — Granes in ports (High load carrying capacity)
— High maintainance

Pneumatic Drives — (not for moving robots)

2. Workspace

Locus of all points to which an end effector can reach.
Defined by major axis (3)

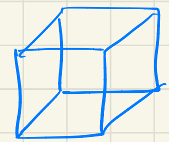
Joints :- i) Revolute (R)  (Rotational motion)

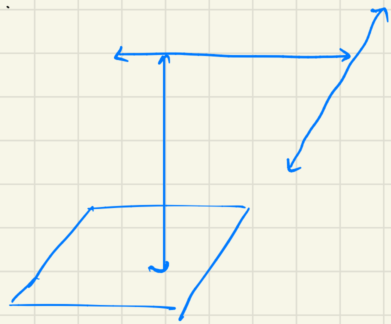
ii) Prismatic (P)  (linear motion)

i) Suppose 3 Axis :- 1 2 3

P P P

This means that the robot is a cartesian robot. It's workspace will be a cube.

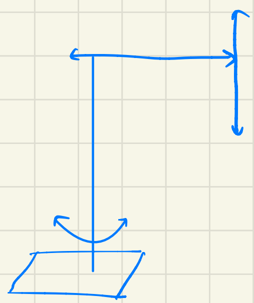




Ex :- CNC, 3D printers

ii) Axis :- 1 2 3

R P P

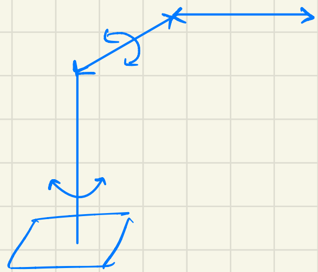


Cylindrical robot.

Workspace = cylinder

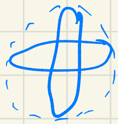
iii) Axis :- 1 2 3

R R P



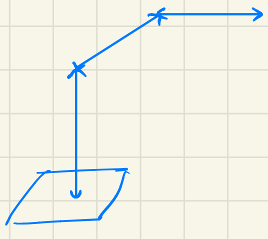
Spherical robot

Workspace = sphere



iv) Axis :- 1 2 3

R R R



Articulated robot

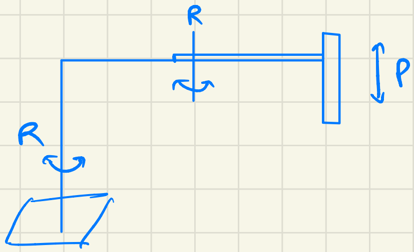
Can reach max points in a defined workspace. So it can also damage itself.

v) Axis :- 1 2 3

R R P

SCARA robot

special purpose robot



high vertical precision

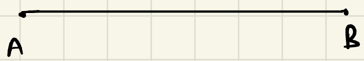
used in PCB manufacturing.

Selective Compliance articulated robotic arm.

Type of motion

Continuous Path motion

(tool move in a prescribed path and speed in 3 dimensional space)

→ 

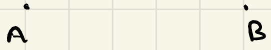
In welding, the tip should be continuously touching from start to end.

→ Direction (trajectory), timing

Point-to-point Operation

(tool moves to a sequence of discrete points)

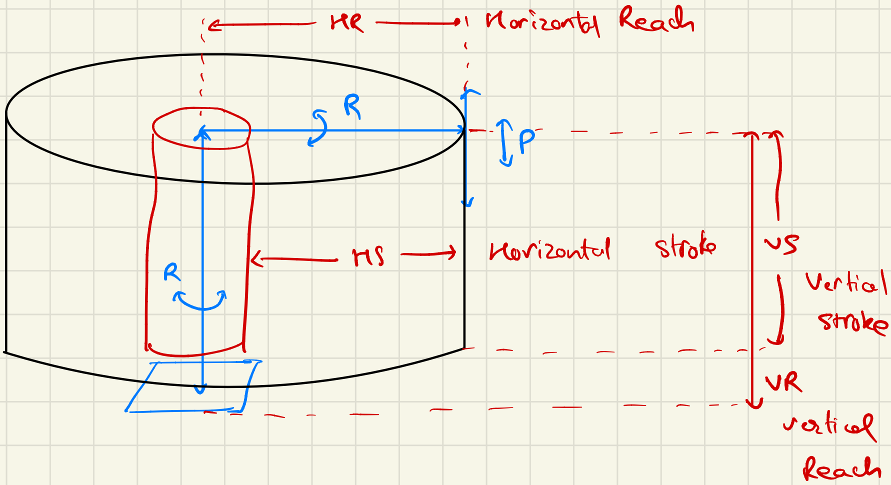
→ Spot welding



→ Direction doesn't matter.
Welding is done at the start and end point only.

Specification of Robots

Reach and Stroke



$$S \leq R$$

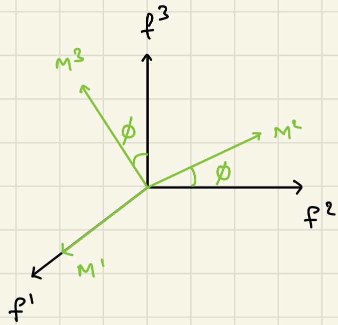
stroke   reach

In case of Articulated (RRR),
$S = R$

∴ We program the robot, to have software breakpoints.

Hardware breakpoints ⇒ limit switch

Yaw

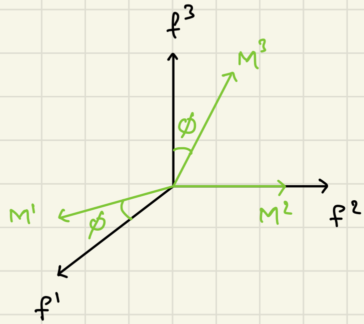


$$R(\phi) = \begin{bmatrix} f^1m^1 & f^1m^2 & f^1m^3 \\ f^2m^1 & f^2m^2 & f^2m^3 \\ f^3m^1 & f^3m^2 & f^3m^3 \end{bmatrix}$$

$$R_1(\phi) = R(\phi) = \begin{bmatrix} C_0 & C_{90} & C_{90} \\ C_{90} & C_\phi & C(C_{90}+\phi) \\ C_{90} & C(90-\phi) & C_\phi \end{bmatrix}$$

$$R_1(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C_\phi & -S_\phi \\ 0 & S_\phi & C_\phi \end{bmatrix}$$

Pitch

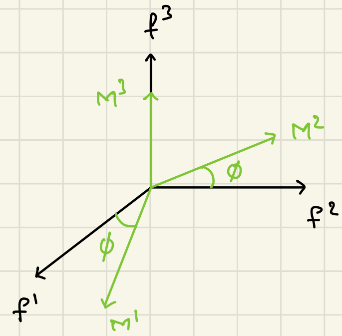


$$R(\phi) = \begin{bmatrix} f^1 m^1 & f^1 m^2 & f^1 m^3 \\ f^2 m^1 & f^2 m^2 & f^2 m^3 \\ f^3 m^1 & f^3 m^2 & f^3 m^3 \end{bmatrix}$$

$$R_2(\phi) = R(\phi) = \begin{bmatrix} C\phi & C_{90} & C_{(90-\phi)} \\ C_{90} & C_0 & C_{90} \\ C_{(90+\phi)} & C_{90} & C\phi \end{bmatrix}$$

$$R_2(\phi) = \begin{bmatrix} C\phi & 0 & S\phi \\ 0 & 1 & 0 \\ -S\phi & 0 & C\phi \end{bmatrix}$$

Roll



$$R(\phi) = \begin{bmatrix} f^1 m^1 & f^1 m^2 & f^1 m^3 \\ f^2 m^1 & f^2 m^2 & f^2 m^3 \\ f^3 m^1 & f^3 m^2 & f^3 m^3 \end{bmatrix}$$

$$R_3(\phi) = \begin{bmatrix} C\phi & -S\phi & 0 \\ S\phi & C\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Q. Consider 2 coordinate frames. Suppose your mobile coordinate is moved by angle $\pi/2$ along fixed axis of the fixed coordinate frame. Find :-

i) The resultant coordinate transformation matrix

ii) Suppose point P is having the coordinate as $[2, 0, 3]^T$ with respect to mobile coordinate frame. Find coordinates of point P with respect to fixed coordinate frame

iii) Suppose Q is a point having fixed coordinate as $[5, 10, 0]^T$. Find the coordinate of point Q with respect to mobile.

A. $[2, 0, 3]^T = [p]^M$

$\longrightarrow$ coordinates of P wrt mobile coordinate frame

$$\underbrace{M \rightarrow F \rightarrow f_1 \rightarrow \pi/2}_{\text{motion}}$$

(write the fundamental matrix in exam)

i) $R_1(\phi) = R_1(\pi/2) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\pi/2 & -S\pi/2 \\ 0 & S\pi/2 & C\pi/2 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

ii)

$$[P]^f = A[P]^M$$

$$= R_1(\phi)[P]^M$$

$$[P]^f = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}$$

$$[P]^f = \begin{bmatrix} 2 & -3 & 0 \end{bmatrix}^T$$

iii)

$$[Q]^M = A^T [P]^f$$

$$[Q]^M = \begin{bmatrix} 5 & 0 & -10 \end{bmatrix}^T$$

R_1 = Yaw

R_2 = Pitch

R_3 = Roll

Q. Suppose we move mobile coordinate frame (M) with respect to f_1 axis by angle π. Then we move mobile coordinate frame (M) with respect to f_2 axis by angle $\pi/2$.

i) Suppose point p having coordinates as $[5, 3, 2]^T$ with respect to mobile coordinate frame. Find point p wrt fixed coordinate frame.

ii) Find $[m']^F$ after the rotations.

iii) Draw the vector diagram.

A) Given :-

$M \to F \to f_1 \to \pi$

$M \to F \to f_2 \longrightarrow \pi/2$

$$[P]^M = [5 \quad 3 \quad 2]^T$$

$$[m']^M = [1 \quad 0 \quad 0]^T$$

$\because M = \{m_1, m_2, m_3\}$

but only m_1 given hence m_2 & $m_3 = 0$

Find :-

$$[p]^F \; \& \; [m']^F$$

$$R_1(\phi) = R_1(\pi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\pi & -S\pi \\ 0 & S\pi & C\pi \end{bmatrix}$$

$$R_1(\phi) = R_1(\pi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$R_2(\phi) = R_2(\pi/2) = \begin{bmatrix} C_{\pi/2} & 0 & S_{\pi/2} \\ 0 & 1 & 0 \\ -S_{\pi/2} & 0 & C_{\pi/2} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

Composite Rotation Algorithm

i) When we have a motion of M wrt F axis then pre multiply the resultant rotation matrix

ii) If mobile coordinate frame M is moved wrt M axis, then post multiply the resultant matrix.

pre multiply $\Rightarrow$ last rotation $\times$ 1^{st} rotation

post multiply $\Rightarrow$

① $R_1(\pi)$ $M \to F$

② $R_2(\pi/2)$ $M \to F$

$$A = R_2(\pi/2)\ R_1(\pi)$$

$\because$ Composite rot^n algorithm (i) point

$$A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

i) $$[P]^F = A\,[P]^M$$

$$= \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 5 \\ 3 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -3 & -5 \end{bmatrix}^T$$

ii) $$[m']^F = A\,[m']^M$$

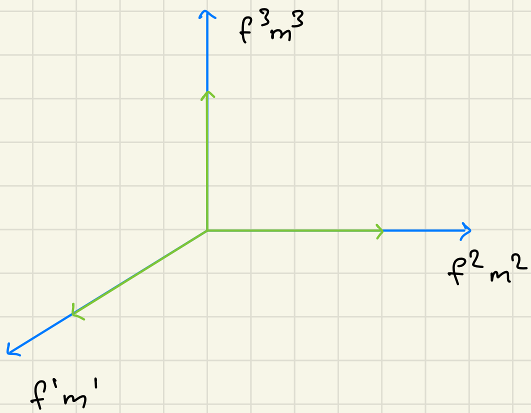
$$= \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$[m']^F = \begin{bmatrix} 0 & 0 & -1 \end{bmatrix}^T$$

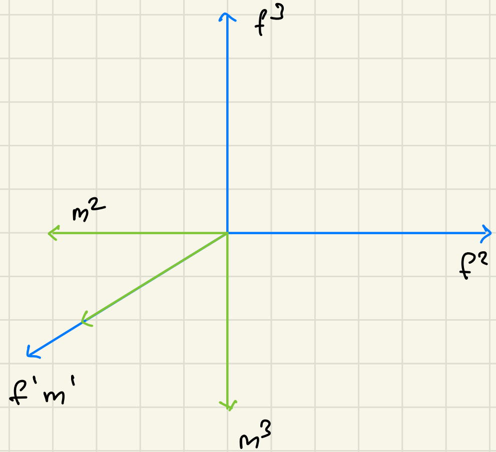
iii) Vector Diagram

(Anticlockwise $\rightarrow$ +ve)

$f_1 \rightarrow \pi$

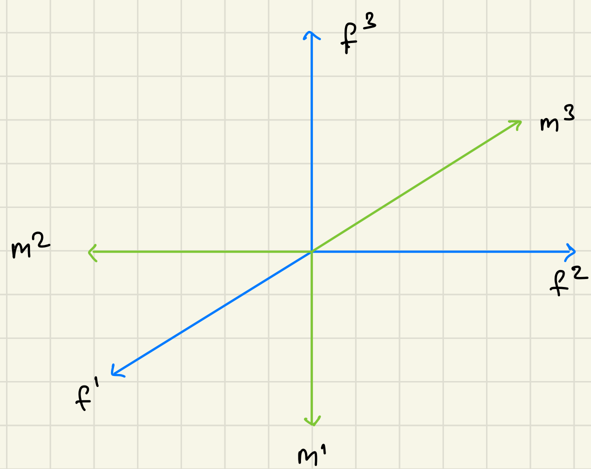


$\Rightarrow$



$\Downarrow$

$f_2 \rightarrow \pi/2$



HW

i) $y \rightarrow \theta_1$

ii) $P \rightarrow \theta_2$

iii) $R \rightarrow \theta_3$

find $YPR(\theta) =$

$R_3(\phi)\, R_2(\phi)\, R_1(\phi)$

$$A \Rightarrow \quad R_1(\theta_1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\theta_1 & -S\theta_1 \\ 0 & S\theta_1 & C\theta_1 \end{bmatrix}$$

$$R_2(\theta_2) = \begin{bmatrix} C\theta_2 & 0 & S\theta_2 \\ 0 & 1 & 0 \\ -S\theta_2 & 0 & C\theta_2 \end{bmatrix}$$

$$R_3(\theta_3) = \begin{bmatrix} C\theta_3 & -S\theta_3 & 0 \\ S\theta_3 & C\theta_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$YPR(\theta) = \begin{bmatrix} C\theta_2 C\theta_3 & C\theta_3 S\theta_1 S\theta_2 - C\theta_1 S\theta_3 & S\theta_1 S\theta_3 + C\theta_1 C\theta_3 S\theta_2 \\ C\theta_2 S\theta_3 & C\theta_1 C\theta_3 + S\theta_1 S\theta_2 S\theta_3 & C\theta_1 S\theta_2 S\theta_3 - C\theta_3 S\theta_1 \\ -S\theta_2 & C\theta_2 S\theta_1 & C\theta_1 C\theta_2 \end{bmatrix}$$

Q 1. Suppose we rotate a tool about the fixed axis starting with yaw of $\pi/2$ radians followed by pitch of π radians and lastly roll of $\pi/2$ radians.

i) Find the resultant matrix using composite algorithm

ii) Find the resultant matrix using YPR transformation matrix.

iii) Suppose a point 'p' at the tool tip has mobile coordinate $[0, 1, 0.6]^T$. Find $[P]^F$ following the yaw, pitch, roll rotations.

iv) Draw the vector diagrams

Q2. yaw angle $= -\pi/2$

pitch $= \pi$

roll $= -\pi$

Find $[m_2]^F$

Draw vector diagram

A1. yaw $= \pi/2$ $R_1(\pi/2)$

pitch $= \pi$ $R_2(\pi)$

roll $= \pi/2$ $R_3(\pi/2)$

$$R_1(\pi/2) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C_{\pi/2} & -S_{\pi/2} \\ 0 & S_{\pi/2} & C_{\pi/2} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$R_2(\pi) = \begin{bmatrix} C_{\pi} & 0 & S_{\pi} \\ 0 & 1 & 0 \\ -S_{\pi} & 0 & C_{\pi} \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$R_3(\pi/2) = \begin{bmatrix} C_{\pi/2} & -S_{\pi/2} & 0 \\ S_{\pi/2} & C_{\pi/2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

i) $A = R_3(\pi/2)\ R_2(\pi)\ R_1(\pi/2)$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

ii) $$YPR(\theta) = \begin{bmatrix} C_2 C_3 & S_1 S_2 C_3 - C_1 S_3 & C_1 S_2 C_3 + S_1 S_3 \\ C_2 S_3 & S_1 S_2 S_3 + C_1 C_3 & C_1 S_2 S_3 - S_1 C_3 \\ -S_2 & S_1 C_2 & C_1 C_2 \end{bmatrix}$$

$$YPR(\theta) = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

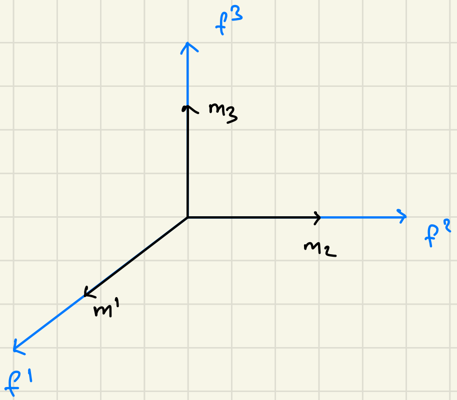
iii) $P = [\,0\ ,\ 1\ ,\ 0.6\,]^T$

$$[P]^F = A\ [P]^M$$

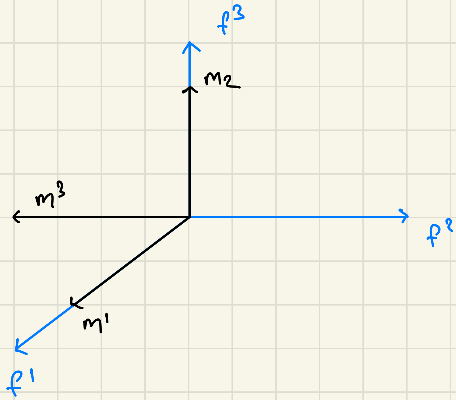
$$= \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0.6 \end{bmatrix}$$

$$= [0.6\quad 0\quad -1]^T$$

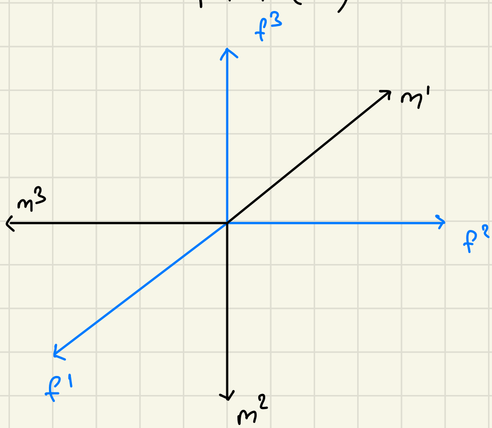
iii) Vector Diagram



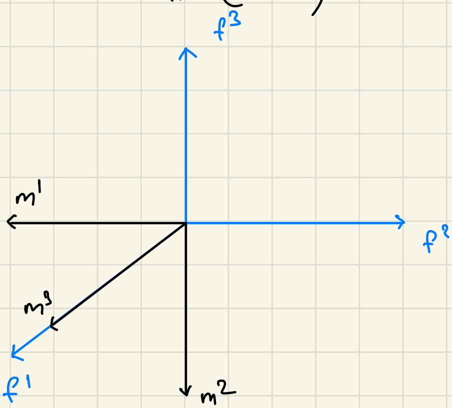
Pitch $(\pi/2)$



pitch (π)



roll $(\pi/2)$



when moving, +ve $\theta \Rightarrow$ anticlock −ve θ = clock

yaw $\Rightarrow$ move about f_1

pitch $\Rightarrow$ move about f_2

roll $\Rightarrow$ move about f_3

translation (Answer +ve → +ve diagram & vice versa)

A2.

$$\text{Yaw}(-\pi/2) = R_1(-\pi/2) = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

$$\text{Pitch}(\pi) = R_2(\pi) = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

$$\text{Roll}(-\pi) = R_3(-\pi) = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

$$A = R_3(-\pi)\, R_2(\pi)\, R_1(-\pi/2)$$

$$= \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

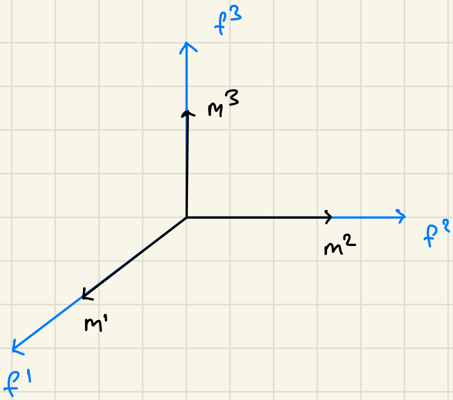
$$[M_2]^M = [0 \quad 1 \quad 0]^T \qquad \because [M]^M = \begin{bmatrix} M_1 \\ M_2 \\ M_3 \end{bmatrix}$$

$$[M_2]^F = A \cdot [M_2]^M$$

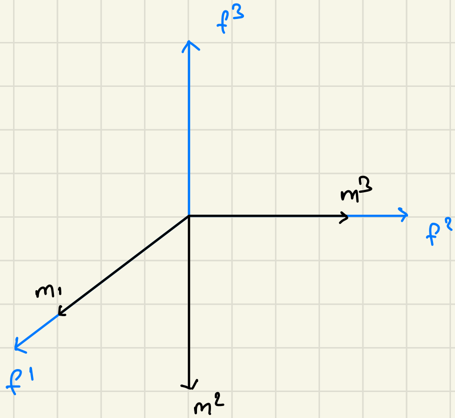
$$= \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$= [0 \quad 0 \quad 1]^T$$

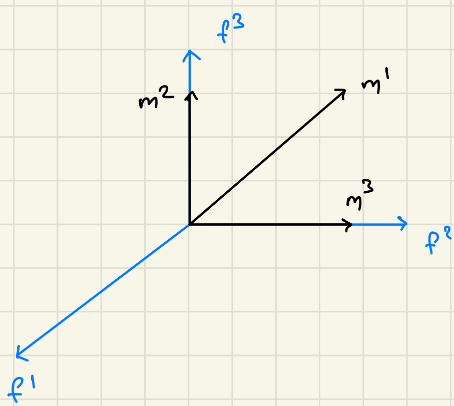
vector diagram :-



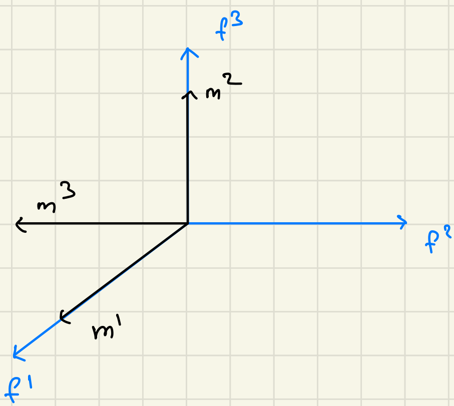
Yaw $(-\pi/2)$



Pitch (π)



Roll $(-\pi)$



Homogeneous Co-ordinate Transformation matrix (HCTM)

$$\left[\begin{array}{ccc|c} & [R(\phi)]_{3\times 3} & & p_1 \\ & & & p_2 \\ & & & p_3 \\ \hline \underbrace{0 \quad 0 \quad 0}_{\text{Perspective } \eta^T} & & & \underbrace{1}_{\text{Scaling factor } (\sigma)} \end{array}\right]_{4\times 4} \quad \left.\begin{matrix} p_1 \\ p_2 \\ p_3 \end{matrix}\right\} p \quad \begin{matrix} x \\ y \\ z \end{matrix}$$

(Always 0 0 0 1)

$$HCTM \cong \left[\begin{array}{c|c} R & p \\ \hline \eta^T & \sigma \end{array}\right]$$

HCTM for pure rotation

$R_1(\phi) \rightarrow$ Yaw

$$\left[\begin{array}{ccc|c} & R_1(\phi) & & \begin{matrix}0\\0\\0\end{matrix} \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \Bigg\} \text{ No translation only rotation}$$

$$\text{HCTM for } R_1(\phi) = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & C\phi & -S\phi & 0 \\ 0 & S\phi & C\phi & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$\text{HCTM for } R_2(\phi) = \left[\begin{array}{ccc|c} C\phi & 0 & S\phi & 0 \\ 0 & 1 & 0 & 0 \\ -S\phi & 0 & C\phi & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$\text{HCTM for } R_3(\phi) = \left[\begin{array}{ccc|c} C\phi & -S\phi & 0 & 0 \\ S\phi & C\phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

HCTM for pure translation

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & P_1 \\ 0 & 1 & 0 & P_2 \\ 0 & 0 & 1 & P_3 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

Q1 Suppose we rotate M axis wrt f_2 axis by angle $\pi/4$ radians. Find $[P^F]$ if $[P^M] = [5, 0, 3, 1]^T$.

Q2. Suppose we translate M axis about f_1 axis by 6 units and then translate M axis about f_3 axis by 10 units. Find $[P^F]$ if $[P^M] = [2, 3, 4]^T$

(Add scaling factor for HCTM)

A1. $f_2 \rightarrow$ pitch $(R_2(\phi))$

HCTM for $R_2(\phi)$ =

$$\left[\begin{array}{ccc|c} C\phi & 0 & S\phi & 0 \\ 0 & 1 & 0 & 0 \\ -S\phi & 0 & C\phi & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$R(\pi/4, f_2) = \left[\begin{array}{ccc|c} 1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 0 & 1 & 0 & 0 \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \begin{bmatrix} 5 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

$$[P]^F = \begin{bmatrix} 4\sqrt{2} & 0 & -\sqrt{2} & 1 \end{bmatrix}^T$$

$$[P]^F = \begin{bmatrix} 5.656 & 0 & -1.414 & 1 \end{bmatrix}^T$$

A2.

HCTM for translation $= \left[\begin{array}{ccc|c} 1 & 0 & 0 & P_1 \\ 0 & 1 & 0 & P_2 \\ 0 & 0 & 1 & P_3 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$

$$\text{Trans}(6, 0, 10) = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \begin{bmatrix} 2 \\ 3 \\ 4 \\ 1 \end{bmatrix}$$

$$[P]^F = \begin{bmatrix} 8 & 3 & 14 & 1 \end{bmatrix}^T$$

Q. Suppose we rotate M axis wrt f_3 axis by angle $\pi/2$ followed by translation of 3 units along f_2 axis. i) Find $[M']^F$. ii) Draw the vector diagram after rotation and translation. iii) Find $[M']^F$ if the order of rotation & translation is reversed. Also draw the vector diagram.

A. Given :-

i) $\text{Rot}^n \quad M \to F \; (f_3) \longrightarrow \pi/2$

$\text{Trans}^n \quad M \to F \; (f_2) \to 3 \text{ unit}$

$$R(\pi/2, 3) = \left[\begin{array}{ccc|c} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$\text{Trans}^n(0, 3, 0) = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

(Rotation & Translation, use Composite Algo)

$M \to F$ (pre)

$R = \text{Trans}(0,3,0), \; R(\pi/2, 3)$

$$R = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$[m']^F = A\,[m']^M$$

$$[M']^M = [1 \; 0 \; 0 \; 1]^T$$

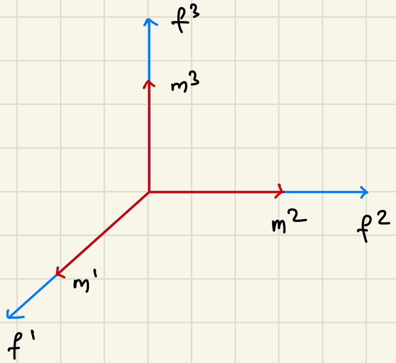
$$[M']^F = [0 \; 4 \; 0 \; 1]^T$$

ii) $R = \text{Rot}^n \times \text{Trans}^n$

$$= \begin{bmatrix} 0 & -1 & 0 & -3 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

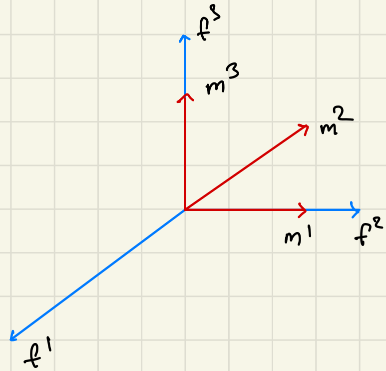
$$= [-3 \; 1 \; 0 \; 1]^T$$

i) Vector Diagram



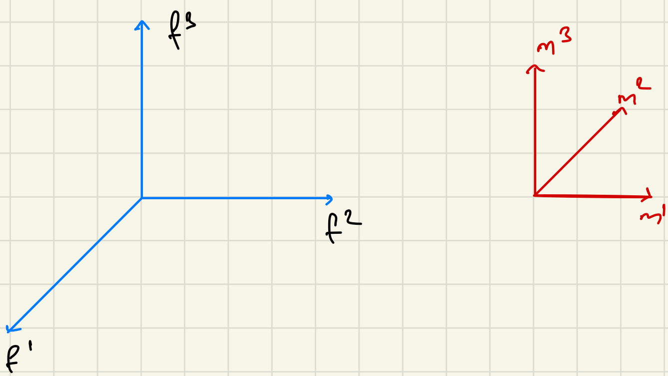
$\simeq$

$f^3(\pi/2)$



$\Downarrow$

translation $f_2 \to 3$ units



Q. Suppose we translate M along f^2 by 3 units & then rotate M about f^3 by π radians. i) Find $[M']^f$ and draw the vector diagrams.
ii) Repeat the above if order of rotation & translation is reversed.

A. i) Given :- $\text{Trans}^n\,(0,3,0)$

$\text{Rot}^n\,(\pi, f^3)$

$$\text{Trans}\,(0,3,0) = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$\text{HCTM for } R_2(\phi) = \left[\begin{array}{ccc|c} C\phi & 0 & S\phi & 0 \\ 0 & 1 & 0 & 0 \\ -S\phi & 0 & C\phi & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$\text{Rot}^n\,(\pi, f^3) = \left[\begin{array}{ccc|c} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$R = \mathrm{Rot}^n(\pi, f_3) \times \mathrm{Tran}^3(0, 3, 0)$$

$$= \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

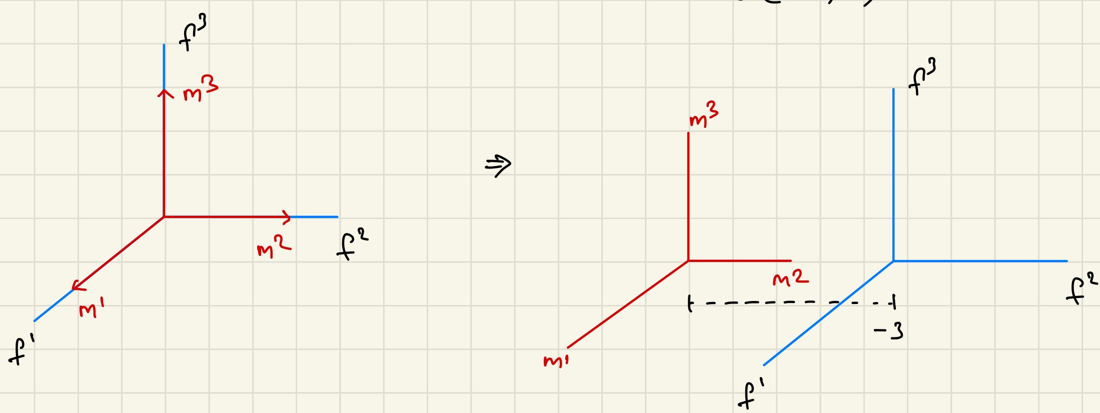
$$[M']^F = [A]\,[M']^M$$

$$= \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$[M']^F = \begin{bmatrix} -1 & -3 & 0 & 1 \end{bmatrix}^T$$

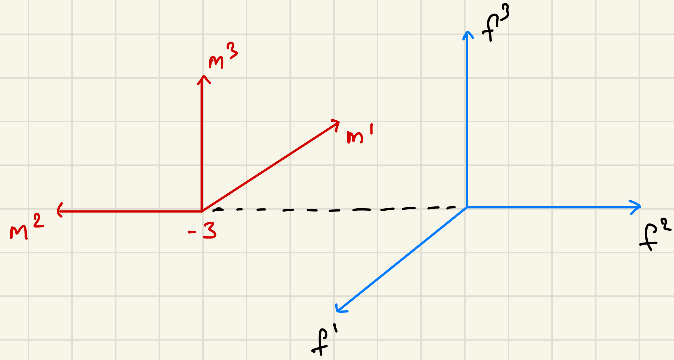
Vector Diagram :-

Trans (0,3,0) f^2



$$\mathrm{Rot}^n(\pi, f^3)$$

$\Rightarrow$



ii) $\mathrm{Rot}^n(\pi, f^3) \longrightarrow \mathrm{Trans}^n(0, 3, 0)$

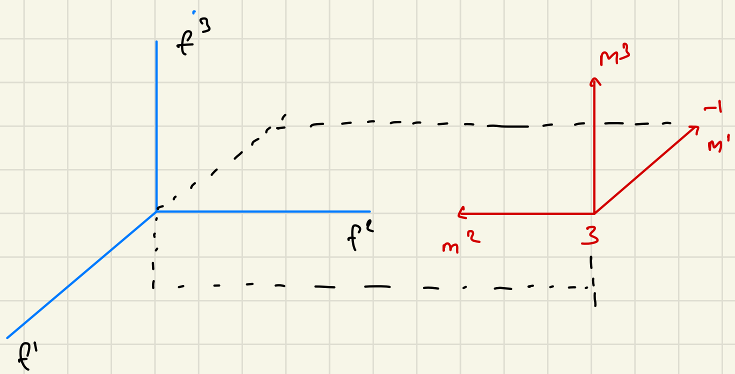
$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[M']^F = [A][M']^M$$

$$= \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 3 & 0 & 1 \end{bmatrix}^T$$



(Translation about f^2 axis, check sign of f^2 value in answer if +ve move towards +ve & if -ve move towards -ve)

Q. Suppose we rotate M axis about f^2 axis by $\pi/2$ radians and then translate along f^2 axis by 3 units. i) Find $[M^3]^F$. ii) Reverse the order and find $[M^3]^F$. Draw the vector diagram.

A. i) Given :- $\text{Rot}^n(\pi/2, f^2)$

$$\downarrow$$

$\text{Trans}^n(0,3,0)$

$$\text{Trans}^n(0,3,0) = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$\text{HCTM for } R_2(\phi) = \left[\begin{array}{ccc|c} C\phi & 0 & S\phi & 0 \\ 0 & 1 & 0 & 0 \\ -S\phi & 0 & C\phi & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$\text{Rot}^n(\pi/2, f^2) = \left[\begin{array}{ccc|c} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$A = \text{Trans}^n(0,3,0) \times \text{Rot}^n(\pi/2, f^1)$$

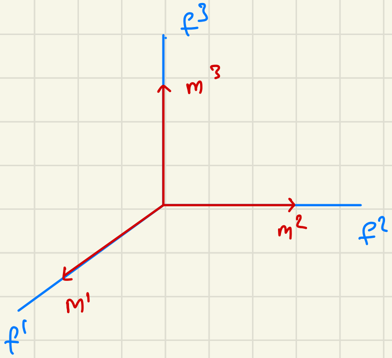
$$= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 3 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[M^3]^F = [A] \; [M^3]^M$$

$$= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 3 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

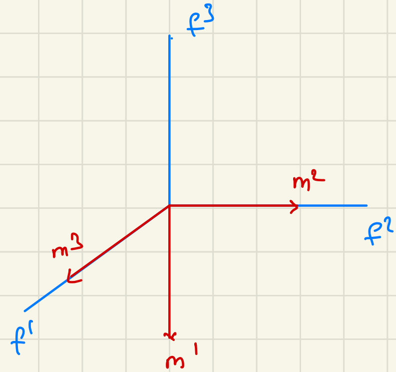
$$= \begin{bmatrix} 1 & 3 & 0 & 1 \end{bmatrix}^T$$

Vector Diagram :-



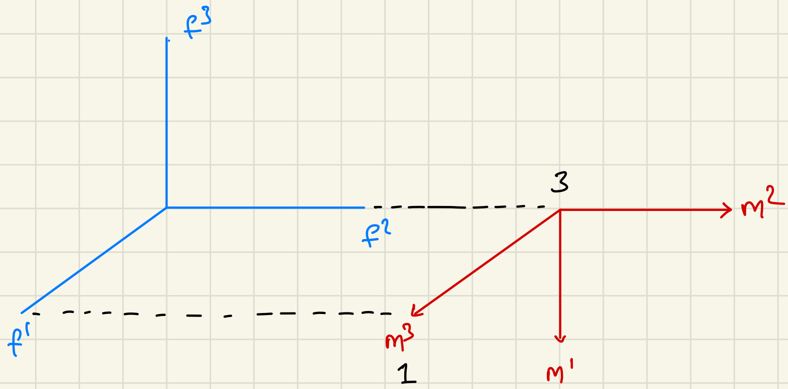
$\Rightarrow$

$\text{Rot}^n(\pi/2, f^2)$



$\mathrm{Trans}^n\ (0,3,0)$

$\Rightarrow$



ii) $\mathrm{Trans}^n\ (0,3,0) \longrightarrow \mathrm{Rot}^n\ (\pi/2\ , f^1)$

$$A = \mathrm{Rot}^n\ (\pi/2\ , f^1) \times \mathrm{Trans}^n (0,3,0)$$

$$= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 3 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[M^3]^F = [A]\ [M^3]^M$$

$$= \begin{bmatrix} 1 & 3 & 0 & 1 \end{bmatrix}^T$$

Same vector diagram as Case I due to screw transformation.

If we get same A in both cases,
the case is called **Screw Transformation**.
Don't draw case II vector diagram and
mention ' same vector diagram cause of
screw transformation.

M-1

$$R_1(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\phi & -S\phi \\ 0 & S\phi & C\phi \end{bmatrix}$$

$$R_2(\phi) = \begin{bmatrix} C\phi & 0 & S\phi \\ 0 & 1 & 0 \\ -S\phi & 0 & C\phi \end{bmatrix}$$

$$R_3(\phi) = \begin{bmatrix} C\phi & -S\phi & 0 \\ S\phi & C\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[P]^F = A\,[P]^M$$

$$[P]^M = A^T\,[P]^F$$

$$[M^1]^M = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T$$

$$HCTM = \left[\begin{array}{c|c} R & P \\ \hline \eta^T & \sigma \end{array}\right]$$

$R =$ rotational
$P =$ positional
$\eta^T =$ perspective
$\sigma =$ scaling factor

$$HCTM = \left[\begin{array}{ccc|c} & R(\phi) & & P \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

HCTM $R_1(\phi) = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & C\phi & -S\phi & 0 \\ 0 & S\phi & C\phi & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$

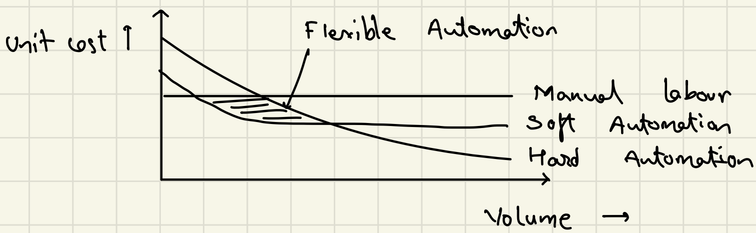
HCTM $R_2(\phi) = \left[\begin{array}{ccc|c} C\phi & 0 & S\phi & 0 \\ 0 & 1 & 0 & 0 \\ -S\phi & 0 & C\phi & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$

HCTM $R_3(\phi) = \left[\begin{array}{ccc|c} C\phi & -S\phi & 0 & 0 \\ S\phi & C\phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$

HCTM for pure translation $= \left[\begin{array}{ccc|c} 1 & 0 & 0 & P_1 \\ 0 & 1 & 0 & P_2 \\ 0 & 0 & 1 & P_3 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$

Robot = software controllable mechanical device that uses sensors to guide one or more end-effectors through programmable motions in a workspace in order to manipulate physical objects.

Categories :-
i) Hard automation (one application)
ii) Soft automation (multiple applications)



Classification of Robots :-

i) Drives → motors — precise, tiny
→ hydraulic drives — heavy load
→ pneumatic drives — no damage, no deformation

ii) Workspace

Locus of all points to which an end-effector can reach.

Joints → Revolute R (Rotational)
→ Prismatic P (Linear)

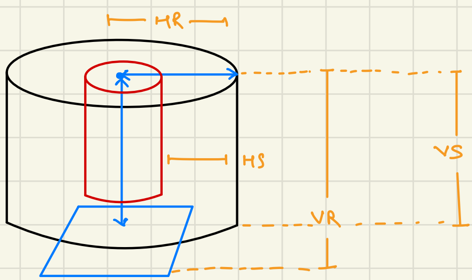
iii) Motion control methods

- Continuous path (prescribed path & speed)
- point to point (sequence of discrete points)

Specifications of robots

HR → maximum radial distance
HS → total radial distance
VR → maximum elevation
VS → total elevation

$S \leq R$

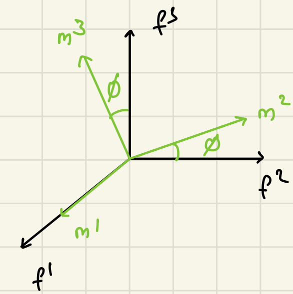


Repeatability → position tool tip same place repeatedly.

Precision → spacial resolution with which the tool can be positioned

Accuracy → ability of robot to place tool tip at an arbitrarily prescribed location

<u>Yaw</u>

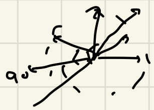


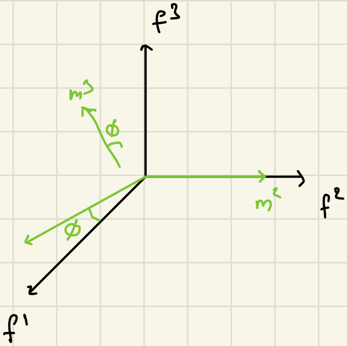
$$R(\phi) = \begin{bmatrix} f^1 m^1 & f^1 m^2 & f^1 m^3 \\ f^2 m^1 & f^2 m^2 & f^2 m^3 \\ f^3 m^1 & f^3 m^2 & f^3 m^3 \end{bmatrix}$$

$$= \begin{bmatrix} C_0 & C_{90} & C_{90} \\ C_{90} & C\phi & C_{(90+\phi)} \\ C_{90} & C_{(90-\phi)} & C\phi \end{bmatrix}$$

$$R'(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\phi & -S\phi \\ 0 & S\phi & C\phi \end{bmatrix}$$

Pitch





$$R(\phi) = \begin{bmatrix} f^1 m^1 & f^1 m^2 & f^1 m^3 \\ f^2 m^1 & f^2 m^2 & f^2 m^3 \\ f^3 m^1 & f^3 m^2 & f^3 m^3 \end{bmatrix}$$

$$= \quad C_\phi \quad C_{90}$$

$$R_1(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C_\phi & -S_\phi \\ 0 & S_\phi & C_\phi \end{bmatrix} \qquad R_2(\phi) = \begin{bmatrix} C_\phi & 0 & S_\phi \\ 0 & 1 & 0 \\ -S_\phi & 0 & C_\phi \end{bmatrix}$$

$$R_1(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C_\phi & -S_\phi \\ 0 & S_\phi & C_\phi \end{bmatrix} \qquad R_2(\phi) = \begin{bmatrix} C_\phi & 0 & S_\phi \\ 0 & 1 & 0 \\ -S_\phi & 0 & C_\phi \end{bmatrix}$$

$$R_1(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C_\phi & -S_\phi \\ 0 & S_\phi & C_\phi \end{bmatrix} \qquad R_2(\phi) = \begin{bmatrix} C_\phi & 0 & S_\phi \\ 0 & 1 & 0 \\ -S_\phi & 0 & C_\phi \end{bmatrix}$$

$$R_3(\phi) = \begin{bmatrix} C\phi & -S\phi & 0 \\ S\phi & C\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_3(\phi) = \begin{bmatrix} C\phi & -S\phi & 0 \\ S\phi & C\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_3(\phi) = \begin{bmatrix} C\phi & -S\phi & 0 \\ S\phi & C\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_1(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\phi & -S\phi \\ 0 & S\phi & C\phi \end{bmatrix} \qquad R_2(\phi) = \begin{bmatrix} C\phi & 0 & S\phi \\ 0 & 1 & 0 \\ -S\phi & 0 & C\phi \end{bmatrix}$$

$$R_3(\phi) = \begin{bmatrix} C\phi & -S\phi & 0 \\ S\phi & C\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_1(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\phi & -S\phi \\ 0 & S\phi & C\phi \end{bmatrix} \qquad R_3(\phi) = \begin{bmatrix} C\phi & -S\phi & 0 \\ S\phi & C\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_2(\phi) = \begin{bmatrix} C\phi & 0 & S\phi \\ 0 & 1 & 0 \\ -S\phi & 0 & C\phi \end{bmatrix}$$

Q. $M \to F \to f_1 \to \pi/2$

i) A?

ii) $[P]^M = [2 \quad 0 \quad 3]^T$, $[P]^F$?

iii) $[Q]^F = [5 \quad 10 \quad 0]^T$, $[Q]^M$?

A. i) $R_1(\pi/2) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\phi & -S\phi \\ 0 & S\phi & C\phi \end{bmatrix}$

$A = R_1(\pi/2) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$

$A = R_1(\pi/2)$

ii) $[P]^F = A[P]^M$

$= R_1(\pi/2) \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}$

iii) $[Q]^M = A^T[Q]^F$

Q. $M \to F \to f_1 \to \pi$

$M \to F \to f_2 \to \pi/2$

i) $[P]^M = [5 \quad 3 \quad 2]^T \qquad [P]^F$?

ii) $[M^1]^F$?

iii) Vector

$$R_1(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\phi & -S\phi \\ 0 & S\phi & C\phi \end{bmatrix} \qquad R_2(\phi) = \begin{bmatrix} C\phi & 0 & S\phi \\ 0 & 1 & 0 \\ -S\phi & 0 & C\phi \end{bmatrix}$$

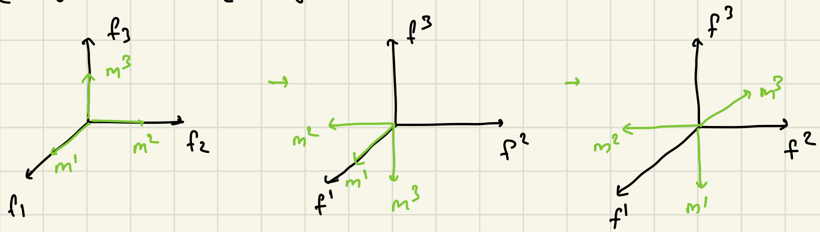
$$R_1(\pi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \qquad R_2(\pi/2) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$$A = R_2(\pi/2) \times R_1(\pi/2) \qquad [\because \text{Composite Algorithm}]$$

i) $[P]^F = A\,[P]^M$

ii) $[M^1]^F = A\,[M^1]^M$

iii)



Q. $Y \rightarrow \theta_1$

$P \rightarrow \theta_2$

$R \rightarrow \theta_3$

$YPR(\theta) = ?$

$$R_3(\phi) \times \underbrace{R_2(\phi) \times R_1(\phi)}$$

A. $YPR(\theta) = R(\theta_3) \times P(\theta_2) \times Y(\theta_1)$

$$YPR(\theta) = \begin{bmatrix} C\phi_3 & -S\phi_3 & 0 \\ S\phi_3 & C\phi_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\phi_2 & 0 & S\phi_2 \\ 0 & 1 & 0 \\ -S\phi_2 & 0 & C\phi_2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\phi_1 & -S\phi_1 \\ 0 & S\phi_1 & C\phi_1 \end{bmatrix}$$

$$YPR(\theta) = \begin{bmatrix} C\phi_3 & -S\phi_3 & 0 \\ S\phi_3 & C\phi_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_2 & S_1S_2 & C_1S_2 \\ 0 & C_1 & -S_1 \\ -S_2 & S_1C_2 & C_1C_2 \end{bmatrix}$$

$$YPR(\phi) = \begin{bmatrix} C_2C_3 & S_1S_2C_3 - C_1S_3 & C_1C_3S_2 + S_1S_3 \\ C_2S_3 & S_1S_2S_3 + C_1C_3 & C_1S_2S_3 - S_1C_3 \\ -S_2 & S_1C_2 & C_1C_2 \end{bmatrix}$$

Q. $Y \to F \to f_1 \to \pi/2$

$P \to F \to f_2 \to \pi$

$R \to f \to f_3 \to \pi/2$

i) A ? composite Algorithm

ii) A ? YPR Transformation

iii) $[P]^M = [0 \quad 1 \quad 0.6]^T \qquad [P]^f = ?$

iv) vector diagram ?

A. i)

$$R_1(\pi/2) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$R_2(\pi) = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

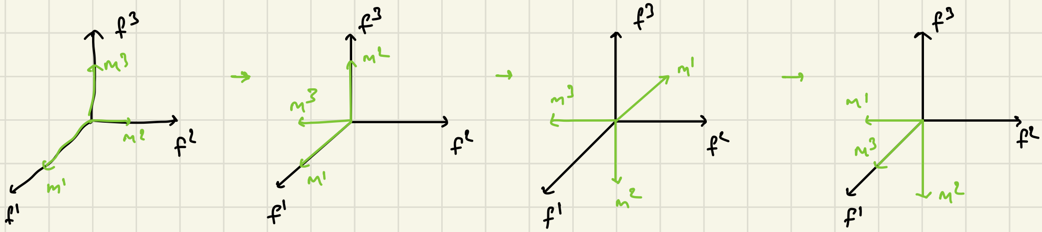
$$R_3(\pi/2) = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

i) $A = R_3(\pi/2) \times R_2(\pi/2) \times R_1(\pi/2)$ [∵ composite Algo]

ii) $A =$ YRR (θ) ... formula

iii) $[P]^f = A \, [P]^M$

iv) Vector Diagram :-



Q. $Y = -\pi/2$

$P = \pi$

$R = -\pi$

i) $[M^2]^F$

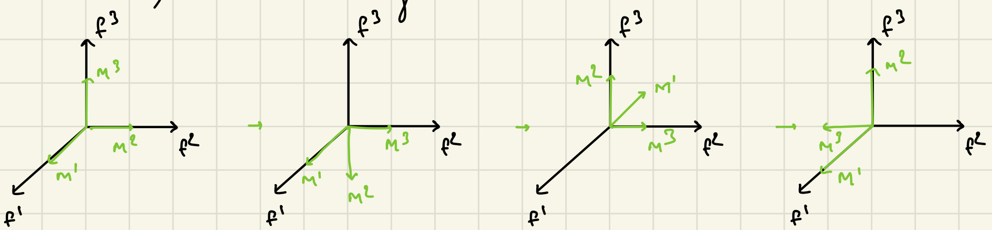
ii) Vector diagram

A. $A = R_3(-\pi) \times R_2(\pi) \times R_1(\pi/2)$

i) $[M^2]^F = A \, [M^2]^M$

$= A \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

ii) Vector diagram :-



Q. $M \to F \to f_2 \to \pi/4$

$[P]^M = [5 \quad 0 \quad 3 \quad 1]^T$

$[P]^F = ?$

A. HCTM $R_2(\pi/4) = \begin{bmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 0 & 1 & 0 & 0 \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = A$

$[P]^F = A\,[P]^M$

Q. $M \to F \to f_1 \to 6$

$M \to F \to f_3 \to 10$

$[P]^M = [2 \quad 3 \quad 4]^T$

$[P]^F = ?$

A. HCTM $= \begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} = A$

$[P]^F = A\,[P]^M = A \begin{bmatrix} 2 \\ 3 \\ 4 \\ 1 \end{bmatrix}$

Q. $M \to F \to f_3 \to \pi/2$

$M \to F \to f_2 \to 3$

i) $[M^1]^F$

ii) Vector Diagram

iii) order reversed. $[M']^F = ?$

iv) Vector diagram

A. $$R(\pi/2, 3) = \begin{bmatrix} C\pi/2 & -S\pi/2 & 0 & 0 \\ S\pi/2 & C\pi/2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

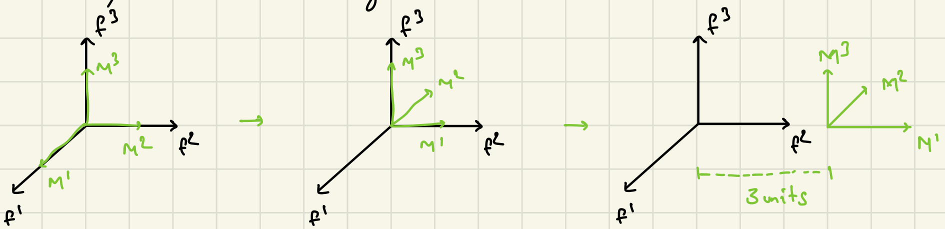
$$\text{Trans}^n(0,3,0) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

i) $$A = \text{Trans}^n(0,3,0) \times R(\pi/2, 3)$$

($\because$ Composite Algorithm)

$$[M']^F = A\,[M']^M = A \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

ii) Vector diagram



Q. $M \to f \to f^2 \to 3$

$M \to f \to f^3 \to \pi$

i) $[M']^f$? Vector diagram.

ii) Order reversed. $[M']^f$? Vector diagram.

A. $$\text{Trans}^n(0,3,0) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R(\pi, 3) = \begin{bmatrix} C\pi & -S\pi & 0 & 0 \\ S\pi & C\pi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

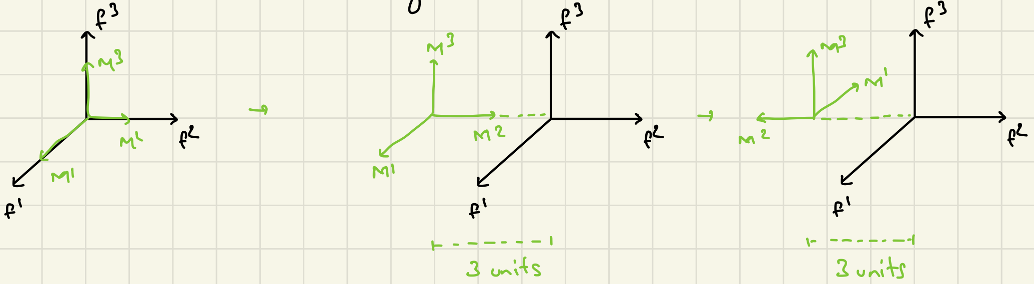
i) $A = R(\pi,3) \times \text{Trans}^n(0,3,0)$

$[M']^M = [1 \quad 0 \quad 0 \quad 1]^T$

$[M']^f = A[M']^M$

$[M']^f = [-1 \quad -3 \quad 0 \quad 1]^T$

Vector Diagram :-



Q. $M \to F \to f_2 \to \pi/2$

$M \to F \to f_2 \to 3$

i) $[M^3]^F$? Vector diagram

ii) Order reverse. $[M^3]^F$? Vector diagram

$$R(\pi/2, 2) = \begin{bmatrix} C\pi/2 & 0 & S\pi/2 & 0 \\ 0 & 1 & 0 & 0 \\ -S\pi/2 & 0 & C\pi/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

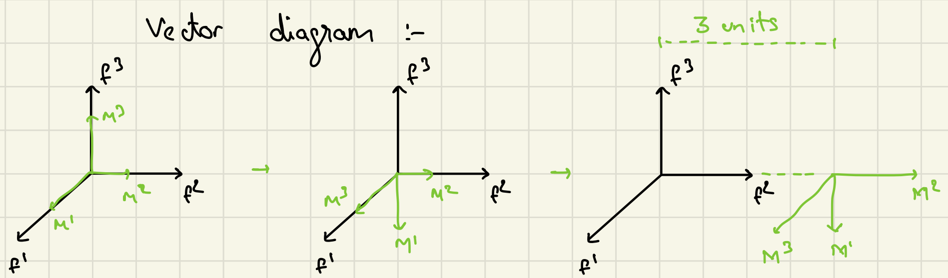
$$\text{Trans}^n(0, 3, 0) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

i) $A = \text{Trans}^n(0,3,0) \times R(\pi/2, 2)$

$$[M^3]^F = A\,[M^3]^M$$
$$= A \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

$$[M^3]^F = [1 \quad 3 \quad 0 \quad 1]^T$$

Vector diagram :-



iii) $A = R(\pi/2, 2) \times \text{Trans}^n (0, 3, 0)$

$$[M^3]^F = A [M^3]^M$$

$$= A \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}^M$$

$$[M^3]^F = \begin{bmatrix} 1 & 3 & 0 & 1 \end{bmatrix}^T$$

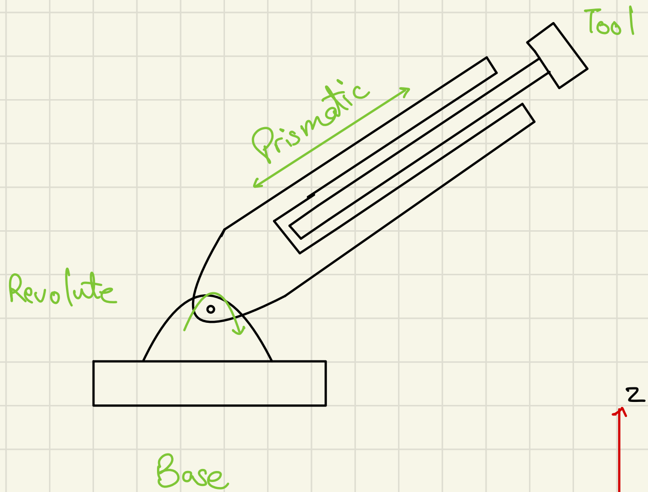
This is case of screw transformation.

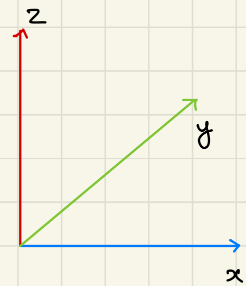
$\therefore$ Same vector diagram as above.

D-H ALGORITHM

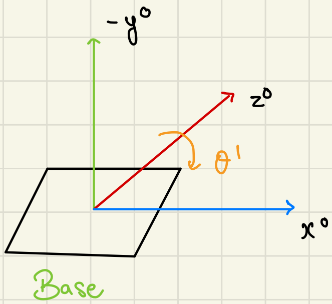
(ARM Matrix)

→ z = joint axis



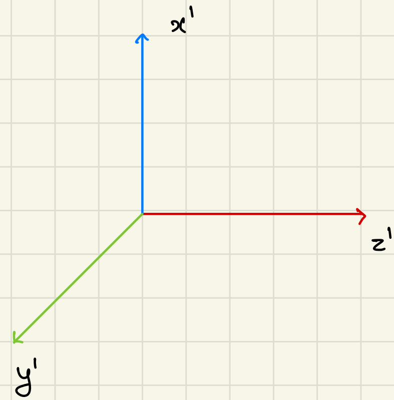


Step 1 :-

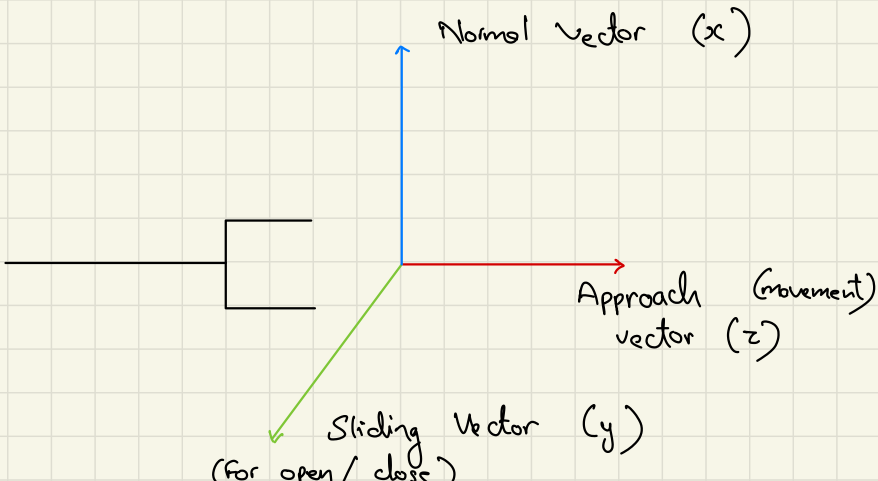


→ x^k should be $\perp$ to z^1 & z^0

Step 2 :-

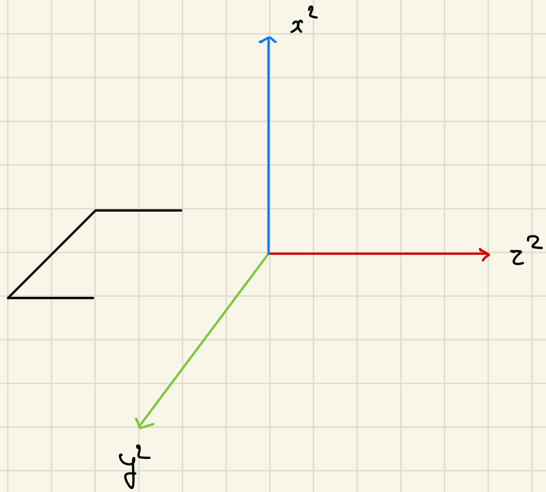


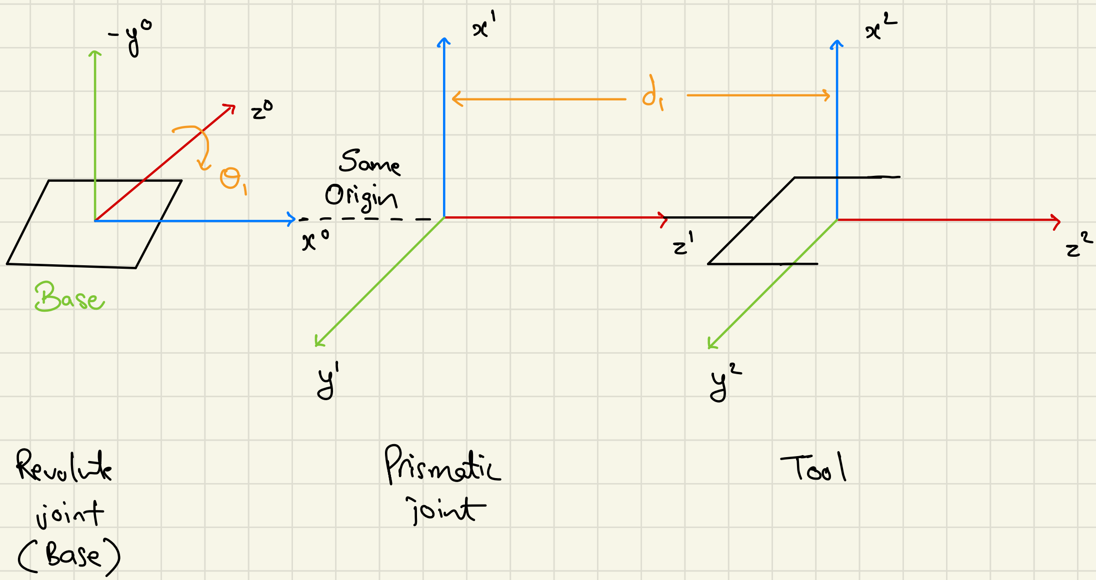
<u>Tool</u> :-



Step 3 :-

$x \rightarrow$ Yaw
$y \rightarrow$ Pitch
$z \rightarrow$ Roll





Kinematic Parameters

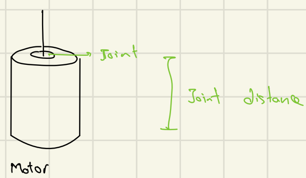
d → Joint distance
θ → Joint angle
} Joint Parameters

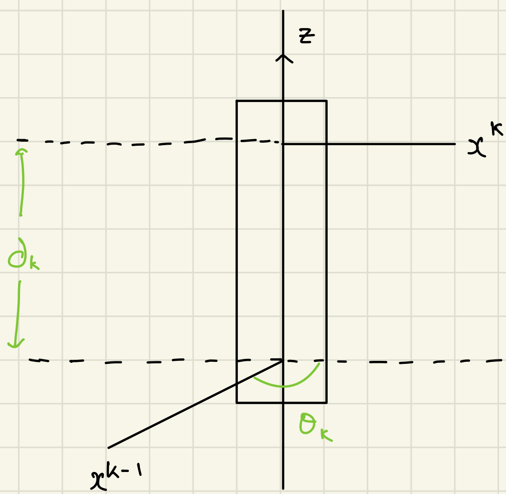
a → Link angle
α → Link Twist angle
} Link Parameters

(Link = mechanical structure which joins 2 joints)

→ d_k Joint distance

Fixed → Revolute
Variable → Prismatic



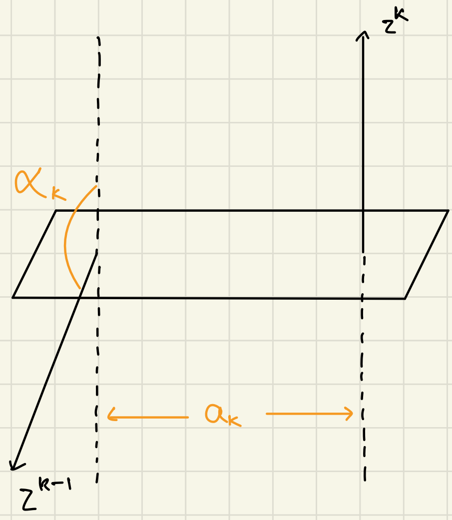


→ θ_k Joint Angle

fixed → Prismatic
variable → joint / revolute

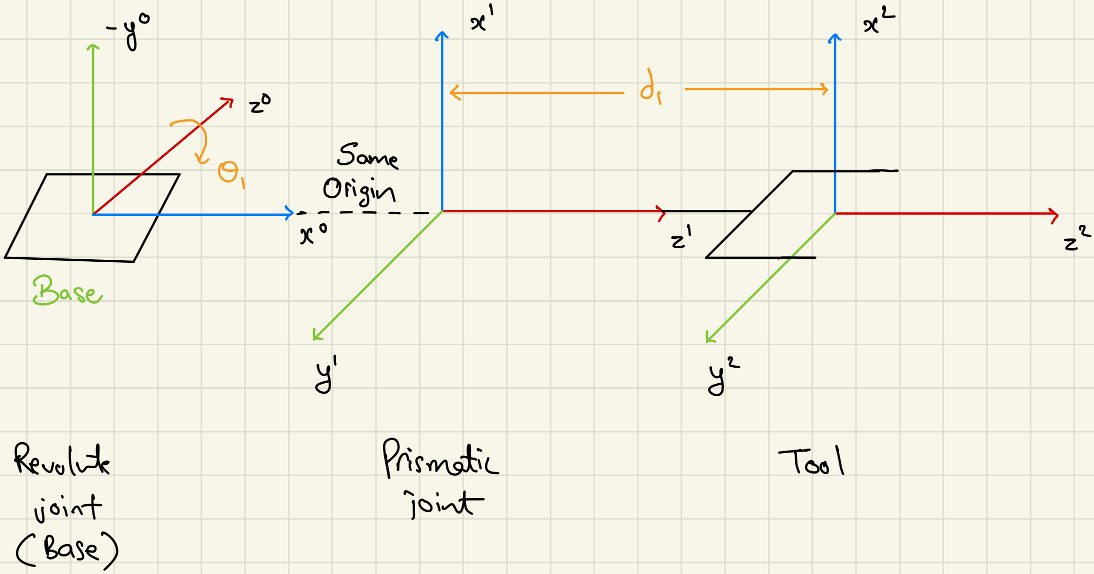
Angle required to make x^{k-1} parallel to x^k

→ Link Parameters



a_k Link length :- always constant

α_k Link Twist angle :- always constant



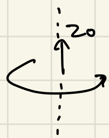
Link Parameter Table :-

Axis	θ	d	a	α	Variable
1	θ_1	0	0	$-\pi/2$	θ_1
2	0	d_1	0	0	d_1

D-H Tricks

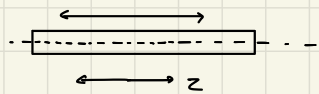
i) Align z such that

Revolute joint



$\perp$

Prismatic joint



ii) $x^k \rightarrow \perp$ z^k & z^{k-1}

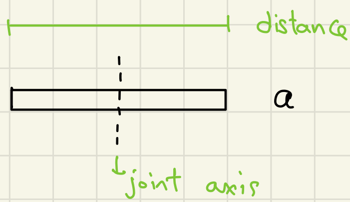
Right hand

z → thumb

x → start of curl

iii) when d & a ?

d



a

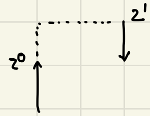
Link Parameter Table

i) $\alpha \rightarrow$ angle b/w now z and previous z

$\downarrow$

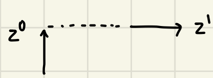
z & z^{k+1}

Ex:- i)



$\alpha = \pi$ or $-\pi$

ii)

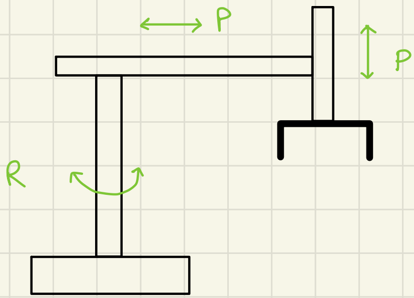


$\pi/2$ for anticlockwise

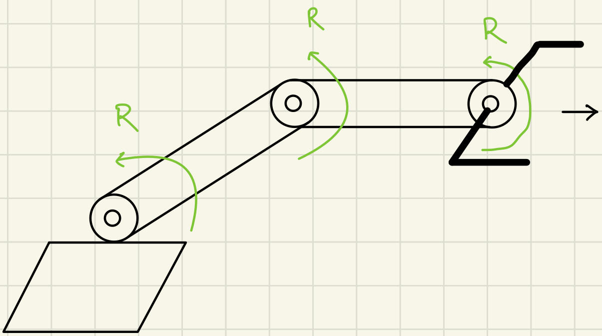
$-\pi/2$ for clockwise

Here $\alpha = -\pi/2$

Q1. Cylindrical Robot

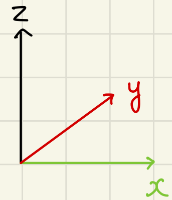


Q2. 3 Axis

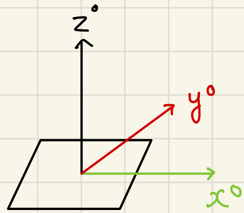


A1.

(Always align z axis)



Reference axis



Base (Align z axis)

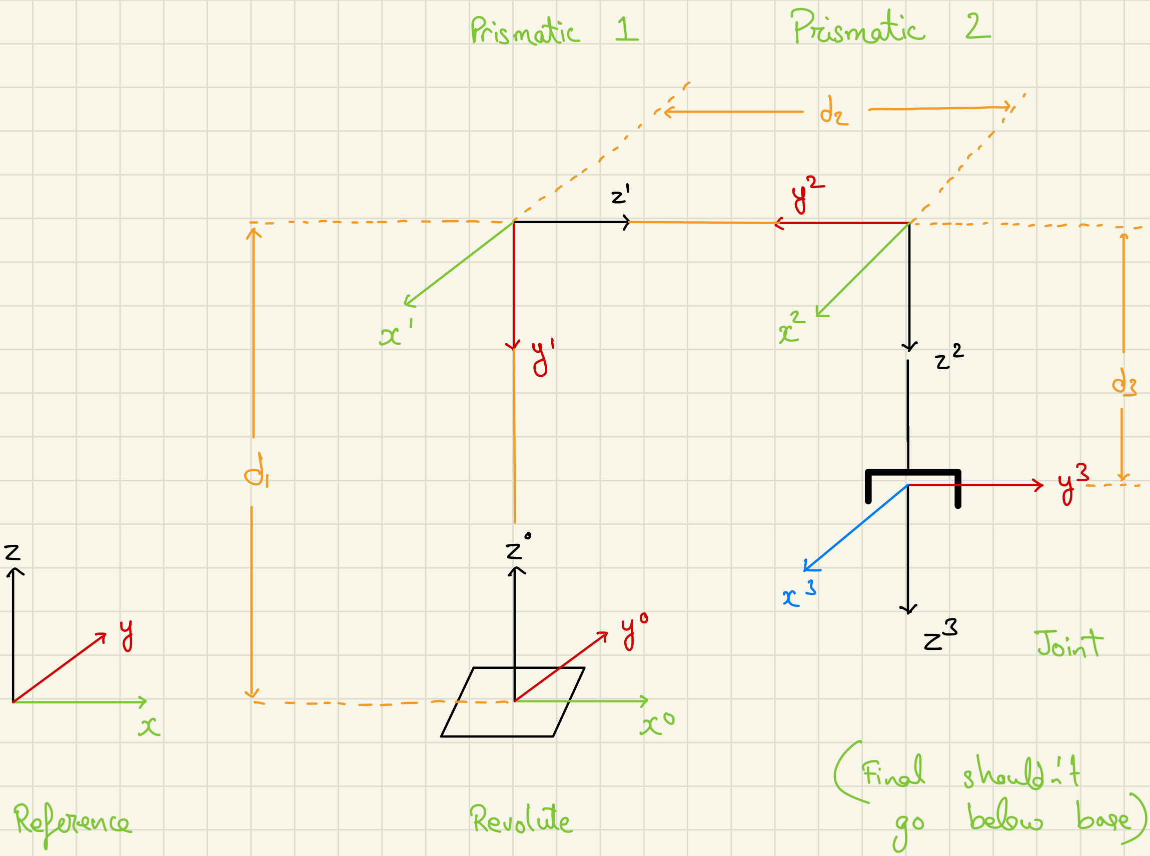
3 axis robot

orientation

$\therefore \; 3+1 = 4$ vectors

Next joint ⇒ Prismatic

but align x axis so that they are perpendicular



Link Parameter table :-

Axis	θ_k	d_k	a_k	α_k
1	θ_1	d_1	0	$-\pi/2$
2	0	d_2	0	$-\pi/2$
3	0	d_3	0	0

→ angular difference b/w current z & previous z

Link Cordinate Transformation matrix :-

Arm matrix

$$T_{k-1}^{k} = \begin{bmatrix} C_{\theta k} & -C_{\alpha k} S_{\theta k} & S_{\alpha k} S_{\theta k} & a_k C_{\theta k} \\ S_{\theta k} & C_{\alpha k} C_{\theta k} & -S_{\alpha k} C_{\theta k} & a_k S_{\theta k} \\ 0 & S_{\alpha k} & C_{\alpha k} & d_k \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^{1} = \begin{bmatrix} C_1 & -C_{-\pi/2} S_1 & S_{-\pi/2} S_1 & a_1 C_1 \\ S_1 & C_{-\pi/2} C_1 & -S_{-\pi/2} C_1 & a_1 S_1 \\ 0 & S_{-\pi/2} & C_{-\pi/2} & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^{1} = \begin{bmatrix} C_1 & 0 & -S_1 & 0 \\ S_1 & 0 & C_1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_1^{2} = \begin{bmatrix} C_0 & -C_{-\pi/2} S_0 & S_{-\pi/2} S_0 & a_2 C_0 \\ S_0 & C_{-\pi/2} C_0 & -S_{-\pi/2} C_0 & a_2 S_0 \\ 0 & S_{-\pi/2} & C_{-\pi/2} & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_1^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^3 = \begin{bmatrix} C_0 & -C_0 S_0 & S_0 S_0 & a_0 C_0 \\ S_0 & C_0 C_0 & -S_0 C_0 & a_0 S_0 \\ 0 & S_0 & C_0 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

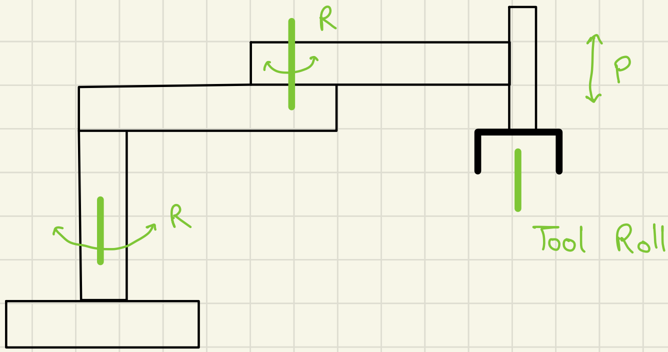
$$T_2^3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^3 = \underbrace{T_0^1 \cdot T_1^2 \cdot T_2^3}_{T_0^3}$$

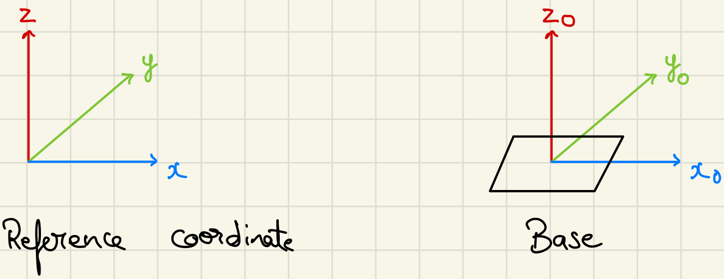
$$= \begin{bmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_1 & s_1 & 0 & -s_1 d_2 \\ s_1 & -c_1 & 0 & c_1 d_2 \\ 0 & 0 & -1 & d_1 - d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Q. SCARA Robot

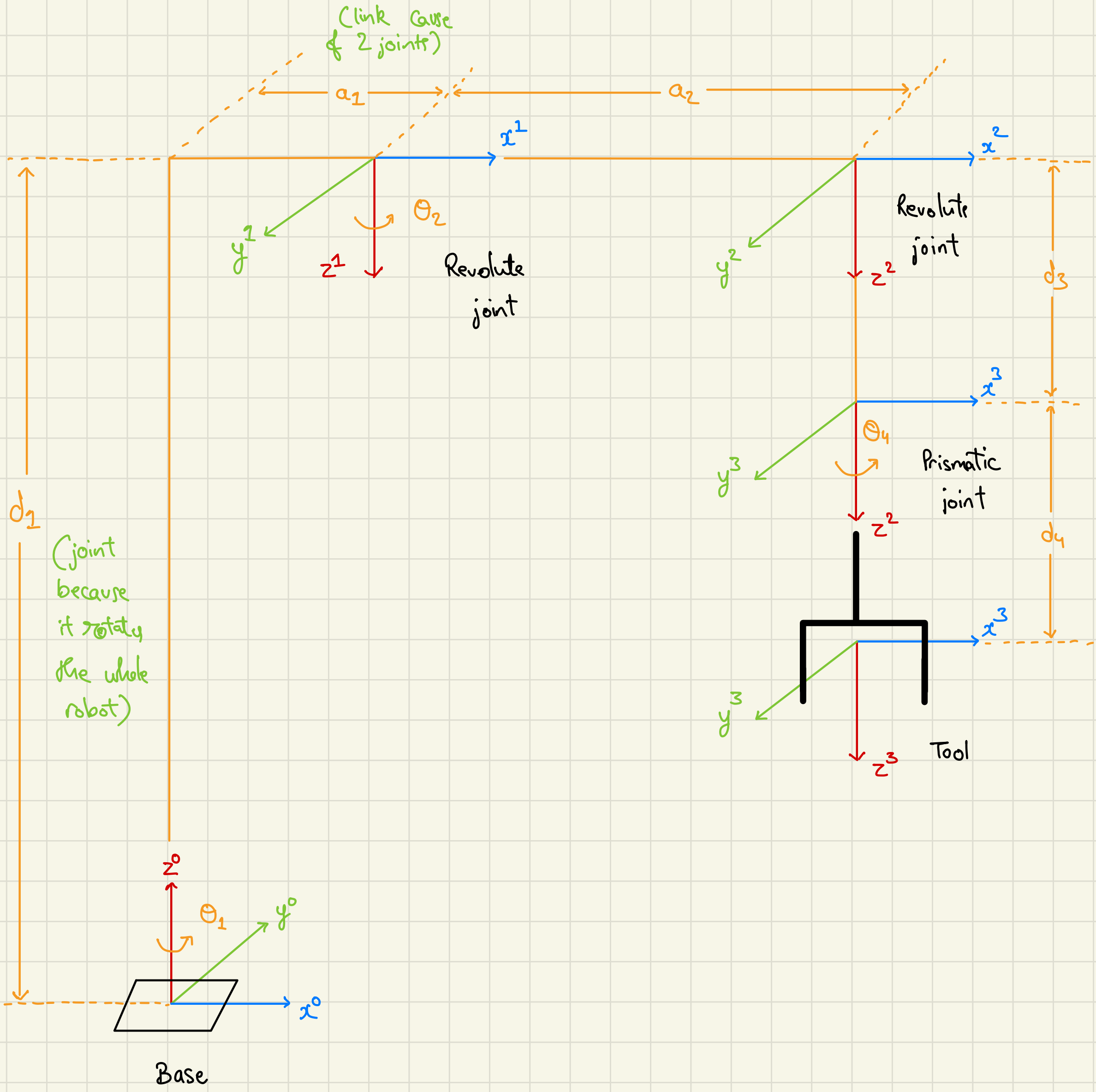


A. Link Coordinate Diagram



no of axis = 4

no. of vectors = 4 + 1 = 5



(link cause of 2 joints)
a_1
a_2
x^1
x^2
θ_2
y^1
z^1
Revolute joint
y^2
z^2
Revolute joint
d_3
x^3
θ_4
y^3
Prismatic joint
z^2
d_4
x^3
y^3
z^3
Tool
d_1
(joint because it rotates the whole robot)
z^0
θ_1
y^0
x^0
Base

Link parameter table

Axis	θ_k	d_k	a_k	α_k	variable
1	θ_1	d_1	a_1	π	θ_1
2	θ_2	0	a_2	0	θ_2
3	0	d_3	0	0	d_3
4	θ_4	d_4	0	0	θ_4

(what values will change, in prismatic d will var & in revolute θ will be var)

Link Coordinate transformation matrix

$$T_{k-1}^{k} = \begin{bmatrix} C\theta_k & -C\alpha_k S\theta_k & S\alpha_k S\theta_k & a_k C\theta_k \\ S\theta_k & C\alpha_k C\theta_k & -S\alpha_k C\theta_k & a_k S\theta_k \\ 0 & S\alpha_k & C\alpha_k & d_k \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^1 = \begin{bmatrix} C_1 & -C_\pi S_1 & S_\pi S_1 & a_1 C_1 \\ S_1 & C_\pi C_1 & -S_\pi C_1 & a_1 S_1 \\ 0 & S_\pi & C_\pi & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^1 = \begin{bmatrix} C_1 & S_1 & 0 & a_1 C_1 \\ S_1 & -C_1 & 0 & a_1 S_1 \\ 0 & 0 & -1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_1^2 = \begin{bmatrix} C_2 & -C_0 S_2 & S_0 S_2 & a_2 C_2 \\ S_2 & C_0 C_2 & -S_0 C_2 & a_2 S_2 \\ 0 & S_0 & C_0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_1^2 = \begin{bmatrix} C_2 & -S_2 & 0 & a_2 C_2 \\ S_2 & C_2 & 0 & a_2 S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^3 = \begin{bmatrix} C_0 & -C_0 S_0 & S_0 S_0 & a_0 C_0 \\ S_0 & C_0 C_0 & -S_0 C_0 & a_0 S_0 \\ 0 & S_0 & C_0 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^4 = \begin{bmatrix} C_4 & -C_0 S_4 & S_0 S_4 & a_0 C_4 \\ S_4 & C_0 C_4 & -S_0 C_4 & a_0 S_4 \\ 0 & S_0 & C_0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^4 = \begin{bmatrix} C_4 & -S_4 & 0 & 0 \\ S_4 & C_4 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{Base}^{Tool} = T_0^4$$

$$T_{Base}^{Wrist} = T_0^3$$

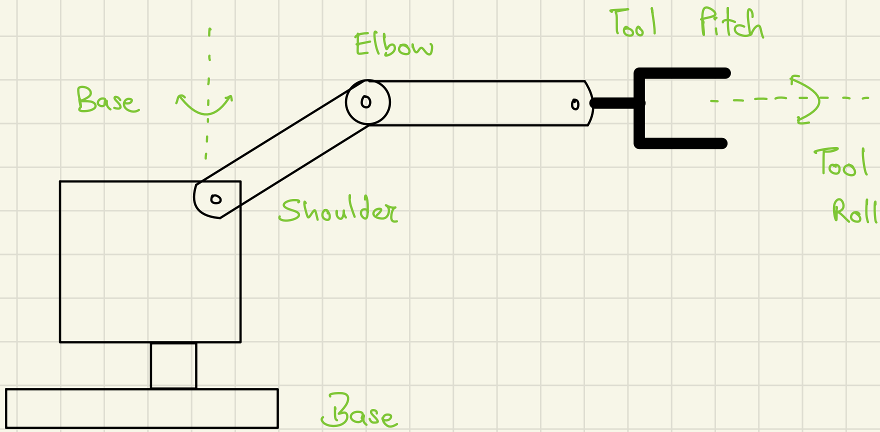
$$T_{WRIST}^{TOOL} = T_3^4$$

$$T_0^4 = T_0^1 \; T_1^2 \; T_2^3 \; T_3^4$$

(multiplication)

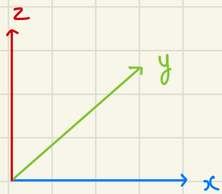
Arm matrix

Q.



5 axis Microbot Alpha II

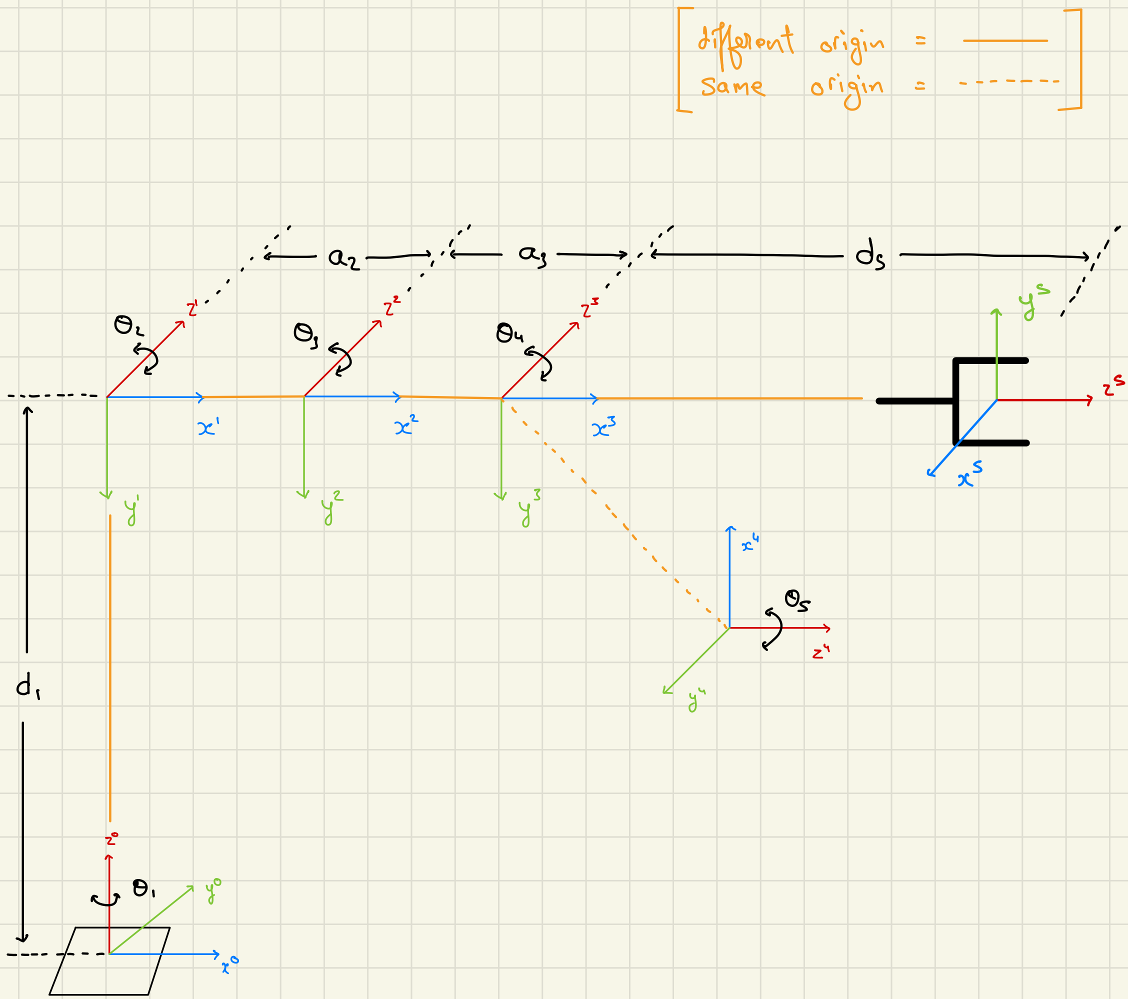
Reference coordinate diagram



no. of axis = 5

no. of vectors = 5 + 1 = 6

Link Coordinate Diagram



Link Parameter table

Axis	θ_k	d_k	a_k	α_k	variable
1	θ_1	d_1	0	$-\pi/2$	θ_1
2	θ_2	0	a_2	0	θ_2
3	θ_3	0	a_3	0	θ_3
4	θ_4	0	0	$-\pi/2$	θ_4
5	θ_5	d_5	0	0	θ_5

Link Coordinate transformation matrix

$$T_{k-1}^{k} = \begin{bmatrix} C\theta_k & -C\alpha_k S\theta_k & S\alpha_k S\theta_k & a_k C\theta_k \\ S\theta_k & C\alpha_k C\theta_k & -S\alpha_k C\theta_k & a_k S\theta_k \\ 0 & S\alpha_k & C\alpha_k & d_k \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^5 = T_0^1 \times T_1^2 \times T_2^3 \times T_3^4 \times T_4^5$$

(Arm matrix)

$$T_{Base}^{Wrist} = T_0^3 = T_0^1 \times T_1^2 \times T_2^3$$

$$T_{Wrist}^{Tool} = T_3^5 = T_3^4 \times T_4^5$$

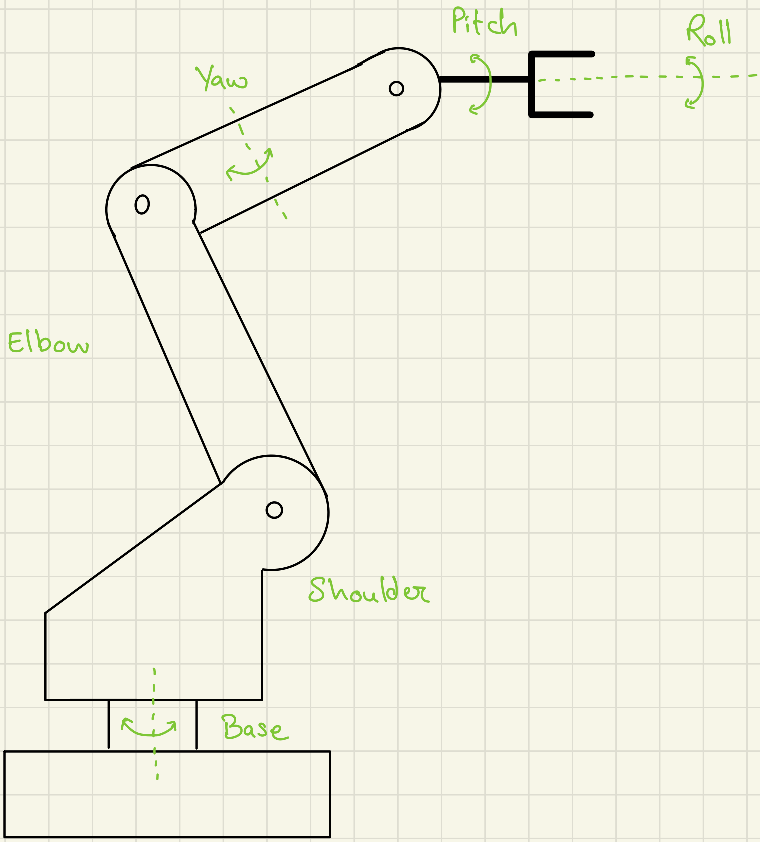
$$T_0^1 = \begin{bmatrix} C_1 & 0 & S_1 & 0 \\ S_1 & 0 & C_1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_1^2 = \begin{bmatrix} c_2 & -S_2 & 0 & a_2 C_2 \\ S_2 & C_2 & 0 & a_2 S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

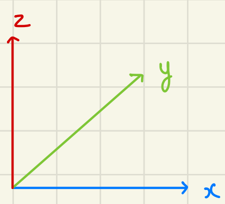
$$\vdots$$

$$T_4^5$$

Q. Yaskawa Motoman

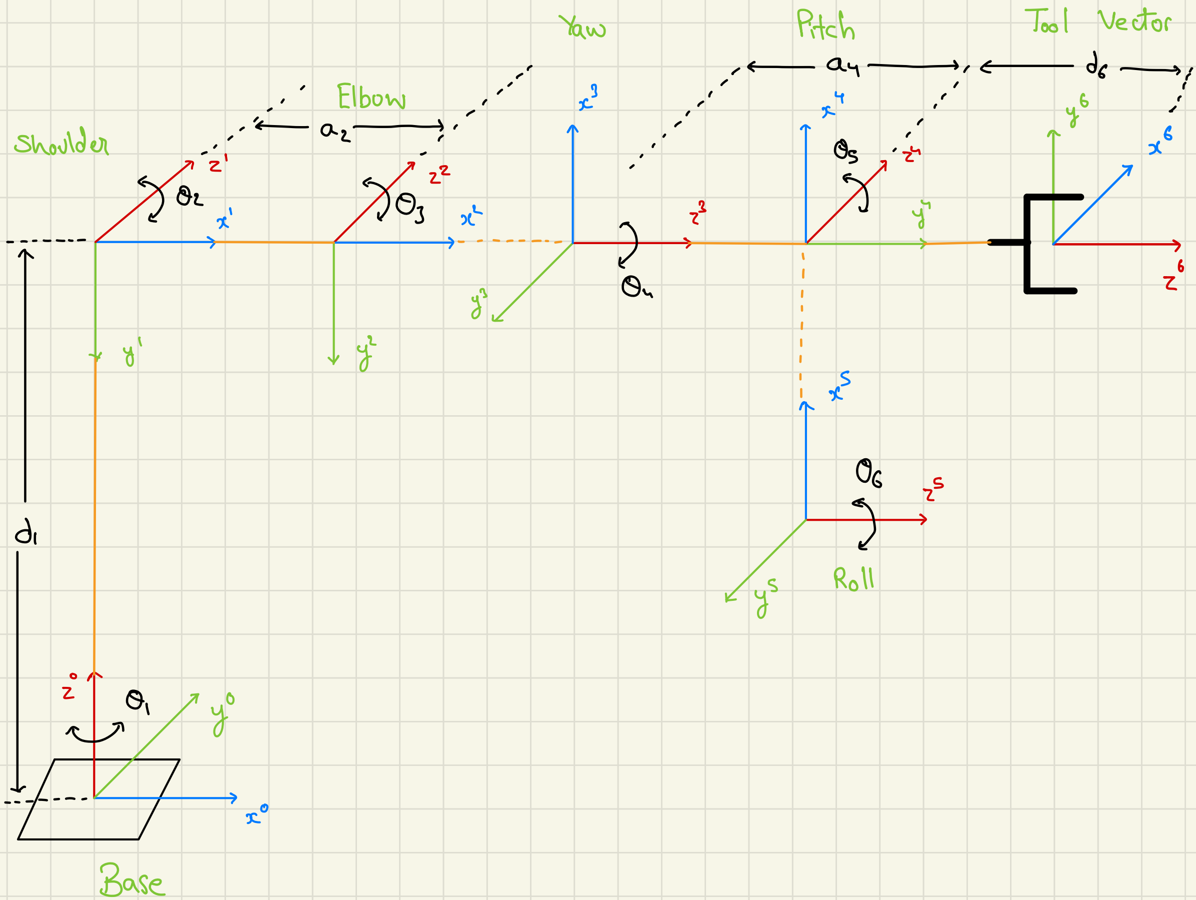


A. Reference coordinate diagram



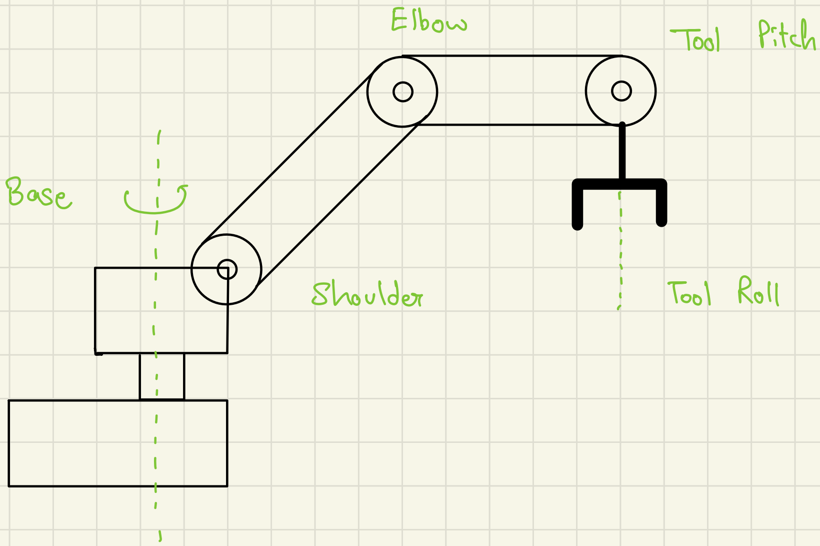
no. of axis = 6 no. of vectors = 6 + 1 = 7

Link Coordinate Diagram

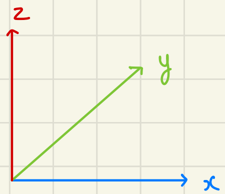


Axis	θ_k	d_k	a_k	α_k	variable
1	θ_1	d_1	0	$-\pi/2$	θ_1
2	θ_2	0	a_2	0	θ_2
3	θ_3	0	0	$-\pi/2$	θ_3
4	θ_4	0	a_4	$\pi/2$	θ_4
5	θ_5	0	0	0	θ_5
6	θ_6	d_6	0	0	θ_6

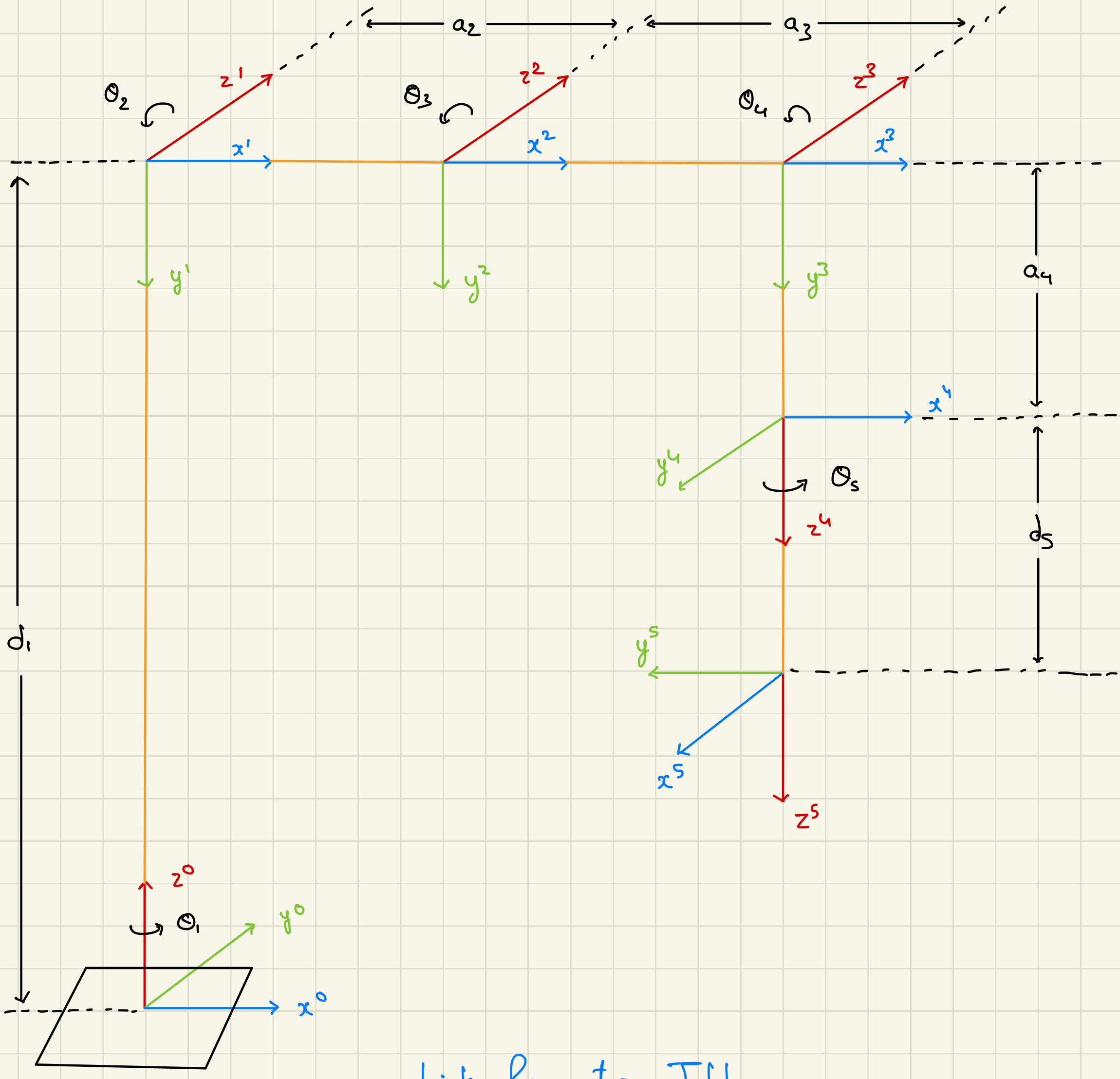
Q. Rhino XR3



A. Reference coordinate diagram



no. of axis = 5 no. of vectors = 5+ 1 = 6



Link Parameter Table

Axis	θ_k	d_k	a_k	α_k	variable
1	θ_1	d_1	0	$-\pi/2$	θ_1
2	θ_2	0	a_2	0	θ_2
3	θ_3	0	a_3	0	θ_3
4	θ_4	0	a_4	$-\pi/2$	θ_4
5	θ_5	d_5	0	0	θ_5

INVERSE KINEMATICS

exam = (cylindrical , SCARA , para)
→ 3 axis

Direct Kinematics → Final position of tool & Arm matrix

Inverse Kinematics → Given location of any object in the envelope, the robot should calculate the joint angles

Arm matrix :-
(HCTM)

Columns

Y = Yaw
P = Pitch
R = Roll

$$\left[\begin{array}{ccc|c} & R & & P \\ \hline 0 & 0 & 0 & 1 \end{array}\right] = R^6 = \begin{array}{cccc} \uparrow YR & \uparrow PR & \uparrow YP & \\ \left[\begin{array}{ccc|c} R_{11} & R_{12} & R_{13} & P_1 \\ R_{21} & R_{22} & R_{23} & P_2 \\ R_{31} & R_{32} & R_{33} & P_3 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \end{array}$$

(Roll least important)
↓
not completely neglected

12 eqⁿs (R_{11}, P_1, ...)

High Priority ⇒ → 3 eqⁿs of Position
→ 3 eqⁿs of YP

⇓

6 eqⁿs

Tool Configuration Vector (TCV)

Arrangement of 6 eqⁿs

$$TCV = \begin{bmatrix} W_1 \\ \hline W_2 \end{bmatrix}$$

W_1 → Position → P_1, P_2, P_3

W_2 → Rotation → $(R_{13})\exp(q_n/\pi)$, $(R_{23})\exp(q_n/\pi)$, $(R_{33})\exp(q_n/\pi)$

q_n, where n = no. of axis of robot

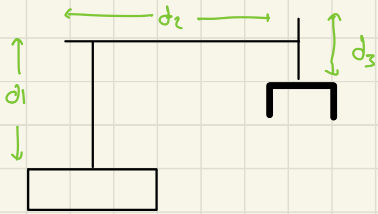
DK		IK
θ_1	$\longrightarrow$	q_1
θ_2	$\longrightarrow$	q_2
θ_3	$\longrightarrow$	q_3

$$TCV = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ \hline (R_{13})\exp(q_n/\pi) \\ (R_{23})\exp(q_n/\pi) \\ (R_{33})\exp(q_n/\pi) \end{bmatrix} = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \hline \omega_4 \\ \omega_5 \\ \omega_6 \end{bmatrix}$$

↓ current position

↓ desired orientation

Q. Inverse Kinematics of Cylindrical robot



A. $d_1 \rightarrow$ constant
$d_2 \rightarrow$ var
$d_3 \rightarrow$ var
$\theta_1 \rightarrow$ var

Arm matrix $= T_0^3$

$$T_0^3 = \begin{bmatrix} c_1 & s_1 & \boxed{0} & \boxed{-s_1 d_2} \\ s_1 & -c_1 & \boxed{0} & \boxed{c_1 d_2} \\ 0 & 0 & \boxed{-1} & \boxed{d_1 - d_3} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$TCV = \begin{bmatrix} W_1 \\ W_2 \end{bmatrix} = \begin{bmatrix} -s_1 d_2 \\ c_1 d_2 \\ d_1 - d_3 \\ 0 \\ 0 \\ -\exp(q_3/\pi) \end{bmatrix}$$

$w_1 = -s_1 d_2$ $\qquad$ $w_4 = 0$

$w_2 = c_1 d_2$ $\qquad$ $w_5 = 0$

$w_3 = d_1 - d_3$ $\qquad$ $w_6 = -\exp(q_3/\pi)$

i) d_2 (var) in w_1 & w_2

$$w_1^2 + w_2^2 = (-s_1 d_2)^2 + (c_1 d_2)^2$$
$$= (\underbrace{s_1^2 + c_1^2}_{1}) d_2^2$$

$$w_1^2 + w_2^2 = d_2^2$$

$$d_2 = \sqrt{w_1^2 + w_2^2}$$

(hypotenuse of right triangle)

ii) $$\frac{w_1}{w_2} = \frac{-s_1 d_2}{c_1 d_2} = \frac{-s_1}{c_1}$$

$$\frac{s\theta_1}{c\theta_1} = \tan\theta_1$$

but $\theta_1 \rightarrow q_1$ in IK

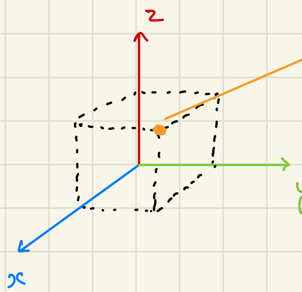
$$\frac{w_1}{w_2} = -\tan(q_1)$$

$$q_1 = \pm \tan^{-1}\left(\frac{w_1}{w_2}\right)$$

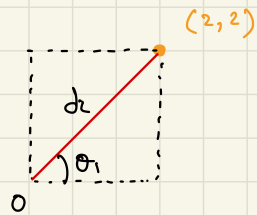
iii) $$w_3 = d_1 - d_3$$

$$d_3 = d_1 - w_3$$

Example :- $(x, y, z) = (2, 2, 2)$



If remove height :-



$w_1 = 2 \qquad w_2 = 2 \qquad w_3 = 2$

i) $d_2 = \sqrt{2^2 + 2^2} = 2\sqrt{2} = 2.82$ units

$d_2 = 2.82$ units

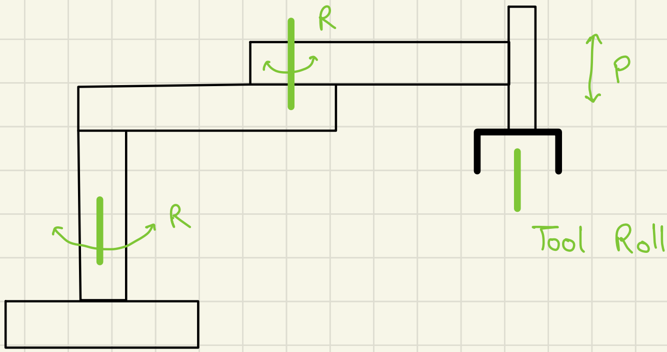
ii) $q_1 = \pm \tan^{-1}\left(\frac{2}{2}\right) = \tan^{-1}(1) = \pi/4$

$q_1 = 45^\circ$ or $\pi/4$ rad

iii) $d_3 = 10 - 2 = 8$

$d_3 = 8$ units

Q. Inverse Kinematics of SCARA robot



$C_{1-2} = \cos(\theta_1 - \theta_2)$

A. Arm matrix :-

$$T_{base}^{Tool} = \left[\begin{array}{ccc|c} C_{1-2-4} & S_{1-2-4} & \boxed{0} & a_1C_1 + a_2C_{1-2} \\ S_{1-2-4} & -C_{1-2-4} & 0 & a_1S_1 + a_2S_{1-2} \\ 0 & 0 & -1 & d_1 - q_3 - d_4 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

IK	DK	
$q_1 \longrightarrow$	$\theta_1 \rightarrow$	var
$q_2 \longrightarrow$	$\theta_2 \rightarrow$	var
$q_3 \longrightarrow$	$d_3 \longrightarrow$	var
$q_4 \longrightarrow$	$\theta_3 \longrightarrow$	var

$$T_{CV} = \begin{bmatrix} W_1 \\ \hdashline W_2 \end{bmatrix} = \begin{bmatrix} a_1 c_1 + a_2 c_{1-2} \\ a_1 s_1 + a_2 s_{1-2} \\ d_1 - q_{v_3} - d_4 \\ \hdashline 0 \\ 0 \\ -\exp(q_{v_4}/\pi) \end{bmatrix} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \end{bmatrix}$$

$$w_1 = a_1 c_1 + a_2 c_{1-2}$$
$$w_2 = a_1 s_1 + a_2 s_{1-2}$$
$$w_3 = d_1 - q_{v_3} - d_4$$
$$w_4 = 0$$
$$w_5 = 0$$
$$w_6 = -\exp(q_{v_4}/\pi)$$

Solve for $q_{v_1} \downarrow \theta_1$ $\quad q_{v_2} \downarrow \theta_2$ $\quad q_{v_3} \downarrow d_3$ $\quad q_{v_4} \downarrow \theta_4$

i) $$w_1^2 + w_2^2 = (a_1 c_1 + a_2 c_{1-2})^2 + (a_1 s_1 + a_2 s_{1-2})^2$$

$$= a_1^2 c_1^2 + a_2^2 c_{1-2}^2 + 2 a_1 a_2 c_1 c_{1-2} + a_1^2 s_1^2 + a_2^2 s_{1-2}^2 + 2 a_1 a_2 s_1 s_{1-2}$$

$$= a_1^2 + a_2^2 + 2 a_1 a_2 [s_1 s_{1-2} + c_1 c_{1-2}]$$
$$= a_1^2 + a_2^2 + 2 a_1 a_2 [c_{1-(1-2)}]$$
$$= a_1^2 + a_2^2 + 2 a_1 a_2 c_2$$

$$c_2 = \frac{w_1^2 + w_2^2 - a_1^2 - a_2^2}{2 a_1 a_2}$$

$$\theta_2 = \cos^{-1}\left[\frac{w_1^2 + w_2^2 - a_1^2 - a_2^2}{2a_1a_2}\right]$$

$$\boxed{q_2 = \cos^{-1}\left[\frac{w_1^2 + w_2^2 - a_1^2 - a_2^2}{2a_1a_2}\right]}$$

ii) $$w_3 = d_1 - q_3 - d_4$$

$$\boxed{q_3 = d_1 - d_4 - w_3}$$

iii) $$w_1 = a_1C_1 + a_2C_{1-2}$$

$$= a_1C_1 + a_2\left[C_1C_2 + S_1S_2\right]$$

$$= a_1C_1 + a_2C_1C_2 + a_2S_1S_2$$

$$w_1 = (a_1 + a_2C_2)C_1 + (a_2S_2)S_1 \longrightarrow ①$$

Now,

$$w_2 = a_1S_1 + a_2S_{1-2}$$

$$= a_1S_1 + a_2(S_1C_2 - C_1S_2)$$

$$= a_1S_1 + a_2S_1C_2 - a_2C_1S_2$$

$$w_2 = (-a_2S_2)C_1 + (a_1 + a_2C_2)S_1 \longrightarrow ②$$

Trick :- From ① & ②

$$S_1 = \frac{(a_2S_2)w_1 + (a_1+a_2C_2)w_2}{(a_2S_2)^2 + (a_1+a_2C_2)^2} \longrightarrow ③$$

$$c_1 = \frac{(a_1 + a_2 c_2)\omega_1 - (a_2 s_2)\omega_2}{(a_1 + a_2 c_2)^2 + (a_2 s_2)^2} \longrightarrow (4)$$

By (3) / (4)

$$\tan_1 = \frac{(a_2 s_2)\omega_1 + (a_1 + a_2 c_2)\omega_2}{\omega_1 (a_1 + a_2 c_2) - (a_2 s_2)\omega_2}$$

$$\boxed{q_1 = \theta_1 = \tan^{-1}\left[\frac{(a_2 s_2)\omega_1 + (a_1 + a_2 c_2)\omega_2}{(a_1 + a_2 c_2)\omega_1 - (a_2 s_2)\omega_2}\right]}$$

iv) $$w_6 = -\exp\left(q_4/\pi\right)$$

$$-\ln w_6 = \frac{q_4}{\pi}$$

$$q_4 = -\left[\pi \ln(w_6)\right]$$

[Take − linearly
∵ rotation from −π or π is same]

Q. $T_{base}^{Tool} = \begin{bmatrix} C_{123} & -S_{123} & 0 & a_1C_1 + a_2C_{12} \\ S_{123} & C_{123} & 0 & a_1S_1 + a_2S_{12} \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

3 Axis Para $(q_1\ q_2\ q_3)$

$C_{12} = C(\theta_1 + \theta_2)$
$C_{1-2} = C(\theta_1 - \theta_2)$

A. $T_{base}^{Tool} = \left[\begin{array}{ccc|c} C_{123} & -S_{123} & \boxed{0} & a_1C_1 + a_2C_{12} \\ S_{123} & C_{123} & 0 & a_1S_1 + a_2S_{12} \\ 0 & 0 & 1 & d_3 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$

$TCU = \begin{bmatrix} W_1 \\ \hdashline W_2 \end{bmatrix} = \begin{bmatrix} a_1C_1 + a_2C_{12} \\ a_1S_1 + a_2S_{12} \\ d_3 \\ \hdashline 0 \\ 0 \\ \exp(q_3/\pi) \end{bmatrix} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \end{bmatrix}$

$$w_1 = a_1c_1 + a_2c_{12}$$
$$w_2 = a_1s_1 + a_2s_{12}$$
$$w_3 = d_3$$
$$w_4 = 0$$
$$w_5 = 0$$
$$w_6 = \exp(q_3/\pi)$$

i) $w_1^2 + w_2^2 = (a_1c_1 + a_2c_{12})^2 + (a_1s_1 + a_2s_{12})^2$

$$= a_1^2c_1^2 + a_2^2c_{12}^2 + 2a_1a_2c_1c_{12} + a_1^2s_1^2 + a_2^2s_{12}^2 + 2a_1a_2s_1s_{12}$$
$$= a_1^2 + a_2^2 + 2a_1a_2(c_1c_{12} + s_1s_{12})$$

c_{1+2}

$c(\theta_1+\theta_2)$

$$w_1^2 + w_2^2 = a_1^2 + a_2^2 + 2a_1a_2(c_{1-1-2})$$
$$w_1^2 + w_2^2 = a_1^2 + a_2^2 + 2a_1a_2\,c_2$$

$$c_2 = \frac{w_1^2 + w_2^2 - a_1^2 - a_2^2}{2a_1a_2}$$

$$q_2 = \cos^{-1}\left[\frac{w_1^2 + w_2^2 - a_1^2 - a_2^2}{2a_1a_2}\right]$$

ii) $w_1 = a_1c_1 + a_2c_{12}$

$$= a_1c_1 + a_2[c_1c_2 - s_1s_2]$$
$$= a_1c_1 + a_2c_1c_2 - a_2s_1s_2$$
$$w_1 = [a_1 + a_2c_2]c_1 - [a_2s_2]s_1 \longrightarrow ①$$

$\sin(1+2)$

$$w_2 = a_1s_1 + a_2s_{12}$$
$$= a_1s_1 + a_2[s_1c_2 + c_1s_2]$$
$$= a_1s_1 + a_2s_1c_2 + a_2c_1s_2$$
$$= [a_1 + a_2c_2]s_1 + [a_2s_2]c_1 \longrightarrow ②$$

Trick from ①, ②,

$$s_1 = \frac{-[a_2 s_2]\, w_1 + (a_1 + a_2 c_2]\, w_2}{(a_2 s_2)^2 + (a_1 + a_2 c_2)^2}$$

$$c_1 = \frac{(a_1 + a_2 c_2)\, w_1 - (a_2 s_2)\, w_2}{(a_2 s_2)^2 + (a_1 + a_2 c_2)^2}$$

$$\tan_1 = \frac{-[a_2 s_2]\, w_1 + [a_1 + a_2 c_2]\, w_2}{(a_1 + a_2 c_2)\, w_1 - (a_2 s_2)\, w_2}$$

$$q_1 = \tan^{-1}\left[\frac{-[a_2 s_2] w_1 + [a_1 + a_2 c_2] w_2}{(a_1 + a_2 c_2) w_1 - (a_2 s_2) w_2}\right]$$

iii)

$$w_6 = \exp\left(q_3/\pi\right)$$

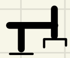
$$\ln w_6 = q_3/\pi$$

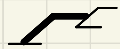
$$q_3 = \pi \ln(w_6)$$

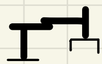
M-2

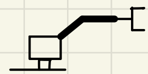
D-H

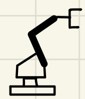
Questions :-

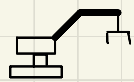
i) cylindrical 

ii) 3 axis Para 

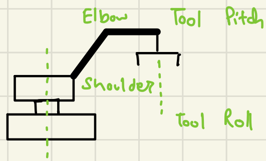
iii) SCARA 

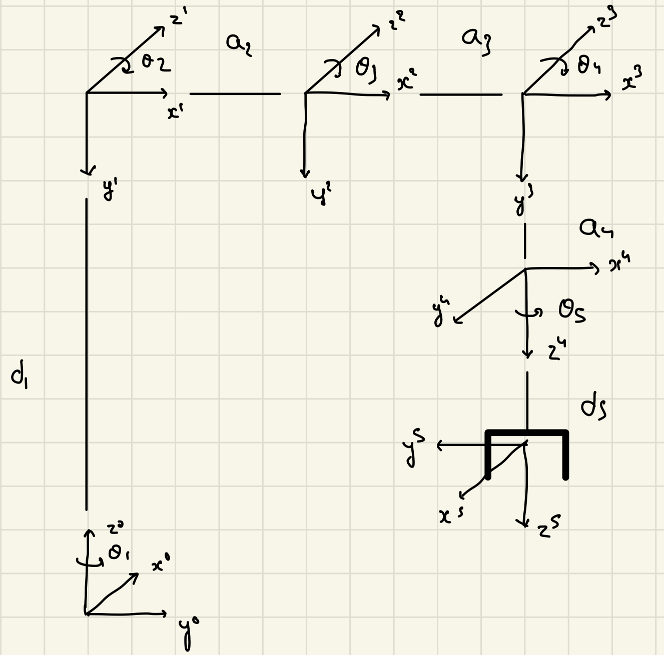
iv) 5 axis Microbot Alpha II 

v) Yaskawa Motoman 

vi) Rhino XR 3 

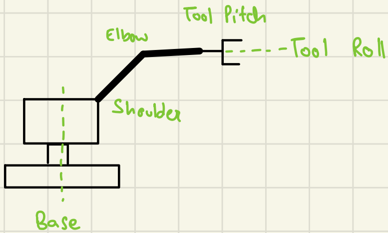
Q. Rhino XR 3

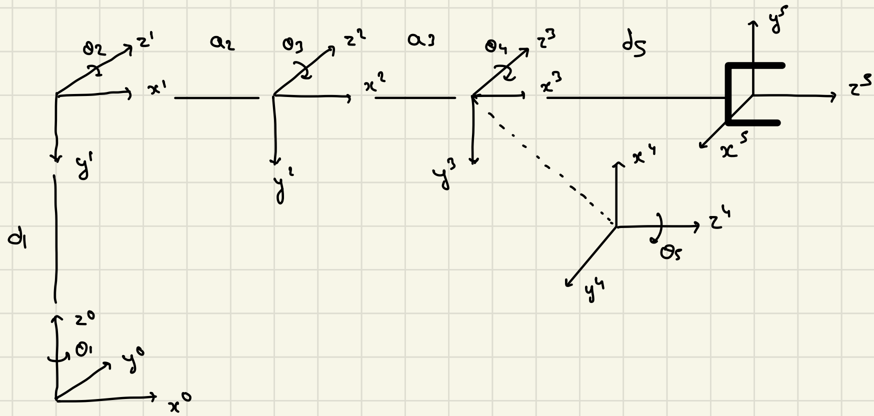




Axis	θ_k	a_k	d_k	α_k	var
1	θ_1	0	d_1	$-\pi/2$	θ_1
2	θ_2	a_2	0	0	θ_2
3	θ_3	a_3	0	0	θ_3
4	θ_4	a_4	0	0	θ_4
5	θ_5	0	d_5	$-\pi/2$	θ_5

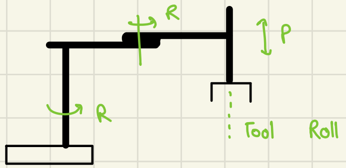
Q. 5 axis Microbot Alpha II

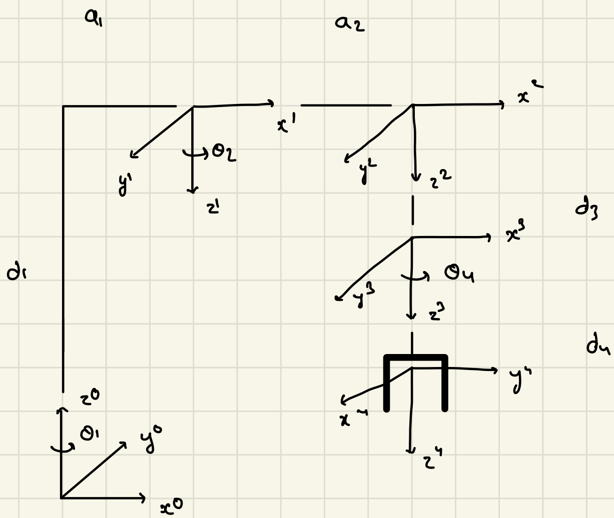




Axis	θ_k	d_k	a_k	α_k	var
1	θ_1	d_1	0	$-\pi/2$	θ_1
2	θ_2	0	a_2	0	θ_2
3	θ_3	0	a_3	0	θ_3
4	θ_4	0	0	$-\pi/2$	θ_4
5	θ_5	d_5	0	0	θ_5

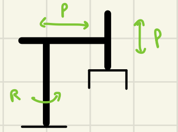
Q. SCARA Robot

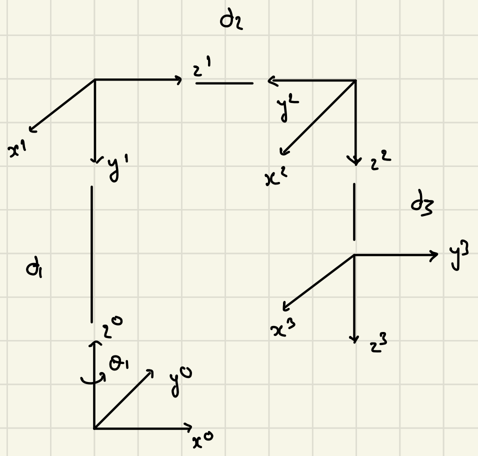




Axis	θ_k	d_k	a_k	α_k	var
1	θ_1	d_1	a_1	π	θ_1
2	θ_2	0	a_2	0	θ_2
3	0	d_3	0	0	d_3
4	θ_4	d_4	0	0	θ_4

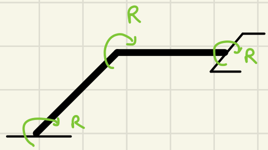
Q. Cylindrical Robot

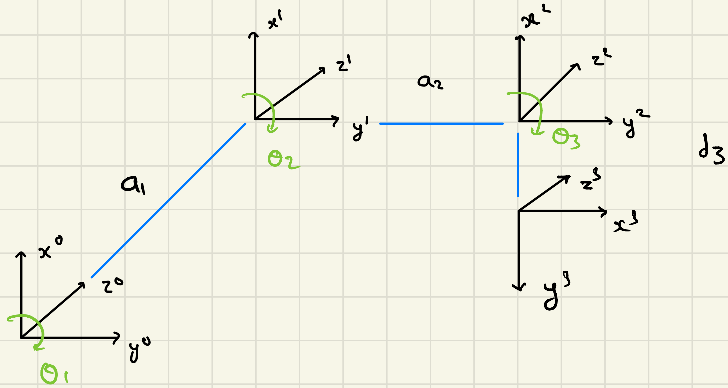




Axis	θ_k	d_k	a_k	α_k	var
1	θ_1	d_1	0	$-\pi/2$	θ_1
2	0	d_2	0	$-\pi/2$	d_2
3	0	d_3	0	0	d_3

Q. 3 axis Para


R
R
R


x^1
z^1
y^1
θ_2
a_2
x^2
z^2
y^2
θ_3
d_3
a_1
z^3
x^3
y^3
x^0
z^0
y^0
θ_1

Theory

DK :- 2-5 51

IK :- 3-1 , 3-2 , 3-3

81 84 87

Q. Joint angle (θ_k)

A. Rotation about z^{k-1} needed to make x^{k-1} parallel with axis x^k

Q. Joint distance (d_k)

A. Translation along z^{k-1} needed to make axis x^{k-1} intersect with axis x^k

Revolute :- θ_k = var

d_k = fixed

Prismatic :- θ_k = fixed

d_k = var

Q. Link Length (a_k)

A. Translation along x^k needed to make axis z^{k-1} intersect with axis z^k

Q. Twist Angle

A. Rotation about x^k needed to make axis z^{k-1} parallel with axis z^k

Q. Kinematic simple manupulators

A. Robots designed to have many constant kinematic parameters equal to zero.

Q. Approach vector

A. aligned in direction of tool roll axis and points away from wrist

Q. Sliding vector

A. Aligned with open-close axis of tool & orthogonal to approach vector

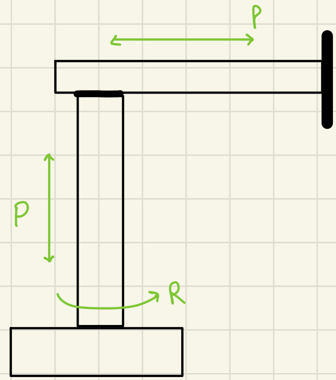
Q. Normal vector

A. Orthogonal to plane defined by approach & sliding vector

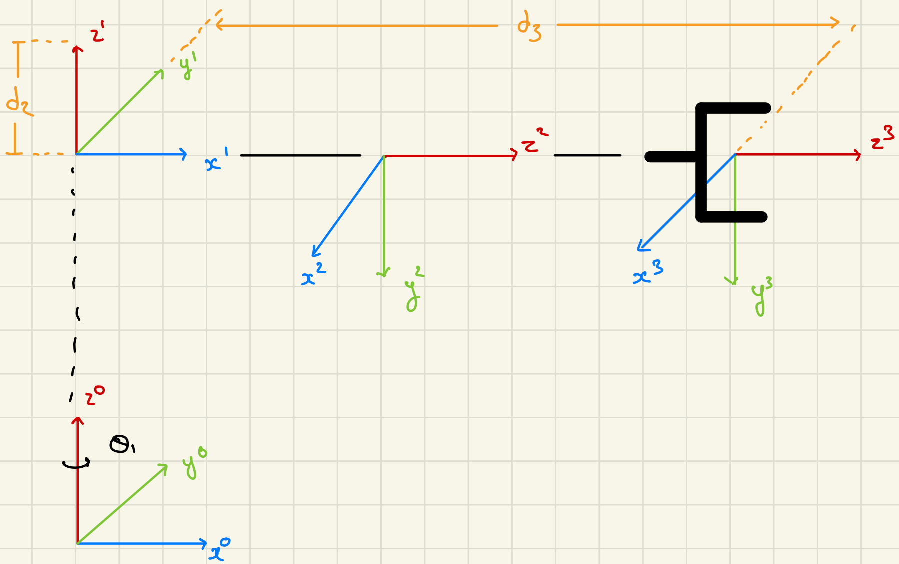
ARM MATRIX

$$T_{k-1}^{k} = \begin{bmatrix} C_{\theta k} & -C_{\alpha_k} S_{\theta k} & S_{\alpha k} S_{\theta k} & a_k C_{\theta k} \\ S_{\theta k} & C_{\alpha_k} C_{\theta k} & -S_{\alpha k} C_{\theta k} & a_k S_{\theta k} \\ 0 & S_{\alpha k} & C_{\alpha k} & d_k \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Q. Direct & Inverse of variation of cylindrical robot



A.



Axis	θ_k	d_k	a_k	α_k
1	θ_1	0	0	0
2	0	d_2	0	$-\pi/2$
3	0	d_3	0	0

$$T_{k-1}^{k} = \begin{bmatrix} C_{\theta k} & -C_{\alpha k} S_{\theta k} & S_{\alpha k} S_{\theta k} & a_k C_{\theta k} \\ S_{\theta k} & C_{\alpha k} C_{\theta k} & -S_{\alpha k} C_{\theta k} & a_k S_{\theta k} \\ 0 & S_{\alpha k} & C_{\alpha k} & d_k \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^{1} = \begin{bmatrix} C_1 & -S_1 & 0 & 0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_1^{2} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^{3} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^3 = T_0^1 \cdot T_1^2 \cdot T_2^3 \qquad \text{(Arm matrix)}$$

$$T_0^3 = \begin{bmatrix} c_1 & 0 & -s_1 & -s_1 d_3 \\ s_1 & 0 & c_1 & c_1 d_3 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Inverse :-

$$T_{CV} = \begin{bmatrix} W_1 \\ \hdashline W_2 \end{bmatrix} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \end{bmatrix} = \begin{bmatrix} -s_1 d_3 \\ c_1 d_3 \\ d_2 \\ -s_1 \exp(q_3/\pi) \\ c_1 \exp(q_3/\pi) \\ 0 \end{bmatrix}$$

$w_1 = -s_1 d_3$

$w_2 = c_1 d_3$

$w_3 = d_2$

$w_4 = -s_1 \exp(q_3/\pi)$

$w_5 = c_1 \exp(q_3/\pi)$

$w_6 = 0$

To find q_1, d_2, d_3

Q. Given M $\rightarrow$ Find F $\quad [P]^F = A[P]^M$

Q. Given F $\rightarrow$ Find M $\quad [Q]^M = A^T [Q]^F$

Composite Algo

M $\rightarrow$ F $\quad \rightarrow$ pre multiply $\quad 2 \times 1$

move $\quad$ wrt

M $\rightarrow$ M $\quad \rightarrow$ post multiply $\quad 1 \times 2$

move $\quad$ wrt