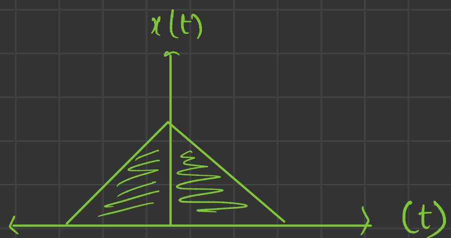
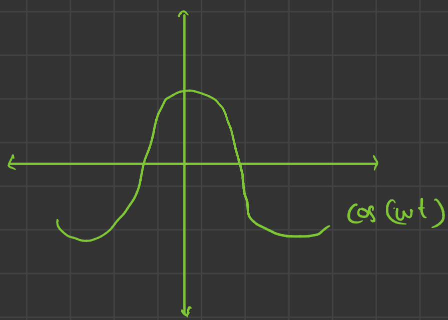
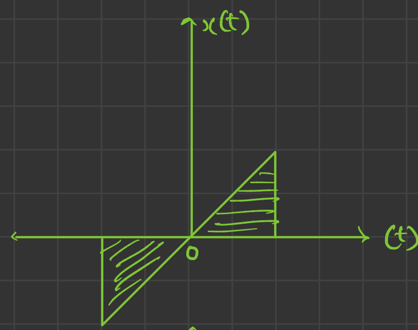
# * Even & Odd Signal

| Even signal | Odd Signal |
|---|---|
| $x(-t) = x(t)$ | $x(-t) \neq x(t)$ |
| Remain identical under folding operation. | $x(-t) = -x(-t)$ |
| | Doesn't remain identical under folding operations. |




Unit 1 :-

i) Even / Odd signal
ii) Energy / Power signal
iii) Signal transformation
iv) Periodic / aperiodic signal

Unit 2 :-

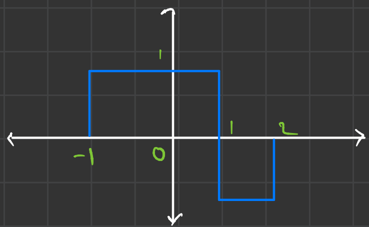
i) Linear / Non-linear system
ii) Stable / Unstable system
iii) Convolution

## Even and Odd Signal :-

Even part $\frac{1}{2}\left[x(t) + x(-t)\right]$

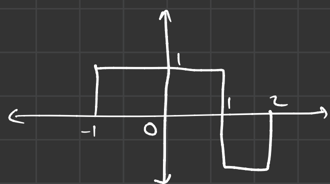
Odd part $\frac{1}{2}\left[x(t) - x(-t)\right]$

Q.

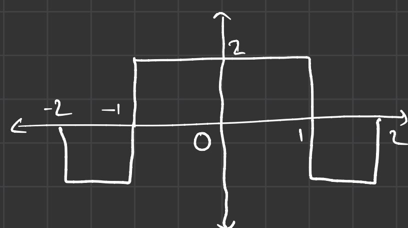



Even part $\Rightarrow$ $\frac{1}{2}\left[x(t) + x(-t)\right]$

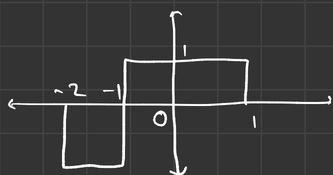
$x(t) \Rightarrow$



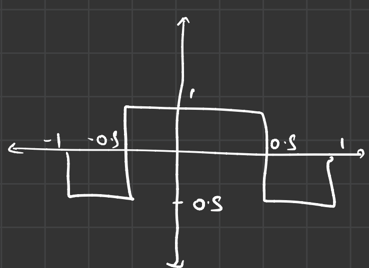

$x(t) + x(-t) \Rightarrow$




$x(-t) \Rightarrow$




$\frac{1}{2}\left(x(t) + x(-t)\right) \Rightarrow$



## Energy / Power Signal :-

$$E = \sum_{n=\infty}^{\infty} |x(n)|^2$$

$$E = 0 + 0 + 0 + \dots + 1^2 + \dots 0 + 0$$

$$E = 1$$

$$P = \lim_{n \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

Even :- $\frac{1}{2}\left[x(t) + x(-t)\right]$

Odd :- $\frac{1}{2}\left[x(t) - x(-t)\right]$

$$E = \lim_{t \to \infty} \int_{-T}^{T} |x(t)|^2 \, dt$$

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$$P = \lim_{t \to \infty} \frac{1}{2T} \int_{-T}^{T} x(t)^2 \, dt$$

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# Periodic & Aperiodic Signal

$$x(t) = \sin 1.2\pi t - 2\cos 3\pi t$$

Ans :-

$$x_1(t) = \sin 1.2\pi t$$
$$x_1(t+T) = \sin 1.2\pi (t+T)$$
$$= \sin 1.2\pi t \cos 1.2\pi T + \cos 1.2\pi t \sin 1.2\pi T$$

$$x_1(t) = x_1(t+T) \text{ if}$$

$$\cos 1.2\pi T = 1$$
$$\sin 1.2\pi T = 0$$

$$1.2\pi T = 2\pi$$
$$T_1 = \frac{2}{1.2} = \frac{20}{12} = \frac{5}{3}$$

$$x_2(t) = \cos 1.2\pi T$$
$$x_2(t+T) = \cos 1.2\pi (t+T)$$
$$= \cos 1.2\pi t \cos 1.2\pi T - \sin 1.2\pi t \sin 1.2\pi T$$

$$x_2(t) = x_2(t+T) \text{ if}$$
$$\cos 1.2\pi T = 1$$
$$\sin 1.2\pi T = 0$$

$$1.2\pi T = 2\pi$$

irration ratio $\left(\frac{T_1}{T_2}, \frac{T_2}{T_3}, \frac{T_1}{T_3}\right) \Rightarrow$ aperiodic

rational ratio ( " ) $\Rightarrow$ Periodic

## Stable / Unstable

$$\int_{-\infty}^{\infty} |h(t)|\, dt$$

$$\sum_{n=-\infty}^{\infty} h(t)$$

Q. $h[n] = (0.3)^n\, u(n)$

$$\sum_{n=0}^{\infty} (0.3)^n$$

$$\sum_{n=0}^{\infty} \alpha^n = \frac{1}{1-\alpha}$$

$$\sum_{n=1}^{\infty} \alpha^n = \frac{1-\alpha^{n+1}}{1-\alpha}$$

Show that

i) $Re\{z\} = \dfrac{z+z^*}{2}$

A⇒ $z = \cos\theta + i\sin\theta$

$z^* = \cos\theta - i\sin\theta$

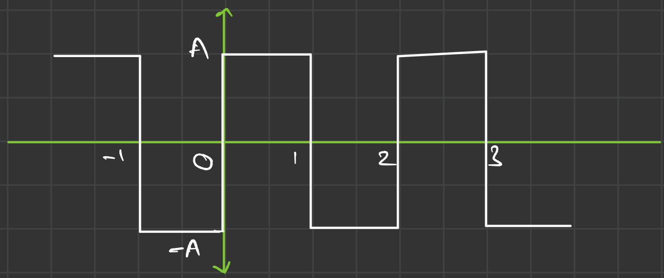
$Re\{z\} = \dfrac{2\cos\theta}{2} = \cos\theta$ = real part of $z$

ii) $img\{z\} = \dfrac{z-z^*}{2}$

$img\{z\} = \dfrac{2i\sin\theta}{2} = i\sin\theta$ = img part of $z$

# Fourier Series

Q. Find fourier series of square wave given below :-




i) signal is symmetric $\longrightarrow$ $a_0 = 0$ $\left( \begin{matrix} 1/2 \text{ +ve} \\ 1/2 \text{ -ve} \end{matrix} \right)$

ii) Signal is odd $x(-t) = -x(t)$

$$x(t) = A \qquad 0 < t < 1$$
$$x(t) = -A \qquad -1 < t < 0$$

$\therefore$ Odd signal

iii) $$x(t) = \sum_{n=1}^{\infty} b_n \sin(n\omega_0)t$$

$$b_n = \frac{2}{T_0} \int_{T_0} x(t) \sin n\omega_0 t \, dt$$

$$b_n = \frac{2}{2}\left\{\int_0^1 A \sin n\omega_0 t \, dt + \int_1^2 -A \sin n\omega_0 t \, dt\right\}$$

$$= 1\left\{A\left(-\frac{\cos n\omega_0 t}{n\omega_0}\right)_0^1 + (-A)\left(-\frac{\cos n\omega_0 t}{n\omega_0}\right)_1^2\right\}$$

Note $\int \sin x \, dx = -\cos x$ & $\cos n\pi = (-1)^n$

$$= 1\left\{\frac{A}{n\omega_0}\left(-\cos n\pi + \cos 0 + \cos 2n\pi - \cos n\pi\right)\right\}$$

$$= \frac{A}{n\omega_0}\left(-(-1)^n + 1 + 1 - (-1)^n\right)$$

$$= \frac{A}{n\omega_0}\left[2 - 2(-1)^n\right]$$

$$\therefore b_n = \frac{2A}{n\pi}\left[1 - (-1)^n\right], \quad \omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{2}$$

$$\omega_0 = \pi$$

$$x(t) = \cancel{a_0} + \sum_{n=1}^{\infty} \cancel{a_n \cos n\omega_0 t} + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t$$

odd

$$a_0 = 0, \quad a_n = 0$$

$$\therefore x(t) = \sum_{n=1}^{\infty} b_n \sin n\omega_0 t$$

$$x(t) = \sum_{n=1}^{\infty} \frac{2A}{\pi} \left[ \frac{+1 - (-1)^n}{n} \right] \sin n\omega_0 t$$

Consider $n = 2, 4, 6, \ldots$ even nos

$n = 1, 3, 5, \ldots$ odd nos

when $n$ is even $(1) - (-1)^n = 0$

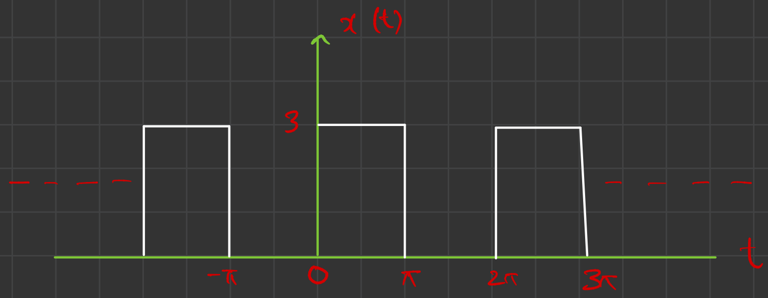
$n$ is odd $(1) - (-1)^n = 2$

$$x(t) = \frac{2A}{\pi} \left[ \frac{2}{n} \sin \pi t \right], \quad n = \text{odd}$$

$$x(t) = \frac{2A}{\pi} \left[ 2 \left( \frac{\sin \pi t}{1} + \frac{\sin 3\pi t}{3} + \frac{\sin 5\pi t}{5} + \ldots \right) \right]$$

$$x(t) = \frac{4A}{\pi} \left[ \frac{\sin \pi t}{1} + \frac{\sin 3\pi t}{3} + \frac{\sin 5\pi t}{5} + \ldots \right]$$

2.




odd Component

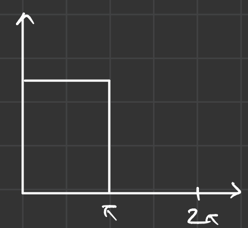
A. $x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t$

DC component

even Component

i) Signal is not symmetric

$$a_0 = \frac{1}{T_0} \int_{T_0} x(t)\, dt$$




$T_0 = 2\pi$

$\omega_0 = \frac{2\pi}{T_0} = 1$

$$a_0 = \frac{1}{2\pi} \int_0^{2\pi} x(t)\, dt$$

$$x(t) = 3 \,, \quad 0 < t < \pi$$
$$= 0 \,, \quad \pi < t < 2\pi$$

$$a_0 = \frac{1}{2\pi} \left\{ \int_0^{\pi} 3\, dt + \int_{\pi}^{2\pi} 0 \cdot dt \right\}$$

$$a_0 = \frac{1}{2\pi} \left\{ 3 (t)_0^{\pi} \right\}$$

$$a_0 = \frac{3\pi}{2\pi}$$

$$a_0 = \frac{3}{2}$$

$$a_n = \frac{2}{T_0} \int_{T_0} x(t) \cos n w_0 t \; dt$$

$T_0 = 2\pi$

$w_0 = \frac{2\pi}{T_0}$

$w_0 = \frac{2\pi}{2\pi} = 1$

$$= \frac{2}{2\pi} \int_0^{2\pi} x(t) \cos n(1)t \; dt$$

$$= \frac{1}{\pi} \left\{ \int_0^{\pi} 3 \cos nt \, dt + \int_{\pi}^{2\pi} 0 \cdot \cos nt \, dt \right\}$$

$$= \frac{1}{\pi} \left\{ 3 \left[ \frac{\sin nt}{n} \right]_0^{\pi} \right\}$$

$$= \frac{3}{n\pi} \left[ \sin n\pi - \sin 0 \right]$$

$$= \frac{3}{n\pi} \left[ 0 - 0 \right] = 0$$

$$b_n = \frac{2}{T_0} \int_{T_0} x(t) \sin n w_0 t \; dt$$

$$= \frac{2}{2\pi} \int_0^{2\pi} x(t) \sin nt \; dt$$

$$= \frac{1}{\pi} \left\{ \int_0^{\pi} 3 \sin nt \, dt + \int_{\pi}^{2\pi} 0 \cdot \sin nt \, dt \right\}$$

$$= \frac{1}{\pi} \left\{ \left[ \frac{-3 \cos nt}{n} \right]_0^{\pi} \right\}$$

$$= \frac{-3}{n\pi} \left( \cos n\pi - \cos 0 \right)$$

$$b_n = \frac{-3}{n\pi}\left((-1)^n - 1\right)$$

when $n$ is odd, $n = 1, 3, 5, \ldots$

$$x(t) = \frac{3}{2} + 0 + \left[\frac{6}{\pi}\sin t + \frac{6}{3\pi}\sin 3t + \frac{6}{5\pi}\sin 5t + \ldots\right]$$

when $n$ is even, $n = 0, 2, 4, 6, \ldots$

$$x(t) = \frac{3}{2}$$

# NUMERICALS

Q. Find Fourier Transform of $x(t) = \sin \omega_0 t$

$$\mathcal{F}[x(t)] = \mathcal{F}[\sin \omega_0 t]$$

$$= \mathcal{F}\left[\frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j}\right]$$

$$= \frac{1}{2j}\left[\mathcal{F}\{1 \cdot e^{j\omega_0 t}\} - \mathcal{F}\{1 \cdot e^{-j\omega_0 t}\}\right]$$

Since, $\mathcal{F}(1) = 2\pi \delta(\omega)$

and $\mathcal{F}[1 \times e^{j\omega_0 t}] = 2\pi \delta(\omega - \omega_0)$

$\therefore$ $\mathcal{F}[1 \times e^{j\omega_0 t}] = 2\pi \delta(\omega - \omega_0)$

$\mathcal{F}[1 \times e^{-j\omega_0 t}] = 2\pi \delta(\omega + \omega_0)$

$$\mathcal{F}[x(t)] = \frac{1}{2j}\left[2\pi \delta(\omega - \omega_0) - \delta(\omega + \omega_0)\right]$$

$$\boxed{X(\omega) = j\pi\left[\delta(\omega + \omega_0) - \delta(\omega - \omega_0)\right]}$$

Q. Find Fourier Transform of

$$x(t) = \cos w_0 t \; u(t)$$

A. $\mathcal{F}[\cos w_0 t \; u(t)]$

$$= \mathcal{F}\left[\left(\frac{e^{jw_0t} + e^{-jw_0t}}{2}\right) u(t)\right]$$

$$= \frac{1}{2}\left[\mathcal{F}[e^{jw_0t} u(t)] + \mathcal{F}[e^{-jw_0t} u(t)]\right]$$

$$\mathcal{F}[u(t)] = \pi \delta(\omega) + \frac{1}{j\omega}$$

$$\mathcal{F}[x(t) e^{jw_0t}] = X(\omega - w_0)$$

$$\mathcal{F}[u(t) e^{jw_0t}] = \pi \delta(\omega - w_0) + \frac{1}{j(\omega - w_0)}$$

$$\mathcal{F}[u(t) e^{-jw_0t}] = \pi \delta(\omega + w_0) + \frac{1}{j(\omega + w_0)}$$

Missing a Hi Hi my name is Kaa!

# Z- Transform

$$\boxed{z = u + jv = re^{j\omega}}$$

$u$ = Real part of $z$

$v$ = imaginary part of $z$

$r = \sqrt{u^2 + v^2}$ = Magnitude of $z$

$\omega = \tan^{-1} \frac{v}{u}$ = Phase or argument of $z$

Discrete :-
$$\boxed{x(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}}$$

$$z = re^{j\omega}$$

$$x(z) = \sum_{n=-\infty}^{\infty} [x(n) r^{-n}] e^{-j\omega n}$$

$$x(z) = FT [x(n) r^{-n}]$$

$\therefore$ Z transform is FT of $(x(n) r^{-n})$

$x(z) = FT$ (when $r = 1$)

## Discrete Time Fourier Transform

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

## Numerical 1

$x(n) = \{\underset{\uparrow}{1}, 2, 5, 7, 0, 1\}$

$$x(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \qquad x(z) = \sum_{n=0}^{5} x(n) z^{-n}$$

$$x(z) = x(0)z^{-0} + x(1)z^{-1} + x(2)z^{-2} + x(3)z^{-3} + x(4)z^{-4} + x(5)z^{-5}$$

$$x(z) = 1 + 2z^{-1} + 5z^{-2} + 7z^{-3} + z^{-5}$$

$$\boxed{x(z) = 1 + \frac{2}{z} + \frac{5}{z^2} + \frac{7}{z^3} + \frac{1}{z^5}}$$

Region of Convergence

## Numerical 2

$x(n) = \{7, \underset{\uparrow}{3}, 4, 9, 5\}$

$$x(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= \sum_{n=-1}^{3} x(n) z^{-n}$$

$$= x(-1)z + x(0) + x(1)z^{-1} + x(2)z^{-2} + x(3)z^{-3}$$

$$= 7z + 3 + 4z^{-1} + 9z^{-2} + 5z^{-3}$$

$$x(n) = 7Z + 3 + \frac{4}{z} + \frac{9}{z^2} + \frac{5}{z^3}$$

# FOURIER SERIES

## Fourier Series

Any arbitrary real or complex signal which is periodic with fundamental frequency $f_0$ can be expressed as sum of sinusoids with various amplitudes, phases at frequency $f_0$ and it's harmonics ($2f_0$, $3f_0$, $4f_0$, ...)

$$x(t) = a_0 + a_1 \cos(\omega_0)t + a_2 \cos(2\omega_0)t + a_3 \cos(3\omega_0)t \ldots$$
$$+ b_1 \sin(\omega_0)t + b_2 \sin(2\omega_0)t + b_3 \sin(3\omega_0)t + \ldots$$

$\omega_0 \rightarrow$ fundamental frequency

$2\omega_0, 3\omega_0 \rightarrow$ harmonics

## Even Symmetry

$$x(t) = x(-t) \Rightarrow a_0 = \frac{1}{T_0}\int_0^{T_0} x(t)\,dt$$

$$a_n = \frac{2}{T_0}\int_0^{T_0} x(t)\cos(n\omega_0)t\,dt$$

$$b_n = 0$$

## Odd Symmetry

$$x(t) = -x(-t) \Rightarrow a_0 = 0$$

$$a_n = 0$$

$$b_n = \frac{2}{T_0}\int_0^{T_0} x(t)\sin(n\omega_0)t\,dt$$

# Polar Fourier Series

Any arbitrary signal can be expressed as linear combination of harmonically related sine and cosine components.

$$x(t) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos(n\omega_0)t + b_n \sin(n\omega_0)t\right) \quad \text{(1)}$$

Since, $A\cos x + B\sin x = C\cos(x-\theta)$

$A\cos x - B\sin x = C\cos(x+\theta)$

Eq (1) can be written as,

$$x(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t - \theta_n)$$

$C_0 = a_0$

$|C_n| = \sqrt{a_n^2 + b_n^2}$

$\theta_n = \tan^{-1}\left(\frac{b_n}{a_n}\right)$

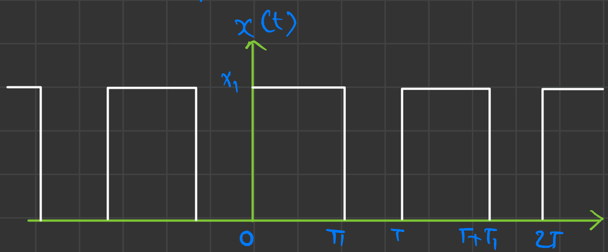
$$x(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \theta_n)$$

$C_0 = a_0$

$|C_n| = \sqrt{a_n^2 + b_n^2}$

$\theta_n = \tan^{-1}\left(\frac{-b_n}{a_n}\right)$

Time Domain Signal :




Amplitude Spectrum : Plot amplitude $|C_n|$ versus n.f.

Phase Spectrum : Plot phase $\theta_n$ versus n.f.

# Exponential Fourier Series

Any arbitrary signal can be expressed as linear combination of harmonically related sine and cosine components.

$$x(t) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos(n\omega_0)t + b_n \sin(n\omega_0)t\right)$$

$$e^{j\phi} = \cos\phi + j\sin\phi$$

The FS can be expressed in terms of $e^{\pm jn\omega_0 t}$

$$x(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t}$$

where, $D_n = \frac{1}{T_0} \int_{T_0} x(t)\, e^{-jn\omega_0 t}\, dt$

and, $D_0 = \frac{1}{T_0} \int_{T_0} x(t)\, dt$

# Numericals

$b_n = \frac{2}{T_0} \int_{T_0} x(t) \sin n\omega_0 t \, dt$ , $\omega_0 = \frac{2\pi}{T_0}$ , $a_0 = \frac{1}{T_0} \int_{T_0} x(t) dt$

$x(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)$ , $a_n = \frac{2}{T_0} \int_{T_0} x(t) \cos n\omega_0 t \, dt$

$\cos n\pi = (-1)^n$ , $\sin n\pi = 0$ , $\int x \cos ax \, dt = \frac{1}{a^2} [\cos ax + ax \sin ax]$

$\cos 2\pi n = 1$ , $\sin 2\pi n = 0$ , $\int x \sin ax \, dt = \frac{1}{a^2} [\sin ax - ax \cos ax]$

$x(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \phi_n)$

$\int u \, dv = uv - \int v \, du$

# Fourier Transform

A mathematical tool / procedure to convert a periodic signal $x(t)$ in the time domain to a complex number $X(\omega)$ in the frequency domain.

Numericals

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$L^{-1}\left[\frac{1}{j\omega + a}\right] = e^{-at}u(t)$$

$$L^{-1}\left[\frac{1}{(j\omega + a)^2}\right] = t e^{-at}u(t)$$

# Z TRANSFORM

$$Z[a^n u(n)] = \frac{z}{z-a}, \quad ROC :- |z| > a$$

$$Z[-a^n u(-n-1)] = \frac{z}{z-a}, \quad ROC :- |z| < a$$

$$Z[\bar{a}^n u(-n-1)] = \frac{-z}{z-a^{-1}}$$

Q. $x(n) = 3^n u(n) + 3^{-n} u(-n-1)$

A. $$3^n u(n) = \frac{z}{z-3}$$

$$3^{-n} u(n-1) = (3^{-1})^n u(-n-1)$$

$$= \frac{z}{z-(3^{-1})}$$

$$x(z) = \frac{z}{z-3} + \frac{z}{z-(3^{-1})}$$

$\downarrow$ ROC :- $|z| > 3$

$\downarrow$ ROC :- $|z| < \frac{1}{3}$

Q. $x(n) = \left(\frac{1}{5}\right)^n [u(n) - u(n-5)]$

$$X(z) = Z\left[\left(\frac{1}{5}\right)^n u(n)\right] - Z\left[\left(\frac{1}{5}\right)^n u(n-5)\right]$$

$$= \frac{z}{z - 1/5} -$$

$$Z\left[\left(1/5\right)^n u(n-5)\right] = \left(\frac{1}{5}\right)^5 z^{-5} \left(\frac{z}{z - 1/5}\right)$$

Q. $Z\left[\left(1/3\right)^{-n} u(-n-1)\right]$

$$Z\left[\left(1/3\right)^{-n} u(-n-1)\right]$$

$$Z\left[a^{-n} u(-n-1)\right] = \frac{-z}{z + a^{-1}}$$

# Energy & Power Signals

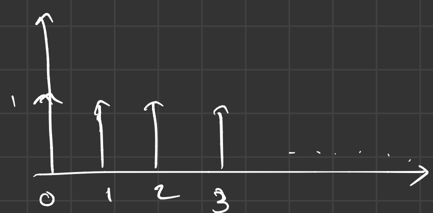
$$E = \sum_{n=\infty}^{\infty} |x(n)|^2 \qquad E = \sum_{n=\infty}^{\infty} |x(n)|^2 \qquad E = \sum_{n=\infty}^{\infty} |x(n)|^2$$

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2 \qquad E = \sum_{n=\infty}^{\infty} |x(n)|^2$$

$$P = \lim_{N \to \infty} \frac{1}{2N+N} \sum_{n=-N}^{N} |x(n)|^2$$

$$E = \sum_{n=\infty}^{\infty} |x(n)|^2 \qquad P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

Q.




$$E = \sum_{n=\infty}^{\infty} |x(n)|^2$$

$$= 0 + 0 + \ldots + 1^2 + 1^2 + 1^2 + \ldots$$

$$E = \infty$$

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=0}^{N} |u(n)|^2$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=0}^{N} (1)$$

$$= \lim_{N \to \infty} \frac{N+1}{2N+1} = \lim_{N \to \infty} \frac{1 + 1/N}{2 + 1/N} = \underline{\underline{\frac{1}{2}}}$$

(3) $x(n) = \left(\frac{1}{4}\right)^n \quad , \quad n \geq 0$

$2^n \quad , \quad n < 0$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$E = \sum_{n=-\infty}^{-1} (2^n)^2 + \sum_{n=\infty}^{0} \left(\frac{1}{4}^n\right)^2$$

$$E = \sum_{n=-\infty}^{-1} 2^{2n} + \sum_{n=\infty}^{0} \left(\frac{1}{4}\right)^{2n}$$

$$\sum_{n=0}^{\infty} \alpha^n = \frac{1}{1-\alpha} \quad , \quad \sum_{n=1}^{\infty} \alpha^n = \frac{\alpha}{1-\alpha}$$

Let $n = -x$

$n = -\infty \quad , \quad x = \infty$

$n = -1 \quad , \quad x = 1$

$$E = \sum_{x=1}^{\infty} (4)^{-x} + \sum_{x=-\infty}^{0} \left(\frac{1}{16}\right)^{-x}$$

Q. $x(n) = (0.5)^n u(n)$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$= \sum_{n=0}^{\infty} |(0.5)^n|^2$$

$$= \sum_{n=0}^{\infty} 0.25^n$$

$$E = \frac{1}{1-0.25}$$

$$E = \frac{1}{0.75} = \underline{\underline{\frac{4}{3}}}$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} |x(n)|^2 = 0$$

Q. $x(n) = 5e^{2jn}$

$$e^{j2n} = \cos 2n + j\sin 2n$$

$$e^{j2n} = \cos 2n + j\sin 2n$$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$E = \sum_{n=\infty}^{\infty} 25 (\cos 2n + j\sin 2n)^2$$

$$E = \sum_{n=-\infty}^{\infty} 25 = \infty$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} 25$$

$$\sum_{n=-N}^{N} A = (2N+1)A$$

$$= \lim \frac{1}{\phantom{x}} \times (2N+1)\, 25$$

$$N \to \infty \quad 2N+1$$

$$= \lim_{N \to \infty} 25$$

$$\underline{\underline{P = 25}}$$

$\therefore$ Power signal

Q. $x(t) = e^{-|t|} \sin 2t$

i.e., $x(t) = e^{-t} \sin 2t , \quad t \geq 0$
$\qquad\qquad e^{t} \sin 2t , \quad t \leq 0$

It is a aperiodic signal

$$E = \sum_{n=-\infty}^{\infty}$$

# TEE

**Energy** $E = \int_{-\infty}^{\infty} |x(n)|^2$  **Power** $P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$

**Periodic / Aperiodic** :- $T = \frac{2\pi}{\omega}$ , Rational , Common

**Even** :- $x_e(t) = \frac{x(t) + x(-t)}{2}$  **Odd** :- $x_o(t) = \frac{x(t) - x(-t)}{2}$

**Transformation** :-
+ve → left
−ve → right
$2t+3$ → left , ÷ signal by 2
$-t+1$ → left , mirror

**Linear / Non - linear** :-
i) $x(t) = 0 \rightarrow y(t) = 0$
ii) $x_1(t) \rightarrow$
iii) $x_2(t) \rightarrow$
iv) $y_1(t) + y_2(t)$
v) Put input as $x_1(t) + x_2(t)$ & compare

$\int uv = u\int v - \int(du \int v)$

$e^{j\omega t} = \cos\omega t + j\sin\omega t$

**Stability** :- $\int_{-\infty}^{\infty} |h(t)|\,dt < \infty$ — for stable system

**Gibb's Phenomenon** :- Oscillations compressed towards either side of discontinuity.

**Fourier series** :- $x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t$

where, $a_0 = \frac{1}{T_0} \int_{T_0} x(t)\,dt$

$\cos n\pi = (-1)^n$

$\int x \cos ax \, dx =$

$\frac{1}{a^2}\left[\cos ax + ax \sin ax\right]$

$T_0 = \frac{2\pi}{\omega_0}$

$a_n = \frac{2}{T_0} \int_{T_0} x(t) \cos n\omega_0 t \, dt$

$b_n = \frac{2}{T_0} \int_{T_0} x(t) \sin n\omega_0 t \, dt$

Amplitude & Phase Spectrum :-

$x(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \phi)$

where, $C_0 = a_0$

Amplitude $\rightarrow C_n = \sqrt{a_n^2 + b_n^2}$

Phase $\longrightarrow \phi = \tan^{-1}\left(\frac{b_n}{a_n}\right)$

$\int e^{ax} \cos bx \, dx = \frac{e^{ax}}{a^2+b^2}\left[a \cos bx + b \sin bx\right]$

$\int e^{ax} \sin bx \, dx = \frac{e^{ax}}{a^2+b^2}\left[a \sin bx - b \cos bx\right]$

Fourier Transform :- $X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} \, dt$

Magnitude $|X(\omega)| = \sqrt{R^2 + I^2}$

Phase $\theta = \tan^{-1}(I/R)$

IFT :- $Y(W) = X(W) \cdot H(W)$

$f^{-1}\left[\frac{1}{a+j\omega}\right] = e^{at}$

Q. $x(t) = e^{-|t|} \sin 2t$

$$x(t) = \begin{cases} e^{-t} \sin 2t, & x \geq 0 \\ e^{t} \sin 2t, & x \leq 0 \end{cases}$$

Signal is aperiodic

$$E = \int_{-\infty}^{\infty} |x(n)|^2$$

$$E = \int_{-\infty}^{0} (e^{t} \sin 2t)^2 dt + \int_{0}^{\infty} (e^{-t} \sin^2 t)^2 dt$$

$$= \int_{-\infty}^{0} e^{2t} \sin^2 2t \, dt + \int_{0}^{\infty} e^{-2t} \sin^2 2t \, dt$$

$$= \int_{-\infty}^{0} e^{2t} \left( \frac{1 - \cos 4t}{2} \right) dt + \int_{0}^{\infty} e^{-2t} \left( \frac{1 - \cos 4t}{2} \right) dt$$

$$= \int_{-\infty}^{0} \frac{e^{2t}}{2} dt - \int_{-\infty}^{0} \frac{e^{2t} \cos 4t}{2} dt + \int_{0}^{\infty} \frac{e^{-2t}}{2} dt - \int_{0}^{\infty} \frac{e^{-2t} \cos 4t}{2} dt$$

Q. $x(n) = 10 e^{-j 0.125 \pi n} u(n)$

$e^{j(-0.125 \pi n)} \rightarrow$ omplex no.

$$E = \sum_{n=-\infty}^{\infty} |10(1)|^2$$

$$E = \sum_{n=-\infty}^{\infty} 100 = \infty$$

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=\infty}^{\infty} (10)^2$$

$$P = \lim_{N \to \infty} \frac{100(2N+1)}{(2N+1)} = \underline{\underline{100}}$$

## Periodic & Aperiodic

Q. $x(t) = \underline{\underline{\sin 1.2\pi t}} - 2\cos 3\pi t$

$x_1(t) = \sin 1.2\pi t$

$x_1(t+T) = \sin 1.2\pi(t+T)$

$= (\sin 1.2\pi t \cos 1.2\pi T) + (\cos 1.2\pi t \sin 1.2\pi T)$

$x_1(t) = x_1(t+T)$ if

$\cos 1.2\pi T = 1$ &

$\sin 1.2\pi T = 0$

This happens at $1.2\pi T = 2\pi$

$T = \frac{2}{1.2} = \underline{\underline{\frac{5}{3}}}$

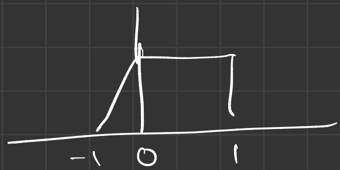
$\omega = 1.2\pi$

$\omega = \frac{2\pi}{T_0}$

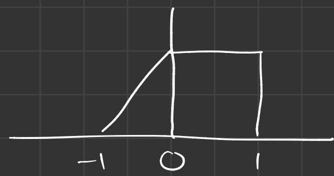
$T = \frac{2\pi}{\omega}$

# Even & Odd part

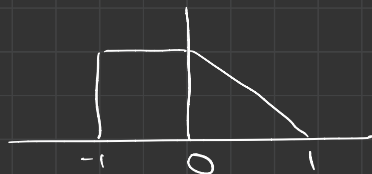
Q.



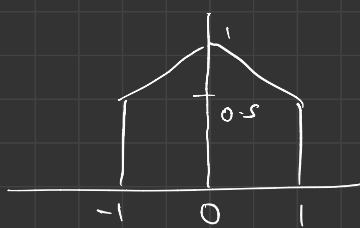

$x(t) =$




$x(-t) =$




$$x_e(t) = \frac{x(t) + x(-t)}{2}$$




$t \quad x(t)$

# Linearity

Q. $y(t) = e^{x(t)}$

i) $x_1(t) \rightarrow \quad y_1(t) = e^{x_1(t)}$

ii) $x_2(t) \rightarrow \quad y_2(t) = e^{x_2(t)}$

iii) $y_1(t) + y_2(t) = e^{[x_1(t) + x_2(t)]}$

iv) if $x_1(t) + x_2(t) \rightarrow y(t) = e^{[x_1(t) + x_2(t)]}$

$= e^{x_1(t)} \cdot e^{x_2(t)}$

$y(t) \neq y_1(t) + y_2(t)$

Q. $y(t) = x^2(t)$

i) $x(t) = 0 \rightarrow y(t) = 0$

ii) $x_1(t) \longrightarrow y_1(t) = x_1^2(t)$

iii) $x_2(t) \rightarrow y_2(t) = x_2^2(t)$

iv) $y_1(t) + y_2(t) = x_1^2(t) + x_2^2(t)$

v) $y(t) \rightarrow (x_1 + x_2)^2 t)$

As $y(t) \neq y_1(t) + y_2(t)$

$\therefore$ Non linear

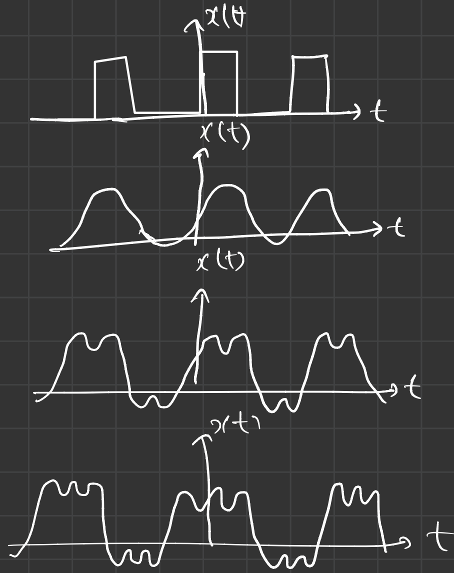
# Stability

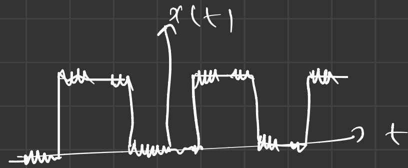
Q. $h(t) = A e^{j\omega t}$

$\int_{-\infty}^{\infty} |h(t)|$

# Gibbs Phenomenon

$x(t)$ is reconstructed with only N no. of terms of the $\infty$ series, the reconstructed signal exhibits oscillations and the oscillations are compressed towards points of discontinuities with increasing value of N.
At the point of discontinuity, the fourier series converges to avg. value of the signal on either side of the discontinuity.
This phenomenon was named after famous mathematician Josiah Gibbs, as Gibbs Phenomenon & oscillations are called Gibbs Oscillations.



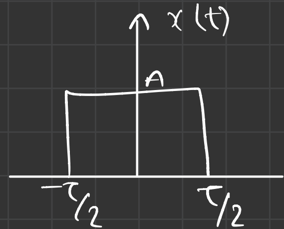



## Fourier Series

i) $a_0 = \frac{1}{T_0} \int_{T_0} x(t)\, dt$ $\qquad a_n = \frac{2}{T_0} \int_{T_0} x(t) \cos n\omega_0 t\, dt$

$a_0 = \frac{1}{T_0} \int_{T_0} x(t)\, dt$

## Fourier of Rectangular Pulse




$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t} dt$$

$$= \int_{-\tau/2}^{\tau/2} A e^{-j\omega t} dt$$

$$= A\left( \frac{e^{-j\omega t}}{-j\omega} \right)_{-\tau/2}^{\tau/2}$$

$$= \frac{A}{-j\omega}\left( e^{-j\omega \tau/2} - e^{+j\omega \tau/2} \right)$$

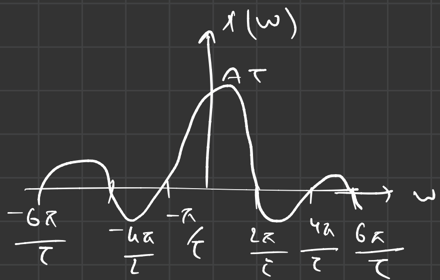
$$= \frac{2A}{\omega}\left(\frac{e^{j\omega\tau/2} - e^{-j\omega\tau/2}}{2j}\right)$$

$$= \frac{2A}{\omega}\left[\sin\frac{\omega\tau}{2}\right]$$

$$X(\omega) = \frac{2A}{\omega}\left[\frac{\sin \omega\tau/2}{\omega\tau/2}\right] \times \frac{\omega\tau}{2}$$

$$X(\omega) = A\tau \, Sa\left(\frac{\omega\tau}{2}\right)$$

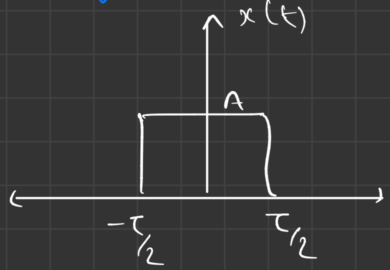
$\therefore$ FT of rectangular pulse $\rightarrow$ $X(\omega) = A\tau \, \text{rect}\left(\frac{t}{\tau}\right)$

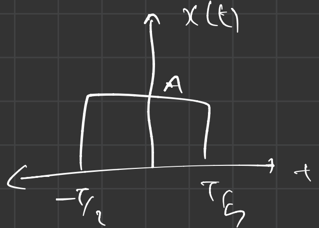



$$e^{at} = \frac{1}{s-a}$$

$$e^{-at} = \frac{1}{s+a}$$

## FT of rectangular pulse




$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\tau/2}^{\tau/2} A\, e^{-j\omega t}\, dt$$

$$= A \left[ \frac{e^{-j\omega t}}{-j\omega} \right]_{-\tau/2}^{\tau/2}$$

$$= \frac{A}{-j\omega} \left[ e^{-j\omega \tau/2} - e^{j\omega \tau/2} \right]$$

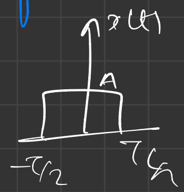
$$= \frac{2A}{\omega} \left[ \frac{e^{j\omega \tau/2} - e^{-j\omega \tau/2}}{2j} \right]$$

$$= \frac{2A}{\omega} \frac{\sin(\omega \tau/2)}{\omega \tau/2} \times \frac{\omega \tau}{2}$$

$= A\tau \; Sa\left(\omega\tau/2\right)$

$\therefore$ FT of rect pulse :- $x(t) = A \; rect\left(t/T\right)$

FT of rect pulse

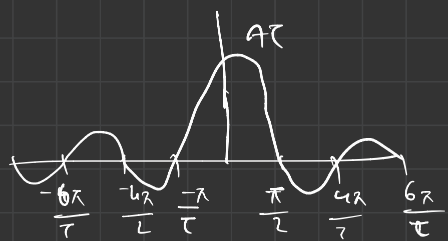



$$x(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

= . . . . .

$\sin \frac{\omega t}{2}$ $\quad$ $Sa\; \omega t/2$

FT :- $x(\omega) = A \; rect\left(t/T\right)$



$$x(t) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt \rightarrow FT$$

sin

$$\int ax \cos ax = \frac{1}{a^2}(\cos ax + ax \sin ax)$$

$$\int ax \sin ax = \frac{1}{a^2}(\sin ax - ax \cos ax)$$

$$\sin A \cos B = \frac{1}{2}(\sin(A+B) + \sin(A-B))$$

$x_1(t)$ + $x_2(t)$

$\sin n\pi = 0$ $\cos n\pi = (-1)^n$

$\sin 2\pi n = 0$ $\cos 2\pi n = 1$

$$\int e^{ax} \cos bx = \frac{e^{ax}}{a^2+b^2}(a \cos bx + b \sin bx)$$

$e^{-\infty} = 0$ $e^{-j2\pi n} = 1$

$e^{\infty} = \infty$ $E = \int_{-\infty} x^2\, dt$

$\frac{1}{2}$

$y(n-2) =$

$z^{-2} X(z) + z^{-1}(-1) + z^{0} (x(-2))$

$(1-z)^{+1}$

$F(s) = \int_{-\infty}^{\infty} f(t) e^{-st} \, dt \qquad z \to 1$

IVT $\to \lim_{z \to \infty} x$

FVT $\to \lim_{z \to 1} (1 - z^{-1}) X$

FVT $\to$

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n w_0 t + \sum_{n=1}^{\infty} b_n \sin w_0 t$$

$$a_0 = \frac{1}{T_0} \int_{T_0} x(t)\, dt$$

$$a_n = \frac{2}{T_0} \int_{T_0} x(t) \cos n w_0 t\, dt$$

$$b_n = \frac{2}{T_0} \int_{T_0} x(t) \sin n w_0 t\, dt$$

$$w_0 = \frac{2\pi}{T}$$

$$\cos n\pi = (-1)^n \qquad \sin n\pi = 0$$

$$\int x \cos ax = \frac{1}{a^2} [\cos ax + ax \sin ax]$$

$$\int x \sin ax = \frac{1}{a^2} [\sin ax - ax \cos ax]$$

$$x(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n w_0 t + \phi)$$

$$C_0 = a_0$$

$$C_n = \sqrt{a_n^2 + b_n^2}$$

$$\phi = \tan^{-1}\left(\frac{b_n}{a_n}\right)$$

$$\int e^{ax} \cos bx = \frac{e^{ax}}{a^2 + b^2} [a \cos bx + b \sin bx]$$

$$\int e^{ax} \sin bx = \frac{e^{ax}}{a^2+b^2}\left[a \sin bx - b \cos bx\right]$$

<u>FT</u>

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$|X(\omega)| = \sqrt{R^2 + I^2}$$

$$\phi = \tan^{-1}\left(\frac{I}{R}\right)$$

| $\omega$ | $\lvert X \rvert$ | $\phi$ |
|---|---|---|
| $-\infty$ | | |
| $-a$ | | |
| $0$ | | |
| $a$ | | |
| $\infty$ | | |

$$\frac{e^{j\omega} - e^{-j\omega}}{2j} = \sin \omega$$

$$\frac{e^{j\omega} + e^{-j\omega}}{2j} = \cos \omega$$

$$F\left[1 \cdot e^{j\omega_0 t}\right] = 2\pi\delta(\omega-\omega_0)$$

$$F\left[u(t)\, e^{j\omega_0 t}\right] = \pi\delta(\omega-\omega_0) + \frac{1}{j(\omega-\omega_0)}$$

$$F\left[1 \cdot e^{j\omega_0 t}\right] = 2\pi\delta(\omega-\omega_0)$$

$$F\left[1 \cdot e^{j\omega_0 t}\right] = 2\pi\delta(\omega-\omega_0)$$

$$F\left(u(t)\, e^{j\omega t}\right) = \pi\delta(\omega-\omega_0) + \frac{1}{j(\omega-\omega_0)}$$

IFT :-

$$F^{-1}\left[\frac{1}{a+j\omega}\right] = e^{-at}u(t)$$

$$F^{-1}\left[\frac{1}{a+j\omega}\right] = e^{-at}u(t)$$

$$F^{-1}\left[\frac{1}{a+j\omega}\right] = e^{-at}u(t)$$

$$F^{-1}\left[\frac{1}{a+j\omega}\right] = e^{-at}u(t)$$

$$t^n e^{-at}u(t) \xleftrightarrow{F} \frac{1}{(a+j\omega)^{n+1}}$$

$$F^{-1}\left[\frac{1}{a+j\omega}\right] = e^{-at}u(t)$$

$$t^n e^{-at} u(t) = \frac{1}{(a+j\omega)^{n+1}}$$

$$t^n e^{-at} u(t) = \frac{1}{(a+j\omega)^{n+1}}$$

LT :-

$$F(s) = \int_{-\infty}^{\infty} f(t) e^{-st} dt$$

$$F(s) = \int_{-\infty}^{\infty} f(t) e^{-st} dt$$

LT :-

$$F(s) = \int_{-\infty}^{\infty} f(t) e^{-st} dt$$

$$F(s) = \int_{-\infty}^{\infty} f(t) e^{-st} dt$$

$$F(s) = \int_{-\infty}^{\infty} f(t) e^{-st} dt$$

$$L[\cos wt] = \frac{s}{s^2 + w^2}$$

$$L[\sin wt] = \frac{w}{s^2 + w^2}$$

Energy :- $E = \lim_{T \to \infty} \int_{-T}^{T} (x(t))^2 \, dt$

$$E = \lim_{T \to \infty} \int_{-T}^{T} x(t)^2 \, dt$$

$$E = \lim_{T \to \infty} \int_{-T}^{T} x(t)^2 \, dt$$

$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x(t)^2 \, dt$$