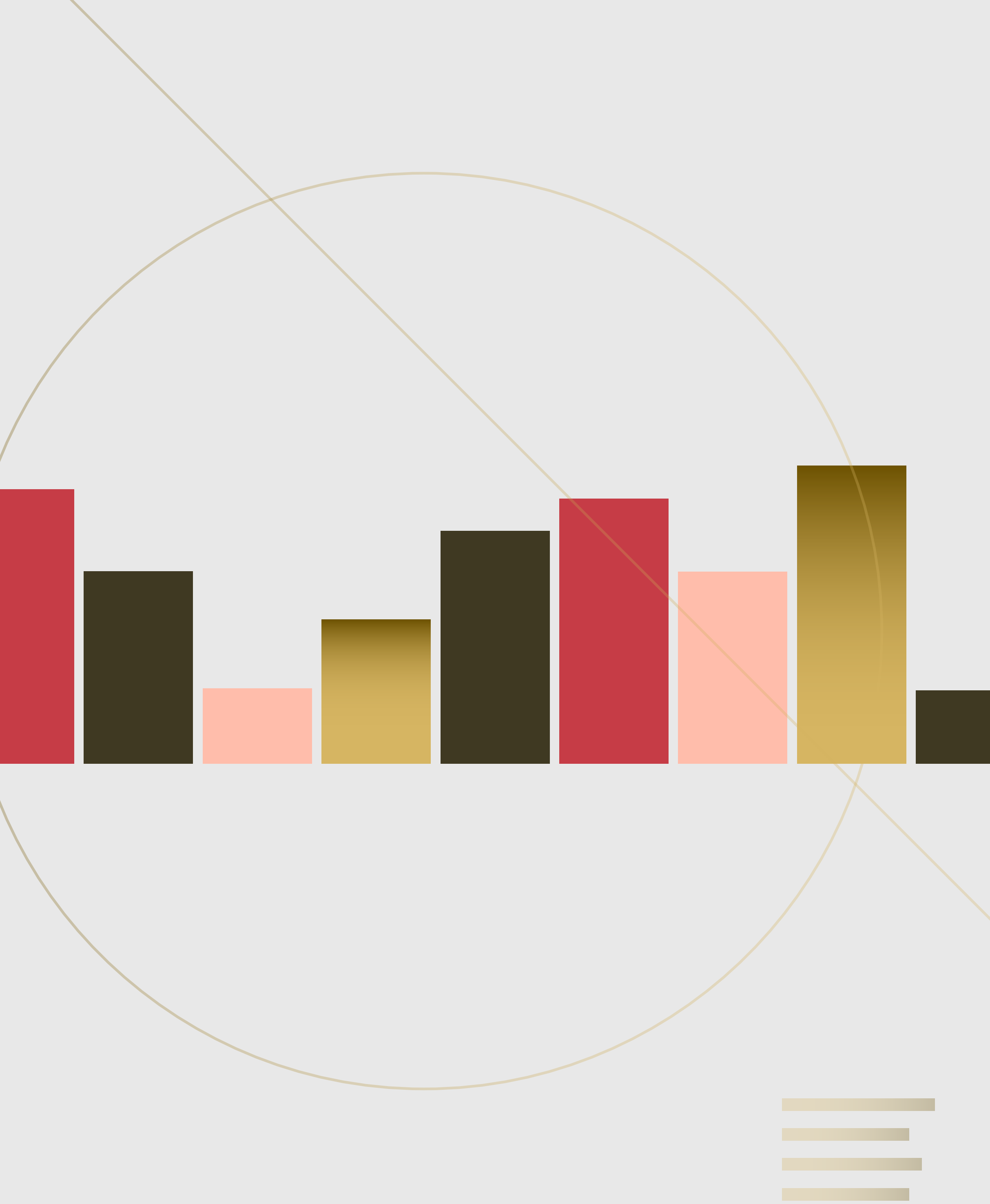


FUNDAMENTAL CONCEPTS OF GRAPHS

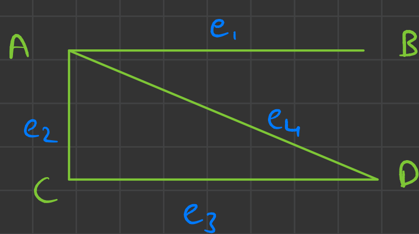
Graph consists of 2 things :-

i) A set $V = V(G)$ whose elements are called vertices, points or nodes of G

ii) A set $E = E(G)$ of unordered pairs of distinct vertices called edges of G.

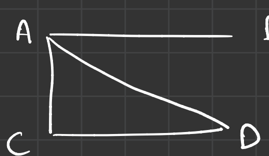
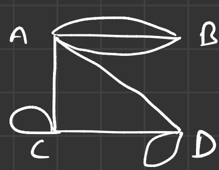
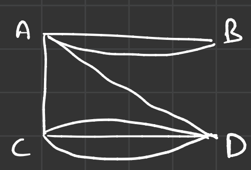
Denoted by $G(V, E)$

Example

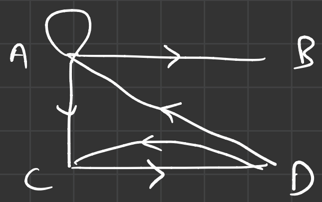


$e_1 = \{A, B\}$, $e_2 = \{A, C\}$,

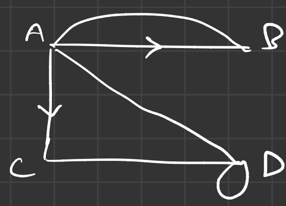
$e_3 = \{C, D\}$, $e_4 = \{A, D\}$

Simple Graphs	Pseudo Graphs	Multi Graphs
		

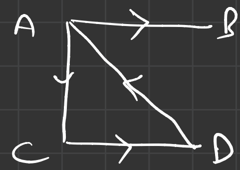
Directed Multigraphs



Mixed Graphs



Simple directed



Terminologies

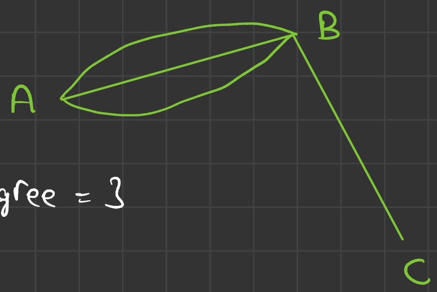
Type	Edges	Multiple Edges	Loops
Simple Graphs	Undirected	No	No
Pseudo Graphs	Undirected	Yes	Yes
Multi Graphs	Undirected	Yes	No
Directed Multigraphs	Directed	Yes	Yes
Simple directed Graphs	Directed	No	No
Mixed Graphs	Directed & Undirected	Yes	Yes

Adjacent / incidence = Two vertex connected with edge

Degree = No. of edges connecting vertex

Hand shaking Lemma :-

The sum of degrees of vertices is equal to twice the number of edges



degree = 4

degree = 3

degree = 1

Pendant Vertex = degree 1

Isolated Vertex = degree 0

Even / Odd Vertex = even / odd degree

Q.

Problems:

Q1. Draw graph models, to represent airline routes where everyday there are four flights from Boston to Newark, two flights from Newark to Boston, three flights from Newark to Miami, two flights from Miami to Newark, one flight from Newark to Detroit, two flights from Detroit to Newark, three flights from Newark to Washington, two flights from Washington to Newark, and one flight from Washington to Miami, with

a) an edge between vertices representing cities that have a flight between them (in either direction).

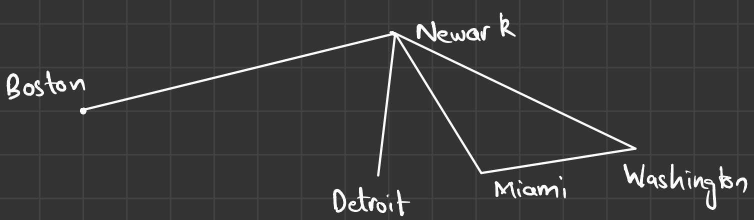
b) an edge between vertices representing cities for each flight that operates between them (in either direction).

c) an edge between vertices representing cities for each flight that operates between them (in either direction), plus a loop for a special sightseeing trip that takes off and lands in Miami.

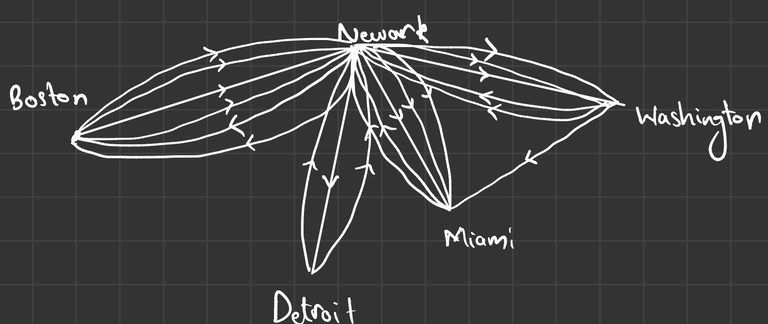
d) an edge from a vertex representing a city where a flight starts to the vertex representing the city where it ends.

e) an edge for each flight from a vertex representing a city where the flight begins to the vertex representing the city where the flight ends.

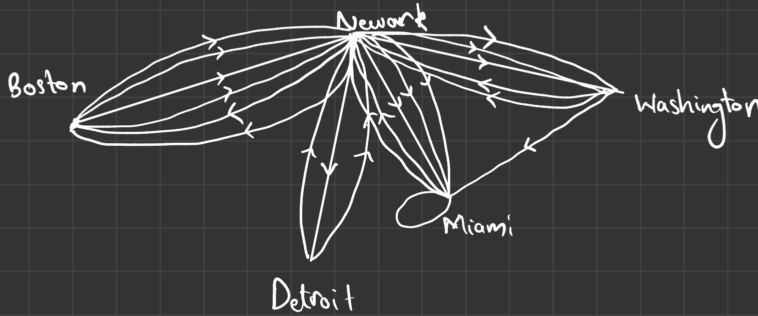
A. a)



b)



c)



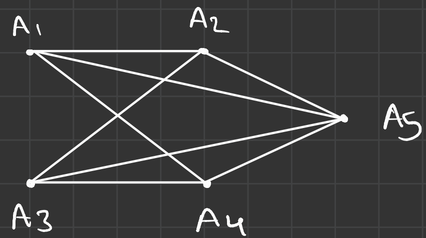
Q4. The **intersection graph** of a collection of sets $A1, A2, \ldots, An$ is the graph that has a vertex for each of these sets and has an edge connecting the vertices representing two sets if these sets have a nonempty intersection. Construct the intersection graph of these collections of sets.

a) $A1 = \{0, 2, 4, 6, 8\}$, $A2 = \{0, 1, 2, 3, 4\}$, $A3 = \{1, 3, 5, 7, 9\}$, $A4 = \{5, 6, 7, 8, 9\}$, $A5 = \{0, 1, 8, 9\}$

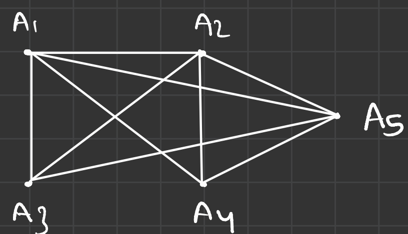
b) $A1 = \{\ldots, -4, -3, -2, -1, 0\}$, $A2 = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$, $A3 = \{\ldots, -6, -4, -2, 0, 2, 4, 6, \ldots\}$, $A4 = \{\ldots, -5, -3, -1, 1, 3, 5, \}$, $A5 = \{\ldots, -6, -3, 0, 3, 6, \ldots\}$

c) $A1 = \{x \mid x < 0\}$, $A2 = \{x \mid -1 < x < 0\}$, $A3 = \{x \mid 0 < x < 1\}$, $A4 = \{x \mid -1 < x < 1\}$, $A5 = \{x \mid x > -1\}$, $A6 = \mathbf{R}$

a)

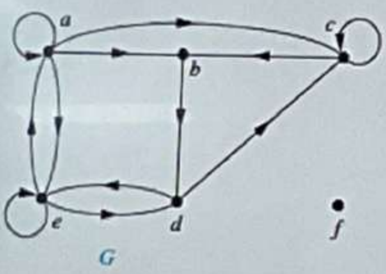


b)



Total indegree = Total out degree = Total no. of edges

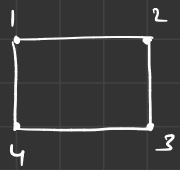
Q5. Find the in-degree and out-degree of each vertex in the graph G with directed edges shown following figure:



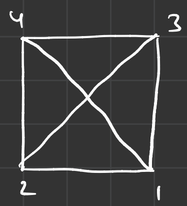
Vertices	In degree	Out degree
a	2	4
b	2	1
c	3	2
d	2	2
e	3	3
f	0	0
	12	12

Adjacency Matrix of a graph

Example :-

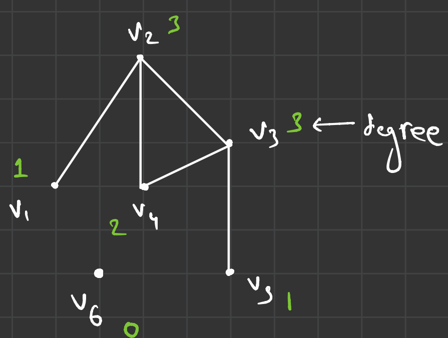


$$= \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$



$$= \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

Degree Sequence



Degree sequence :- (Arrange in descending order)

3, 3, 2, 1, 1, 0

Q. Is the degree sequence graphic?

can a graph be drawn?

Theorem 2 (Havel, Hakimi)

Ex:- 4, 3, 2, 2, 1, 0 sequence 1

$\downarrow$ ignore first digit & subtract 1 from others

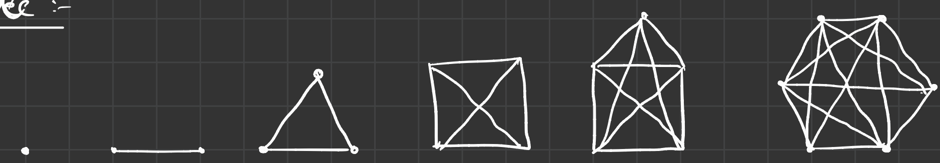
2, 1, 1, 0, 0 sequence 2

Sequence (1) is graphic if and only if sequence (2) is graphic

Complete Graph :-

A graph is complete if every vertex is connected to every other vertex.

Example :-



K_1 K_2 K_3 K_4 K_5

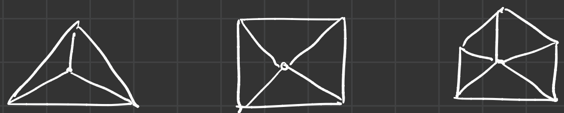
Cycle

Graph with consecutive edges & vertices connected

C_3 C_4 C_5 C_6

Wheel

A wheel is obtained by taking a cycle and adding a vertex, and then connecting all vertices to the added vertex.

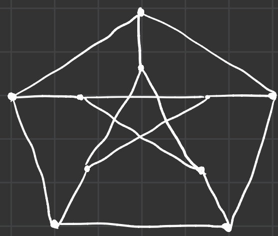


W_4 W_5 W_6

Path

P_1 P_2 P_3 P_4

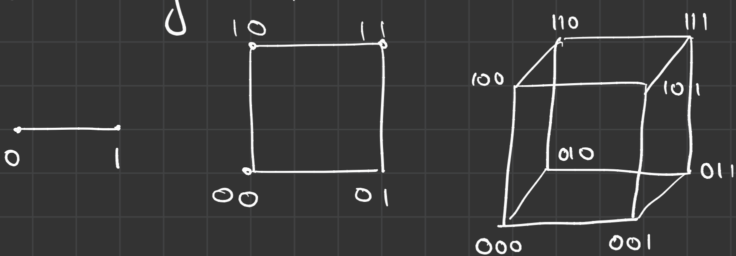
Peterson's Graph



n- Cubes

A graph whose vertices can be represented as 2^n.

Denoted by Q_n

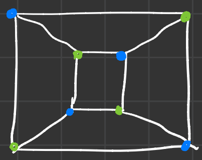


Regular Graph

A graph where each vertex has same degree as every other vertex.

Bipartite Graph

Example :-



Criteria :- is not odd cycle graphs

Complete Bipartite Graph

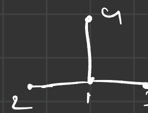
Every vertex is connected to every other vertex from other set.

Question 1

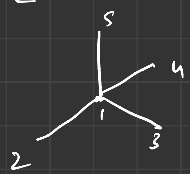
1) For C_n to be bipartite, $n \geq 4$ & even.
2) W_n can never be bipartite
3) Q_n can always be bipartite
4) For K_n to be bipartite, $n = 2$

K_{1n} Graph

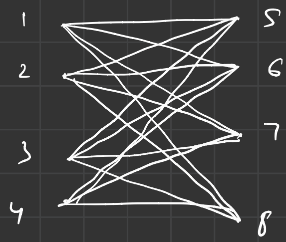
K_{13}



K_{14}



$K_{4,4}$ →



Q3

a) K_n → K_2
b) C_n → 2 regular
c) W_n → Only W_3 which is 3 regular
d) Q_n → n Regural

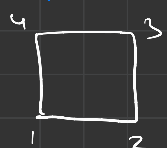
Sub Graph

subsets of vertices & edges

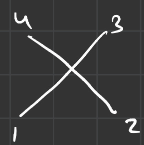
Complement Graph

Ex :-

1)

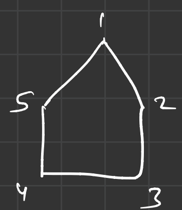


C_4



Complement of C_4

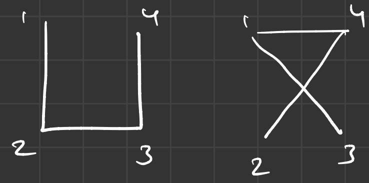
2)



C_5

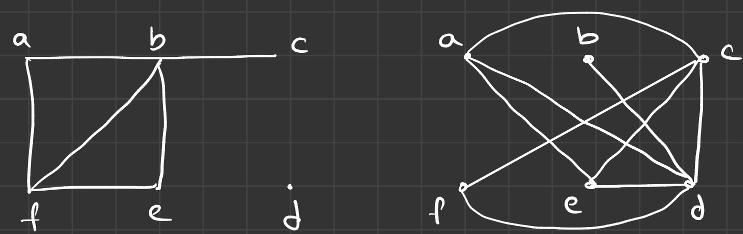
Self Complementary

3)



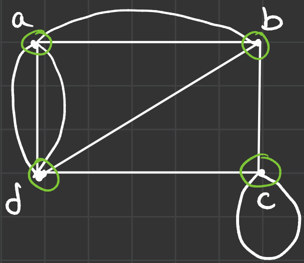
self complementary

4)



Q. Write the adjacency matrix of the following graphs.

i)



$$A_G = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 2 & 0 & 3 \\ 2 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 3 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

Incidence Matrix

Let $G = (V, E)$ be an undirected graph. Suppose $v_1, \dots, v_n$ are the vertices and $e_1, e_2, \dots, e_m$ are the edges of G. Then the incidence matrix with respect to this ordering of V and E is an $n \times m$ matrix denoted by $M = [m_{ij}]$, where

$$m_{ij} = \begin{cases} 1 & \text{when edge } e_j \text{ is incident on } v_i \\ 0 & \text{otherwise} \end{cases}$$

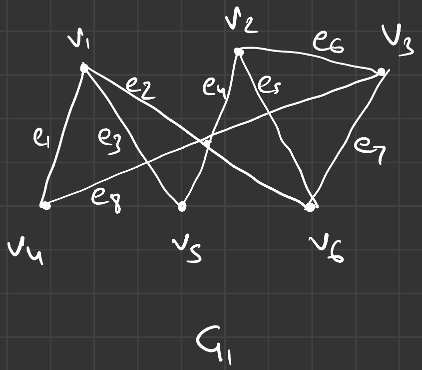
edges $\longrightarrow$

vertex $\downarrow$

$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ \vdots \end{array} \begin{array}{c} e_1 \; e_2 \; e_3 \; \cdots \\ \begin{bmatrix} & & & \\ & & & \\ & & & \end{bmatrix} \end{array}$$

sum of row = no. of edges = degree

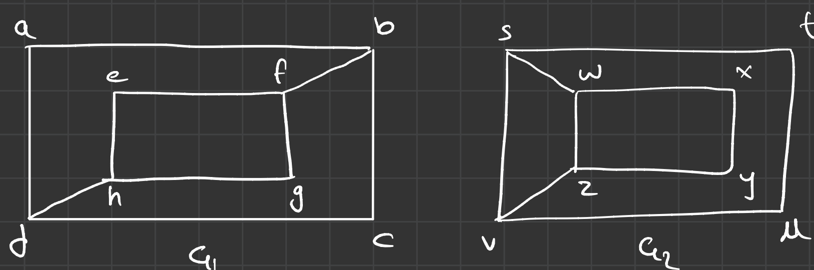
Q. Give incidence matrix.



$$A_{G_1} = \begin{array}{c|cccccccc} & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 \\ \hline v_1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ v_2 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ v_3 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ v_4 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ v_5 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ v_6 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \end{array}$$

Q. Are the following graphs isomorphic?

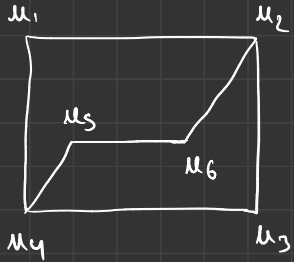
i)



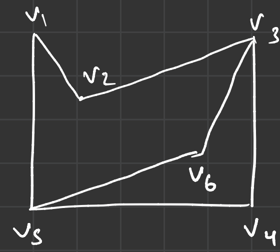
A :- Degree of a = 2 in G_1 and it must correspond to degree 2 vertex in G_2 (t, u, x, y) but vertex a is not adjacent to degree 2 vertex in G_1. Hence they are not isomorphic.

(Can contradict any one point from PPT)

ii)



G



H

Mapping :-

$$G = \{ u_1 , u_2 , u_3 , u_4 , u_5 , u_6 \}$$

$$H = \{ v_6 , v_3 , v_4 , v_5 , v_1 , v_2 \}$$

Ans :- Let $f : (G, V) \to (H, V)$

such that
$$f(u_1) = v_6$$
$$f(u_2) = v_3$$
$$f(u_3) = v_4$$
$$f(u_4) = v_5$$
$$f(u_5) = v_1$$
$$f(u_6) = v_2$$

Adjacency matrix :-

$$A_G = \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{array} \begin{array}{c} \begin{array}{cccccc} u_1 & u_2 & u_3 & u_4 & u_5 & u_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$A_H = \begin{array}{c} \\ v_6 \\ v_3 \\ v_4 \\ v_5 \\ v_1 \\ v_2 \end{array} \begin{array}{c} \begin{array}{cccccc} v_6 & v_3 & v_4 & v_5 & v_1 & v_2 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$\therefore$ They are isomorphic graphs

Cut Vertex

A vertex v in a graph G is a cut vertex if G- v has more no. of components than G.

Example :-

v is a cut vertex in a connected graph if G-v is disconnected graph.

Example 2 :-

G

Cut vertex :- b, c, e

Bridge Edge

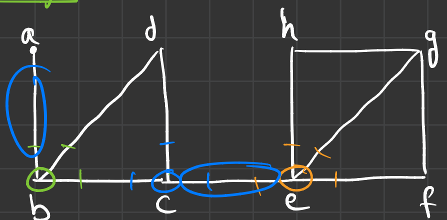
An edge e in a graph G is a bridge of G if G-e has more components than G.

Note :-

→ For vertex v in graph G, G-v is obtained by removing v and all edges incident with v.

→ For an edge e in G, G-e has the same vertex set as G and contains all edges of G except e.

Bridge Edge :-

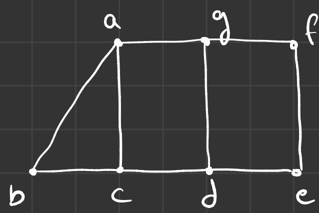


G

Bridge edge → ce & ab

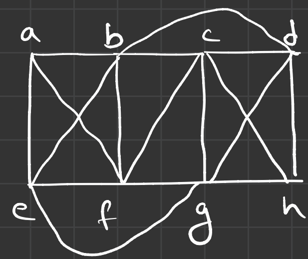
Q. Find cut vertex & bridge edge.

i)



G_{13}

ii)

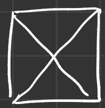


Adjacency matrix :-

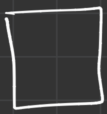
$$G_{13} = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \\ f \\ g \end{array} \begin{array}{c} \begin{array}{ccccccc} a & b & c & d & e & f & g \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

Q. Draw adjacency matrix of following graphs

i) K_4



iii) C_4



ii) $K_{2,3}$

iv) P_3

Theorem 1 :-

An edge e of a connected graph G is a bridge if and only if e is not on any cycle of G.

Theorem 2 :-

Every non-trivial connected graph has atleast 2 vertices which are not cut vertices.

(Trivial graph :- $\bullet_{v_1}$ [only one point])

Result 1 :-

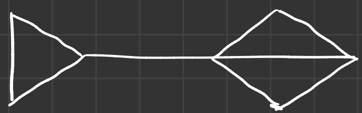
If $x = uv$ is a bridge for a connected graph $G \neq K_2$ then either 'u' or 'v' is a cut vertex.

Blocks

A connected non-trivial graph having no cut vertex is a block.
A block of a graph is a sub-graph that is maximal with respect to this property.

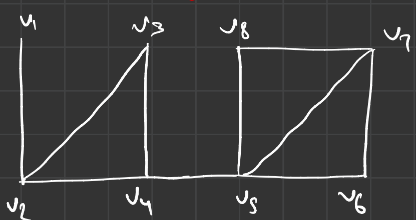
Example :-

Q. Write blocks of this graph

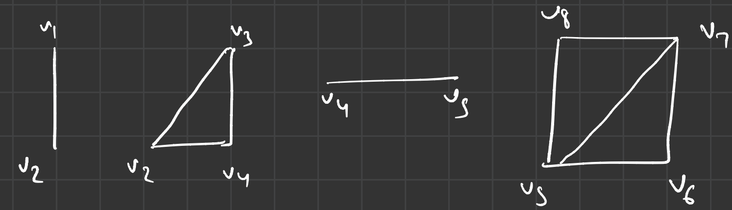


A:- Blocks :-

Q Write blocks of the graph



Ans :-



Distances

In a connected graph G, the distance from a vertex u to vertex v is the length of the shortest uv path in G and we denote distance in G as $d(u, v)$.

If there exists no path joining the vertices u and v, then distance b/w u & v is infinite
$d(u,v) = \infty$

Distance satisfies all metric properties.

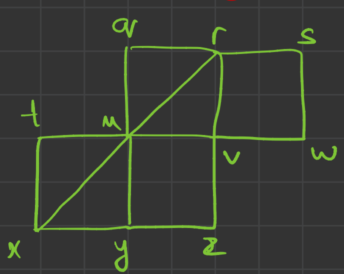
i) $d(u, v) \geq 0$
ii) $d(u,v) = 0$ ~~iff~~ if and only if $u = v$
iii) $d(u,v) = d(v,u)$
iv) $d(u,v) \leq d(u,w) + d(w,v)$

Eccentricity of a vertex

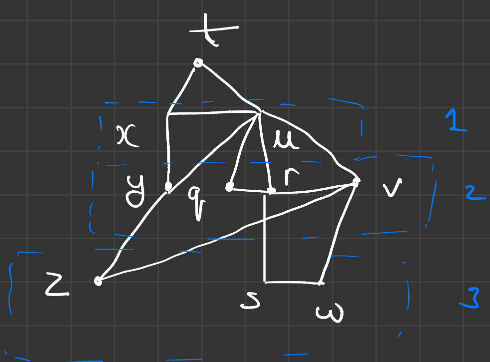
For a given vertex v, eccentricity is denoted by e(v) & is defined to be greatest distance from v to any other vertex.

$$e(v) = \max\{d(v,x) : x \in V(G)\}$$

Q. Find eccentricity e(t)

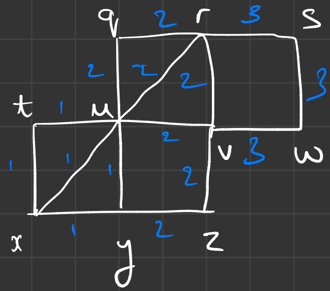


A:-



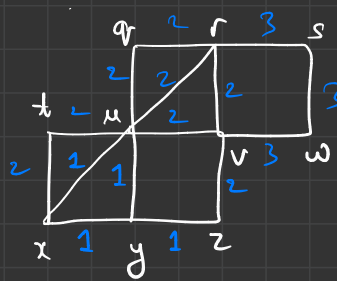
$$\therefore e(t) = \underline{\underline{3}}$$

$e(x) = ?$



$e(x) = 3$

$e(y) = ?$



Radius of Graph

In a connected graph G, the radius of G is denoted by

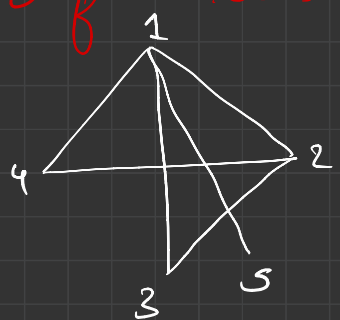
$$\text{rad}(G)$$

and it is the value of the smallest eccentricity.

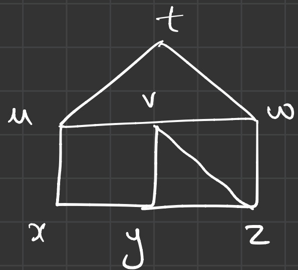
Diameter of Graph

It is denoted by diam(G) and it is value of the greatest eccentricity.

Q1. Find the eccentricity of each vertex of the graph, also find radius and diameter.



Q2. Find the distance b/w two vertices x and w in the following graph



A1.

vertex	eccentricity
1	1
2	2
3	2
4	2
5	2

$$\text{rad}(G) = \min\{e(v) : v \in V\}$$
$$= 1$$
$$\text{diam}(G) = \max\{e(v) : v \in V\}$$
$$= 2$$

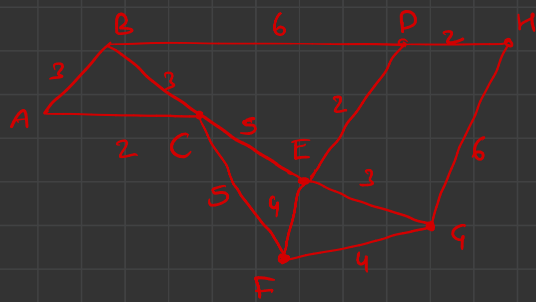
A2. path b/w x & w

i) x u v w
ii) u v t w
iii) x y v w
iv) x y z w
v) x y v u t w
vi) x y v z w
vii) x y z v u t w

path b/w x & w :- min distance

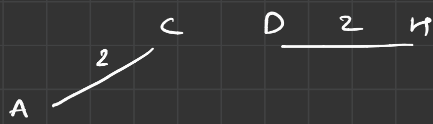
→ $\underline{\underline{3}}$

Q. Find MST using Krushkal's algorithm.

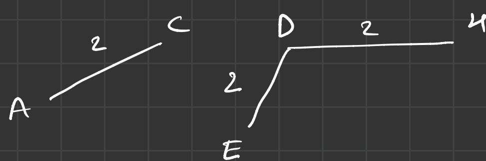


Step 1 :-

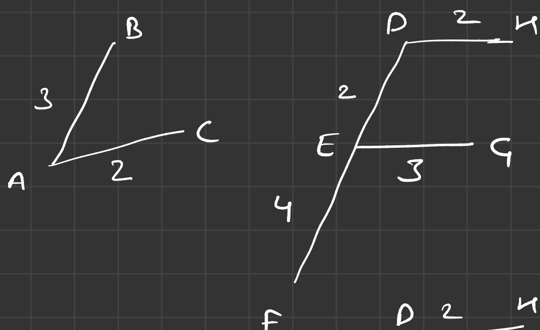
Step 2 :-



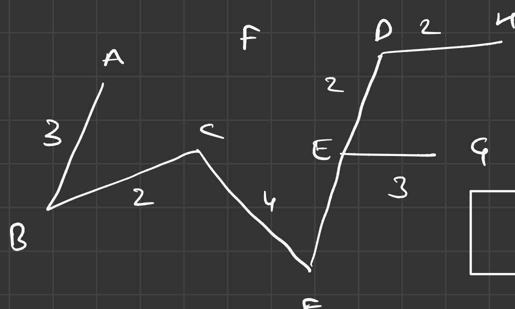
Step 3 :-



Step 4 :-

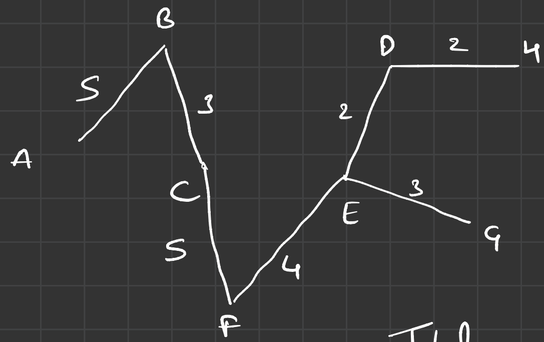


Step 5 :-



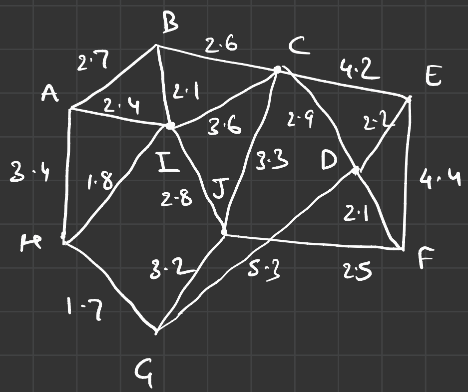
$\boxed{\text{Total weight} = 21}$

Ex :-



Total weight = 24

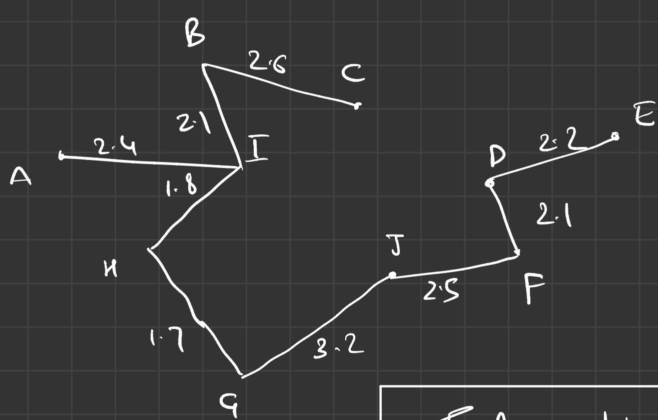
Q. Find MST using Krushkal's Algorithm



In exam, write steps :-

Ex :- 1. V = { B, C }
2. V = { B, C, D }
3. V = { B, C, D, E . . . }
.
.
.

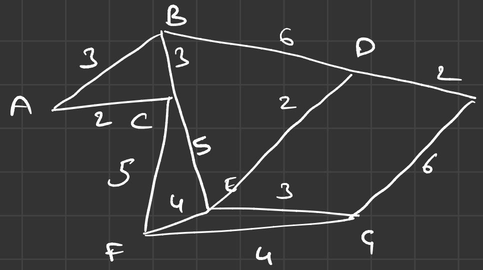
Answer :-



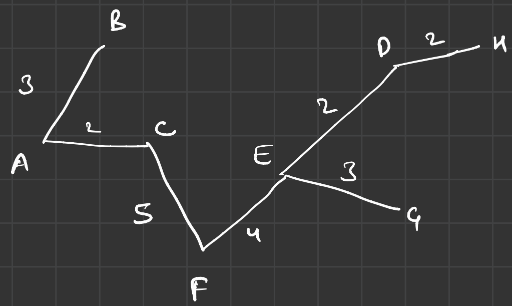
Total weight = 20.2

Prim's Algorithm

Q.



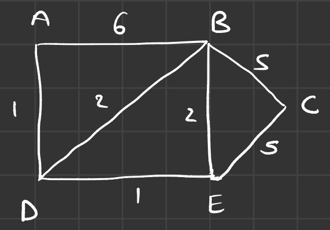
A:-



Total weight = 21

Dijikstra's Algorithm

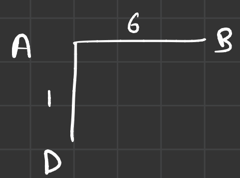
Ex:-



Vertex	A	B	C	D	E
shortest distance from A	0	∞	∞	∞	∞
Previous vertex					

Visited = $\{\}$ Not visited = $\{A, B, C, D, E\}$

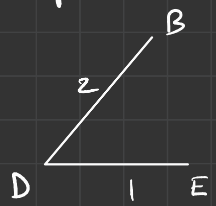
Step 2 :-



Vertex	A	B	C	D	E
shortest distance from A	0	6	∞	1	∞
previous vertex		A		A	

visited = $\{A\}$ not visited = $\{B, C, D, E\}$

Step 3 :-



Vertex	A	B	C	D	E
shortest distance from A	0	3	∞	1	2
previous vertex		D		A	D

visited = {A, D}   not visited = {B, C, E}

Vertex	A	B	C	D	E
shortest distance from A	0	3	7	1	2
Previous vertex		D	E	A	D

Planar Graph

Euler's Formula

Let G be a connected planar graph and consider a planar representation and let

V = no. of vertices

E = no. of edges

F = no. of regions or faces

then,

$$\boxed{V + F = E + 2}$$

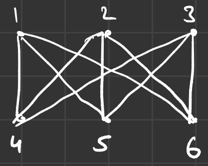
Corollary

If $V \geq 3$, and G is a planar graph then

$$\boxed{E \leq 3V - 6}$$

Q. Check if $K_{3,3}$ is planar or not

A.



$V = 6$

$E = 9$

using Euler's formula,

$V + F = E + 2$

$F = E + 2 - V$

$F = 5$

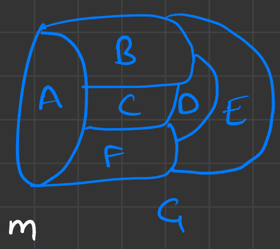
$E \geq \frac{4F}{2}$ → specific to prob

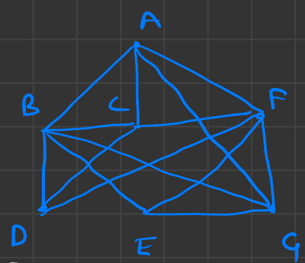
$9 \geq \frac{4(5)}{2}$

$9 \geq 10$

$\therefore$ False

$\therefore$ Non-planar





Dual graph

vertices = region

edge = border

The map m can be coloured properly if and only if a proper vertex colouring exist for the dual graph of m

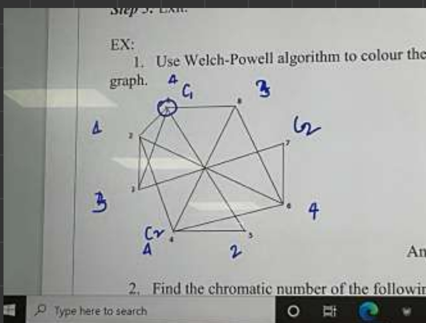
Vertex Colouring

min colours = $\chi(G)$ [Kai]

Welch - Powell Algorithm

For colouring a graph G :-

Q.



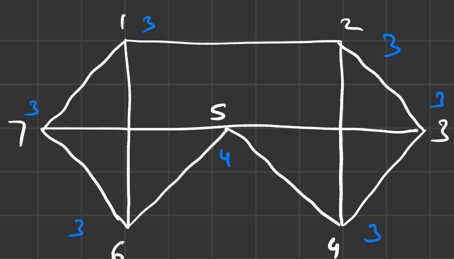
Deg seq → 4, 4, 4, 4, 3, 3, 2, 2

C_1 – 1 4 7

C_2 – 2 5 8

C_3 – 3 6

Q.



deg seq :- 4, 3, 3, 3, 3, 3, 3

C_1 :- 5

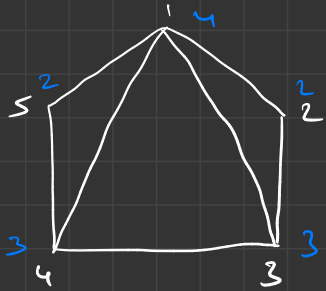
C_2 :- 6, 4

C_3 :- 3, 7

C_4 :- 1, 2

$\chi(G) = 4$

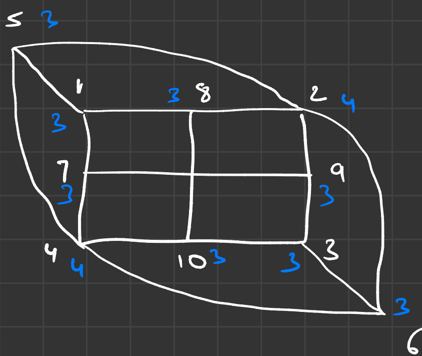
Q.



deg seq :-

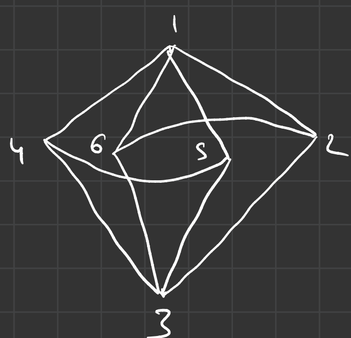
deg seq :-	4	3	3	2	2
c_1 :-	1				
c_2 :-			4	2	
c_3 :-		3			5

iii)



deg seq :-	4	4	3	3	3	3	3	3	3	3
c_1 :-	2	4	1	3						
c_2 :-					9	10	7	8	5	6

iv)



Chromatic number (χ)

$$\chi(K_n) = n$$

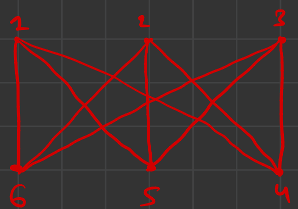
$$\chi(\overline{K_n}) = 1 \qquad \overline{K_n} = \text{isolated } n \text{ vertices}$$

$$\chi(C_n) = \begin{cases} 3, & n \text{ is odd} \\ 2, & n \text{ is even} \end{cases}$$

bipartite graph has chromatic number = 2

Edge chromatic number

Q. i)



C_1 :- 16, 34, 25
C_2 :- 15, 24, 36
C_3 :- 14, 26, 35

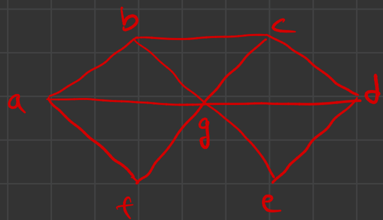
$$\therefore \chi'(G) = \underline{\underline{3}}$$

ii)

$C_1 :- 13, 24$
$C_2 :- 12, 34$
$C_3 :- 14, 23$

$\therefore \quad \chi'(G) = \underline{\underline{3}}$

iii)

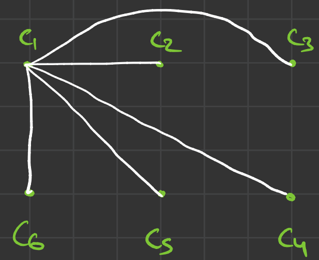


$C_1 :- af, bg$
$C_2 :- ag, ed$
$C_3 :- bc, gf$
$C_4 :- cd, ab, eg$
$C_5 :- gd$
$C_6 :- gc$

$\therefore \quad \chi'(G) = \underline{\underline{6}}$

2. The mathematics department has six committees, each meeting once a month. How many different meeting times must be used to ensure that no member is scheduled to attend two meetings at the same time if the committees are $C1 = \{$Arlinghaus, Brand, Zaslavsky$\}$, $C2 = \{$Brand, Lee, Rosen$\}$, $C3 = \{$Arlinghaus, Rosen, Zaslavsky$\}$, $C4 = \{$Lee, Rosen, Zaslavsky$\}$, $C5 = \{$Arlinghaus, Brand$\}$, and $C6 = \{$Brand, Rosen, Zaslavsky$\}$?

A. Let committees C_1 C_2 C_3 .... be verticees and let edge be the person common in the committee.



Planar Graph

Let $P(G, k)$ denote the no. of ways to colour the vertices of G with k colours, such that adjacent vertices have different colour.

$P(G, k)$

$P(G, x)$ denotes a polynomial function that returns $P(G, k)$ for $x = k$

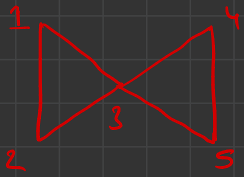
i) <u>Empty graph on n vertices</u>

$$P(G, k) = k^n$$
$$P(G, x) = x^n$$

ii) K_n

$$P(G, k) = k \times (k-1) \times (k-2) \times \ldots\ldots \times (k-(n-1))$$
$$P(G, x) = x \times (x-1) \times (x-2) \times \ldots\ldots \times (x-n+1)$$

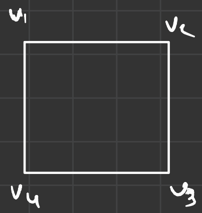
iii)



vertices	choices
1	k
2	$(k-1)$
3	$(k-2)$
4	$(k-1)$
5	$(k-2)$

$$P(G, x) = x(x-1)^2(x-2)^2$$

iv) $P(C_4, x)$



case I :- $v_2 = v_4$ (colour)

Case I :- v_2 & v_4 have same colour

vertex	choices
v_1	k
v_2	$k-1$
v_4	$k-1$
v_3	$k-1$

$\}$ $v_2 = v_4$

$$P(G, k) = k(k-1)^2$$

Case II :- v_2 & v_4 have different colours

vertices	choices
v_1	k
v_2	$(k-1)$
v_3	$(k-2)$
v_4	$(k-2)$

$$P(G, k) = k(k-1)(k-2)^2$$

$$P(G, k) = k(k-1)^2 + k(k-1)(k-2)^2$$

Q. Chromatic polynomial of C_5

Deletion

Let G be a graph & e be edge of G, the $G-e$ is called deletion & of graph is obtained by deleting the edge

Contraction

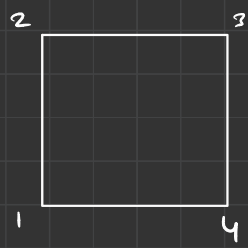
G/e (joining the vertices)

Deletion Contraction Lemma

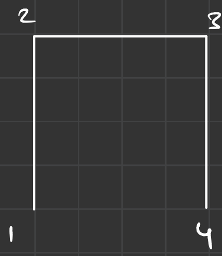
Let G be a simple graph & e be any edge in G, then

$$P(G,x) = P(G-e,x) - P(G/e,x)$$

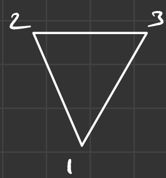
Q. Find chromatic polynomial of C_4 by deletion-contraction.



C_4



$C_4 - e$



C_4 / e

chromatic polynomial: $k(k-1)^3$ $k(k-1)(k-2)$

$\therefore$ chromatic polynomial of C_4 $= k(k-1)^3 - k(k-1)(k-2)$

$$P(C_4, x) = x(x-1)^3 - x(x-1)(x-2)$$

$$x(x-1)\left[(x-1)^2 - (x-2)\right]$$

Network flow

Max flow - min cut Theorem

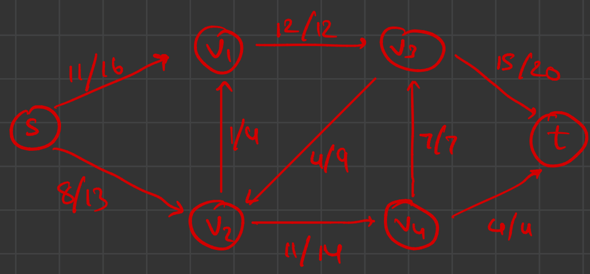
If F is a flow in a flow network G with source s and sink t, then the following are equivalent:

i) f is max flow
ii) residual network with respect to the flow f contains no augmenting path
iii) the flow value $|f|$ is equal to the capacity of the cut for some cut $(S, \bar{S})$.

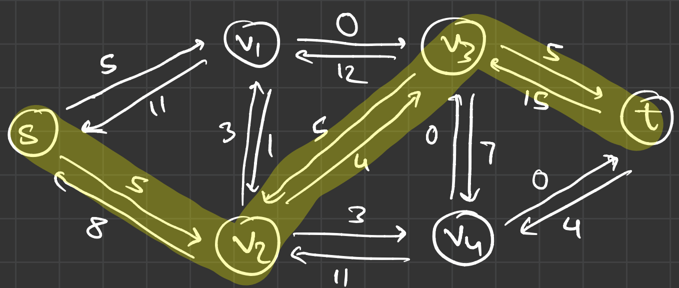
$$|f| = c(S, \bar{S})$$

The set S is a set of all vertices which are accessible from the source node.

Q.



A. Step 1:- Make residual network

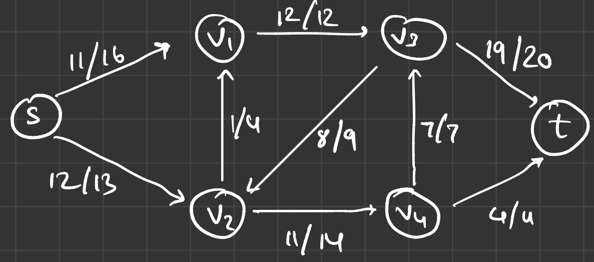


Step 2 :- Augmenting Path

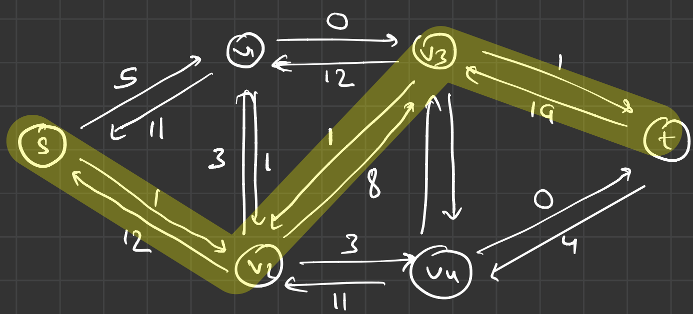
$$P = \{s, v_2, v_3, t\}$$

$$SP = \min\{5, 4, 5\} = 4$$

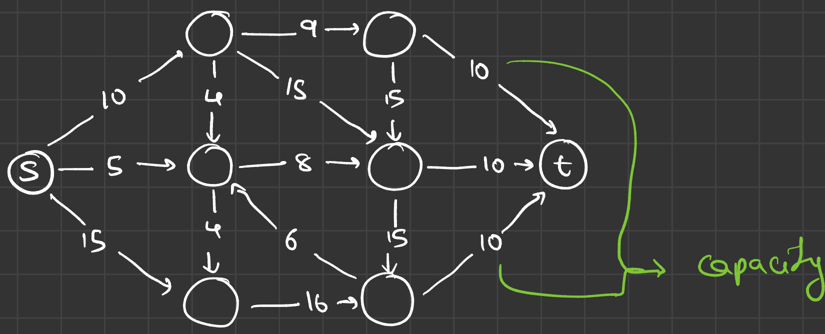
(Add 4 to flow of path)



Residual path :-

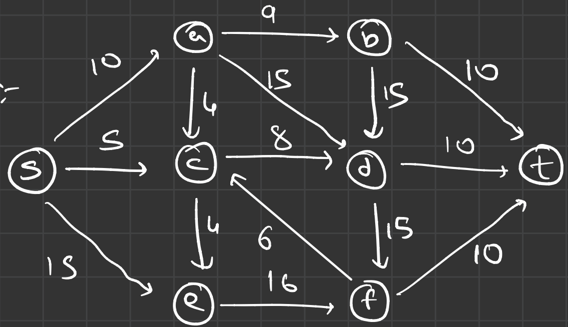


Q. Apply max flow min cut theorem to find min cut.



A :- Apply Ford's Algo :-

Residual Network :-



Augmenting path :-

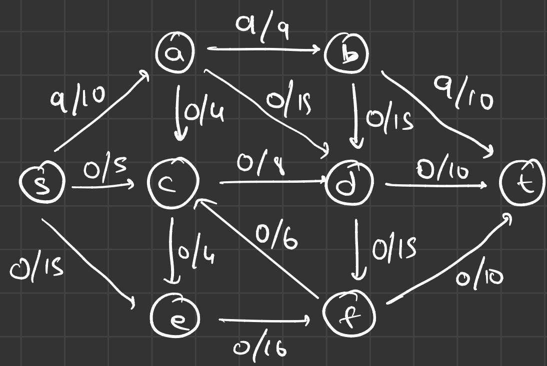
$$\delta(P) = \min\{c(s,a),\ c(a,b),\ c(b,t)\}$$

$$= \min\{10, 9, 10\}$$

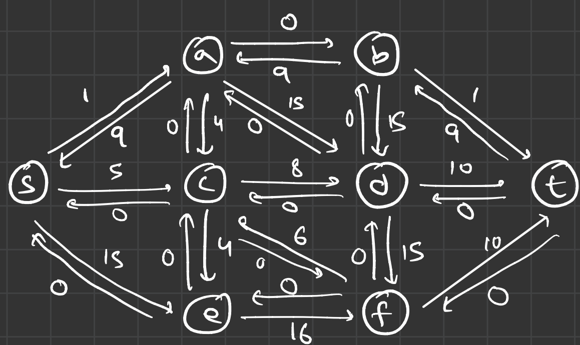
$$= 9$$

Now increase flow by 9

Residual Network :-



Residual flow :-



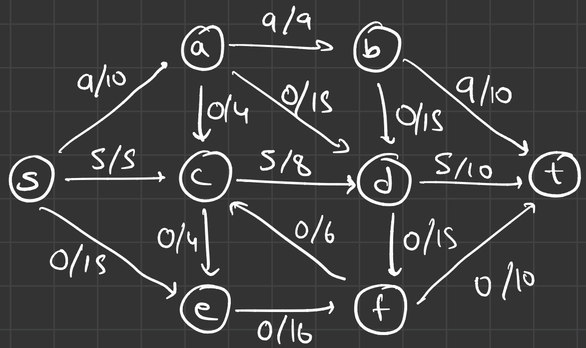
$$\delta(P) = \min\{c(s,c),\ c(c,d),\ c(d,t)\}$$

$$= \min\{5,\ 8,\ 10\}$$

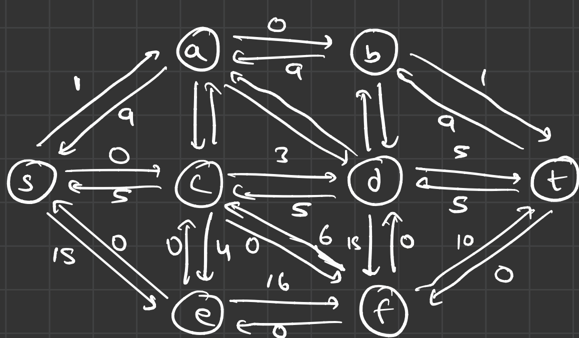
$$= 5$$

increase flow by 5

Residual network :-



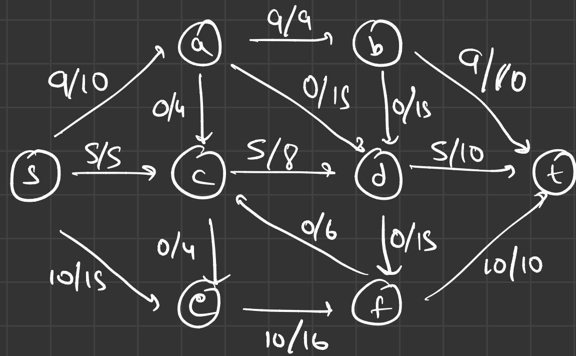
Residual flow :-

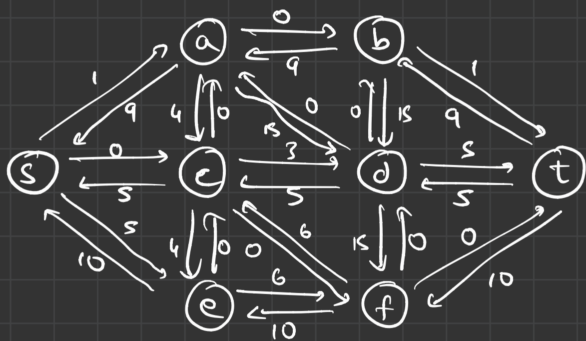


$$\delta(s) = \min\{c(s,e),\ c(e,f),\ c(f,t)\}$$

$$= \min\{15,\ 16,\ 10\}$$

$$= 10$$



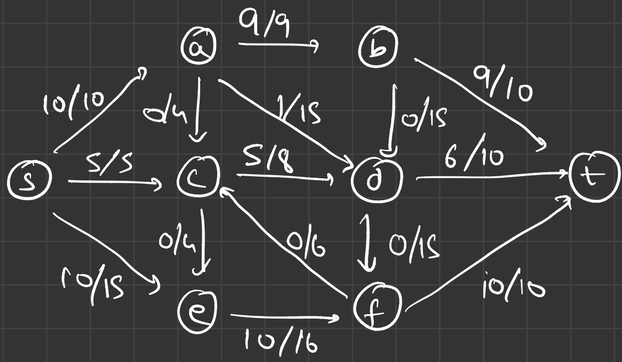


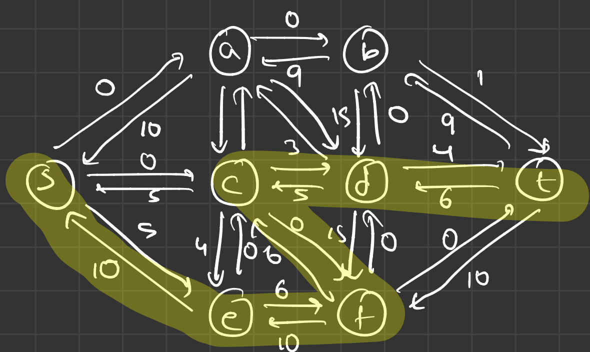
$$\delta(s) = \min\{c(s,a),\ c(a,d),\ c(d,t)\}$$

$$= \min\{1,\ 15,\ 5\}$$

$$= 1$$

increase flow by 1





$$\delta(G) = \min\{c(s,e),\ c(e,f),\ c(f,c),\ c(c,d),\ c(d,t)\}$$

$$= \min\{10,\ 10,\ 6,\ 3,\ 4\}$$

$$= 4$$

H002 Danish
70042000109

Assignment 1

Q1.

$C_1 = \{$ Arlinghaus, Brand, Zaslavsky $\}$

$C_2 = \{$ Brand, Lee, Rosen $\}$

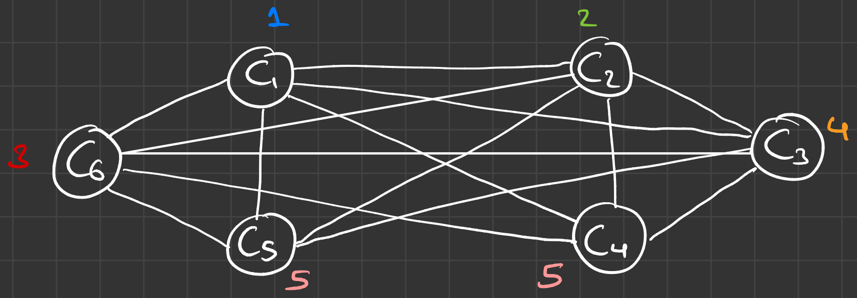
$C_3 = \{$ Arlinghaus, Rosen, Zaslavsky $\}$

$C_4 = \{$ Lee, Rosen, Zaslavsky $\}$

$C_5 = \{$ Arlinghaus, Brand $\}$

$C_6 = \{$ Brand, Rosen, Zaslavsky $\}$

Let $C_1, C_2 \ldots C_6$ be vertices of the graph.



By vertex coloring :-

$\eta = 5$

$\therefore$ 5 meeting times are needed.

Q2.

Vertices = animals

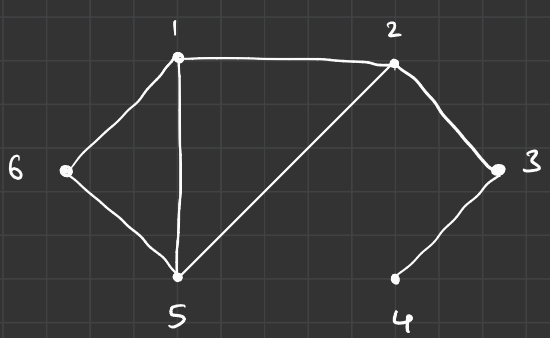
edge (u, v) = if animal u eats animal v

∴ Chromatic number of graph will provide the minimum number of habitats needed in the zoo.

Q3.

	1	2	3	4	5	6
1	—	85	175	200	50	100
2	85	—	125	175	100	160
3	175	125	—	100	200	250
4	200	175	100	—	210	220
5	50	100	200	210	—	100
6	100	160	250	220	100	—

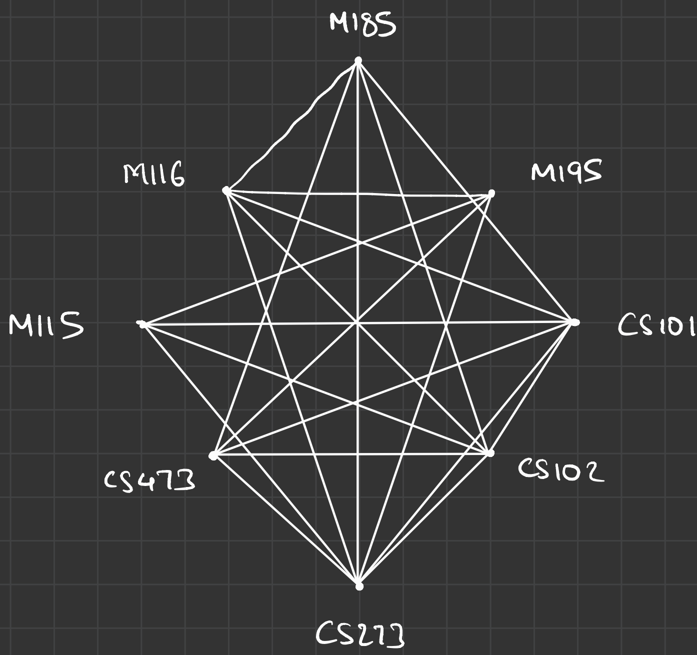
when distance < 150 miles, two station cannot use same channel



edge chromatic no. = 3

∴ atleast 3 channels are needed

Q4.

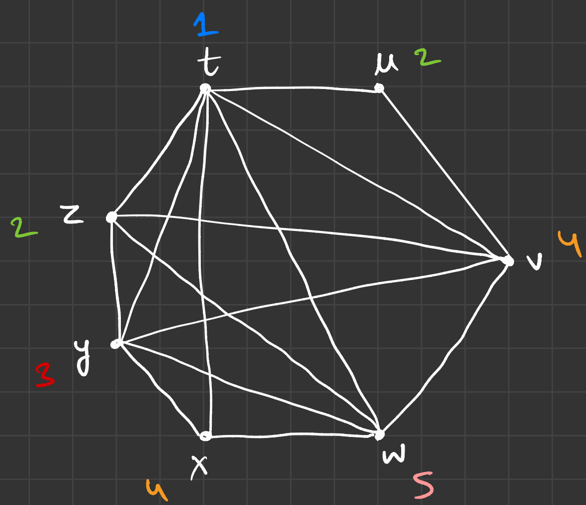


chromatic no. = 5

∴ fewest no. of different time slots = $\underline{\underline{5}}$

Q5. vertices = variables

edge = if variable stored in same program



Chromatic no. = 5

∴ different index register = 5

Q6. Since in k- tuple colouring adjacent edges do not share the same colour, in this question, if the vertices of the graph are assigned to frequencies and edge are the zones sharing same frequencies then the chromatic no. can signify the k frequencies to each mobile radio zone in a region.