Laboratory work 10
Extended Kalman filter for navigation and tracking

Performance -Thursday, October 18, 2018
Due to submit a performance report –Monday, October 22, 2018

The objective of this laboratory work is to develop Extended Kalman filter for tracking a moving object when measurements and motion model are in different coordinate systems. This will bring about a deeper understanding of main difficulties of practical Kalman filter implementation for nonlinear models.

This laboratory work is performed in the class by students as in teams of 3 on October 18, 2018 and the team will submit one document reporting about the performance till October 22, 2018. Within your group, you may discuss all issues openly, and discuss and debate until you reach a consensus.

1. ***Here is the recommended procedure:***
   Generate a true trajectory X_i of an object motion disturbed by normally distributed random acceleration

$$x_i = x_{i-1} + V_{i-1}^x T + \frac{a_{i-1}^x T^2}{2}$$
$$V_i^x = V_{i-1}^x + a_{i-1}^x T$$
$$y_i = y_{i-1} + V_{i-1}^y T + \frac{a_{i-1}^y T^2}{2}$$
$$V_i^y = V_{i-1}^y + a_{i-1}^y T$$

   Initial conditions to generate trajectory
   (a) Size of trajectory is $N = 500$ points.

   (b) $T = 1$ – interval between measurements.

   (c) Initial coordinates
   $x_0 = 1000;\ y_0 = 1000$

   (a) Initial components of velocity V
   $V_x = 10; V_y = 10;$

   (b) Variance of noise a_i, $\sigma_a^2 = 0.3^2$ for both a_i^x, a_i^y

2. Generate also true values of range D and azimuth β

$$D_i = \sqrt{x_i^2 + y_i^2}$$
$$\beta_i = arctg\left(\frac{x}{y}\right)$$

3. Generate measurements D^m and β^m of range D and azimuth β

$$D_i^m = D_i + \eta_i^D$$
$$\beta_i^m = \beta_i + \eta_i^\beta$$

   Variances of measurement noises η_i^D, η_i^β are given by

$$\sigma_D^2 = 50^2$$
$$\sigma_\beta^2 = 0.004^2$$

4. Initial conditions for Extended Kalman filter algorithm
   Initial filtered estimate of state vector $X_{0,0}$

$$X_0 = \begin{vmatrix} D_i^m(1)sin\beta_i^m(1) \\ 0 \\ D_i^m(1)cos\beta_i^m(1) \\ 0 \end{vmatrix}$$

   Initial filtration error covariance matrix $P_{0,0}$
   First use great initial filtration error covariance matrix

$$P_{0,0} = \begin{vmatrix} 10^{10} & 0 & 0 & 0 \\ 0 & 10^{10} & 0 & 0 \\ 0 & 0 & 10^{10} & 0 \\ 0 & 0 & 0 & 10^{10} \end{vmatrix}$$

5. Create the transition matrix Φ
   Consult charts, page 27

6. Calculate state noise covariance matrix Q

$$Q = GG^T\sigma_a^2$$

7. Create the measurement noise covariance matrix R

$$R = \begin{vmatrix} \sigma_D^2 & 0 \\ 0 & \sigma_\beta^2 \end{vmatrix}$$

8. At every filtration step in the algorithm you should linearize measurement equation by determining

$$\frac{dh(\hat{X}_{i+1,i})}{dX_{i+1}}$$

   Consult charts, page 32

9. Develop Kalman filter algorithm to estimate state vector X_i (extrapolation and filtration). Using extrapolated and filtered estimates at every extrapolation and filtration step you will need to calculate
   (a) range D
   (b) azimuth β

10. Run Kalman filter algorithm over $M = 500$ runs.
    Calculate true estimation errors of
    (a) Errors of extrapolation and filtration estimates of range D
    (b) Errors of extrapolation and filtration estimates of azimuth β

11. Compare estimation results with measurement errors of D and β.

12. Make general conclusions.

13. Prepare performance report and submit to Canvas:
    Performance report should include 2 documents:
    1) A report (PDF) with performance of all the items listed above

2) Code

The code should be commented. It should include:

-Title of the laboratory work;

-The names of a team, indication of Skoltech, and date;

-Main procedures also should be commented, for example

%13-month running mean

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…here comes the code