# Midterm

Due 17 Dec 2018 at 4:00 p.m.

Quantum Information Theory 2018*[1]

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Those interested in doing a theory thesis are personally encouraged to solve all of these problems analytically. Numerical examples are required. Working with others is cheating and will earn you a grade of zero.

**Remark 0.1.** We adhere to the notation conventions given in the attachment to the first homework sheet.

## 1 Matrix Product States

**Problem 1.** Write down the matrix product state for the n-party GHZ and n-party W-states. Let us define the two-point correlation as

$$C_{i,j}(\psi, A_i, A_j) = \langle\psi| A_i A_j |\psi\rangle - \langle\psi| A_i |\psi\rangle \langle\psi| A_j |\psi\rangle \tag{1}$$

Find $C(j \leq n) := C_{1,j}(\psi, X_1, X_j)$ for both GHZ- and W- where $X$ is the familiar Pauli matrix. Using Mathematica, make a publication quality plot for some fixed $n$ (e.g. label everything and create a caption explaining the plot).

## 2 Werner States

**Problem 2.** Define the Werner states acting on $\mathbb{C}_2 \otimes \mathbb{C}_2$ as

$$\rho_r = r|\phi^+\rangle\langle\phi^+| + \frac{1-r}{4}\mathbb{1} \tag{2}$$

where $\sqrt{2}\,|\phi^+\rangle = |00\rangle + |11\rangle$ is the standard Bell state and $r$ takes only values in the real interval $[0, 1]$. Find (i) the matrix trace of $\rho_r$ and (ii) the eigenvalues of $\rho_r$. The concurrence is

$$C(\rho) = \max\{\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4, 0\} \tag{3}$$

where $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4$ Using part (ii) or otherwise, find $C(\rho_r)$.

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## 3 Entanglement Time-Capacity

**Definition 3.1.** We will be working with the two-parameter Hamiltonian operator on $\mathcal{L}(\mathbb{C}_4)$

$$H(\beta, \gamma) = \beta X \otimes X - \gamma Y \otimes Y \tag{4}$$

where $X$, $Y$ are the standard Pauli matrices.

**Definition 3.2** (Entanglement velocity)**.** Let $|\psi\rangle$ act on the bipartite complex euclidean space $\mathcal{A} \otimes \mathcal{B}$ for $\mathcal{A}$ isomorphic to $\mathcal{B}$. The state-dependent-entanglement-velocity is

$$\Gamma(\tau) = \frac{d}{d\tau} E(\exp\{-\imath H \tau\} |\psi(0)\rangle) \tag{5}$$

where

$$E(|\psi\rangle) = S(\mathrm{Tr}_{\mathcal{B}} |\psi\rangle\langle\psi|) \tag{6}$$

with $S$ the standard von Neumann Entropy.

**Problem 3.** Now consider the Hamiltonian (4) time-evolving the initial state $|\psi(0)\rangle = |0\rangle \otimes |0\rangle$. Create a heat map $H(1, \gamma) \mapsto \Gamma(\tau)$ for $\gamma \in [0, 1]$, $\tau \in [0, 2\pi]$.

Compare this with a heat map found from acting on the initial state $\cos\theta |00\rangle + \sin\theta |11\rangle$ and hence fix $\gamma = 1/2$ and consider $\theta \in [0, 2\pi]$.

Compare and contrast these two maps, pointing out regions where entanglement is increasing/decreasing.

**Problem 4** (Entanglement limit of $H$)**.** We then consider the general form of single qubit states as

$$|\psi(\theta, \phi)\rangle = \cos\theta |0\rangle + e^{\imath\phi} \sin\theta |1\rangle \tag{7}$$

where $\theta$ and $\phi$ take only real values. Calculate the following limit numerically for $H(\beta, 1)$

$$\max_{|\psi(\theta_1,\phi_1)\rangle \otimes |\psi(\theta_2,\phi_2)\rangle} \Gamma(\tau)|_{\tau \to 0} \tag{8}$$

and show numerically that it is upper bounded by 1.92.