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Line and Curves Detection

Given an edge image, find line or curve segments present in the image.

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Issues with Curve Detection

- Grouping (e.g., the Canny hysteresis thresholding procedure)
- Model fitting

They can be performed sequentially or simultaneously.

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The Hough Transform

- The Hough Transform (HT) is a popular technique for detecting straight lines and curves on gray-scale images.
- It maps image data from image space to a parameter space, where curve detection becomes peak detection problem.

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The HT for Line detection

1. **Parameterize the line**

$$y = ax + b$$

In the parameter spaces determined by a and b, each parameter pair (a, b) represents a line. For example (0,4) represents a horizontal line. This line representation can not represent a vertical line.

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Alternatively, we can also parameterize a line in polar coordinates as

$$x\cos\theta + y\sin\theta - \rho = 0$$

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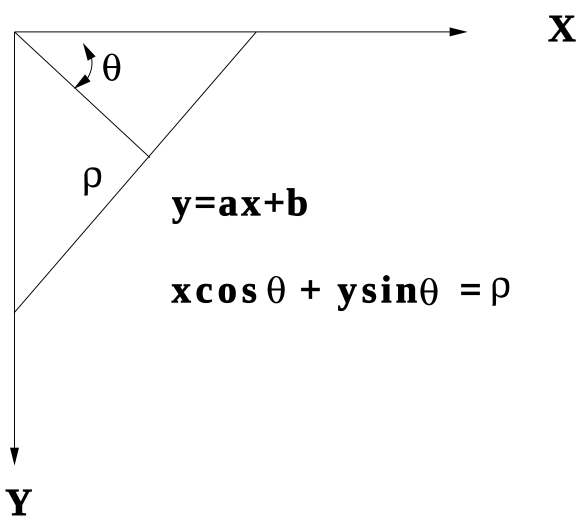

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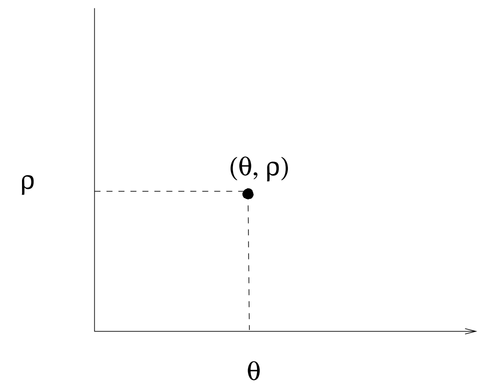

In parameter space determined by θ and ρ, each pair of (θ, ρ) represents a line. For example, a vertical line my be represented as $(0, 4)$.

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An image point (x_0, y_0) represents the intersection of all lines that satisfy $x_0 \cos\theta + y_0 \sin\theta - \rho = 0$, each of which can be represent by a pair of (θ, ρ). In the parameter space, the image point is represented by a sinusoid curve

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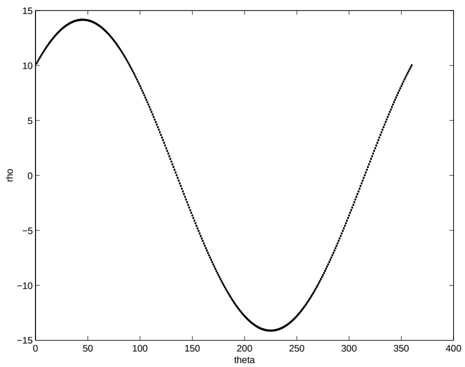

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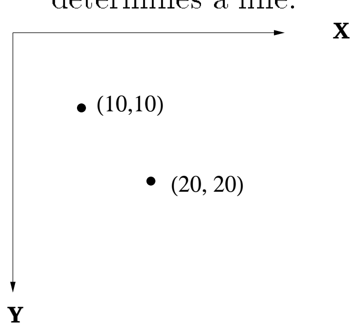
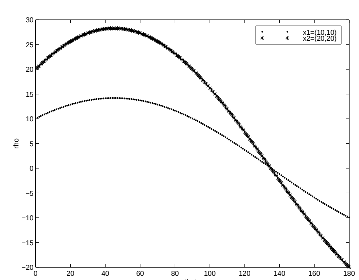
The intersection of two sinusoid curves representing the line going through two image points (e.g., (10,10) and (20,20)) uniquely determines a line.





If lines are parameterized using a and b, then an

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image point is represented by as a line in the parameter space (fig. 5.1).

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2. **Quantize parameters ρ and θ**

   To keep the parameter space finite, it is often necessary to quantize the parameter space. By quantization, the parameter space is divided into finite grid of cells, the dimensions of each cell are determined by the quantization intervals. Each cell is indexed by a pair of quantized (θ, ρ).

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3. **Build an accumulator array $A(\theta, \rho)$**

   The accumulator array is used to accrue the contribution of each image point to a quantized (θ, ρ). The standard HT scheme for updating the accumulator array A(θ, ρ) is

   $$A(\theta,\rho) = \#\{(x,y) \in X \times Y | x\cos\theta + y\sin\theta + \rho = 0\}$$

   where $X \times Y$ are the image domain.

   For example, if we have N image points lying on a line with parameter pair (θ_0, ρ_0), then we have $A(\theta_0, \rho_0) = N$ and the accumulator array values for all other parameter pairs are 1.

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For each quantized θ, we use the line equation to compute the corresponding ρ. The computed ρ is then quantized. The accumulator corresponding to the quantized θ and ρ are then incremented by 1.

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4. **Search $A(\theta, \rho)$ for peaks**

   The image lines are therefore detected by the peaks of A(ρ, θ). To detect multiple lines, look for all local maxima of $A(\theta, \rho)$.

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HT Algorithm Outline

For an input gray scale image I of $M \times N$.

1. Locate the HT coordinate system
2. Identify the ranges for θ and ρ. Let the range for for θ be between θ_l and θ_h, and the range for ρ between ρ_l and ρ_h.
3. Choose quantization intervals $\delta\theta$ and $\delta\rho$ for θ and ρ respectively.
4. Discrete the parameter space of ρ and θ using sampling steps $\delta\rho$ and $\delta\theta$. Let θ_d and ρ_d be 1D

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arrays containing the discretized θ and ρ.

5. Let A(T, R) be an array of integer counter; initialize all elements of A to zero, where $R = \frac{\rho_h - \rho_l}{\delta\rho}$ and $T = \frac{\theta_h - \theta_l}{\delta\theta}$.
6. Let L(T,R) be an array of a list of 2D coordinates.
7. For each image pixel (c, r), if its gradient magnitude $g(c, r) > \tau$, where τ is a gradient magnitude threshold.

   for i=1 to T

   - $$\rho = c * \cos(\theta_d(i)) + r * \sin(\theta_d(i))$$

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   - find the index k, for the element of ρ_d closest to ρ
   - increment A(i,k) by one.
   - Add point (c,r) to L(i,k)

8. Find all local $A(i_p, k_p)$ such that $A(i_p, k_p) > t$, where t is a threshold.
9. The output is a set of pairs $(\theta_d(i_p), \rho_d(k_p))$ and lists of points in $L(i_p, k_p)$, describing the lines detected in the image, along with points located on these lines

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HT Coordinate Systems

- The HT coordinate system coincides with the row-column coordinate system.

  Ranges of ρ and θ

  $0 < \rho < \sqrt{M^2 + N^2}$

  $-90 < \theta < 180$

  or

  -$M < \rho < \sqrt{M^2 + N^2}$

  $0 < \theta < 180$

- The HT coordinate system locates at the center of

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the image

  Ranges of ρ and θ

  $0 < \rho < \frac{\sqrt{M^2+N^2}}{2}$

  $0 < \theta < 360$

  or

  $-\frac{\sqrt{M^2+N^2}}{2} < \rho < \frac{\sqrt{M^2+N^2}}{2}$

  $0 < \theta < 180$

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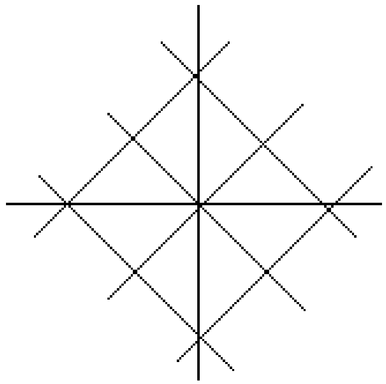
An Example



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Incorporating Gradient Direction

If gradient direction at each image point is available, it can be used to restrict the range of values of θ that the image point may vote for. Instead of checking for all possible θ s as in the standard HT, we only increase the accumulators for those lines whose orientations are close to the gradient direction of the image point, therefore significantly reducing the required computations.

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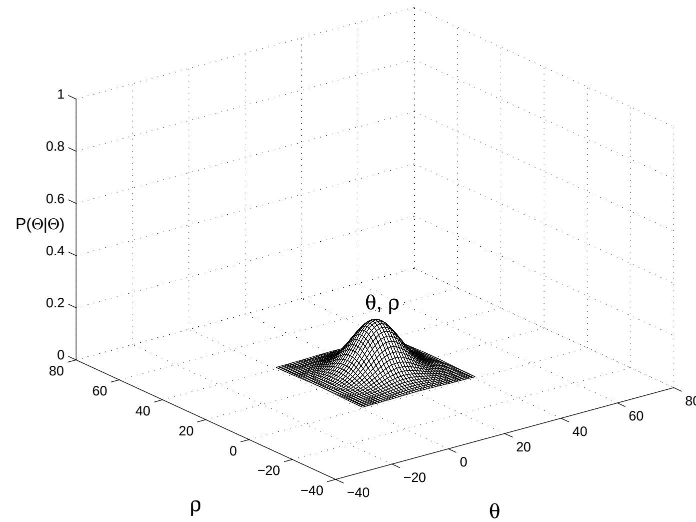

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Incorporating Gradient Magnitude

Instead of updating the accumulator array by an unit, the contribution of each point to the accumulator array can be determined based on the gradient magnitude of the point.

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HT for Curves

The HT can be generalized to detect any parameterizable curves such as circles and ellipse. For example for circle detection, a circle can be parameterized as

$$(x - x_0)^2 + (y - y_0)^2 = R^2$$

where the parameters are (x_0, y_0), and R. We can therefore construct a 3D accumulator $A(x_0, y_0, R)$. For each edge point vary (x_0, y_0), compute the corresponding R using the circle equation, and then

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update A. In the end, search A for peaks.

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If we know the gradient direction of each point, we may use a different parameterizable representation for circle to take advantage of the gradient direction

$$\begin{aligned} x &= R\cos\theta + x_0 \\ y &= R\sin\theta + y_0 \end{aligned}$$

where θ represents the gradient direction of point (x, y). We can then reduce search from 2D to 1D. We can vary R and then use the above equation to compute x_0 and y_0, then updating the corresponding accumulator accordingly.

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Another HT Technique for Circle Detection

- Detect circle center first
- Determine circle radius

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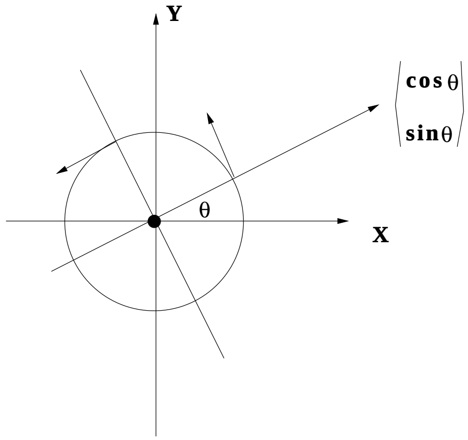

For the parametric circle equation, we have the

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tangent of the circle at (x, y) is

$$\frac{dx}{dy} = -\tan\theta$$

From circle equation $(x - x_0)^2 + (y - y_0)^2 = R^2$, we have

$$\frac{dx}{dy} = -\frac{y - y_0}{x - x_0}$$

Equating the above two equations yields

$$\frac{y - y_0}{x - x_0} = \tan\theta$$

where θ represents the gradient direction at (x, y).

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This equation does not contain circle radius R, therefore allowing to find circle centers first.

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Another HT Technique for Circle Detection

Given (x_0, y_0), we can then use equation $(x - x_0)^2 + (y - y_0)^2 = R^2$ to determine Rs for each (x_0, y_0). Note different Rs may be identified for each (x_0, y_0), indicating concentric circles.

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HT for Ellipse Detection

Let an ellipse equation be

$$\frac{(x-x_0)^2}{a^2}+\frac{(y-y_0)^2}{b^2}=1$$

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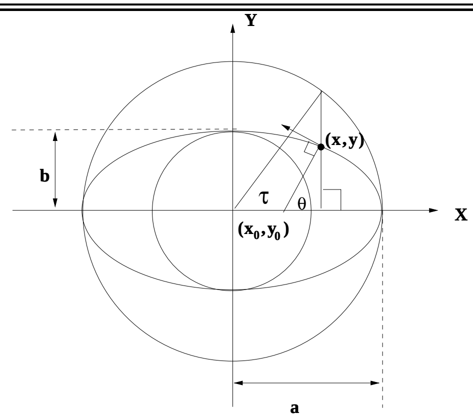
HT for Ellipse Detection (cont'd)



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$$\begin{aligned} x &= x_0 + && a\cos\tau \\ y &= y_0 + && b\sin\tau \end{aligned}$$

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HT for Ellipse Detection (cont'd)

From the tangent definition,

$$\frac{dx}{dy}=-\tan\theta$$

From the parametric ellipse equation, we have

$$\frac{dx}{dy}=-\frac{a}{b}\tan\tau$$

As a result, we have

$$\tan\tau=\frac{b}{a}\tan\theta$$

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Substituting the above equation to the parametric ellipse equation yields the parametric representation of an ellipse using θ

$$
\begin{aligned}
x &= x_0 \pm \frac{a}{\sqrt{1+\frac{b^2}{a^2}\tan^2\theta}} \\
y &= y_0 \pm \frac{b}{\sqrt{1+\frac{a^2}{b^2\tan^2\theta}}}
\end{aligned}
$$

This still, however, involves 4 parameters.

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HT for Ellipse Detection (cont'd)

From the previous equations, we have

$$\frac{y-y_0}{x-x_0} = \frac{b}{a}\tan\tau$$

We also know $\tan\tau = \frac{b}{a}\tan\theta$ As a result, we have

$$\frac{y-y_0}{x-x_0} = \frac{b^2}{a^2}\tan\theta$$

In the above equation, we can treat $\frac{b^2}{a^2}$ as a single unknown, effectively reducing to three dimensions. We

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can therefore construct an accumulator array involving x_0, y_0, and $\frac{b^2}{a^2}$.

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Detection of Ellipse with an Unknown Orientation

Let $X'-Y'$ represent the coordinate system where the major and minor axes align with X' and Y' respectively as shown in the figure.

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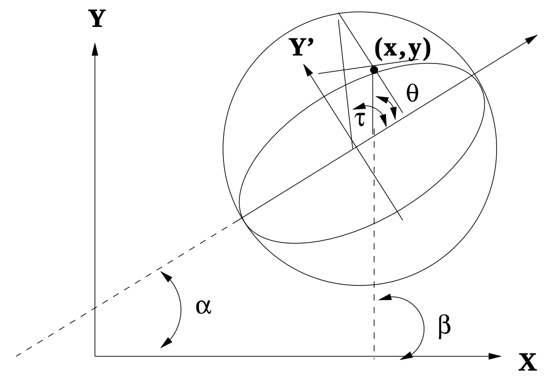

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} + \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} a\cos\tau \\ b\sin\tau \end{pmatrix}$$

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From the previous discussion, we have

$$\tan\tau = \frac{b}{a}\tan\theta$$

Since $\theta = \beta - \alpha$, we therefore have

$$\tan\tau = \frac{b}{a}\tan(\beta - \alpha)$$

where β is the gradient direction of point (x, y) in X-Y coordinate frame.

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Detection of Ellipse with an Unknown Orientation

From the above equation, we have

$$\frac{(x - x_0)\sin\alpha + (y - y_0)\cos\alpha}{(x - x_0)\cos\alpha - (y - y_0)\sin\alpha} = \frac{b^2}{a^2}\tan(\beta - \alpha)$$

When $\alpha = 0$, the above equation reduces to the equation for a nominal ellipse, with $\theta = \beta$. Again, we reduce the dimension of parameter array from 5 to 4.

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Other Efficient Ellipse Detection using HT

One method is to scan the image horizontally and identify any possible edge points on each row. For each row, identify the centroid of any two selected edge points. The idea is that if the two points happen to be on an ellipse, then their centroid must be on the vertical center line of the ellipse. So, by voting, we may detect the vertical center line. Similarly, if scan vertically, the horizontal centerline can be detected. The ellipse center is the intersection of the vertical and horizontal center lines. Given the center of the ellipse, other parameters are then determined.

Generalized HT

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HT transform can be generalized to detect any shapes, even for those without analytic (parametric) representation.

HT Summary

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- Advantages:

  Robust to image noise and occlusion

  Can detect multiple curves

- Disadvantages

  computational intense

  noise and quantization errors may introduce spurious peaks

  standard HT only detects line not line segments