The mathematics of Musical Improvisation

Acknowledgements

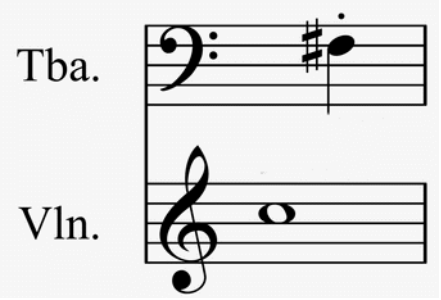
東京工業大学
Tokyo Institute of Technology

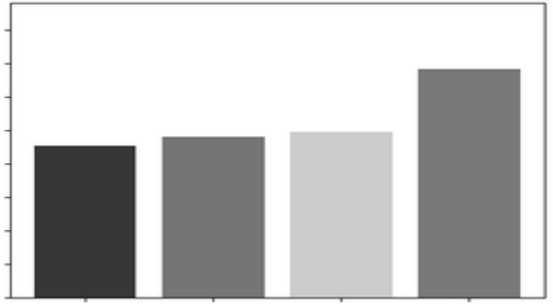
GUILD SCHOOL HALL

Dr Takayuki Nozawa
Portorius Quartet
Dr David Dolan
Dr Hardik Rajpal
Dr Pedro Mediano etc

This Talk

Improvisation
Experiment
Music
Synchrony
Math





Improvisation

19th century

20th century

21st century

Group experience

“In all societies, a primary function of music is collective and communal, to bring and bind people together. People sing together and dance together in every culture […] in such a situation, there seems to be a binding of nervous systems accomplished by rhythm.”

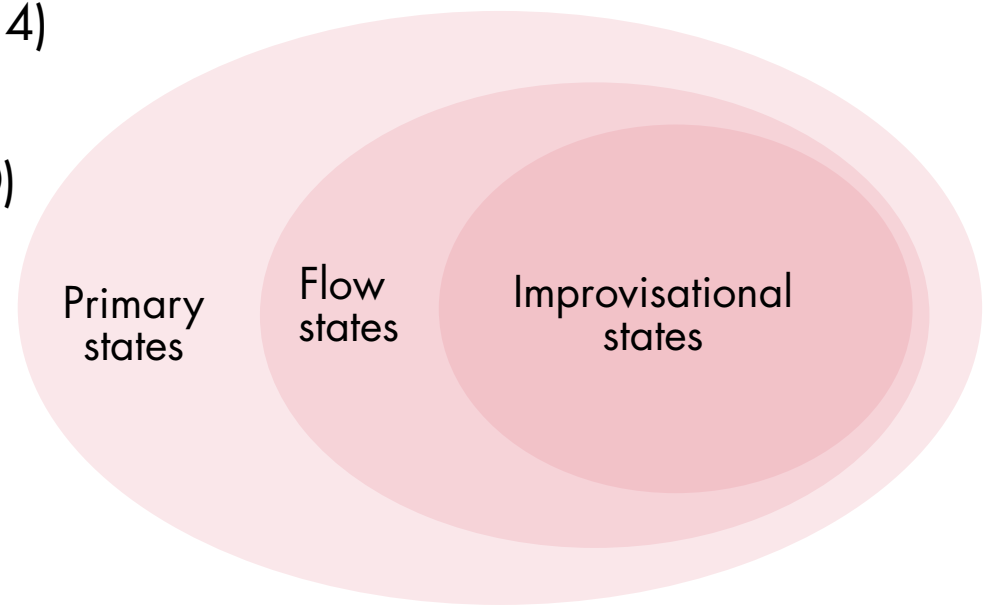
— Oliver Sacks

Improvisational state of mind

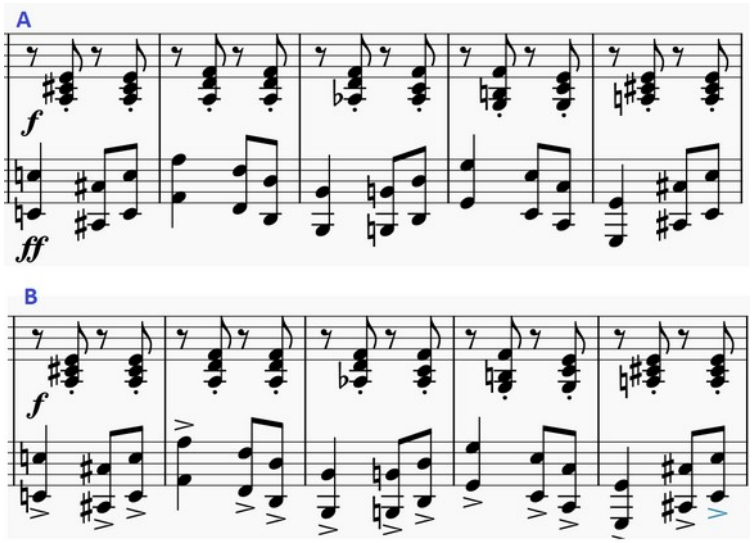
- more **entropic** brain signals in both musicians and audiences (*Dolan et al,* 2018)

- a type of **flow state** (*Csikszentmihalyi,* 1975)
- a **primary state** (*Carhart-Harris et al,* 2014)

- a type of **team flow**? (*Shehata et al*, 2020)



Modes of performance



Strict

controlling technical precision, timing co-ordination, accuracy of the score's details, avoiding risks, while at the same time creating the most convincing and expressive performance possible

Let-go

play freely, spontaneously, not putting an imperative focus on “no wrong notes”, deviate from the written text in an unplanned coordination with the other ensemble partners

Subjective ratings by performers

… the let-go version puts us on an awareness edge, and suddenly we are able to see past the music and look out for these spontaneous cues for the changes in both the music itself and the way we perform it…

… a very high level of mind-reading and synchronicity …

Experiment

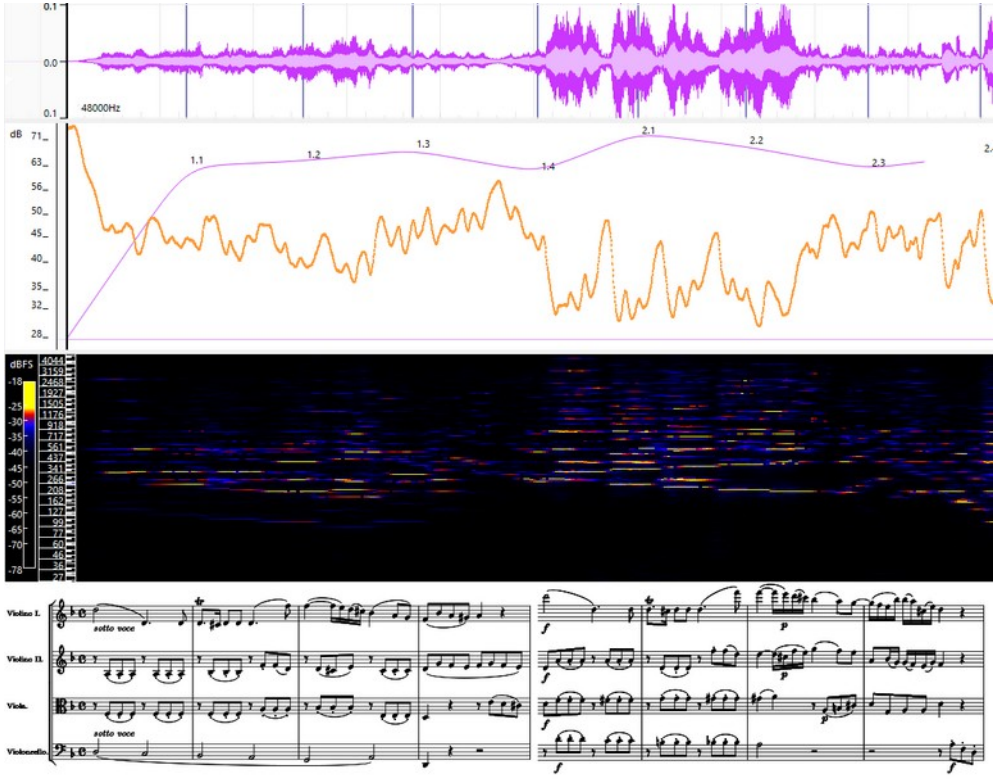
- Each piece played twice in strict and let-go conditions
  - Mozart String Quartet KV. 421
  - Haydn Op. 76 Minuet

- musicians equipped with head motion sensors
- ~50 audience members with accelerometers

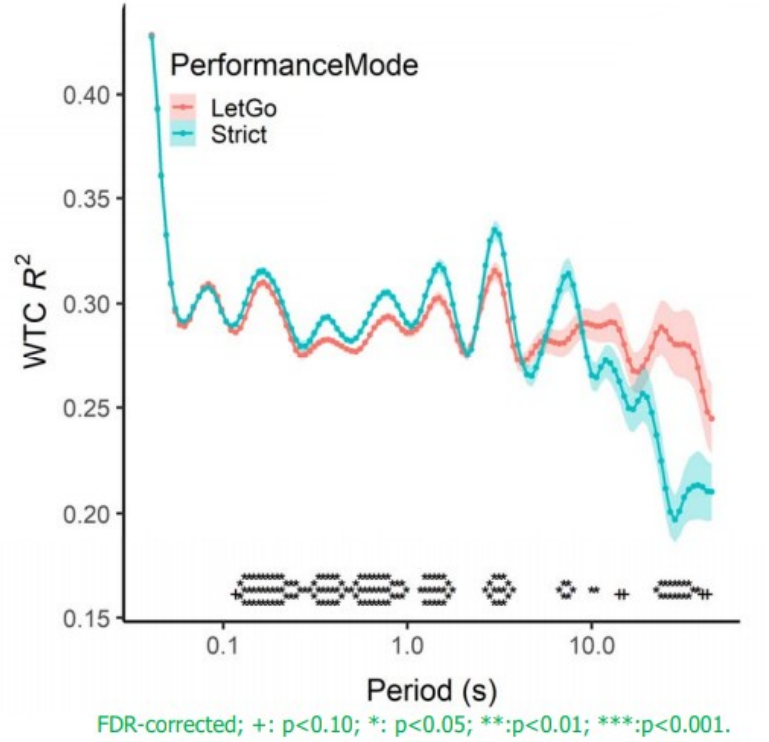
Methods

Audience synchrony

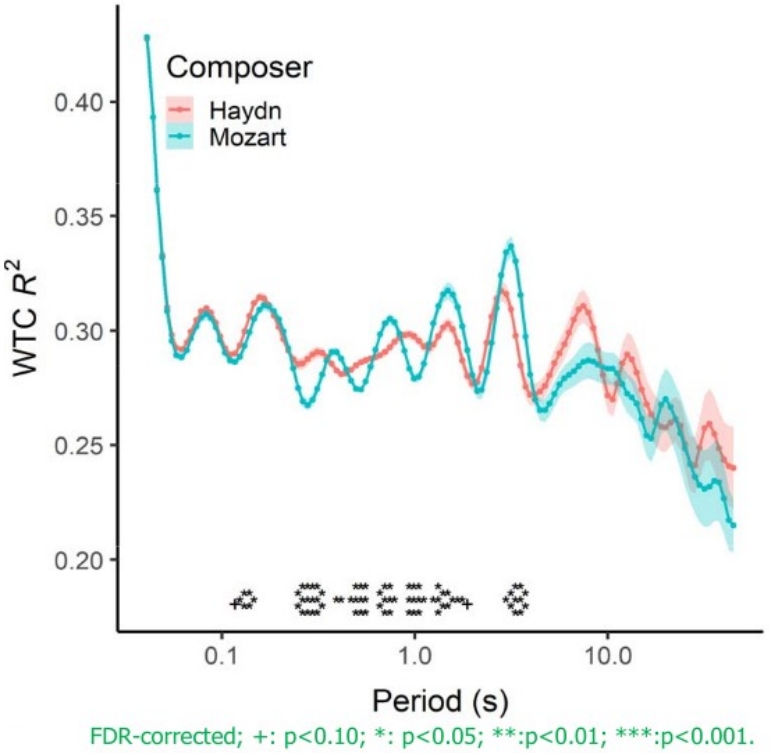
Music performance analysis



Musician-Audience physical sync



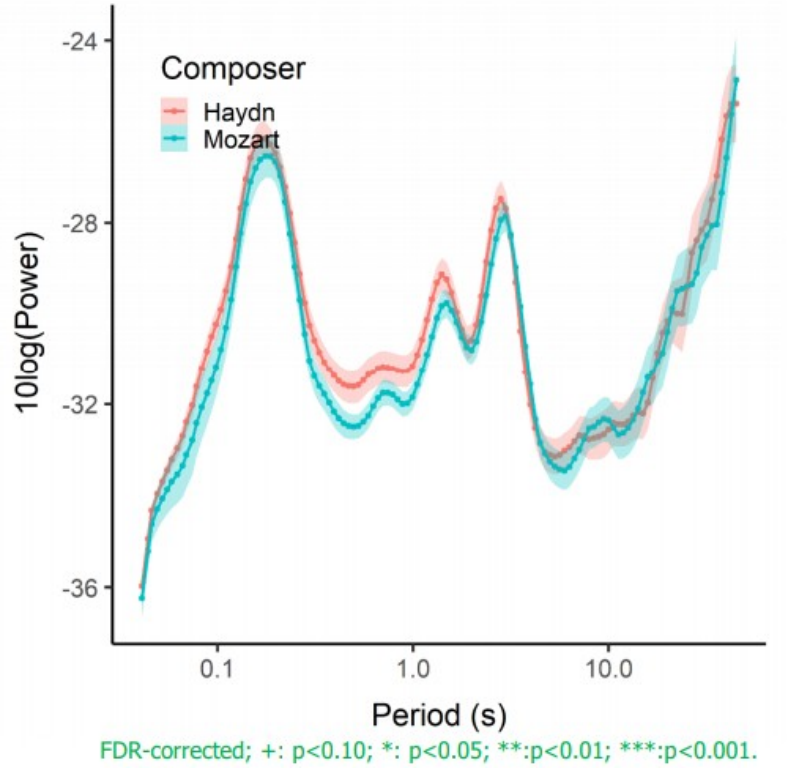
FDR-corrected; +: p<0.10; *: p<0.05; **:p<0.01; ***:p<0.001.

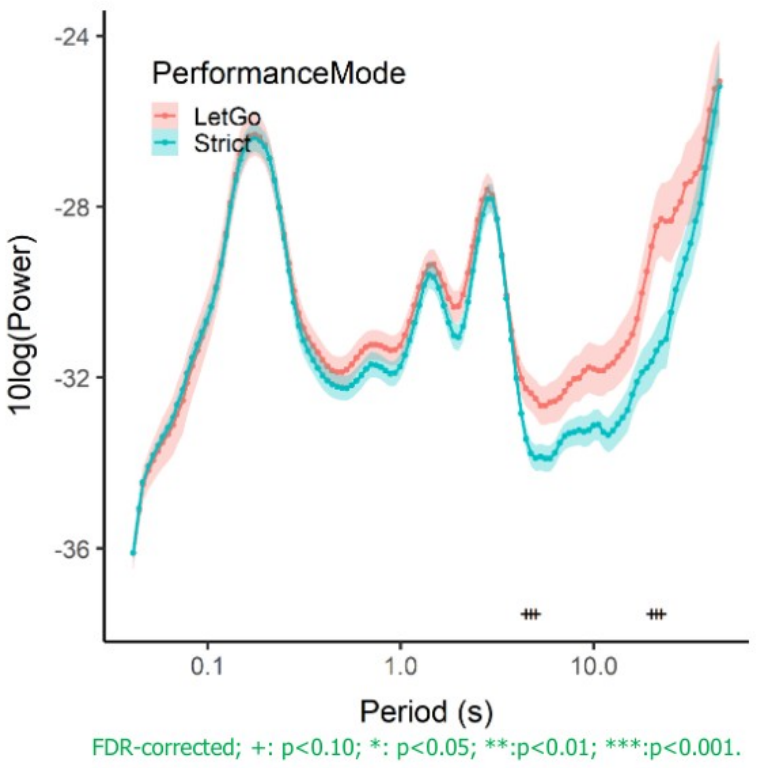


FDR-corrected; +: p<0.10; *: p<0.05; **:p<0.01; ***:p<0.001.

analysis by Dr. Takayuki Nozawa

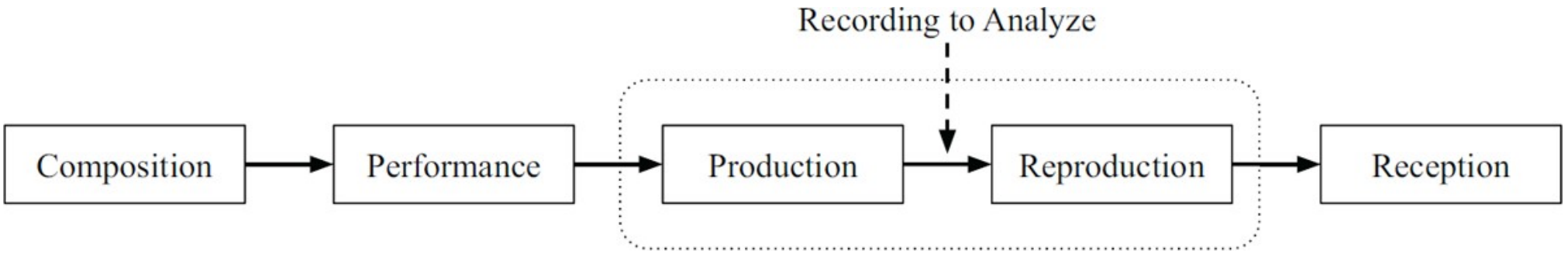
Audience physical activity power





analysis by Dr. Takayuki Nozawa

Musical analysis



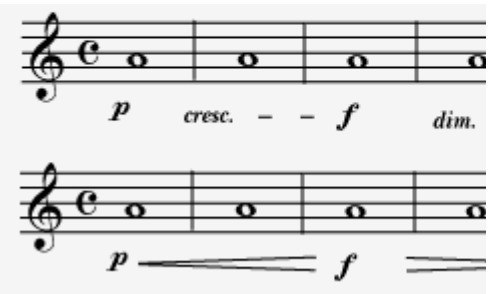
- Musicology: focus on symbolic music (sheet music or MIDI)
- Music information retrieval: focus on a single piece, using only one recording
- Music performance analysis: focus on recordings of a specific performance

(*Lerch et al*, 2020)

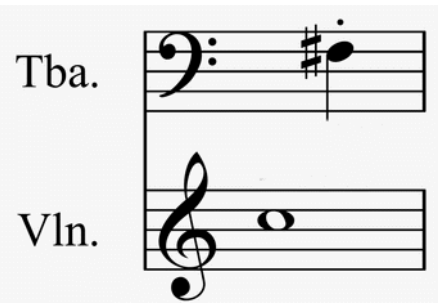
Performance related parameters

tempo

dynamics



timbre



(*Bowen*,1996; *Lerch et al*, 2020)

Hypothesis

Let-go

- More variation in tempo
- Less harmonics
- More fluctuations in dynamics
- Audience synchronises more on longer time intervals, corresponding to a musical phrase (**music-sync**)

Strict

- Less variation in tempo
- More harmonics
- Less fluctuations in dynamics
- Audience synchronises more on shorter time intervals corresponding to a beat (**beat-sync)**

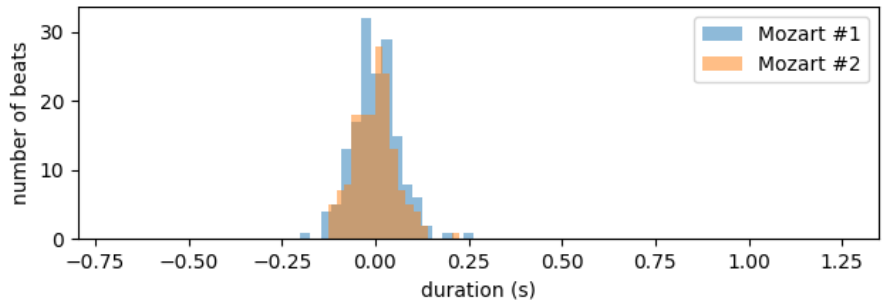
How exactly do we quantify these effects?

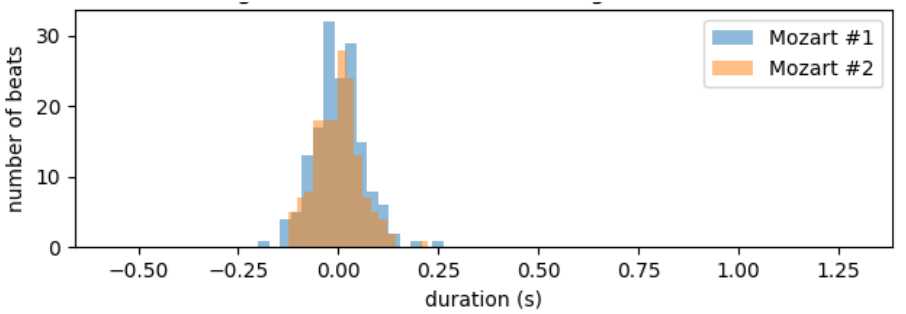
How do they influence the audience?

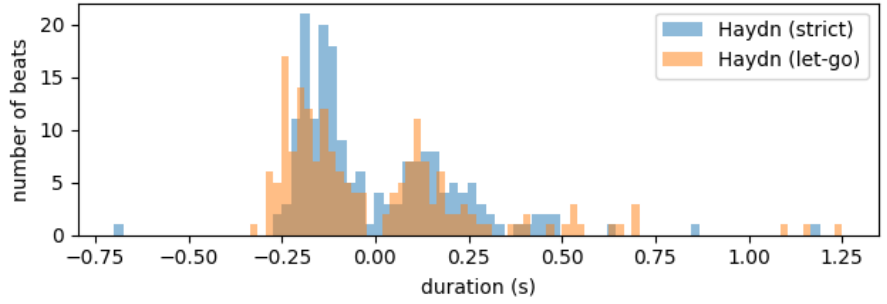
Ideas

- Tempo variations associated with correlations on the longer term and anti-correlations on the shorter term (*Rasanen et al*, 2014; *Hennig et al*, 2014)
- Consistent with sensorimotor synchronisation (tapping) literature (*Repp et al*, 2005)
- Long term correlations in amplitude give the sense of ‘groove’ or ‘danceability’ (*Rasanen et al*, 2014; *Streich et al*, 2005)

Fluctuations around mean beat duration







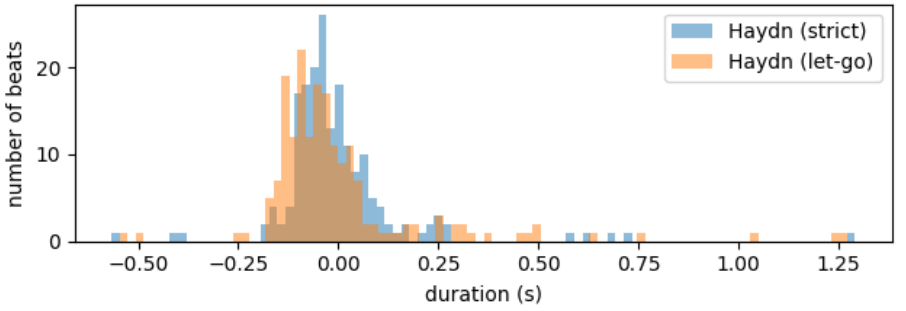


Fig (a), (b): Multimodal distribution reveals song structure

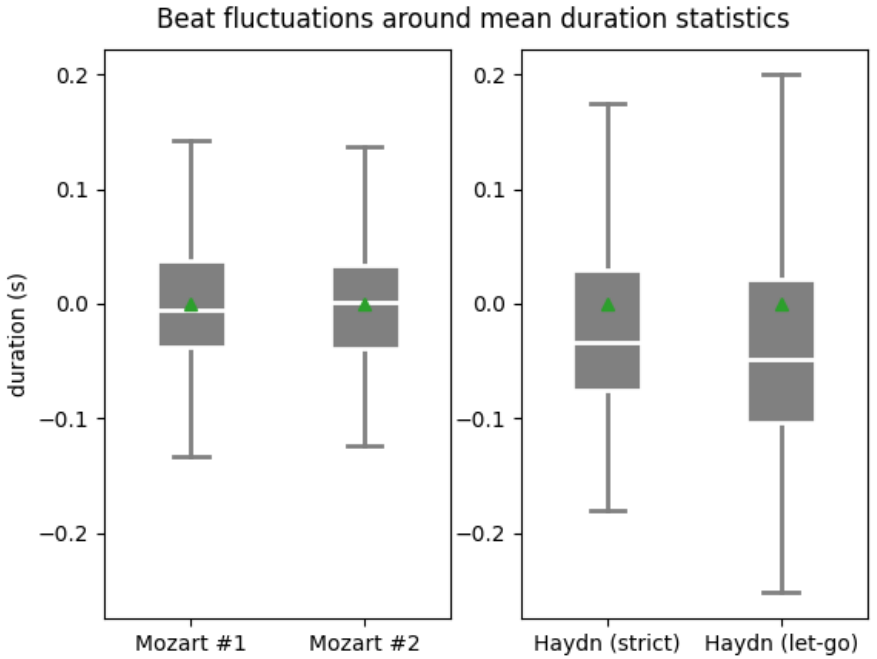
Fig (c), (d): When averaging across the sections in the song the distribution of beat intervals becomes unimodal and appears more skewed in let-go

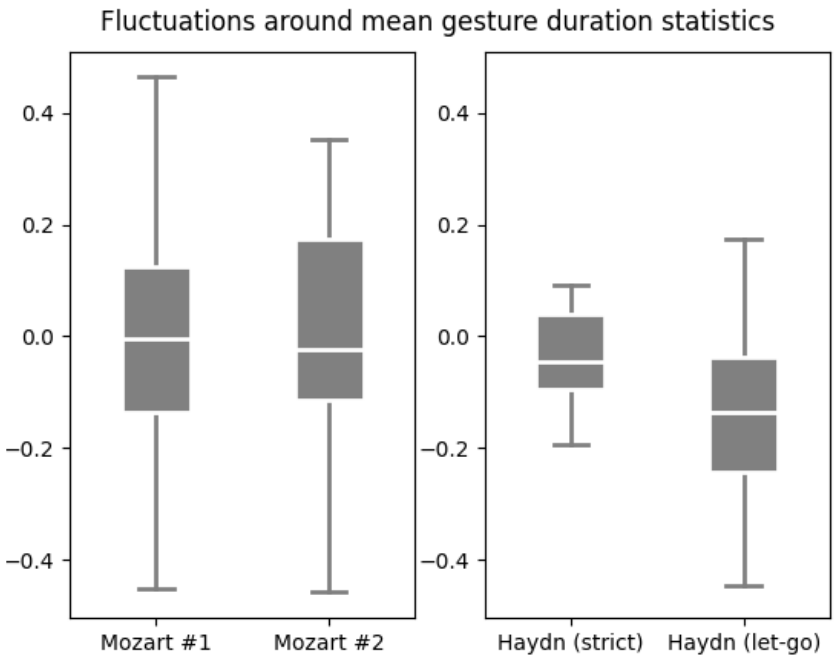
We need to consider tempo in our analysis of audience motion

Temporal scales

- **Beat-sync**: intervals smaller than 1 bar, can be 1 up to 4 beats depending on tempo
  - ~1s, or 1 beat in Mozart, 3 beats in Haydn
- **Gesture-sync**: intervals larger than 2-3 bars – a "*musical gesture*"
  - ~5-6s, or 2 bars in Mozart, 8 bars in Haydn
- **Music-sync**: intervals larger than 2-3 gestures – a "*musical gesture*" (e.g. 4 bars in Mozart, 5-6s)
  - > 20s, or 8 bars in Mozart, 20 bars in Haydn

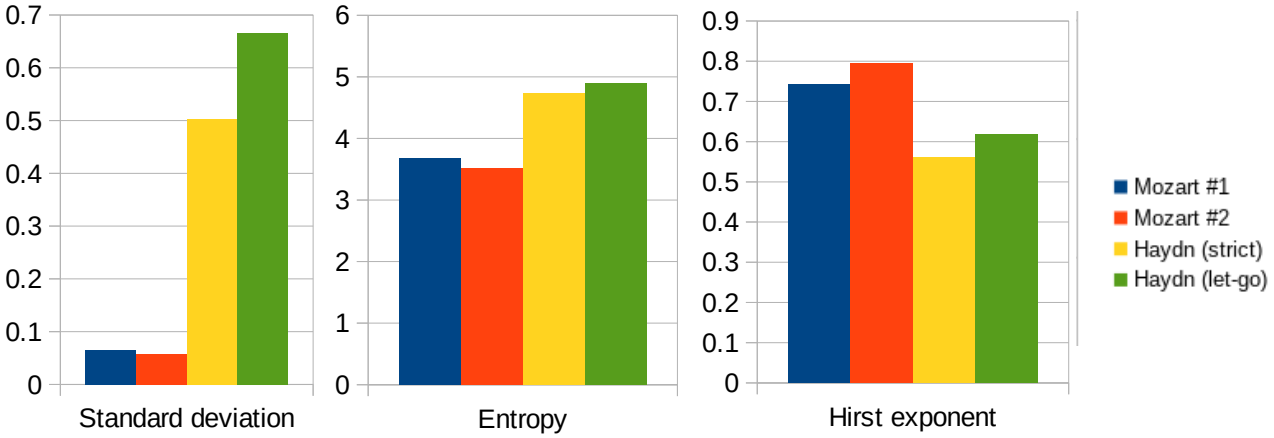
Fluctuations around mean beat duration





- Hypothesis: distribution of beat intervals is broader for let-go than strict
- The effect is consistent in let-go across different temporal scales, both for individual beats and longer musical gestures
- Suggests Mozart #1 is more like let-go w.r.t. tempo

Tempo

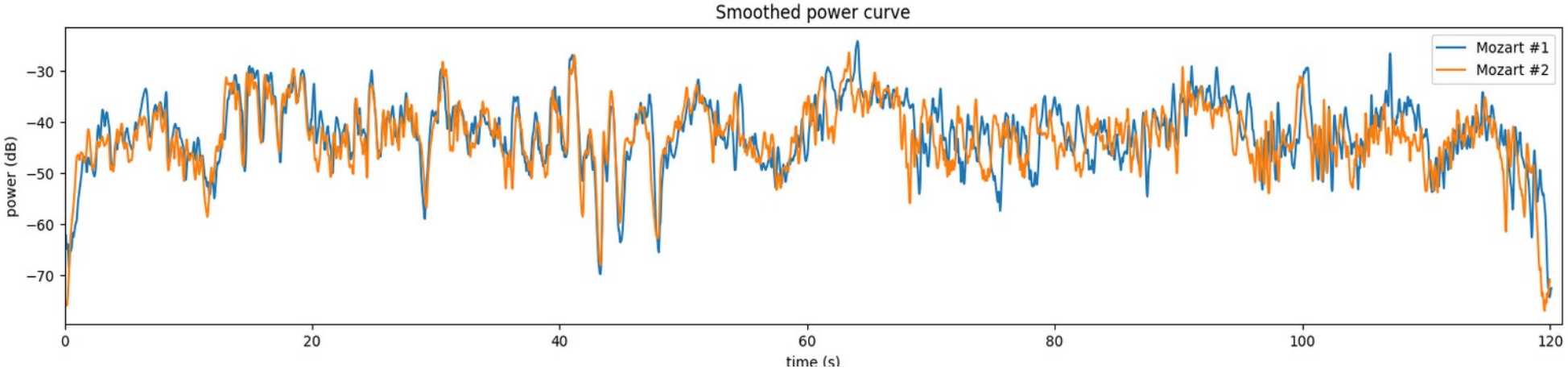


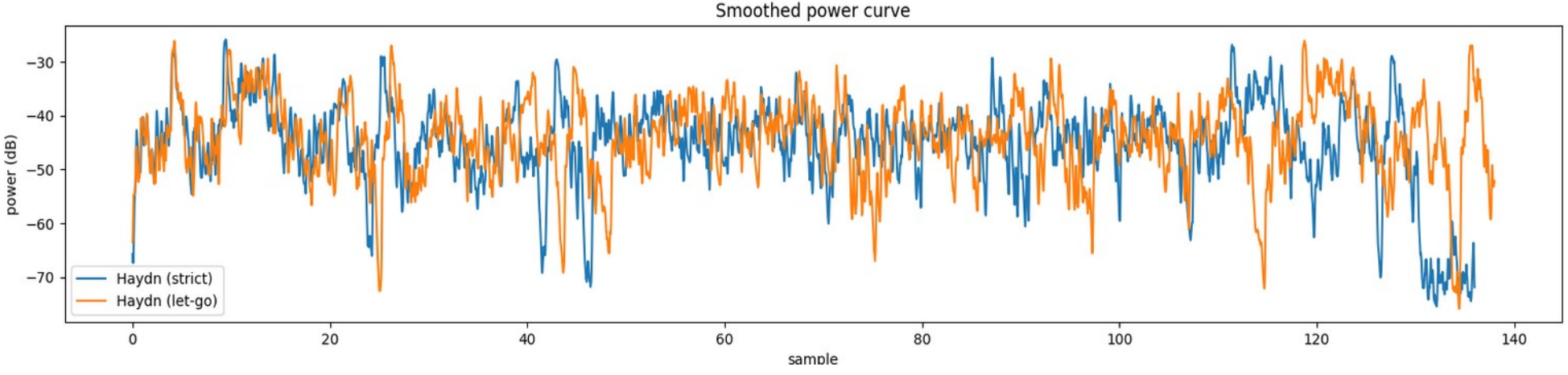
- Strict > letgo
- Mozart consistent in standard deviation and entropy, but not Hirst

- Hypothesis: distribution of beat intervals is broader for let-go than strict => higher entropy, larger standard deviation
- Suggests Mozart #1 is more let-go
- Hirst exponent suggests self-similarity if >0.5 and <1.5
- Hypothesis: closer to 1 in let-go
- Suggests Mozart #2 is more let-go

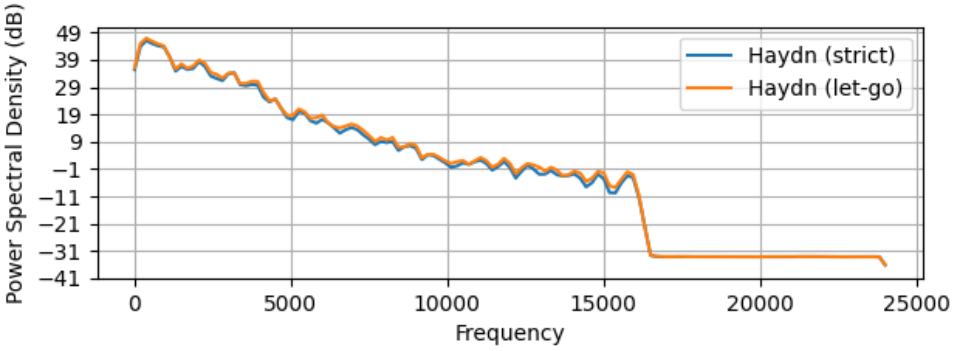
Dynamics

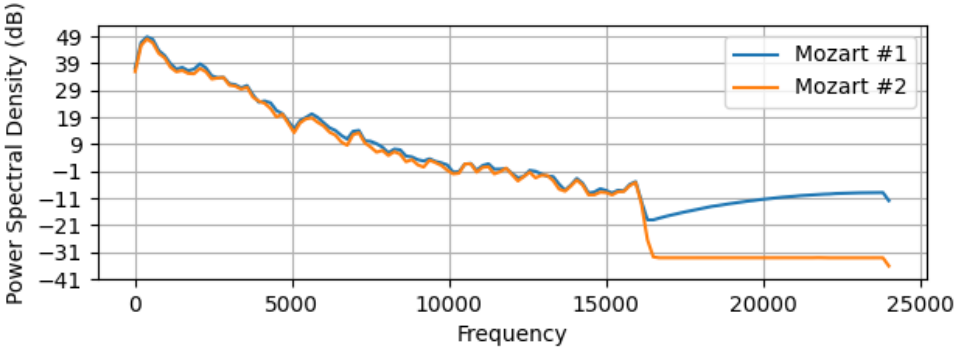
$$P_{\text{avg}} = 10 \log_{10} \left(\frac{1}{N} \sum_{n} x_n^2 \right)$$

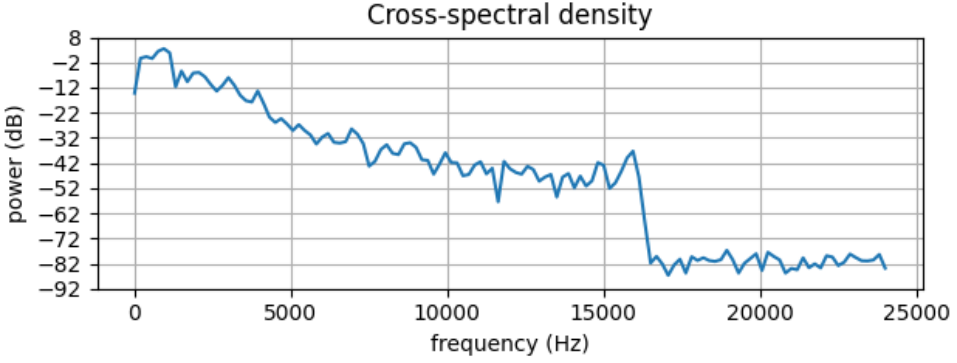


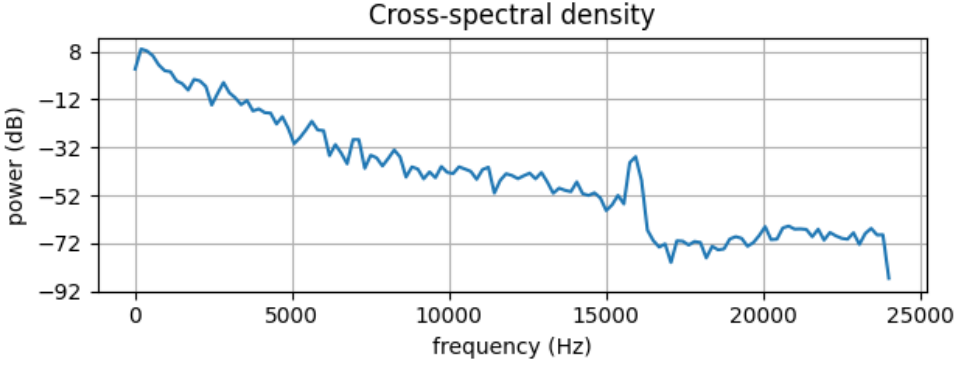


Timbre

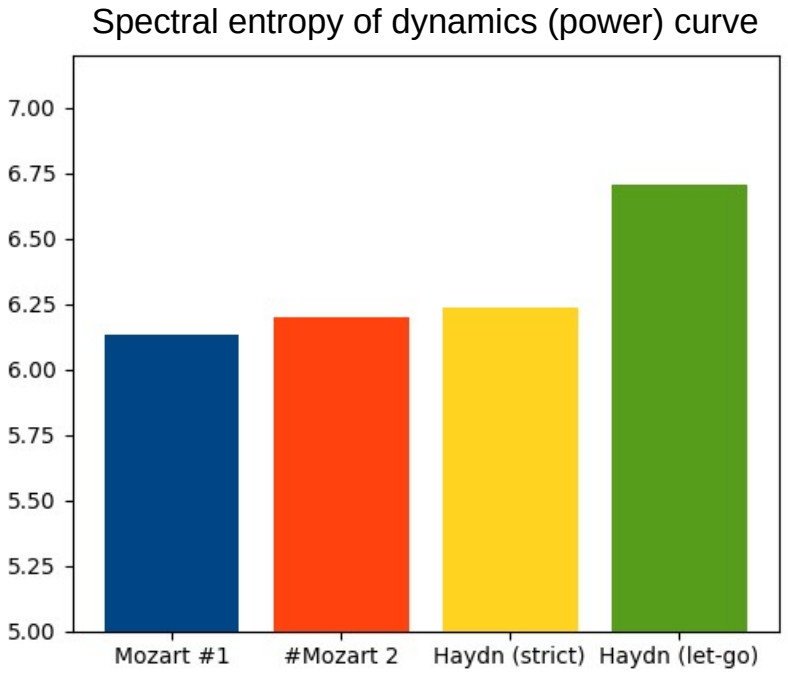

Power Spectral Density (dB)
49
39
29
19
9
−1
−11
−21
−31
−41
0
5000
10000
15000
20000
25000
Frequency
Haydn (strict)
Haydn (let-go)

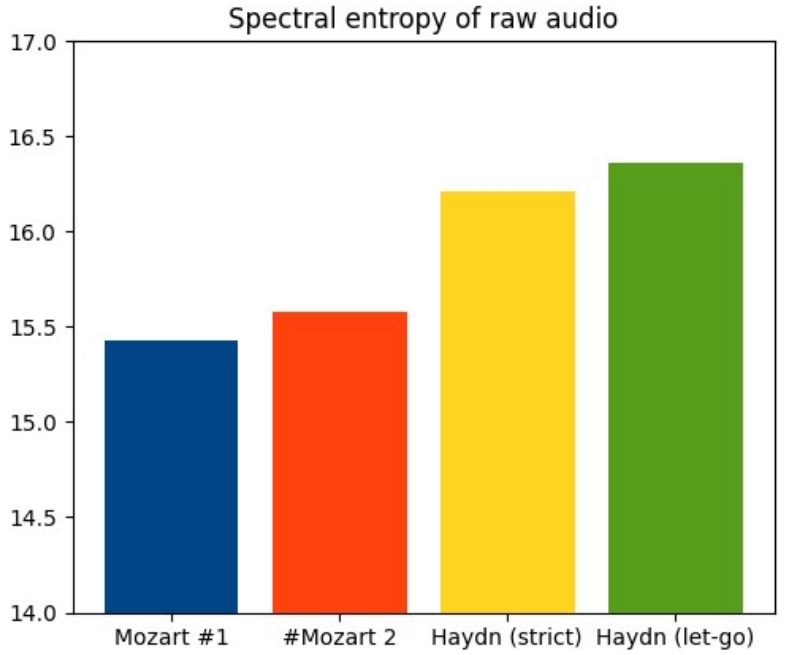

Power Spectral Density (dB)
49
39
29
19
9
−1
−11
−21
−31
−41
0
5000
10000
15000
20000
25000
Frequency
Mozart #1
Mozart #2


Cross-spectral density
power (dB)
8
−2
−12
−22
−32
−42
−52
−62
−72
−82
−92
0
5000
10000
15000
20000
25000
frequency (Hz)


Cross-spectral density
power (dB)
8
−12
−32
−52
−72
−92
0
5000
10000
15000
20000
25000
frequency (Hz)

Entropic dynamics & timbre





- Hypothesis: let-go produces more entropic music

Closing thoughts

- Larger tempo variations in let-go performance
- Long term correlations in amplitude in let-go performance
- Song structure affects statistics: minuet (Haydn) has a three-part structure, in which each part requires investigation

- Future work: use an AI model to generate a ‘null’ piece, that perfectly follows the score, and compare with strict and let-go