Syed Measum
FA21- BEE-252

Q1 Solve:

(a) $A(4,-4,-1)$ , $B(-4,3,-4)$ , $C(4,-1,-2)$

$$AB = \begin{bmatrix} -4 \\ 3 \\ -4 \end{bmatrix} - \begin{bmatrix} 4 \\ -4 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} -4-4 \\ 3-(-4) \\ -4-(-1) \end{bmatrix} = \begin{bmatrix} -8 \\ 7 \\ -5 \end{bmatrix}$$

$$AC = \begin{bmatrix} 4 \\ -1 \\ -2 \end{bmatrix} - \begin{bmatrix} 4 \\ -4 \\ +1 \end{bmatrix}$$

$$= \begin{bmatrix} 4-4 \\ +4+1 \\ -1+2 \end{bmatrix} = \begin{bmatrix} 0 \\ +3 \\ -3 \end{bmatrix}$$

$$n = AB \times AC$$
$$= i(-21+15) - j(24) + k(-24)$$
$$= -6i - 24j - 24k$$
$$= i + 4k + 4j$$

$$d = a \cdot n$$
$$= (4i - 4j + k) \cdot (i + 4j + 4k)$$
$$= 4 - 16 + 4$$
$$= -8$$

b)

$$\perp.\text{Distance} = \frac{d}{|n|} = \frac{8}{\sqrt{1+16+16}} = \frac{8}{\sqrt{33}}$$

(c) line AD - $r = a + \lambda b$

$$r = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \lambda \begin{bmatrix} 2 \\ 3 \\ -3 \end{bmatrix}$$

for some value of $\lambda$

$$r \cdot n = d$$

$$\begin{bmatrix} 2\lambda \\ 3\lambda \\ -3\lambda \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 4 \\ 4 \end{bmatrix} = -8$$

$$2\lambda + \cancel{12\lambda} - \cancel{12\lambda} = -8$$

$$2\lambda = -8$$

$$\lambda = \frac{-8}{2} = -4$$

$$r = \begin{bmatrix} 2(-4) \\ 3(-4) \\ -3(-4) \end{bmatrix}$$

$$r = \begin{bmatrix} -8 \\ -12 \\ 12 \end{bmatrix}$$

Q2 Solve:

$A : (7i + 4j - k)$ , $B : (11i + 3j)$

$C : (3i + 6j + 3k)$ , $D : (2i + 7j + \lambda k)$

A.T.G C

$AB : B - A$

$: 4i - 1j + k$

$CD = D - C$

$= 0i - j + (3 - \lambda)k$

Line$_1$ : $l_1$ $\cancel{AB}$ $= OA + \lambda AB$

line$_2$ : $l_2$ $CD$ $= OC + \lambda CD$

$$l_1 = \vec{r}_1 = \begin{bmatrix} 7 \\ 4 \\ -1 \end{bmatrix} + \lambda \begin{bmatrix} 4 \\ -1 \\ 1 \end{bmatrix}$$

$$l_2 = \vec{r}_2 = \begin{bmatrix} 2 \\ 6 \\ -3 \end{bmatrix} + \lambda \begin{bmatrix} 0 \\ -1 \\ 3+\lambda \end{bmatrix}$$

$$D = \frac{(b_1 \times b_2) \cdot (a_2 \times a_1)}{|(b_1 \times b_2)|}$$

$$b_1 \times b_2 = \begin{bmatrix} -3+\lambda \\ 2-4\lambda \\ -4 \end{bmatrix}$$

$$|b_1 \times b_2| = \sqrt{\lambda^2 + 9 - 2\lambda + 144 + 16\lambda^2 - 24\lambda + 16}$$

$$= \sqrt{17\lambda^2 - 26\lambda + 169}$$

$$a_2 - a_1 = \begin{bmatrix} 7 \\ 4 \\ -1 \end{bmatrix} - \begin{bmatrix} 2 \\ 6 \\ -3 \end{bmatrix}$$

$$= \begin{pmatrix} 5 \\ -2 \\ 2 \end{pmatrix}$$

$$3 = \frac{(-3+\lambda).5 + 2(12-4\lambda) - 2(4)}{\sqrt{17\lambda^2 - 26\lambda + 169}}$$

$$9(17\lambda^2 - 26\lambda + 169) = (7\lambda + 29 - 23)^2$$

$$= (7\lambda - 1)^2$$

$\lambda =$ something

Then $\lambda^2 - 5\lambda + 4 = 0$

(b) $\Pi_1 \; \lambda = 1$

$\Pi_2 \; \lambda = 4$

Plane ABD when $\lambda = 1$

So $AD = 2i + 7j + k$

$$AB \times CD = \begin{bmatrix} i & j & k \\ 4 & -1 & 1 \\ -5 & 3 & 2 \end{bmatrix}$$

Q2 Continued:

$AB \times CD = -5i - 13j + 7k$

Equation: $-5(2) - 13(7) + 7.(1)$

$= -10 - 91 + 7$

For Plane $\pi_2$ where $\lambda = 4$

$AB \times AD = i(-5-3) - j(20-5) + k(12+5)$

$= -8i - 15j + 17k$

Equation $= -8(x-11) - 15(y-3) + 17(z-6)$

$= -8x + 88 - 15y + 45 + 17z$

$= 8x + 15y - 17z - 13z$

(c) $\cos Q = \dfrac{(n_1 \cdot n_2)}{|n_1||n_2|}$

$= \dfrac{4 + 195 + 119}{\sqrt{25+169+49}\ \sqrt{64+225+283}}$

$Q = \cos^{-1}\left(\dfrac{318}{374.56}\right)$

$Q = 31.89°$

Q.3 (a) Find Values of $t$

$l_1 : ti + j \quad (-2i - j)$

$l_2 : j + tk \quad (-2j + k)$

Shortest distance b/w two lines is $\sqrt{21}$

$\vec{r}_1 = OA + \lambda AB$

$\vec{r}_2 = OA + \mu AB$

$$L_1 = \vec{r}_1 = \begin{bmatrix} t \\ 1 \\ 0 \end{bmatrix} + \lambda \begin{bmatrix} -2 \\ -1 \\ 0 \end{bmatrix}$$

$$L_2 = \vec{r}_2 = \begin{bmatrix} 0 \\ 1 \\ t \end{bmatrix} + \lambda \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$$

$$D = \frac{(b_1 \times b_2) \cdot (a_2 \times a_1)}{|b_1 \times b_2|}$$

$$b_1 \times b_2 = \begin{vmatrix} i & j & k \\ -2 & 1 & 0 \\ 0 & -2 & 1 \end{vmatrix}$$

$$= i + 2j + 4k$$

$$a_2 - a_1 = -ti + kt$$

$$\sqrt{21} = \frac{(i + 2j + 4k)(-ti + 0j + kt)}{\sqrt{21}}$$

$21 = -t + 4t$

$t = 7$

(b) $\pi_1 = \vec{r} = OA + AB\lambda + \mu AC$

$= -7i + j + \lambda(-2i - j) + \mu(2j + k)$

$= 5x - 6y + 7z = 0$

$\theta = ?$ b/w $l_2$ & $\pi_2$

$n_1 = (0, -2, 1)$

$n_2 = (5, -6, 7)$

$$\theta = \cos^{-1} \frac{n_1 \cdot n_2}{|n_1||n_2|}$$

$$= \cos^{-1} \frac{19}{\sqrt{5}\sqrt{110}}$$

$$= \cos^{-1} \frac{19}{23.43}$$

$\theta = 35.813$

(d) $\pi_1$ $n = (7, 1, 0)$, $\pi_2 = [5, -6, 7]$

$$\theta = \cos^{-1}\left(\frac{35 - 6 + 0}{\sqrt{50}\ \sqrt{110}}\right)$$

$$\theta = \cos^{-1}\left(\frac{29}{\sqrt{50}\sqrt{110}}\right) = 67.69^\circ$$

Q5 Solve:

(a) $P(-2,-1)$, $Q(-6,-3)$ are end Pt's of Circle

$$M.P = \frac{-6-2}{2}, \frac{-3-1}{2}$$

$$M.P = -4, -2$$

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(-2+4)^2 + (1+2)^2 = r^2$$

$$r^2 = 5$$

OR $$(x+4)^2 + (y+2)^2 = 5$$

(b) let y be centre.

at Pt. 4, 0

$$(4)^2 + (0-b)^2 = r^2$$

$$16 + b = r^2$$

at Pt $(0, 2)$

$$(0)^2 + (2-b)^2 = r^2$$

$$r^2 = 4 - b^2 - 4b$$

$$16 + \cancel{b^2} = 4 + \cancel{b^2} - 4b$$

$$b = -3$$

$$16 + 9 = r^2$$

$$r = 5$$

(c) $y = 100x$

$y^2 = 4ax$

$4ax = 100x$

$a = 25$

$x = -a$

$x = -25$

(d) $x^2 = 24y$

$x^2 = 24y$

$x^2 = 4ay$

$4ay = 24y$

$a = +6$

$x = -6$

(e) $\left(\frac{x}{25}\right)^2 + \left(\frac{y}{16}\right)^2 = 1$

$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$

$a = 5, \; b = 4$

$c = \sqrt{a^2 - b^2}$

$= \sqrt{25-16}$

$c = +3$

$F_1 = (-3, 0), \; F_2 = 3, 0)$

length $= 2a \Rightarrow 2(5) = 10$