Assignment # 2

Multi Variable Calculus
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Q1: (a) $\lim_{(x,y)\to(2,1)} \dfrac{x^2-2xy}{x^2-4y^2}$

$x^2 = 4y^2$

$\sqrt{x^2} = \sqrt{4y^2} \Rightarrow x = 2y$

$y = x/2$

$\Rightarrow$ Put in eq -

$$= \frac{x^2 - 2(x)(x/2)}{x^2 - 4(x/2)^2}$$

$$= \frac{x^2 - \cancel{2}(x^2/\cancel{2})}{x^2 - \cancel{4}(x^2/\cancel{4})} = \frac{x^2-x^2}{x^2-x^2}$$

$$= \frac{0}{0}$$

which is undefined or is not allowed in Mathematics
So, limit doesn't exist.

(b) $\lim_{(x,y)\to(0,0)} \dfrac{x-4y}{6y+7x}$

let

$6y - 7x = 0$

OR $6y = 7x$

$y = \dfrac{7x}{6}$

$\Rightarrow$ Put y in Question

$$= \frac{x-{}^{2}\cancel{4}(7x/\cancel{6}_3)}{7x+7x} = \frac{x - 14x/3}{7x+7x} = \frac{\frac{3x - 14x}{3}}{14x}$$

$$= \frac{-11x}{52x}$$

$$= \frac{-11}{52}$$

(c) $\lim_{(x,y) \to (0,0)} \frac{x^2 - y^6}{xy^3}$

let $y = mx$

$$= \frac{x^2 - (mx)^6}{x(mx)^3}$$

$$= \frac{x^2 - m^6x^6}{m^3x^4} = \frac{\cancel{x^2}(1 - m^6x^3)}{m^3x^{\cancel{4}2}}$$

$$= \frac{1 - m^6x^3}{m^3x^2}$$

(d) $\lim_{(x,y,z) \to (-1,0,4)} \frac{x^3 - ze^2y}{6x + 2y - 3z}$

$$= \frac{(-1)^3 - 4(0)e^2}{6(-1) + 2(0) - 12}$$

$$= \frac{-1 - 0}{-6 + 2 - 12}$$

$$= \frac{-1}{-16} = \frac{1}{16}$$

Answer

Q2: $f(x,y) = \cos(x/y)$ in $v = (3,-4)$

$$\nabla f = \frac{\partial}{\partial x}(\cos(x/y)i) + \frac{\partial}{\partial y}(\cos(x/y)j)$$

$$= \frac{-1}{y}\sin x/y\, i + \frac{x}{y^2}\sin(x/y)\, j$$

Unit: $\frac{3i-4j}{\sqrt{9+16}} = \frac{3}{\sqrt{25}}i + \left(-\frac{4}{25}\right)j$

$$D\vec{v}f = \frac{-3}{5y}\sin(x/y) + \frac{4x}{5y^2}\sin(x/y)$$

$$= \frac{1}{5y}\sin(x/y)(4x/y^{-3})$$

(b) $f(x,y,z) = x^2y^3 - 4xz$

$$\nabla f = ((2y^3x)i + ((-4z)j) + 3y^2x^2y^3 j - 4xk$$

$$\hat{v} = \frac{-i + 2j + 0k}{\sqrt{1+4}}$$

$$= \frac{-1}{\sqrt{5}}i + \frac{2}{\sqrt{5}}j + 0k$$

$$D\vec{v}f = \frac{-1}{\sqrt{5}}(2y^3x - 4z) - \frac{2}{\sqrt{5}}(3y^2x^3) + 0$$

$\vec{v}$: $f(x,y,z) = 4x - y^2 e^{3xz}$

$\nabla f = 4 - y^2 e^{3xz}(3z)\,i - 2y e^{3xz}\,j - y^2 e^{3xz} \cdot 3x\,k$

$\nabla f_{(3,-1,0)} = 4 - (-1)^2 (e)^{3(3)(0)} (3(0))\,i - 2(-1) e^{3(0)} j - (-1)^2 e^{2(3)(0)}$

$= (4-0)\,i - 2j - 9k$

for $v = (-1, 4, 2)$

$\hat{v} = \dfrac{-1i + 4j + 2k}{\sqrt{1+16+4}} = \dfrac{-1}{\sqrt{21}} i + \dfrac{4}{\sqrt{21}} j + \dfrac{2k}{\sqrt{21}}$

$= \dfrac{-4}{\sqrt{21}} - \dfrac{8}{\sqrt{21}} \dfrac{-18}{\sqrt{21}} = \dfrac{-4-8-18}{\sqrt{21}}$

$= \dfrac{-32}{\sqrt{21}}$

Answer

Q4 (a) $f(x,y) = \sqrt{x^2+y^3}$ at $(-2, 3)$

$\nabla f = \frac{1}{\cancel{2}}(x^2+y^3)^{-1/2}(\cancel{2}x)\,i + \frac{1}{2}(x^2+y^3)^{-1/2}(3y^2)\,j$

$\nabla f(-2,3) = (4+9)^{-1/2}(-2)\,i + \frac{1}{2}(4+9)^{-1/2} 3(9)\,j$

$= \dfrac{-2}{\sqrt{13}}\,i + \dfrac{27}{2\sqrt{13}}\,j$

Q4 (b) $f(x, y, z) = e^{2x}\cos(y - 2z)$ at $(4, -2, 0)$

$\nabla f = (e^{2x} \cdot 2\cos(y - 2z))i + e^{2x}(-\sin(y - 2z))j + e^{2x}(-\sin(y - 2z))(-2)k$

$\nabla f_{(4,-2,0)} = e^{4x} \cdot 2\cos(-2 - 0)i + e^{4x}(-\sin(-2))j + e^{4x}(-\sin(+4))$

$\nabla f_{(4,-2,0)} = e^{4x}(-2\cos 2i) - \sin(-2)j + 2\sin(-2)k)$

Answer

Q5 (a) $F = x^2yi - (z^3 - 3x)j + 4y^2zk$

$\nabla F = 2xyi - 0j + 0$

$\text{Div} f = \nabla f \cdot f$

$= (2xyi) \cdot (x^3yi + (z^3 - 3x)j + 4y^2k)$

$= 2x^3y^2 + f$

$$\text{curve } \nabla f \times f = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2y & +(z^3 - x^3) & 4y^2 \end{vmatrix}$$

$= i(8y + 3z^3) - j(0 - 0) + k(3x^2 - x^2)$

$= (8y + 3z^2)i - 0j + 2x^2k$

$f = (2x + 2z^2)i + \frac{x^2y^2}{z}j - (z \cdot 7x)k$

div $= \nabla f \cdot f$

$$= \left(\frac{\partial}{\partial x} i + \frac{y}{\partial y} j + \frac{z}{\partial z} k\right) \cdot \left((2x + x^2) i + \frac{x^3}{z} y^2 j - (7zx) k\right)$$

$$= 2 + \frac{2x^3}{z} y - 1$$

$$= 2 + \frac{2x^3 y}{z} - 1$$

Curve $= \nabla f \times f$

$$= \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 3x + 2x^2 & \frac{x^3 y^2}{z} & -(z7x) \end{vmatrix}$$

$$= i\left(0 + \frac{x^3 y^2}{z^2}\right) - j(7 - 4x) + k\left(3\frac{x^3}{z} y^2 + 0\right)$$

$$= \frac{x^3 y^2}{z^2} i - (7 - 4z) j + \left(3 \frac{x^3}{z} y^2\right) k$$

Q6 (a) $f = \left(4y^2 + \frac{3x^2y}{z^2}\right) i + \left(3xy + \frac{x}{z^3}\right) j + \left(11 - \frac{2x}{z}\right) k$

$$\frac{\partial M}{\partial P} = \frac{\partial N}{\partial x}, \quad \frac{\partial N}{\partial z} = \frac{\partial P}{\partial y}, \quad \frac{\partial M}{\partial z} = \frac{\partial P}{\partial x}$$

$$\frac{\partial P}{x} = -\frac{6x^3y}{z^3}, \quad \frac{\partial M}{\partial z} = -\frac{6x^3y}{z^3}$$

$$\frac{\partial M}{\partial y} = 8y + \frac{3x^2}{z^3}, \quad \frac{\partial N}{\partial x} = 8y + \frac{3x^2}{z^2}$$

$$\frac{\partial P}{\partial z} = -\frac{3x^2y}{z^2}$$

Hence vectors are Conservative.

Q7 (a) : $z = \frac{x^2 - w}{y^4}, \; x = t^3 + 7 \; ; \; y = \cos(2t)$

$zw = 4t$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial t} + \frac{\partial z}{\partial w} \cdot \frac{\partial w}{\partial t}$$

$$= \left(\frac{2x}{y^4} \cdot 3t^2\right) + 4\left(-\frac{1}{y^5}(x^2 - w)\right) + (-\sin 2t)(2)$$

$$= -\frac{1}{y^4} \cdot 4$$

$$= \frac{6xt^2}{y^4} + \frac{8\sin 2t\,(x^2 - w)}{y^5} - \frac{4}{t^4}$$

$$= \frac{2}{y}\left(3xt^2 + \frac{4\sin 2t\,(x^2 - w)}{y^5} - 2\right)$$

(b) $z = x^2 y^4 - 2y \quad , \quad y = \sin(x^2)$

$$\frac{dz}{\partial x} = \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial x}$$

$$= (4y^3 x^2 - 2) \cdot \cos(x)) \cdot (2x)$$

$$= 8 y^3 x^2 \cos x^2 - 4x \cos x^2$$

Answer

End