# A Design Study Approach to Classical Control

Randal W. Beard Timothy W. McLain
Brigham Young University

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## Homework D.2

**(a)** Using the configuration variable $z$, write an expression for the kinetic energy of the system.

**(b)** Create an animation of the mass-spring-damper system in Matlab, Python, or Simulink. The input should be a variable $z$. Turn in a screen capture of the animation.

## Solution

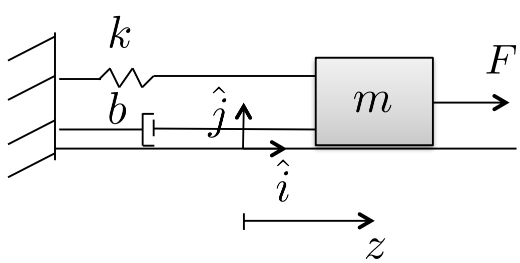


Figure 1: Computing the kinetic energy for the mass-spring-damper.

Define the inertial coordinate frame as in Figure 1, with $\hat{k}$ out of the page. The horizontal position of the mass $m$ is given by

$$\mathbf{p} = \begin{pmatrix} z(t) \\ 0 \\ 0 \end{pmatrix}.$$

Differentiating to obtain the velocity of $m$

$$\mathbf{v} = \begin{pmatrix} \dot{z} \\ 0 \\ 0 \end{pmatrix}.$$

Since there is no rotational motion of the mass, the kinetic energy of the system is given by

$$\begin{aligned} K &= \frac{1}{2} m \mathbf{v}^\top \mathbf{v} \\ &= \frac{1}{2} m \begin{pmatrix} \dot{z} & 0 & 0 \end{pmatrix} \begin{pmatrix} \dot{z} \\ 0 \\ 0 \end{pmatrix} \\ &= \frac{1}{2} m \dot{z}^2. \end{aligned} \tag{1}$$