A Design Study Approach to Classical Control

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Homework B.17

For this problem, use the gains found in HW B.10.

(a) For the inner loop of the inverted pendulum, use the Matlab `bode` and `margin` commands to find the phase and gain margin for the inner loop system under PD control. On the same graph, plot the open loop Bode plot and the closed loop Bode plot. What is the bandwidth of the inner loop, and how does this relate to the crossover frequency?

(b) For the outer loop of the inverted pendulum, use the Matlab `bode` and `margin` commands to find the phase and gain margin for the outer loop system under PID control. Plot the open and closed loop Bode plots for the outer loop on the same plot as the open and closed loop for the inner loop. What is the bandwidth of the outer loop, and how does this relate to the crossover frequency?

(c) What is the bandwidth separation between the inner (fast) loop, and the outer (slow) loop. For this design, is successive loop closure justified?

Solution

The Matlab code used to generate the plots is shown below.

```
1  % transfer function for inverted pendulum
2  P_in  = tf([-2/P.m2/P.ell],...
3             [1, 0, -2*(P.m1+P.m2)*P.g/P.m2/P.ell]);
4  P_out = tf([-P.m1*P.g/P.m2],[1, P.b/P.m2, 0]);
5  C_in = tf([(P.kd_th+P.sigma*P.kp_th), P.kp_th], [P.sigma, 1]);
6  C_out = tf([(P.kd_z+P.kp_z*P.sigma),...
7              (P.kp_z+P.ki_z*P.sigma),P.ki_z],[P.sigma,1,0]);
8
9  % margin and bode plots
10 figure(1), clf, margin(P_in*C_in), grid on, hold on
11 bode(P_in*C_in/(1+P_in*C_in))
12 margin(P_out*C_out)
13 bode(P_out*C_out/(1+P_out*C_out))
14 legend('Open Loop-Inner', 'Closed Loop-Inner',...
15        'Open Loop-Outer', 'Closed Loop-Outer')
```

The transfer functions for the inner and outer loop plants and controller are defined in Lines 2–7. For this problem we plot both the inner and outer loop frequency response on the same Bode plot, as implemented in Lines 10–15. The results of this code are shown in Figure 1.

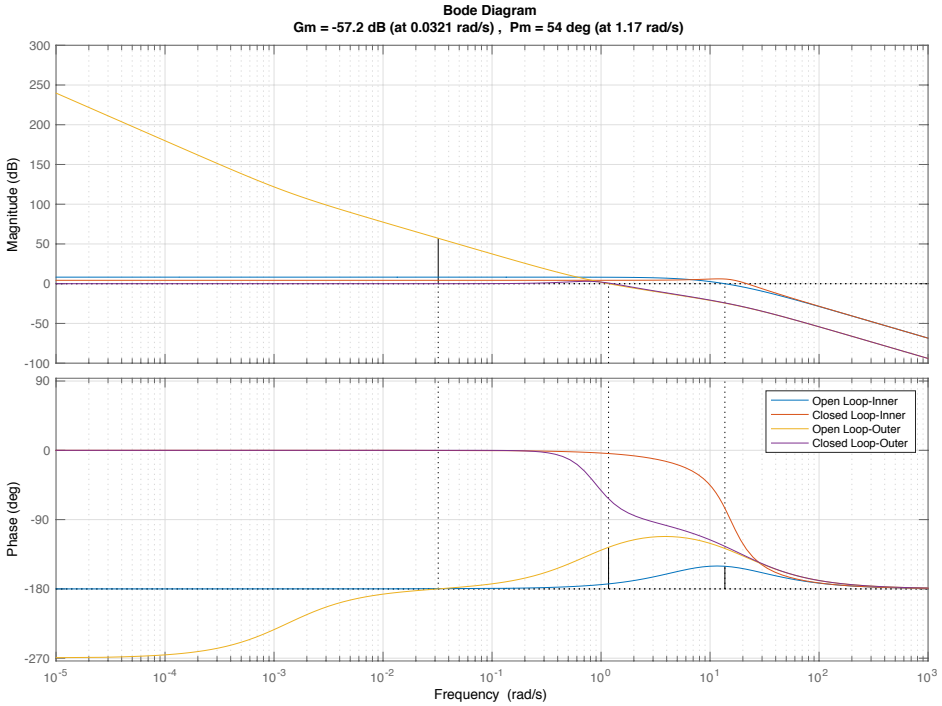

Figure 1: The `margin` and `bode` plots for the the open and closed loop systems of both the inner and outer loops of the inverted pendulum system.

As seen from Figure 1 the bandwidth of the inner loop is approximately 22 rad/sec, which is slightly larger than the cross over frequency of 14 rad/sec. Similarly, Figure 1 indicates that the bandwidth of the outer loop is approximately 1.4 rad/sec, which is slightly smaller than the cross over frequency of 1.2 rad/sec with a phase margin of $PM = 54$ degrees.

The bandwidth separation between the inner and outer loop is about one decade justifying the successive loop closure design approach.