A Design Study Approach to Classical Control

Randal W. Beard   Timothy W. McLain
Brigham Young University

Updated: December 28, 2020

Homework D.16

For the mass spring, use the `bode` command (from Python or Matlab) to create a graph that simultaneously displays the Bode plots for (1) the plant, and (2) the plant under PID control, using the control gains calculated in Homework D.10.

(a) What is the tracking error to a unit ramp under PID control?

(b) If the frequency content of the input disturbance $d_{in}(t)$ is below $\omega_{d_{in}} = 0.1$ radians per second, what percentage of the input disturbance shows up in the output z under PID control?

(c) If all of the frequency content of the noise $n(t)$ is greater than $\omega_{no} = 100$ radians per second, what percentage of the noise shows up in the output signal z?

Solution

The Bode plot of the plant $P(s)$, and the loop gain with PD control $P(s)C_{PD}(s)$, and the loop gain with PID control $P(s)C_{PID}(s)$ are shown in Figure 1.

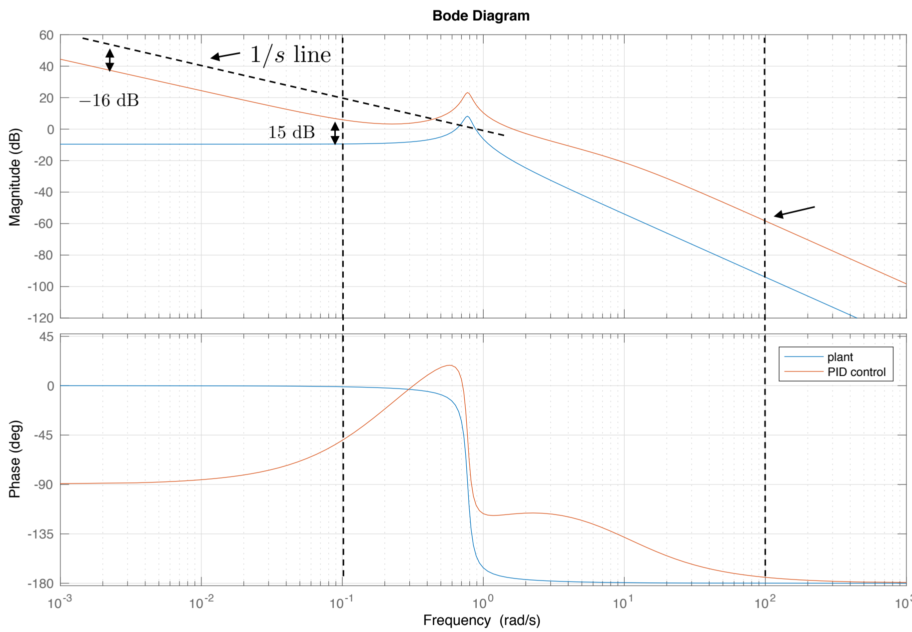

Figure 1: Bode plot for mass spring damper, plant only, and under PID control.

(a) From Figure 1 we see that as $\omega \to 0$, the Bode magnitude for PID control satisfies $20\log_{10}|P(j\omega)C(j\omega)| - 20\log_{10}\left|\frac{1}{j\omega}\right| \to B_1 = -16$ dB. Therefore, the steady state tracking error is

$$\lim_{t\to\infty} |e(t)| = \frac{1}{M_v},$$

where $M_v = 10^{B_1/20} = 0.1585$, which implies that the steady state error is 6.3096.

(b) For $\omega \leq \omega_{d_{in}} = 0.1$ rad/s, we have

$$20\log_{10}|P(j\omega)C_{PD}(j\omega)| - 20\log_{10}|P(j\omega)| \to B_{d_{in}} = 15 \text{ dB}.$$

Therefore,

$$\gamma_{d_{in}} = 10^{-15/20} = 0.1778,$$

implying that 18% of the input disturbance will show up in the output.

(c) For $\omega \geq \omega_n = 100$ rad/sec, we see from Figure 1 that $B_{no} = -58$ dB. Therefore, $\gamma_n = 10^{-58/20} = 0.0013$ which implies that 0.13% of the noise will show up in the output signal.