A Design Study Approach to Classical Control

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Homework D.8

For the mass spring, do the following:

(a) Suppose that the design requirements are that the rise time is $t_r \approx$ 2 seconds, with a damping ratio of $\zeta = 0.7$. Find the desired closed loop characteristic polynomial $\Delta_{cl}^d(s)$, and the associated pole locations. Find the proportional and derivative gains k_P and k_D to achieve these specifications, and modify the simulation from HW D.7 to verify that the step response satisfies the requirements.

(b) Suppose that the size of the input force is limited to $F_{\max} = 6$ N. Modify the simulation to include a saturation block on the force F. Using the rise time t_r as a tuning parameter, tune the PD control gains so that the input just saturates when a step of size of 1 meter is placed on z^r. Plot the step response showing that saturation does not occur for a 1 meter step input for the new control gains.

Solution

A rise time of $t_r \approx 2$ seconds, implies that the natural frequency is

$$\omega_n = 2.2/2 = 1.1.$$

Therefore, the desired characteristic polynomial is

$$\Delta_{cl}^d = s^2 + 2\zeta\omega_n s + \omega_n^2 = s^2 + 1.54s + 1.21. \tag{1}$$

From HW D.7 the actual closed loop characteristic polynomial is

$$\Delta_{cl}(s) = s^2 + (0.1 + 0.2k_D)s + (0.6 + 0.2k_P). \tag{2}$$

Equating Equations (1) and (2) and solving for the gains gives

$$k_P = 3.05$$
$$k_D = 7.2.$$

For the gains selected above, we can see that saturation does not occur if we limit the maximimum control force to be 6 N. By lowering the rise time requirement slightly, we get a higher proportional control gain. As it turns out, if k_P is less than or equal to 6, saturation will not occur. For tuning your control design to avoid saturation, you can use a "guess and check" approach by varying the desired rise time and checking to see if saturation occurs in your simulation, or you can follow the more rigorous approach below.

Rather than using the rise time design requirement to choose the proportional gain k_P, the gain can instead be chosen using the saturation constraint. At the instant a step of 1 m is applied, there error will be 1 m. If the control force is limited to be 6 N, then the maximum value for the control gain is $k_P = 6$ N/m.

We must now find the value of k_D to ensure $\zeta = 0.7$. Recall that the characteristic polynomial for canonical second-order system is given by

$$\Delta_{\text{2nd}}(s) = s^2 + 2\zeta\omega_n s + \omega_n^2. \tag{3}$$

Comparing coefficients of Equations (3) and (2), we find that for $k_P = 6$, $\omega_n = \sqrt{1.8}$ rad/s. Equating s coefficients, we have

$$2(0.7)\sqrt{1.8} = 1.878 = 0.1 + 0.2k_D,$$

giving $k_D = 8.89$.

The code files associated with this problem is included on the wiki associated with the book.