A Design Study Approach to Classical Control

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Homework F.16

For the altitude hold loop of the VTOL system, use the `bode` command (from Matlab or Python) to create a graph that simultaneously displays the Bode plots for (1) the plant, and (2) the plant under PID control using the control gains calculated in Homework F.10.

(a) What is the tracking error if the reference input is a parabola with curvature 5?

(b) If all of the frequency content of the noise $n(t)$ is greater than $\omega_{no} = 30$ radians per second, what percentage of the noise shows up in the output signal h?

For the inner loop of the lateral controller for the VTOL system, use the `bode` command to create a graph that simultaneously displays the Bode plots for (1) the plant, and (2) the plant under PD control, using the control gains calculated in Homework F.8.

(c) If the frequency content of the input disturbance is all contained below $\omega_{d_{in}} = 2$ radians per second, what percentage of the input disturbance shows up in the output?

(d) If a sensor for θ is to be selected, what are the characteristics of the sensor (frequency band and size) that would result in measurement noise for θ that is less than 0.1 degrees?

For the outer loop of the lateral controller for the VTOL system, use the `bode` command to create a graph that simultaneously displays the Bode plots for (1) the plant, and (2) the plant under PID control, using the control gains calculated in Homework F.10.

(e) To what percent error can the closed loop system track the desired input if all of the frequency content of $z_r(t)$ is below $\omega_r = 0.1$ radians per second?

(f) If the frequency content of an output disturbance is contained below $\omega_{d_{out}} = 0.01$ radian/sec, what percentage of the output disturbance will be contained in the output?

Solution

The Bode plot of the altitude loop $P_{lon}(s)$ and the loop gain with PID control $P_{lon}(s)C_{PID}(s)$,are shown in Figure 1.

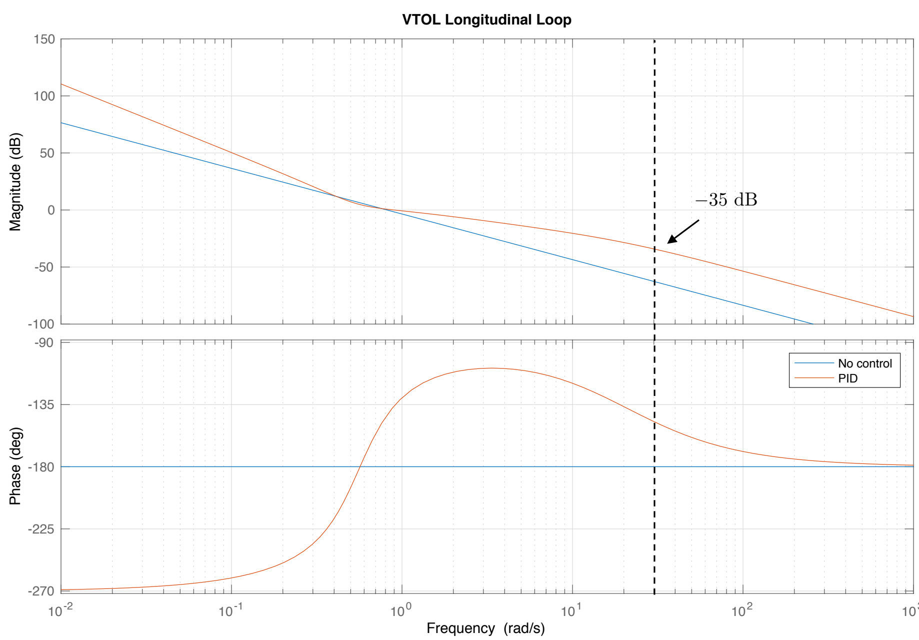

Figure 1: Bode plot for altitude loop of the VTOL system, plant only and under PID control.

(a) For PID control, the slope of the Bode plot is -60 dB/dec as $\omega \to 0$. Therefore, the system is type 3 and will track a parabola with zero steady state error.

(b) For $\omega \geq \omega_{no} = 30$ rad/sec, we see from Figure 1 that $B_{no} = -35$ dB. Therefore, $\gamma_n = 10^{-35/20} = 0.0178$ which implies that 1.78% of the noise will show up in the output signal.

The Bode plot of the lateral inner loop $P_{lat,in}(s)$, and the loop gain with PD control $P_{lat,in}(s)C_{PD}(s)$, are shown in Figure 2.

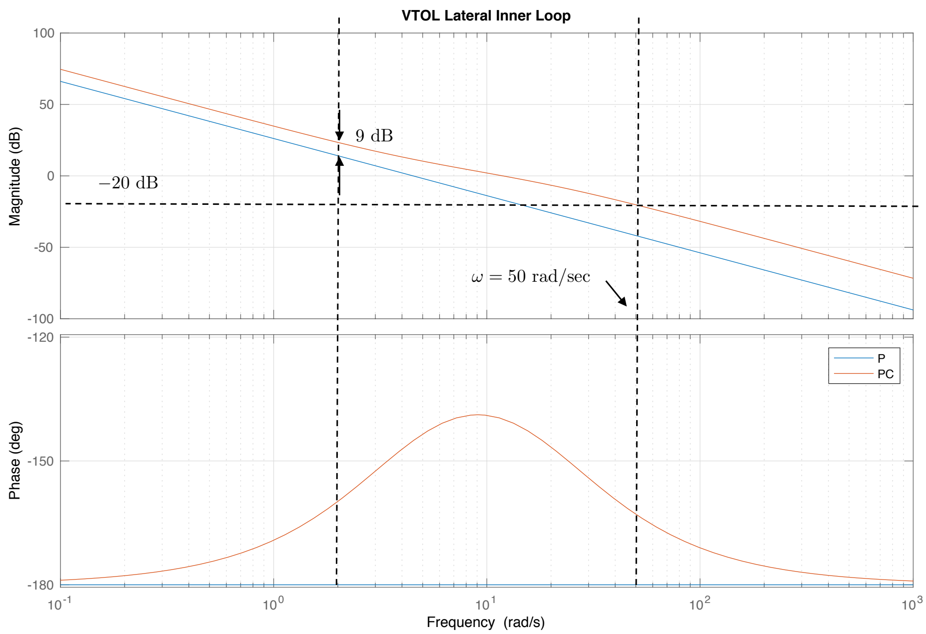

Figure 2: Bode plot for lateral inner loop of the VTOL system, plant only, and under PD control.

(c) From Figure 2 we see that for $\omega \leq \omega_{d_{in}} = 2$ dB, the Bode magnitude plot satisfies

$$20\log_{10}|P(j\omega)C(j\omega)| - 20\log_{10}|P(j\omega)| \geq B_{d_{in}} = 9 \text{ dB}.$$

Therefore, the contribution of the input disturbance to the error satisfies

$$|e(t)| \leq 10^{-9/20}\,|d_{in}(t)| = 0.3548\,|d_{in}(t)|\,,$$

which implies that 35% of the input disturbance shows up in the output θ.

(d) For an error of 0.1 degrees we need

$$|y(t)| \leq 10^{B_{no}/20} |n(t)| = 0.1 \text{ degrees}$$

If the sensor can produce 1 degree of noise, then we need $B_{no} = -20$ dB. From Figure 2 this would require that the frequency content of the noise should be greater than or equal to $\omega_n = 50$ rad/sec.

The Bode plot of the lateral outer loop $P_{lat,out}(s)$, and the loop gain with PD control $P_{lat,out}(s)C_{PD}(s)$, are shown in Figure 3.

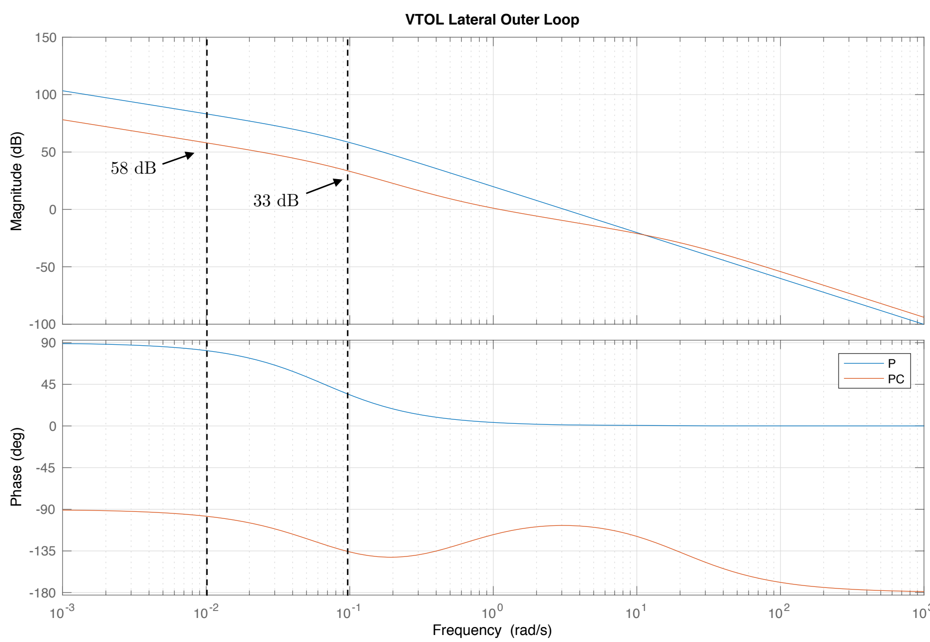

Figure 3: Bode plot for lateral outer loop of the VTOL system, plant only, and under PD control.

(e) From Figure 3 we see that for $\omega < \omega_r = 0.1$ rad/sec, the loop gain is greater than $B_r = 33$ dB. Therefore, the tracking error satisfies

$$|e(t)| \leq 10^{-33/20} |r(t)| = 0.0224 |r(t)| ,$$

which implies that tracking accuracy is to within 2.24%.

(f) For $\omega \leq \omega_{d_{out}} \leq 0.01$ rad/sec, we see from Figure 3 that $B_{d_{out}} = 58$ dB. Therefore, $\gamma_{d_{in}} = 10^{-58/20} = 0.0013$, which implies that 0.13% of the output disturbance will show up in the output signal.