

A Design Study Approach to Classical Control

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Homework E.18

For this homework assignment we will use the loopshaping design technique to design a successive loop closure controller for the ball and beam problem.

- (a) First consider the inner loop, where $P_{in}(s)$ is the transfer function of the inner loop derived in HW [E.5](#). Design the inner control system $C_{in}(s)$ to stabilize the system with a phase margin close to 60 degrees, and to ensure that reference inputs with frequency below $\omega_r = 1$ radians/sec have tracking error $\gamma_r = 0.0032$, and to ensure that noise above $\omega_n = 1000$ radians/second is rejected by $\gamma_n = 0.0032$.
- (b) Now consider the design of the controller for the outer loop system. The 'plant' for the design of the outer loop controller is

$$P = P_{out} \frac{P_{in} C_{in}}{1 + P_{in} C_{in}}.$$

Design the outer loop controller C_{out} so that the system is stable with phase margin close to $PM = 60$ degrees, and so that constant input disturbances are rejected, reference signals with frequency below $\omega_r = 0.1$ radians/sec are tracked with error $\gamma_r = 0.01$, and noise with frequency content above $\omega_n = 100$ radians/sec are rejected with $\gamma_n = 0.001$. Add a prefilter to reduce peaking in the closed loop transfer function.

Solution

Figure 1 shows the Bode plot of the plant $P_{in}(s)$ together with the design specification on tracking and noise attenuation.

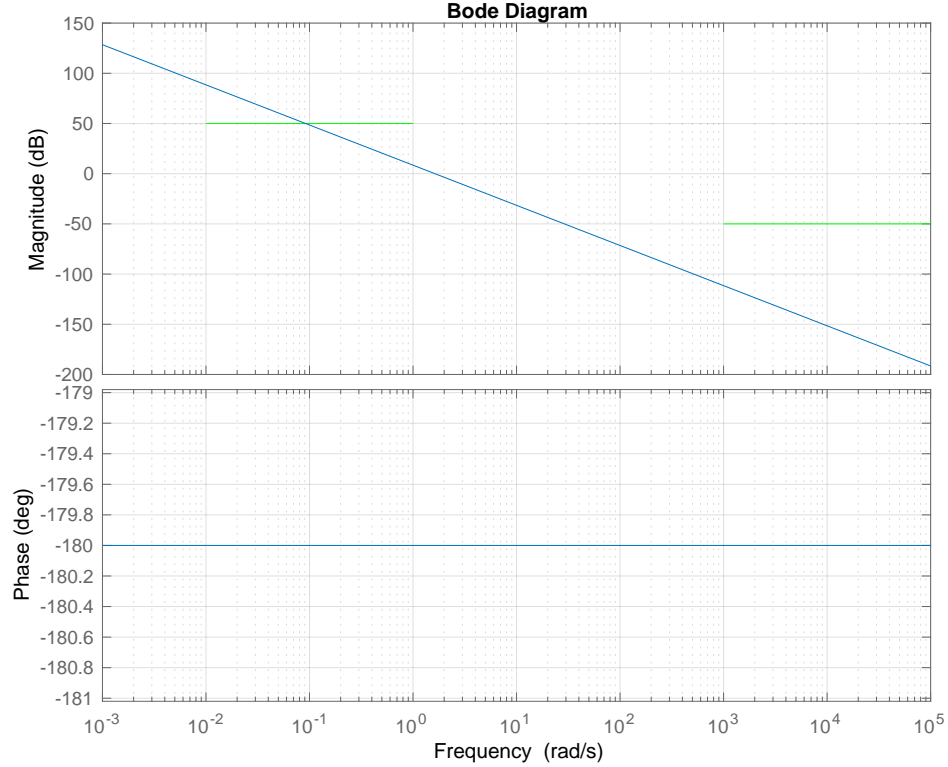


Figure 1: The Bode plot for the inner loop plant in HW E.18, together with the design specification.

The first step is to add a proportional gain so that the open loop response below $\omega_r = 1$ radians/sec is above $20 \log_{10} \gamma_r^{-1} = 50$ dB. Figure 2 shows the corresponding Bode plot for a proportional gain of $k_p = 150$.

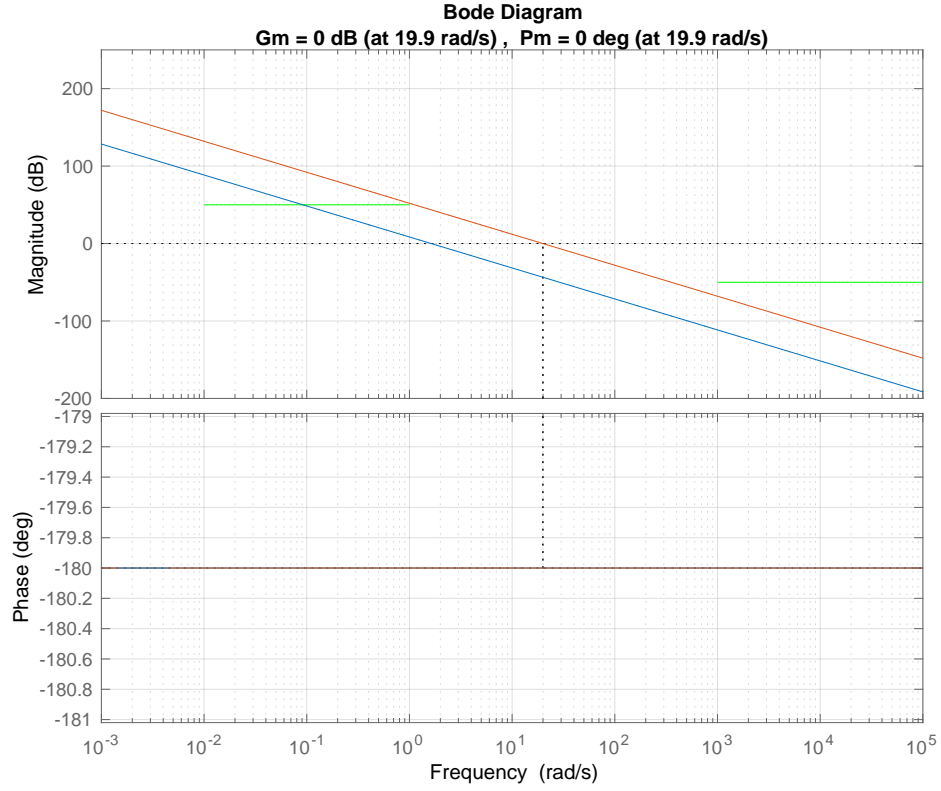


Figure 2: The Bode plot for the inner loop plant in HW E.18, with proportional gain.

To stabilize the system, a phase lead compensator needs to be added around the cross over frequency. Figure 3 shows the addition of the phase lead filter

$$C_{lead} = \frac{15(s + \frac{40}{\sqrt{15}})}{(s + \frac{40}{\sqrt{15}})},$$

which has a phase margin of $PM = 61$ degrees.

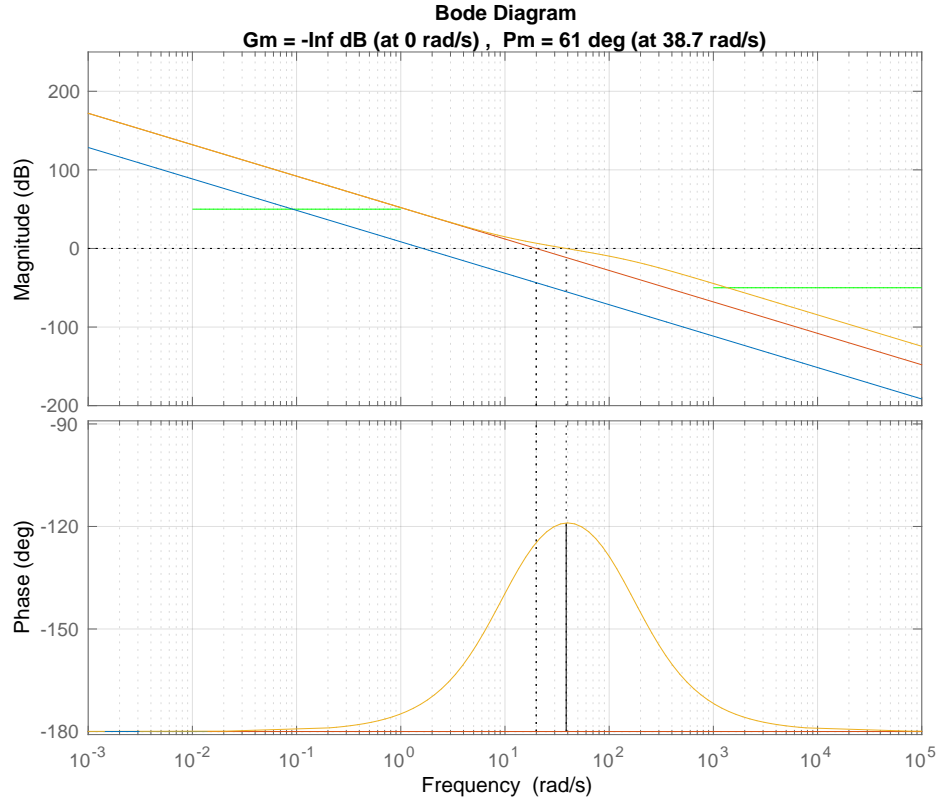


Figure 3: The Bode plot for the inner loop system in HW [E.18](#), with proportional gain and phase lead compensation.

The noise specification is met by adding the low pass filter

$$C_{lpf} = \frac{500}{s + 500}$$

as shown in Figure [4](#).

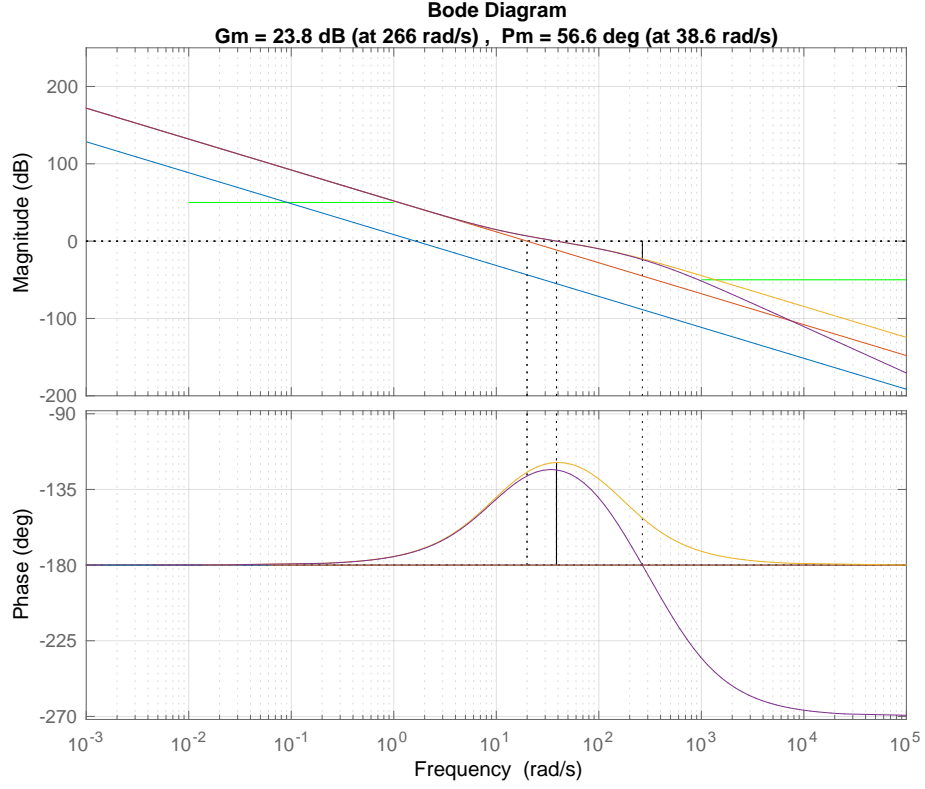


Figure 4: The Bode plot for the inner loop system in HW E.18, with proportional gain, phase lead compensation, and a low pass filter.

The compensator for the inner loop is given by

$$C_{in}(s) = 150 \left(\frac{15(s + \frac{40}{\sqrt{15}})}{(s + \frac{40}{\sqrt{15}})} \right) \left(\frac{500}{s + 500} \right).$$

The closed loop response for the inner subsystem

$$\frac{P_{in}C_{in}}{1 + P_{in}C_{in}},$$

as well as the unit step response for the output and control signal are all show in Figure 5.

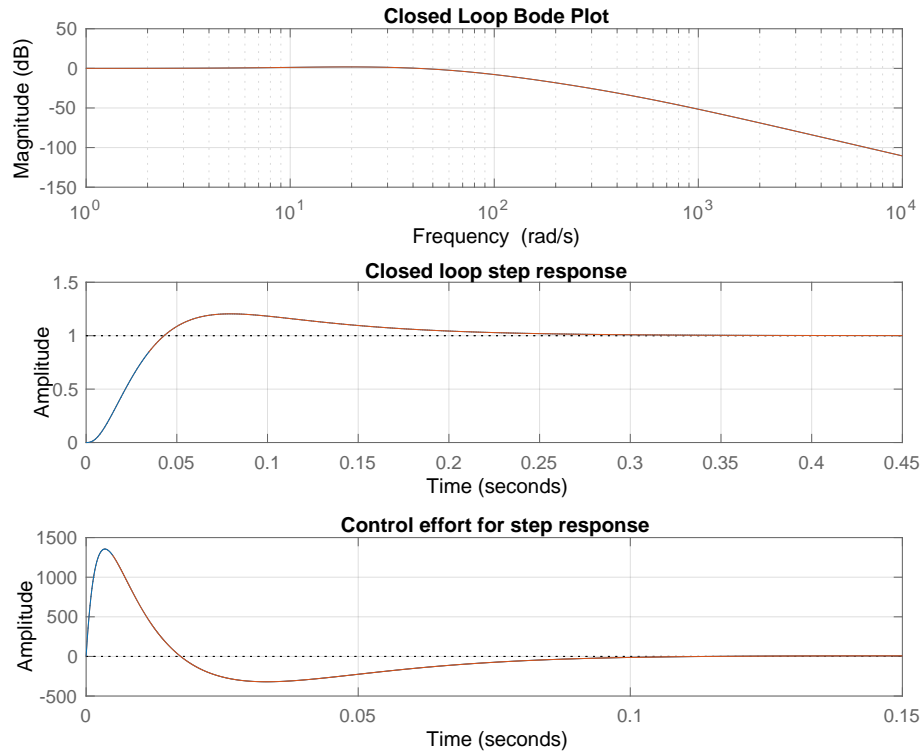


Figure 5: The closed loop bode response, the unit step response for the output, and the unit step response for the input of the inner loop design in HW E.18.

The Matlab code used to design the inner loop is shown below.

```

1 Plant = P_in;
2
3 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
4 % Define Design Specifications
5 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
6
7 %—— general tracking specification ——
8     omega_r = 10^0; % track signals below this frequency
9     gamma_r = 10^(-50/20); % tracking error below this value
10    w = logspace(log10(omega_r)-2,log10(omega_r));
11
12 %—— noise specification ——

```

```

13     omega_n = 10^3; % attenuate noise above this frequency
14     gamma_n = 10^(-50/20); % attenuate noise by this amount
15     w = logspace(log10(omega_n),2+log10(omega_n));
16     plot(w,20*log10(gamma_n)*ones(size(w)),'g')
17
18     %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
19     % Control Design
20     C = tf(1,1);
21     %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
22
23     % proportional control: change cross over frequency
24     kp = 150;
25     C = C*kp;
26
27     % phase lead: increase PM (stability)
28     wmax = 40; % location of maximum frequency bump
29     M = 15; % separation between zero and pole
30     Lead =tf(M*[1,wmax/sqrt(M)],[1,wmax*sqrt(M)]);
31     C = C*Lead;
32
33     % low pass filter: decrease gain at high frequency (noise)
34     p = 500;
35     LPF = tf(p,[1,p]);
36     C = C*LPF;
37
38     %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
39     % Convert controller to state space equations
40     %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
41     [num,den] = tfdata(C,'v');
42     [P.Ain_C,P.Bin_C,P.Cin_C,P.Din_C]=tf2ss(num,den);
43     C_in = C;

```

For the outer loop design, Figure 6 shows the Bode plot of the plant $P = P_{out} \frac{P_{in} C_{in}}{1 + P_{in} C_{in}}$ together with the design specification on reference tracking and noise attenuation. To reject constant input disturbances, we need to add an integrator to the controller.

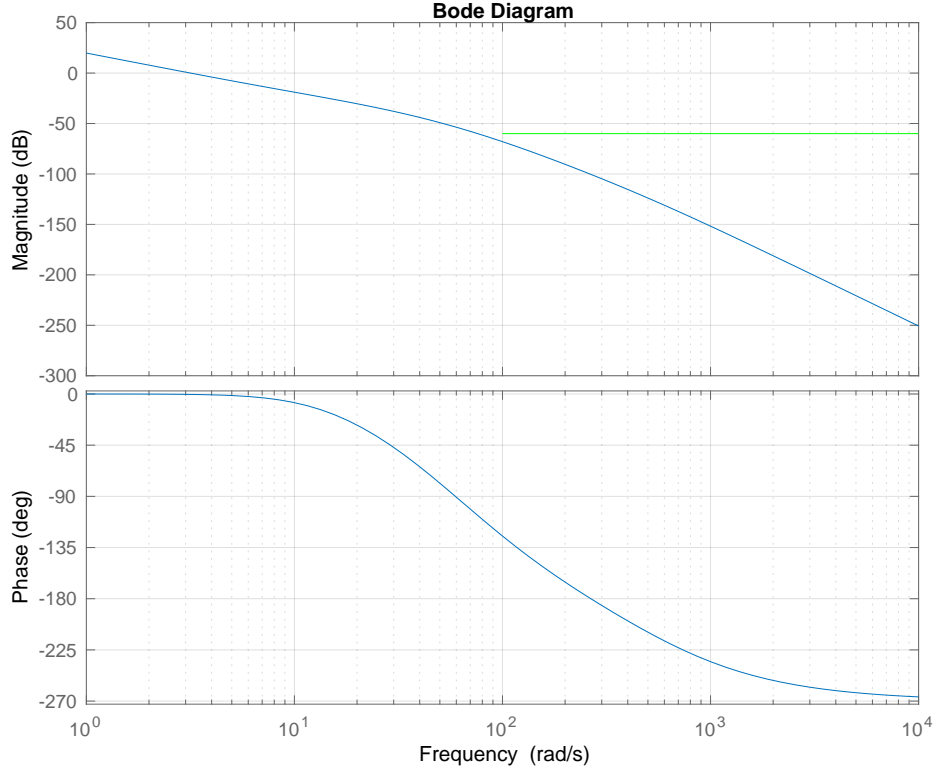


Figure 6: The Bode plot for the outer loop plant in HW E.18, together with the design specification.

The first step is to add a negative proportional gain to lower the bandwidth to reduce the size of the control input. The result is shown in Figure 7 where $k_p = -0.1$.

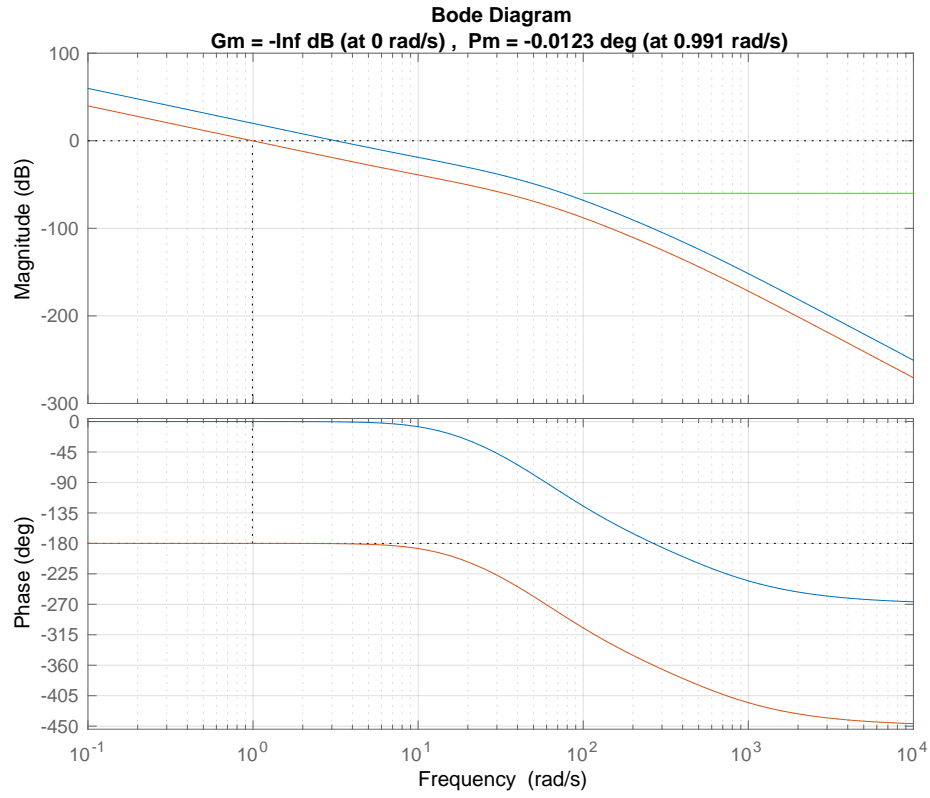


Figure 7: The Bode plot for the outer loop system in HW E.18, with negative proportional gain.

To reject constant input disturbances, integral control is added. Integral control is also used to satisfy the tracking specification. Figure 8 shows the loopgain after adding

$$C_{int} = \frac{s + 0.3}{s}.$$

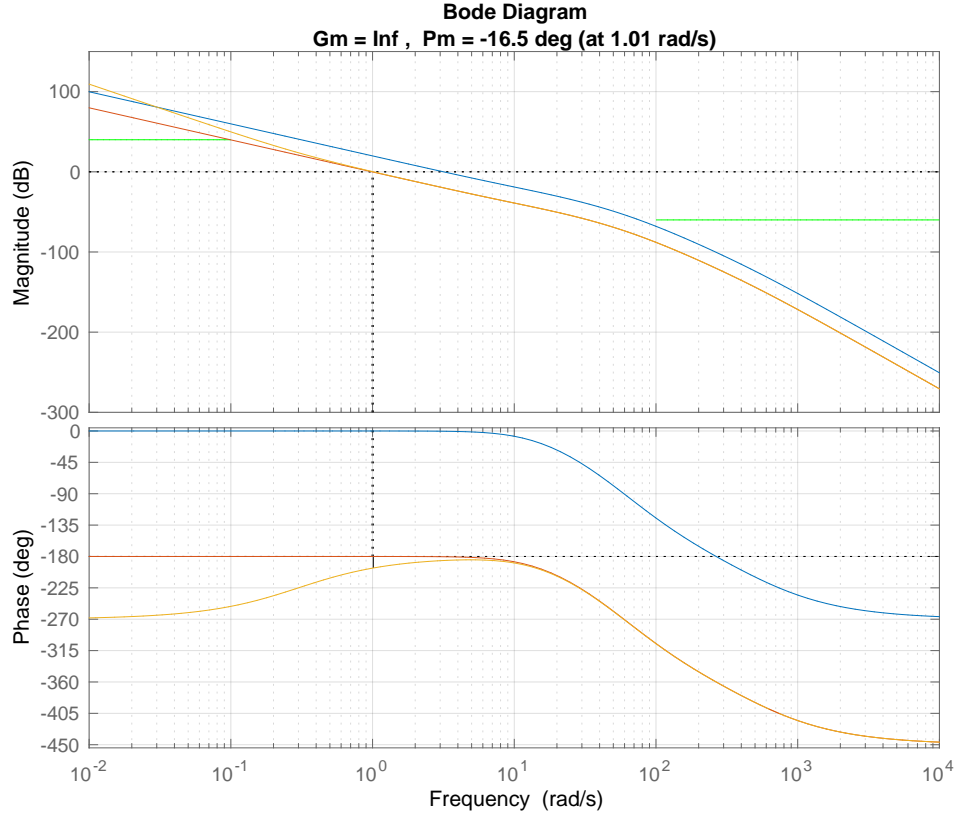


Figure 8: The Bode plot for the outer loop system in HW E.18, with proportional and integral control.

Note that the phase margin is negative which implies that the closed loop system is currently unstable. To stabilize the system, a phase lead compensator needs to be added around the cross over frequency. Figure 4 shows the addition of the phase lead filter

$$C_{lead} = \frac{25(s + \frac{2}{\sqrt{25}})}{(s + \frac{2}{\sqrt{25}})},$$

which results in a phase margin of $PM = 59.8$ degrees.

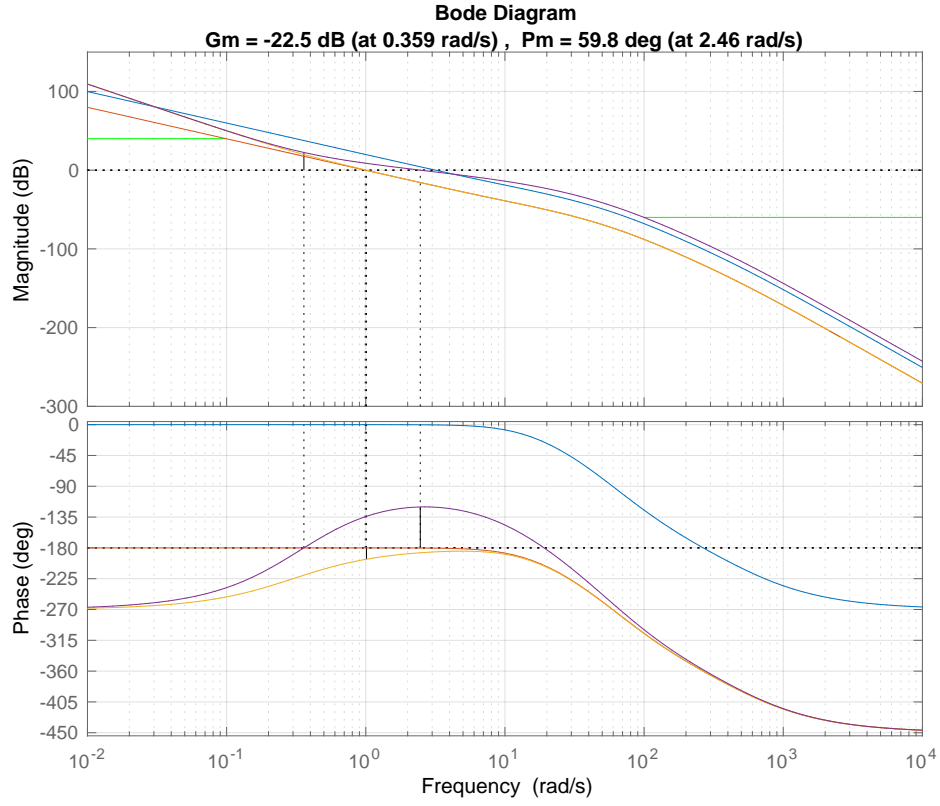


Figure 9: The Bode plot for the outer loop system in HW E.18, with proportional gain, integral control, and phase lead compensation.

To meet the noise specification, the low pass filter

$$C_{lpf} = \frac{50}{s + 50}$$

is added to the compensator, and the resulting loopgain is shown in Figure 10.

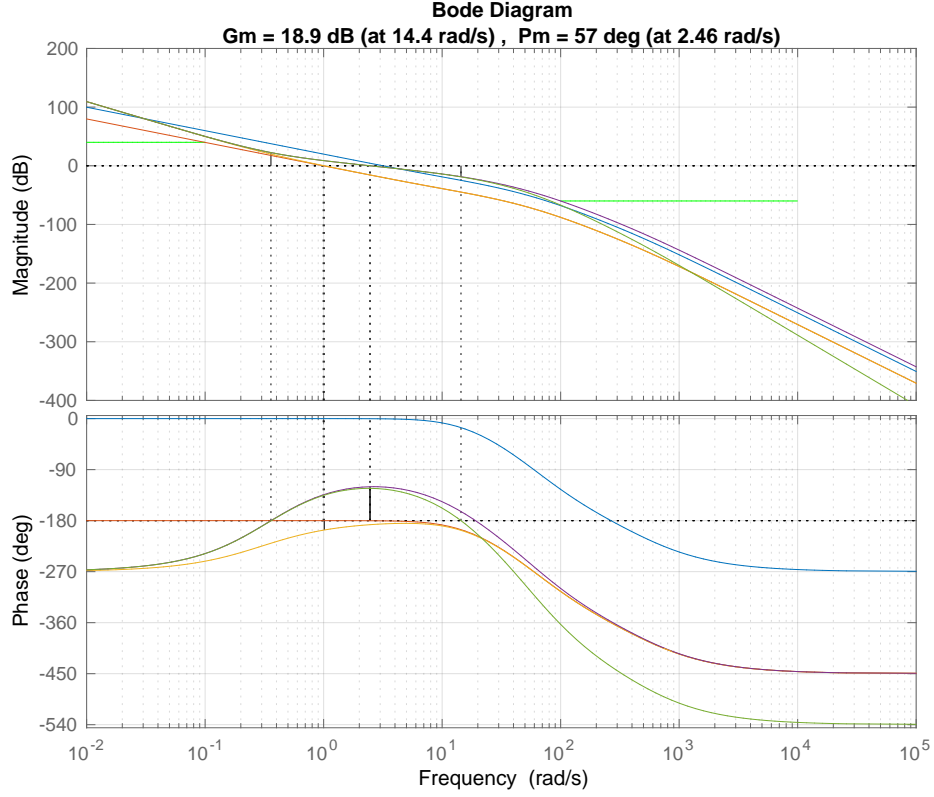


Figure 10: The Bode plot for the outer loop system in HW E.18, with proportional gain, integral control, phase lead compensation, and a low pass filter.

The resulting compensator is

$$C_{out}(s) = -0.1 \left(\frac{s + 0.3}{s} \right) \left(\frac{25(s + \frac{2}{\sqrt{25}})}{(s + \frac{2}{\sqrt{25}})} \right) \left(\frac{50}{s + 50} \right).$$

The closed loop response for both the inner and outer loop systems, as well as the unit step response for the output and control signal of the outer loops are all show in Figure 11.

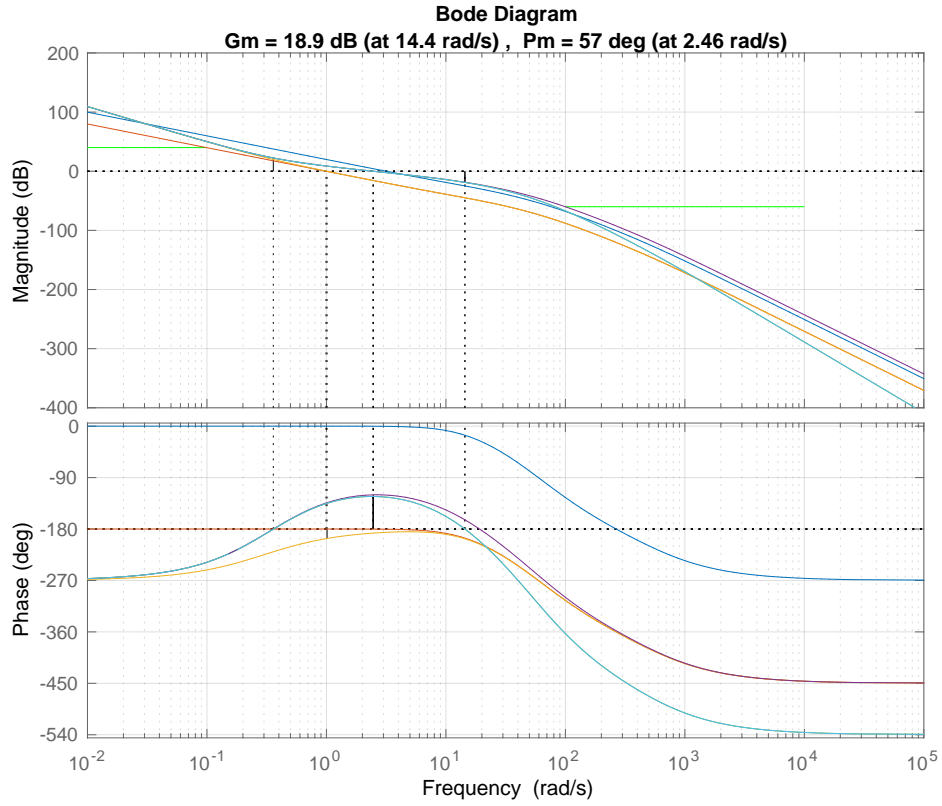


Figure 11: The closed loop bode response, the unit step response for the output, and the unit step response for the input of the inner loop design in HW E.18.

The Matlab code used to design the outer loop is shown below.

```

1 Plant = minreal(P_out*(P_in*C_in/(1+P_in*C_in)));
2
3 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
4 % Define Design Specifications
5 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
6
7 %—— general tracking specification ——
8 omega_r = 10^-1; % track signals below this frequency
9 gamma_r = 10^(-40/20); % tracking error below this value
10 w = logspace(log10(omega_r)-2,log10(omega_r));

```

```

11
12 %—— noise specification ——
13 omega_n = 10^2; % attenuate noise above this frequency
14 gamma_n = 10^(-60/20); % attenuate noise by this amount
15 w = logspace(log10(omega_n),2+log10(omega_n));
16
17 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
18 % Control Design
19 C = tf(1,1);
20 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
21
22 % proportional control: change cross over frequency
23 kp = -0.1;
24 C = C*kp;
25
26 % integral control: increase steady state tracking and dist rejection
27 k_I = .3; % frequency at which integral action ends
28 Integrator = tf([1,k_I],[1,0]);
29 C = C*Integrator;
30
31 % phase lead: increase PM (stability)
32 wmax = 2; % location of maximum frequency bump
33 M = 25; % separation between zero and pole
34 Lead = tf(M*[1,wmax/sqrt(M)], [1,wmax*sqrt(M)]);
35 C = C*Lead;
36
37 % low pass filter: decrease gain at high frequency (noise)
38 p = 50;
39 LPF = tf(p, [1,p]);
40 C = C*LPF;
41
42 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
43 % Prefilter Design
44 F = tf([1],[1]);
45 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
46
47 % low pass filter
48 p = 1; % frequency to start the LPF
49 LPF = tf(p, [1,p]);
50 F = F*LPF;
51
52
53 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Convert controller t
54 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
55 C=minreal(C);

```

```
56 [num,den] = tfdata(C,'v');
57 [P.Aout_C,P.Bout_C,P.Cout_C,P.Dout_C]=tf2ss(num,den);
58
59 [num,den] = tfdata(F,'v');
60 [P.Aout_F, P.Bout_F, P.Cout_F, P.Dout_F] = tf2ss(num,den);
61
62 C_out=C;
```

The complete Simulink files are on the wiki associated with the book.