A Design Study Approach to Classical Control

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Homework A.3

(a) Find the potential energy for the system.

(b) Define the generalized coordinates.

(c) Find the generalized forces and damping forces.

(d) Derive the equations of motion using the Euler-Lagrange equations.

Solution

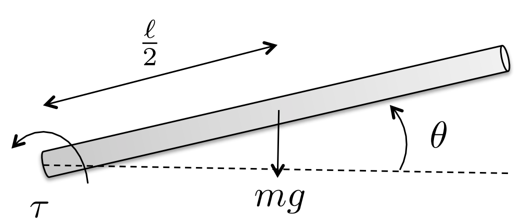

Figure 1: Energy calculation for the single link robot arm

Let P_0 be the potential when $\theta = 0$. The height of the center of mass is $\ell/2 \sin\theta$, which is zero when $\theta = 0$. Accordingly, the potential energy is given by

$$P = P_0 + mg\frac{\ell}{2}\sin\theta.$$

The generalized coordinate is

$$q_1 = \theta.$$

The generalized force is

$$\tau_1 = \tau,$$

and the generalized damping is

$$-B\dot{\mathbf{q}} = -b\dot{\theta}.$$

From Homework **??**.1 we found that the kinetic energy is given by

$$K = \frac{1}{2}\left(\frac{m\ell^2}{3}\right)\dot{\theta}^2.$$

Therefore the Lagrangian is

$$L = K - P = \frac{1}{2}\left(\frac{m\ell^2}{3}\right)\dot{\theta}^2 - P_0 + mg\frac{\ell}{2}\sin\theta.$$

For this case, the Euler-Lagrange equation is given by

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = \tau_1 - b\dot{\theta}$$

where

$$\begin{aligned}\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) &= \frac{d}{dt}\left(\frac{m\ell^3}{3}\dot{\theta}\right) = \frac{m\ell^3}{3}\ddot{\theta} \\ \frac{\partial L}{\partial \theta} &= -mg\frac{\ell}{2}\cos\theta.\end{aligned}$$

Therefore, the Euler Lagrange equation becomes

$$\frac{m\ell^2}{3}\ddot{\theta} + mg\frac{\ell}{2}\cos\theta = \tau - b\dot{\theta}. \tag{1}$$