A Design Study Approach to Classical Control

Randal W. Beard Timothy W. McLain
Brigham Young University

Updated: December 28, 2020

Homework D.5

For the equations of motion for the mass spring damper,

(a) Using the Laplace transform, convert from time domain to the s-domain.

(b) Find the transfer function from the input force F to the mass position z.

(c) Draw the associated block diagram.

Solution

As shown in homework D.3, the equations of motion for the mass-spring-damper are given by

$$m\ddot{z} + b\dot{z} + kz = F \tag{1}$$

These equations are linear and do not require a linearization step prior to finding the corresponding transfer function. Taking the Laplace transform of Equation (1) and setting all initial conditions to zero we get

$$(ms^2 + bs + k)Z(s) = F(s).$$

Solving for $Z(s)$ and putting the transfer function in monic form gives

$$Z(s) = \left(\frac{\frac{1}{m}}{s^2 + \frac{b}{m}s + \frac{k}{m}} \right) F(s). \tag{2}$$

where the expression in the parenthesis is the transfer function from F to z. The block diagram associated with Equation (2) is shown in Figure 1

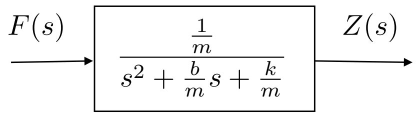

Figure 1: A block diagram of the mass spring damper system.