A Design Study Approach to Classical Control

Randal W. Beard   Timothy W. McLain
Brigham Young University

Updated: December 28, 2020

Homework F.7

For this problem we will only consider the longitudinal (up-down) dynamics of the VTOL system.

(a) Given the open-loop transfer function from total force $\tilde{F}$ to altitude $\tilde{h}$ found in homework F.5, find the open-loop poles of the system.

(b) Using PD control architecture shown in Figure **7-2**, find the closed-loop transfer function from $\tilde{h}_r$ to $\tilde{h}$ and find the closed-loop poles as a function of k_P and k_D.

(c) Select k_P and k_D to place the closed-loop poles at $p_1 = -0.2$ and $p_2 = -0.3$.

(d) Using the gains from part (c), implement the PD control for the altitude control in simulation and plot the step response.

Solution

The open-loop transfer function from homework F.5 is

$$\tilde{H}(s) = \left(\frac{\frac{1}{m_c + 2m_r}}{s^2} \right) \tilde{F}(s).$$

Using the parameters from Section F gives

$$\tilde{H}(s) = \left(\frac{0.667}{s^2}\right)\tilde{F}(s).$$

The open-loop poles are therefore the roots of the open loop polynomial

$$\Delta_{ol}(s) = s^2,$$

which are given by

$$p_{ol} = 0, 0.$$

Using PD control, the closed-loop system is therefore shown in Figure 1.

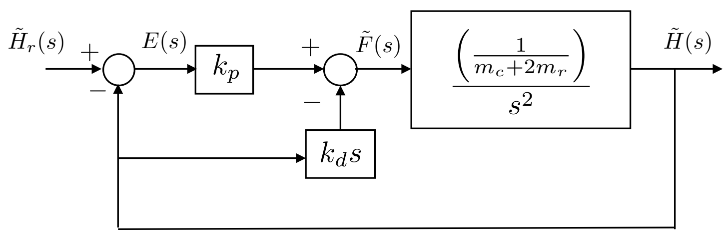

Figure 1: PD control of the altitude control for the planar VTOL system.

The transfer function of the closed-loop system is given by

$$\begin{aligned}
\tilde{H}(s) &= \left(\frac{0.667}{s^2}\right)\left(k_P(\tilde{H}_r(s) - \tilde{H}(s)) - k_D s\tilde{H}(s)\right) \\
\implies (s^2)\tilde{H}(s) &= \left(0.667k_P(\tilde{H}_r(s) - \tilde{H}(s)) - 0.667k_D s\tilde{H}(s)\right) \\
\implies (s^2 + 0.667k_D s + 0.667k_P)\tilde{H}(s) &= 0.667k_P\tilde{H}_r(s) \\
\implies \tilde{H}(s) &= \frac{0.667k_P}{s^2 + 0.667k_D s + 0.667k_P}\tilde{H}_r(s).
\end{aligned}$$

Therefore, the closed-loop poles are given by the roots of the closed-loop characteristic polynomial

$$\Delta_{cl}(s) = s^2 + 0.667k_D s + 0.667k_P.$$

If the desired closed-loop poles are at -0.3 and -0.2, then the desired closed loop characteristic polynomial is

$$\begin{aligned}
\Delta_{cl}^d &= (s + 0.3)(s + 0.2) \\
&= s^2 + 0.5s + 0.06.
\end{aligned}$$

Equating the actual closed-loop characteristic polynomial Δ_{cl} with the desired characteristic polynomial Δ_{cl}^d gives

$$s^2 + 0.667k_D s + 0.667k_P = s^2 + 0.5s + 0.06,$$

or by equating each term we get

$$\begin{aligned} k_P &= 0.09 \\ k_D &= 0.75. \end{aligned}$$

The Simulink implementation is contained on the wiki associated with this book.