A Design Study Approach to Classical Control

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Homework C.e

Adding an integrator to obtain PID control for the outer loop of the satellite system, put the characteristic equation in Evan's form and use the Matlab `rlocus` command to plot the root locus verses the integrator gain k_I. Select a value for k_I that does not significantly change the other locations of the closed loop poles.

Solution

The closed loop block diagram for the outer loop including an integrator is shown in Figure 1. The closed loop transfer function is given by

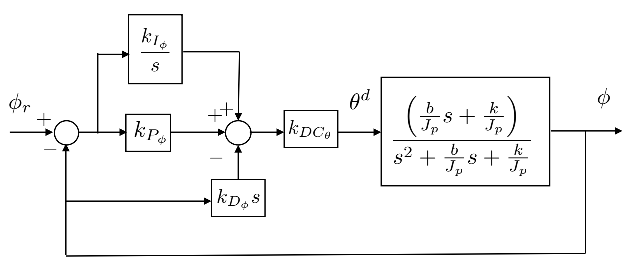

Figure 1: PID control for the outer loop of the satellite system.

$$\Phi(s) = \frac{b_2 s^2 + b_1 s + b_0}{a_3 s^3 + a_2 s^2 + a_1 s + a_0} \Phi_r(s),$$

where

$$
\begin{aligned}
b_2 &= \frac{bk_{DC_\theta}k_{P_\phi}}{J_p} \\
b_1 &= \frac{bk_{DC_\theta}k_{I_\phi}}{J_p} + \frac{kk_{DC_\theta}k_{P_\phi}}{J_p} \\
b_0 &= \frac{kk_{DC_\theta}k_{I_\phi}}{J_p} \\
a_3 &= 1 + \frac{bk_{DC_\theta}k_{D_\phi}}{J_p} \\
a_2 &= \frac{b}{J_p} + \frac{kk_{DC_\theta}k_{D_\phi}}{J_p} + \frac{bk_{DC_\theta}k_{P_\phi}}{J_p} \\
a_1 &= \frac{k}{J_p} + \frac{bk_{DC_\theta}k_{I_\phi}}{J_p} + \frac{kk_{DC_\theta}k_{P_\phi}}{J_p} \\
a_0 &= \frac{kk_{DC_\theta}k_{I_\phi}}{J_p}.
\end{aligned}
$$

The characteristic equation is therefore

$$
\begin{aligned}
\left(1 + \frac{bk_{DC_\theta}k_{D_\phi}}{J_p}\right) s^3 + \left(\frac{b}{J_p} + \frac{kk_{DC_\theta}k_{D_\phi}}{J_p} + \frac{bk_{DC_\theta}k_{P_\phi}}{J_p}\right) s^2 \\
+ \left(\frac{k}{J_p} + \frac{bk_{DC_\theta}k_{I_\phi}}{J_p} + \frac{kk_{DC_\theta}k_{P_\phi}}{J_p}\right) s + \frac{kk_{DC_\theta}k_{I_\phi}}{J_p} = 0.
\end{aligned}
$$

In Evan's form we have

$$
1+k_{I_\phi}\left(\frac{\frac{bk_{DC_\theta}}{J_p} s + \frac{kk_{DC_\theta}}{J_p}}{\left(1 + \frac{bk_{DC_\theta}k_{D_\phi}}{J_p}\right) s^3 + \left(\frac{b}{J_p} + \frac{kk_{DC_\theta}k_{D_\phi}}{J_p} + \frac{bk_{DC_\theta}k_{P_\phi}}{J_p}\right) s^2 + \left(\frac{k}{J_p} + \frac{kk_{DC_\theta}k_{P_\phi}}{J_p}\right) s}\right) = 0.
$$

The appropriate Matlab command is therefore

```
>> L = tf([b*kDC/Jp, k*kDC/Jp],...
          [1+b*kDC*kD/Jp, b/Jp+k*kDC*kD/Jp+b*kDC*kP/Jp,...
           k/Jp+k*kDC*kP/Jp, 0]);
>> figure(1), clf, rlocus(L);
```