A Design Study Approach to Classical Control

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Homework F.17

For this problem, use the gains found in Homework F.10.

(a) For the altitude hold loop of the VTOL system, use the `bode` and `margin` commands (from Matlab or Python) to find the phase and gain margin for the closed loop system under PID control. On the same graph, plot the open loop Bode plot and the closed loop Bode plot. What is the bandwidth of the closed loop system, and how does this relate to the crossover frequency?

(b) For the inner loop of the lateral control loop, use the `bode` and `margin` commands to find the phase and gain margin for the inner loop system under PD control. On the same graph, plot the open loop Bode plot and the closed loop Bode plot. What is the bandwidth of the closed loop system, and how does this relate to the crossover frequency?

(c) For the outer loop of the lateral control loop, use the `bode` and `margin` commands to find the phase and gain margin for the outer loop system under PD control. Plot the open and closed loop Bode plots for the outer loop on the same plot as the open and closed loop for the inner loop. What is the bandwidth of the closed loop system, and how does this relate to the crossover frequency?

(d) What is the bandwidth separation between the inner (fast) loop, and the outer (slow) loop? For this design, is successive loop closure justified?

Solution

The Matlab code used to generate the plots is shown below.

```
1  % transfer functions for plants and controllers
2  P_lon = tf([1/(P.mc+2*P.mr)],[1,0,0]);
3  P_lat_in = tf([1/(P.Jc+2*P.mr*P.d^2)],[1,0,0]);
4  P_lat_out = tf([-P.Fe/(P.mc+2*P.mr)],[1,P.mu/(P.mc+2*P.mr),0]);
5  C_lon = tf([(P.kd_h+P.kp_h*P.sigma),...
6              (P.kp_h+P.ki_h*P.sigma),P.ki_h],[P.sigma,1,0]);
7  C_lat_in = tf([(P.kd_th+P.sigma*P.kp_th), P.kp_th],...
8                [P.sigma, 1]);
9  C_lat_out = tf([(P.kd_z+P.sigma*P.kp_z), P.kp_z],...
10                [P.sigma, 1]);
11
12 % margin and bode plots for longitudinal controller
13 figure(1), clf, margin(P_lon*C_lon), grid on, hold on
14 bode(P_lon*C_lon/(1+P_lon*C_lon))
15 legend('Open Loop-Inner', 'Closed Loop-Inner')
16
17 % margin and bode plots for lateral controller
18 figure(2), clf, margin(P_lat_in*C_lat_in), grid on, hold on
19 bode(P_lat_in*C_lat_in/(1+P_lat_in*C_lat_in))
20 margin(P_lat_out*C_lat_out)
21 bode(P_lat_out*C_lat_out/(1+P_lat_out*C_lat_out))
22 legend('Open Loop-Inner', 'Closed Loop-Inner',...
23        'Open Loop-Outer', 'Closed Loop-Outer')
```

The transfer functions for the longitudinal plant and controller are defined in Lines 2 and 5–6. The `margin` and `bode` plots of the open and closed loop system respectively, for the longitudinal system are generated in Lines 12–15. The results of this code are shown in Figure 1.

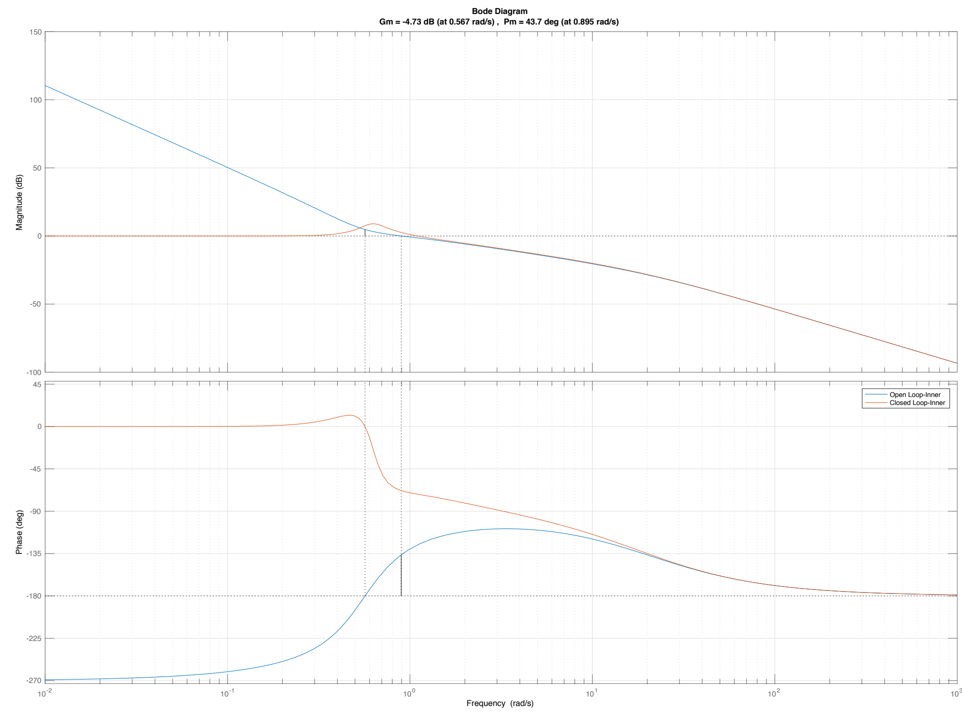

Figure 1: The `margin` plot of the open loop system and the `bode` plot of the closed loop system, of the longitudinal control for the VTOL system under PID control.

As seen from Figure 1 the bandwidth of the closed loop system approximately 1.5 whereas the cross over frequency is 0.9 rad/sec. We also see from Figure 1 that there is a resonance in the closed loop system at crossover. This is due to the relatively small phase margin of $PM = 43.7$ degrees.

The transfer functions for the lateral inner and outer loop plants and controller are defined in Lines 3–4 and 7–10. For this problem we plot both the inner and outer loop frequency response on the same Bode plot, as implemented in Lines 17–23. The results of this code are shown in Figure 2.

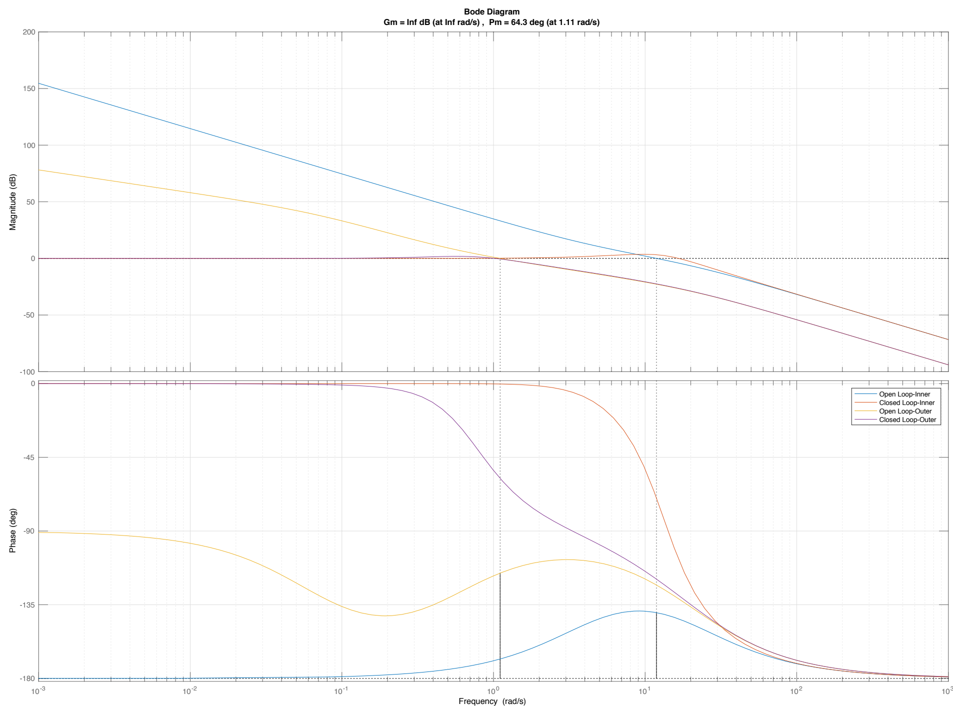

Figure 2: The `margin` and `bode` plots for the the open and closed loop systems of both the inner and outer loops of the VTOL lateral system.

As seen from Figure 2 the bandwidth of the inner loop is approximately 21 rad/sec, which is larger than the cross over frequency of 12 rad/sec. The larger bandwidth is due to the small phase margin of $PM = 41$ degrees. Similarly, Figure 2 indicates that the bandwidth of the outer loop is approximately equal to 1.4 where the cross over frequency is approximately 1.1 rad/sec, due to the excellent phase margin of $PM = 64.3$ degrees.

The bandwidth separation between the inner and outer loop is almost a decade and successive loop closure is justified by the fact that the bode plot of the inner loop is approximately one far beyond the cross over frequency of the outer loop.