A Design Study Approach to Classical Control

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Homework E.16

For the inner loop of the ball & beam system, use the Matlab `bode` command to create a graph that simultaneously displays the Bode plots for (1) the plant, and (2) the plant under PD control, using the control gains calculated in Homework E.8.

(a) To what percent error can the closed loop system under PD control track the desired input if all of the frequency content of $\theta_r(t)$ is below $\omega_r = 1.0$ radians per second?

(b) If the frequency content of the input disturbance is all contained below $\omega_{d_{in}} = 0.8$ radians per second, what percentage of the input disturbance shows up in the output?

(c) If all of the frequency content of the noise $n(t)$ is greater than $\omega_{no} = 300$ radians per second, what percentage of the noise shows up in the output signal θ?

For the outer loop of the ball & beam, use the Matlab `bode` command to create a graph that simultaneously displays the Bode plots for (1) the plant, and (2) the plant under PID control using the control gains calculated in Homework E.10.

(d) If the frequency content of an output disturbance is contained below $\omega_{d_{out}} = 0.1$ radian/sec, what percentage of the output disturbance will be contained in the output under PID control?

(e) If the reference signal is $y_r(t) = 2\sin(0.6t)$ what is the output error under PID control?

Solution

The Bode plot of the inner loop $P_{in}(s)$, and the loop gain with PD control $P_{in}(s)C_{PD}(s)$, are shown in Figure 1.

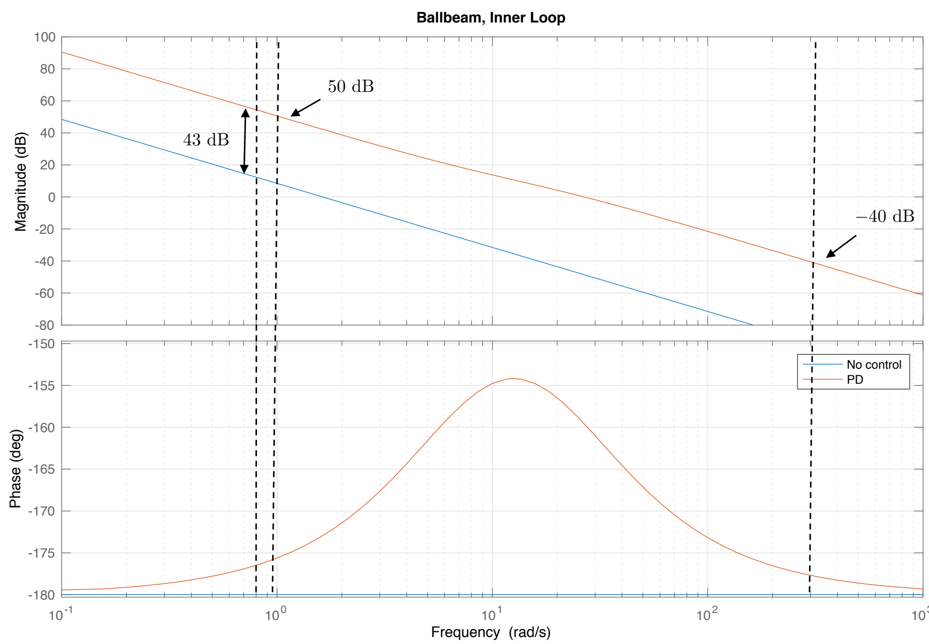

Figure 1: Bode plot for ball and beam inner loop, plant only, and under PD control.

(a) From Figure 1 we see that for $\omega < \omega_r = 1$ rad/sec, the loop gain is greater than $B_r = 50$ dB. Therefore, the tracking error satisfies

$$|e(t)| \leq 10^{-50/20} |r(t)| = 0.0032 |r(t)|,$$

which implies that tracking accuracy is to within 0.32%.

(b) From Figure 1 we see that for $\omega \leq \omega_{d_{in}} = 0.8$ dB, the Bode magnitude plot satisfies

$$20\log_{10}|P(j\omega)C(j\omega)| - 20\log_{10}|P(j\omega)| \geq B_{d_{in}} = 43 \text{ dB}.$$

Therefore, the contribution of the input disturbance to the error satisifies

$$|e(t)| \leq 10^{-43/20} |d_{in}(t)| = 0.0071 |d_{in}(t)|,$$

which implies that 0.71% of the input disturbance shows up in the output θ.

(c) For $\omega \geq \omega_{no} = 300$ rad/sec, we see from Figure 1 that $B_{no} = -40$ dB. Therefore, $\gamma_n = 10^{-40/20} = 0.01$ which implies that 1% of the noise will show up in the output signal.

The Bode plot of the outer loop $P_{out}(s)$, and the loop gain with PD control $P_{in}(s)C_{PD}(s)$, and with PID control $P_{in}(s)C_{PID}(s)$ are shown in Figure 2.

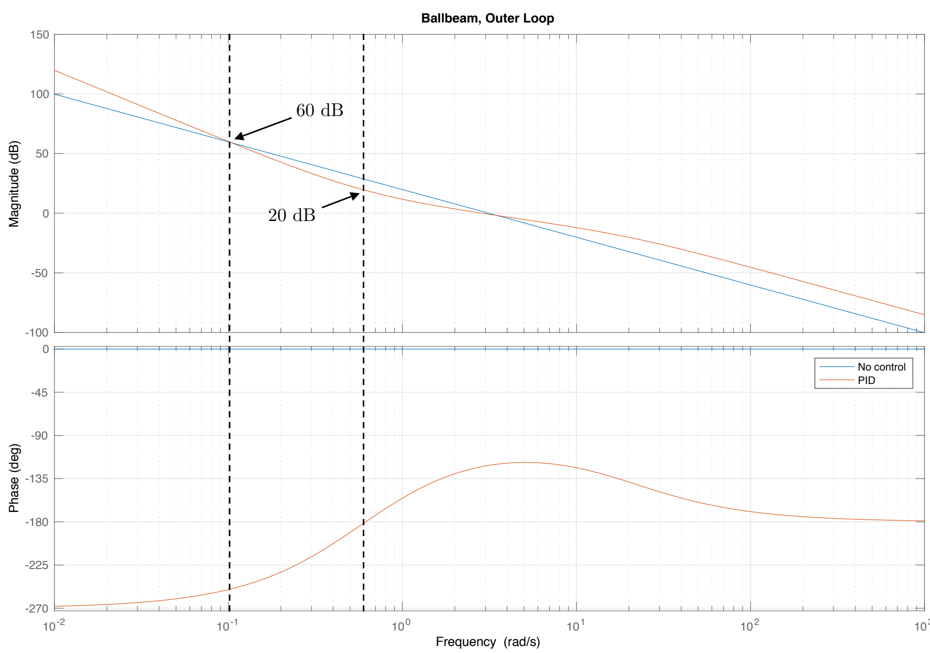

Figure 2: Bode plot for ball and beam outer loop, plant only, under PD control, and under PID control.

(d) For $\omega \leq \omega_{d_{out}} = 0.1$ rad/sec, we see from Figure 2 that for PID control $B_{d_{out}} = 60$ dB. Therefore, $\gamma_{d_{in}} = 10^{-60/20} = 0.001$, which implies that 0.1% of the output disturbance will show up in the output signal.

(e) At $\omega_0 = 0.6$ rad/sec, we see from Figure 2 that the loop gain is

$20\log_{10}|P(j\omega_0)C_{PD}(j\omega_0)| = 20$ dB. Therefore, the tracking error will be

$$|e(t)| = \left|\frac{1}{1+P(j\omega_0)C(j\omega_0)}\right| \cdot 2 \approx \left|\frac{1}{P(j\omega_0)C(j\omega_0)}\right| \cdot 2 = \frac{2}{10^{20/20}} = 0.2.$$