A Design Study Approach to Classical Control

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Homework B.15

(a) Draw by hand the Bode plot of the inner loop transfer function from force F to angle θ for the inverted pendulum. Use the Matlab `bode` command and compare your results.

(b) Draw by hand the Bode plot of the outer loop transfer function from angle θ to position z for the inverted pendulum. Use the Matlab `bode` command and compare your results.

Solution

From HW B.5, the transfer function for the inner loop of the inverted pendulum is

$$P_{in}(s) = \frac{-2/m_2\ell}{s^2 - \frac{2(m_1+m_2)g}{m_2\ell}} = \frac{-4}{s^2-49} = \frac{-4}{(s+7)(s-7)}. \tag{1}$$

In Bode canonical form we have

$$P_{in}(j\omega) = \frac{0.0816}{(1+j\frac{\omega}{7})(1-j\frac{\omega}{7})}$$

Therefore

$$20\log_{10}|P_{in}(j\omega)| = 20\log_{10} 0.0816 - 20\log_{10}\left|1+j\frac{\omega}{7}\right| - 20\log_{10}\left|1-j\frac{\omega}{7}\right|. \tag{2}$$

Therefore, the Bode plot for magnitude will be the graphical addition of a constant gain, a right half plane pole, and a left half plane pole. Similarly, the phase is given by

$$\angle P_{in}(j\omega) = \angle 0.0816 - \angle(1 + j\frac{\omega}{7}) - \angle(1 - j\frac{\omega}{7}).$$

The straight line approximation as well as the Bode plot generated by Matlab are shown in Figure 1.

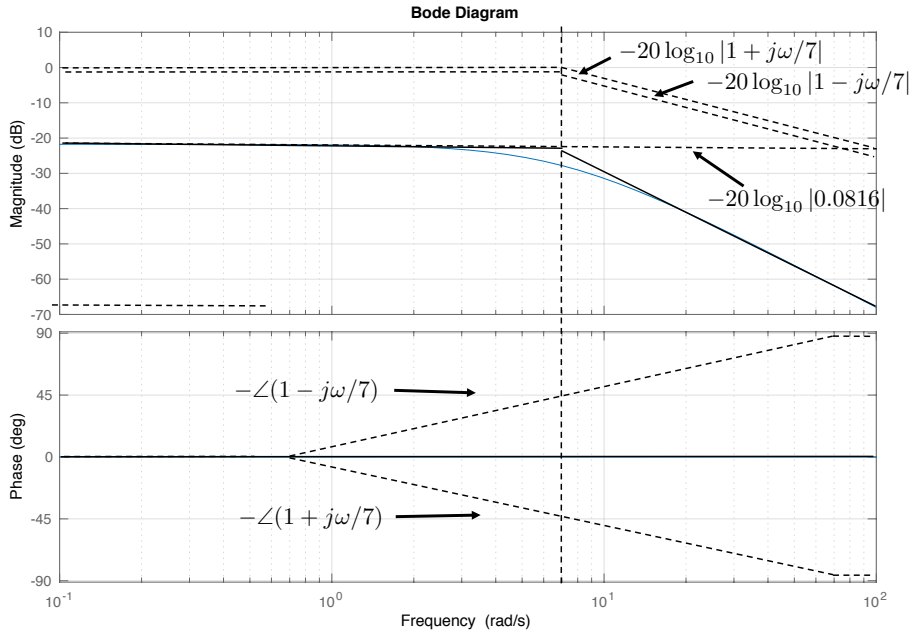

Figure 1: Bode plot for the transfer function given in Equation (2).

The Matlab command to generate the Bode plot is

```
>> Pin = tf([-4], [1, 0, -49]);
>> figure(1), clf, bode(Pin), grid on
```

From HW B.5, the transfer function for the outer loop of the inverted pendulum is

$$P_{out}(s) = \frac{g}{s^2} = \frac{9.8}{s^2}. \tag{3}$$

In Bode canonical form we have

$$P_{out}(j\omega) = \frac{9.8}{(j\omega)^2}.$$

Therefore

$$20\log_{10}|P_{out}(j\omega)| = 20\log_{10}9.8 - 20\log_{10}|\omega|^2. \quad (4)$$

Similarly, the phase is given by

$$\angle P_{out}(j\omega) = \angle 9.8 - \angle(j\omega) - \angle(j\omega).$$

The straight line approximation as well as the Bode plot generated by Matlab are shown in Figure 2.

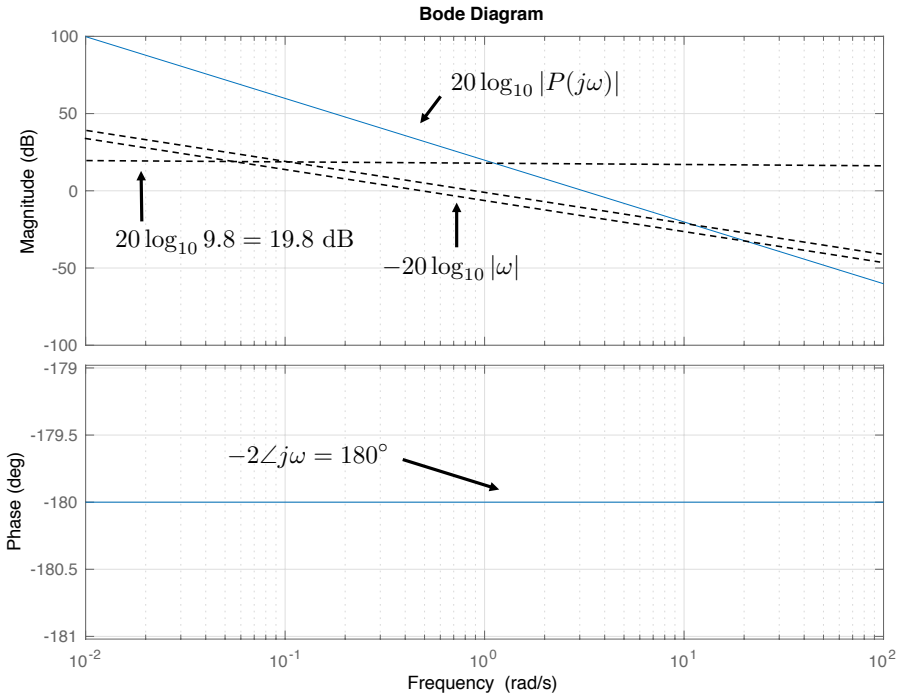

Figure 2: Bode plot for the transfer function given in Equation (4).

The Matlab command to generate the Bode plot is

```
>> Pout = tf([9.8], [1, 0, 0]);
>> figure(1), clf, bode(Pout), grid on
```