A Design Study Approach to Classical Control

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Updated: December 28, 2020

Homework E.f

For the ball-beam system, you will use root-locus methods to analyze and design a PD controller for the outer ball-position loop of the system.

(a) Using the gains calculated in homework problem E.8 (for $t_{r_z} = 5$ sec) for the inner loop (k_{p_θ}, k_{d_θ}), calculate the inner closed-loop transfer function from θ_d to θ. Draw the block diagram for the outer z-position control loop with the inner closed-loop transfer function represented, as well as the transfer function for the dynamics from θ to z, and the transfer function for your outer-loop PD controller. To facilitate your root-locus design, implement your PD control in this form

$$KC(s) = K(s/a + 1),$$

where a is used to shape the locus and K is used to place poles at desired positions along the locus. (Note that $k_{p_z} = K$ and $k_{d_z} = K/a$.)

(b) Use root-locus methods to determine suitable values for K and a to ensure that $\zeta_z > 0.7$ for all closed-loop poles (four of them). Utilize the addition design freedom made available by the root-locus analysis to maximize the speed of the inner-loop poles.

(c) Implement your root-locus-designed PD controller in Simulink using the successive-loop closure implementation from homework problem E.8 with the outer-loop PD gains replaced by those designed in part (b) above. Apply a step input of 0.25 m (e.g., from $z = 0.125$ m to 0.375 m) and observe the response. Is the response of your root-locus design faster that the PD design of homework E.8?