A Design Study Approach to Classical Control

Randal W. Beard   Timothy W. McLain
Brigham Young University

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Homework C.8

(a) **For this problem and future problems**, change the satellite spring constant to $k = 0.1$ N m. For the simple satellite system, using the principle of successive loop closure, draw a block diagram that uses PD control for the inner and the outer loop control. The input to the outer loop controller is the desired angle of the solar panel ϕ_r and the output is the desired angle of the satellite θ_r. The input to the inner loop controller is θ_r and the output is the torque on the satellite τ.

(b) Focusing on the inner loop, find the PD gains k_{P_θ} and k_{D_θ} so that the rise time of the inner loop is $t_{r_\theta} = 1$ second, and the damping ratio is $\zeta_\theta = 0.9$.

(c) Find the DC gain k_{DC_θ} of the inner loop.

(d) Replacing the inner loop by its DC-gain, find the PD gains k_{P_ϕ} and k_{D_ϕ} so that the rise time of the outer loop is $t_{r_\phi} = 10 t_{r_\theta}$ with damping ratio $\zeta_\phi = 0.9$.

(f) Implement the successive loop closure design for the satellite system in Simulink where the commanded solar panel angle is given by a square wave with magnitude 15 degrees and frequency 0.015 Hz.

(g) Suppose that the size of the input torque on the satellite is limited to $\tau_{\max} = 5$ Nm. Modify the Simulink diagram to include a saturation block on the torque τ. Using the rise time of the outer loop as a tuning parameter, tune the PD control law to get the fastest possible response without input saturation when a step of size 30 degrees is placed on ϕ^r.

Solution

The block diagram for the inner loop is shown in Figure **??**.

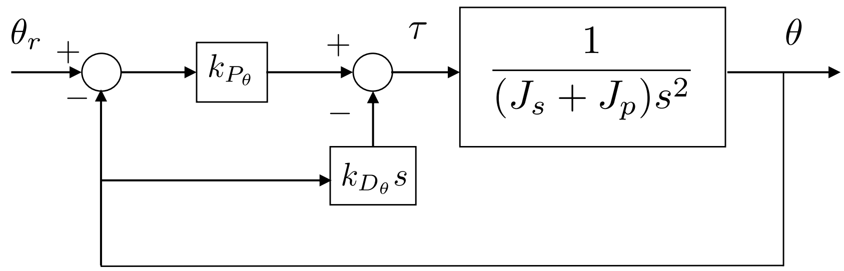

Figure 1: Block diagram for inner loop of satellite control

The closed loop transfer function from Θ^d to Θ is given by

$$\Theta(s) = \frac{\frac{k_{P_\theta}}{J_s+J_p}}{s^2 + \left(\frac{k_{D_\theta}}{J_s+J_p}\right) s + \left(\frac{k_{P_\theta}}{J_s+J_p}\right)} \Theta^d(s).$$

Therefore the closed loop characteristic equation is

$$\Delta_{cl}(s) = s^2 + \left(\frac{k_{D_\theta}}{J_s + J_p}\right) s + \left(\frac{k_{P_\theta}}{J_s + J_p}\right).$$

The desired closed loop characteristic equation is

$$\Delta_{cl}^d(s) = s^2 + 2\zeta_\theta \omega_{n_\theta} s + \omega_{n_\theta}^2,$$

where

$$\omega_{n_\theta} = \frac{2.2}{t_{r_\theta}} = 2.2$$
$$\zeta_\theta = 0.9.$$

Therefore

$$k_{P_\theta} = \omega_{n_\theta}^2 (J_s + J_p) = 29.04$$
$$k_{D_\theta} = 2\zeta_\theta \omega_{n_\theta} (J_s + J_p) = 23.76.$$

The DC gain of the inner lofop is given by

$$k_{DC_\theta} = 1.$$

Replacing the inner loop by its DC gain, the block diagram for the outer loop is shown in Figure **??**.

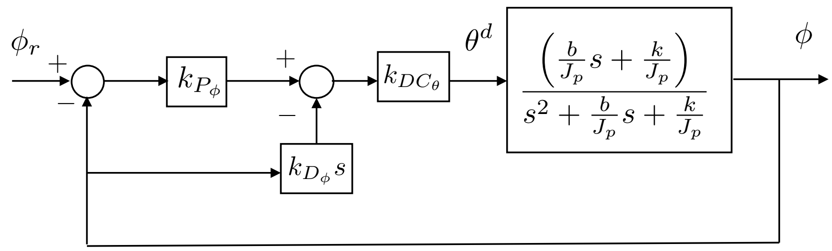

Figure 2: Block diagram for outer loop of satellite attitude control

To find the closed loop transfer function from ϕ^d to ϕ, follow the loop backwards from ϕ to obtain

$$\Phi(s) = \left(\frac{\frac{b}{J_p}s + \frac{k}{J_p}}{s^2 + \frac{b}{J_p}s + \frac{k}{J_p}} \right) \left[k_{DC_\theta} k_{P_\theta} (\Phi_r - \Phi) - k_{DC_\theta} k_{D_\theta} s \Phi \right].$$

After some manipulation we get

$$\begin{aligned} \left[(J_p + b k_{DC_\theta} k_{D_\theta}) s^2 + (b + b k_{DC_\theta} k_{P_\theta} + k k_{DC_\theta} k_{D_\theta}) s + (k + k k_{DC_\theta} k_{P_\theta}) \right] \Phi \\ = \left[b k_{DC_\theta} k_{P_\theta} s + k k_{DC_\theta} k_{P_\theta} \right] \Phi_r, \end{aligned} \tag{1}$$

which results in the monic transfer function

$$\Phi(s) = \frac{\left(\frac{b k_{DC_\theta} k_{P_\theta}}{J_p + b k_{DC_\theta} k_{D_\theta}} \right) s + \left(\frac{k k_{DC_\theta} k_{P_\theta}}{J_p + b k_{DC_\theta} k_{D_\theta}} \right)}{s^2 + \left(\frac{b + b k_{DC_\theta} k_{P_\theta} + k k_{DC_\theta} k_{D_\theta}}{J_p + b k_{DC_\theta} k_{D_\theta}} \right) s + \left(\frac{k + k k_{DC_\theta} k_{P_\theta}}{J_p + b k_{DC_\theta} k_{D_\theta}} \right)}.$$

The desired closed loop characteristic equation for the outer loop is

$$\Delta_{cl}^d(s) = s^2 + 2\zeta_\phi \omega_{n_\phi} s + \omega_{n_\phi}^2,$$

where

$$\begin{aligned} t_{r_\phi} &= 10 t_{r_\theta} = 10 \\ \omega_{n_\phi} &= \frac{2.2}{t_{r_\phi}} = 0.22 \\ \zeta_\phi &= 0.9. \end{aligned}$$

Therefore the gains k_{P_ϕ} and k_{P_θ} satisfy

$$\frac{k + kk_{DC_\theta}k_{P_\theta}}{J_p + bk_{DC_\theta}k_{D_\theta}} = \omega_{n_\phi}^2$$
$$\frac{b + bk_{DC_\theta}k_{P_\theta} + kk_{DC_\theta}k_{D_\theta}}{J_p + bk_{DC_\theta}k_{D_\theta}} = 2\zeta_\phi\omega_{n_\phi}.$$

Expressing these equations in matrix form gives

$$\begin{pmatrix} kk_{DC_\theta} & -J_p bk_{DC_\theta} \\ bk_{DC_\theta} & kk_{DC_\theta} - 2bk_{DC_\theta}\zeta_\phi\omega_{n_\phi} \end{pmatrix} \begin{pmatrix} k_{P_\phi} \\ k_{D_\phi} \end{pmatrix} = \begin{pmatrix} -k + J_p\omega_{n_\phi}^2 \\ -b + 2J_p\zeta_\phi\omega_{n_\phi} \end{pmatrix},$$

implying that

$$\begin{pmatrix} k_{P_\phi} \\ k_{D_\phi} \end{pmatrix} = \begin{pmatrix} kk_{DC_\theta} & -J_p bk_{DC_\theta} \\ bk_{DC_\theta} & kk_{DC_\theta} - 2bk_{DC_\theta}\zeta_\phi\omega_{n_\phi} \end{pmatrix}^{-1} \begin{pmatrix} -k + J_p\omega_{n_\phi}^2 \\ -b + 2J_p\zeta_\phi\omega_{n_\phi} \end{pmatrix} = \begin{pmatrix} 0.7946 \\ 2.621 \end{pmatrix}$$

The DC gain of the outer loop is given by

$$k_{DC_\phi} = \frac{kk_{DC_\theta}k_{P_\phi}}{k + kk_{DC_\theta}k_{P_\phi}} = 0.4428.$$

Note that the DC gain of the outer loop is not equal to one, so there will be significant steady state error. To remedy this, a feedforward term can be used to ensure that θ and ϕ are made to be equal in steady state, resulting in an overall DC gain of one. This configuration is depicted in Figure **??**. Feedforward control will be discussed in a later chapter.

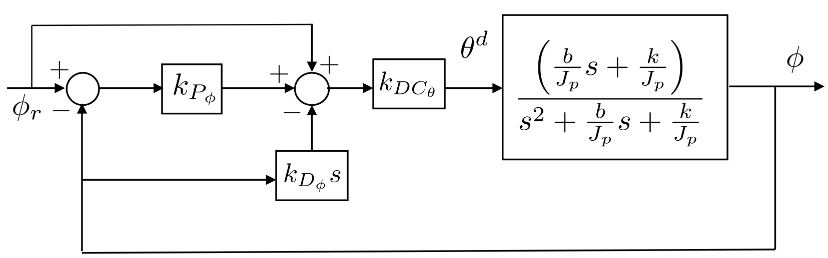

Figure 3: Block diagram for outer loop of satellite attitude control with a feedforward term

See http://controlbook.byu.edu for the complete solution, including a Simulink implementation with and without feedforward control.