A Design Study Approach to Classical Control

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Homework C.15

(a) Draw by hand the Bode plot of the inner loop transfer function from torque τ to angle θ for the satellite attitude problem. Use the Matlab `bode` command and compare your results.

(b) Draw by hand the Bode plot of the outer loop transfer function from body angle θ to panel angle ϕ for the satellite attitude problem. Use the Matlab `bode` command and compare your results.

Solution

From HW C.5, the transfer function for the inner loop of the satellite attitude problem is

$$P_{in}(s) = \frac{1/J_s}{s^2 + \frac{b}{J_s}s + \frac{k}{J_s}} = \frac{0.2}{s^2 + 0.01s + 0.03}. \tag{1}$$

In Bode canonical form we have

$$P_{in}(j\omega) = \frac{6.67}{1 + 0.33j\omega + \left(j\frac{\omega}{0.173}\right)^2}$$

Therefore

$$20\log_{10}|P_{in}(j\omega)| = 20\log_{10} 6.67 - 20\log_{10}\left|1 + 0.33j\omega + \left(j\frac{\omega}{0.173}\right)^2\right| \tag{2}$$

Therefore, the Bode plot for magnitude will be the graphical addition of a constant gain, and a complex pole. Similarly, the phase is given by

$$\angle P_{in}(j\omega) = \angle 6.67 - \angle\left(1 + 0.33j\omega + \left(j\frac{\omega}{0.173}\right)^2\right).$$

The straight line approximation as well as the Bode plot generated by Matlab are shown in Figure 1.

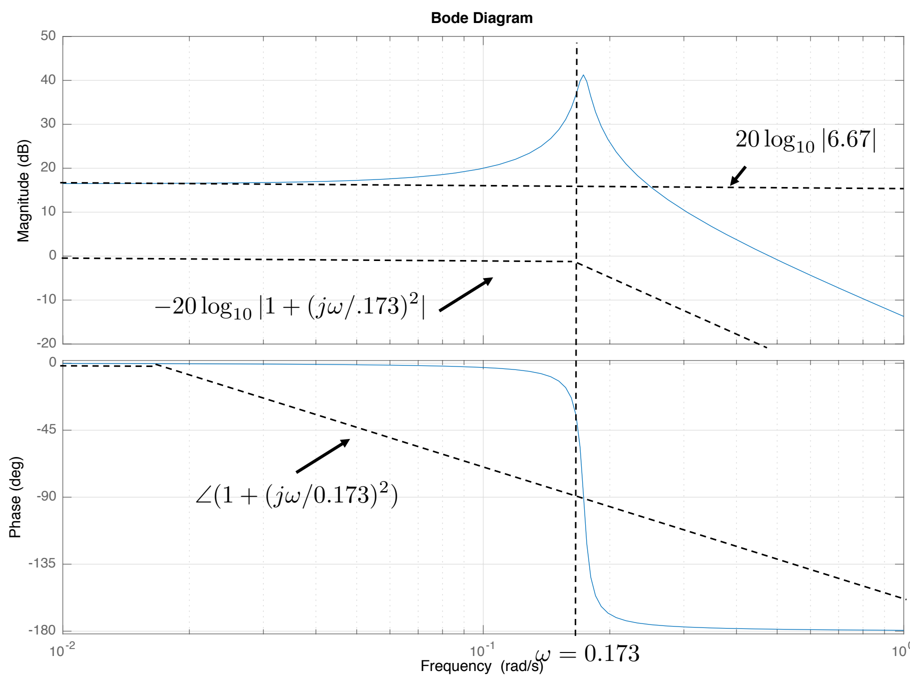

Figure 1: Bode plot for the transfer function given in Equation (15.21).

The Matlab command to generate the Bode plot is

```
>> Pin = tf([0.2], [1, 0.01, 0.03]);
>> figure(1), clf, bode(Pin), grid on
```

From HW C.5, the transfer function for the outer loop of the satellite is

$$P_{out}(s) = \frac{\frac{b}{J_p}s + \frac{k}{J_p}}{s^2 + \frac{b}{J_p}s + \frac{k}{J_p}} = \frac{0.05s + 0.15}{s^2 + 0.05s + 0.15}. \tag{3}$$

In Bode canonical form we have

$$P_{out}(j\omega) = \frac{1 + j\frac{\omega}{3}}{1 + j\frac{\omega}{3} + \left(j\frac{\omega}{0.3873}\right)^2}$$

Therefore

$$20\log_{10}|P_{out}(j\omega)| = 20\log_{10}|1 + j\omega/3| - 20\log_{10}\left|1 + j\omega/3 + \left(j\frac{\omega}{0.3873}\right)^2\right| \tag{4}$$

Therefore, the Bode plot for magnitude will be the graphical addition of a zero, and a complex pole. Similarly, the phase is given by

$$\angle P_{out}(j\omega) = \angle(1 + j\omega/3) - \angle\left(1 + j\omega/3 + \left(j\frac{\omega}{0.3873}\right)^2\right).$$

The straight line approximation as well as the Bode plot generated by Matlab are shown in Figure 2.

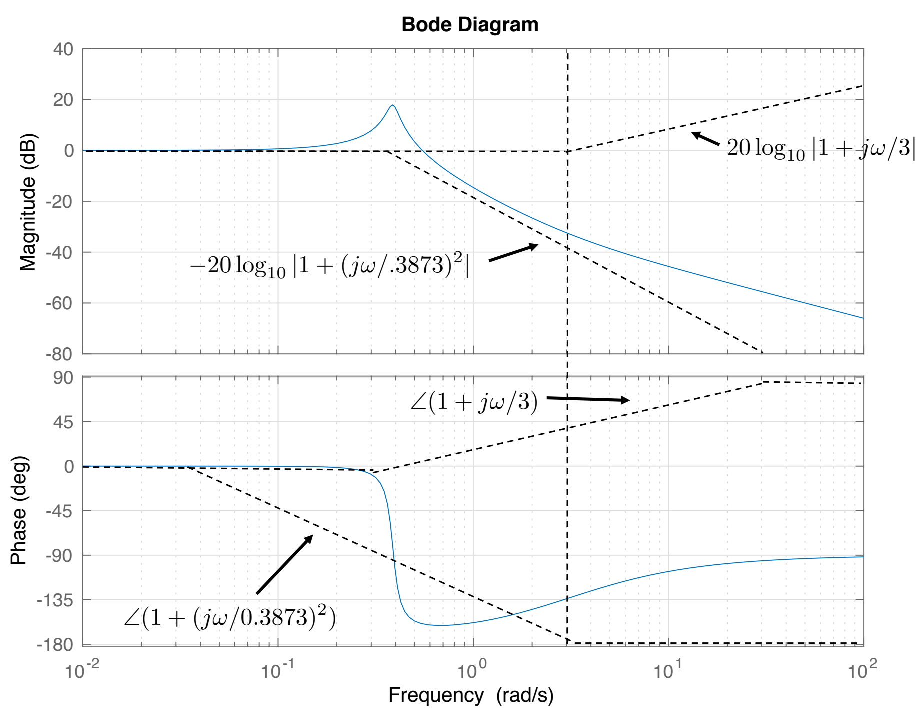

Figure 2: Bode plot for the transfer function given in Equation (15.23).

The Matlab command to generate the Bode plot is

```
>> Pout = tf([0.05, 0.15], [1, 0.05, 0.15]);
>> figure(1), clf, bode(Pout), grid on
```