A Design Study Approach to Classical Control

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Homework C.9

(a) When the inner loop controller of the satellite system is PD control, what is the system type of the inner loop with respect to the reference input, and with respect to the disturbance input? Characterize the steady state error when the reference input is a step, a ramp, and a parabola. Characterize the steady state error when the input disturbance is a step, a ramp, and a parabola.

(b) With PD control for the outer loop, what is the system type of the outer loop with respect to the reference input and with respect to the disturbance input? Characterize the steady state error when the reference input and disturbance input is a step, a ramp, and a parabola. How does this change if you add an integrator?

Solution

The block diagram for the inner loop is shown in Figure 1.

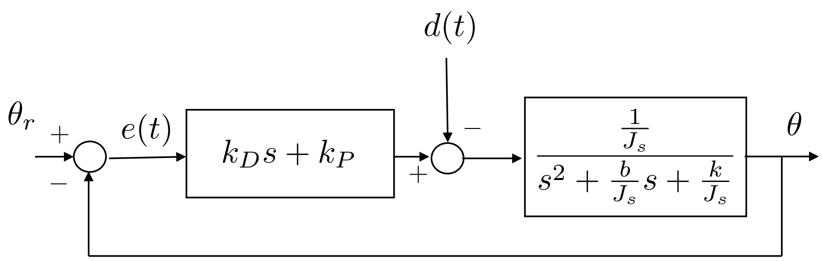

Figure 1: Inner loop system for problem HW **??**.**??**.

The open loop system is given by

$$P(s)C(s) = \left(\frac{\frac{1}{J_s}}{s^2 + \frac{b}{J_s}s + \frac{k}{J_s}} \right) (k_D s + k_P).$$

Since there are no free integrators, the system is type 0, and from Table **??** the tracking error when the input is a step is

$$\lim_{t\to\infty} e(t) = \frac{1}{1+M_p} = \frac{1}{1 + \lim_{s\to 0} sP(s)C(s)} = \frac{1}{1+\frac{k_P}{k}}.$$

The tracking error when the input is a ramp, or higher order polynomial, is ∞.

For a disturbance input of $D(s) = 1/s^{q+1}$, the steady state error is

$$\begin{aligned}
\lim_{t\to\infty} e(t) &= \lim_{s\to 0} s \frac{P(s)}{1+P(s)C(s)} \frac{1}{s^{q+1}} \\
&= \lim_{s\to 0} \frac{\left(\frac{\frac{1}{J_s}}{s^2+\frac{b}{J_s}s+\frac{k}{J_s}} \right)}{1 + \left(\frac{\frac{1}{J_s}}{s^2+\frac{b}{J_s}s+\frac{k}{J_s}} \right)(k_D s + k_P)} \frac{1}{s^q} \\
&= \lim_{s\to 0} \frac{\frac{1}{J_s}}{\left(s^2 + \frac{b}{J_s}s + \frac{k}{J_s}\right) + \frac{1}{J_s}(k_D s + k_P)} \frac{1}{s^q} \\
&= \lim_{s\to 0} \frac{\frac{1}{J_s}}{\frac{k}{J_s} + \frac{k_P}{J_s}} \frac{1}{s^q}
\end{aligned}$$

which is finite and equal to $1/(k+k_P)$ when $q = 0$. Therefore, the system is type 0 with respect to the input disturbance, and the steady state error to a step on $d(t)$ is $1/(k+k_P)$, and is infinite to a ramp and higher order polynomials.

The block diagram for the outer loop is shown in Figure 2.

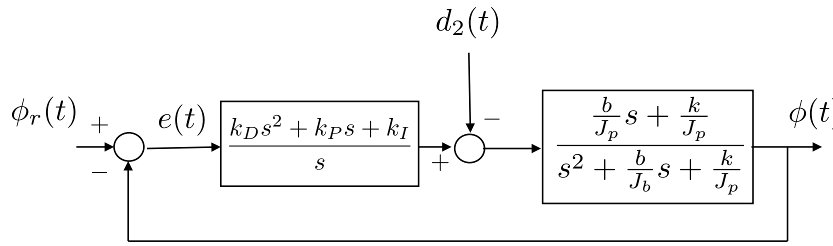

Figure 2: Outer loop system for problem HW **??**.**??**.

The open loop system is given by

$$P(s)C(s) = \left(\frac{\frac{b}{J_P}s + \frac{k}{J_p}}{s^2 + \frac{b}{J_p}s + \frac{k}{J_p}}\right)\left(\frac{k_D s^2 + k_P s + k_I}{s}\right).$$

When $k_I = 0$ there are no free integrator in $P(s)C(s)$ and the system is type 0, and from Table **??** the tracking error when the input is a step is

$$\lim_{t\to\infty} e(t) = \frac{1}{1+M_p} = \frac{1}{1+\lim_{s\to 0} P(s)C(s)} = \frac{1}{1+k_P}.$$

When the reference is a ramp or higher order polynomial, the tracking error is infinite. When $k_I \neq 0$, there is one free integrators in $P(s)C(s)$ and the system is type 1. From Table **??** the tracking error when the input is a ramp is

$$\lim_{t\to\infty} e(t) = \frac{1}{M_v} = \frac{1}{\lim_{s\to 0} sP(s)C(s)} = \frac{1}{k_I}.$$

The tracking error when ϕ_r is a step is zero, and it is infinite when ϕ_r is a parabola or higher order polynomial.

For the disturbance input, the steady state error when $D(s) = \frac{1}{s^{q+1}}$ is

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} s\frac{P(s)}{1+P(s)C(s)}\frac{1}{s^{q+1}} = \lim_{s\to 0} \frac{\left(\frac{\frac{b}{J_P}s+\frac{k}{J_p}}{s^2+\frac{b}{J_p}s+\frac{k}{J_p}}\right)}{1+\left(\frac{\frac{b}{J_P}s+\frac{k}{J_p}}{s^2+\frac{b}{J_p}s+\frac{k}{J_p}}\right)\left(\frac{k_D s^2+k_P s+k_I}{s}\right)}\frac{1}{s^q}.$$

Without the integrator, i.e., when $k_I = 0$ we have

$$\begin{aligned}
\lim_{t\to\infty} e(t) &= \lim_{s\to 0} \frac{\frac{b}{J_p}s + \frac{k}{J_p}}{(s^2 + \frac{b}{J_p}s + \frac{k}{J_p}) + (\frac{b}{J_P}s + \frac{k}{J_p})(k_D s + k_P)} \frac{1}{s^q} \\
&= \lim_{s\to 0} \frac{1}{1 + k_P} \frac{1}{s^q}
\end{aligned}$$

which is finite when $q = 0$. Therefore, the system is type 0 with respect to the input disturbance and the steady state error when $d_2(t)$ is a unit step is $1/(1 + k_P)$.

When $k_I \neq 0$, we have

$$\begin{aligned}
\lim_{t\to\infty} e(t) &= \lim_{s\to 0} \frac{s\left(\frac{b}{J_p}s + \frac{k}{J_p}\right)}{s\left(s^2 + \frac{b}{J_p}s + \frac{k}{J_p}\right) + \left(\frac{b}{J_p}s + \frac{k}{J_p}\right)(k_D s^2 + k_P s + k_I)} \frac{1}{s^q} \\
&= \lim_{s\to 0} \frac{1}{k_I} \frac{1}{s^{q-1}}
\end{aligned}$$

which is finite when $q = 1$. Therefore, the system is type 1 with respect to the input disturbance and the steady state error when $d_2(t)$ is a unit ramp is $1/k_I$. It is zero when d_2 is a step, and it is infinite when d_2 is a parabola or higher order polynomial.