A Design Study Approach to Classical Control

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Homework C.18

For this homework assignment we will use the loopshaping design technique to design a successive loop closure controller for the satellite attitude problem.

(a) First consider the inner loop, where $P_{in}(s)$ is the transfer function of the inner loop derived in HW C.5. The Bode plot for this system shows an underdamped resonate mode. We have seen in previous chapters that this resonate mode can be removed by introducing rate feedback of the form
$$u = -k_D \dot{y} + u',$$
where y is the output signal and u' is the additional control signal to be designed. Using the derivative gain k_{D_θ} found in HW C.10, find the revised transfer function from τ' to θ, and use this transfer function for $P_{in}(s)$. Letting
$$\tau'(s) = C_{in}(s)E(s),$$
where $e(t) = \theta_r(t) - \theta(t)$, design $C_{in}(s)$ to meet the following specifications:

- The inner loop will track θ_r with frequency content below $\omega_r = 0.01$ radians/sec to within $\gamma_r = 0.01$.
- The inner loop will attenuate noise on the measurement of θ for all frequencies above $\omega_n = 20$ rad/sec by $\gamma_n = 0.01$.
- The phase margin of the inner loop should be around $PM = 60$ degrees.

(b) Now consider the design of the controller for the outer loop system. As seen from HW C.5, the transfer function for the outer loop also has a strong resonate mode. Similar to the inner loop, use the derivative gain found in HW C.10 to introduce rate damping, and find the associated transfer function $P_{out}(s)$. The 'plant' for the design of the outer loop controller is

$$P = P_{out}\frac{P_{in}C_{in}}{1+P_{in}C_{in}}.$$

Design the outer loop controller C_{out} to meet the following specs:

- Track steps in ϕ_r with zero steady state error.
- Reject input disturbances with frequency content below $\omega_{d_{in}} = 0.01$ by $\gamma_{d_{in}} = 0.1$.
- Attenuate noise on the measurement of ϕ with frequency content above $\omega_n = 10$ rad/sec by $\gamma_n = 10^{-4}$.
- The phase margin of the outer loop should be around $PM = 60$ degrees.
- Use a prefilter to reduce any peaking in the closed loop response of the outer loop.

Solution

From HW C.5, the transfer function model for the inner loop is given by

$$\Theta(s) = \frac{\frac{1}{J_s}}{s^2 + \frac{b}{J_s}s + \frac{k}{J_s}}\tau(s).$$

Rate damping is added by designing the control signal so that

$$\tau(s) = -k_D\frac{s}{\sigma s+1}\Theta(s) + \tau'(s).$$

Solving for $\Theta(s)$ in terms of $\tau'(s)$ gives

$$\Theta(s) = \frac{\sigma s+1}{\sigma J_s s^3 + (\sigma b + J_s)s^2 + (\sigma k + b + k_D)s + k}\tau'(s).$$

Therefore, for the inner loop we will set

$$P_{in} = \frac{\sigma s+1}{\sigma J_s s^3 + (\sigma b + J_s)s^2 + (\sigma k + b + k_D)s + k}.$$

Figure 1 shows the Bode plot of the plant $P_{in}(s)$ together with the design specification on input tracking and noise attenuation.

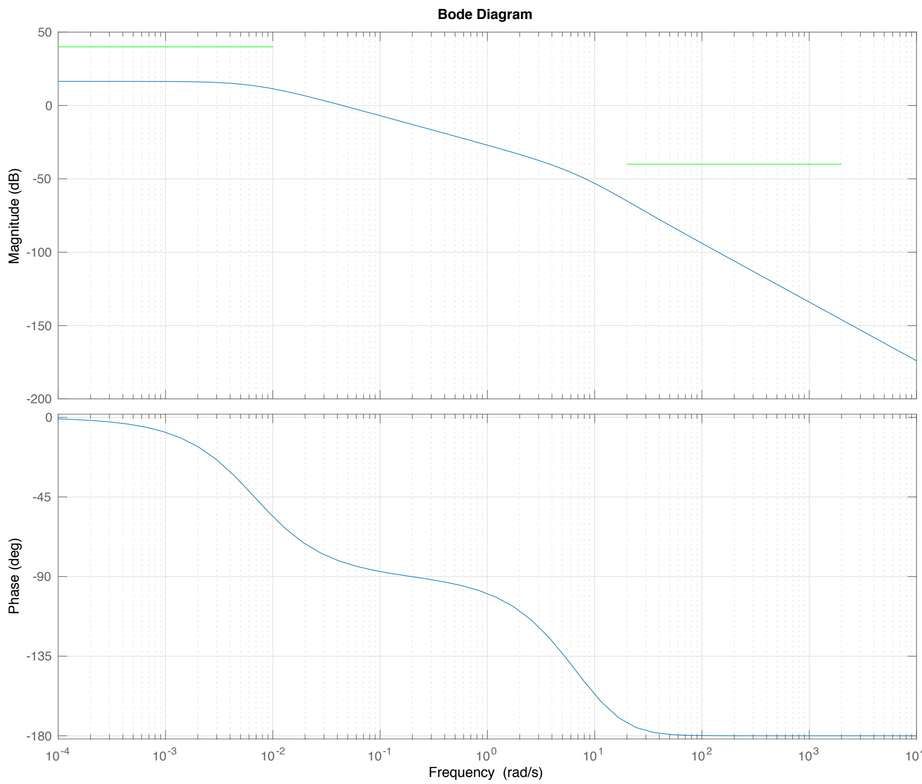

Figure 1: The Bode plot for the inner loop plant in HW C.18, together with the design specification.

The first step is to add a proportional gain to satisfy the tracking requirement. Figure 2 shows the corresponding Bode plot for a proportional gain of $k_P = 30$.

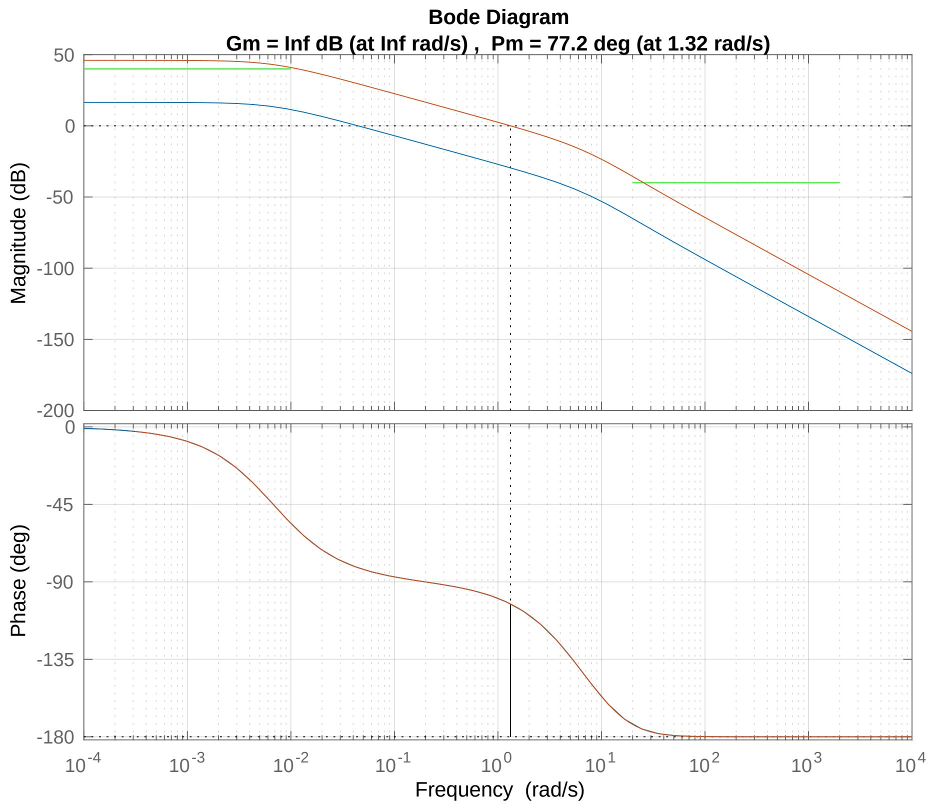

Figure 2: The Bode plot for the inner loop plant in HW C.18, with proportional gain.

Since the phase margin is over $PM = 60$ degrees, we add a low pass filter. Figure 3 shows the loop gain with the addition of the low pass filter

$$C_{LPF} = \frac{10}{s+10},$$

which has a phase margin of $PM = 69.8$ degrees, and satisfies the noise specification.

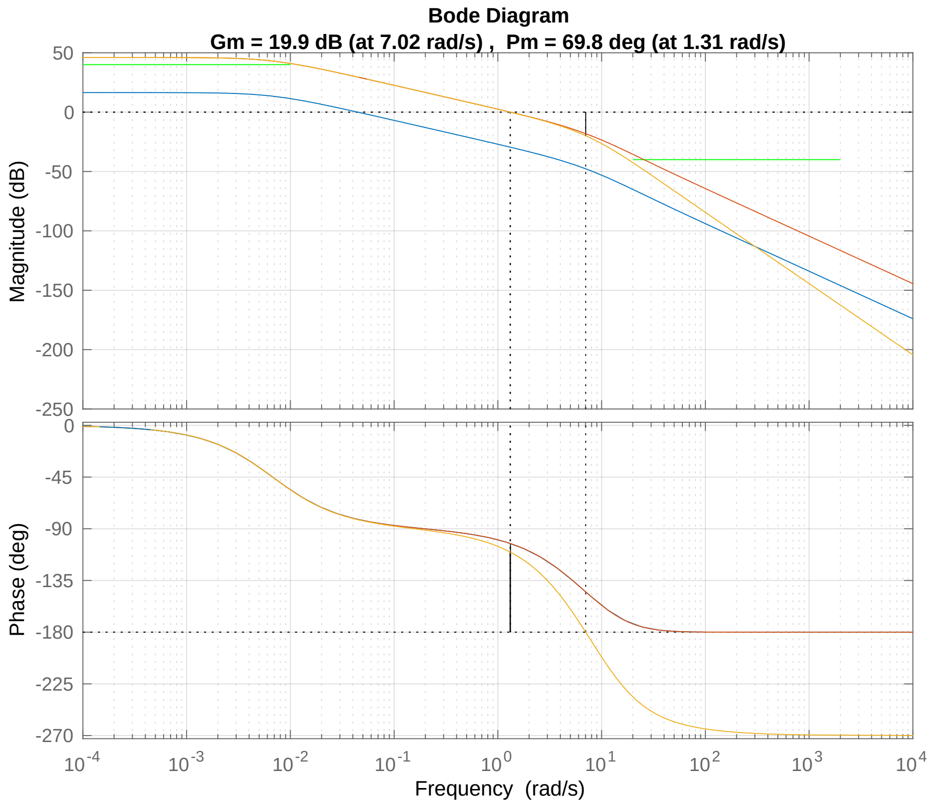

Figure 3: The Bode plot for the inner loop system in HW **??**.18, with proportional gain and low pass filter.

The resulting compensator is

$$C_{in}(s) = 30\left(\frac{10}{s+10}\right).$$

Notice that the DC-gain is not equal to one. The closed loop response for the inner subsystem

$$\frac{P_{in}C_{in}}{1+P_{in}C_{in}},$$

as well as the unit step response for the output and control signal are all show in Figure 4.

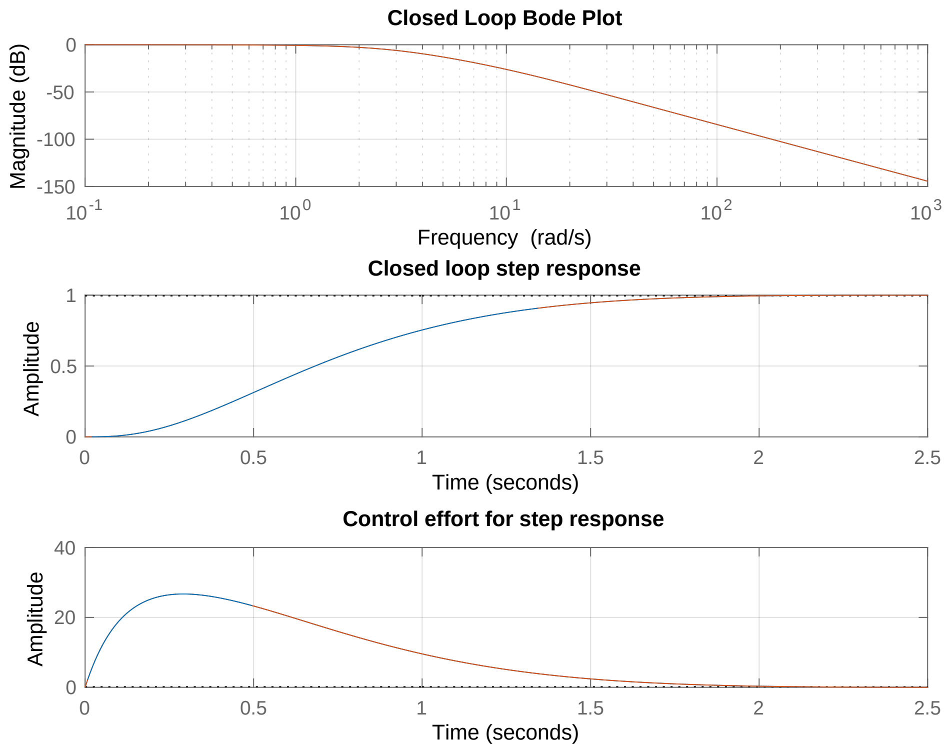

Figure 4: The closed loop bode response, the unit step response for the output, and the unit step response for the input of the inner loop design in HW C.18.

The Matlab code used to design the inner loop is shown below.

```
Plant = P_in;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%  Define Design Specifications
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
   %--- general tracking specification ---
   omega_r = 10^-2;  % track signals below this frequency
   gamma_r = 10^(-40/20);  % tracking error below this value
   w = logspace(log10(omega_r)-2,log10(omega_r));

   %--- noise specification ---
   omega_n = 2*10^1;  % attenuate noise above this frequency
   gamma_n = 10^(-40/20);   % attenuate noise by this amount
```

```
    w = logspace(log10(omega_n),2+log10(omega_n));

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Control Design
  C = 1;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% proportional control: change cross over frequency
    kp = 30;
    C = C*kp;

% low pass filter: decrease gain at high frequency (noise)
    p = 10;
    LPF = tf(p,[1,p]);
    C = C*LPF;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Convert controller to state space equations
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
[num,den] = tfdata(C,'v');
[P.Ain_C,P.Bin_C,P.Cin_C,P.Din_C]=tf2ss(num,den);

C_in = C;
```

The transfer function model for the outer loop was derived in HW C.5 to be

$$\Phi(s) = \frac{\frac{b}{J_p}s + \frac{k}{J_p}}{s^2 + \frac{b}{J_p}s + \frac{k}{J_p}}\Theta^r(s).$$

Rate damping is added by designing the control signal so that

$$\Theta^r(s) = -k_D\frac{s}{\sigma s + 1}\Phi(s) + \Theta^{r'}(s).$$

Solving for $\Phi(s)$ in terms of $\Theta^{r'}(s)$ gives

$$\Phi(s) = \frac{\sigma b s^2 + (\sigma k + b)s + k}{\sigma J_p s^3 + (\sigma b + J_p + bk_D)s^2 + (\sigma k + b + kk_D)s + k}\Theta^{r'}(s).$$

The plant for the outer loop is therefore

$$P_{out} = \frac{\sigma s + 1}{\sigma J_s s^3 + (\sigma b + J_s)s^2 + (\sigma k + b + k_D)s + k}\frac{P_{in}C_{in}}{1 + P_{in}C_{in}}.$$

Figure 5 shows the Bode plot for P_{out} together with the design specification on rejecting input disturbances and attenuating the noise. The requirement that steady state error to a step requires that the slope as $s \to 0$ is -20 dB/dec, and is not shown in Figure 5.

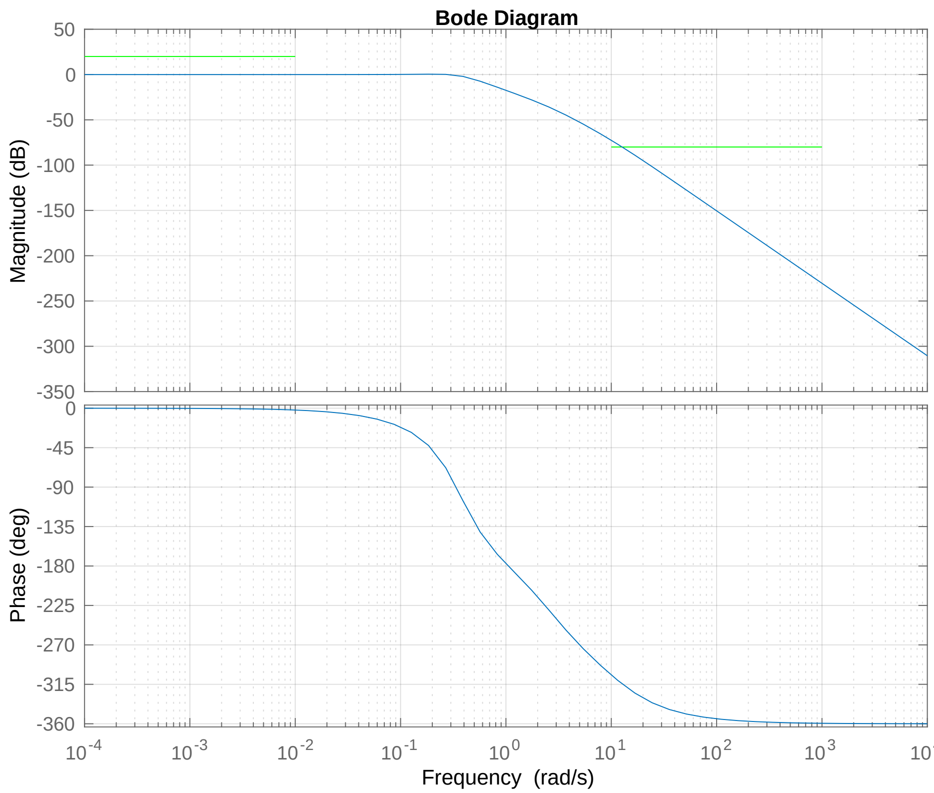

Figure 5: The Bode plot for the outer loop plant in HW C.18, together with the design specification.

The first step is to add an integrator to meet the steady state tracking requirement. Figure 6 shows the loop gain after adding the integral control

$$C_{int} = \frac{s + 0.4}{s}.$$

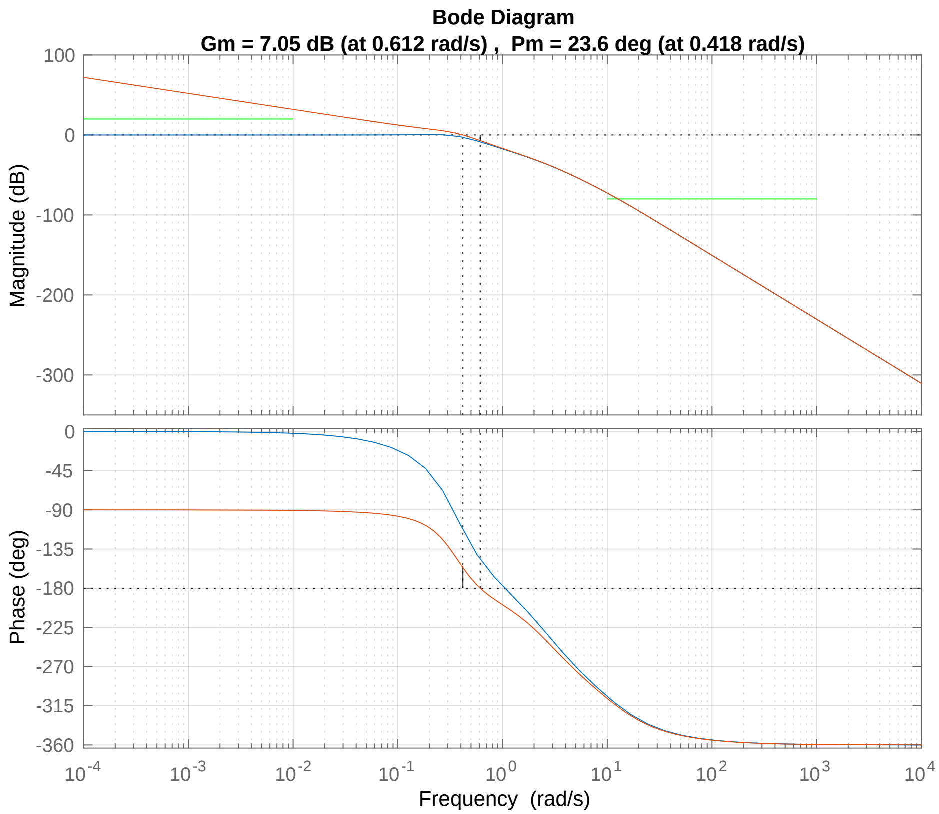

Figure 6: The Bode plot for the outer loop system in HW C.18, with integral control.

It is clear from Figure 6 that the input disturbance requirement is satisfied with a large buffer. To reduce the bandwidth of the system, and to make it easier to satisfy the noise attenuation requirement, a proportional gain of $k_P = 0.5$ is added to the compensator. The result is shown in Figure 7

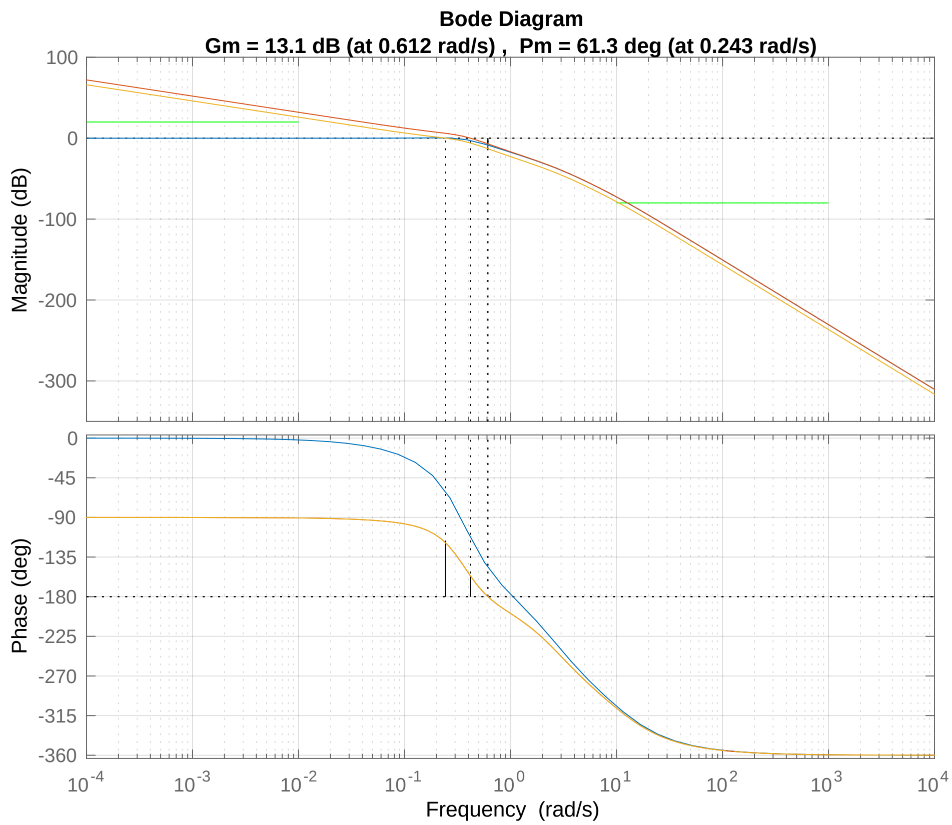

Figure 7: The Bode plot for the outer loop system in HW C.18, with proportional and integral control.

To meet the noise specification, the low pass filter

$$C_{lpf} = \frac{3}{s+3}$$

is added to the compensator, and the resulting loopgain is shown in Figure 8.

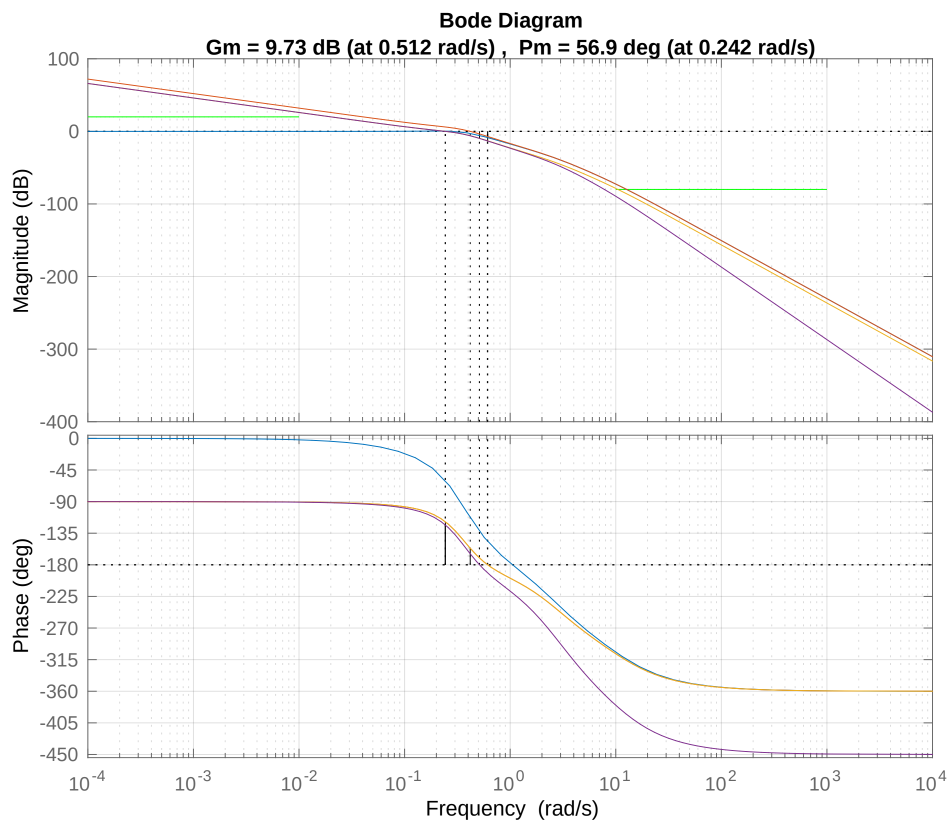

Figure 8: The Bode plot for the outer loop system in HW C.18, with proportional gain, integral control, and a low pass filter.

Since the phase margin is $PM = 56.9$ degrees, we consider the phase margin specification to be satisfied. The resulting compensator is

$$C_{out}(s) = 0.5\left(\frac{s+0.4}{s}\right)\left(\frac{3}{s+3}\right).$$

The closed loop response for both the inner and outer loop systems, as well as the unit step response for the output and control signal of the outer loops are all show in Figure 9, where the prefilter

$$F(s) = \frac{0.8}{s+0.8}$$

has been added to reduce the peaking in the closed loop response.

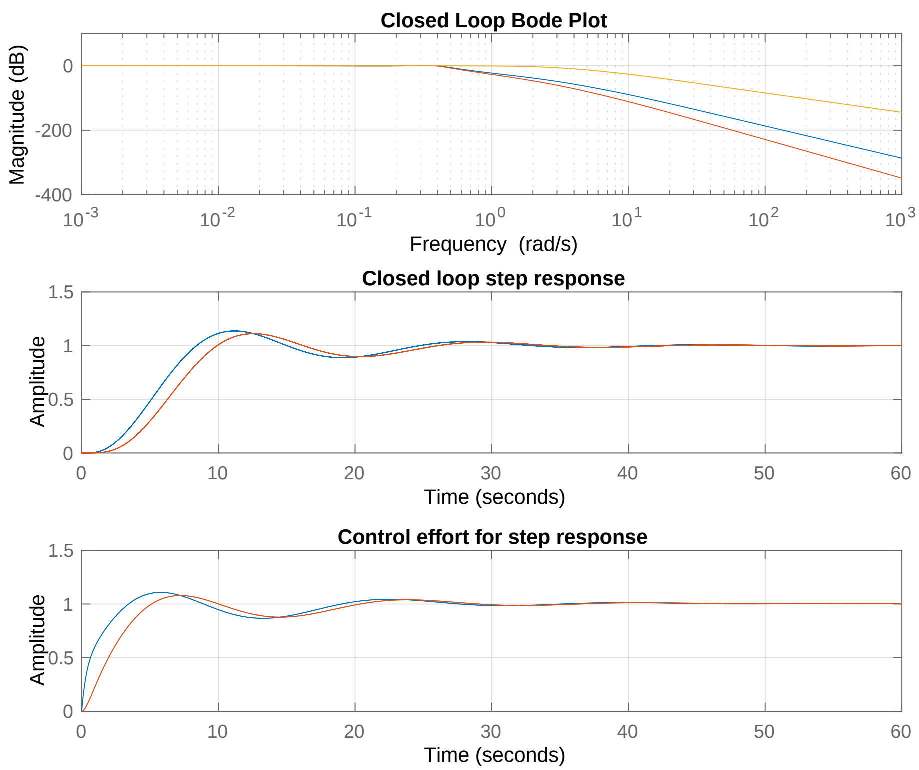

Figure 9: The closed loop bode response, the unit step response for the output, and the unit step response for the input of the inner loop design in HW C.18.

The Matlab code used to design the outer loop is shown below.

```
Plant = minreal(P_out*(P_in*C_in/(1+P_in*C_in)));

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%  Define Design Specifications
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

        %--- input disturbance specification ---
        omega_din = 10^-2;
        gamma_din = 0.1;
        w = logspace(log10(omega_din)-2,log10(omega_din));
        Pmag=bode(Plant,w);
```

```
        for i=1:size(Pmag,3), Pmag_(i)=Pmag(1,1,i); end

        %--- noise specification ---
        omega_n = 10^1;
        gamma_n = 10^(-80/20);
        w = logspace(log10(omega_n),2+log10(omega_n));
        plot(w,20*log10(gamma_n)*ones(size(w)),'g')

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Control Design
  C = tf([1],[1]);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% integral control:
     k_I = .4; % frequency at which integral action ends
     Integrator = tf([1,k_I],[1,0]);
     C = C*Integrator;

% proportional control: change cross over frequency
     kp = .5;
     C = C*kp;

% low pass filter: decrease gain at high frequency (noise)
     p = 3;
     LPF = tf(p,[1,p]);
     C = C*LPF;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Prefilter Design
  F = tf([1],[1]);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% low pass filter
    p = 0.8;  % frequency to start the LPF
    LPF = tf(p, [1,p]);
    F = F*LPF;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Convert controller to state space equations
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
C=minreal(C);
[num,den] = tfdata(C,'v');
[P.Aout_C,P.Bout_C,P.Cout_C,P.Dout_C]=tf2ss(num,den);

[num,den] = tfdata(F,'v');
```

```
[P.Aout_F, P.Bout_F, P.Cout_F, P.Dout_F] = tf2ss(num,den);

C_out=C;
```

The complete Simulink files are on the wiki associated with the book.