A Design Study Approach to Classical Control

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Homework F.15

(a) Draw by hand the Bode plot of the altitude transfer function from force $\tilde{F}$ to altitude $\tilde{h}$. Use the `bode` command (from Matlab or Python) and compare your results.

(b) Draw by hand the Bode plot of the inner loop transfer function for the lateral dynamics from torque τ to angle θ. Use the `bode` command and compare your results.

(c) Draw by hand the Bode plot of the outer loop transfer function for the lateral dynamics from angle θ to position $z(t)$. Use the `bode` command and compare your results.

Solution

(a) From HW F.5, the transfer function for the VTOL altitude loop is

$$P_{in}(s) = \frac{0.667}{s^2}. \tag{1}$$

In Bode canonical form we have

$$P_{in}(j\omega) = \frac{0.667}{(j\omega)^2}$$

Therefore

$$20\log_{10}|P_{in}(j\omega)| = 20\log_{10}0.667 - 40\log_{10}|\omega|.$$

Therefore, the Bode plot for magnitude will be the graphical addition of a constant gain, and a line with slope of -40 dB/decade. Similarly, the phase is given by

$$\angle P_{in}(j\omega) = \angle 0.667 - \angle(j\omega) - \angle(j\omega) = 0 - 90 - 90 = -180 \text{ degrees}.$$

The straight line approximation as well as the Bode plot generated by Matlab are shown in Figure 1.

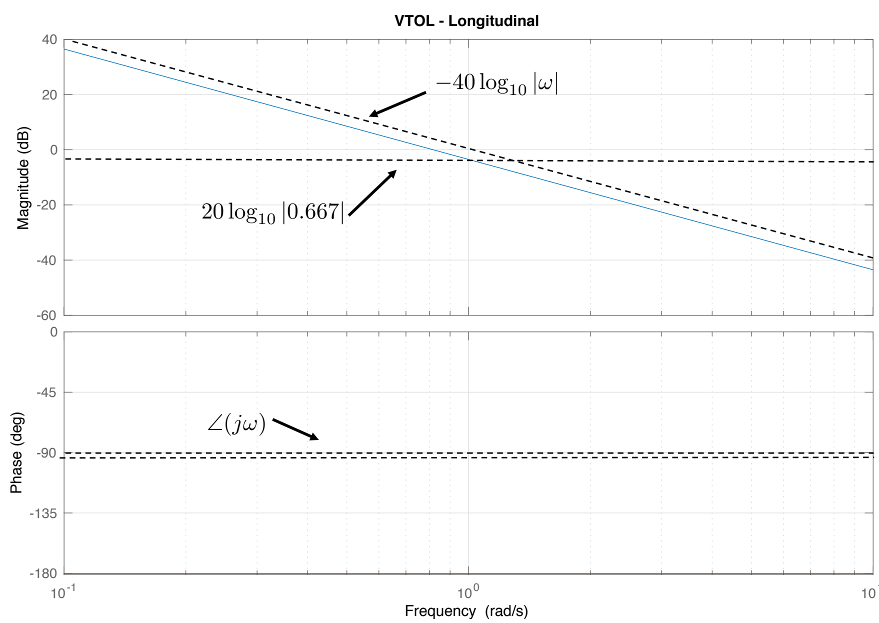

Figure 1: Bode plot for the transfer function given in Equation (1).

The Python command to generate the Bode plot is

```
>> import matplotlib.pyplot as plt
>> import control as cnt
>> P = tf([0.667], [1, 0, 0]);
>> plt.figure(1), clf, cnt.bode_plot(P), grid on
```

(b) From HW F.5, the transfer function for the inner loop of the VTOL lateral system is

$$P_{in}(s) = \frac{20.33}{s^2}. \tag{2}$$

In Bode canonical form we have

$$P_{in}(j\omega) = \frac{20.33}{(j\omega)^2}$$

Therefore

$$20\log_{10}|P_{in}(j\omega)| = 20\log_{10} 20.33 - 40\log_{10}|\omega|.$$

Therefore, the Bode plot for magnitude will be the graphical addition of a constant gain, and a line with slope of -40 dB/decade. Similarly, the phase is given by

$$\angle P_{in}(j\omega) = \angle 20.33 - \angle(j\omega) - \angle(j\omega) = 0 - 90 - 90 = -180 \text{ degrees}.$$

The straight line approximation as well as the Bode plot generated by Matlab are shown in Figure 2.

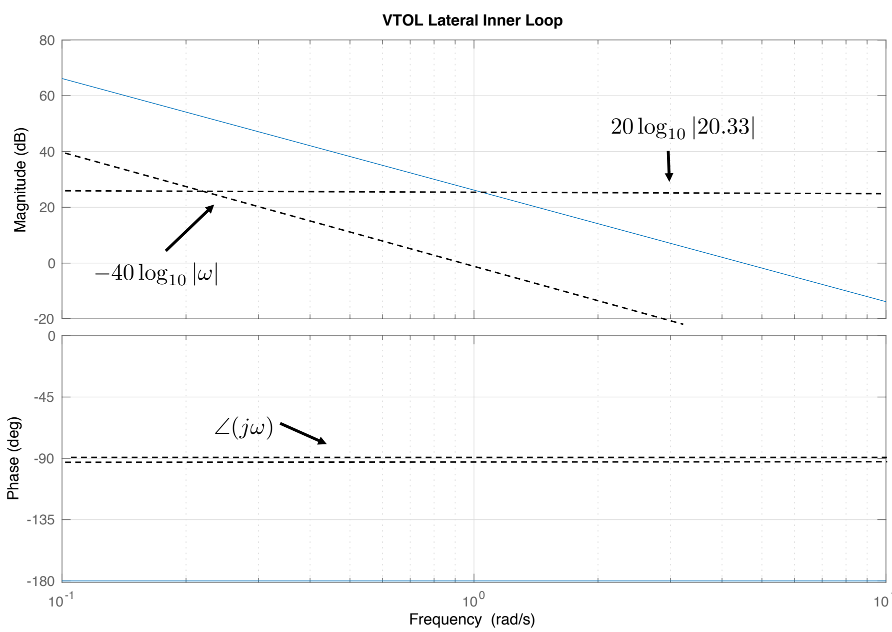

Figure 2: Bode plot for the transfer function given in Equation (2).

The Python command to generate the Bode plot is

```
>> import matplotlib.pyplot as plt
>> import control as cnt
>> Pin = tf([20.33], [1, 0, 0]);
>> plt.figure(1), clf, cnt.bode_plot(Pin), grid on
```

(c) From HW F.5, the transfer function for the VTOL lateral system is

$$P(s) = \frac{-a}{s(s+b)} = \frac{-9.81}{s(s+0.0667)}, \quad (3)$$

where

$$a = -\frac{F_e}{m_e + 2m_r}$$
$$b = \frac{\mu}{m_c + 2m_r}.$$

In Bode canonical form we have

$$P(j\omega) = \frac{-147.1}{(j\omega)(1 + j\frac{\omega}{0.0667})}$$

Therefore

$$20\log_{10}|P(j\omega)| = 20\log_{10}147.1 - 20\log_{10}|j\omega| - 20\log_{10}\left|1 + j\frac{\omega}{0.0667}\right|.$$

Therefore, the Bode plot for magnitude will be the graphical addition of a constant gain, an integrator, and a pole. Similarly, the phase is given by

$$\angle P(j\omega) = \angle - 147.1 - \angle(j\omega) - \angle(1 + j\frac{\omega}{0.1}).$$

The straight line approximation as well as the Bode plot generated by Matlab are shown in Figure 3.

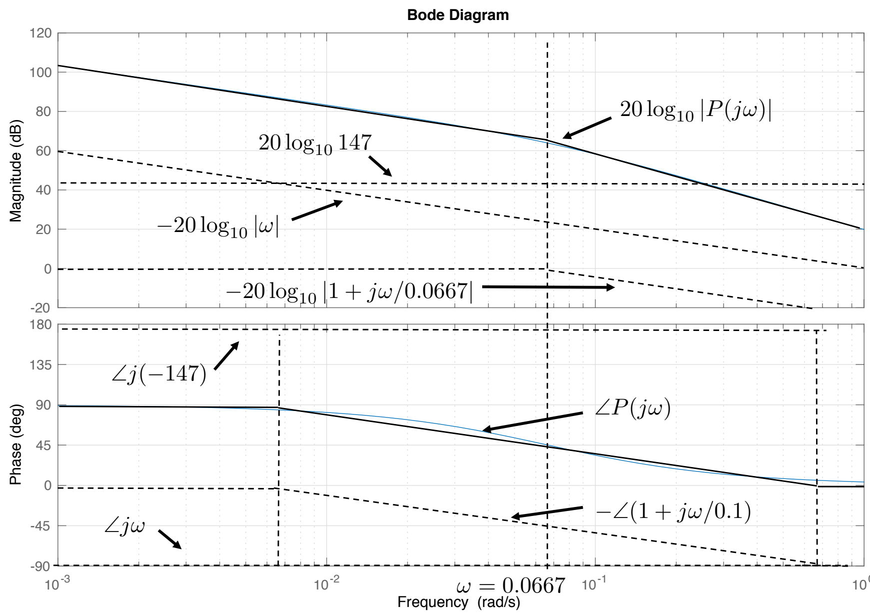

Figure 3: Bode plot for the transfer function given in Equation (3).

The Python command to generate the Bode plot is

```
>> import matplotlib.pyplot as plt
>> import control as cnt
>> Pout = tf([-9.8],[1, 0.0667, 0]);
>> plt.figure(1), clf, cnt.bode_plot(Pout), grid on
```