A Design Study Approach to Classical Control

Randal W. Beard Timothy W. McLain
Brigham Young University

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Homework D.17

For the mass spring damper, use the `bode` and `margin` commands (from Matlab or Python) to find the phase and gain margin for the closed loop system under PID control. On the same graph, plot the open loop Bode plot and the closed loop Bode plot. What is the bandwidth of the closed loop system, and how does this relate to the crossover frequency? Use the gains found in HW D.10.

Solution

The Matlab code used to generate the plots is shown below.

```
1 % transfer function for robot arm
2 Plant = tf([1/P.m],[1, P.b/P.m, P.k/P.m]);
3 C_pid = tf([(P.kd+P.kp*P.sigma),(P.kp+P.ki*P.sigma),P.ki],...
4             [P.sigma,1,0]);
5 % margin and bode plots
6 figure(1), clf, margin(Plant*C_pid), grid on, hold on
7 bode(Plant*C_pid/(1+Plant*C_pid))
8 legend('Open Loop-Inner', 'Closed Loop-Inner')
```

The transfer functions for the plant and controller are defined in Lines 2–3. The `margin` and `bode` plots of the open and closed loop system respectively, are generated in Lines 6–8. The results of this code are shown in Figure 1.

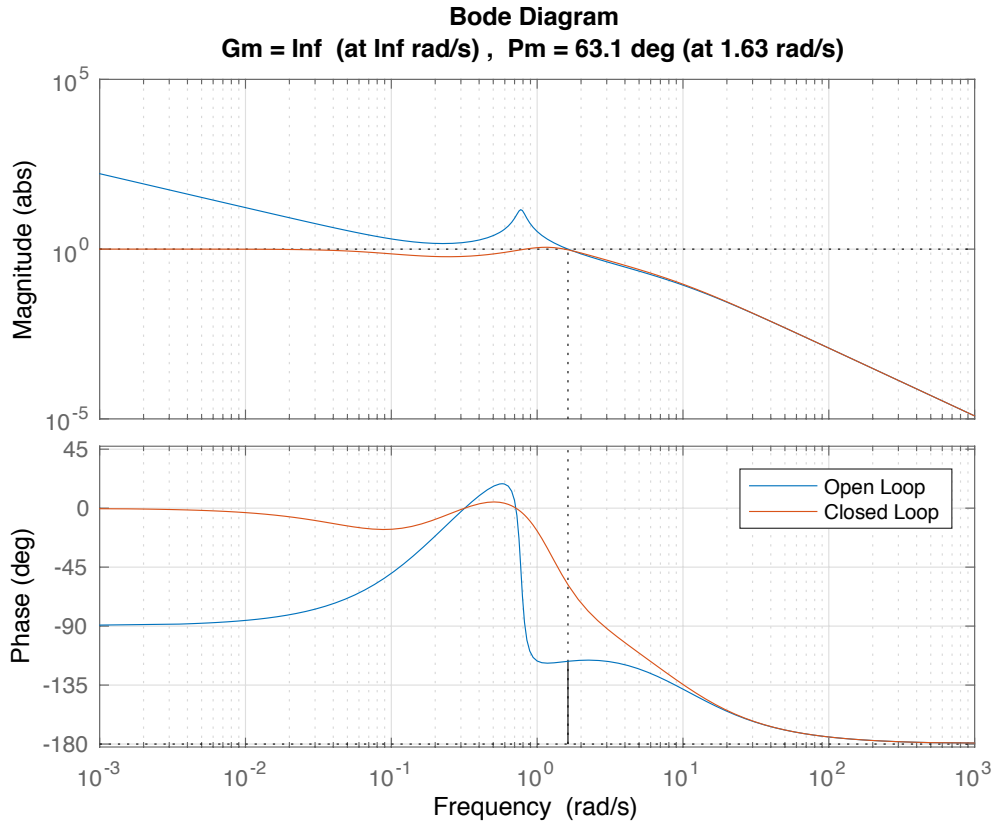

Figure 1: The `margin` plot of the open loop system and the `bode` plot of the closed loop system, of the mass spring damper under PID control.

As seen from Figure 1 the bandwidth of the closed-loop system is approximately 0.11 rad/s according to the definition of bandwidth (-3 dB or 0.71 magnitude), which is well below the crossover frequency. This sag in the closed-loop response is due to the low gain in the open-loop system at this frequency and could be remedied by turning up the proportional gain of the PID controller. The second crossing of the -3 dB line occurs at 2.1 rad/s which is in line with expectations for the closed-loop bandwidth given the crossover frequency of the open-loop system that occurs at 1.6 rad/s with a phase margin of 63 deg.