A Design Study Approach to Classical Control

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Homework A.8

For the single link robot arm, do the following:

(a) Suppose that the design requirements are that the rise time is $t_r \approx 0.8$ seconds, with a damping ratio of $\zeta = 0.707$. Find the desired closed loop characteristic polynomial $\Delta_{cl}^d(s)$, and the associated pole locations. Find the proportional and derivative gains k_P and k_D to achieve these specifications, and modify the Simulink simulation from HW **??**.8 to verify that the step response satisfies the requirements.

(b) Suppose that the input torque for the ball and beam system is limited to $|\tau| \leq \tau_{\max} = 1$ Nm. Modify the Simulink diagram to include a saturation block on the torque τ. Using the rise time t_r and damping ratio ζ as tuning parameters, tune the PD control law so that the input just saturates when a step of size 50 degrees is placed on $\tilde{\theta}^r$.

Solution

A rise time of $t_r \approx 0.8$ seconds, implies that the natural frequency is

$$\omega_n = 2.2/0.8 = 2.75.$$

Therefore, the desired characteristic polynomial is

$$\Delta_{cl} = s^2 + 2\zeta\omega_n s + \omega_n^2 = s^2 + 3.885s + 7.5625. \tag{1}$$

From HW **??**.8 the actual closed loop characteristic polynomial is

$$\Delta_{cl}(s) = s^2 + (0.667 + 66.67k_D)s + 66.67k_P. \tag{2}$$

Equating Equations (1) and (2) and solving for the gains gives

$$k_P = 0.1134$$
$$k_D = 0.0483.$$

The Simulink file associated with this problem is included on the wiki associated with the book.