A Design Study Approach to Classical Control

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Homework A.7

(a) Given the open loop transfer function found in problem **??**.6, find the open loop poles of the system, when the equilibrium angle is $\theta_e = 0$.

(b) Using the PD control architecture shown in Figure **??**, find the closed loop transfer function from θ_r to θ and find the closed loop poles as a function of k_P and k_D.

(c) Select k_P and k_D to place the closed loop poles at $p_1 = -3$ and $p_2 = -4$.

(d) Using the gains from part (c), implement the PD control for the single link robot arm in Simulink and plot the step response.

Hint: Change the s-function that defines the dynamics so that the output is the configuration variable θ. The Simulink block should look like that shown in Figure 1. Recall that the torque in the transfer function model is the feedback linearized torque $\tilde{\tau} = \tau - \tau_e$. Therefore, the torque applied to the robot arm is $\tau = \tau_e + \tilde{\tau}$. The feedback linearizing torque must be added to the torque computed by the PD controller as shown in Figure 1.

Solution

The open loop transfer function from homework **??**.6 is

$$\Theta(s) = \left(\frac{\frac{3}{m\ell^2}}{s^2 + \frac{3b}{m\ell^2} s} \right) \tilde{\tau}(s).$$

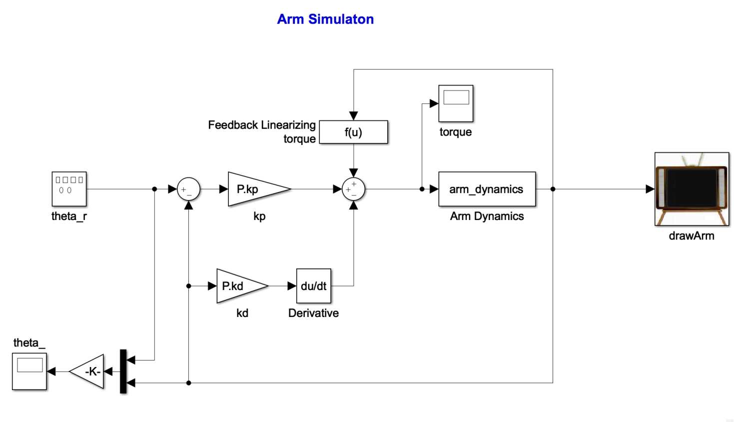

Figure 1: Simulink file for Homework **??**.8

Using the parameters from Section **??** gives

$$\Theta(s) = \left(\frac{66.67}{s^2 + 0.667s} \right) \tilde{\tau}(s).$$

The open loop poles are therefore the roots of the open loop polynomial

$$\Delta_{ol}(s) = s^2 + 0.667s,$$

which are given by

$$p_{ol} = 0, -0.667.$$

Using PD control, the closed loop system is therefore shown in Figure 2.

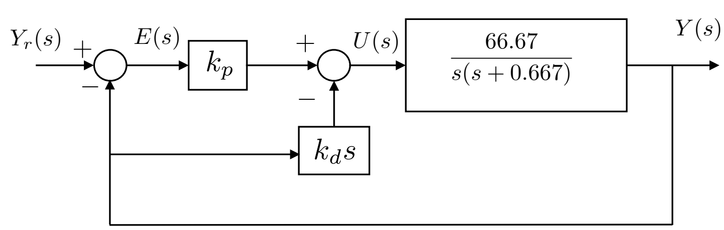

Figure 2: PD control of the single link robot arm.

The transfer function of the closed loop system is given by

$$
\begin{aligned}
\Theta(s) &= \left(\frac{66.67}{s^2 + 0.667s}\right)(k_P(\Theta^c(s) - \Theta(s)) - k_D s\Theta(s)) \\
\implies (s^2 + 0.667s)\Theta(s) &= (66.67k_P(\Theta^c(s) - \Theta(s)) - 66.67k_D s\Theta(s)) \\
\implies \left[(s^2 + 0.667s) + (66.67k_D s)\right]\Theta(s) &= 66.67k_P\Theta^c(s) \\
\implies (s^2 + (0.667 + 66.67k_D)s + 66.67k_P)\Theta(s) &= 66.67k_P\Theta^c(s) \\
\implies \Theta(s) &= \frac{66.67k_P}{s^2 + (0.667 + 66.67k_D)s + 66.67k_P}\Theta^c(s).
\end{aligned}
$$

Therefore, the closed loop poles are given by the roots of the closed loop characteristic polynomial

$$\Delta_{cl}(s) = s^2 + (0.667 + 66.67k_D)s + 66.67k_P$$

which are given by

$$p_{cl} = -\frac{(0.667 + 66.67k_D)}{2} \pm \sqrt{\left(\frac{(0.667 + 66.67k_D)}{2}\right)^2 - 66.67k_P}$$

If the desired closed loop poles are at -3 and -4, then the desired closed loop characteristic polynomial is

$$
\begin{aligned}
\Delta_{cl}^d &= (s+3)(s+4) \\
&= s^2 + 7s + 12.
\end{aligned}
$$

Equating the actual closed loop characteristic polynomial Δ_{cl} with the desired characteristic polynomial Δ_{cl}^d gives

$$s^2 + (0.667 + 66.67k_D)s + 66.67k_P = s^2 + 7s + 12,$$

or by equating each term we get

$$
\begin{aligned}
0.667 + 66.67k_D &= 7 \\
66.67k_P &= 12.
\end{aligned}
$$

Solving for k_P and k_D gives

$$
\begin{aligned}
k_P &= 0.18 \\
k_D &= 0.095.
\end{aligned}
$$

The Simulink implementation is contained on the wiki associated with this book.