A Design Study Approach to Classical Control

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Homework E.10

The objective of this problem is to implement the PID controller using only measured outputs of the system.

(a) Modify the system dynamics file so that the parameters m_1, m_2, and ℓ vary by up to 20% of their nominal value each time they are run (uncertainty parameter = 0.2).

(b) Change the simulation files so that the input to the controller is the output and not the state. Implement the nested PID loops in Problems E.8 in a separate class. Use the dirty derivative gain of $\tau = 0.05$. Tune the integrators so that there is no steady state error. The controller should only assume knowledge of the position z, the angle θ, and the reference position z_r.

(c) Note that the integrator gain will need to be negative which will cause problems for the anti-windup scheme that we have been implementing. Remove the old anti-windup scheme and implement a new scheme where the integrator only winds up when $|\dot{z}|$ is small.

Solution

The solution is on the wiki page associated with the book.

The Matlab code for the controller is shown below.

```python
import sys
sys.path.append('..')  # add parent directory
import ballbeamParam as P
import ballbeamParamHW10 as P10
from PIDControl import PIDControl

class ballbeamController:
    '''
        This class inherits other controllers in order to organize multiple contr
    '''

    def __init__(self):
        # Instantiates the SS_ctrl object
        self.zCtrl = PIDControl(P10.kp_z, P10.ki_z, P10.kd_z, P10.theta_max, P.be
        self.thetaCtrl = PIDControl(P10.kp_th, 0.0, P10.kd_th, P.Fmax, P.beta, P.

    def update(self, z_r, y):
        z = y.item(0)
        theta = y.item(1)
        # the reference angle for theta comes from the outer loop PD control
        theta_r = self.zCtrl.PID(z_r, z, flag=False)
        # the force applied to the cart comes from the inner loop PD control
        F_tilde = self.thetaCtrl.PID(theta_r, theta, flag=False)
        # feedback linearizing force
        F_fl = P.m1*P.g*(z/P.length) + P.m2*P.g/2.0
        # total force
        F = F_tilde + F_fl
        return F
```

```python
import numpy as np

class PIDControl:
    def __init__(self, kp, ki, kd, limit, beta, Ts):
        self.kp = kp                 # Proportional control gain
        self.ki = ki                 # Integral control gain
        self.kd = kd                 # Derivative control gain
        self.limit = limit           # The output will saturate at this limit
        self.beta = beta             # gain for dirty derivative
        self.Ts = Ts                 # sample rate

        self.y_dot = 0.0              # estimated derivative of y
        self.y_d1 = 0.0              # Signal y delayed by one sample
        self.error_dot = 0.0          # estimated derivative of error
```

```
        self.error_d1 = 0.0          # Error delayed by one sample
        self.integrator = 0.0        # integrator
        self.diff_flag = 0

    def PID(self, y_r, y, flag=True):
        '''
            PID control,

            if flag==True, then returns
                u = kp*error + ki*integral(error) + kd*error_dot.
            else returns
                u = kp*error + ki*integral(error) - kd*y_dot.

            error_dot and y_dot are computed numerically using a dirty derivative
            integral(error) is computed numerically using trapezoidal approximati
        '''

        # Compute the current error
        error = y_r - y
        # integral needs to go before derivative to update error_d1 correctly
        self.integrateError(error)
        # differentiate error and y
        self.differentiateError(error)
        self.differentiateY(y)

        # PID Control
        if flag is True:
            u_unsat = self.kp*error + self.ki*self.integrator + self.kd*self.erro
        else:
            u_unsat = self.kp*error + self.ki*self.integrator - self.kd*self.y_do
        # return saturated control signal
        u_sat = self.saturate(u_unsat)
        self.integratorAntiWindup(u_sat, u_unsat)
        return u_sat

    def PD(self, y_r, y, flag=True):
        '''
            PD control,

            if flag==True, then returns
                u = kp*error + kd*error_dot.
            else returns
                u = kp*error - kd*y_dot.

            error_dot and y_dot are computed numerically using a dirty derivative
```

```
        '''

        # Compute the current error
        error = y_r - y
        # differentiate error and y
        self.differentiateError(error)
        self.differentiateY(y)

        # PD Control
        if flag is True:
            u_unsat = self.kp*error + self.kd*self.error_dot
        else:
            u_unsat = self.kp*error - self.kd*self.y_dot
        # return saturated control signal
        u_sat = self.saturate(u_unsat)
        return u_sat

    def differentiateError(self, error):
        '''
            differentiate the error signal
        '''
        self.error_dot = self.beta*self.error_dot + (1-self.beta)*((error - self.
        self.error_d1 = error

    def differentiateY(self, y):
        '''
            differentiate y
        '''
        if self.diff_flag==0:
            self.y_d1 = y
            self.diff_flag = 1
        self.y_dot = self.beta*self.y_dot + (1-self.beta)*((y - self.y_d1) / self
        self.y_d1 = y

    def integrateError(self, error):
        '''
            integrate error
        '''
        self.integrator = self.integrator + (self.Ts/2)*(error+self.error_d1)

    def integratorAntiWindup(self, u_sat, u_unsat):
        # integrator anti - windup
        if self.ki != 0.0:
            self.integrator = self.integrator + self.Ts/self.ki*(u_sat-u_unsat);

```

```
    def saturate(self,u):
        if abs(u) > self.limit:
            u = self.limit*np.sign(u)
        return u
```