A Design Study Approach to Classical Control

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Homework F.18

For this homework assignment we will use the loopshaping design technique to design a controller for the VTOL flight control problem.

(a) First consider the longitudinal or altitude control problem. For this part, the force command will be

$$F = F_e + C_{lon}(s)E_h(s),$$

where F_e is the equilibrium force as found in other homework problems, and the error signal is $e_h = h_f^r - h_m$, where h_f^r is the prefiltered reference altitude command, and h_m is the measured altitude. Find the longitudinal controller $C_{lon}(s)$ so that the closed loop system meets the following specifications.

- Reject constant input disturbances.
- Track reference signals with frequency content below $\omega_r = 0.1$ rad/s to within $\gamma_r = 0.01$.
- Attenuate noise on the measurement of h for all frequencies above $\omega_n = 200$ rad/s by $\gamma_n = 10^{-4}$.
- The phase margin is close to $PM = 60$ deg.
- Use a prefilter to improve the closed loop response.

(b) Now consider the inner loop of the lateral, or position, dynamics. Find the inner loop controller $C_{lat_{in}}(s)$ so that the inner loop is stable with phase margin approximately $PM = 60$ deg, and with closed loop bandwidth approximately $\omega_{co} = 10$ rad/s. Add a low pass filter to enhance noise rejection.

(c) For the outer loop of the lateral system, find the controller $C_{lat_{out}}(s)$ to meet the following specifications:

- Cross over frequency is around $\omega_{co} = 1$ rad/s.
- Reject constant input disturbances.
- Attenuate noise on the measurement of z for all frequencies above $\omega_{no} = 100$ rad/s by $\gamma_{no} = 10^{-5}$.
- The phase margin is close to $PM = 60$ deg.
- Use a prefilter to reduce overshoot in the step response.

Solution

(a) From HW F.5, the transfer function model is

$$H(s) = \frac{\left(\frac{1}{m_c+2m_r}\right)}{s^2}\tilde{F}(s) = P_{lon}(s)\tilde{F}(s).$$

Figure 1 shows the Bode plot for P_{lon} together with the design specification on tracking and attenuating the noise. The requirement to reject constant input disturbances requires the inclusion of an integrator which is not shown in Figure 1.

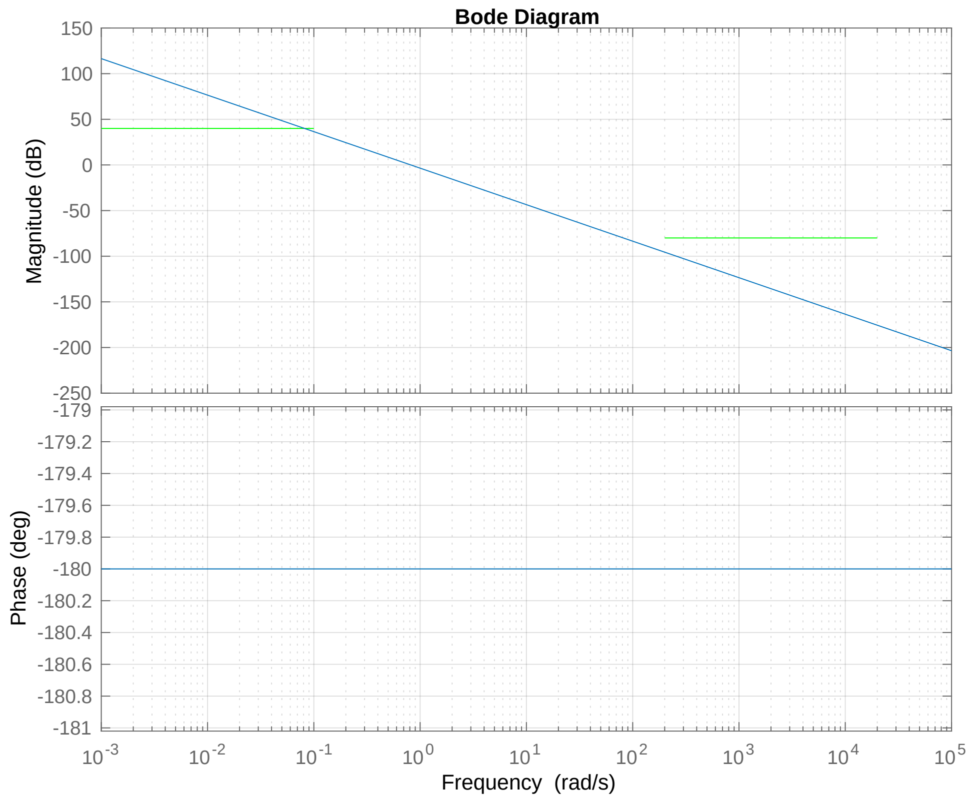

Figure 1: The Bode plot for the plant in HW F.18, together with the design specifications.

The first step is to add a phase lead element to stabilize the system. Figure 2 shows the loopgain with the addition of the phase lead filter

$$C_{lead} = \frac{45(s + 1/\sqrt{45})}{s + 1/\sqrt{45}},$$

and a proportional gain of $k_P = 0.4$.

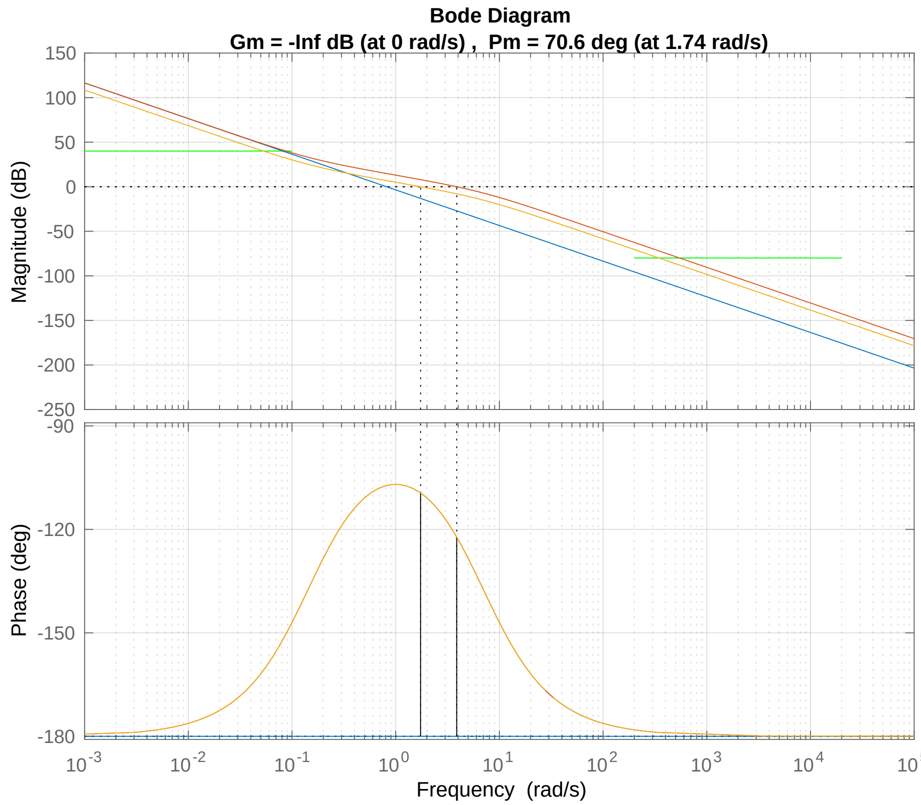

Figure 2: The Bode plot for the system in HW F.18, with phase lead, and proportional control.

To meet the requirement on rejecting input disturbances, an integrator is added, as shown in Figure 3 where

$$C_{int} = \frac{s + 0.5}{s}.$$

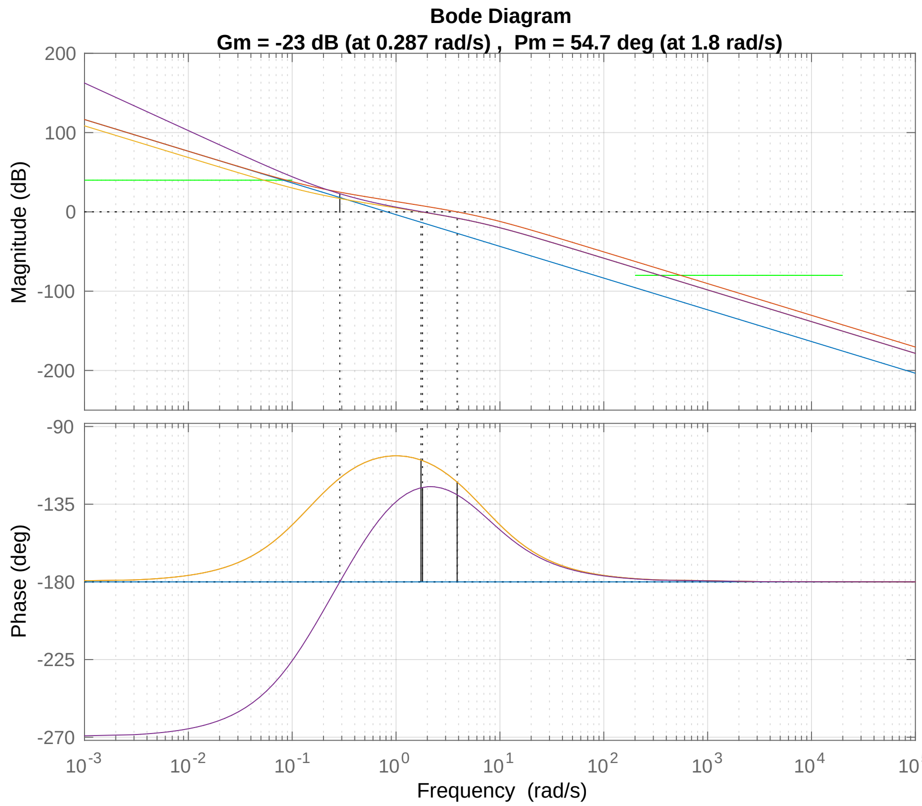

Figure 3: The Bode plot for the outer loop system in HW F.18, phase lead, proportional, andintegral control.

To meet the spec on noise attenuation, the low pass filter

$$C_{lpf} = \frac{50}{s+50}$$

and the result is shown in Figure 4.

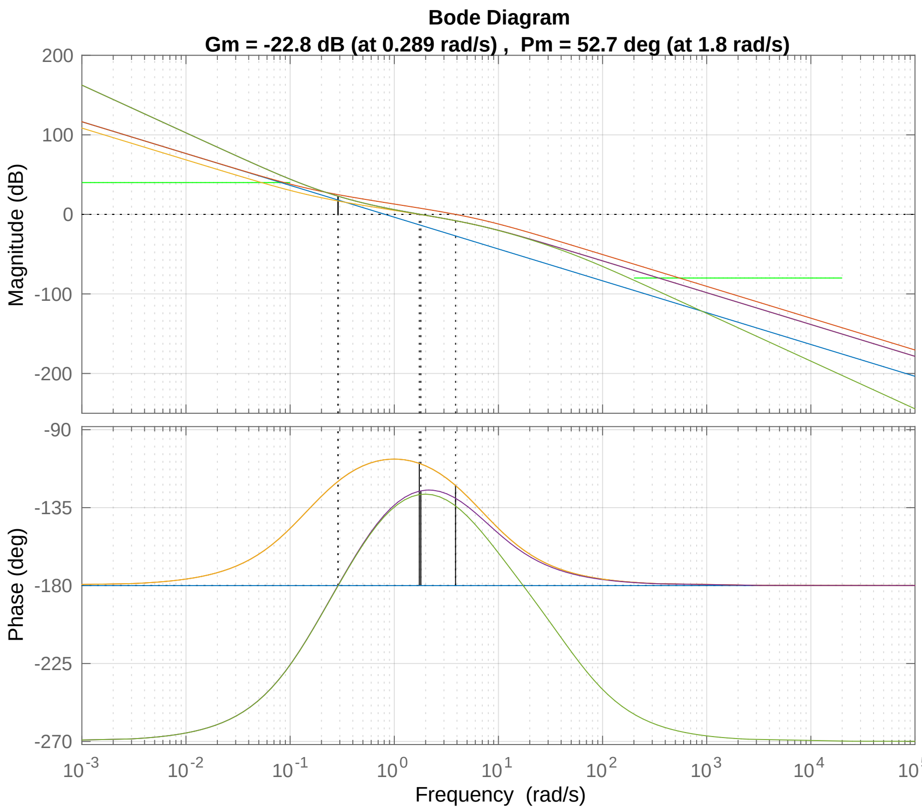

Figure 4: The Bode plot for the system in HW F.18, with phase lead, proportional, integral control, and a low pass filter.

Since the phase margin is $PM = 52.7$ degrees, we consider the phase margin specification to be satisfied. The resulting compensator is

$$C(s) = 0.4\left(\frac{45(s + 1/\sqrt{45})}{s + 1/\sqrt{45}}\right)\left(\frac{s + 0.5}{s}\right)\left(\frac{50}{s + 50}\right).$$

The closed loop response, as well as the unit step response for the output and control signal are all show in Figure 5, where the prefilter

$$F(s) = \frac{1}{s + 1}$$

has been added to reduce the peaking in the closed loop response.

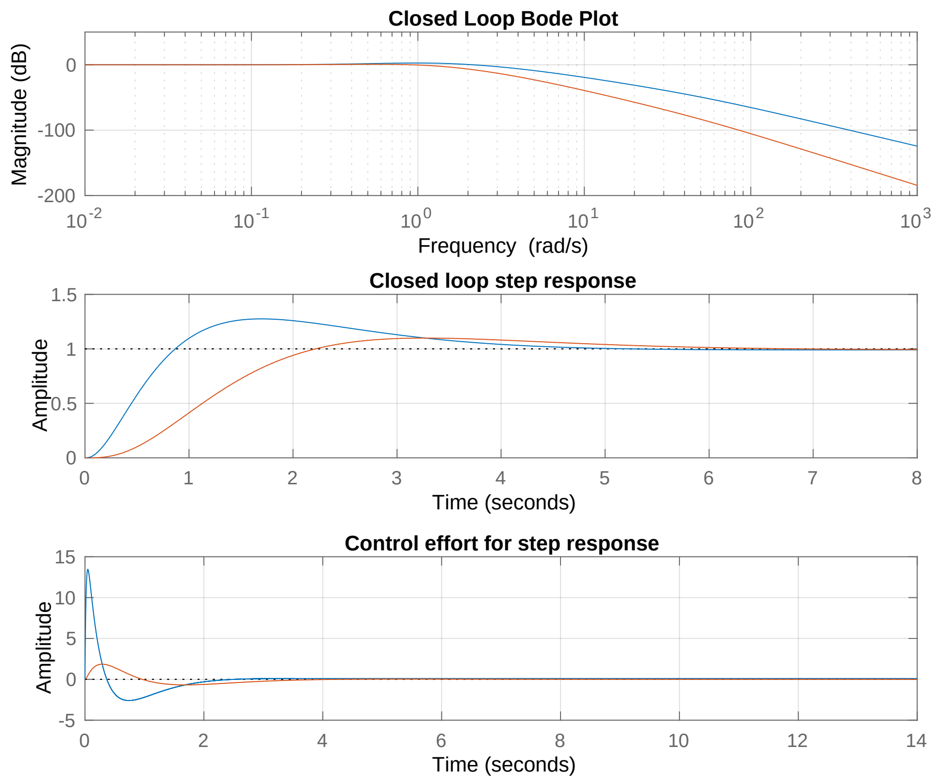

Figure 5: The closed loop bode response, the unit step response for the output, and the unit step response for the input for the design in HW F.18.

The Matlab code used to design the outer loop is shown below.

```
Plant = P_lon;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%  Define Design Specifications
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%--- general tracking specification ---
    omega_r = 10^(-1);  % track signals below this frequency
    gamma_r = 10^(-40/20);  % tracking error below this value
    w = logspace(log10(omega_r)-2,log10(omega_r));

```

```
%--- noise specification ---
    omega_n = 2*10^2;  % attenuate noise above this frequency
    gamma_n = 10^(-80/20);   % attenuate noise by this amount
    w = logspace(log10(omega_n),2+log10(omega_n));

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Control Design
  C = tf(1,1);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% phase lead: increase PM (stability)
    wmax = 1.0; % location of maximum frequency bump
    M    = 45; % separation between zero and pole
    Lead =tf(M*[1,wmax/sqrt(M)],[1,wmax*sqrt(M)]);
    C = C*Lead;

% proportional control: change cross over frequency
     kp = 0.4;
     C = C*kp;

% integral control:
     k_I = 0.5; % frequency at which integral action ends
     Integrator = tf([1,k_I],[1,0]);
     C = C*Integrator;

% low pass filter: decrease gain at high frequency (noise)
     p = 50;
     LPF = tf(p,[1,p]);
     C = C*LPF;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Prefilter Design
  F = tf(1,1);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% low pass filter
    p = 1;  % frequency to start the LPF
    LPF = tf(p, [1,p]);
    F = F*LPF;



%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Convert controller to state space equations
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
[num,den] = tfdata(C,'v');
[P.A_lon_C,P.B_lon_C,P.C_lon_C,P.D_lon_C]=tf2ss(num,den);

[num,den] = tfdata(F,'v');
[P.A_lon_F, P.B_lon_F, P.C_lon_F, P.D_lon_F] = tf2ss(num,den);

C_lon = C;
```

(b) From HW F.5, the transfer function model for the inner loop is

$$\tilde{\Theta}(s) = \frac{\left(\frac{1}{a_2}\right)}{s^2}\tilde{\tau}(s) = P_{lat_{in}}(s)\tilde{\tau}(s).$$

Figure 6 shows the Bode plot for $P_{lat_{in}}$.

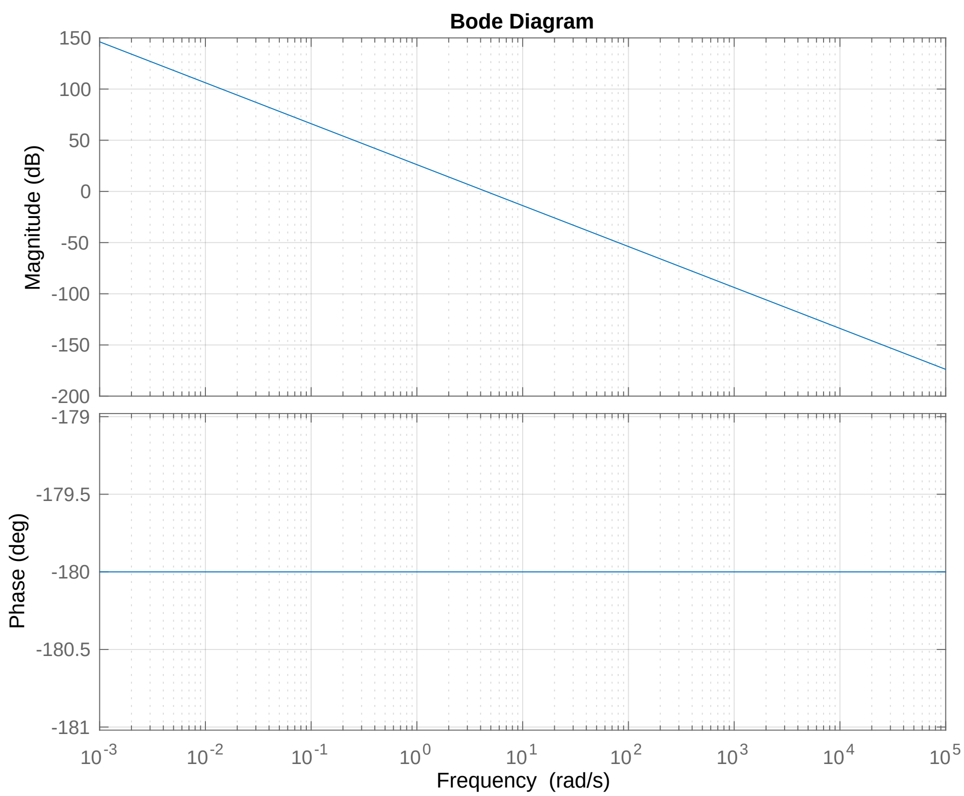

Figure 6: The Bode plot for $P_{lat_{in}}$ in HW F.18.

To stabilize the system, we add a phase lead element around the cross-over frequency. Figure 7 shows the loopgain with the addition of the phase lead filter

$$C_{lead} = \frac{15(s + 8/\sqrt{15})}{s + 8/\sqrt{15}}.$$

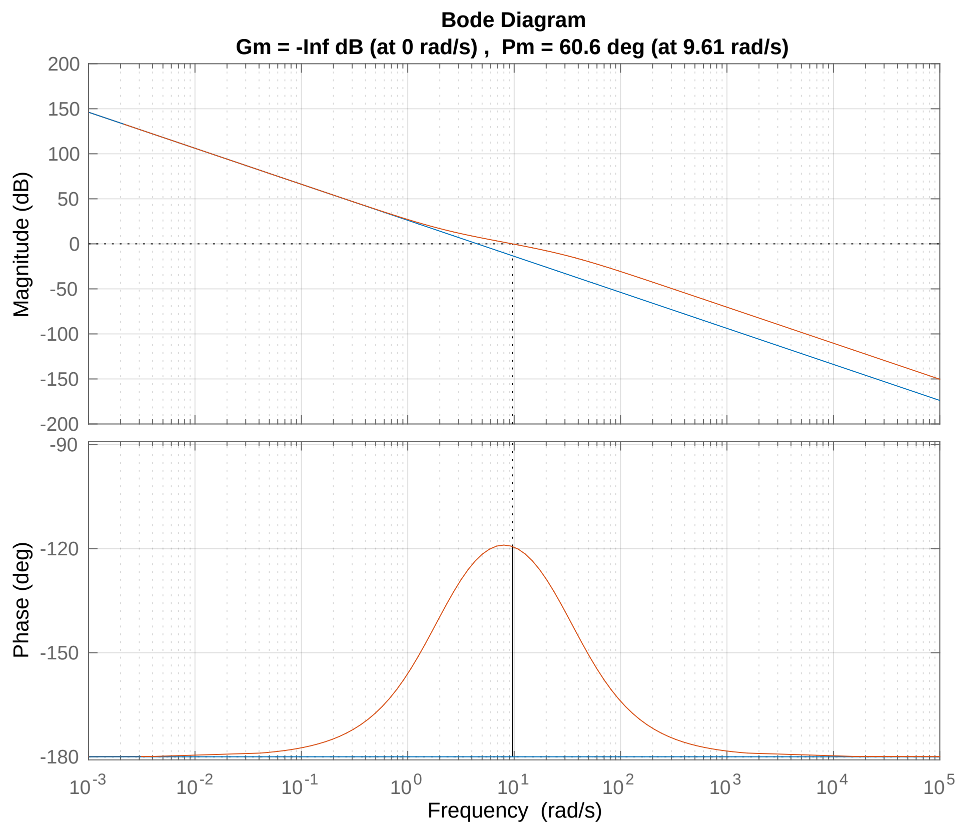

Figure 7: The Bode plot for the inner loop of the lateral system in HW F.18, with phase lead control.

Since the bandwidth and phase margin specifications are satisfied, we complete the design by adding the low pass filter

$$C_{lpf} = \frac{200}{s + 200}$$

and the result is shown in Figure 8.

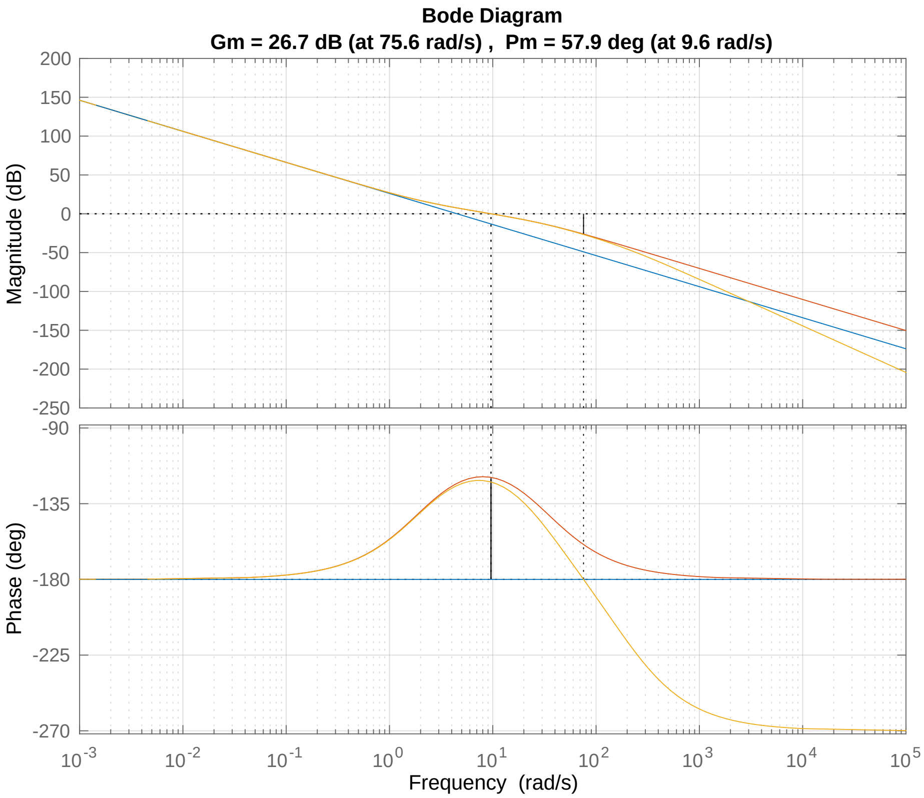

Figure 8: The Bode plot for the inner loop of the lateral system in HW F.18, with phase lead control, and a low pass filter.

The resulting compensator is

$$C(s) = \left(\frac{15(s + 8/\sqrt{15})}{s + 8\sqrt{15}}\right)\left(\frac{200}{s + 200}\right).$$

The closed loop response, as well as the unit step response for the output and control signal are all show in Figure 9.

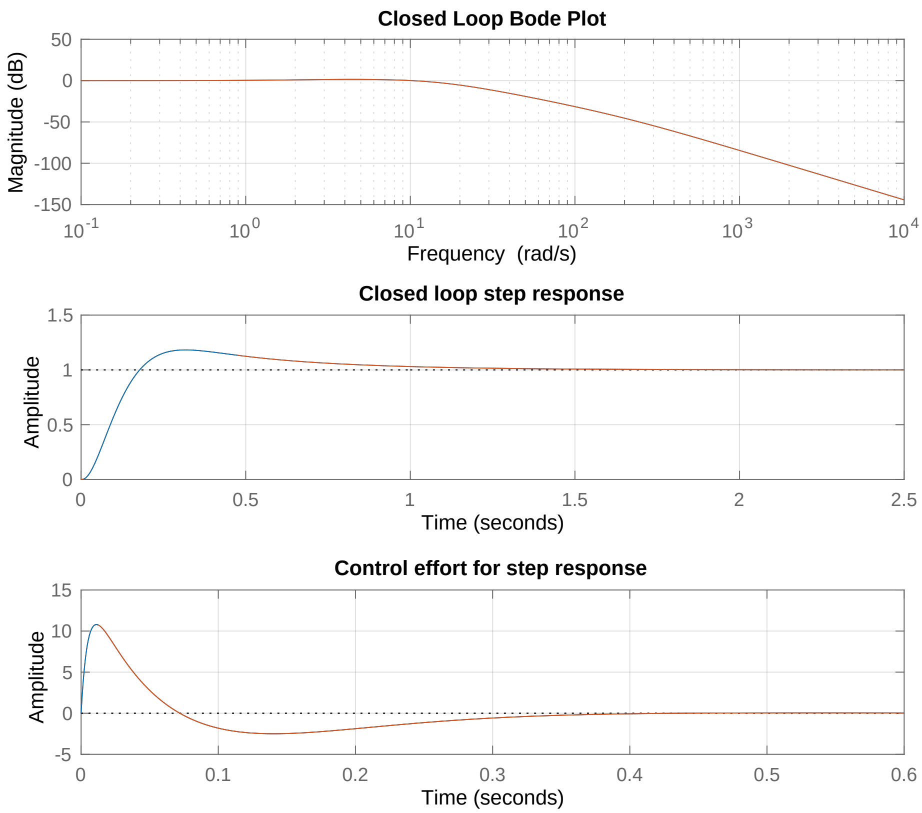

Figure 9: The closed loop bode response, the unit step response for the output, and the unit step response for the input for the inner loop lateral design in HW F.18.

The Matlab code used to design the outer loop is shown below.

```
Plant = P_lat_in;


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Control Design
  C = tf(1,1);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% phase lead: increase PM (stability)
    wmax = 8.0; % location of maximum frequency bump
```

```
    M    = 15; % separation between zero and pole
    Lead =tf(M*[1,wmax/sqrt(M)],[1,wmax*sqrt(M)]);
    C = C*Lead;

% low pass filter: decrease gain at high frequency (noise)
     p = 200;
     LPF = tf(p,[1,p]);
     C = C*LPF;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Convert controller to state space equations
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
[num,den] = tfdata(C,'v');
[P.A_lat_in_C,P.B_lat_in_C,P.C_lat_in_C,P.D_lat_in_C]=...
    tf2ss(num,den);

C_lat_in = C;
```

(c)

From HW F.5, the transfer function model is

$$Z(s) = \frac{-\left(\frac{F_0}{a_1}\right)}{s^2 + \frac{\mu}{a_1}} \tilde{\Theta}(s) = P_{lat_{out}}(s)\tilde{\Theta}(s).$$

Figure 10 shows the Bode plot for $P_{lat_{out}}$ together with the design specification on attenuating the noise. The requirement to reject constant input disturbances requires the inclusion of an integrator which is not shown in Figure 10.

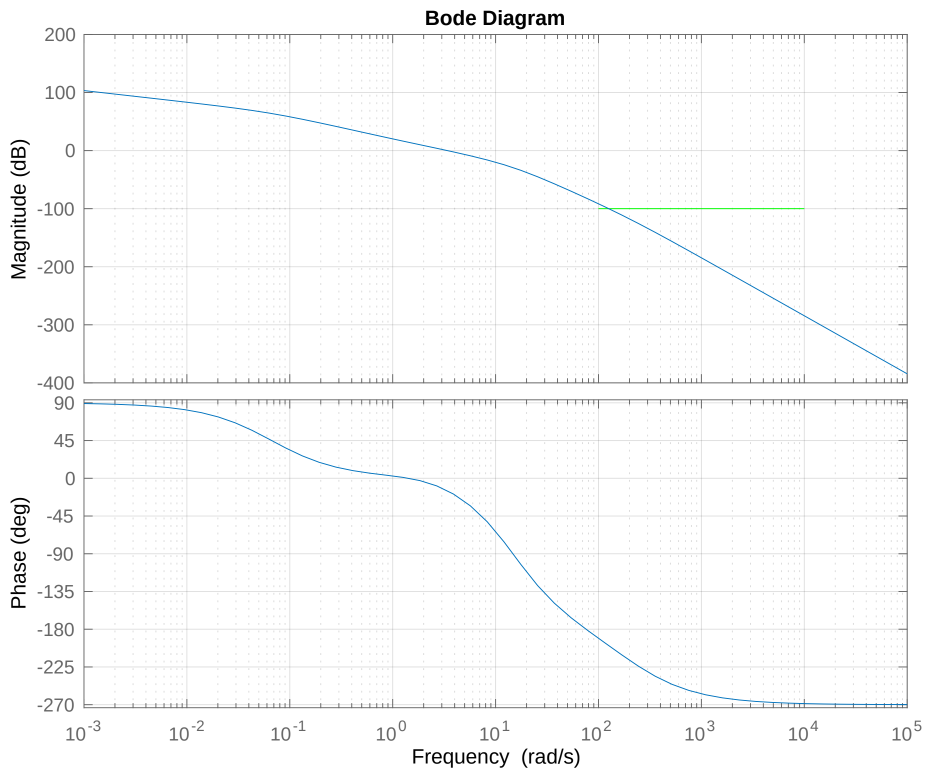

Figure 10: The Bode plot for the outer loop of the lateral system in HW F.18, together with the design specifications.

Since the gain of the open loop plant is negative, a negative proportional gain is needed to stabilize the system. To place the cross over frequency at the correct location, a proportional gain of $k_P = -0.03$ is selected. The resulting loop gain is shown in Figure 11

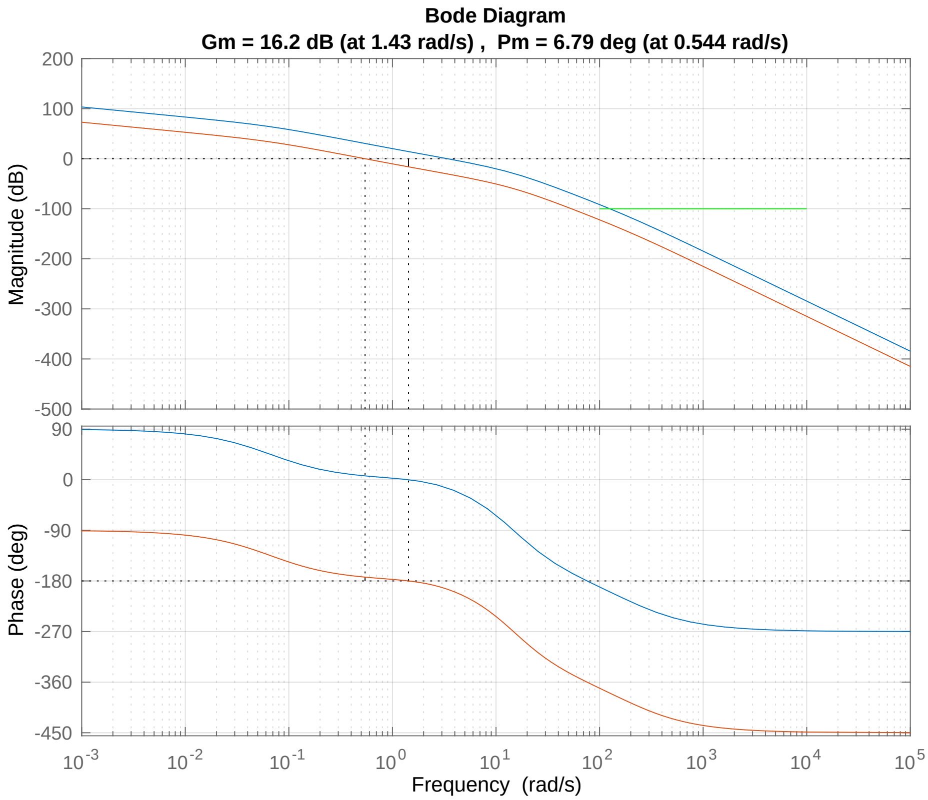

Figure 11: The Bode plot for the outer loop of the lateral system in HW F.18 with proportional gain $k_P = -0.03$.

To increase the phase margin, the phase lead element

$$C_{lead} = \frac{45(s + 1.3/\sqrt{45})}{s + 1.3\sqrt{45}},$$

is added, and the resulting loop gain is shown in Figure 12

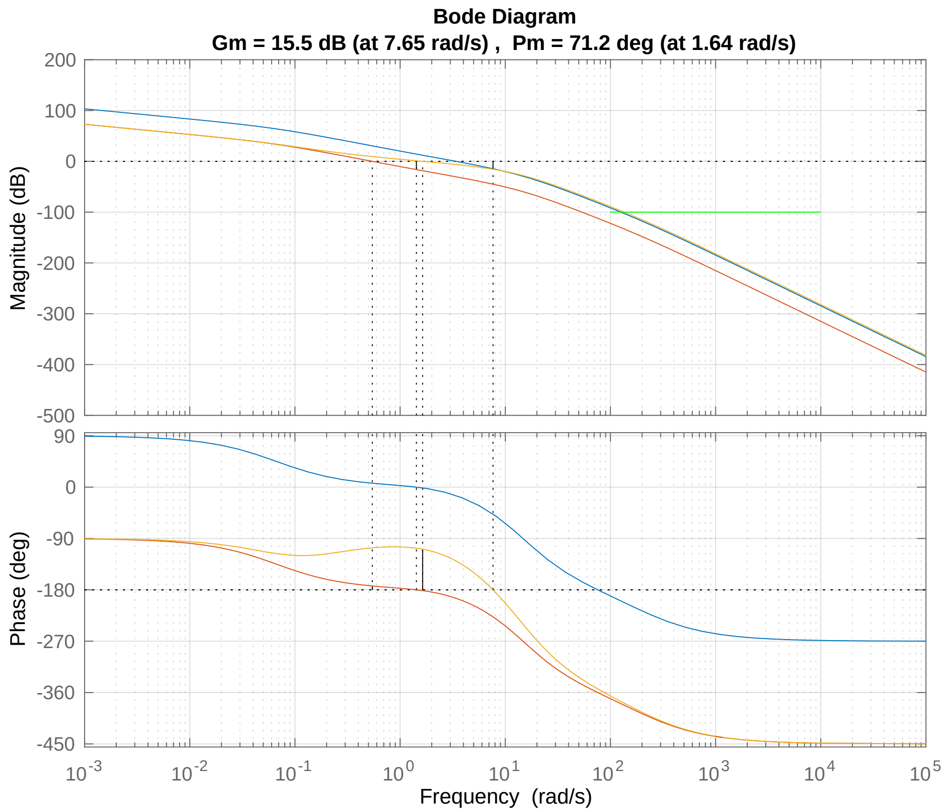

Figure 12: The Bode plot for the outer loop of the lateral system in HW F.18 with proportional gain, and phase lead control.

To meet the requirement on rejecting input disturbances, an integrator is added, as shown in Figure 13 where

$$C_{int} = \frac{s + 0.4}{s}.$$

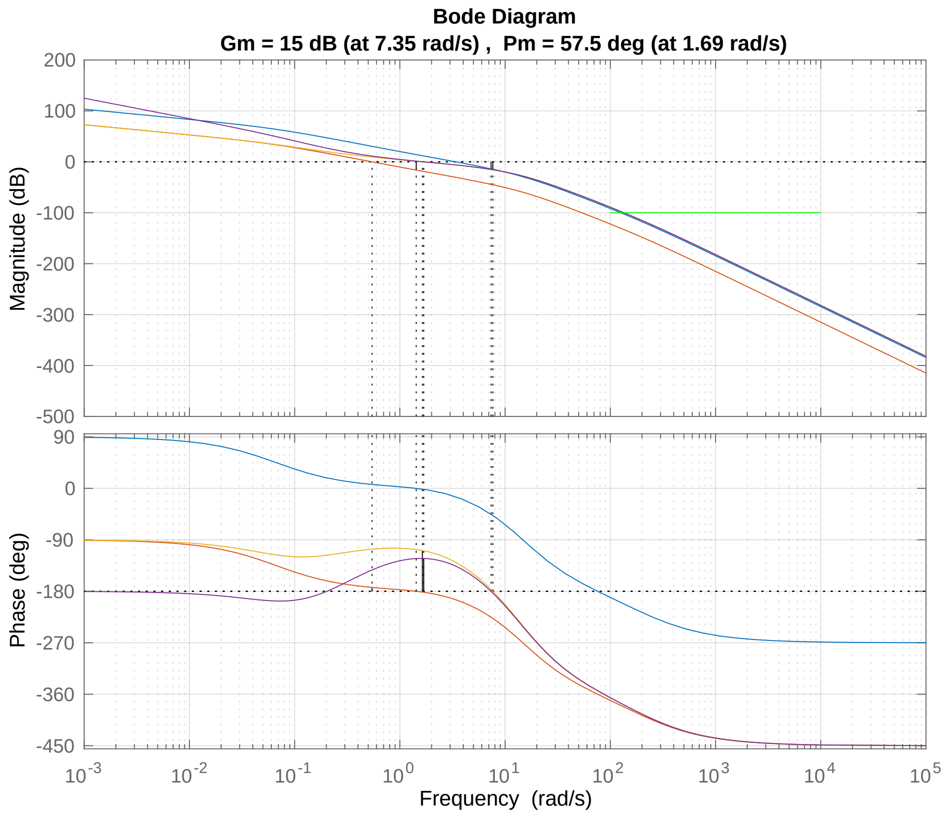

Figure 13: The Bode plot for the outer loop of the lateral system in HW F.18 with proportional gain, phase lead, and integral control.

To meet the spec on noise attenuation, the low pass filter

$$C_{lpf} = \frac{20}{s + 20}$$

and the result is shown in Figure 14.

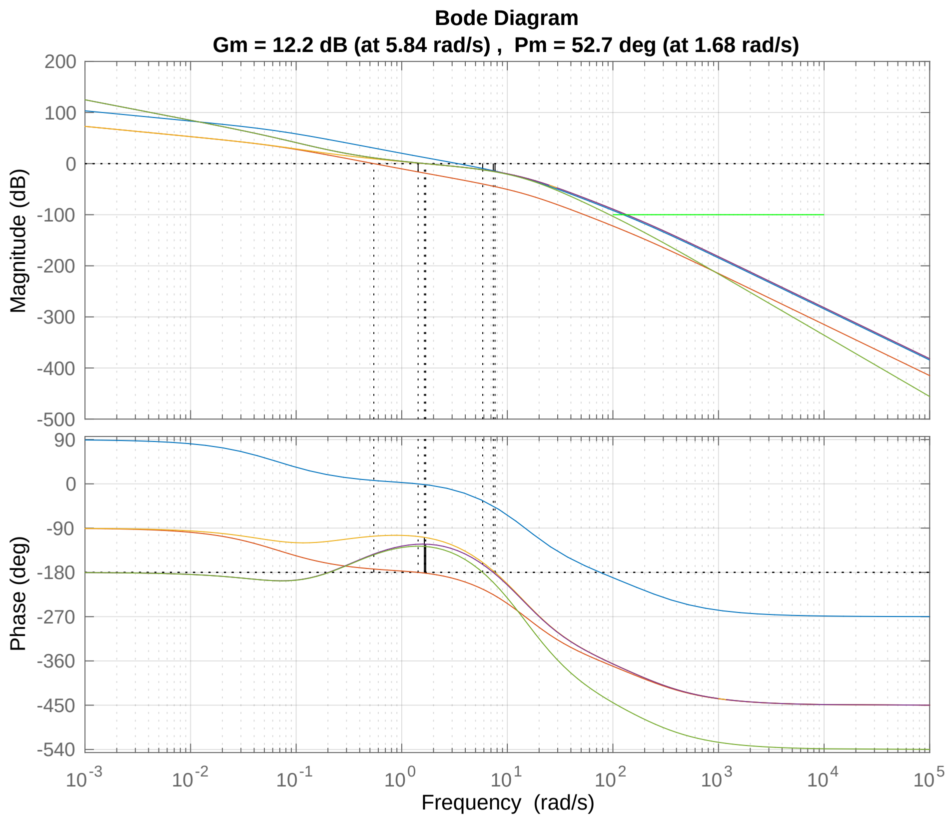

Figure 14: The Bode plot for the outer loop of the lateral system in HW F.18 with proportional gain, phase lead, integral control, and low pass filter.

Since the phase margin is $PM = 52.7$ degrees, we consider the phase margin specification to be satisfied. The resulting compensator is

$$C(s) = -0.03 \left(\frac{45(s + 1.3/\sqrt{45})}{s + 1.3\sqrt{45}} \right) \left(\frac{s + 0.4}{s} \right) \left(\frac{20}{s + 20} \right).$$

The closed loop response, as well as the unit step response for the output and control signal are all show in Figure 15, where the prefilter

$$F(s) = \frac{0.5}{s + 0.5}$$

has been added to reduce the peaking in the closed loop response.

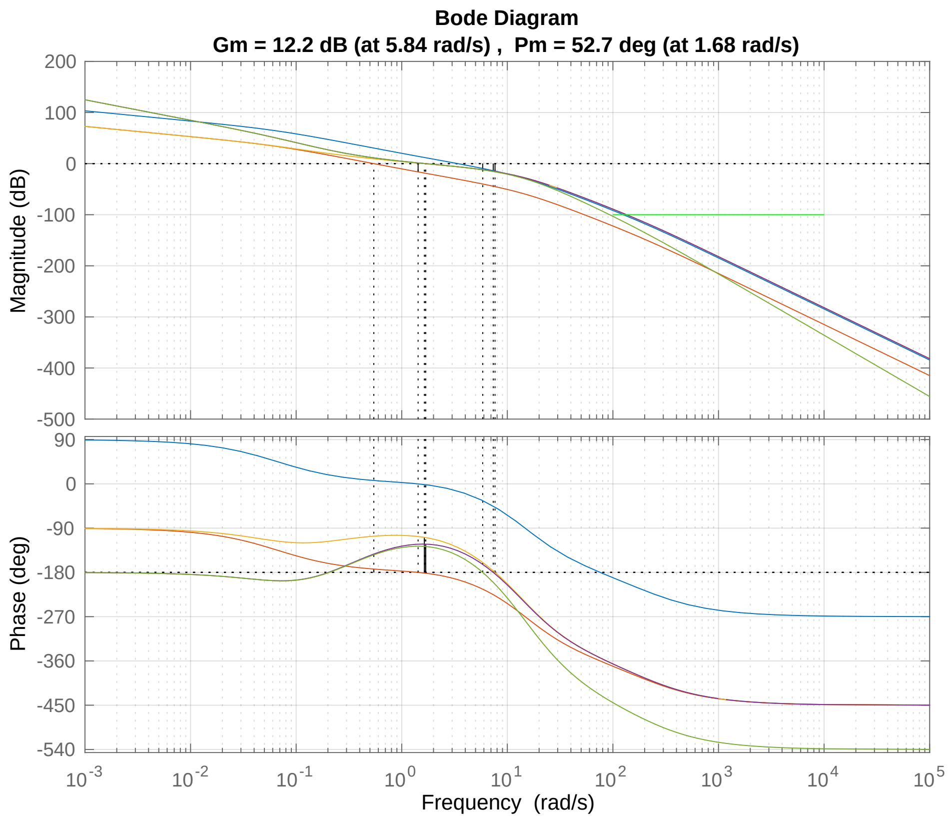

Figure 15: The closed loop bode response, the unit step response for the output, and the unit step response for the outer loop of the lateral control for the design in HW F.18.

The complete Simulink files are on the wiki associated with the book.