A Design Study Approach to Classical Control

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Homework C.6

Suppose that a star tracker is used to measure θ and a strain gage is used to approximate $\phi - \theta$. Defining the states as $x = (\theta, \phi, \dot{\theta}, \dot{\phi})^\top$, the input as $u = \tau$, and the measured output as $y = (\theta, \phi - \theta)^\top$, find the linear state space equations in the form

$$
\begin{aligned}
\dot{x} &= Ax + Bu \\
y &= Cx + Du.
\end{aligned}
$$

Solution

The equations of motion from HW **??**.3 are given by

$$
\begin{aligned}
\ddot{\theta} &= \frac{1}{J_s}\tau - \frac{b}{J_s}\dot{\theta} + \frac{b}{J_s}\dot{\phi} - \frac{k}{J_s}\theta + \frac{k}{J_s}\phi \\
\ddot{\phi} &= \frac{b}{J_p}\dot{\theta} - \frac{b}{J_p}\dot{\phi} + \frac{k}{J_p}\theta - \frac{k}{J_p}\phi.
\end{aligned}
$$

Suppose that a star tracker is used to measure θ and an on-board strain sensor is used to measure $\psi - \theta$. Defining the state $x \triangleq (x_1, x_2, x_3, x_4)^\top = (\theta, \phi, \dot{\theta}, \dot{\phi})^\top$, the input $u \triangleq \tau$, and the output $y \triangleq (y_1, y_2)^\top = (\theta, \phi - \theta)^\top$, the state space equations are given by

$$\dot{x} = \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{pmatrix} = \begin{pmatrix} \dot{\theta} \\ \dot{\phi} \\ \ddot{\theta} \\ \ddot{\phi} \end{pmatrix} = \begin{pmatrix} x_3 \\ x_4 \\ \frac{1}{J_s}u - \frac{b}{J_s}x_3 + \frac{b}{J_s}x_4 - \frac{k}{J_s}x_1 + \frac{k}{J_s}x_2 \\ \frac{b}{J_p}x_3 - \frac{b}{J_p}x_4 + \frac{k}{J_p}x_1 - \frac{k}{J_p}x_2 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k}{J_s} & \frac{k}{J_s} & -\frac{b}{J_s} & \frac{b}{J_s} \\ \frac{k}{J_p} & -\frac{k}{J_p} & \frac{b}{J_p} & -\frac{b}{J_p} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \frac{1}{J_s} \\ 0 \end{pmatrix} u.$$

Similarly

$$y = \begin{pmatrix} \theta \\ \phi - \theta \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 - x_1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix} u.$$

Therefore, the state space model is

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du,$$

where

$$A = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k}{J_s} & \frac{k}{J_s} & -\frac{b}{J_s} & \frac{b}{J_s} \\ \frac{k}{J_p} & -\frac{k}{J_p} & \frac{b}{J_p} & -\frac{b}{J_p} \end{pmatrix}$$

$$B = \begin{pmatrix} 0 \\ 0 \\ \frac{1}{J_s} \\ 0 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \end{pmatrix}$$

$$D = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$