A Design Study Approach to Classical Control

Randal W. Beard   Timothy W. McLain
Brigham Young University

Updated: December 2, 2016

Homework A.17

For the single link robot arm, use the Matlab `bode` and `margin` commands to find the phase and gain margin for the closed loop system under PID control. On the same graph, plot the open loop Bode plot and the closed loop Bode plot. What is the bandwidth of the closed loop system, and how does this relate to the crossover frequency? Use the gains found in HW A.10.

Solution

The Matlab code used to generate the plots is shown below.

```
% transfer functions for plant and controller
Plant = tf([2/P.m/P.ell^2],[1, 2*P.b/P.m/P.ell^2, 0]);
C_pid = tf([(P.kd+P.kp*P.sigma),(P.kp+P.ki*P.sigma),P.ki],...
            [P.sigma,1,0]);

% margin and bode plots for PID control
figure(1), clf, margin(Plant*C_pid), grid on, hold on
bode(Plant*C_pid/(1+Plant*C_pid))
legend('Open Loop', 'Closed Loop')
```

The transfer function for the plant is defined in Line 2. The transfer function for the PID controller is

$$C_{PID}(s) = k_P + \frac{k_I}{s} + \frac{k_D s}{\sigma s + 1} = \frac{s(\sigma s+1)k_P + (\sigma s+1)k_I + k_D s^2}{s(\sigma s+1)}$$
$$= \frac{(k_D + \sigma k_P)s^2 + (k_P + \sigma k_I)s + k_I}{s(\sigma s+1)},$$

and is defined in Line 3-4. The `margin` command is similar to the `bode` command except that it also annotates the Bode plot with the phase and gain margins. The open loop transfer functions are PC as defined in Line 7, and the closed loop transfer functions are $PC/(1+PC)$ as defined in Line 8. The results of this code are shown in Figure 1.

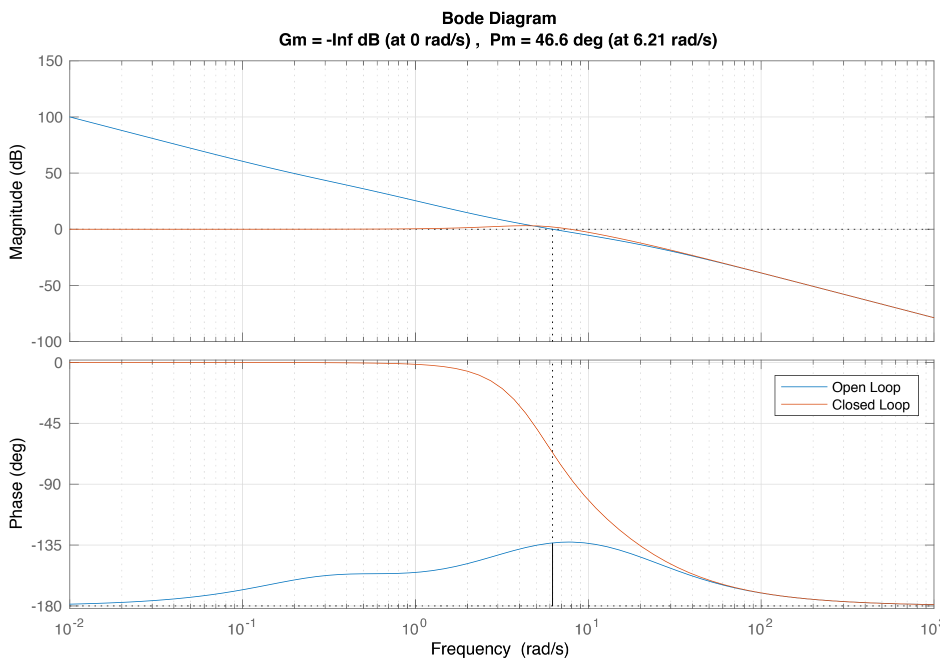

Figure 1: The `margin` plot of the open loop system and the `bode` plot of the closed loop system, of the single link robot arm under PID control.

As seen from Figure 1 the bandwidth for PID control is approximately 10 rad/sec, which is slightly larger than the cross over frequency of 6.21 rad/sec. The larger bandwidth is due to the small phase margin of $PM = 45.6$ degrees.