A Design Study Approach to Classical Control

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Homework D.e

Adding an integrator to obtain PID control for the mass spring damper, put the characteristic equation in Evan's form and use the Matlab `rlocus` command to plot the root locus verses the integrator gain k_I. Select a value for k_I that does not significantly change the other locations of the closed loop poles.

Solution

The closed loop block diagram including an integrator is shown in Figure 1. The characteristic equation is given by

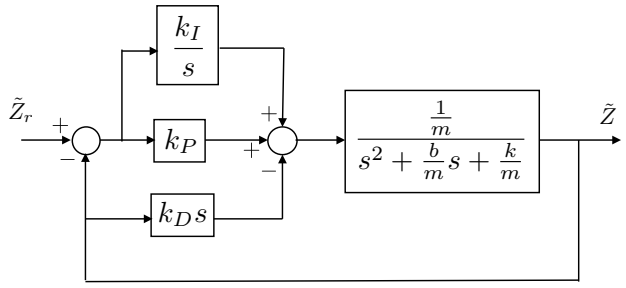

Figure 1: PID control for the mass spring damper.

$$1 + P(s)C(s) = 1 + \left(\frac{\frac{1}{m}}{s^2 + \frac{b}{m}s + \frac{k}{m}}\right)\left(\frac{k_D s^2 + k_P s + k_I}{s}\right) = 0.$$

Multiplying by the denominator and simplifying gives

$$s^3 + \left(\frac{b + k_K}{m}\right) s^2 + \left(\frac{k + K_P}{m}\right) s + \frac{k_I}{m} = 0.$$

In Evan's form we have

$$1 + k_I \left(\frac{\frac{1}{m}}{s^3 + \left(\frac{b+k_D}{m}\right) s^2 + \frac{k+k_P}{m} s} \right) = 0.$$

The appropriate Matlab command is therefore

```
>> L = tf([1/m],[1, (b+kd)/m, (k+kp)/m,0]);
>> figure(1), clf, rlocus(L);
```