A Design Study Approach to Classical Control

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Homework A.e

For the single link robot arm, use the PD gains derived in HW A.8. Add an integrator to the controller to get PID control. Put the resulting closed loop characteristic equation in Evan's form and use the Matlab `rlocus` command to plot the root locus verses the integrator gain k_I. Select a value for k_I that does not significantly change the other locations of the closed loop poles.

Solution

The closed loop block diagram including an integrator is shown in Figure 1. The closed loop transfer function is given by

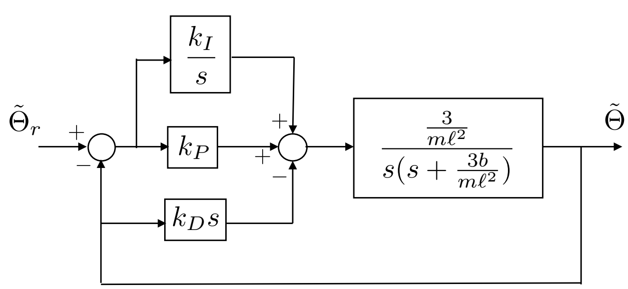

Figure 1: PID control for the single link robot arm.

$$\tilde{\Theta}(s) = \frac{\frac{3k_P}{m\ell^2}s + \frac{3k_I}{m\ell^2}}{s^3 + \left(\frac{3b+3k_D}{m\ell^2}\right)s^2 + \frac{3k_P}{m\ell^2}s + \frac{3k_I}{m\ell^2}}\tilde{\Theta}^d(s).$$

The characteristic equation is therefore

$$s^3 + \left(\frac{3b + 3k_D}{m\ell^2}\right) s^2 + \frac{3k_P}{m\ell^2} s + \frac{3k_I}{m\ell^2} = 0.$$

In Evan's form we have

$$1 + k_I \left(\frac{\frac{3}{m\ell^2}}{s^3 + \left(\frac{3b+3k_D}{m\ell^2}\right) s^2 + \frac{3k_P}{m\ell^2} s} \right) = 0.$$

The appropriate Matlab command is therefore

```
>> L = tf([3/m/L^2],[1,(3*b+3*kd)/m/L^2,3*kp/m/L^2,0]);
>> figure(1), clf, rlocus(L);
```