A Design Study Approach to Classical Control

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Homework E.15

(a) Draw by hand the Bode plot of the inner loop transfer function from force $\tilde{F}$ to angle $\tilde{\theta}$ for the ball beam system. Use the `bode` command from Matlab or Python and compare your results.

(b) Draw by hand the Bode plot of the outer loop transfer function from angle $\tilde{\theta}$ to position $\tilde{z}(t)$ for the ball beam system. Use the `bode` command and compare your results.

Solution

From HW E.5, the transfer function for the inner loop of the ball & beam is

$$P_{in}(s) = \frac{a}{s^2} = \frac{2.652}{s^2}, \tag{1}$$

where

$$a = \frac{\ell}{\frac{m_2\ell^2}{3} + m_1 z_e^2}.$$

In Bode canonical form we have

$$P_{in}(j\omega) = \frac{2.652}{(j\omega)^2}$$

Therefore

$$20\log_{10}|P_{in}(j\omega)| = 20\log_{10} 2.652 - 40\log_{10}|\omega|.$$

Therefore, the Bode plot for magnitude will be the graphical addition of a constant gain, and a line with slope of -40 dB/decade. Similarly, the phase is given by

$$\angle P_{in}(j\omega) = \angle 2.652 - \angle(j\omega) - \angle(j\omega) = 0 - 90 - 90 = -180 \text{ degrees.}$$

The straight line approximation as well as the Bode plot generated by Matlab are shown in Figure 1.

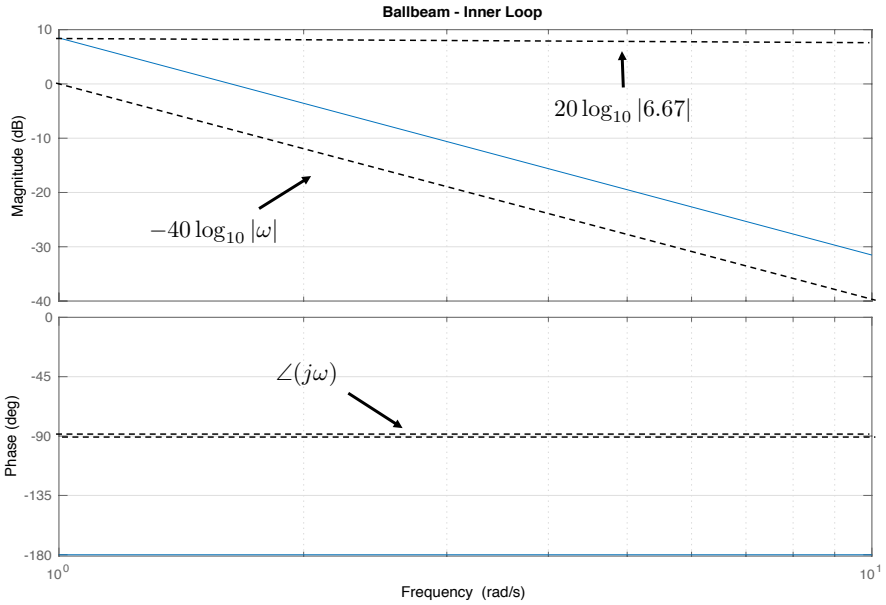

Figure 1: Bode plot for the transfer function given in Equation (1).

The Python command to generate the Bode plot is

```
>> import matplotlib.pyplot as plt
>> import control as cnt
>> Pin = tf([2.652], [1, 0, 0]);
>> plt.figure(1), clf, cnt.bode_plot(Pin), grid on
```

From HW E.5, the transfer function for the outer loop of the ball & beam is

$$P_{out}(s) = \frac{-g}{s^2} = \frac{-9.8}{s^2}. \tag{2}$$

In Bode canonical form we have

$$P_{out}(j\omega) = \frac{-9.8}{(j\omega)^2}.$$

Therefore

$$20\log_{10}|P_{out}(j\omega)| = 20\log_{10} 9.8 - 40\log_{10}|omega|$$

Similarly, the phase is given by

$$\angle P_{out}(j\omega) = \angle -9.8 - \angle(j\omega) - \angle(j\omega) = 180 - 90 - 90 = 0 \text{ degrees.}$$

The straight line approximation as well as the Bode plot generated by Matlab are shown in Figure 2.

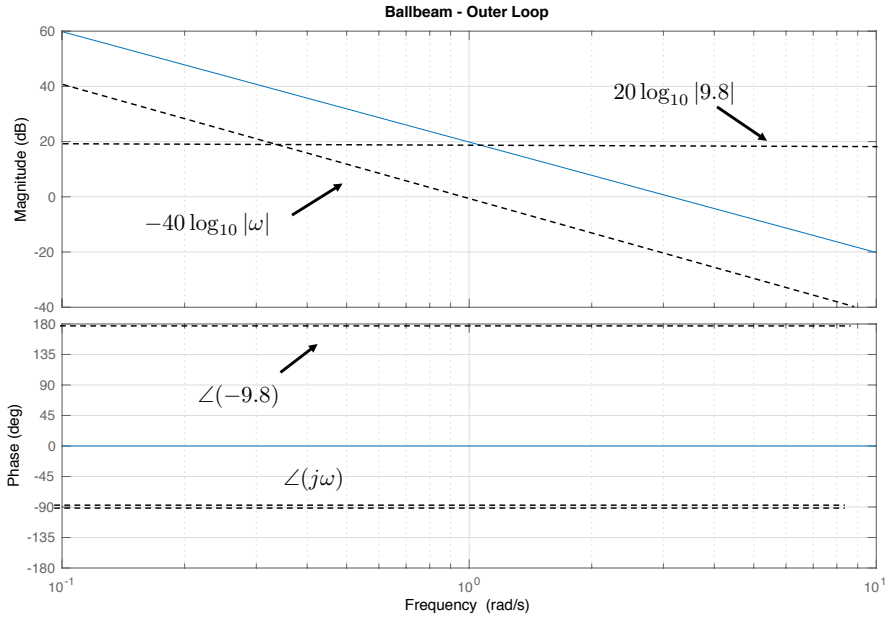

Figure 2: Bode plot for the transfer function given in Equation (2).

The Python command to generate the Bode plot is

```
>> import matplotlib.pyplot as plt
>> import control as cnt
>> Pout = tf([−9.8], [1, 0, 0]);
>> plt.figure(1), clf, cnt.bode_plot(Pout), grid on
```