A Design Study Approach to Classical Control

Randal W. Beard Timothy W. McLain
Brigham Young University

Updated: August 29, 2016

Homework B.16

For the inner loop of the inverted pendulum, use the Matlab `bode` command to create a graph that simultaneously displays the Bode plots for (1) the plant, and (2) the plant under PD control, using the control gains calculated in Homework B.10.

(a) To what percent error can the closed loop system track the desired input if all of the frequency content of $\theta_r(t)$ is below $\omega_r = 1.0$ radians per second?

(b) If all of the frequency content of the sensor noise on the inner $n(t)$ is greater than $\omega_{no} = 200$ radians per second, what percentage of the noise shows up in the output signal θ?

For the outer loop of the inverted pendulum, use the Matlab `bode` command to create a graph that simultaneously displays the Bode plots for the plant, and the plant under PID, using the control gains calculated in Homework B.10.

(c) If the reference signal $y_r(t)$ has frequency content below $\omega_r = 0.001$ radians/second, what is the tracking error under PID control if $|r(t)| \leq 50$?

Solution

(a) The Bode plot of the plant $P(s)$, and the loop gain with PD control $P(s)C_{PD}(s)$ are shown in Figure 1.

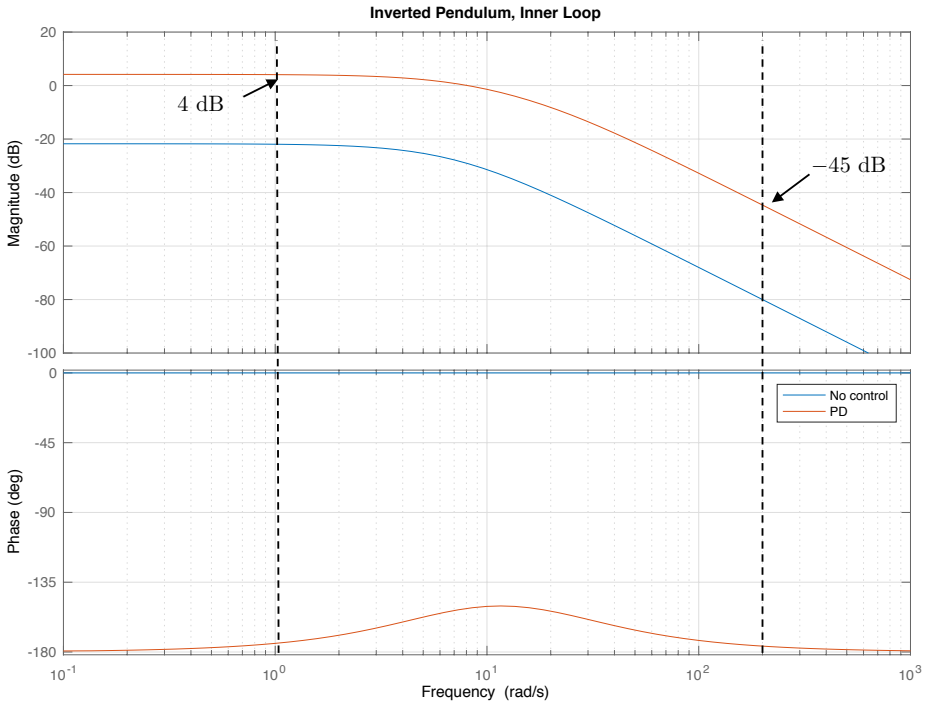

Figure 1: Bode plot for inner loop of the inverted pendulum, plant only, and under PD control.

From Figure 1 we see that below $\omega_r = 1.0$ rad/sec, the loop gain is above $B_r = 4$ dB. Therefore, from Equation (16.5) we have that

$$|e(t)| \leq \gamma_r \, |r(t)| \,,$$

where $\gamma_r = 10^{-4/20} = 0.63$, which implies that the tracking error will be 63% of the magnitude of the input.

(b) For noise greater than $\omega_{no} = 200$ rad/sec, we see from Figure 1 that $B_n = 45$ dB. Therefore, $\gamma_n = 10^{-45/20} = 0.0056$ which implies that 0.56% of the noise will show up in the output signal.

(c) The Bode plot of the outer loop $P(s)$, and the loop gain with PID control $P(s)C_{PID}(s)$ is shown in Figure 2.

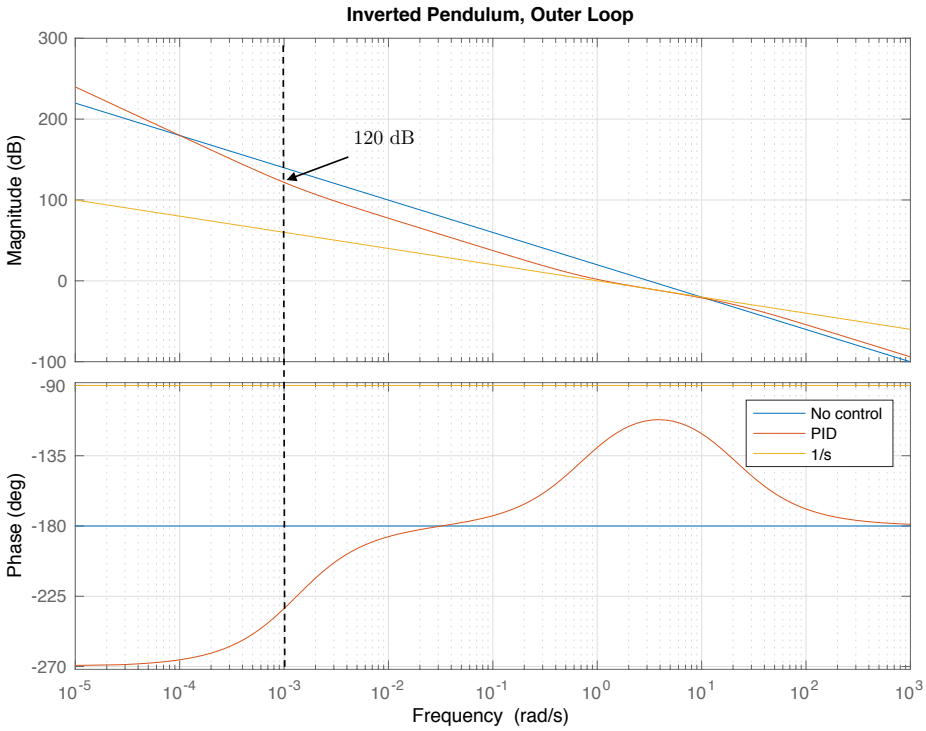

Figure 2: Bode plot for outer loop of the inverted pendulum, plant only, and under PID control.

From Figure 2 it can be seen that the loopgain under PID control for the outer loop is above $B_r = 120$ dB for all $\omega < \omega_r = 0.001$ radians/second. Therefore, from Equation (16.5) the tracking error satisfies

$$|e(t)| \leq \gamma_r \, |r(t)| \, ,$$

where $\gamma_r = 10^{-120/20} = 1.0e-06$. Therefore, if $|r(t)| \leq 50$, then $|e(t)| \leq 50e-06$.