A Design Study Approach to Classical Control

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Homework F.9

(a) When the longitudinal controller for the VTOL system is PD control, what is the system type? Characterize the steady state error when h^d is a step, a ramp, and a parabola. How does this change if you add an integrator? What is the system type with respect to an input disturbance for both PD and PID control?

(b) When the inner loop of the lateral controller for the VTOL system is PD control, what is the system type of the inner loop? Characterize the steady state error when $\tilde{\theta}^d$ is a step, a ramp, and a parabola. What is the system type with respect to an input disturbance?

(c) When the outer loop of the lateral controller for the VTOL system is PD control, what is the system type of the outer loop? Characterize the steady state error when z^d is a step, a ramp, and a parabola. How does this change if you add an integrator? What is the system type with respect to an input disturbance for both PD and PID control?

Solution

The closed loop system is shown in Figure 1.

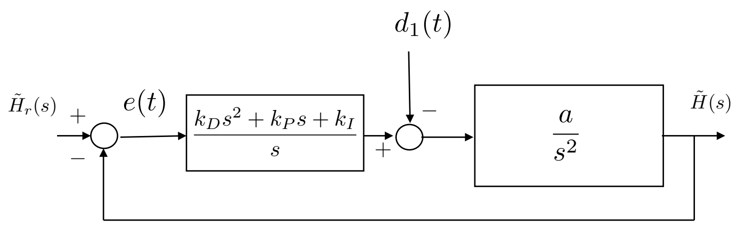

Figure 1: Closed loop system for problem HW F.9.

The open-loop transfer function is given by

$$P(s)C(s) = \left(\frac{a}{s^2}\right)\left(\frac{k_D s^2 + k_P s + k_I}{s}\right),$$

where

$$a = \frac{1}{m_c + 2m_r}.$$

Without the integrator ($k_I = 0$) the system has two free integrators and is therefore type 2, which, from Table **9-1** implies that the tracking error when the input is a step and ramp is zero and when it is a paraboloa is

$$\lim_{t\to\infty} e(t) = \frac{1}{1 + M_a} = \frac{1}{1 + \lim_{s\to 0} s^2 P(s)C(s)} = \frac{1}{ak_P}.$$

With the integrator, the system is type 3 and there is zero steady state error to a step, ramp, and parabola.

For the input disturbance, the transfer function from $D(s)$ to $E(s)$ is given by

$$\begin{aligned} E(s) &= \frac{P(s)}{1 + P(s)C(s)} D(s) \\ &= \frac{as}{s^3 + (ak_D)s^2 + (ak_P)s + (ak_I)}. \end{aligned}$$

When $D(s) = \frac{A}{s^{q+1}}$, the steady state error is given by

$$\begin{aligned} \lim_{t\to\infty} e(t) &= \lim_{s\to 0} \frac{sP}{1 + PC} \frac{A}{s^{q+1}} \\ &= \lim_{s\to 0} \frac{as}{s^3 + (ak_D)s^2 + (ak_P)s + (ak_I)} \frac{A}{s^q}. \end{aligned}$$

Without the integrator we have

$$\lim_{t\to\infty} e(t) = \frac{A}{k_P}.$$

when $q = 0$. Therefore, to an input disturbance the system is type 0, and the steady state error to a constant step of size A on $d(t)$ is $\frac{A}{k_P}$. With the integrator we have

$$\lim_{t\to\infty} e(t) = \frac{A}{k_I},$$

when $q = 1$. Therefore, to an input disturbance the system is type 1, and the steady state error to a constant ramp of size A on $d(t)$ is $\frac{A}{k_I}$.

The inner loop for the lateral control is identical to part (a) with only a difference in the constant a.

The block diagram for the outer loop of the lateral controller is shown in Figure 2.

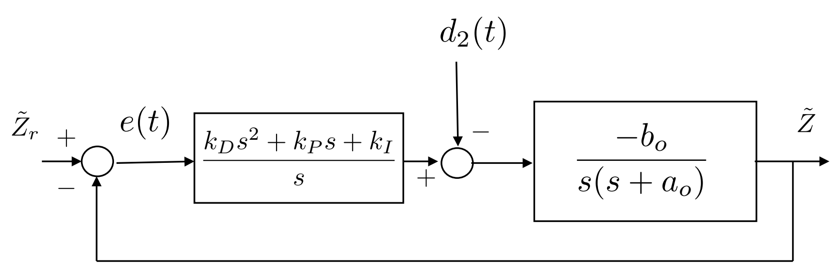

Figure 2: Outer loop system for problem HW F.9.

The open loop system is given by

$$P(s)C(s) = \left(\frac{-b_o}{s(s+a_o)}\right)\left(\frac{k_D s^2 + k_P s + k_I}{s}\right),$$

where

$$a_o = \frac{\mu}{m_c + 2m_r}$$
$$b_o = \frac{F_e}{m_c + 2m_r}.$$

When $k_I = 0$ there is one free integrator in $P(s)C(s)$ and the system is type 1, and from Table 9-1 the tracking error when the input is a step is

$$\lim_{t\to\infty} e(t) = \frac{1}{M_v} = \frac{1}{\lim_{s\to 0} sP(s)C(s)} = -\frac{a_o}{b_o k_P}.$$

When $k_I \neq 0$, there are two free integrators in $P(s)C(s)$ and the system is type 2. From Table 9-1 the tracking error when the input is a parabola is

$$\lim_{t\to\infty} e(t) = \frac{1}{M_a} = \frac{1}{\lim_{s\to 0} s^2 P(s)C(s)} = -\frac{a_o}{b_o k_I}.$$

For the input disturbance, the transfer function from $D(s)$ to $E(s)$ is given by

$$\begin{aligned} E(s) &= \frac{P(s)}{1 + P(s)C(s)} D(s) \\ &= \frac{-b_o s}{s^3 + (a_o - b_o k_D)s^2 + (-b_o k_P)s + (-b_o k_I)}. \end{aligned}$$

When $D(s) = \frac{A}{s^{q+1}}$, the steady state error is given by

$$\begin{aligned} \lim_{t\to\infty} e(t) &= \lim_{s\to 0} \frac{sP}{1 + PC} \frac{A}{s^{q+1}} \\ &= \lim_{s\to 0} \frac{-b_o s}{s^3 + (a_o - b_o k_D)s^2 + (-b_o k_P)s + (-b_o k_I)} \frac{A}{s^q}. \end{aligned}$$

Without the integrator we have

$$\lim_{t\to\infty} e(t) = \frac{A}{k_P}$$

when $q = 0$. Therefore, to an input disturbance the system is type 0, and the steady state error to a constant step of size A on $d(t)$ is $\frac{A}{k_P}$. With the integrator we have

$$\lim_{t\to\infty} e(t) = \frac{A}{k_I},$$

when $q = 1$. Therefore, to an input disturbance the system is type 1, and the steady state error to a constant ramp of size A on $d(t)$ is $\frac{A}{k_I}$.