A Design Study Approach to Classical Control

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Homework E.d

Adding an integrator to obtain PID control for the outer loop of the ballbeam system, put the characteristic equation in Evan's form and use the Matlab `rlocus` command to plot the root locus verses the integrator gain k_I. Note that since the integrator gain will be negative, the transfer function in Evan's form must be negated since the Matlab `rlocus` command only plots the return for $k_I > 0$. Select a value for k_I that does not significantly change the other locations of the closed loop poles.

Solution

The closed loop block diagram for the outer loop of the ball on beam system including an integrator is shown in Figure 1. The characteristic equation is

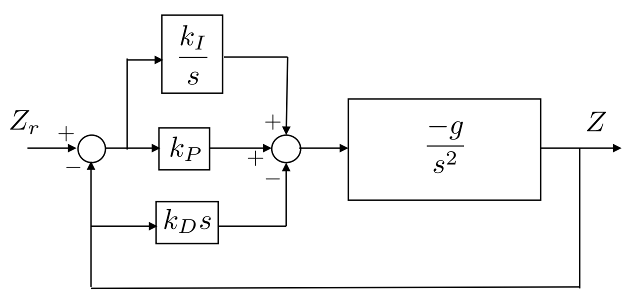

Figure 1: PID control for the outer loop of the ball on beam system.

given by

$$1 + P(s)C(s) = 1 + \left(\frac{-g}{s^2}\right)\left(\frac{k_D s^2 + k_P s + k_I}{s}\right) = 0.$$

Rearranging gives

$$s^3 + (-gk_D)s^2 + (-gk_P)s + (-gk_I) = 0.$$

Therefore, in Evan's form we have

$$1 + k_I\left(\frac{-g}{s^3 + (-gk_D)s^2 + (-gk_P)s}\right) = 0.$$

The appropriate Matlab command is therefore

```
>> L = tf([-g], [1, -g*kd, -g*kp, 0]);
>> figure(1), clf, rlocus(L);
```