A Design Study Approach to Classical Control

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Homework D.4

For the mass spring damper:

(a) Find the equilibria of the system.

(b) Linearize the system about the equilibria using Jacobian linearization.

(c) Linearize the system using feedback linearization.

Solution

The differential equation describing the mass spring damper derived in HW D.3 is

$$m\ddot{z} + b\dot{z} + kz = F. \tag{1}$$

Defining $x = (z, \dot{z})$, and $u = F$ we get

$$\dot{x} = \begin{pmatrix} \dot{z} \\ \ddot{z} \end{pmatrix} = \begin{pmatrix} \dot{z} \\ \frac{1}{m}F - \frac{b}{m}\dot{z} - \frac{k}{m}z \end{pmatrix} \stackrel{\triangle}{=} f(x, u).$$

The equilibrium is when $f(x_e, u_e) = 0$, or in other words when

$$z_e = \text{ anything }, \qquad \dot{z}_e = 0, \qquad F_e = kz_e. \tag{2}$$

Equivalently, at equilibrium there is no motion in the system, which implies that $\ddot{z}_e = \dot{z}_e = 0$, which from Equation (1) implies that $F_e = kz_e$. Any pair (z_e, F_e) satisfying Equation (2) is an equilibria.

The equations of motion for this system are linear and do not require linearization. With that said, the techniques of feedback linearization can be helpful for systems requiring a non-zero control effort (input) to acheive a specific equilibrium output. In the case of the mass-spring-damper system, a non-zero force is required to hold the mass at any non-zero equilibrium position, due to the spring force generated when the position of the mass is not zero.