A Design Study Approach to Classical Control

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Homework A.16

For the single link robot arm, use the Matlab `bode` command to create a graph that simultaneously displays the Bode plots for (1) the plant, (2) the plant under PID control, using the control gains calculated in HW A.10.

(a) To what percent error can the closed loop system under PID control track the desired input if all of the frequency content of $\theta_r(t)$ is below $\omega_r = 0.4$ radians per second?

(b) If the reference input is $\theta_r(t) = 5t^2$ for $t \geq 0$, what will be the steady state tracking error to this input when using PID control?

(c) If all of the frequency content of the input disturbance $d_{in}(t)$ is below $\omega_{d_{in}} = 0.01$ radians per second, what percentage of the input disturbance shows up in the output θ under PID control?

(d) If all of the frequency content of the noise $n(t)$ is greater than $\omega_{no} = 100$ radians per second, what percentage of the noise shows up in the output signal θ?

Solution

(a) The Bode plot of the plant $P(s)$, and the loop gain with PID control $P(s)C_{PID}(s)$ is shown in Figure 1.

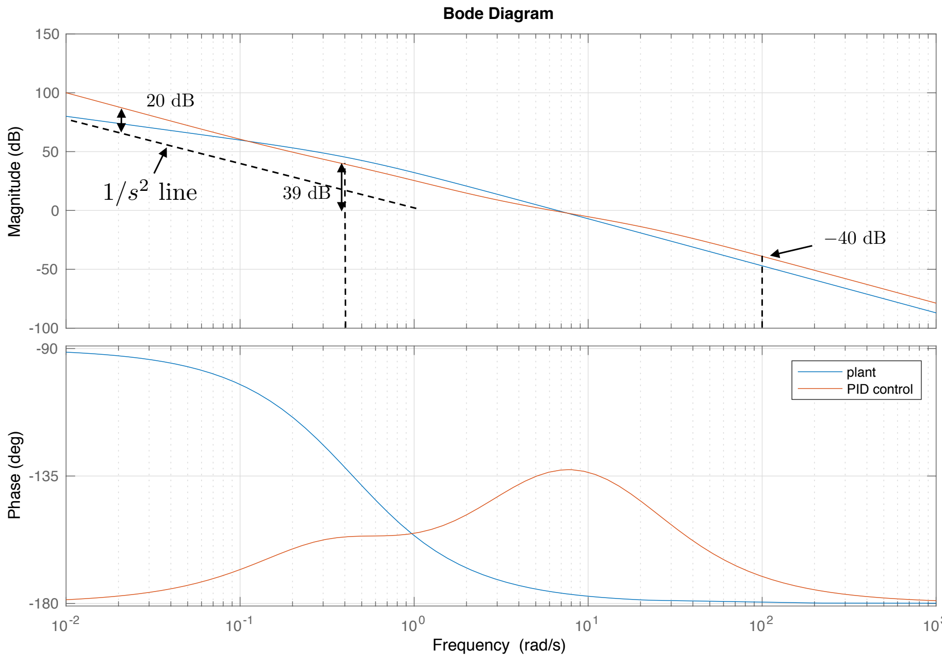

Figure 1: Bode plot for single link arm, plant only and under PID control.

From Figure 1 we see that below $\omega_r = 0.4$ rad/sec, the loop gain is above $B_r = 39$ dB. Therefore, from Equation (16.5) we have that

$$|e(t)| \leq \gamma_r \, |r(t)| \, ,$$

where $\gamma_r = 10^{-39/20} = 0.1059$, which implies that the tracking error will be 10.6% of the magnitude of the input.

(b) Suppose now that the desired reference input is $\theta^d(t) = 5t^2$. Under PID control, the slope of the loop gain as $\omega \to 0$ is -40 dB/dec, which implies that the system is type 2 and will track a step and a ramp with zero steady state error. For a parabola, there will be a finite error. Since the loop gain under PID control is $B_2 = 20$ dB above the $1/s^2$ line, from Equation (**??**) the steady state error satisfies

$$\lim_{t\to\infty} |e(t)| \leq \frac{A}{M_a} = 5 \cdot 10^{-20/20} = 0.5.$$

(c) If the input disturbance is below $\omega_{d_{in}} = 0.01$ rad/s, then the difference between the loop gain and the plant is $B_{d_{in}} = 20$ dB at $\omega_{d_{in}} = 0.01$ rad/s, therefore, from Equation (16.9) we see that

$$\gamma_{d_{in}} = 10^{-20/20} = 0.1,$$

implying that 10% of the input disturbance will show up in the output.

(d) For noise greater than $\omega_{no} = 100$ rad/sec, we see from Figure 16.9 that $B_n = 40$ dB. Therefore, $\gamma_n = 10^{-40/20} = 0.01$ which implies that 1% of the noise will show up in the output signal.201