A Design Study Approach to Classical Control

Randal W. Beard Timothy W. McLain
Brigham Young University

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Homework A.5

For the single link robot arm, find the transfer function of the system from the torque τ to the angle θ.

Solution

As shown in Section **??**, the feedback linearized model for the single link robot arm is given by Equation (**??**) as

$$\frac{m\ell^2}{3}\ddot{\theta} = \tilde{\tau} - b\dot{\theta}. \tag{1}$$

Taking the Laplace transform of Equation (1) and setting all initial conditions to zero we get

$$\frac{m\ell^2}{3}s^2\Theta(s) + bs\Theta(s) = \tilde{\tau}(s).$$

Solving for $\Theta(s)$ gives

$$\Theta(s) = \left(\frac{1}{\frac{m\ell^2}{3}s^2 + bs}\right)\tilde{\tau}(s).$$

The canonical form for transfer functions is for the leading coefficient in the denominator polynomial to be unity. This is called monic form. Putting the transfer function in monic form results in

$$\Theta(s) = \left(\frac{\frac{3}{m\ell^2}}{s^2 + \frac{3b}{m\ell^2}s}\right)\tilde{\tau}(s), \tag{2}$$

where the expression in the parenthesis is the transfer function from $\tilde{\tau}$ to θ, where $\tilde{\tau}$ indicate that we are working with feedback linearized control in Equation (**??**). The block diagram associated with Equation (2) is shown in Figure 1

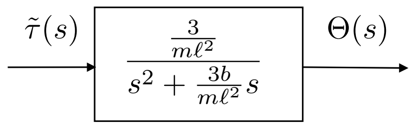

Figure 1: A block diagram of the single link robot arm.