A Design Study Approach to Classical Control

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Homework F.e

(a) Adding an integrator to obtain PID control for the longitudinal controller for the VTOL system, put the characteristic equation in Evan's form and use the Matlab/Python `rlocus` command to plot the root locus verses the integrator gain k_{I_h}. Select a value for k_{I_h} that does not significantly change the other locations of the closed loop poles.

(b) Adding an integrator to obtain PID control for the outer loop of the lateral controller for the VTOL system, put the characteristic equation of the outer loop in Evan's form and use the Matlab/Python `rlocus` command to plot the root locus verses the integrator gain k_{I_z}. Select a value for k_{I_z} that does not significantly change the other locations of the closed loop poles.

Solution

For the longitudinal controller, the closed loop block diagram including an integrator is shown in Figure 1. The characteristic equation is given by

$$1 + P(s)C(s) = 1 + \left(\frac{a}{s^2}\right)\left(\frac{k_D s^2 + k_P s + k_I}{s}\right) = 0,$$

where

$$a = \frac{1}{m_c + 2m_r}.$$

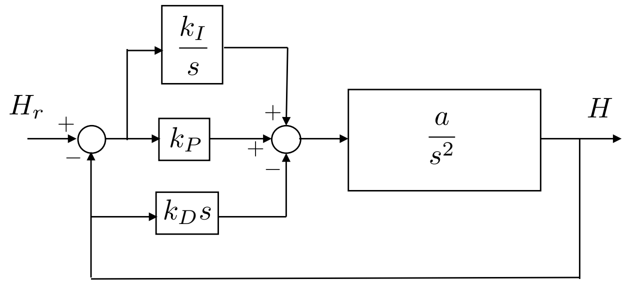

Figure 1: PID control for the longitudinal control of the VTOL system.

Multiplying by the denominator and simplifying gives

$$s^3 + ak_D s^2 + ak_P s + ak_I = 0.$$

Therefore, in Evan's form we have

$$1 + k_I \left(\frac{a}{s^3 + ak_D s^2 + ak_P s} \right) = 0.$$

The appropriate Matlab command is therefore

```
>> L = tf([a],[1, a*kd, a*kp,0]);
>> figure(1), clf, rlocus(L);
```

For the outer loop of the lateral controller, the closed loop block diagram including an integrator is shown in Figure 2. The characteristic equation is

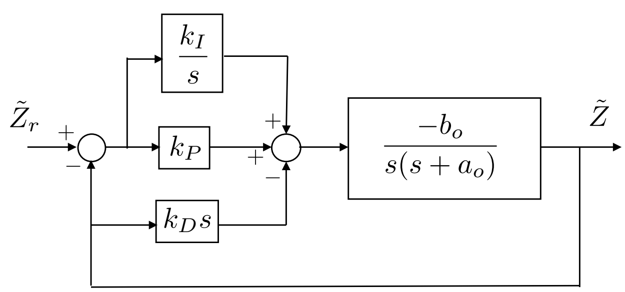

Figure 2: PID control for the outer loop of the lateral control of the VTOL system.

given by

$$1 + P(s)C(s) = 1 + \left(\frac{-b_o}{s(s+a_o)}\right)\left(\frac{k_D s^2 + k_P s + k_I}{s}\right) = 0,$$

where

$$\begin{aligned} a_o &= \frac{\mu}{m_c + 2m_r} \\ b_o &= \frac{F_e}{m_c + 2m_r}. \end{aligned}$$

Multiplying by the denominator and simplifying gives

$$s^3 + (a_o - b_o k_D)s^2 + (-b_o k_P)s + (-b_o k_I) = 0.$$

Therefore, in Evan's form we have

$$1 + k_I \left(\frac{-b_o}{s^3 + (a_o - b_o k_D)s^2 + (-b_o k_P)s}\right) = 0.$$

The appropriate Matlab command is therefore

```
>> L = tf([-bo],[1, (ao-bo*kd), -bo*kp,0]);
>> figure(1), clf, rlocus(L);
```