## Case Study I (Arm)

The Linearized model is

$$\frac{ml^2}{3}\ddot{\tilde{\theta}} - mg\frac{l}{2}\sin\theta_0\,\tilde{\theta} = \tilde{\tau} - b\dot{\tilde{\theta}}$$

Rearranging to get $\tilde{\theta}$'s on the LHS and $\tilde{\tau}$'s on the RHS gives

$$\frac{ml^2}{3}\ddot{\tilde{\theta}} + b\dot{\tilde{\theta}} - mg\frac{l}{2}\sin\theta_0\,\tilde{\theta} = \tilde{\tau}$$

Dividing by $\frac{ml^2}{3}$ to get $\ddot{\tilde{\theta}}$ by itself gives

$$\ddot{\tilde{\theta}} + \frac{3b}{ml^2}\dot{\tilde{\theta}} - \frac{3g}{2l}\sin\theta_0\,\tilde{\theta} = \frac{3}{ml^2}\tilde{\tau}$$

Taking the Laplace transform and collecting terms gives

$$\left[s^2 + \frac{3b}{ml^2}s - \frac{3g}{2l}\sin\theta_0\right]\tilde{\theta}(s) = \left(\frac{3}{ml^2}\right)\tilde{\tau}(s)$$

Solving for $\tilde{\theta}(s)$ gives

$$\tilde{\theta}(s) = \left(\frac{3/ml^2}{s^2 + \frac{3b}{ml^2}s - \frac{3g}{2l}\sin\theta_0}\right)\tilde{\tau}(s)$$

Block Diagram