1.1 $\boxed{K = \frac{1}{2} m \dot{x}_1^2}$

1.2 $\boxed{P = \frac{1}{2} k_1 x_1^2 + \frac{1}{4} k_2 x_1^4 - mg x_1}$

1.3 $L = K - P$

$$\boxed{L = \frac{1}{2} m \dot{x}_1^2 - \frac{1}{2} k_1 x_1^2 - \frac{1}{4} k_2 x_1^4 + mg x_1}$$

1.4 $F - B\dot{q}$

$$\boxed{F - c \cdot \text{sign}(\dot{x}_1) - b\dot{x}_1}$$

(Assuming the linear damper has damping coefficient b)

1.5 $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) - \frac{\partial L}{\partial x_1} = F - c \cdot \text{sign}(\dot{x}_1) - b\dot{x}_1$

$$\frac{d}{dt}(m\dot{x}_1) - (mg - k_1 x_1 - k_2 x_1^3) = F - c \cdot \text{sign}(\dot{x}_1) - b\dot{x}_1$$

$$m\ddot{x}_1 - mg + k_1 x_1 + k_2 x_1^3 = F - c \cdot \text{sign}(\dot{x}_1) - b\dot{x}_1$$

$$\boxed{\ddot{x}_1 = \frac{1}{m}\left(mg - k_1 x_1 - k_2 x_1^3 + F - c \cdot \text{sign}(\dot{x}_1) - b\dot{x}_1\right)}$$

2.1 z_e = anything

$\dot{z}_e = 0$

$\ddot{z}_e = 0$

$$\cancel{m\ddot{z}}^{0} + k_1 z + k_2 z^3 - \frac{1}{\sqrt{2}} mg = F - \cancel{b\dot{z}}^{0}$$

$$\boxed{F_e = k_1 z_e + k_2 z_e^3 - \frac{1}{\sqrt{2}} mg}$$

2.2 See Word Document

2.3 $m_1 \ddot{z} \approx m_1 \ddot{\tilde{z}}$

$k_1 z \approx k_1 \tilde{z}$

$$z^3 \approx z_e^3 + \frac{\partial z^3}{\partial z}\bigg|_e (z - z_e)$$

$$= z_e^3 + 3z_e^2 z - 3z_e^3$$

$$= 3z_e^2 \tilde{z} - 2z_e^3$$

$b\dot{z} \approx b\dot{\tilde{z}}$

$$\hookrightarrow \boxed{m\ddot{\tilde{z}} + k_1 \tilde{z} + k_2 (3z_e^2 \tilde{z} - 2z_e^3) - \frac{1}{\sqrt{2}} mg = F_e + \tilde{F} - b\dot{\tilde{z}}}$$

2.4 When $z_e = 0$ ...

$$m\ddot{\tilde{z}} + k_1 \tilde{z} + \cancel{k_2 (0)} - \cancel{\frac{1}{\sqrt{2}} mg} = \cancel{k_1(0)} + \cancel{k_2(0)} - \cancel{\frac{1}{\sqrt{2}} mg} + \tilde{F} - b\dot{\tilde{z}}$$

$$m\ddot{\hat{z}} + k_1 \hat{z} = \tilde{F} - b\dot{\tilde{z}}$$

$$m s^2 \tilde{Z}(s) + k_1 \tilde{Z}(s) = \tilde{F}(s) - bs\tilde{Z}(s)$$

$$\boxed{\tilde{Z}(s) = \frac{1/m}{s^2 + \frac{b}{m}s + \frac{k_1}{m}} \tilde{F}(s)}$$

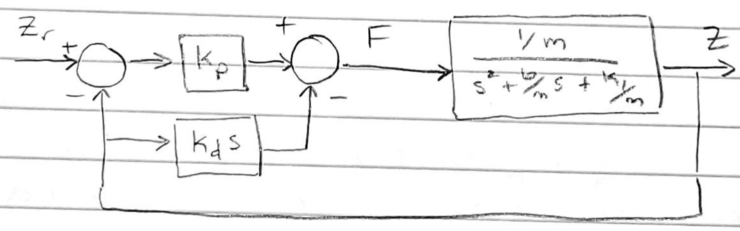
2.5 Around $z_e = 0$ ...

$$m\ddot{z} + k_1 z = F - b\dot{z}$$

$$\boxed{\begin{pmatrix}\dot{\tilde{z}}\\ \ddot{\tilde{z}}\end{pmatrix} = \begin{pmatrix}0 & 1\\ -k_1/m & -b/m\end{pmatrix}\begin{pmatrix}\tilde{z}\\ \dot{\tilde{z}}\end{pmatrix} + \begin{pmatrix}0\\ 1/m\end{pmatrix}F}$$

2.6 I used my own. See word Doc.

3.1



3.2

$$Z(s) = \left[-k_d s Z(s) + k_p\left(Z_r(s) - Z(s)\right)\right]\frac{1/m}{s^2 + \frac{b}{m}s + k_1/m}$$

$$\boxed{Z(s) = \frac{\frac{1}{m}\cdot k_p}{s^2 + \left(\frac{b}{m} + \frac{1}{m}k_d\right)s + \left(\frac{k_1}{m} + \frac{1}{m}k_p\right)} Z_r(s)}$$

3.3

$$k_p = \frac{5\,N}{1\,m} = \boxed{5\,N/m}$$

3.4 $s^2 + 2\zeta\omega_n + \omega_n^2 = s^2 + \left(\frac{b}{m} + \frac{1}{m}k_d\right)s + \left(\frac{k_i}{m} + \frac{1}{m}k_p\right)$

$$2\zeta\omega_n = \frac{b}{m} + \frac{1}{m}k_d$$
$$\omega_n^2 = k_i/m + 1/m\, k_p$$

$\Rightarrow \boxed{\omega_n = 3.178}$

$k_d = m\left(2\zeta\omega_n - \frac{b}{m}\right) = \boxed{2.147}$

3.5 See Word Doc.

3.6 See Word Doc as well as included note.
$K_i = 2.0$

4.1 $K = [\ 13.95\ ,\ 4.4\ ]$
$k_i = -20.0$

See Param File for implementation.

4.2 $L^T = [[\ 19.8,\ 195.94\ ]]$

4.3 I used poles at $-5 \pm 0.1j$ and integrator pole at -5. I used eigenvalues for A-LC that were 10 times greater than A-BK. See Word Doc for performance.

The new gains: $K = [\ 37.455,\ 7.4\ ]$
$K_i = [\ -62.52\ ]$
$L^T = [\ 99.8,\ 2480.94\ ]$

4.4 See Word Doc

4.5 See Word Doc

4.6 See Word Doc

5.1 Noise attenuation γ_n = Mag. @ 20 rad/s

$= \boxed{0.2234}$ See loopshape_mass.py

Tracking accuracy $\gamma_r = \frac{1}{\text{Mag @ } 0.2 \text{ rad/s}}$

$\gamma_r = \frac{1}{138.85} = \boxed{0.0072}$

5.2 See Loopshaping Code

5.3 See Loopshaping Code

5.4 $F(s) = \dfrac{0.8}{1 + 0.8}$, See Loopshaping Code

5.5 See Files

5.6–5.8 See Word Doc.