# R program for flexible Cox models including time-dependent (TD) and/or non-linear (NL) effects:

CoxFlex

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## R script

- Function CoxFlex allows to estimate a Cox model with time-dependent (TD) and/or non-linear (NL) effects for one or several variables
  - Can include variables without TD and NL effects
- CoxFlex can handle:
  - a) Time-invariant data (one observation per subject)

id	time	dose	event
1	56	3.0	1
2	365	0.5	0
3	283	0	0

b) Time-dependent (or time-varying) data (several observations per subject)

id	start	stop	dose	event
1	0	14	1.0	0
1	14	28	2.0	0
1	28	56	3.0	1
2	0	180	1.0	0
2	180	365	0.5	0

- The data must be a data frame
- 2. The <u>first column</u> of data *must* be a <u>numeric</u> ID variable identifying the subjects (with the name of your choice)
- 3. No missing data are allowed (otherwise the function will crash)
- 4. All string characters or factors *must* be recorded as numeric values
  - E.g., gender: (0, 1) instead of ("male", "female")
- 5. All binary variables must be coded (0, 1)

- 5. Categorical variables, with more than 2 categories, *must* be recorded as dummy variables (not as factor)
  - E.g., if 4 age groups (<18, 18-39, 40-64, ≥65) with reference <18, then create 3 dummy (binary) variables to include in the model

age.gr		age18.39	age40.64	age65
<18		0	0	0
18-39		<b>1</b>	0	0
40-64	-	0	1	0
≥65		0	0	1

- 6. Negative values of continuous covariates are *not* a problem, as opposed to when using fractional polynomials
- 7. Include in dataset passed to CoxFlex only the variables used in the model. This will greatly improve the efficiency (speed) of the program.

#### 8. For time-varying data:

- Each line can be for time intervals with length of 1 (e.g. 1 day) or longer
- The 'start' of a line must be the same as the 'stop' of the previous line (for the same subject), i.e. no gap and no overlap in time intervals
- No intervals with 'start' = 'stop'

id	start	stop	dose	event
1	0	14	1.0	0
1	14	28	2.0	0
1	28	56	3.0	1
2	0	180	1.0	0
2	180	365	0.5	0

The 1<sup>st</sup> start value of each subject must be 0 (delayed entry not allowed)

- 9. For time-invariant data (1 line per subject):
  - Event times must be > 0

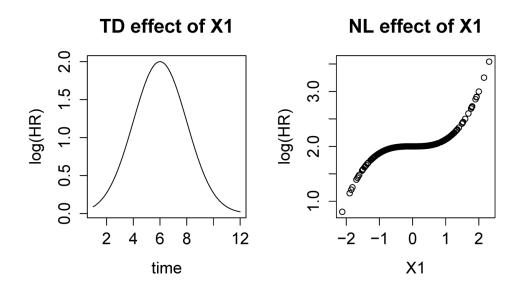
## Example of a dataset (dat) with 3 time-varying covariates (available with the R script)

Id	Event	Fup	Start	Stop	<b>x1</b>	x2	х3
1	0	2	0	1	-0.23549	1.541651	0.87255
1	1	2	1	2	-0.86123	-1.27724	0.87255
2	0	2	0	1	0.737676	-0.17344	1.959211
2	1	2	1	2	1.345106	-0.60027	1.959211
3	0	8	0	1	0.863421	1.182985	3.862302
3	0	8	1	2	0.914108	0.784081	3.862302
3	0	8	2	3	0.933757	-0.38651	3.862302
3	0	8	3	4	1.19725	-0.77683	3.862302
3	0	8	4	5	1.139094	1.371349	3.862302
3	0	8	5	6	0.518455	0.056261	3.862302
3	0	8	6	7	0.575675	-0.71817	3.862302
3	1	8	7	8	0.632166	0.67002	3.862302

## Example

- 300 patients followed for up to 12 months (in dat)
- $X_1(t)$ ,  $X_2(t)$ ,  $X_3(t)$  are continuous time-varying variables
- True model (data generated from it):
  - X<sub>1</sub>: TD and NL effects
  - X<sub>2</sub> and X<sub>3</sub>: constant-over-time and linear effects

$$\lambda(t \mid X_1(t), X_2(t), X_3(t)) = \lambda_0 \exp\{\beta_1(t)g_1(X_1(t)) + \beta_2X_2(t) + \beta_3X_3(t)\}$$



## Download CoxFlex R script & data

- https://github.com/mebeauchamp/CoxFlex
  - Content:
    - CoxFlex 20220330 to share.R: R script including CoxFlex and related functions
    - dat.Rdata: Dataset used for the example in the tutorial

### Code

```
# Source the program in current R session (not a package yet):
source("C:/.../CoxFlex - 20220330 - to share.R")
  # No need to look at the code in this file
# Load data
load("C:/.../dat.RData")
head(dat)
 # Id Event Fup Start Stop
                                  x1
                                            x2
                                                      x3
 # 1
                   0 1 -0.2354852 1.5416514 0.8725503
          0
  # 1 1 2 1 2 -0.8612346 -1.2772441 0.8725503
          0 2
   2
                0 1 0.7376760 -0.1734419 1.9592107
   2
      1 2
                   1 2 1.3451063 -0.6002743 1.9592107
  #
          0 8
                   0 1 0.8634209 1.1829845 3.8623023
    3
              8
                   1
          0
                        2 0.9141075 0.7840808 3.8623023
# Select only variables relevant for model estimation
dat.red <- dat[, c('Id', 'Event', 'Start', 'Stop', 'x1', 'x2', 'x3')]</pre>
```

```
# Check if data are in a data frame
is.data.frame(dat.red)
    # [1] TRUE
# Check if the ID variable is numeric (must be 1st column)
is.numeric(dat.red[, 1])
   # [1] TRUE
# Display structure of data (all variables must be numeric, 1st one is ID)
str(dat.red)
# 'data.frame': 2307 obs. of 7 variables:
   $ Id : num 1 1 2 2 3 3 3 3 3 3 ...
   $ Event: num 0 1 0 1 0 0 0 0 0 0 ...
   $ Start: num 0 1 0 1 0 1 2 3 4 5 ...
   $ Stop: num 1 2 1 2 1 2 3 4 5 6 ...
   $ x1
           : num -0.235 -0.861 0.738 1.345 0.863 ...
   $ x2 : num 1.542 -1.277 -0.173 -0.6 1.183 ...
   $ x3
           : num 0.873 0.873 1.959 1.959 3.862 ...
# Check no missing values in any variables used for the model
sum(is.na(dat.red))
   # [1] 0
# Distribution of 1st Start value across subjects (check no delayed entry)
table(by(dat.red$Start, dat.red$Id, min))
    #
        0
                                                                            11
    # 300
```

## Estimation of a *predefined* model with CoxFlex

#### **ARGUMENTS:**

- data: Your dataset (data frame). 1st column must be an ID variable of individuals.
- <u>Type</u>: Variables in data indicating the start and stop of time intervals, and the event (1=event, 0=censored).

```
If time-varying data: Type=c ("Start", "Stop", "Event").
```

If time-invariant data: Type=c ("Time", "Event").

Start, Stop, and Time do *not* have to be integers.

- variables: All independent variables in the model (exposure and covariates)
- <u>TD</u>: Indicate for each independent variable if the TD effect is modeled (0/1)
- NL: Indicate for each independent variable if the NL effect is modeled (0/1). <u>Can be 1 only for continuous variables.</u>

## Estimation of a *predefined* model with CoxFlex

#### **ARGUMENTS:**

- $\underline{m}$ : Number of interior knots (the same for all TD and NL effects). By default  $\underline{m}=1$ .
- p: Order of splines (the same for all TD and NL effects). By default p=2.
  - p=0: step functions
  - p=1: linear splines
  - p=2: quadratic splines
  - p=3: cubic splines
- knots: Position of interior knots. Default knots=-999, which indicates that the knots are automatically allocated.

To specify the position of interior knots, specify a matrix with (length (variables) +1) rows by m columns. There is one row per variable (add NA if no NL effect for a variable) and one for time. E.g., for this model it could be:

```
knots = matrix(c(-1, NA, NA, 4), nrow=4, ncol=1).
```

## Output of the model (more on next slides)

```
# Type the name of the object of results to see the output
m1
$Partial Log Likelihood
[1] -1013.063
                                                      To calculate AIC use:
                                                      AIC =
                                                      -2 * m1$Partial Log Likelihood
$Number of parameters
                                                      + 2 * m1$Number of parameters
[1] 9
$Number events
[1] 202
$Number knots
[1] 1
$Degree of splines
[1] 2
```

#### \$knots covariates

[,2] [,1] [,3] [,4] x1 -3.229684 -3.229684 -3.229684 -0.06910736 x2NA NA NA NA x3NA NA NA NA [,5] [,6] [,7]

x1 3.687497 4.687497 5.687497

x2 NA NA NA NA X3

#### \$knots time

[1] 0 0 0 5 12 13 14

Position of interior and exterior knots, for each variable with a NL effect

Position of interior and exterior knots for time

#### \$coefficients

x1

x2

x3

NA 0.3267542 0.1293491

#### \$Standard Error

[1] NA 0.07258465 0.01504884

#### \$coefficients splines NL

x1 x2 x3

[1,] 0.000000 NA NA

[2,] 2.568096 NA NA

[3,] 2.445147 NA NA

[4,] 6.327139 NA NA

#### \$coefficients\_splines\_TD

x1 x2 x3

[1,] 0.5779502 NA NA

[2,] 1.3659911 NA NA

[3,] 1.5177027 NA NA

[4,] -0.4379260 NA NA

Coefficients (log hazard) and SE for variables <u>without</u> TD nor NL effects

Coefficients of splines for NL and TD effects requested:

3 splines for a NL effect (m+p):

First NL spline coefficient always set 0 for technical reasons.

4 splines for a TD effect (m+p+1).

```
$variables
```

[1] "x1" "x2" "x3"

\$coef

[1] NA NA 0.327 0.129

\$var

[1] NA NA 0.005329 0.000225

\$pvalue

[1] 0.264 0.398 0.000 0.000

#### \$variables

[1] "x1" "x2" "x3"

For <u>each variable</u> above, the values shown below are, respectively, for:

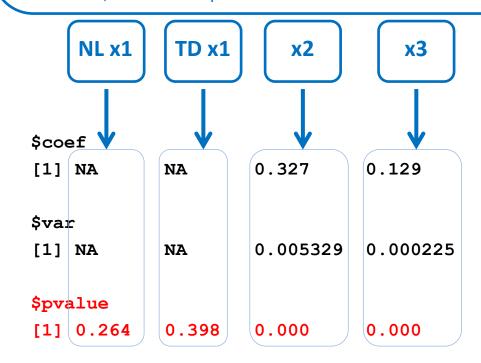
- 1) NL effect (when applicable), and/or
- 2) TD effect (when applicable), or
- 3) "Standard" effect when no NL nor TD effects were requested.

Then, move to the next variable.

For this model, the request was:

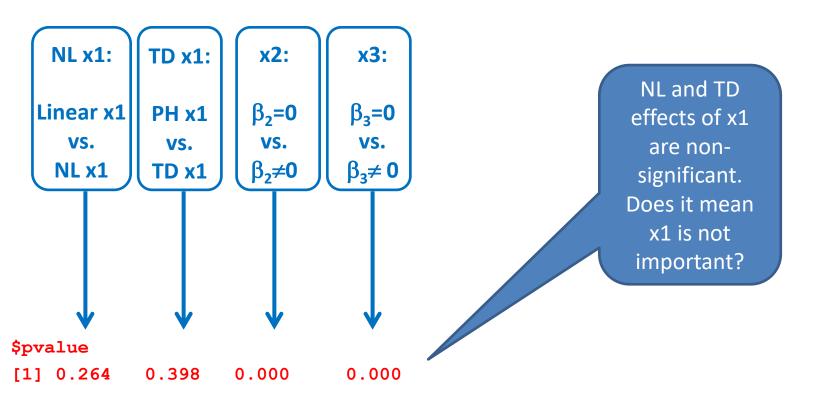
variables=c("x1","x2","x3"), TD=c(1,0,0), NL=c(1,0,0)

Therefore, the values reported below are for:



#### \$variables

#### Likelihood ratio tests (LRTs):



### Standard Cox PH model

```
library(survival)
m.cox <- coxph(Surv(Start, Stop, Event) ~ x1 + x2 + x3, data=dat.red)</pre>
m.cox
   #Call:
   #coxph(formula = Surv(Start, Stop, Event) ~ x1 + x2 + x3, data = dat.red)
   # coef exp(coef) se(coef) z
                                          p
   #x1 0.401
                 1.49 0.0728 5.51 3.6e-08
   #x2 0.316 1.37 0.0721 4.38 1.2e-05
   #x3 0.136 1.15 0.0151 8.99 0.0e+00
   #Likelihood ratio test=89.1 on 3 df, p=0 n= 2307, number of events= 202
# Significant effect for x1, even though the NL and TD effects were not
# significant in the flexible model.
# Don't discard a variable because NL and/or TD effects are non-significant!
AIC (m.cox)
   # [1] 2037.618
BIC (m.cox)
   # [1] 2047.543
```

## AIC/BIC for a model estimated with CoxFlex

## Order of p-values in \$pvalue for another example of model

• If the model requested in the CoxFlex function was:

```
variables=c("x1", "x2", "x3"), TD=c(0,1,0), NL=c(0,1,1)
```

For <u>each variable</u>, the p-values shown are, respectively, for:

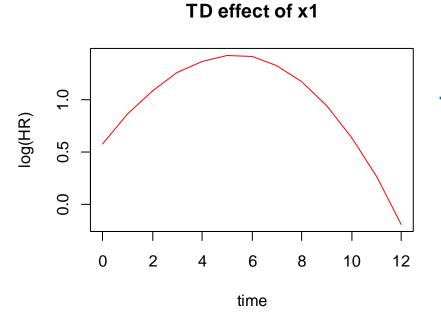
- 1) NL effect (when applicable), and/or
- 2) TD effect (when applicable), or
- 3) "Standard" effect when no NL nor TD effects were requested.

Then, move to the next variable.

- Then, the p-values in vector \$pvalue would be for:
  - Significance of "standard" effect for x1 (β₁≠0)
  - NL effect of x2
  - TD effect of x2
  - NL effect of x3

## Plot the NL/TD effects: plot.FlexSurv

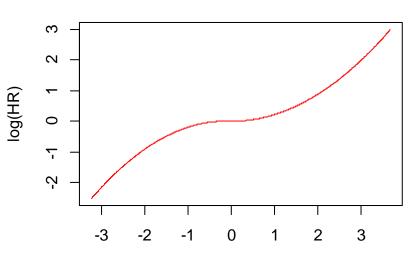
```
# To plot two graphs on top of each other
par(mfrow=c(2,1))
                                             Only 1 variable
                                               at the time
# Plot for TD effect of x1
plot.FlexSurv (model.FlexSurv = m1, variable="x1", TD=1, NL=0,
   col="red", xlab="time", ylab="log(HR)",
   main="TD effect of x1", type="1")
                                                             NL effect plotted with
                                                            respect to this reference
                                                             value of the variable
                                                                  (default 0)
# Plot for NL effect of x1
plot.FlexSurv (model.FlexSurv = m1, variable="x1", TD=0, NL=1, ref.value.NL=0,
   col="red", xlab="x1", ylab="log(HR)",
   main="NL effect of x1", type="1")
```



Shows how the strength of the effect of x1 varies over time.



**x**1

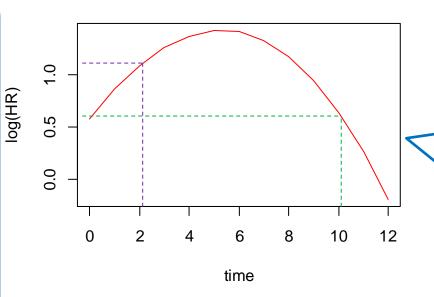


Shows the log(HR) comparing each value of x1 (numerator) to the reference value x1=0 (denominator)

However, in the current model estimated, TD and NL effects for x1 are multiplied by each other:  $\beta_1(t) \cdot g_1(X_1(t))$ 

Consequently, shapes of NL and TD effects are good on these independent graphs, but not log(HR) on y axes

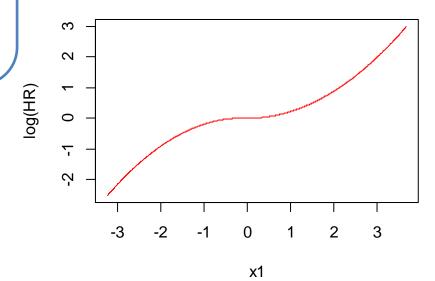
#### TD effect of x1



E.g., at t=2 the NL effect of x1 has to be multiplied by 1.1

But at *t*=10 the NL effect has to be multiplied by 0.6

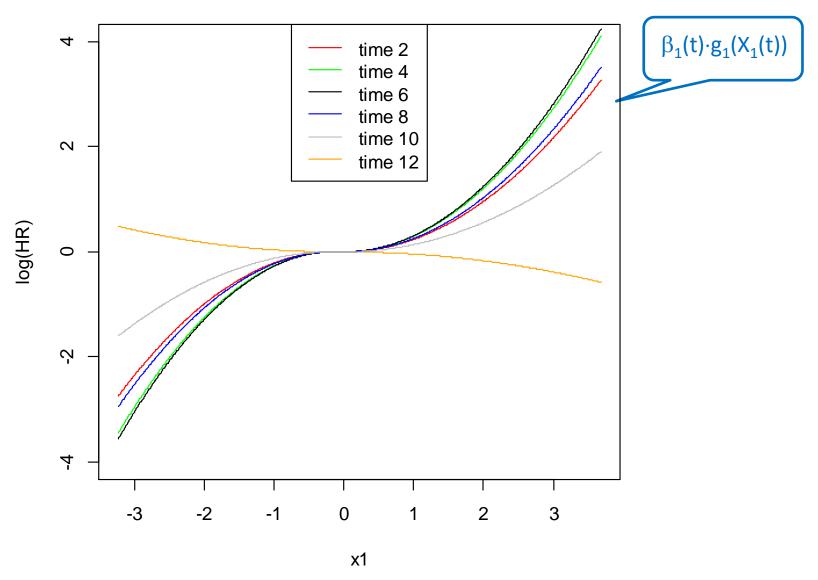
NL effect of x1



## NL effect at fixed time points (when a TD effect is also modeled)

```
par(mfrow=c(1,1))
plot.FlexSurv(m1, variable="x1", TD=1, NL=1, TimePoint=2, ref.value.NL=0,
   vlim=c(-4,4), xlab="x1", vlab="log(HR)", type="l", col="red",
   main="Total effect: NL effect of x1 at fixed time points")
lines.FlexSurv(m1, variable="x1", TD=1, NL=1, TimePoint=4, ref.value.NL=0,
   col="green")
lines.FlexSurv(m1, variable="x1", TD=1, NL=1, TimePoint=6, ref.value.NL=0,
   col="black")
lines.FlexSurv(m1, variable="x1", TD=1, NL=1, TimePoint=8, ref.value.NL=0,
   col="blue")
lines.FlexSurv(m1, variable="x1", TD=1, NL=1, TimePoint=10, ref.value.NL=0,
   col="gray")
lines.FlexSurv(m1, variable="x1", TD=1, NL=1, TimePoint=12, ref.value.NL=0,
   col="orange")
legend("top", c("time 2","time 4","time 6","time 8","time 10","time 12"),
   lty=c(1,1,1,1,1,1), col=c("red", "green", "black", "blue", "gray", "orange"))
```

#### Total effect: NL effect of x1 at fixed time points



## Backward selection of NL/TD effects

#### **ARGUMENTS:**

- continuous: Indicate whether each variable is continuous (1=yes, 0=no)
- TD=1 / NL=1: Force TD/NL effect of corresponding variable
- <u>TD=0</u> / <u>NL=0</u>: Do not force the effect (TD/NL effects are evaluated, and a variable may be excluded from final model)
- TD=-1 / NL=-1: Exclude TD/NL effect of corresponding variable (i.e. <u>force</u> the PH/LL effect)
- <u>alpha back</u>: Alpha value used to select effects

```
# Command to see model output for the final model
m2
                                                       Here, better likelihood,
$final model$Partial Log Likelihood
                                                       with fewer parameters,
[1] -999.011
                                                       than the predefined model
                                                       (-1013.063, with 9)
$final model$Number of parameters
                                                      parameters)
[1] 5
$final model$Number events
[1] 202
$final model$Number knots
[1] 1
$final model$Degree of splines
[1] 2
$final model$knots covariates
         [,1]
                     [,2]
                                [,3]
                                         [,4]
                                                   [,5]
                                                            [,6]
                                                                     [,7]
x1
           NA
                      NA
                                  NA
                                           NA
                                                    NA
                                                              NA
                                                                       NA
x2
           NA
                      NA
                                  NA
                                           NA
                                                    NA
                                                              NA
                                                                       NA
x3 0.04792073 0.04792073 0.04792073 0.846845 35.50922 36.50922 37.50922
$final model$knots time
     0 0 0 5 12 13 14
[11]
```

```
$final model$coefficients
                 x2
       x1
                           x3
0.3743083 0.3481198
                           NA
$final model$Standard Error
[1] 0.07211582 0.07257598
                                  NA
$final model$coefficients splines NL
     x1 x2
                  x3
[1,] NA NA 0.0000000
[2,] NA NA 0.2563118
[3,] NA NA 7.4415084
[4,] NA NA 3.0269701
$final model$coefficients splines TD
     x1 x2 x3
[1,] NA NA NA
[2,] NA NA NA
[3,] NA NA NA
[4,] NA NA NA
```

NL effect selected for x3, but no TD effects

```
$final model$variables
[1] "x1" "x2" "x3"
$final_model$coef
[1] 0.374 0.348
                   NA
$final_model$var
[1] 0.005184 0.005329
                            NA
$final model$pvalue
[1] 0 0 0
```

Showing respectively, p-value for significance of:

$$-\beta_1 = 0 \text{ vs. } \beta_1 \neq 0$$

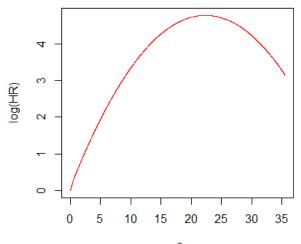
$$-\beta_2 = 0 \text{ vs. } \beta_2 \neq 0$$

$$-\text{Linear x3 vs. NL x3}$$

- 
$$β_2$$
 = 0 vs.  $β_2$  ≠ 0

## Model selected by backward selection

#### NL effect of x3



## Estimation problem: **NL effect** of a variable with *non-negative values* & *frequent 0 values*

- Variable X with all values  $\geq 0$  but with 0 for a majority of observations (i.e. median(X) = 0). E.g.,
  - Number of cigarettes per day, when > 50% of non-smokers
  - Drug dose, when subjects are often unexposed
- Estimation problem for NL effect: the interior knot is placed at median(X) = 0, which is also min(X)
  - Spline estimation crashes because interior knot = one of the exterior knots
- The problem would also occur if median(X) = max(X)
- But no problem if only a TD effect is requested for X

### Solution

1. Create a new binary variable *Z*:

- 1. Create a new variable *X.centered*:
  - Center the <u>non-zero</u> values of original X at 0, i.e. subtract the <u>mean of non-zero</u> X values (say M) to each <u>non-zero</u> value of X
  - Keep X.centered=0 when original X=0
  - Therefore, mean(X.centered) = median(X.centered) = 0, but min(X.centered) < 0</p>
- 2. Estimate the flexible model with:

NL effect of **X.centered** (excluding original X) + **Z** + all other covariates

Now, the NL effect describes the NL effect of non-zero values of X, while HR of Z estimates the HR for X=M vs. X=0

NOTES: You can include TD effects for *X.centered* and *Z* in the model

## Explanation of effects

- **HR of Z** estimates the HR for *X*=M vs. *X*=0:
  - HR for Z =  $\lambda(t|Z=1, X.centered, covariates) / \lambda(t|Z=0, X.centered, covariates)$ 
    - i.e. hazard ratio for Z=1 vs. Z=0 while keeping constant other variables in the model, including X.centered
    - Occurs only for X.centered = M that Z could equal 1 or 0
  - Thus, hazard ratio for Z=1 vs. Z=0 when X.centered = M
    - $\Rightarrow$  Hazard ratio for X=M vs. X=0
- NL effect of X.centered describes the effect of non-zero values of X:
  - It's the NL effect of X.centered, while keeping constant other variables in the model, including Z
    - Occurs only for Z=0 that X.centered could change value, i.e. for non-zero values
      of X

## Code (on mock data)

```
summary(data$X)
  # Min. 1st Ou. Median Mean 3rd Ou.
                                              Max.
  # 0.0000 0.0000 0.0000 0.6597 1.0000
                                             5.0000
M <- mean(data$X[data$X != 0])</pre>
data$X.centered <- ifelse(data$X == 0, 0, data$X - M)</pre>
summary(data$X.centered)
  # Min. 1st Qu. Median Mean 3rd Qu.
                                              Max.
  # -0.614 -0.614 0.000 0.000
                                      0.000
                                               3.386
dat.red$Z <- ifelse(dat.red$X > 0, 1, 0)
                                                                 NL effect of X.centered
m3 <- CoxFlex(data=data,
       Type=c("Start", "Stop", "Event"),
       variables = c("X.centered", "Z", "x1", "x2"),
                                                        log(HR)
       TD=c(1,1,0,0), NL=c(1,0,1,0),
                                                           Ю
       m=1, p=2, knots=-999
                                                           0.0
                                                                              2
                                                                  0
                                                                          time
```

### References

#### References to cite:

- Abrahamowicz M, MacKenzie TA. Joint estimation of time-dependent and non-linear effects of continuous covariates on survival. Statistics in Medicine 2007;26(2):392-408.
- Wynant W, Abrahamowicz M. Impact of the model-building strategy on inference about nonlinear and time-dependent covariate effects in survival analysis. *Statistics in Medicine* 2014; 33: 3318–3337.

#### **Examples of applications:**

- Gagnon B, Abrahamowicz M, Xiao Y, Beauchamp ME, MacDonald N, Kasymjanova G, Kreisman H, Small D. Flexible modeling improves assessment of prognostic value of Creactive protein in advanced non-small cell lung cancer. *British Journal of Cancer* 2010;102(7):1113-1122.
- Le Teuff G, Abrahamowicz M, Wynant W, Binquet C, Moreau M, Quantin C. Flexible modeling of disease activity measures improved prognosis of disability progression in relapsing-remitting multiple sclerosis. *J Clin Epidemiol* 2015;68(3):307-16.
- Isidean SD, Wang Y, Mayrand M-H, Ratnam S, Coutlée F, Franco EL, Abrahamowiz M, for the CCCaST Study Group. Assessing the time-dependence of prognostic values of cytology and human papillomavirus testing in cervical cancer screening. *Int J Cancer* 2019;144(10):2408-2418.

## Help!

• For conceptual questions about your project:

Prof. Michal Abrahamowicz

To obtain the program, and for help with it:

marie-eve.beauchamp@rimuhc.ca