HW8

Madeline Blasingame
205412882

Exercise a

We know:

$$\frac{(SSE_R - SSE_F) / df_R - df_F}{SSE_F / df_F} \sim F_{m,\, df_F}$$

where: $m = df_R - df_F$
$df_F = n-k-1$
$df_R = n-(k-m)-1$

$$\Rightarrow \frac{(SSE_R - SSE_F)}{SSE_F}\left(\frac{df_F}{df_R - df_F}\right) \sim F_{m,\, df_F}$$

Note: in this case, $m = 1$

$$\frac{SSE(x_2, x_3, \ldots x_k) - SSE(x_1, x_2, \ldots, x_k)}{SSE(x_1, x_2, \cdots, x_k)}\left(\frac{n-k-1}{(n-(k-1)-1)-(n-k-1)}\right) \sim F_{1,\, n-k-1}$$

$$= \frac{SSE(x_2, x_3, \ldots x_k) - SSE(x_1, x_2, \ldots, x_k)}{SSE(x_1, x_2, \cdots, x_k)}\left(\frac{n-k-1}{1}\right) \sim F_{1,\, n-k-1}$$

Note: $F_{1,\, n-k-1} = t^2_{n-k-1}$

We know t is the t-statistic used for testing $\beta_1 = 0$ in the regression of Y on $x_1, x_2, \cdots x_k$ which means that t must have $df = n-k-1$

$$r^2_{yx_1} = \frac{t^2_{n-k-1}}{t^2_{n-k-1} + n-k-1} = \frac{F_{1,\, n-k-1}}{F_{1,\, n-k-1} + n-k-1}$$

$$= \frac{\frac{SSE_R - SSE_F}{SSE_F}(n-k-1)}{\frac{SSE_R - SSE_F}{SSE_F}(n-k-1) + (n-k-1)} = \frac{\frac{SSE_R - SSE_F}{SSE_F}(n-k-1)}{(n-k-1)\left[\frac{SSE_R - SSE_F}{SSE_F} + 1\right]}$$

$$= \frac{\frac{SSE_R - SSE_F}{SSE_F}}{\frac{SSE_R - SSE_F + SSE_F}{SSE_F}} = \frac{SSE_R - SSE_F}{SSE_R}$$

$$= \frac{SSE(X_2, X_3, \dots X_k) - SSE(X_1, X_2, \dots, X_k)}{SSE(X_1, X_2, \dots, X_k)}$$

$$\Rightarrow \quad r^2_{yx_1} = \frac{t^2}{t^2 + (n-k-1)} = \frac{SSE(X_2, X_3, \dots X_k) - SSE(X_1, X_2, \dots, X_k)}{SSE(X_1, X_2, \dots, X_k)}$$

Exercise 6

$c = [0, 1, \dots, 0, 0, 0] \qquad \gamma = 0$

$c\beta - \gamma = \beta_1$

Therefore:

$H_0 : \beta_1 = 0$

$H_a : \beta_1 \neq 0$

$$\frac{\hat{\beta}' X'Y - \hat{\beta}_1 X_1' Y}{SSE/(n-k-1)} = \frac{Y'X(X'X)^{-1}X'Y - Y'X_1(X_1'X_1)^{-1}Y}{SSE/(n-k-1)}$$

Note: $\hat{\beta}' = Y'X(X'X)^{-1}$

$\hat{\beta}_1 = Y'X_1(X_1'X_1)^{-1}$

$$= \frac{Y'HY - Y'H_1Y \pm n\bar{Y}^2}{SSE/(n-k-1)} = \frac{(Y'HY - n\bar{Y}^2) - (Y'H_1Y - n\bar{Y}^2)}{SSE/(n-k-1)}$$

Note: $X(X'X)^{-1}X' = H$

$X_1(X_1'X)^{-1}X' = H_1$

Note: $SSR = \Sigma(\hat{Y}_i - \bar{Y})^2 = \Sigma \hat{Y}_i^2 - n\bar{Y}^2$

$$\begin{aligned} Y'HY - n\bar{Y}^2 &= Y'HHY - n\bar{Y}^2 \\ &= (HY)'(HY) - n\bar{Y}^2 \\ &= \hat{Y}'\hat{Y} - n\bar{Y}^2 \\ &= \Sigma \hat{Y}_i^2 - n\bar{Y}^2 \\ &= SSR_F \end{aligned}$$

$$\begin{aligned} Y'H_1Y - n\bar{Y}^2 &= Y'H_1H_1Y - n\bar{Y}^2 \\ &= (H_1Y)'(H_1Y) - n\bar{Y}^2 \\ &= \hat{Y}_1'\hat{Y}_1' - n\bar{Y}^2 \\ & \quad SSR_c \end{aligned}$$

$$\Longrightarrow \frac{\hat{\beta}' X'Y - \hat{\beta}_1 X_1' Y}{SSE_F/(n-k-1)} = \frac{SSR_F - SSR_c}{SSE_F/(n-k-1)}$$

Note: $SSR_F = SST - SSE_F$

$SSR_c = SST - SSE_c$

$$= \frac{(SST - SSE_F) - (SST - SSE_c)}{SSE_F/(n-k-1)} = \frac{SSE_c - SSE_F}{SSE_F/(n-k-1)} = \frac{SSE_c - SSE_F / 1}{SSE_F/(n-k-1)} \sim F_{1,n-k-1}$$

Note: in this case we have 1 constraint so $m = 1$

$$\Longrightarrow \frac{\hat{\beta}' X'Y - \hat{\beta}_1' X_1' Y}{SSE/(n-k-1)} \sim F_{1,n-k-1}$$

Exercise c

$$\hat{\beta} \sim N(\beta, \sigma^2 (X'X)^{-1})$$

Let $V = (X'X)^{1/2} (\hat{\beta} - \beta)$

$$E[V] = (X'X)^{1/2} E[\hat{\beta}] - (X'X)^{1/2}\beta$$
$$= (X'X)^{1/2}\beta - (X'X)^{1/2}\beta = 0$$

$$\begin{aligned} var(V) &= var((X'X)^{1/2}\hat{\beta} - (X'X)^{1/2}\beta) \\ &= var((X'X)^{1/2}\hat{\beta}) \\ &= (X'X)^{1/2} var(\hat{\beta}) (X'X)^{1/2} \\ &= \sigma^2 (X'X)^{1/2} (X'X)^{-1} (X'X)^{1/2} \\ &= \sigma^2 (X'X)^{1/2} (X'X)^{-1/2} (X'X)^{-1/2} (X'X)^{1/2} \\ &= \sigma^2 I_{k+1} \end{aligned}$$

$$\Rightarrow V \sim N(0, \sigma^2 I_{k+1})$$

$$\Rightarrow \frac{V'V}{\sigma^2} \sim \chi^2_{k+1} \Rightarrow \frac{(\hat{\beta}-\beta)' (X'X)^{1/2} (X'X)^{1/2} (\hat{\beta}-\beta)}{\sigma^2} \sim \chi^2_{k+1}$$

$$\Rightarrow \frac{(\hat{\beta}-\beta)' (X'X) (\hat{\beta}-\beta)}{\sigma^2} \sim \chi^2_{k+1}$$

$H_0: \beta = 0$

$H_a: \beta \neq 0$

Note: $\frac{(n-k-1) S_e^2}{\sigma^2} \sim \chi^2_{n-k-1}$

$$\frac{\frac{(\hat{\beta}-\beta)' (X'X) (\hat{\beta}-\beta)}{\sigma^2} \Big/ k+1}{\frac{(n-k-1) S_e^2}{\sigma^2} \Big/ n-k-1} \sim F_{k+1, n-k-1}$$

$$\Rightarrow \frac{(\hat{\beta}-\beta)' (X'X) (\hat{\beta}-\beta)}{(k+1) S_e^2} \sim F_{k+1, n-k-1}$$

Under H_0:

$$\frac{\hat{\beta}' (X'X) \hat{\beta}}{(k+1) S_e^2} \sim F_{k+1, n-k-1}$$

Exercise D

Read in data and define y, x1, x2, x3, x4, and x5

```
a <- read.table("http://www.stat.ucla.edu/~nchristo/statistics100C/body_fat.txt",
header=TRUE)
#Response variable:
y <- a$y
#Predictor variables:
x1 <- a$x11
x2 <- a$x12
x3 <- a$x13
x4 <- a$x14
x5 <- a$x15
ones <- rep(1, length(x1))
```

H_0: $\beta_2 = \beta_4 = \beta_5 = 0$

H_a: At least one of the values $\beta_2, \beta_4, \beta_5$ is not equal to 0

$c\beta = \gamma$

c = [0, 0, 1, 0, 1, 1]

$\gamma = 0$

F statistic: $\frac{(c\hat{\beta}-\gamma)^T[c(x^Tx)^{-1}c^T]^{-1}(c\hat{\beta}-\gamma)}{mS_e^2}(ncp = \frac{(c\beta-\gamma)^T[c(x^Tx)^{-1}c^T]^{-1}(c\beta-\gamma)}{\sigma^2}) \sim F_{3,244}$

Df:

m = 3 because we are using 3 constraints

n-k-1 = 251 - 5 - 1 = 245

```r
c = rbind(c(0,0,1,0,0,0), c(0,0,0,0,1,0), c(0,0,0,0,0,1))
gamma = rbind(0,0,0)

beta = matrix(c(-42.0, 2.4, -0.5, 1.9, 0.1, -1.6))
x = cbind(ones, x1, x2, x3, x4, x5)
beta_hat = solve(t(x) %*% x) %*% t(x) %*% y
H <- x %*% solve(t(x) %*% x) %*% t(x)
I <- diag(251)

e <- (I - H) %*% y
se_2 <- t(e) %*% e /(251 - 5 - 1)

#f_stat = (t(c %*% beta_hat - gamma) %*% solve(c %*% solve(t(x) %*% x) %*% t(c)) %*% (c
 %*% beta_hat - gamma)) / (3 * se_2)
ncp = (t(c %*% beta - gamma) %*% solve(c %*% solve(t(x) %*% x) %*% t(c)) %*% (c %*% beta
- gamma)) / (50)

f_star = qf(0.05, 3, 245, lower.tail = FALSE)
power = pf(f_star, 3, 245, ncp, lower.tail = FALSE)
type_2_error <- 1 - power

power
```

```
## [1] 0.7151795
```

```r
type_2_error
```

```
## [1] 0.2848205
```

Exercise E

$X'X\beta = X'Y$

$$\frac{\hat{\beta}'X'Y}{(k+1)Se^2} = \frac{\hat{\beta}'X'X\beta}{(k+1)Se^2} \sim F_{k+1,\, n-k-1}$$

which we showed in part C. It is testing

$H_0: \beta = 0$ where β is all β_i including β_0

$H_a: \beta \neq 0$

Distribution when H_0 is true:

$$\frac{\beta' X' Y}{(k+1)Se^2} \sim F_{k+1,\, n-k-1}$$

Distribution when H_0 is not true:

$\hat{\beta} \sim N(\hat{\beta}, \sigma^2 (X'X)^{-1})$

let $V = \dfrac{(X'X)^{1/2}\hat{\beta}}{\sigma}$

$$E[V] = \frac{(X'X)^{1/2}}{\sigma} E[\hat{\beta}] = \frac{(X'X)^{1/2}}{\sigma}\beta$$

$$\text{var}(V) = \frac{(X'X)^{1/2}}{\sigma} \text{var}(\hat{\beta}) \frac{(X'X)^{1/2}}{\sigma^2}$$

$$= \frac{\sigma^2}{\sigma^2} (X'X)^{1/2} (X'X)^{-1} (X'X)^{1/2}$$

$$= (X'X)^{1/2} (X'X)^{-1/2} (X'X)^{-1/2} (X'X)^{1/2}$$

$$= I_{k+1}$$

$\Rightarrow V \sim N\left(\frac{(X'X)^{1/2}\beta}{\sigma}, I\right)$

$$\Rightarrow V'V = \frac{\hat{\beta}' (X'X)^{1/2} (X'X)^{1/2} \hat{\beta}}{\sigma^2} = \frac{\hat{\beta}' (X'X) \hat{\beta}}{\sigma^2}$$

Note $\hat{\beta}'X'Y = Y(X'X)^{-1}X'Y = Y'HY$

$= Y'HHY$

$= (HY)'(HY)$

$= \hat{Y}'\hat{Y} = \hat{\beta}'X'X\hat{\beta} \Rightarrow \hat{\beta}'X'X\hat{\beta} = \hat{\beta}'X'Y$

$$\frac{\hat{\beta}'(X'X)\hat{\beta}}{\sigma^2} = \frac{\hat{\beta}'X'Y}{\sigma^2} \sim \chi^2_{k+1} \left(NCP = \frac{\beta'(X'X)\beta}{\sigma^2}\right)$$

$$\Rightarrow \frac{\frac{\hat{\beta}'X'Y}{\sigma^2} \Big/ k+1}{\frac{(n-k-1)Se^2}{\sigma^2} \Big/ n-k-1} = \frac{\hat{\beta}'X'Y}{(k+1)Se^2} \sim F_{k+1,\, n-k-1} \left(NCP = \frac{\beta'(X'X)\beta}{\sigma^2}\right)$$

Exercise F

$H_0: \alpha_1 = \alpha_2$

$y_1 = \alpha_1 + \varepsilon$

$y_2 = 2\alpha_1 - \alpha_2 + \varepsilon_2$

$y_3 = \alpha_1 + 2\alpha_2 + \varepsilon_3$

$$Y = X \beta + \epsilon$$

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & -1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{bmatrix}$$

show that the F-test for $H_0: \alpha_1 = \alpha_2$ is given by $F = \dfrac{(\hat{\alpha}_1 - \hat{\alpha}_2)^2}{\frac{11}{30} Se^2}$

$H_0: \alpha_1 = \alpha_2$ can also be written as $H_0: c\beta = \gamma$

$H_a: \alpha_1 \neq a_2$ $\qquad H_a: c\beta \neq \gamma$

$$c\hat{\beta} - \gamma = \hat{\alpha}_1 - \hat{\alpha}_2$$

$$\underbrace{\begin{bmatrix} 1 & -1 \end{bmatrix}}_{c} \underbrace{\begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix}}_{\beta} = \underbrace{0}_{\gamma}$$

we know:

$$\frac{(c\hat{\beta} - \gamma)' (c(X'X)^{-1}c')^{-1} (c\hat{\beta} - \gamma)}{\sigma^2} \sim \chi^2_m$$

$$X'X = \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & -1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 5 \end{bmatrix}$$

$$\Rightarrow (X'X)^{-1} = \begin{bmatrix} 1/6 & 0 \\ 0 & 1/5 \end{bmatrix} \Rightarrow c(X'X)^{-1}c' = \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} 1/6 & 0 \\ 0 & 1/5 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = 11/30$$

$$\frac{(c\hat{\beta} - \gamma)' [c(X'X)^{-1}c']^{-1} (c\hat{\beta} - \gamma)}{\sigma^2} = \frac{(\hat{\alpha}_1 - \hat{\alpha}_2)' [11/30]^{-1} (\hat{\alpha}_1 - \hat{\alpha}_2)}{\sigma^2}$$

Note:
$c = m \times k+1$
$\Rightarrow m = 1$

$$= \frac{(\hat{\alpha}_1 - \hat{\alpha}_2)^2}{11/30\, \sigma^2} \sim \chi^2_1$$

$$\frac{(n-k-1)Se^2}{\sigma^2} \sim \chi^2_{n-k-1}$$

$$\longrightarrow \frac{\dfrac{(\hat{\alpha}_1 - \hat{\alpha}_2)^2}{11/30\, \sigma^2} \Big/ 1}{\dfrac{(n-k-1)Se^2}{\sigma^2} \Big/ (n-k-1)} \sim F_{1, n-k-1}$$

$$\Longrightarrow \frac{(\hat{\alpha}_1 - \hat{\alpha}_2)^2}{11/30\, \sigma^2} \sim F_{1, n-k-1}$$

Exercise 9

$$Y_1 = \theta_1 + \theta_2 + \epsilon_1 \qquad \epsilon \sim N_3(0, \sigma^2 I)$$

$$Y_2 = 2\theta_2 + \epsilon_2 \qquad \left.\begin{array}{l} H_0: \theta_1 = 2\theta_2 \\ H_a: \theta_1 \neq 2\theta_2 \end{array}\right] \rightarrow \begin{array}{l} H_0: \theta_1 - 2\theta_2 = 0 \\ H_a: \theta_1 - 2\theta_2 \neq 0 \end{array}$$

$$Y_3 = -\theta_1 + \theta + \epsilon_3$$

or $H_0 : c\hat{\ell} = \gamma$ where $c = \begin{bmatrix} 1 & -2 \end{bmatrix}$ and $\gamma = 0$

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} + \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \end{bmatrix}$$

$$Y = X \ell + \epsilon$$

The F statistic needed to solve this problem is:

$$\frac{(SSE_c - SSE_F)/df_R - df_F}{SSE_F / df_F} \sim F_{df_R - df_F,\, df_F}$$

where: $df_F = 3-2 = 1$

$df_R = 3-(2-1) = 2$

$df_R - df_F = 2-1 = 1$

In order to solve for the numerator:

$$SSE_c = e_c' e_c = e'e + (c\hat{\ell} - \gamma)' \left[c(X'X)^{-1}c'\right]^{-1} (c\hat{\ell} - \gamma)$$

$$SSE_F = e'e$$

$$\Rightarrow SSE_c - SSE_F = (c\hat{\ell} - \gamma)' \left[c(X'X)^{-1}c'\right]^{-1} (c\hat{\ell} - \gamma)$$

$$\Rightarrow \frac{(c\hat{\ell} - \gamma)' \left[c(X'X)^{-1}c'\right]^{-1} (c\hat{\ell} - \gamma)}{e'e} \sim F_{1,1}$$

← This is our new F statistic

In order to solve for our F statistic, we would use the matrices c and γ found above. we would also need to use the X matrix shown above and our $\hat{\ell}$ matrix which would be $\hat{\ell} = \begin{bmatrix} \hat{\theta}_1 \\ \hat{\theta}_2 \end{bmatrix}$ to find the numerator.

In order to find the denominator we would need to recall that $\frac{e'e}{df_F} = S_e^2$

Exercise h

$Y = X\beta + \varepsilon$

$Y = \beta_0 1 + X_{(0)}\beta_{(0)} + \varepsilon \qquad X = \begin{bmatrix} 1 & X_{(0)} \end{bmatrix} \qquad \beta = \begin{bmatrix} \beta_0 \\ \beta_{(0)} \end{bmatrix}$

Regression of Y on 1 to get Y^*:

$$Y^* = (I - \tfrac{1}{n}11')Y$$

Regression of $X_{(0)}$ on 1 to get $X_{(0)}^*$

$$X_{(0)}^* = (I - \tfrac{1}{n}11')X_{(0)}$$

Ression of Y^* on $X_{(0)}^*$ to get $\hat{\beta}_{(0)}$

$$\hat{\beta}_{(0)} = (X_{(0)}^{*\prime} X_{(0)}^*)^{-1} X_{(0)}^{*\prime} Y^*$$

$$= (X_{(0)}'(I - \tfrac{1}{n}11')X_{(0)})^{-1} X_{(0)}'(I - \tfrac{1}{n}11')Y$$

Note: $(I - \tfrac{1}{n}11') = (I - \tfrac{1}{n}11')(I - \tfrac{1}{n}11')$

only transform Y:

$$(X_{(0)}'X_{(0)})^{-1} X_{(0)} Y^*$$

$$= (X_{(0)}'X_{(0)})^{-1} X_{(0)}(I - \tfrac{1}{n}11')Y \neq \hat{\beta}_{(0)}$$

=> we do not get the same result if we only transform Y

only transform $X_{(0)}$:

$$(X_{(0)}^{*\prime} X_{(0)}^*)^{-1} X_{(0)}^{*\prime} Y =$$

$$(X_{(0)}(I - \tfrac{1}{n}11')X_{(0)})^{-1} X_{(0)}'(I - \tfrac{1}{n}11')Y = \hat{\beta}_{(0)}$$

=> we do get the same result if we only transform $X_{(0)}$

Exercise i

$Y = X\beta + \epsilon$

Subtract a constant c from each y_i:

$Y - 1c$

Note: this does not impact degrees of freedom

$H_0: \beta_{(0)} = 0$

$H_a: \beta_{(0)} \neq 0$

$$\frac{(SSE_c - SSE_F)/(df_R - df_F)}{SSE_F / df_F} \sim F_{df_R - df_F,\, df_F}$$

Without subtracting a constant:

$SSE_F = Y'(I-H)Y \quad (Y-\hat{Y})'(Y-\hat{Y})$ $\qquad e = (I-H)Y$

subtracting a constant:

$$\begin{aligned}
SSE_F^* &= (Y-1c)'(I-H)(Y-1c) \\
&= (Y-1c)'(I-H)(I-H)(Y-1c) \\
&= (Y'-c1')(I-H)(I-H)(Y-1c) \\
&= (Y'-Y'H-c1'+c1'H)(Y-1c-HY+H1c) \\
&= (Y'-\hat{Y}'-c1'+c1')(Y-\hat{Y}-1c+1c) \\
&= (Y-\hat{Y})'(Y-\hat{Y}) = SSE_F
\end{aligned}$$

Without subtracting a constant:

$SSE_c = Y'(I-H_1)Y = (Y-\hat{Y}_1)'(Y-\hat{Y}_1)$ $\qquad e_c = (I-H_1)Y$

subtracting a constant:

$$\begin{aligned}
SSE_c^* &= (Y-1c)'(I-H_1)(Y-1c) \\
&= (Y-1c)'(I-H_1)(I-H_1)(Y-1c) \\
&= (Y'-c1')(I-H_1)(I-H_1)(Y-1c) \\
&= (Y'-Y'H_1-c1'+c1'H_1)(Y-1c-H_1Y+H_11c) \\
&= (Y'-\hat{Y}_1'-c1'+c1')(Y-\hat{Y}-1c+1c) \\
&= (Y-\hat{Y}_1)'(Y-\hat{Y}_1) = SSE_c
\end{aligned}$$

$$\Rightarrow \frac{(SSE_c^* - SSE_F^*)/df_R - df_F}{SSE_F^*/df_F} = \frac{(SSE_c - SSE_F)/df_R - df_F}{SSE_F/df_F}$$

$$\Rightarrow \quad F^* = F$$

Exercise j

Gauss-Markov:

$cov(\varepsilon) = \sigma^2 I_n$

$\parallel$

$var(\varepsilon)$

1. $\hat{Y}_0 = w'Y$ want: $E[\hat{Y}_0] = X_0'\beta$, find constraint on w

$$E[\hat{Y}_0] = w' E[Y] = w' X\beta$$

$w'X\beta = X_0'\beta$ ← the constraint

$\Rightarrow w'X = X_0'$

$\Rightarrow X_0 = X'w$

2. Want to minimize $E[(Y_0 - \hat{Y}_0)^2]$

show $E[(Y_0 - \hat{Y}_0)^2] = var(Y_0 - \hat{Y}_0)$

$$var(Y_0 - \hat{Y}_0) = E[(Y_0 - \hat{Y}_0)^2] - \overline{E[Y_0 - \hat{Y}_0]^2}^{=0}$$

$$\Rightarrow var(Y_0 - \hat{Y}_0) = E[(Y_0 - \hat{Y}_0)^2]$$

$$var(Y_0 - \hat{Y}_0) = var(Y_0 - w'Y) = var(Y_0) + var(w'Y) - 2cov(Y_0, w'Y)$$

$$= \sigma^2 + w' var(Y) w - 0$$

$$= \sigma^2 + \sigma^2 w'w$$

3. what is the lagrangian?

Minimize $E[(Y_0 - \hat{Y}_0)]^2$ such that $E[\hat{Y}_0] = X_0'\beta$

Minimize $var[(Y_0 - \hat{Y}_0)]$ such that $X'w = X_0$

Minimize $Q(w, \lambda) = \sigma^2 + \sigma^2 w'w - \lambda[X'w - X_0]$

$$\frac{\partial Q}{\partial w} = 2\sigma^2 w - X\lambda' = 0 \rightarrow 2\sigma^2 w = X\lambda' \rightarrow w = \frac{X\lambda'}{2\sigma^2}$$

$$X'w = \frac{X'X\lambda'}{2\sigma^2} = X_0 \rightarrow 2\sigma^2 X_0 = X'X\lambda'$$

$$2\sigma^2 (X'X)^{-1} X_0 = \lambda'$$

$$w = \frac{2\sigma^2 X(X'X)^{-1}X_0}{2\sigma^2} = X(X'X)^{-1}X_0$$

$$\hat{Y}_0 = w'Y = X_0'(X'X)^{-1}X'Y = X_0'\hat{\beta}$$