

## Propositional logic:

① Translate the following sentence into propositional logic-

"You can access the internet from campus only if you are a current student or you are a faculty."

② Show that,  $\neg(P \vee (\neg P \wedge Q))$  is logically equivalent to  $\neg P \wedge \neg Q$ .

③ Express in propositional logic:

"The automated reply cannot be sent when the file system is full."

④ Translate these system specifications into English where,

$F(p)$  is "printer  $p$  is out of service."

$B(p)$  is "Printer  $p$  is busy."

$L(j)$  is "Print job  $j$  is lost"

$Q(j)$  is "print job  $j$  is queued."

Let the domain be all printers.

Ⓐ  $\exists p (F(p) \wedge B(p)) \rightarrow \exists j L(j)$

Ⓑ  $(\forall p B(p)) \rightarrow (\exists j Q(j))$

Ⓒ  $\exists j (Q(j) \wedge L(j)) \rightarrow \exists p F(p)$

Ⓓ  $(\forall p B(p) \wedge \forall j Q(j)) \rightarrow \exists j L(j)$

⑤ Express each of the system specifications using predicate, quantifiers and logical connectives.

① Every user has access to an electronic mailbox.

② The system mailbox can be accessed by everyone in the group if the file system is locked.

③ The firewall is in a diagnostic state only if the proxy server is in a diagnostic state.

④ At least one router is functioning normally if the throughput is between 100 kbps and 500 kbps and the proxy server is not in a diagnostic mode.

(vi) Use rules of inference to show that the hypotheses:

Alice studies well

if Alice studies well, then she became smart.

if Alice is smart, then she will get a good job

imply the conclusion.

Alice will get the job.

(vii) From the collections of premises below, what relevant conclusion can be drawn? Explain the rules of inference used to obtain each conclusion from the premises.

I watch movies if it rains

I watched a movie on Tuesday, or I  
watched a movie on Thursday.

It did not rain on Thursday.

VIII) Use truth tables to verify the propositions below:

(a)  $(p \vee q) \vee r \equiv p \vee (q \vee r)$

(b)  $\neg(p \vee q) \equiv \neg p \vee \neg q$

IX) Show that,  $(p \vee q) \wedge (\neg p \vee r) \rightarrow (q \vee r)$  is a tautology.

X) State the below sentences as converse, contrapositive and inverse of each of these conditional statements:

(a) If it rains today, I will swim tomorrow.

(b) I come to class whenever there is going to be quiz.

(c) A positive integer is a prime only if it has no divisors other than 1 and itself.

ⓧ Let,  $C(x)$ :  $x$  has a cat. Let  $D(x)$ :  $x$  has a dog.  
Let  $P(x)$  has a parrot.

Express each of these statements in terms of  $C(x)$ ,  $D(x)$ ,  $P(x)$ , quantifiers and logical connectives. Let the domain consist of all students in your class.

- ⓐ A student in your class has a cat, a dog and a parrot.
- ⓑ All students in your class have a cat, a dog or a parrot.
- ⓒ Some student in your class has a cat and a parrot, but not a dog.
- ⓓ No student in your class has a cat, a dog, and a parrot.



(XII) Find a counterexample, if possible, to these universally quantified statements, where the domain for all variables consists of all integers.

(a)  $\forall x (x^2 \geq x)$

(b)  $\forall x (x > 0 \vee x < 0)$

(c)  $\forall x (x = 1)$

(XIII) Construct a truth table for each of these compound propositions:

(a)  $(q \rightarrow \neg p) \vee (\neg p \rightarrow \neg q)$

(b)  $(p \vee \neg t) \wedge (p \vee \neg s)$

(c)  $(p \wedge r \wedge s) \vee (q \wedge t) \vee (r \wedge \neg t)$

(XIV) Translate these statements into English, where  $R(x)$ :  $x$  is a rabbit and  $H(x)$ :  $x$  hops, and the domain consists of all animals.

(Remember some of these propositions below aren't tautologies).

- (a)  $\forall x (R(x) \rightarrow H(x))$
- (b)  $\forall x (R(x) \wedge H(x))$
- (c)  $\exists x (R(x) \rightarrow H(x))$
- (d)  $\exists x (R(x) \wedge H(x))$

(XV) Express the negation of these propositions using quantifiers and then express negation in English:

- (a) Some drivers do not obey the speed limit.
- (b) All animation movies are funny.
- (c) No one can keep a secret.
- (d) There is someone in this class who does not have visited Saint-Martin.