Practice Sheet-2 on Number Theory

Question-01:

Use modular exponentiation to find:

i) 7^{644} **mod** 645.
ii) 3^{2003} **mod** 99.
iii) 242^{329} **mod** 243.

Question-02:

i) What is the octal and hexadecimal expansion of $(1\ 1000\ 0110\ 0011)_{2.}$
ii) What is the hexadecimal expansion of $(177130)_{10.}$
iii)What is the decimal expansion of $(7016)_{8.}$
iv) What is the binary expansion of $(BADFACED)_{16.}$
v) What is the octal expansion of $(12345)_{10.}$

Question-03:

Let **a**, **b**,**c,** and **d** be integers, where **a** $\neq$ **0** such that **a | c** and **b | d**, then prove that **ab | cd**.

Question-04:

Prove that if n is an odd positive integer, then $n^2 \equiv 1 \pmod 8$.

Question-05:

Find the integer a such that:

i) $a \equiv -11 \pmod{21}$ and $90 \leq a \leq 110$.
ii) $a \equiv 99 \pmod{41}$ and $100 \leq a \leq 140$.

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iii) $a \equiv 17 \pmod{29}$ and $-14 \leq a \leq 14$.

Question-06:

Find the **quotient** and **remainder** when:

i) 1,234,567 is divided by 1001?
ii) −123 is divided by 19?
iii) 0 is divided by 17?
iv) −2002 is divided by 87?
v) 1001 is divided by 13?

Question-07:

Find the **prime factorization** of the following numbers:

i)909,090.
ii)10!
iii)7007.

Question-08:

Show that if **a** and **b** are positive integers, then **ab = gcd(a, b) · lcm(a, b)**.

Question-09:

Determine whether the integers in each of these sets are **pairwise relatively prime**:

i)14,17,85.
ii)21,34,55.
iii) 25, 41, 49, 64.
iv) 17, 18, 19, 23.

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Question-10:

Express the **greatest common divisor** of each of these pairs of integers as a **linear combination** of these integers.

i) 252,198.
ii) 35,78.
iii) 33,44.

Question-11:

Find the **greatest common divisors** and the **least common multiples** of the following pairs:

i) $3^{13} \cdot 5^{17}$, $2^{12} \cdot 7^{21}$
ii) $2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13$, $2^{11} \cdot 3^{9} \cdot 11 \cdot 17^{14}$
iii) $41 \cdot 43 \cdot 53$, $41 \cdot 43 \cdot 53$

Question-12:

Show that if **a**, **b**, and **m** are integers such that **m ≥ 2** and $a \equiv b \pmod{m}$, then **gcd(a,m) = gcd(b,m)**.

Question-13:

If the product of two integers is $2^{7}3^{8}5^{2}7^{11}$ and their greatest common divisor is $2^{3}3^{4}5$, what is their **least common multiple**?

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Question-14:

Find the greatest common divisor of the following pair of numbers using the Euclidean Algorithm:
i) 11111, 111111.
ii) 1529, 14038.
iii) 750,900.
iv) 414,662.

Question-15:

How many divisions are required to find gcd(21, 34) using the Euclidean algorithm?