BRAC University
CSE230 : Discrete Mathematics
Final Examination

Duration : 1 hour 50 minutes (4:30 pm - 6:20 pm)

Total Marks : 60 Set: A

[Answer any 4 out of 5 questions. Answer all the sub-parts of a question together. Please start each question in a new page]

ID: **Name:** **Sec:**

Q01: [CO2] [15 Marks]

a) Verify using a series of logical equivalences that $[p \rightarrow (q \rightarrow r)] \rightarrow [(p \rightarrow q) \rightarrow (p \rightarrow r)]$ is a tautology. **[6 points]**

b) Determine the truth value of each of these statements if the domain consists of all integers. Justify your answers. **[3 points]**

   i) $\forall n(n + 1 > n)$

   ii) $\exists n(n = -n)$

   iii) $\forall n(3n \leq 4n)$

c) Translate each of these statements into logical expressions using predicates, quantifiers, and logical connectives. **[6 points]**

   i) Not every course is easy.

   ii) All your favorite courses are easy.

   iii) At least one of your favorite courses is easy.

   iv) Every course is your favorite and easy.

Q02: [CO4] [15 Marks]

A television channel holds two contests everyday.

Contest 1: On day 1, the prize money is \$1000. For each day, the prize money increases by 10%, plus an additional \$5 from the chairperson.

Contest 2: On day 1, the prize money is \$100. On day 2, the price money is \$200. On the following days, the prize money will be equal to the sum of prize money of the previous two days.

a) Model recurrence relations for both contests. (Mention the base cases) **[5 points]**

b) Solve the recurrence relation for Contest 1 and find what will be the prize money on the 11th day according to Contest-1. **[5 points]**

c) Solve the recurrence relation for Contest 2 and find what will be the prize money on the 13th day according to Contest-2. **[5 points]**

Q03: [CO1] [15 Marks]

a) Prove, using the principle of mathematical induction, that for all positive integer n:

$$\frac{1}{3*5}+\frac{1}{5*7}+\frac{1}{7*9}+\ldots\ldots+\frac{1}{(2n+1)(2n+3)}=\frac{n}{3(2n+3)} \qquad \textbf{[5]}$$

b) Prove that if a | b and a | c then $a^2 \mid 7bc$ **[3]**

c) Find the remainder when 11^{247} is divided by 392 **[4]**

d) Using the Euclidean Algorithm, find the greatest common divisor of 192 and 258. **[3]**

Q04: [CO3] [15 Marks]

A small Jersey business receives, on average, 8 calls per hour. The probability of selling a jersey in one call is 20%.

a) What is the probability that the business will receive exactly 7 calls in 1 hour? **[4]**

b) What is the probability that the business will receive, at most, 5 calls in one hour? **[3]**

c) What is the probability that the business will receive more than 6 calls in one hour? **[4]**

d) Calculate the probability that the business needs at least 8 calls to sell a jersey? **[4]**

Q05: [CO2] [15 Marks]

a) Use mathematical induction to prove that $7^{n+2}+8^{2n+1}$ is divisible by 19 for every non-negative integer n. **[6 points]**

b) The probability that an archer hits the bull's eye in an attempt is 75%.

   i) What is the probability that he hits the bull's eye for the first time on his 5th trial? **[3 points]**

   ii) How many attempts are expected from him to hit the bull's eye for 12 times? **[3 points]**

c) The archer returned with more training. This time he hits the bull's eye in an attempt with 80% probability and 15% of the time he hits the board and finally only 5% of time he misses the entire board. If he attempts 7 times, what is the probability that he will hit the bull's eye 4 times, hits the board (other than bull's eye) twice and misses the entire board once. **[3 points]**