

BRAC University
CSE230 : Discrete Mathematics

Duration : 1 hour 15 minutes (4:45 pm - 6:00 pm)

Total Marks : 50 Set: B

*[Answer any 5 out of 6 questions. Answer all the sub-parts of a question together.
Please start each question in a new page]*

Student Name:	Student ID:
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Q01: [CO2] [10 Marks]

- (a) Draw a Venn diagram using 3 sets Q, T and P. None of these 3 sets are pairwise disjoint. Moreover, $P \cap Q \cap T \neq \phi$. How many disjoint regions are there? Indicate which regions fall under $P - (Q \cap T)$. **[1+1=2 marks]**
- (b) Find the domain of $f(x) = \sqrt{\frac{3x+x^2}{2-x}}$. Show the domain in a number line. **[4 marks]**
- (c) Find the range of $g(x) = \frac{3x-7}{2+5x}$. What should be the domain of g(x)? **[3+1=4 marks]**

Q02: [CO2] [10 Marks]

A,B,C,D,E,F,G,H and I are nine students of CSE230 Fall 2022. They want to take pictures.

- a) In how many ways can they stand in a straight line to take the picture keeping **F** right in the middle of the picture? **[2 marks]**
- b) Now consider B,C, and D are close friends and they want to stay together in the group picture. How many ways can they take pictures keeping the “close” friends together? **[4 marks]**
- c) Well, D has a “thing” for H. So he(D) wants to stay beside her(H). But he also wants to stay with his friends(B and C). For example, **ABCD**FGEHI is not allowed as D is not beside H although BCD are together, but **ACBD**HEFIG is allowed as D is with his friends and beside H too. Another allowed arrangement can be **AHDCB**FEGI. In how many ways can they stand for taking the picture keeping all these scenarios into consideration? **[4 marks]**

Q03: [CO1] [10 Marks]

- a) How many arrangements of the word “**tictactoe**” are possible so that no two vowels are side-by-side? How many of them start and end with the same letter? **[3+3=6 marks]**
- b) Adnan and Binti are playing a game in which Binti chooses k numbers from the set $\{2,...,20\}$. If Adnan can find 2 numbers from Binti’s chosen numbers whose sum is divisible by 10, then Adnan wins. What is the minimum value of k so that Adnan always wins? **[2 marks]**
- c) In how many ways can you arrange the letters of the word “**ceremony**” so that y always comes after the e’s? For example, “**ermenoyc**” is acceptable, whereas “**yermenoc**” and “**remyonce**” are not. **[2 marks]**

Q04: [CO3] [10 Marks]

Read the following equations.

- 1) ${}^7C_a = {}^7C_5$, a is an integer and $0 < a < 7$
- 2) $(x + y)^n = \sum_{r=0}^n {}^nC_r x^{n-r} y^r$, n is a non-negative integer.

Now, answer the following questions.

- a) Find out all the possible values of a from the equation (1). **[2 marks]**
- b) If $a \geq 4$, show that, the a -th term in the expansion of $(3x + \frac{1}{ax})^8$ is a constant.
[Use value of a from eq. No.1] **[4 marks]**
- c) If $a < 4$, find out the coefficient of $x^{1+a}y^{-3-a}$ in the expansion of $(3x + \frac{1}{ay})^8$
[Use value of a from eq. No.1] **[4 marks]**

Q05: [CO4] [10 Marks]

A deck of DIEZ Cards has 4 different colors which are Red, Green, Yellow and Blue. Each color has 1 Wild Card, 1 Reverse Card, 1 Block Card and 7 Normal Cards numbered from 1-7.

- a) Find the probability of picking a Reverse Card from the deck. **[2 marks]**
- b) If 3 cards are picked at random from the deck, what is the probability of picking at most 2 Wild cards? **[4 marks]**
- c) Now, imagine one card is lost from the deck at random. If we pick a card from the deck, what is the probability of that card being a Block Card? **[4 marks]**

Q06: [CO4] [10 Marks]

The Graduate Record Examinations Test (GRE) is a requirement for all applicants of Msc Programs.

Suppose that a survey of GRE students reveals that among GRE scorers above 310, 52% took Magoosh (An Online Education Company) paid subscription, whereas among GRE scorers of less than 310 only 23% took the subscription. An applicant thinks that in order to get into a certain university he needs more than 310. The chance of obtaining more than 310 is $x\%$

- a) Suppose $x = 10\%$. Given that he took a Magoosh subscription, What is the probability of getting less than 310? **[7 marks]**
- b) If the probability of getting more than 310 given that he has taken the Magoosh Subscription is 40%. Then what is x ? **[3 marks]**