Practice Sheet - Solution
Combinatorics and Pigeonhole Principle
[Made by ST_Keya_T133]

Combinatorics:

1.
   a. Total number of words in MISSISSIPPI = 11
      There are 4 I's, 2 P's, 4 S's.
      So,the number of words that can be formed
      = 11! / (4!2!4!)
      = 34650 ways

   b. Total number of words in NORMAL = 6
      There are no common words here.
      So,the number of words that can be formed
      = 6!
      = 720 ways

   c. Total number of words in ABOMINABLE = 10
      There are 2 A's, 2 B's.
      So,the number of words that can be formed
      = 10! / (2!2!)
      = 907200

   d. Total number of words in MONKEYDLUFFY = 12
      There are 2 Y's, 2 F's.
      So,the number of words that can be formed
      = 12! / (2!2!)
      = 119750400

   e. Total number of words in FMALCHEMIST = 11
      There are 2 M's.
      So,the number of words that can be formed
      = 11! / (2!)
      = 19958400

2.

   a. Let’s keep OM together and consider it one letter. Now we have 12 letters which can be arranged in,
      =12! / (2!2!2!)
      =59875200
   b. Now O and M can interchange their places in 2! Ways. So, the total arrangements would be,
      ={12! / (2!2!2!)} * (2!)
      =119750400

3. If a license plate has 8 letters, where the first 4 letters include only alphabets and last 4 letters include digits only,
   Now,
   [With Repetition]
      My name is SHARMIN, where 7 unique letters are there,
      So, to fill up 4 slots with 7 unique letters, the arrangements will be,
      = 7 * 7 * 7 * 7
      = 2401

      My ID is 19301074, the unique digits are 0,1,3,4,7,9 (total 6 unique digits).
      We have to fill up 4 slots with these 6 unique letters.
      = 6 * 6 * 6 * 6
      = 1296
      Therefore, the number of 8 digit license plate can be made from my name and ID where 1st 4 letters are alphabets and last 4 letters are digits will be,
      = 2401 * 1296
      = 3111696.
   [Without Repetition]
      My name is SHARMIN, where 7 unique letters are there,
      So, to fill up 4 slots with 7 unique letters, the arrangements will be,
      = 7 * 6 * 5 * 4
      = 840

My ID is 19301074, the unique digits are 0,1,3,4,7,9 (total 6 unique digits).
We have to fill up 4 slots with these 6 unique letters.
= 6 * 5 * 4 * 3
= 360
Therefore, the number of 8 digit license plate can be made from my name and ID where 1st 4 letters are alphabets and last 4 letters are digits will be,
= 840 * 360
= 302400.

4. My ID is 19301074
   Here, the unique digits are 0,1,3,4,7,9 (total 6 unique digits)
   We have to make a 3 digit number, So, we have 3 slots to fill up.
   For the first slot, we can’t have 0 as a digit, otherwise it’ll make a 2 digit number. Therefore, for the first slot we only have 5 options.
   The second and third slot has all 6 options.
   [With repetition]
   So, the total possible 3 digit numbers,
   = 5 * 6 * 6
   = 180
   [Without repetition]
   So, the total possible 3 digit numbers,
   = 5 * 5 * 4
   = 100

5. Number of ways to choose 3 black and 2 green marbles,
   =12C3 * 8C2
   = 6160

6.
   a. 8!
      = 40320 ways

   b. 8C1 × 7C1
      = 56 ways

7. Total no of committees
   =11C4 × 9C3
   =330*84
   =27720

8.
   a. There is no restriction on the selection,so,
      = 15C11
      =1365
   b. One Particular Player is always chosen, therefore,
      = 14C10
      =1001
   c. Two Particular Player is never chosen, then,
      = 13C11
      = 78

9.
   a. So, the arrangements would be,
      = 10!
      =3628800 ways
   b. There are 10 slots to fill up with 5 couples who must seat side by side together.
      So, there are 10 options for the first person to seat and 1 option for that person’s mate to seat. (Since the first person’s mate has to be seated next to him/her)
      Then, there are 8 choices for the 2nd couple to seat. (since 2 people have already been seated in 2 seats)
      Then, there are 6 options for the 3rd couple to seat,
      After that, there are 4 options for the 4th couple to seat.
      Lastly, only 2 seats will be empty and the 5th or last couple has to sit on those two seats.
      Therefore, the arrangements here will be,
      = 10 * 8 * 6 * 4 * 2
      = 3840 ways

10.

a. If we think how it look like, BGBGBGBGB [B= Boys, G=Girls]
   After a boy, there'll be a girl or after a girl there will be a boy.
   So, the permutations will be,
   = 5! * 4!
   =2880 ways
b. Total number of boys and girls = 9
   If Anne and Jim wants to stay together, then consider them as one person, so now the total number = 8
   So, the permutations will be,
   = 8!
   = 40320
   But, Anne and Jim can interchange their places in 2! Ways, therefore the total arrangements will be,
   = 2! * 8!
   = 80640

11. There are 3 sisters and 8 other girls.
So, the total number of girls = 11 .
Now,
The number of ways to arrange these 11 girls in a circular manner,
= (11– 1)!
= 10!
= 3628800 ways
If we keep the three sisters always together, then the three sisters can now rearrange themselves in (3!) = 6 Ways.
So, the number of ways that the 3 sisters always come together in the arrangement,
= 8! × 3!
= 241920 ways
Hence, the required number of ways in which the arrangement can take place if none of the 3 sisters is seated together will be,
= 10! – (8! × 3!)
= 3628800 – 241920
= 3386880 ways.

Pigeonhole principle:

1. Each person can have 0 to 19 friends. But if someone has 0 friends, then no one can have 19 friends (since there's mutual friendship) and similarly you cannot have 19 friends and no friends. So, there are only 19 options for the number of friends and 20 people.
   Therefore, by pigeonhole, there exist 2 people who have the same number of friends.
2. There are $\{(10 * 10^6) + 1\}$ different number of lines of code you can write.
   So, there exists a number of line of codes with at least,
   = $(300 * 10^6) \div \{(10 * 10^6) + 1\}$
   = 30 people.

3. By pigeonhole,
   there exists a person who has gotten at least,
   = 202 / 3
   = 68 votes.
   So, someone could win with a 67 − 67 − 68 split.

4. Think about the worst the cat, Briar could do:
   If the cat pulls 3 socks of each color (for a total of 9 socks)
   then it still does not have enough to make a matching set,
   but if a 10th sock is chosen then, it must complete one set of matching socks.
   Therefore 9 socks is the most Briar can pull before finding a match.

5. There are only 3 possible remainders when dividing a whole number by 3, which are 0, 1, or 2.
   No matter which 7 numbers Tammy chooses, they each have one of these 3 remainders.
   Hence, by the pigeonhole principle, there must be at least 3 numbers with the same remainder. If two numbers have the same remainder when divided by 3, their difference is divisible by 3.

6. There exists a time period that will have at least,
   = 677 / 38
   = 18 classes during one time period.
   So 18 different rooms will be needed.