Practice - Sheet - C&P

ST_Nadia_T121

Question 01

a)

6 letters (all small)

a) End with y?

Ans: a) 26^5 = 11881376

b) Start with π and end with b? No repitition

Ans: b) $^{26}P_4$ = 14950

c) Contain only vowel?

Ans: c) 5^6 = 15625

d) Do not contain z, e, π, 0?

Ans: d) 22^6 = 113379904

e) contain "est"?

Ans: e) ~~$^{26}P_3 \times 3!$~~ $26^3 \times 4!$ = 421824

Ques 2

Given,

positive integer < 1000

a) Divisible by 5?

Ans: a) $\frac{999}{5} = 199.8 \simeq$ **199 (Ans)**

b) Divisible by 5 but not 7?

Ans: b) $\frac{999}{5 \times 7} = 28.5 \simeq 28$

So, (199 - 28) = **171 numbers (Ans)**

c) Divisible by both 5 and 7?

Ans: c) $\frac{999}{5 \times 7} = 28.5 \simeq$ **28 (Ans)**

d) Divisible by either 5 or 7?

Ans: d) Divisible by 5 = 199

" " 7 = $\frac{999}{7} = 142.7 \simeq 142$

" " both = 28

" " either 5 or 7 = (199 - 28) + (142 - 28) + 28

= 313

Ans: 313

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e) divisible by only 5 or only 7?

Ans: $(199-28)+(142-28)$

$= 285$

f)

For 1 digit number : $10C_1$

For 2 " " : $9C_1 \times 9C_1$

For 3 " " : $9C_1 \times 9C_1 \times 8C_1$

Total : 739

Question 03

10 blue toys

10 red " toys

a) 3 toys of same color
How many toys?

Ans: a) Number of color = pigeonhole, toys = pigeon

$\frac{\text{pigeon}}{\text{pigeonhole}} = \text{same} \Rightarrow \frac{k}{2} = 3 \quad \therefore k = 5$

Worst case:

OR)

Ans: a) 3 red + 2 blue
2 red + 3 blue

∴ 5 balls

b) At least 3 toys of red

b) How many ~~balls~~? toys?

Ans: $\frac{\text{Worst case}}{}$

10 red ~~balls~~ toys

+ 3 blue "

= 13 toys

c) Arrange all toys.

In how many ways?

Ans: $\frac{20!}{10!\,10!} = 184756$

Question 04

Given, Dhakas population = 22 M

$= 22 \times 1000000$

At most ~~peopl~~ number of people who can have ~~same~~ different initial and ~~same~~ different birthday =

$366 \times {}^{26}P_3$

people who have same initial and same birthday $= \frac{22000000}{366 \times 26^3} = 3.424$

[proven]

Question 5

a) Correction: at least 15 female or 15 male

Ans: if female = 14
male = 14
total = 28 < 30

[The statement is false if both are false]

So number of female or male must have to be >= 15.

~~4 = 4 + 6 = 6~~

b) 17 male students
13 female ii

if male = 16
female = 12
total = 28 < 30

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(c) {1, 2, 3, 4, 5, 6, 7, 8}

{(1,8), (2,7), (3,6), (4,5)}

~~Worst case: {0, 1, 2, 3, 4, 5, 6}~~

Worst case: {1, 2, 3, 4, 5}

So, we need to select at least 5 integers to ensure that there must be a pair whose sum=9

(any)

(d)

NO, if {1, 2, 3, 4} is selected then the conclusion is violated.

Question 6

21 women

10 men

NO man stand next to each other

In how many ways they can stand in a line?

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Ans) Valid position = 21 + 1

= 22.

So, ways $^{22}P_{10}$ = $\boxed{2.35 \times 10^{12}}$

Question 7

(a)

$^{16}C_{10}$ = $\boxed{8008}$ Ans

(b)

$^{16}P_{10}$ = $\boxed{2.9 \times 10^{10}}$ Ans

(c)

No women case: $^{10}C_{10}$

At least one woman case: $^{16}C_{10} - {}^{10}C_{10}$

(d) $\boxed{= 8007}$ Ans

$^{10}C_1 \times {}^{9}C_1 \times {}^{8}C_1 \times 3!$

$\boxed{= 4320 \text{ Ans}}$

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Question 8

(a)

6 women

8 men

8 seats

At least 3 women passengers

Case 1 : 3 women x 5 men

(+) $= {}^{6}C_3 \times {}^{8}C_5$

Case 2 : 4 women x 4 men

(+) $= {}^{6}C_4 \times {}^{8}C_4$

Case 3 : 5 women x 3 men

(+) $= {}^{6}C_5 \times {}^{8}C_3$

Case 4 : 6 women x 2 men

$= {}^{6}C_6 \times {}^{8}C_2$

Ans Total : 2534

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(b)

~~Exactly 4~~ corrections

At least 1 women

" " 4 men

Case 1: 1 women 7 men

(+) $^6C_1 \times {}^8C_7$

Case 2: 2 women 6 men

(+) $^6C_2 \times {}^8C_6$

Case 3: 3 women 5 men

(+) $^6C_3 \times {}^8C_5$

Case 4: 4 women 4 men

(+) $^6C_4 \times {}^8C_4$

Ans: Total = 2638

~~Case 5: 5 women 3 men~~

6

Question 09

(a)

(011) (011) (011) 1 1 1 1 1

$$\frac{8!}{3!\,5!} = \boxed{56}$$

(b)

String length = 14

at least five 0

1 1 1 1 1

500 5 zero 9 one $= \frac{14!}{5!9!} = 2002$

6 11 8 1 $= \frac{14!}{6!8!} = 3003$

7 11 7 11 $= \frac{14!}{7!7!} = 3432$

8 11 6 11 $= \frac{14!}{8!6!} = 3003$

9 11 5 11 $= \frac{14!}{9!5!} = 2002$

$$\boxed{\text{total} = 13492}$$

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Question 10

$$^{48}C_8 \times {}^{35}C_6 \times {}^{44}C_4 = \boxed{8.31 \times 10^{19}}$$

Question 11

(a)

EVERGREEN

if 9 letters $\frac{9!}{4!2!} = 7560$

if 8 letters

4 E
2 R
1 V
1 G
1 N

4 same 2 same 2 d

$(+) \frac{8!}{4!2!} = 840$

4 same 4 diff

$(+) \frac{8!}{4!} = 1680$

3 same 2 same 2 diff

$(+) \frac{8!}{3!2!} = 3360$

Total = 5880

Of 7 letters %

4 same 2 same 1 dif

$$^3C_1 \times \frac{7!}{4!2!} = 315$$

4 same 3 dif

$$^4C_3 \times \frac{7!}{4!} = 840$$

3 same 2 same 2 diff

$$^3C_2 \times \frac{7!}{3!2!} = 1260$$

3 same 4 diff

$$\frac{7!}{3!} = 840$$

2 same 2 same 3 diff

$$\frac{7!}{2!2!} = 1260$$

4515

Total % 7560 + 5880 + 4515

Ques 11 (a)

=17955 (Any)

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(b)

EEEE

V.R.G.R.N

$\frac{6!}{}$ $\frac{5!}{4!} \times \frac{5!}{2!}$

Ans = 300

(c)

000 0 11 0 1 1 1 0

length = 11

zero = 6

one = 5

$\frac{11!}{6!6!} = 462$

Ans = (462 - 1) = 461

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Ques 12

$$(36^6 - 26^6) + (36^7 - 26^7) + (36^8 - 26^8)$$

$$= 2.68 \times 10^{12}$$

Ques 13

correction 3 different sizes

$$4 + 2 + 3$$

$$= 9$$

Ques 14

$$(10-1)! = 9! = 362880$$

Ques 15

$$\frac{(7-1)!}{2} = 20160$$

Ques 16

(a)

$$4C_3 \times 48C_4$$

$$= 778320$$

(b)

$$13C_5 \times 39C_2$$

$$= 953667$$

(c)

$$12C_4 \times 40C_3$$

$$= 4890600$$

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