

BRAC University
CSE230 : Discrete Mathematics
Final Examination

Duration : 1 hour 50 minutes (4:30 pm - 6:20 pm)

Total Marks : 60 Set: B

[Answer any 4 out of 5 questions. Answer all the sub-parts of a question together. Please start each question in a new page]

ID:

Name:

Sec:

Q01: [CO2] [15 Marks]

- a) Verify using a series of logical equivalences that $[a \rightarrow (b \rightarrow c)] \rightarrow [(a \rightarrow b) \rightarrow (a \rightarrow c)]$ is a tautology. **[6 points]**
- b) Determine the truth value of each of these statements if the domain consists of all integers. Justify your answers. **[3 points]**
 - i) $\forall n(n + 4 > n)$
 - ii) $\exists n(2n = 3n)$
 - iii) $\forall n(3n \leq 4n)$
- c) Translate each of these statements into logical expressions using predicates, quantifiers, and logical connectives. **[6 points]**
 - i) All your favorite topics are interesting.
 - ii) At least one of your favorite topics is interesting.
 - iii) Not every topic is interesting.
 - iv) Every topic is your favorite and interesting.

Q02: [CO4] [15 Marks]

Farah's brother proposed two schemes for Eid Salami to her.

Scheme-1: She will get Tk.500 in the first year and Tk.1000 in the second year and after that every year she will get summation of previous two years' salami as her current year Eid salami.

Scheme-2: She will get Tk. 10 on the first Eid. But every year she will get double the previous year and Tk.5 extra.

- a) Model recurrence relations for both the schemes. (Mention the base cases) **[5 points]**
- b) Solve recurrence relation for Scheme-1 and find what will be her Salami on the 10th year according to Scheme-1. **[5 points]**
- c) Solve the recurrence relation for Scheme-2 and find how much Salami will Farah get on her 12th Eid according to Scheme-2. **[5 points]**

Q03: [CO1] [15 Marks]

- a) Prove, using the principle of mathematical induction, that for all positive integer n:

$$\frac{1}{2*4} + \frac{1}{4*6} + \frac{1}{6*8} + \dots + \frac{1}{(2n)(2n+2)} = \frac{n}{4(n+1)} \quad [5]$$

- b) Prove that if $a \mid b$ and $a \mid c$ then $a^2 \mid 7bc$ [3]
c) Find the remainder when 11^{249} is divided by 357 [4]
d) Using the Euclidean Algorithm, find the greatest common divisor of 252 and 198. [3]

Q04: [CO3] [15 Marks]

A small Jersey business receives, on average, 12 calls per hour. The probability of selling a jersey in one call is 20%.

- a) What is the probability that the business will receive exactly 6 calls in 1 hour? [4]
b) What is the probability that the business will receive, at most, 4 calls in one hour? [3]
c) What is the probability that the business will receive more than 5 calls in one hour? [4]
d) Calculate the probability that the business needs at least 9 calls to sell a jersey? [4]

Q05: [CO2] [15 Marks]

- a) Use mathematical induction to prove that $5 \times 2^{3n+1} + 3^{3n+2}$ is divisible by 19 for every non-negative integer n. [6 points]
b) The probability that an archer hits the bull's eye in an attempt is 60%.
i) What is the probability that he hits the bull's eye for the first time on his 5th trial? [3 points]
ii) How many attempts are expected from him to hit the bull's eye for 9 times? [3 points]
c) The archer returned with more training. This time he hits the bull's eye in an attempt with 70% probability and 25% of the time he hits the board and finally only 5% of time he misses the entire board. If he attempts 7 times, what is the probability that he will hit the bull's eye 4 times, hits the board (other than bull's eye) twice and misses the entire board once. [3 points]