

CSE230 Final Questions Solution

Set: B

Q01: [CO4] [20 Marks]

In a box, there are 5 red balls, 6 blue balls and a ball which can either be red or blue with 50-50 chances (Not known with certainty). If you pick two balls at random from the box,

- a) What is the probability that both the balls will be red? [3 marks]
- b) What is the probability that the balls will be of the same colors? [6 marks]
- c) What is the probability that the balls will be of different colors? [6 marks]
- d) What is the probability that at least one ball will be red? [5 marks]

Solution:

Q1 (Solution)

Set B

$$a) \frac{1}{2} \times \frac{{}^6C_2}{{}^{12}C_2} + \frac{1}{2} \times \frac{{}^5C_2}{{}^{12}C_2}$$

$$b) \frac{1}{2} \times \left(\frac{{}^6C_2 + {}^6C_2}{{}^{12}C_2} \right) + \frac{1}{2} \times \left(\frac{{}^5C_2 + {}^7C_2}{{}^{12}C_2} \right)$$

$$c) \frac{1}{2} \times \frac{{}^6C_1 \times {}^6C_1}{{}^{12}C_2} + \frac{1}{2} \times \frac{{}^5C_1 \times {}^7C_1}{{}^{12}C_2} \quad \text{Or, } 1 - (b)$$

$$d) 1 - \left(\frac{1}{2} \times \frac{{}^6C_2}{{}^{12}C_2} + \frac{1}{2} \times \frac{{}^7C_2}{{}^{12}C_2} \right)$$

Or,

~~$\frac{1}{2} \times \frac{{}^6C_1}{{}^{12}C_1}$~~ Ans (a) + Ans (c)

Q02: [CO2] [20 Marks]

- a) A famous restaurant has introduced a combo meal, consisting of 3 types of rice, 5 types of side dishes, 4 types of salads, and some type of desserts. Among each item, the diner can select one for their meal.
- Suppose, 180 different types of combo meals can be prepared in this situation. How many types of desserts does the restaurant offer? [3 marks]
 - On a particular day, the manager instructed the waiters to arrange the rice, salads, and desserts on a table serially. But, he wanted to keep the 3 types of rice separately (so they cannot be placed side by side), keeping a salad on one end and a side dish on the other end. How many ways are there to arrange the foods according to the instructions? [7 marks]
- b) Tom loves playing with integers. He has picked up some distinct positive integers that are not larger than 30. According to Tom, there is an integer that is divisible by either 3 or 5. How many integers must have been picked by him? [5 marks]
- c) Your friend has some fruits. Each of them is of distinct type. He randomly chooses a pair of fruits, each containing fruits of different kinds.
- If the total number of possible combinations were 66, how many fruits does your friend have? [3 marks]
 - Will the number of possible combinations change from 66, if the count of each type of fruit had been more than 1? Give a proper explanation. [2 marks]

Solution:

Q2 (Set B)

a) (i) $3 \times 5 \times 4 \times d = 180 \Rightarrow d = 3$

(ii) $n = 3 + 5 + 4 + 3 = 15$ items

Diagram illustrating the arrangement of items:

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  [ salad ] --- 3 types rice in (10+1) ways --- [ salad or side dish ]
  [ salad or side dish ]
  
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13 items

$4C_1 \times 5C_1 \times 2 \times {}^{11}P_3 \times 10!$

b) $n(\text{div by 3 or 5}) = 10 + 6 - 2 = 14$

Ans: $30 - 14 + 1 = 17$

c) (i) ${}^nC_2 = 66 \therefore n = 12$

(ii) No [give proper explanation]

In 2(c) i - if a student starts with $nC_2 * (n-2)C_2 * (n-4)C_2 * (n-6)C_2 \dots = 66$ and goes a bit deeper into a form like $n! / 2^k$.. he/she should be given marks as well.

Q03: [CO1] [20 Marks]

- a) Sue has three coins. Two are fair, with a head on one side and a tail on the other. The other is a trick coin and has a tail on both sides. Sue picks up one of the coins at random and flips it.

Given that it lands tails up, find the probability that she picked up the fair coin. [6 marks]

- b) In a factory, machines A, B and C are all producing metal rods of the same length. Machine A produces 35% of the rods, machine B produces 25% and the rest are produced by machine C. Of their production of rods, machines A, B and C produce 3%, 6% and 5% defective rods respectively.

Find the probability that a randomly selected rod is:

I. Produced by machine A and is defective [3 marks]

II. Defective. [5 marks]

III. Given that a randomly selected rod is defective, find the probability that it was made by machine C. [6 marks]

Solution:

Q 3 (Set B)

$$\therefore P(C_f | T) = \frac{P(C_f \cap T)}{P(T)}$$

$$= \frac{\frac{2}{3} \times \frac{1}{2}}{\frac{2}{3} \times \frac{1}{2} + \frac{1}{3} \times 1} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$$

(b) (I) $P(A \cap D) = \frac{35}{100} \times \frac{3}{100}$

(II) $P(D) = \frac{35}{100} \times \frac{3}{100} + \frac{25}{100} \times \frac{6}{100} + \frac{40}{100} \times \frac{5}{100}$

(III) $P(C|D) = \frac{P(C \cap D)}{P(D)}$
 $= \frac{\frac{40}{100} \times \frac{5}{100}}{(Ans II)}$

Q04: [CO3] [20 Marks]

Read the following equations:

I. $a^5(b - 3a)^{12}$

II. $(1 + 4a - 3b)^4$

Answer the following questions considering the given equations:

a) If the coefficient of a^8 is -5940, then what is the value of b from equation I where $a, b > 0$ and $a, b \in R$?

[4 marks]

b) Find the value of a , if the 5th term of the expression $a^5(b - 3a)^{12}$ is equal to 120285?

[use the value of b from the previous question]

[6 marks]

c) Determine the total number of terms if we expand equation II.

[3 marks]

d) Determine the value of the constant term in the expansion of $(4p^2 - p^3 + \frac{6}{p^5})^{10}$

[7 marks]

Solution:

Q4 (Set B)

a) a^8 in expansion of $a^5(b-3a)^{12}$ means

a^3 in expansion of $(b-3a)^{12}$

$$\text{coefficient} = {}^{12}C_3 b^{12-3} (-3)^3 = -5940 \quad (\text{A/Q})$$

$$\Rightarrow b^9 = \frac{-5940}{220(-27)} = 1$$

$$\therefore b = 1$$

$$b) \text{ 5th term} = a^5 \cdot {}^{12}C_4 b^8 (-3a)^4 = 120285$$

$$\Rightarrow a^9 = \frac{120285}{495 \times (-3)^4}$$

$$a = \sqrt[9]{3} \quad (\text{Ans})$$

$$c) \text{ Total no. of terms} = {}^{9+2}C_2 = 15$$

$$d) (4p^2)^6 (-p^3)^1 \left(\frac{6}{p^5}\right)^3 \text{ will be constant}$$

$$\therefore \text{Constant term} = \frac{10!}{6!3!} 4^6 (-1) 6^3 \quad (\text{Ans})$$