Number Theory (Practice Sheet I)

1. Does 17 divide each of these numbers?

   a) 68 b) 84 c) 357 d) 1001

2. Show that if a, b, and c are integers, where a = 0 and c = 0, such that ac | bc, then a | b.

3. What time does a 12-hour clock read

   a) 80 hours after it reads 11:00?

   b) 40 hours before it reads 12:00?

   c) 100 hours after it reads 6:00?

4. Suppose that a and b are integers, a ≡ 4 (mod 13), and b ≡ 9 (mod 13). Find the integer c with 0 ≤ c ≤ 12 such that

   a) c ≡ 9a (mod 13).

   b) c ≡ 11b (mod 13).

   c) c ≡ a + b (mod 13).

   d) c ≡ 2a + 3b (mod 13).

   e) c ≡ a2 + b2 (mod 13).

   f) c ≡ a3 − b3 (mod 13).

5. Show that if n and k are positive integers, then n/k = (n − 1)/k + 1.

6. Find a div m and a mod m when

   a) a = 228, m = 119.

   b) a = 9009, m = 223.

   c) a = −10101, m = 333.

d) $a = -765432$, $m = 38271$.

7. List all integers between -100 and 100 that are congruent to -1 modulo 25.

8. Show that if $n \mid m$, where n and m are integers greater than 1, and if $a \equiv b \pmod{m}$, where a and b are integers, then $a \equiv b \pmod{n}$.

9. Determine whether the integers in each of these sets are pairwise relatively prime.

   a) 11, 15, 19

   b) 14, 15, 21

   c) 12, 17, 31, 37

   d) 7, 8, 9, 11

10. What is the greatest common divisor of 17 and 22?

11. Determine whether the integers 10, 17, and 21 are pairwise relatively prime and whether the integers 10, 19, and 24 are pairwise relatively prime.

12. Use the Euclidean algorithm to find

    a) gcd(12, 18).

    b) gcd(111, 201).

    c) gcd(1001, 1331).

    d) gcd(12345, 54321).

e) gcd(1000, 5040).

f) gcd(9888, 6060)

13. How many divisions are required to find gcd(34, 55) using the Euclidean algorithm?

14. What are the greatest common divisors of these pairs of integers?

a) 37 · 53 · 73, 211 · 35 · 59

b) 11 · 13 · 17, 29 · 37 · 55 · 73

c) 2331, 2317

15. Find gcd(92928, 123552) and lcm(92928, 123552), and verify that gcd(92928, 123552) · lcm(92928, 123552) = 92928 · 123552. [Hint: First find the prime factorizations of 92928 and 123552.]