Word Problem:

Consider the word 'DIAPHANOUS'.

1) How many ways are there to rearrange the letters of the word?
2) How many ways are there to rearrange the letters of the word so that <u>the vowels are together</u>?
3) How many ways are there to rearrange the letters of the word so that <u>all the vowels are never together</u>? [D(IOU)PHANAS is allowed; DPH(IAAOU)SN isn't allowed.]
4) How many ways are there to rearrange the letters of the word so that <u>no two vowels are placed consecutively</u>?[D I P A H A N U S O allowed; I D O U P H A N A S is not allowed.]
5) How many ways are there to rearrange the letters of the word so that the vowels <u>take only the places which are taken by vowels in the original word</u>?
6) How many ways are there to rearrange the letters of the word so that the vowels <u>maintain their order as they appear in the original word</u>?

Set of Digits:

Consider the set : {1, 0, 5 , 6, 7, 4}
1) How many ways are there to create odd numbers containing less than 5 digits using from the digits of the set using each digit only once?
2) How many ways are there to create even numbers containing less than 5 digits using from the digits of the set using each digit only once?

Binomial & Multinomial Expansion:

(Collected)

a) $(a + by)^3$

b) $(1 + \frac{3}{q^2} + p + 2r)^{8a+3b}$

Answer the following questions considering the given expressions:

1) If the coefficient of y^2 is 81 and the coefficient of y^3 is 27, what are the values of a and b where $a, b > 0$ and $a, b \in R$?

2) What is the value of y in the expression $(a + by)^3$ if the 3rd term is equal to 121?

3) Using the value of a and b from the previous question, find the coefficient of the term $p^5 r^2 q^{-4}$.

Combinations:

1) There are 7 notes on a table.Of the 7 notes, 3 are green, 2 are red and 2 are blue. How many ways are there to select 4 notes from them?
2) Solve the same problem under the condition that the number of notes of each color is at least 4.

*(Food for thought)

$x_1 + x_2 + x_3 + x_4 = 15;$ *where* x_1, x_2, x_3, x_4 are non-negative integers.

1) How many possible solutions does this equation have?
2) How many possible solutions will this equation have if $1 \leq x_4 \leq 4$ *and* $x_3 \geq 6$

Pigeon Hole Principle:

1) Show that whenever 25 girls and 25 boys are seated around a circular table there is always a person both of whose neighbors are boys.
2) Show that given three integers a_1, a_2, a_3 at least one of the consecutive sums must be divisible by three.
3) During a month with 30 days, a baseball team plays at least one game a day, but no more than 45 games. Show that there must be a period of some number of consecutive days during which the team must play exactly 14 games.
4) Assume that in a group of six people, each pair of individuals consists of two friends or two enemies. Show that there are either three mutual friends or three mutual enemies in the group.
5) A basket of fruit is arranged out of apples, bananas, and oranges. What is the smallest number of fruit that should be put into the basket to guarantee that either there are at least eight apples, or at least six bananas, or at least nine oranges ?
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