'Solutions of Binomial and Multinomial Practice problems'

1. $\frac{7!}{2!\times 4!\times 1!}(3a)^2(5b)^4(d)^1$

$= 105(9a^2)(625b^4)(d)$

$= 590625a^2b^4d$.

2) a) $\underbrace{(x+y+z+t)}_{4}^{10} = {}^{10+4-1}C_{10} = {}^{13}C_{10} = 286$.

b) $(3a+5b-2c+5d)^7 = {}^{7+4-1}C_7 = {}^{10}C_7 = 120$

3) $\frac{10!}{3!\times 2!\times 4!\times 1!}(a)^3\times(-b)^2\times(-c)^4d^1 = 12600$.

4) $\cancel{\boxed{\frac{5!}{3!\times 2!}\times a^2\times(3c)^2 = 90a}}$ 4) $\frac{5!}{3!\times 2!}a^3\times(2b)^2 = 40a^3b^2$

5) a) 3rd term of $(3+y)^6_6 =$ $\cancel{3^4 y^2 15 = 1215y^2}$ ${}^6C_2\times 3^{6-2}\times y^2 = 1215y^2$.

b) z^4 will come in 5th term.

$\therefore$ 5th term of $(5+z)^8 =$ $\cancel{5^4 z^4 70}$ $= 43750z^4$.

c) The term containing x^3 is $\rightarrow (r+1)^{th}$ term $= (3+1)$ [Here $r=3$]
$=$ 4th term.

6)

7. 5th expansion: $(-1)^6 \binom{10}{4} \left(\frac{1}{2\sqrt{x}}\right)^6 \left(\frac{1}{2}\right)^4 = 105$

$$\Rightarrow \binom{10}{4} \frac{1}{64x^3} \cdot \frac{1}{16} = 105$$

$$\Rightarrow x^3 = \frac{210}{105 \cdot 2^{10}}$$

$$\Rightarrow x^3 = \frac{1}{2^9} \Rightarrow x = \frac{1}{8}.$$

8) 9). $T_{r+1} = {}^nC_r\, a^{n-r} b^r$

$\left(ax^2 + \frac{1}{bx}\right)^{11}$

$T_{r+1} = {}^{11}C_r\, x^{22-3r} \cdot a^{11} (ab)^{-r}.$

$\therefore 22 - 3r = 7 \Rightarrow r = 5$ [for co-efficient x^7]

$T_6 = {}^{11}C_5\, x^7 a^{11} (ab)^{-5}$ —(i)

Now, for $\left(ax^2 - \frac{1}{bx^2}\right)^{11}$

$T_{r+1} = (-1)^r\, {}^{11}C_r\, x^{11-3r} a^{11} (ab)^{-r}.$

$\therefore 11 - 3r = -7 \Rightarrow r = 6$ for co-efficient x^{-7}

$T_7 = {}^{11}C_6\, x^{-7} a^{11} ab^{-6}$ —(ii)

(i) = (ii)

$\Rightarrow {}^{11}C_6\, a^{11} (ab)^{-6} = {}^{11}C_5\, a^{11} (ab)^{-5}$

$\Rightarrow ab = 1.$

10. $(1+x^2)^5 (1+x)^4$

$$\Rightarrow \left({}^5C_0 (1) + {}^5C_1 (1) x^2 + {}^5C_2 (1)(x^2)^2 + {}^5C_3 (x^2)^3 + {}^5C_4 (x^2)^4 \right.$$
$$\left. + {}^5C_5 (x^2)^5 \right) \times \left({}^4C_0 (1) + {}^4C_1 (x) + {}^4C_2 (x^2) + {}^4C_3 (x^3) + {}^4C_4 (x)^4 \right)$$

$$= \{ {}^5C_2 x^4 \times {}^4C_1 x + {}^5C_1 x + {}^5C_1 x^2 \times {}^4C_3 x^3 \}$$

$\therefore$ coefficient of x^5

$$= \left(\frac{5\times4}{2\times1} \times 4 \right) x^5 + (5\times4) x^5 = 60x^5$$

11(a). $T_{r+1} = {}^6C_r \left(\frac{3x^2}{2}\right)^{6-r} \left(-\frac{1}{3x}\right)^r$

$$= {}^6C_r \left(3/2\right)^{6-r} \left(-\frac{1}{3}\right)^r x^{12-2r-r}$$

$\therefore x^{12-2r-r} = x^0$

$\Rightarrow r = 4$

$$T_{4+1} = {}^6C_4 \left(3/2\right)^{6-4} \left(-\frac{1}{3}\right)^4 = \frac{5}{12}$$

b) $T_{r+1} = {}^{10}C_r \left(\sqrt{x/3}\right)^{10-r} \left(\frac{\sqrt{3}}{2x^2}\right)^r$

$$= {}^{10}C_r \frac{x^{\frac{10-r}{2}}}{3^{\frac{10-r}{2}}} \cdot \frac{(\sqrt{3})^r}{2^r x^{2r}}$$

$\therefore x^{\frac{10-r}{2} - 2r} = x^0 \Rightarrow r = 2.$

$$\therefore T_{2+1} = {}^{10}C_2 \frac{(\sqrt{3})^2}{3^4 \cdot 2^2} = 5/12$$

12. $(1-2x+x^3)^6$

$\Rightarrow (1+(x^3-2x))^6 = \boxed{1 + {}^6C_1(x^3-2x) + {}^6C_2(x^3-2x)^2} + {}^6C_3(x^3-2x)^3$ → not possible to get x^7.

$+ {}^6C_4(x^3-2x)^4 + {}^6C_5(x^3-2x)^5 + (x^3-2x)^6$

${}^6C_3(x^3-2x)^3 = 20\left({}^3C_1(x^3)^2(-2x)^1\right) = -120x^7$

${}^6C_4(x^3-2x)^4 = 15\left((x^3)\ (-2x)\right) \rightarrow$ not possible to get x^7.

${}^6C_5(x^3-2x)^5 = 5\left({}^5C_4(x^3)^1(-2x)^4\right) = 5(5(16x^7))$
$= 480x^7$

$(x^3-2x)^6 = (x^3)(-2x) \rightarrow$ not possible to get x^7

$\therefore x^7$ term $= -120x^7 + 480x^7 = 360x^7$.

13. $\dfrac{15!}{10! \times 3! \times 2!} \times (0.6)^{10} \times (0.25)^3 \times (0.15)^2 =$

14. ${}^nC_r\,(ax)^{n-r}(b)^r$

$\Rightarrow {}^nC_2\,(5x)^{n-2}(2)^2$

$\Rightarrow \dfrac{n!}{2!(n-2)!}(5x)^{n-2}.\,4 = 70312500$

$\Rightarrow \dfrac{n(n-1)(n-2)!}{2(n-2)!}\,5^{n-2}.\,4 = 70312500$

$\Rightarrow 2n(n-1).\dfrac{5^n}{5^2} = 70312500$

$\Rightarrow \dfrac{2n}{25}(n-1).\,5^n = 70312500$

$\Rightarrow n(n-1)\cdot 5^n = \dfrac{70312500 \times 25}{2}$

$\Rightarrow n = 10$

15. $\dfrac{10!}{3! \times 5! 2!} = 2520$

8. In $(2+x/3)^n$,

coefficient of x^7 is $= {}^nC_7\, 2^{n-7} (x/3)^7$

$= {}^nC_7\, 2^{n-7} \cdot \dfrac{1}{3^7}$ — ①

~~co t~~ co-efficient of x^8 is $= {}^nC_8\, 2^{n-8} (x/3)^8$

$= {}^nC_8\, 2^{n-8} \cdot \dfrac{1}{3^8}$ — ②

Given,

① = ②

$\Rightarrow {}^nC_7\, 2^{n-7} \cdot \dfrac{1}{3^7} = {}^nC_8\, 2^{n-8} \cdot \dfrac{1}{3^8}$

$\Rightarrow \dfrac{n!}{(n-7)!\,7!} \cdot \dfrac{3^8}{3^7} = \dfrac{n!}{(n-8)!\,8!} \cdot \dfrac{2^{n-8}}{2^{n-7}}$

$\Rightarrow \dfrac{(n-8)!\,8!}{(n-7)!\,7!} \cdot 3 = \dfrac{2^{n-8}}{2^{n-7}}$

$\Rightarrow \dfrac{\cancel{(n-8)!}}{(n-7)\cancel{(n-8)!}} \cdot 3 = \dfrac{2^{-1}}{8 \times \cancel{7!}}\, \cancel{7!}$

$\Rightarrow \dfrac{3}{n-7} = \dfrac{1}{16} \Rightarrow n = 55.$