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CSE230
SEC 22

Assignment 01
Assignment on Combinatorics

① There are 4 different colors available for each models of iPhone 14 Pro Max and iPhone 14 Pro.

$$^4C_1 + {^4C_1} = 4+4 = 8$$

Also, there are 5 different colors available for each models of iPhone 14 Plus and iPhone 14.

$$^5C_1 + {^5C_1} = 5+5 = 10$$

So, there are total $(8+10) = 18$ different types of iPhone 14 available this year.

② As, One woman and ~~of~~ one of her children ~~ar~~ will be chosen as mother and child of the year,

~~So,~~ woman can be chosen in $^{10}C_1 = 10$ different ways.

children can be chosen in $^3C_1 = 3$ different ways.

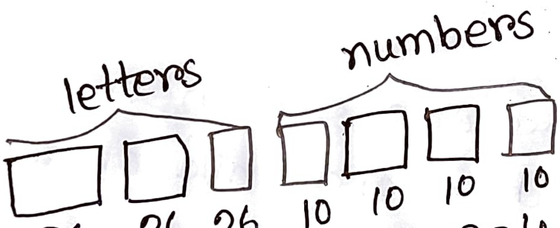
So, totally ~~10x~~ $^{10}C_1 \times {^3C_1} = 10 \times 3 = 30$ different choices are possible for selecting mother and child of the year.

③ College planning committee consists of 3 freshmen, 4 sophomores, 5 juniors, and 2 seniors.

If they create ~~For creating~~ subcommittee of 4 members which consist 1 person from each class, they will be able to make ${}^{3}C_1 \times {}^{4}C_1 \times {}^{5}C_1 \times {}^{2}C_1 = 120$ different subcommittees.

④

④



$n=26, r=3$; $n=10, r=4$

~~In~~ the first 3-places can be occupied by letters in $n^r = 26^3 = 17576$ ways

the final 4-places can be occupied by ~~letter~~ numbers in $n^r = 10^4 = 10000$ ways.

Totally, $26^3 \times 10^4 = 175760000$ different 7-place license plates are possible.

⑤ If repetition among letters or numbers were prohibited, then there would be ${}^{26}P_3 \times {}^{10}P_4 = 7862$-4000 different ~~pl~~ license plates.

⑥ As functional value is either 0 or 1. and it could be defined on 'n' points, totally $n^r = 2^n$ functions are possible.

⑦ COURAGE

Vowels - O, U, A, E

~~Vo~~

□ □ □ □ □ □ □

✓ 6 5 4 3 2 1

One Vowel other 6 letters $6!$

So, possible arrangements of ~~courage~~ COURAGE begin with a vowel $= {}^4P_1 \times 6! = 2880$

⑧ ARRANGE

~~Possible~~ Possible arrangements $= \frac{7!}{2!2!}$

Contains RR $= \frac{6!}{2!}$

$\therefore$ Possible arrangements of ARRANGE do not contain two RR together $= \frac{7!}{2!2!} - \frac{6!}{2!} = 900$

(9) TRIANGLE

Possible arrangements = $8!$

All vowels together = $6! \times 3!$

[$6!$ → all vowels can act as a single letter
$3!$ → all vowels]

So, arrangements of TRIANGLE do not contain all vowels together = $8! - (6! \times 3!) = 36000$

(10) PERMUTATION

Vowels - E, U, A, I, O

So, without changing the position of vowels possible rearrangements of PERMUTATION = $\frac{6!}{2!} - 1 = 359$

(11) PERMUTATIONS

① As, each arrangement starts with P and ends with S,
so, P and S act as a letter.

∴ Possible arrangements = $\frac{10!}{2!} = 1814400$

② ~~If vowels are not all together = 5!~~

If vowels are all tog.

② If all vowels act as a ~~letter~~ single letter $= \frac{8!}{2!}$

If don't act as a single letter $= 5!$

So, vowels are all together $= 5! \times \frac{8!}{2!}$

$= 2419200$

③ If there are always 4 letters between P and S, then possible cases

When, P comes first,

$P(1) \rightarrow S(6)$

$P(2) \rightarrow S(7)$

$P(3) \rightarrow S(8)$

$P(4) \rightarrow S(9)$

$P(5) \rightarrow S(10)$

$P(6) \rightarrow S(11)$

$P(7) \rightarrow S(12)$

So, there are total 7 ~~ways~~ cases. Again if S comes first there also be 7 cases.

So, total possible arrangements $= (7+7) \times \frac{10!}{2!}$

$= 14 \times \frac{10!}{2!}$

$= 25401600$

(14)

CAMBRIDGE

$\square\square\square\square\square$

~~If all vowels are present =~~ $^3C_3$

Vowels - A, I, E (3)

Constant - C, M, B, R, D, G (6)

3 vowels can be chosen = $^3C_3$ ways

6 constants can be chosen = $^6C_2$

Now, 5-letter arrangements of CAMBRIDGE contain all vowels = $5! \times {}^3C_3 \times {}^6C_2 = 1800$

(15) DIAPHANOUS

vowels - I, A, O, U

If vowels don't change positions = $\frac{5!}{2!}$

Now, with all other constants the arrangement will be = $6! \times \frac{5!}{2!}$

$= 43200$

(21) Set = $\{1, 2, 3, \ldots, 200\}$

Numbers divisible by 7 = $\left\lfloor \frac{200}{7} \right\rfloor = \lfloor 28.57 \rfloor = 28$

Numbers divisible by 9 = $\left\lfloor \frac{200}{9} \right\rfloor = \lfloor 22.\dot{2} \rfloor = 22$

LCM(7, 9) = 63

Numbers divisible by 63 = $\left\lfloor \frac{200}{63} \right\rfloor = \lfloor 3.175 \rfloor = 3$

As this 3 numbers are present in the set of 7 and 9 both.

So, total numbers which are divisible by either 7 or 9 but not both = $28 + 22 - (3 \times 2) = 44$

(22) set = $\{1, 2, 3, \ldots 200\}$

Numbers divisible by 3 = $\left\lfloor \frac{200}{3} \right\rfloor = \lfloor 66.67 \rfloor = 66$

Numbers divisible by 4 = $\left\lfloor \frac{200}{4} \right\rfloor = \lfloor 50 \rfloor = 50$

Numbers divisible by 12 = $\left\lfloor \frac{200}{12} \right\rfloor = \lfloor 16.67 \rfloor = 16$

As, 12 is the LCM of 3 & 4 So, the 16 numbers are present in the set of 3 & 4 both.

So, total numbers ~~wich~~ which are divisible by neither 3 nor 4 nor 12 = $200 - [(66 - 16) + (50 - 16)] = 116$

(34)

DADDY DID A DEADLY DEED

Present letters - A, D, E, L, Y, I

So, the possible combination,

$$^6C_1 + {}^6C_2 + {}^6C_3 + {}^6C_4 + {}^6C_5 + {}^6C_6$$

$$= 63$$