BRAC University
CSE230 : Discrete Mathematics
Midterm Examination

Duration : 75 minutes (4:45 pm - 6:00 pm)

Total Marks : 45 Set: A

[Answer any 3 out of 4 questions. Answer all the sub-parts of a question together. Please start each question in a new page]

ID: **Name:** **Sec:**

Q01: [CO1] [15 Points]

a) Verify using a truth table that $\neg((\neg a \wedge b) \rightarrow (a \vee b))$ is a contradiction. **[5 points]**

b) Write the converse, inverse and contrapositive of the following statement: **[5 points]**

"I will pass CSE230 if I study"

c) Let P(x), Q(x), R(x), S(x) and T(x) be the statements "x is a hummingbird," "x is rich in color," "x lives on honey," "x is large," and "x can fly fast," respectively. Express each of these statements using quantifiers; logical connectives; and P(x), Q(x), R(x), S(x) and T(x). **[5 points]**

i) All hummingbirds are rich in color.
ii) Some large birds live on honey.
iii) Not all hummingbirds can fly fast.
iv) No hummingbird is large.
v) Some large birds cannot fly fast.

Q02: [CO4] [15 Points]

a) Show that the following mathematical statement is true for all positive integers n,

$$\frac{5}{1\times2\times3}+\frac{6}{2\times3\times4}+\frac{7}{3\times4\times5}+\ldots\ldots.+\frac{n+4}{n(n+1)(n+2)}=\frac{n(3n+7)}{2(n+1)(n+2)}$$ **[8 points]**

b) Prove by mathematical induction that 10 divides $3^n + 7^n$ for all positive odd integers of n.

[7 points]

Q03: [CO6] [15 Points]

a) Nayel drops a ping pong ball from the top of a 100 meter tall building. After each drop on the ground, the ball jumps up to the two-third of its previous height. Find a recurrence relation expressing the total distance covered by the ball before its n^{th} drop on the ground. **[5 points]**

b) Solve the following recurrence relation: **[8 points]**

$2a_{n+2} = 4a_{n+1} + 126a_n + 2.\ 5^n$

Here $a_0 = 0$, $a_1 = 5$

c) $a_8 - a_6 = ?$ **[2 points]**

Q04: [CO7] [15 Points]

a) Find the closest integer to 3^{731} which is divisible by 7. **(Answer will be in $\boldsymbol{a^b + c}$ format)**. **[5 points]**

b) We know that the following congruences are true: $a \equiv a$ **mod** $\boldsymbol{m}$ (mod $\boldsymbol{m}$), $b \equiv b$ **mod** $\boldsymbol{m}$ (mod $\boldsymbol{m}$). From this, show that ab **mod** $m = ((a$ **mod** $m)(b$ **mod** $m))$ **mod** m.

(**Note that** '(mod m)' denotes congruency and **'mod** m' denotes the mod function.) **[5 points]**

c) Find the least common multiple between 3528 and 524 with the help of the Euclidean algorithm. **[5 points]**