part 1

Read the instructions carefully.

There are 3 types of questions -- MCQ, MSQ & NV.

For Multiple **choice** Questions (**MCQ**)
Each question should have exactly one correct answer.
If you select the correct answer, you will get 1 mark.
Otherwise you will get 0 mark.

For Multiple **selection** Questions (**MSQ**)
Each question may have one or multiple correct answer(s). Say, we have *t* correct answers.
If you select any of the incorrect answers, you will get 0 mark.
Otherwise your mark will be p/t upon selecting *p* correct answers.

For Numerical Value questions (**NVQ**)
There might be multiple correct answers, however you **must** submit exactly one of them.
If your answer is correct, you will get 1.
Otherwise you will get 0.

Each question, be it **MCQ**, **MSQ** or **NVQ**, carries 1 mark.

MCQ

Let p, q be two propositions. Then find the truth value in "??" .

$p \vee q$	$p \wedge q$	$p \oplus q$
T	F	??

- (●) True
- ( ) False
- ( ) Insufficient information is given

MSQ

Let x, y, z be the last three digits of your Student ID.
[For example, if your student ID is 12345678, then $x = 6, y = 7, z = 8$]
Also consider the following propositions:
p: $(y + z)$ is **even**
q: $(y \times z)$ is **odd**
Which of the following proposition(s) is/are true?

- [ ] $p \rightarrow q$
- [x] converse of $p \rightarrow q$
- [ ] contrapositive of $p \rightarrow q$
- [x] inverse of $p \rightarrow q$

MSQ

Let the domain of x consists of all the people in the world.
Consider the following predicates:
P(x): x is miser
Q(x): x is rich
Which of the following statements represent the **negation** of: "Every rich person is miser"?

- [ ] $\neg\forall x\,(P(x) \wedge Q(x))$
- [ ] $\neg\forall x\,(P(x) \rightarrow Q(x))$
- [ ] $\neg\exists x\,(P(x) \wedge Q(x))$
- [ ] $\exists x\,(P(x) \wedge Q(x))$
- [ ] $\exists x\,(P(x) \wedge \neg Q(x))$

NVQ

How many values amount to **False** (F) in the last column of the following truth table?

p	q	r	$(p \rightarrow (r \oplus q))$
F	F	F	
F	F	T	
F	T	F	
F	T	T	
T	F	F	
T	F	T	
T	T	F	
T	T	T	

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NVQ

The following information is given about three integers p, q and t:

- $GCD(p, q) = 12$
- $GCD(q, t) = 16$
- $LCM(p, q) = 336$
- $LCM(q, t) = 240$
- $p \times t = 6720$

Find the value of q.

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MSQ

Let x, y, z be the last three digits of your Student ID.
[For example, if your student ID is 12345678, then $x = 6, y = 7, z = 8$]
Let a, b, c be three positive integers and let $a|b$ and $b|c$. Then which of the following is/are true?

a. $a \mid ax - by + cz$
b. $a^2 \mid bc$
c. $ab \mid 3c^2 - bcy$
d. $c^2 \mid ax + by + cz$

☐ (a)

☐ (b)

☐ (c)

☐ (d)

NVQ

Let x, y, z be the last three digits of your Student ID.
[For example, if your student ID is 12345678, then $x = 6, y = 7, z = 8$]
Let $v = 40 + z$

Then find the value of $11^v (mod\ 13)$.

....

MSQ

Let x, y, z be the last three digits of your Student ID.
[For example, if your student ID is 12345678, then $x = 6, y = 7, z = 8$]
Let $w = 10\times y + z + 99$
What is/are the possible remainder(s) when $\left(w^{300} - w^{100} + w\right)$ is divided by 3?

- [ ] 0
- [ ] 1
- [ ] 2
- [ ] 3

NVQ

Let x, y, z be the last three digits of your Student ID.
[For example, if your student ID is 12345678, then $x = 6, y = 7, z = 8$]
What is the degree of the following recurrence relation?

$$5a_{n+x+1} + 6a_{n+y} = 3a_n$$

8

MSQ

Which of the following recurrence relations is/are equivalent to $a_n = a_{n-1} + 2^n$?

a. $a_n = a_{n-2} + 2^{n+1}$

b. $a_n = a_{n-2} + 2^n$

c. $a_n = a_{n-2} + 2^{n-1}$

d. $a_n = a_{n-2} + 2^n + 2^{n-1}$

- [ ] (a)
- [ ] (b)
- [ ] (c)
- [ ] (d)

MSQ

Let x, y, z be the last three digits of your Student ID.
[For example, if your student ID is 12345678, then $x = 6, y = 7, z = 8$]
Which statement(s) is/are correct about the following recurrence relation?

$$12a_{n+4} + 2a_{n+2} - z \times n^{y} = 12a_{n+1} - 38a_{n+3}$$

a. The relation is non-homogeneous
b. The relation is non-linear
c. Homogeneous part of the relation has 3 characteristics roots
d. 3 is a characteristics root of the homogeneous part of the relation

- [x] (a)
- [x] (b)
- [x] (c)
- [ ] (d)

part 2

Read the instructions carefully.

There are 3 types of questions -- MCQ, MSQ & NV.

For multiple **choice** questions (**MCQ**)
Each question should have exactly one correct answer.
If you select the correct answer, you will get 1 mark.
Otherwise you will get 0 mark.

For multiple **selection** questions (**MSQ**)
Each question may have one or multiple correct answer(s). Say, we have *t* correct answers.
If you select any of the incorrect answers, you will get 0 mark.
Otherwise your mark will be p/t upon selecting *p* correct answers.

For numerical value questions (**NVQ**)
There might be multiple correct answers, however you **must** submit exactly one of them.
If your answer is correct, you will get 1.
Otherwise you will get 0.

Each question, be it **MCQ**, **MSQ** or **NVQ**, carries 1 mark.

MSQ

In the Basis step, There is/are error(s) in:

Prove by induction that,

$P(n)$: $1 \times 2^1 + 2 \times 2^2 + 3 \times 2^3 + \cdots\cdots\cdots + n \times 2^n = 2 + (n-1) \times 2^{n+1}$

Proof:

Basis Step:

To prove the base, we should start with $n = 2$ ________________ **(Line 1)**

for $n = 2$, L.H.S of $P(n = 2) = 2 \times 2^2 = 8$ ________________ **(Line 2)**

for $n = 2$, R.H.S of $P(n = 2) = (2 - 1) \times 2^{2+1} = 8$ ________________ **(Line 3)**

$\Rightarrow P(n = 2)$ is true.

Inductive Step:

Let's assume that, $P(n = m)$ is true for some $m \in R$ ________________ **(Line 4)**

$\Rightarrow 1 \times 2^1 + 2 \times 2^2 + 3 \times 2^3 + \cdots + m \times 2^m = 2 + (m-1) \times 2^{m+1}$ ....**(i)**

Using **(i)**, we have to show that , $p(n = m + \square)$ is also true ________________ **(Line 5)**

$\Rightarrow 1 \times 2^1 + 2 \times 2^2 + 3 \times 2^3 + \ldots\ldots + m \times 2^m + (m+1) \times 2^{m+1} = 2 + (m+1) \times 2^{m+2}$ ......**(ii)** __**(Line 6)**

Now,

L.H.S of **(ii)** $= 1 \times 2^1 + 2 \times 2^2 + 3 \times 2^3 + \ldots\ldots + m \times 2^m + (m+1) \times 2^{m+1}$

L.H.S of **(ii)** $= 2 + (m-1) \times 2^{m+1} + (m+1) \times 2^{m+1}$ [from **(i)**] ________________ **(Line 7)**

L.H.S of **(ii)** $= 2 + 2^{m+1}(m - 1 + m + 1)$ ________________ **(Line 8)**

L.H.S of **(ii)** $= 2 + (m+1) \times 2^{m+2}$ ________________ **(Line 9)**

L.H.S of **(ii)** $=$ R.H.S of **(ii)**

So, Our claim is true for all positive integers n.

- [ ] Line 1
- [ ] Line 2
- [x] Line 3

NVQ

In line 5, determine the numerical value in the empty box.

Prove by induction that,

$P(n)$: $1 \times 2^1 + 2 \times 2^2 + 3 \times 2^3 + \cdots\cdots\cdots + n \times 2^n = 2 + (n - 1) \times 2^{n+1}$

Proof:

Basis Step:

To prove the base, we should start with $n = 2$ ________________(Line 1)

for $n = 2$, L.H.S of $P(n = 2) = 2 \times 2^2 = 8$ ________________(Line 2)

for $n = 2$, R.H.S of $P(n = 2) = (2 - 1) \times 2^{2+1} = 8$ ________________(Line 3)

$\Rightarrow P(n = 2)$ is true.

Inductive Step:

Let's assume that, $P(n = m)$ is true for some $m \in R$ ________________(Line 4)

$\Rightarrow 1 \times 2^1 + 2 \times 2^2 + 3 \times 2^3 + \cdots + m \times 2^m = 2 + (m - 1) \times 2^{m+1}$ ....(i)

Using **(i)**, we have to show that, $p(n = m + \square)$ is also true ________________(Line 5)

$\Rightarrow 1 \times 2^1 + 2 \times 2^2 + 3 \times 2^3 + \ldots\ldots + m \times 2^m + (m + 1) \times 2^{m+1} = 2 + (m + 1) \times 2^{m+2}$ ......(ii) __(Line 6)

Now,

L.H.S of **(ii)** $= 1 \times 2^1 + 2 \times 2^2 + 3 \times 2^3 + \ldots\ldots + m \times 2^m + (m + 1) \times 2^{m+1}$

L.H.S of **(ii)** $= 2 + (m - 1) \times 2^{m+1} + (m + 1) \times 2^{m+1}$ [from **(i)**] ____________(Line 7)

L.H.S of **(ii)** $= 2 + 2^{m+1}(m - 1 + m + 1)$ ________________(Line 8)

L.H.S of **(ii)** $= 2 + (m + 1) \times 2^{m+2}$ ________________(Line 9)

L.H.S of **(ii)** = R.H.S of **(ii)**

So, Our claim is true for all positive integers n.

1

....

MCQ

In the Inductive step, between Line 4 and Line 6:

Prove by induction that,

$P(n)$: $1 \times 2^1 + 2 \times 2^2 + 3 \times 2^3 + \cdots\cdots\cdots + n \times 2^n = 2 + (n - 1) \times 2^{n+1}$

Proof:

Basis Step:

To prove the base, we should start with $n = 2$ ________________(**Line 1**)

for $n = 2$, L.H.S of $P(n = 2) = 2 \times 2^2 = 8$ ________________(**Line 2**)

for $n = 2$, R.H.S of $P(n = 2) = (2 - 1) \times 2^{2+1} = 8$ ________________(**Line 3**)

$\Rightarrow P(n = 2)$ is true.

Inductive Step:

Let's assume that, $P(n = m)$ is true for some $m \in R$ ________________(**Line 4**)

$\Rightarrow 1 \times 2^1 + 2 \times 2^2 + 3 \times 2^3 + \cdots + m \times 2^m = 2 + (m - 1) \times 2^{m+1}$ ....**(i)**

Using **(i)**, we have to show that , $p(n = m + \square)$is also true ________________(**Line 5**)

$\Rightarrow 1 \times 2^1 + 2 \times 2^2 + 3 \times 2^3 + \ldots\ldots + m \times 2^m + (m + 1) \times 2^{m+1} = 2 + (m + 1) \times 2^{m+2}$ ......**(ii)** __(**Line 6**)

Now,

L.H.S of **(ii)** $= 1 \times 2^1 + 2 \times 2^2 + 3 \times 2^3 + \ldots\ldots + m \times 2^m + (m + 1) \times 2^{m+1}$

L.H.S of **(ii)** $= 2 + (m - 1) \times 2^{m+1} + (m + 1) \times 2^{m+1}$ [from **(i)**] ________________(**Line 7**)

L.H.S of **(ii)** $= 2 + 2^{m+1}(m - 1 + m + 1)$ ________________(**Line 8**)

L.H.S of **(ii)** $= 2 + (m + 1) \times 2^{m+2}$ ________________(**Line 9**)

L.H.S of **(ii)** = R.H.S of **(ii)**

So, Our claim is true for all positive integers n.

- Line 4 has error, Line 6 does not
- Line 6 has error, Line 4 does not
- Both Line 4 and Line 6 have errors
- Both Line 4 and Line 6 are correct

MCQ

Is there any error in Line 7?

Prove by induction that,

$P(n)$: $1 \times 2^1 + 2 \times 2^2 + 3 \times 2^3 + \dots\dots\dots + n \times 2^n = 2 + (n - 1) \times 2^{n+1}$

Proof:

Basis Step:

To prove the base, we should start with $n = 2$ ______________________ **(Line 1)**

for $n = 2$, L.H.S of $P(n = 2) = 2 \times 2^2 = 8$ ______________________ **(Line 2)**

for $n = 2$, R.H.S of $P(n = 2) = (2 - 1) \times 2^{2+1} = 8$ ______________________ **(Line 3)**

$\Rightarrow P(n = 2)$ is true.

Inductive Step:

Let's assume that, $P(n = m)$ is true for some $m \in R$ ______________________ **(Line 4)**

$\Rightarrow 1 \times 2^1 + 2 \times 2^2 + 3 \times 2^3 + \cdots + m \times 2^m = 2 + (m - 1) \times 2^{m+1}$ ....**(i)**

Using **(i)**, we have to show that, $p(n = m + \square)$ is also true ______________________ **(Line 5)**

$\Rightarrow 1 \times 2^1 + 2 \times 2^2 + 3 \times 2^3 + \dots\dots + m \times 2^m + (m + 1) \times 2^{m+1} = 2 + (m + 1) \times 2^{m+2}$ ......**(ii)** ___**(Line 6)**

Now,

L.H.S of **(ii)** $= 1 \times 2^1 + 2 \times 2^2 + 3 \times 2^3 + \dots\dots + m \times 2^m + (m + 1) \times 2^{m+1}$

L.H.S of **(ii)** $= 2 + (m - 1) \times 2^{m+1} + (m + 1) \times 2^{m+1}$ [from **(i)**] ______________________ **(Line 7)**

L.H.S of **(ii)** $= 2 + 2^{m+1}(m - 1 + m + 1)$

L.H.S of **(ii)** $= 2 + (m + 1) \times 2^{m+2}$ ______________________ **(Line 8)**

L.H.S of **(ii)** $=$ R.H.S of **(ii)** ______________________ **(Line 9)**

So, Our claim is true for all positive integers n.

- ○ Yes
- ◉ No

MCQ

Choose the correct answer after comparing the right hand sides of Line 8 and Line 9:

Prove by induction that,

$P(n)$: $1 \times 2^1 + 2 \times 2^2 + 3 \times 2^3 + \cdots\cdots\cdots + n \times 2^n = 2 + (n - 1) \times 2^{n+1}$

Proof:

Basis Step:

To prove the base, we should start with $n = 2$ ____________ **(Line 1)**

for $n = 2$, L.H.S of $P(n = 2) = 2 \times 2^2 = 8$ ____________ **(Line 2)**

for $n = 2$, R.H.S of $P(n = 2) = (2 - 1) \times 2^{2+1} = 8$ ____________ **(Line 3)**

$\Rightarrow P(n = 2)$ is true.

Inductive Step:

Let's assume that, $P(n = m)$ is true for some $m \in R$ ____________ **(Line 4)**

$\Rightarrow 1 \times 2^1 + 2 \times 2^2 + 3 \times 2^3 + \cdots + m \times 2^m = 2 + (m - 1) \times 2^{m+1}$ ....**(i)**

Using **(i)**, we have to show that , $p(n = m + \square)$ is also true ____________ **(Line 5)**

$\Rightarrow 1 \times 2^1 + 2 \times 2^2 + 3 \times 2^3 + \ldots\ldots + m \times 2^m + (m + 1) \times 2^{m+1} = 2 + (m + 1) \times 2^{m+2}$ ......**(ii)** __ **(Line 6)**

Now,

L.H.S of **(ii)** $= 1 \times 2^1 + 2 \times 2^2 + 3 \times 2^3 + \ldots\ldots + m \times 2^m + (m + 1) \times 2^{m+1}$

L.H.S of **(ii)** $= 2 + (m - 1) \times 2^{m+1} + (m + 1) \times 2^{m+1}$ [from **(i)**] ____________ **(Line 7)**

L.H.S of **(ii)** $= 2 + 2^{m+1}(m - 1 + m + 1)$ ____________ **(Line 8)**

L.H.S of **(ii)** $= 2 + (m + 1) \times 2^{m+2}$ ____________ **(Line 9)**

L.H.S of **(ii)** = R.H.S of **(ii)**

So, Our claim is true for all positive integers n.

- ○ (R.H.S. of Line 8) > (R.H.S. of Line 9)
- ○ (R.H.S. of Line 8) = (R.H.S. of Line 9)
- ○ (R.H.S. of Line 8) ≥ (R.H.S. of Line 9)
- ○ (R.H.S. of Line 8) < (R.H.S. of Line 9)

ALL ANSWERS ARE NOT CORRECT