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CSE230
SEC 22

1. Coefficient of $x^8 y^9$ in the equation of $(3x^2 - 2y^3)^7$

$T_{r+1} = {}^7C_r (3x^2)^{7-r} (-2y^3)^r$

$= {}^7C_r \, 3^{7-r} \, x^{14-2r} \, (-2)^r \, y^{3r}$

As, we have to find the coefficient of x^8y^9

$\therefore 14 - 2r = 8$

$\Rightarrow 2r = 6$

$\therefore r = 3$

$T_{3+1} = {}^7C_3 \, 3^{7-3} \, x^{14-6} \, (-2)^3 \, y^9$

$= 35 \times 3^4 \times (-2)^3 \times x^8 y^9$

$= 35 \times 81 \times (-8) \times x^8 y^9$

$= -22680 x^8 y^9$

$\therefore$ Coefficient of x^8y^9 in the equation of $(3x^2 - 2y^3)^7$
is -22680 Ans

(2) Coefficient of x^6y^6 in the expansion of $(4x^2-y^3)^5$

$$T_{r+1} = {}^5C_r (4x^2)^{5-r} (-y^3)^r$$

$$= {}^5C_r\, 4^{5-r}\, x^{10-2r} \cdot (-y^{3r})$$

As, we have to find the coefficient of x^6y^6

$\therefore$ $10-2r=6$

$\Rightarrow$ $2r=4$

$\therefore$ $r=2$

$$T_{2+1} = {}^5C_2\, 4^{5-2}\, x^{10-4} (-y^6)$$

$$= -10 \times 64 \times x^6y^6$$

$$= -640x^6y^6$$

$\therefore$ Coefficient of x^6y^6 in the expansion of $(4x^2-y^3)^5$ is -640. Ans

(3) $a^2 = 10a - 21$

$\Rightarrow a^2 - 10a + 21 = 0$

$\Rightarrow a^2 - 7a - 3a + 21 = 0$

$\Rightarrow a(a-7) - 3(a-7) = 0$

$\Rightarrow (a-7)(a-3) = 0$

$a - 7 = 0$	$a - 3 = 0$
$\therefore a = 7$	$\therefore a = 3$

$\therefore$ All the possible values of a from the equation are 7 and 3.

(4) $(a+2)$-th term in the expansion of $\left(5x + \frac{1}{ax}\right)^8$

$$T_{(a+1)+1} = {}^8C_{a+1} (5x)^{8-(a+1)} \left(\frac{1}{ax}\right)^{a+1}$$

$$= {}^8C_{a+1} (5x)^{7-a} \left(\frac{1}{ax}\right)^{a+1}$$

$$= {}^8C_{a+1}\, 5^{7-a} \left(\frac{1}{ax}\right)^{a+1} x^{7-a}$$

$$= {}^8C_{a+1}\, 5^{7-a} \left(\frac{1}{a}\right)^{a+1} x^{-(a+1)} \cdot x^{7-a}$$

$$= {}^8C_{a+1} \cdot 5^{7-a} \left(\frac{1}{a}\right)^{a+1} x^{7-a-(a+1)}$$

$$= {}^8C_{a+1}\, 5^{7-a} \left(\frac{1}{a}\right)^{a+1} x^{6-2a}$$

A·

Now, for the constant term,

$$x^{6-2a} = x^0$$

$$6-2a = 0$$

$$\Rightarrow 2a = 6$$

$$\therefore a = 3$$

$\therefore$ In the expansion of $\left(5x+\frac{1}{ax}\right)^8$; the $(a+2)$-th term will be constant only for $a=3$ which is less than 4. But, if $a=1,2$ the $(a+2)$-th term will not be constant.

(5) ① Given expression, $a^5(b-3a)^{12}$

The coefficient of a^8 in the expression $a^5(b-3a)^{12}$ n is -5940.

If we cancel out a^5 from the expression, then -5940 will be the coefficient of $\frac{a^8}{a^5} = a^{8-5} = a^3$ in the expression $(b-3a)^{12}$

$$\therefore T_{r+1} = {}^{12}C_r\, b^{12-r}\,(-3a)^r$$

$$= {}^{12}C_r\, b^{12-r}\, a^r\,(-3)^r$$

Now, $a^r = a^3$

$$\therefore r = 3$$

$\therefore T_{3+1} = {}^{12}C_3 \, b^{12-3} \, a^3 (-3)^3$

$= 220 \times (-27) \times a^3 b^9$

$= -5940 \, a^3 b^9$

As the coefficient of a^3 in the expression of $(b-3a)^{12}$ is -5940

$\therefore -5940 b^9 = -5940$

$\Rightarrow b^9 = 1$

$\therefore b = 1$

$\therefore$ If the coefficient of a^8 in the given expression is -5940, then the value of b is 1 ~~Ans~~

(11) 5th term of the expression $a^5(b-3a)^{12}$

$T_{4+1} = {}^{12}C_4 \, b^{12-4} (-3a)^4$

$= 495 \times (-3)^4 \times a^4 b^8$

$= 40095 a^4 b^8$

$\therefore$ 5th term of $a^5(b-3a)^{12}$ is $= 40095 a^4 b^8 \times a^5$

$= 40095 a^9 b^8$

From (i), $b = 1$ $\therefore$ 5th term of $a^5(b-3a)^{12}$ is $40095 a^9$

Now, according to the question,

$40095a^9 = 120285$

$\Rightarrow a^9 = \frac{120285}{40095}$

$\Rightarrow a^9 = 3$

$\Rightarrow a = \sqrt[9]{3}$

$\therefore$ The value of a in the expression $a^5(b-3a)^{12}$ is $\sqrt[9]{3}$ Ans

6 Given expression, $(Ax-4y^2)^7$
1

$T_{r+1} = {}^nC_r\, a^{n-r}\, b^r$

$= {}^7C_r\, (Ax)^{7-r}\, (-4y^2)^r$

$= {}^7C_r\, A^{7-r}\, x^{7-r}\, (-2)^r\, (2y^2)^r$

$= {}^7C_r\, (-4)^r\, A^{7-r}\, x^{7-r}\, y^{2r}$

Now, $7-r = 2$

$\Rightarrow r = 5$

$T_{5+1} = {}^{7}C_5 (-4)^5 A^{7-5} x^{7-5} y^{2\times5}$

$= 21 \times (-1024) \times A^2 x^2 y^{10}$

$= -21504 A^2 x^2 y^{10}$

$= -10752 A^2 \times x^2 2y^{10}$

According to the question coefficient of $x^2 2y^{10}$ in the given expression is -86016

$\therefore -10752A^2 = -86016$

$\Rightarrow A^2 = \frac{-86016}{-10752}$

$\Rightarrow A^2 = 8$

$\Rightarrow A = \sqrt{8}$

$\therefore A = 2\sqrt{2}$ $[\because x, y > 0$ and $x, y \in \mathbb{R}]$

$\therefore$ If the coefficient of $x^2 2y^{10}$ in the given expression is -86016, the value of A is $2\sqrt{2}$ Ans

11 ~~from~~ from (1), value of A is $2\sqrt{2}$

$\therefore$ The expression is $(2\sqrt{2}x - 4y^2)^{13}$

~~The~~ We know, the binomial r-th term formula is,

$$T_{r+1} = {}^{n}C_{r}\, a^{n-r} b^{r}$$

$\therefore$ 2nd last term of the given expression

$$T_{12+1} = {}^{13}C_{12} (2\sqrt{2}x)^{13-12} (-4y^2)^{12}$$

$$= 13 \times 2\sqrt{2} \times x \times (-4)^{12} \times y^{24}$$

$$= 616890726.6\, xy^{24}$$

$\therefore$ The 2nd last term of $(Ax - 4y^2)^{13}$ is ~~616890726.6~~ $616890726.6 xy^{24}$

Ans

Multinomial expansion problems:

$$\left(2z^2 - 1 + \frac{2}{z^2}\right)^7$$

$$\frac{n!}{m_1!\, m_2!\, m_3!\, \cancel{m_4!}} \times a^{m_1} \times b^{m_2} \times c^{m_3}$$

$$= \frac{7!}{m_1!\, m_2!\, m_3!} (2z^2)^{m_1} \times (-1)^{m_2} \times \left(\frac{2}{z^2}\right)^{m_3}$$

$$= \frac{7!}{m_1!\, m_2!\, m_3!} 2^{m_1} z^{2m_1} \times (-1)^{m_2} \times (2z^{-2})^{m_3}$$

$$= \frac{7!}{m_1!\, m_2!\, m_3!} 2^{m_1} \times (-1)^{m_2} \times 2^{m_3} \times z^{2m_1 - 2m_3}$$

As, we have to find the constant term

$$2m_1 - 2m_3 = 0$$

$$\Rightarrow 2m_1 = 2m_3$$

$$\therefore m_1 = m_3$$

$$m_1 + m_2 + m_3 = 7$$

$$\Rightarrow m_1 + m_1 + m_2 = 7$$

$$\Rightarrow 2m_1 + m_2 = 7$$

Now, the possible values of m_1, m_2 & m_3

$2m_1+m_2=7$

$m_1=0$, $2\times 0+m_2 = 7 \Rightarrow m_2=7$ $\therefore m_3=0$ $[\because m_1=m_3]$

$m_1=1$, $2\times 1+m_2=7 \Rightarrow m_2=5$

$\therefore m_1=1, m_2=5, m_3=1$ $[\because m_1=m_3]$

$m_1=2$, $2\times 2+m_2=7 \Rightarrow m_2=3$

$\therefore m_1=2, m_2=3, m_3=2$ $[\because m_1=m_3]$

$m_1=3$, $2\times 3+m_2=7 \Rightarrow m_2=1$

$\therefore m_1=3$, $m_2=1$, $m_3=3$ $[\because m_1=m_3]$

$m_1=4$, $2\times 4+m_2=7 \Rightarrow m_2=-1$ [Not Acceptable]

$\therefore$ The constant term of $\left(2z^2-1+\frac{2}{z^2}\right)^7$ is

$$\frac{7!}{1!\,5!\,1!}\,2^1\times(-1)^5\times 2^1\times z^{2.1-2.1}+\frac{7!}{2!\,3!\,2!}\,2^2\times(-1)^3\times 2^2\times z^0$$

$$+\frac{7!}{3!\,1!\,3!}\,2^3\times(-1)^1\times 2^3\times z^0 = -12488$$

Ans

$\therefore$ The constant term of $\left(2z^2-1+\frac{2}{z^2}\right)^7$ is

$$\frac{7!}{0!\,7!\,0!}\,2^0\times(-1)^7\times 2^0\times z^{2.0-2.0}+\frac{7!}{1!\,5!\,1!}\times 2^1\times(-1)^5\times 2^1\times z^0$$

$$+\frac{7!}{2!\,3!\,2!}\,2^2\times(-1)^3\times 2^2\times z^0+\frac{7!}{3!\,1!\,3!}\,2^3\times(-1)^1\times 2^3\times z^0$$

$$=-12489$$

Ans

(11) $\left(3z-5+\frac{1}{2z}\right)^7$

$$\frac{n!}{m_1!\,m_2!\,m_3!}\,a^{m_1}\,b^{m_2}\,c^{m_3}$$

$$= \frac{7!}{m_1!\,m_2!\,m_3!}\,(3z)^{m_1}\,(-5)^{m_2}\left(\frac{1}{2z}\right)^{m_3}$$

$$= \frac{7!}{m_1!\,m_2!\,m_3!}\,3^{m_1}\,(-5)^{m_2}\left(\frac{1}{2}\right)^{m_3}\,z^{m_1}\cdot z^{-m_3}$$

$$= \frac{7!}{m_1!\,m_2!\,m_3!}\,3^{m_1}\,(-5)^{m_2}\left(\frac{1}{2}\right)^{m_3}\,z^{m_1-m_3}$$

As we have to find constant term,

$\therefore m_1 - m_3 = 0 \Rightarrow m_1 = m_3$

$m_1+m_2+m_3 = 7 \Rightarrow 2m_1 + m_2 = 7$

Now, the possible values of m_1, m_2 & m_3

$2m_1+m_2=7$

$m_1 = 0$, $2\times 0 + m_2 = 7 \Rightarrow m_2 = 7$ $\therefore m_3 = 0$ $[\because m_1 = m_3]$

$m_1 = 1$, $2\times 1 + m_2 = 7 \Rightarrow m_2 = 5$

$\therefore m_1 = 1,\ m_2 = 5,\ m_3 = 1$ $[\because m_1 = m_3]$

$m_1 = 2$, $2\times 2 + m_2 = 7 \Rightarrow m_2 = 3$

$\therefore m_1 = 2$, $\cancel{m_2 = 3}$, $m_2 = 3,\ m_3 = 2$ $[\because m_1 = m_3]$

$m_1 = 3, \quad 2\times 3 + m_2 = 7 \Rightarrow m_2 = 1$

$\therefore m_1 = 3, \quad m_2 = 1, \quad m_3 = 3 \quad [\because m_1 = m_3]$

$m_1 = 4 \quad 2\times 4 + m_2 = 7 \Rightarrow m_2 = -1$ [Not Acceptable]

$\therefore$ The constant term of $\left(3z - 5 + \frac{1}{2z}\right)^7$ is

$\frac{7!}{0!7!0!} 3^0 (-5)^7 \left(\frac{1}{2}\right)^0 z^0 + \frac{7!}{1!5!1!} 3^1 (-5)^5 \left(\frac{1}{2}\right)^1 z^0 + \frac{7!}{2!3!2!} 3^2 (-5)^3$
$\left(\frac{1}{2}\right)^2 z^0 + \frac{7!}{3!1!3!} 3^3 (-5)^1 \left(\frac{1}{2}\right)^3 z^0 = -336425$ Ans

(iii) $\left(4p^2 - p^3 + \frac{6}{p^5}\right)^{10}$

$$= \frac{n!}{m_1! m_2! m_3!} a^{m_1} b^{m_2} c^{m_3}$$

$$= \frac{10!}{m_1! m_2! m_3!} (4p^2)^{m_1} (-p^3)^{m_2} \left(\frac{6}{p^5}\right)^{m_3}$$

$$= \frac{10!}{m_1! m_2! m_3!} 4^{m_1} 6^{m_3} p^{2m_1} (-p)^{3m_2} \left(\frac{1}{p^5}\right)^{m_3}$$

$$= \frac{10!}{m_1! m_2! m_3!} 4^{m_1} (-1)^{3m_2} 6^{m_3} p^{2m_1 + 3m_2 - 5m_3}$$

As we have to find the constant term,

$2m_1 + 3m_2 - 5m_3 = 0$ --- (i)

$m_1 + m_2 + m_3 = 10$ --- (ii)

$\Rightarrow m_1 = 10 - m_2 - m_3$ --- (ii)

(i) $\Rightarrow$

$2(10 - m_2 - m_3) + 3m_2 - 5m_3 = 0$ [From (ii)]

$\Rightarrow 20 - 2m_2 - 2m_3 + 3m_2 - 5m_3 = 0$

$\Rightarrow m_2 - 7m_3 + 20 = 0$

$\Rightarrow m_2 = 7m_3 - 20$ --- (iii)

(iii) $\Rightarrow$

$m_3 = 2$, $m_2 = 7.2 - 20 = -6$, $m_1 = 10 + 6 - 2 = 14$ [Not Acceptable]

$m_3 = 3$, $m_2 = 7.3 - 20 = 21 - 20 = 1$, $m_1 = 10 - 1 - 3 = 6$

$m_3 = 4$, $m_2 = 28 - 20 = 8$, $m_1 = 10 - 8 - 4 = -2$ [Not Acceptable]

$\therefore$ The constant term of $\left(3p^3 - 2p^2 + \frac{6}{p^5}\right)^{10}$ is

$\frac{10!}{6!\,1!\,3!} 4^6 (-1)^{3\times1} 6^3 \; p^{2\times6 + 3\times1 - 5\times3}$

$= 840 \times 4096 \times (-1) \times 216 \times p^0$

$= -743178240$

Ans

(iv) $\left(3p^3 - 2p^2 + \frac{5}{p^5}\right)^{10}$

$$= \frac{n!}{m_1!\, m_2!\, m_3!}\, a^{m_1} b^{m_2} c^{m_3}$$

$$= \frac{10!}{m_1!\, m_2!\, m_3!} (3p^3)^{m_1} (-2p^2)^{m_2} \left(\frac{5}{p^5}\right)^{m_3}$$

$$= \frac{10!}{m_1!\, m_2!\, m_3!}\, 3^{m_1}\, p^{3m_1}\, (-2)^{m_2}\, p^{2m_2}\, 5^{m_3}\, p^{-5m_3}$$

$$= \frac{10!}{m_1!\, m_2!\, m_3!}\, 3^{m_1} (-2)^{m_2}\, 5^{m_3}\, p^{3m_1+2m_2-5m_3}$$

As we have to find the constant term.

$3m_1 + 2m_2 - 5m_3 = 0$ --- (1)

$m_1 + m_2 + m_3 = 10$

$\Rightarrow m_2 = 10 - m_1 - m_3$ --- (ii)

(1) $\Rightarrow 3m_1 + 2(10 - m_1 - m_3) - 5m_3 = 0$

$\Rightarrow 3m_1 + 20 - 2m_1 - 2m_3 - 5m_3 = 0$

$\Rightarrow m_1 - 7m_3 + 20 = 0$

$\Rightarrow m_1 = 7m_3 - 20$ --- (iii)

$\Rightarrow$

(iii) $\Rightarrow$

$m_3 = 3, \quad m_1 = 7.3 - 20 = 21 - 20 = 1 \quad m_2 = 10 - 1 - 3 = 6$

$m_3 = 4, \quad m_1 = 28 - 20 = 8, \quad m_2 = 10 - 8 - 4 = -2$ [Not Acceptable]

The constant term of $\left(3p^3 - 2p^2 + \frac{5}{p^5}\right)^{10}$ is

$$\frac{10!}{1!\,6!\,3!} 3^1 \cdot (-2)^6 \cdot 5^3 \cdot p^{3.1 + 2.6 - 5.3}$$

$= 20160000$ Ans

(v) $\left(5x + \frac{3}{y} + \frac{4}{x} + \frac{5}{7}y\right)^8$

$$= \frac{n!}{m_1!\,m_2!\,m_3!\,m_4!} (5x)^{m_1} \left(\frac{3}{y}\right)^{m_2} \left(\frac{4}{x}\right)^{m_3} \left(\frac{5y}{7}\right)^{m_4}$$

$$= \frac{8!}{m_1!\,m_2!\,m_3!\,m_4!} 5^{m_1} 3^{m_2} 4^{m_3} \left(\frac{5}{7}\right)^{m_4} x^{m_1} \left(\frac{1}{y}\right)^{m_2} \left(\frac{1}{x}\right)^{m_3} y^{m_4}$$

$$= \frac{8!}{m_1!\,m_2!\,m_3!\,m_4!} 5^{m_1} 3^{m_2} 4^{m_3} \left(\frac{5}{7}\right)^{m_4} x^{m_1 - m_3} y^{m_4 - m_2}$$

As, we have to find the constant term

$m_1 - m_3 = 0 \qquad m_4 - m_2 = 0$

$m_1 = m_3 \qquad \therefore m_4 = m_2$

$m_1 + m_2 + m_3 + m_4 = 8$

$\Rightarrow 2m_1 + 2m_2 = 8$

$\Rightarrow m_1 + m_2 = 4$

$\Rightarrow m_2 = 4 - m_1 \cdots (1)$

Possible case,

$m_1 = 0,\ m_3 = 0,\ m_2 = 4-0 \Rightarrow m_2 = 4,\ m_4 = 4$

$m_1 = 1,\ m_3 = 1,\ m_2 = 4-1 \Rightarrow m_2 = 3,\ m_4 = 3$

$m_1 = 2,\ m_3 = 2,\ m_2 = 4-2 \Rightarrow m_2 = 2,\ m_4 = 2$

$m_1 = 3,\ m_3 = 3,\ m_2 = 4-3 \Rightarrow m_2 = 1,\ m_4 = 1$

$m_1 = 4,\ m_3 = 4,\ m_2 = 4-4 \Rightarrow m_2 = 0,\ m_4 = 0$

$m_1 = 5,\ m_3 = 5,\ m_2 = 4-5 \Rightarrow m_2 = -1,\ m_4 = -1$ [Not Acceptable]

$\therefore$ The constant term in the expansion of $\left(5x + \frac{3}{y} + \frac{4}{x} + \frac{5}{7}y\right)^8$ is

$$\frac{8!}{0!\,4!\,0!\,4!} 5^0 . 3^4 . 4^0 . \left(\frac{5}{7}\right)^4 x^0 y^0 + \frac{8!}{1!\,3!\,1!\,3!} 5^1 3^3 . 4^1 . \left(\frac{5}{7}\right)^3 x^0 y^0$$

$$+ \frac{8!}{2!\,2!\,2!\,2!} 5^2 3^2 4^2 \left(\frac{5}{7}\right)^2 x^0 y^0 + \frac{8!}{3!\,1!\,3!\,1!} 5^3 3^1 4^3 \left(\frac{5}{7}\right)^1 x^0 y^0$$

$$+ \frac{8!}{4!\,0!\,4!\,0!} 5^4 3^0 4^4 \left(\frac{5}{7}\right)^0 x^0 y^0 = 35250455.54$$

Ans

(vi) $(2a-3b+4c-d)^{14}$

$$^{14+4-1}C_{4-1}$$

$= 680$

$\therefore$ There are 680 terms in the expansion of $(2a-3b+4c-d)^{14}$ Ans

Co-efficient of $a^{p+1}.b^{q+1}.c^{r-2}.d^{s+2}$

We know,

$$\frac{14!}{m_1!m_2!m_3!m_4!} \times (2a)^{m_1} \times (-3b)^{m_2} \times (4c)^{m_3} \times (-d)^{m_4}$$

$$= \frac{14!}{m_1!m_2!m_3!m_4!} \times 2^{m_1} \times (-3)^{m_2} \times 4^{m_3} \times a^{m_1} b^{m_2} c^{m_3} d^{m_4} \times (-1)^{m_4}$$

Now, according to the question,

$$a^{m_1} b^{m_2} c^{m_3} d^{m_4} = a^{p+1} b^{q+1} c^{r-2} d^{s+2} \quad \text{---} (1)$$

Given,

$3p = 2q$

$\Rightarrow 3p = s+1$

$\Rightarrow p = \frac{s+1}{3}$

$2q = \cancel{r+2}$

$\Rightarrow 2q = s-\cancel{3}+2$

$\Rightarrow 2q = s+1$

$2q = s+1$

$\therefore q = \frac{s+1}{2}$

$r+2 = s+1$

$\Rightarrow r = s+1-2$

$\therefore r = s-1$

$$p+1+q+1+r-2+s+2=14$$

$$\Rightarrow p+q+r+s+2=14$$

$$\Rightarrow p+q+r+s=12$$

$$\Rightarrow \frac{s+1}{3}+\frac{s+1}{2}+s-1+s=12$$

$$\Rightarrow \frac{2(s+1)+3(s+1)+6(s-1)+6s}{6}=12$$

$$\Rightarrow 2s+2+3s+3+6s-6+6s=72$$

$$\cancel{\Rightarrow 17+1} \Rightarrow 17s-1=72$$

$$\Rightarrow \cancel{8}\ 17s=73 \Rightarrow s=\frac{73}{17}$$

$$r=s-1 = \frac{73}{17}-1 = \frac{73-17}{17} = \frac{56}{17}$$

$$p=\frac{s+1}{3} = \frac{\frac{73}{17}+1}{3} = \frac{30}{17}$$

$$q=\frac{s+1}{2} = \frac{\frac{73}{17}+1}{2} = \frac{45}{17}$$

Now, $m_1 = p+1 = \frac{30}{17}+1 = \frac{47}{17}$

$$m_2 = q+1 = \frac{45}{17}+1 = \frac{62}{17}$$

$$m_3 = r-2 = \frac{56}{17}-2 = \frac{22}{17}$$

$$m_4 = s+2 = \frac{73}{17}+2 = \frac{107}{17}$$

As, 'm_i' cannot be fractional number, there exists no terms that contains $a^{p+1} b^{q+1} c^{r-2} d^{s+2}$.

$\therefore$ The coefficient of the term that contains $a^{p+1} b^{q+1} c^{r-2} d^{s+2}$ is 0. Ans

(vii) $(2a-3b+4c-d)^{14}$

$$^{n+k-1}C_{k-1} = {}^{14+4-1}C_{4-1} = 680$$

$\therefore$ There are 680 terms in the expansion of $(2a-3b+4c-d)^{14}$. Ans

Co-efficient of $a^{p-1} b^{q+1} c^{r-2} d^{s+2}$

We know,

$$\frac{14!}{m_1!\,m_2!\,m_3!\,m_4!} \times (2a)^{m_1} \times (-3b)^{m_2} \times (4c)^{m_3} \times (-d)^{m_4}$$

$$= \frac{14!}{m_1!\,m_2!\,m_3!\,m_4!}\, 2^{m_1} (-3)^{m_2} 4^{m_3} (-1)^{m_4} \times a^{m_1} b^{m_2} c^{m_3} d^{m_4}$$

Now, according to the question,

$$a^{m_1} b^{m_2} c^{m_3} d^{m_4} = a^{p-1} b^{q+1} c^{r-2} d^{s+2}$$

Given,

$2p = 3q$
$\Rightarrow 2p = s+2$
$\Rightarrow p = \frac{s+2}{2}$

$3q = s+2$
$q = \frac{s+2}{3}$

$r+1 = s+2$
$\Rightarrow r = s+2-1$
$\Rightarrow r = s+1$

$p-1+q+1+r-2+s+2 = 14$

$\Rightarrow p+q+r+s = 14$

$\Rightarrow \frac{s+2}{2} + \frac{s+2}{3} + s+1+s = 14$

$\Rightarrow \frac{3s+6+2s+4+6s+6+6s}{6} = 14$

$\Rightarrow 17s+16 = 84$

$\Rightarrow 17s = 84-16$

$\Rightarrow 17s = 68$

$\therefore s = 4$

$r = s+1 = 4+1 = 5$

$q = \frac{s+2}{3} = \frac{4+2}{3} = 2$

$p = \frac{s+2}{2} = \frac{4+2}{2} = 3$

Now,

$m_1 = p-1 = 3-1 = 2$

$m_2 = q+1 = 2+1 = 3$

$m_3 = r-2 = 5-2 = 3$

$m_4 = s+2 = 4+2 = 6$

So, the co-efficient of the term that contains

$a^{p-1} b^{q+1} c^{r-2} d^{s+2}$ is $\frac{14!}{2!\,3!\,3!\,6!} \times 2^2 . (-3)^3 . 4^3 . (-1)^6$

$= -1.162377216 \times 10^{10}$

Ans