BRAC University
CSE230 : Discrete Mathematics
Midterm Examination

Duration : 75 minutes (4:45 pm - 6:00 pm)

Total Marks : 45 Set: B

[Answer any 3 out of 4 questions. Answer all the sub-parts of a question together. Please start each question in a new page]

ID: **Name:** **Sec:**

Q01: [CO1] [15 Points]

a) Verify using a truth table that $(\neg a \wedge b) \rightarrow (a \vee b)$ is a tautology. **[5 points]**

b) Write the converse, inverse and contrapositive of the following statement: **[5 points]**

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c) Let P(x), Q(x), R(x), S(x) and T(x) be the statements "x is a hummingbird," "x is rich in color," "x lives on honey," "x is large," and "x can fly fast," respectively. Express each of these statements using quantifiers; logical connectives; and P(x), Q(x), R(x), S(x) and T(x). **[5 points]**

i) Some hummingbirds are rich in color.
ii) All large birds live on honey.
iii) Not all hummingbirds are large.
iv) No hummingbird can fly fast.
v) Some fast flying birds are not large.

Q02: [CO4] [15 Points]

a. Show that the following mathematical statement is true for all positive integers n,

$$\frac{\mathbf{1}}{\mathbf{1\times2\times3}}+\frac{\mathbf{1}}{\mathbf{2\times3\times4}}+\frac{\mathbf{1}}{\mathbf{3\times4\times5}}+\ldots\ldots.+\frac{\mathbf{1}}{\boldsymbol{n}(\boldsymbol{n}+\mathbf{1})(\boldsymbol{n}+\mathbf{2})}=\frac{\boldsymbol{n}(\boldsymbol{n}+\mathbf{3})}{\mathbf{4}(\boldsymbol{n}+\mathbf{1})(\boldsymbol{n}+\mathbf{2})}$$ **[8 points]**

b. Prove by mathematical induction that 10 divides $3^n + 7^n$ for all positive odd integers of n. **[7 points]**

Q03: [CO6] [15 Points]

a) Nisho drops a ping pong ball from the top of a 200 meter tall building. After each drop on the ground, the ball jumps up to three-fourths of its previous height. Find a recurrence relation expressing the total distance covered by the ball before its n^{th} drop on the ground. **[5 points]**

b) Solve the following recurrence relation: **[8 points]**

$3a_{n+2} = 6a_{n+1} + 189a_n + 3.\ 5^n$

Here $a_0 = 0$, $a_1 = 3$

c) $a_9 - a_7 = ?$ **[2 points]**

Q04: [CO7] [15 Points]

- **a)** Find the closest integer to 3^{729} which is divisible by 7. **[5 points]**
- **b)** We know that the following congruences are true: $a \equiv a \textbf{ mod } \boldsymbol{m} \pmod{\boldsymbol{m}}$, $b \equiv b \textbf{ mod } \boldsymbol{m} \pmod{\boldsymbol{m}}$. From this, show that $ab \textbf{ mod } m = ((a \textbf{ mod } m)(b \textbf{ mod } m)) \textbf{ mod } m$.
  (Note that '(mod m)' denotes congruency and **'mod** m' denotes the mod function.) **[5 points]**
- **c)** Find the least common multiple between 4552 and 624 with the help of the Euclidean algorithm. **[5 points]**