Solutions

ProbabilityPractice sheet 3

1. 2/27
2. a) 1/12   b) 5/18   c) 5/9
3. 0.38
4. 4/36

5.

Total number of trials = 300 (Since there are 300 students all together).

Number of times a cricket player is chosen = 95 (Since 95 students play cricket).

Number of times a football player is chosen = 120.

Number of times a volleyball player is chosen = 80.

Number of times a student is chosen who plays no games = 5.

(i) Therefore, the probability of getting a player who plays volleyball

$$= \frac{\text{Number of Times a Volleyball Player can be Chosen}}{\text{Total Number of Trials}}$$

$$= \frac{80}{300}$$

$$= \frac{4}{15}.$$

(ii) The probability of getting a player who plays either cricket or volleyball

$$= \frac{\text{Number of Times a Cricket or a Volleyball Player can be Chosen}}{\text{Total Number of Trials}}$$

$$= \frac{95+80}{300}$$

$$= \frac{175}{300}$$

$$= \frac{7}{12}.$$

(iii) The probability of getting a player who plays neither football nor volleyball $= \frac{\text{Number of Times a Student can be Chosen who do not Play Football or Volleyball}}{\text{Total Number of Trials}}$

$$= \frac{300-120-80}{300}$$

$$= \frac{100}{300}$$

$$= \frac{1}{3}.$$

6.

Solution:

Let E_1 be the event of choosing bag I, E_2 the event of choosing bag II, and A be the event of drawing a black ball.

Then,

$$P(E_1) = P(E_2) = \frac{1}{2}$$

Also, $P(A|E_1)$ = P(drawing a black ball from Bag I) = 6/10 = 3/5

$P(A|E_2)$ = P(drawing a black ball from Bag II) = 3/7

By using Bayes' theorem, the probability of drawing a black ball from bag I out of two bags,

$$P(E_1|A) = \frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)}$$

$$= \frac{\frac{1}{2}\times\frac{3}{5}}{\frac{1}{2}\times\frac{3}{5}+\frac{1}{2}\times\frac{3}{7}}$$

$$= \frac{7}{12}$$

7.

Let H be the head and T be the tail of the coin. The sample space S of the experiment

described in question 5 is as follows

S = { (1,H),(2,H),(3,H),(4,H),(5,H),(6,H)

(1,T),(2,T),(3,T),(4,T),(5,T),(6,T)}

Let E be the event "the die shows an odd number and the coin shows a head". Event E

may be described as follows

E={(1,H),(3,H),(5,H)}

The probability P(E) is given by

P(E) = n(E) / n(S) = 3 / 12 = 1 / 4

8. Ans: 5/12

Here n(S) = (6 x 6) = 36

Let E = event of getting a total more than 7

= {(2,6),(3,5),(3,6),(4,4),(4,5),(4,6),(5,3),(5,4),(5,5),(5,6),(6,2),(6,3),(6,4),(6,5),(6,6)}

Therefore, P(E) = n(E)/n(S) = 15/36 = 5/12.

9. 19/90

Explanation:

$ASSISTANT \rightarrow AAINSSSTT$

$STATISTICS \rightarrow ACIISSSTTT$

Here N and C are not common and same letters can be A, I, S, T.

Therefore

Probability of choosing A = $\frac{^2C_1}{^9C_1} \times \frac{^1C_1}{^{10}C_1}$ = 1/45

Probability of choosing I = $\frac{1}{^9C_1} \times \frac{^2C_1}{^{10}C_1}$ = 1/45

Probability of choosing S = $\frac{^3C_1}{^9C_1} \times \frac{^3C_1}{^{10}C_1}$ = 1/10

Probability of choosing T = $\frac{^2C_1}{^9C_1} \times \frac{^3C_1}{^{10}C_1}$ = 1/15

Hence, Required probability = $\frac{1}{45} + \frac{1}{45} + \frac{1}{10} + \frac{1}{15} = \frac{19}{90}$

10.

Solution: Let A be the event the second marble is red, and B the event that the first one is red. $\mathbb{P}(B) = 3/5$, while $\mathbb{P}(A \cap B)$ is the probability both are red, or is the probability that we chose 2 red out of 3 and 0 black out of 2. Then $\mathbb{P}(A \cap B) = \binom{3}{2}\binom{2}{0}/\binom{5}{2}$, and so $\mathbb{P}(A \mid B) = \frac{3/10}{3/5} = 1/2$.

11.

Solution: Let D be the families that own a dog, and C the families that own a cat. We are given $\mathbb{P}(D) = 0.36, \mathbb{P}(C) = 0.30, \mathbb{P}(C \mid D) = 0.22$ We want to know $\mathbb{P}(D \mid C)$. We know

$\mathbb{P}(D \mid C) = \mathbb{P}(D \cap C)/\mathbb{P}(C)$. To find the numerator, we use $\mathbb{P}(D \cap C) = \mathbb{P}(C \mid D)\mathbb{P}(D) = (0.22)(0.36) = 0.0792$. So $\mathbb{P}(D \mid C) = 0.0792/0.3 = 0.264 = 26.4\%$.

12. Answer is 1/2

The possible outcomes are as follows :

5H, 5T, (H, 4T), (T, 4H), (2H, 3T) (3H, 2T), i.e. 6 outcomes in all.

Therefore the probability that head appears an odd number of times = 3/6 =1/2 (In only three outcomes out of the six outcomes, head appears an odd number of times).

13.

Solution: denote by B the event that *Phan takes Biology*, and by C the event that *Phan takes Chemistry*, and by $A =$ the event that the *score is an A*. Then, since $\mathbb{P}(B) = \mathbb{P}(C) = \frac{1}{2}$ we have

$$\mathbb{P}(A \cap C) = \mathbb{P}(C)\,\mathbb{P}(A \mid C) = \frac{1}{2} \cdot \frac{1}{7} = \frac{1}{14}.$$

The identity $\mathbb{P}(E \cap F) = \mathbb{P}(E)\mathbb{P}(F \mid E)$ from Proposition 4.2(i) can be generalized to any number of events in what is sometimes called the *multiplication rule*.

14.

Sol: Total number of cases = ${}^{52}C_3$

One card each should be selected from a different suit. The three suits can be chosen in ${}^{4}C_3$ was

The cards can be selected in a total of $({}^{4}C_3) \times ({}^{13}C_1) \times ({}^{13}C_1) \times ({}^{13}C_1)$

Probability = ${}^{4}C_3 \times ({}^{13}C_1)^3 / {}^{52}C_3$

$= 4 \times (13)^3 / {}^{52}C_3$

15.

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$= 4 \times (13)^3 / {}^{52}C_3$

16. Ans: 4/13

Explanation:

Number of spades in a standard deck of cards=13

Number of aces in a standard deck of cards=4

And,one of the aces is a spade.

So, 13 + 4 - 1 = 16 spades or aces to choose from.

Therfore,probabiltiy of getting a spade or an ace=16/52=4/13

17.

Solution: Let D is the event that a person is HIV positive, and T is the event that the person tests positive.

$$\mathbb{P}(D \mid T) = \frac{\mathbb{P}(D \cap T)}{\mathbb{P}(T)} = \frac{(0.99)(0.003)}{(0.99)(0.003) + (0.01)(0.997)} \approx 23\%.$$

18. 5/12