1. Answer:

Let p be the proposition "you can access the internet from campus", q be the proposition "you are a current student", and r be the proposition "you are a faculty".

Then the given sentence can be translated into propositional logic as:

$p \longrightarrow (q \lor r)$

where $\longrightarrow$ represents the conditional "if...then".

In words, the propositional statement is "If you can access the internet from campus, then either you are a current student or you are a faculty."

2. Answer:

First, we will use De Morgan's laws to distribute the negation symbol ¬ over the disjunction $(p \lor (\neg p \land q))$ as follows:

$\neg(p \lor (\neg p \land q)) \Leftrightarrow \neg p \land \neg(\neg p \land q)$

Next, we will apply De Morgan's laws again to the second term $\neg(\neg p \land q)$ as follows:

$\neg(p \lor (\neg p \land q)) \Leftrightarrow \neg p \land (\neg\neg p \lor \neg q)$

Since $\neg\neg p$ is logically equivalent to p, we can simplify the expression further:

$\neg(p \lor (\neg p \land q)) \Leftrightarrow \neg p \land (p \lor \neg q)$

Using the distributive property of $\land$ over $\lor$, we can write this as:

$\neg(p \lor (\neg p \land q)) \Leftrightarrow (\neg p \land p) \lor (\neg p \land \neg q)$

The expression $\neg p \land p$ is a contradiction, so we can simplify it to false:

$\neg(p \lor (\neg p \land q)) \Leftrightarrow \text{false} \lor (\neg p \land \neg q)$

Finally, we can simplify false $\vee$ ($\neg p \wedge \neg q$) to just $\neg p \wedge \neg q$, since false $\vee$ anything is logically equivalent to just that thing:

$\neg(p \vee (\neg p \wedge q)) \Leftrightarrow \neg p \wedge \neg q$

Therefore, we have shown that $\neg(p \vee (\neg p \wedge q))$ is logically equivalent to $\neg p \wedge \neg q$.

3. <u>Answer:</u>

Let p be the proposition "the Automated reply can be sent" and q be the proposition "the file system is full".

Then the given sentence can be expressed in propositional logic as:

$\neg p \longrightarrow q$

4. <u>Answer:</u>

4. a

The system specification can be translated into English as follows:

If there exists a printer 'p' such that it is both out of service (F(p)) and busy (B(p)), then there exists a print job 'J' that has been lost (L(J)). In other words, if there is at least one printer that is not working properly and is also busy, it implies that there is a print job that has been lost.

Additionally, the notation $\exists p(F(p) \wedge B(p))$ represents the existence of a printer 'p' that is both out of service (F(p)) and busy (B(p)), while $\exists J\ L(J)$ indicates the existence of a print job 'J' that has been lost (L(J)).

4. b

The system specification can be translated into English as follows:

If all printers are busy (represented by the statement $\forall pB(p)$), then there exists at least one print job 'J' that is queued (represented by the statement $\exists J\ Q(J)$). In other words, if every printer in the domain is currently in use, then there must be at least one print job that is waiting to be printed.

Here, the notation $\forall pB(p)$ represents the condition that all printers are busy (B(p)), and $\exists J\ Q(J)$ indicates the existence of a print job 'J' that is queued (Q(J)).

4. c

The system specification can be translated into English as follows:

If there exists a print job 'J' that is both queued (Q(J)) and lost (L(J)), then there exists a printer 'p' that is out of service (F(p)). In other words, if there is at least one print job that has been queued but has subsequently been lost, it implies that there must be a printer that is currently not working.

Here, the notation $\exists J(Q(J) \land L(J))$ represents the existence of a print job 'J' that is both queued (Q(J)) and lost (L(J)), while $\exists pF(p)$ indicates the existence of a printer 'p' that is out of service (F(p)).

Therefore, the system specification can be interpreted as a condition where if a print job is queued but lost, it suggests that there may be a problem with the printer and it is currently out of service.
4. d
The system specification can be translated into English as follows:

If all printers are busy (represented by the statement $\forall pB(p)$) and all print jobs are queued (represented by the statement $\forall JQ(J)$), then there exists at least one print job 'J' that has been lost (represented by the statement $\exists JL(J)$). In other words, if every printer in the domain is currently in use and all print jobs are waiting to be printed, then there must be at least one print job that has been lost.

Here, the notation $\forall pB(p)$ represents the condition that all printers are busy (B(p)), $\forall JQ(J)$ indicates that all print jobs are queued (Q(J)), and $\exists JL(J)$ indicates the existence of a print job 'J' that has been lost (L(J)).

Therefore, the system specification can be interpreted as a condition where if all printers are busy and all print jobs are queued, it implies that there may be an issue with the printing system, and at least one print job has been lost.

5. <u>Answer:</u>

   a. Let U(x) be the proposition "x is a user" and M(x) be the proposition "x has access to an electronic mailbox".

      Then the given system specification "Every user has access to an electronic mailbox" can be expressed using universal quantification and implication as:

$\forall x(U(x) \longrightarrow M(x))$

b. Let G(x) be the proposition "x is a member of the group" and L be the proposition "the file system is locked".

   Let S(x) be the proposition "x can access the System box".

   Then the given system specification "The System box can be accessed by everyone in the group if the file system is locked" can be expressed using universal quantification, implication, and conjunction as:

   $\forall x(G(x) \longrightarrow (L \longrightarrow S(x)))$

   This can be read as "For all x, if x is a member of the group, then if the file system is locked, x can access the System box." In other words, if the file system is locked, then everyone in the group can access the System box.

c. Let's define the following predicates:

   F: "The firewall is in a diagnostic state"
   P: "The proxy server is in a diagnostic state"

   Then, we can express the system application using logical connections and quantifiers as follows:

   "The firewall is in a diagnostic state only if the proxy server is in a diagnostic state" can be written as:

   $\forall x\ [(F(x) \rightarrow P(x))]$

   This means that for all x (representing any possible state of the system), if the firewall is in a diagnostic state (F(x)), then the proxy server must also be in a diagnostic state (P(x)). In other words, if the firewall is not in a diagnostic state, then the proxy server can be in any state. But if the firewall is in a diagnostic state, the proxy server must also be in a diagnostic state.

d. Let's define the following predicates:

R(x): "Router x is functioning normally"
T(x): "The throughput is between 100 kbps and 500 kbps"
P: "The proxy server is not in diagnostic mode"
Then, we can express the system application using logical connections and quantifiers as follows:

"At least one router is functioning normally if the throughput is between 100 kbps and 500 kbps and the proxy server is not in diagnostic mode" can be written as:

$\exists x\ [(R(x)\ \land\ T(x)\ \land\ P)]$

This means that there exists at least one x (representing a router in the system) such that it is functioning normally (R(x)), the throughput is between 100 kbps and 500 kbps (T(x)), and the proxy server is not in diagnostic mode (P). In other words, if the throughput is not between 100 kbps and 500 kbps or the proxy server is in diagnostic mode, then none of the routers need to be functioning normally. But if the throughput is between 100 kbps and 500 kbps and the proxy server is not in diagnostic mode, then there must be at least one router that is functioning normally.

6. <u>Answer:</u>

Using rules of influence, we can show that if the hypothesis "Alice studies well" is true, then the conclusion "Alice will get the job" must also be true:

If Alice studies well, then she became smart. (premise)
If Alice is smart, then she will get a good job. (premise)
Therefore, if Alice studies well, then she will get a good job. (1,2, Modus Ponens)
Alice studies well. (hypothesis)
Therefore, Alice will get a good job. (3,4, Modus Ponens)
So, we can conclude that if Alice studies well, then she will get the job. However, we cannot infer that Alice studies well just from the premises given; we would need additional evidence or premises to support that claim

7. <u>Answer:</u>

From the given premises, we can draw the following relevant conclusion:

I did not watch a movie on Thursday.
Explanation:
Let's define the following predicates:

R: "It rains"
M_T: "I watched a movie on Tuesday"
M_{Th}: "I watched a movie on Thursday"
Using these predicates, we can write the premises as:

$R \rightarrow M$
$M_T \lor M_{Th}$
$\neg R_{Th}$
where M is shorthand for "I watched a movie" and $\neg R_{Th}$ means "It did not rain on Thursday".

To obtain the conclusion "I did not watch a movie on Thursday", we can use the following rules of inference:

Disjunctive Syllogism: If we have a disjunction (such as "$M_T \lor M_{Th}$") and we know one of the disjuncts is false (such as "$\neg R_{Th}$"), we can infer that the other disjunct must be true. So we can conclude that "M_T" must be true since we know that it is either true or "M_{Th}" is true, but we know that "M_{Th}" is false. Therefore, we can conclude that "I did not watch a movie on Thursday", since we know that if I watched a movie on Thursday, it must have been raining (by the contrapositive of the first premise), and we know that it did not rain on Thursday (by the third premise).

8. <u>Answer:</u>
(a)
To verify the proposition $(p \lor q) \lor r \equiv p \lor (q \lor r)$ using truth tables, we need to consider all possible truth values for p, q, and r, and evaluate both sides of the equation for each combination of truth values.

p	q	r	$p \lor q$	$(p \lor q) \lor r$	$q \lor r$	$p \lor (q \lor r)$	$(p \lor q) \lor r \equiv p \lor (q \lor r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	T	T	T	T	T
T	F	F	T	T	F	T	T
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	F	T	F	T	T	T	T
F	F	F	F	F	F	F	T

As we can see from the truth table, both sides of the equation have the same truth values for all possible combinations of p, q, and r. Therefore, we can conclude that $(p \lor q) \lor r \equiv p \lor (q \lor r)$ is a tautology, and the proposition is true for all possible truth values of p, q, and r.

8. <u>Answer:</u>

(b)

To verify the proposition $\neg(p \lor q) \equiv \neg p \lor \neg q$ using truth tables, we need to consider all possible truth values for p and q, and evaluate both sides of the equation for each combination of truth values.

p	**q**	**p ∨ q**	**¬(p ∨ q)**	**¬p**	**¬q**	**¬p ∨ ¬q**	**¬(p ∨ q) ≡ ¬p ∨ ¬q**
T	T	T	F	F	F	F	T
T	F	T	F	F	T	T	T
F	T	T	F	T	F	T	T
F	F	F	T	T	T	T	T

As we can see from the truth table, both sides of the equation have the same truth values for all possible combinations of p and q. Therefore, we can conclude that $\neg(p \lor q) \equiv \neg p \lor \neg q$ is a tautology, and the proposition is true for all possible truth values of p and q.

12. Answer:

a) The statement "$\forall x\ (x^2 >= x)$" states that for all integers x, x squared is greater than or equal to x.
   This statement is true for all non-negative integers (0, 1, 2, 3, ...) since the square of any non-negative integer is greater than or equal to the integer itself.
   However, it is false for negative integers. For example, if x = -1, then x^2 = 1 which is not greater than or equal to -1. Therefore, the statement is false for all negative integers.
   Thus, the counterexample to the statement is x = -1.
b) The statement "$\forall x\ (x > 0 \lor x < 0)$" states that for all integers x, either x is greater than 0 or x is less than 0.
   This statement is actually a tautology, meaning it is always true for any integer x. Every integer is either greater than 0 or less than 0 (except for 0 itself, which is not included in the domain of this statement). There are no counterexamples to this statement.
c) The statement "$\forall x\ (x = 1)$" states that for all integers x, x is equal to 1.
   This statement is false, as there are many integers that are not equal to 1. For example, 0, -1, and 2 are all integers that are not equal to 1. Therefore, the counterexample to this statement is any integer that is not equal to 1.

14. Answer:

a) The statement "$\forall x\ (R(x) \rightarrow H(x))$" translated into English is "For all animals x, if x is a rabbit, then x hops."
b) The statement "$\forall x\ (R(x) \land H(x))$" translated into English is "For all animals x, x is a rabbit and x hops."
c) The statement "$\exists x\ (R(x) \rightarrow H(x))$" translated into English is "There exists an animal x such that if x is a rabbit, then x hops."
d) The statement "$\exists x\ (R(x) \land H(x))$" translated into English is "There exists an animal x such that x is a rabbit and x hops."

15. Answer:

The negation of the proposition "some drivers do not obey the speed limit" using quantifiers is:

a) $\neg(\exists x)(\text{Driver}(x) \land \neg\text{ObeysSpeedLimit}(x))$

In English, the negation is "It is not the case that there exists a driver who does not obey the speed limit", or equivalently, "Every driver obeys the speed limit."

b) $\neg(\forall x)(\text{AnimationMovie}(x) \rightarrow \text{Funny}(x))$

In English, the negation is "It is not the case that every animation movie is funny", or equivalently, "There exists an animation movie that is not funny."

c) $\exists x\, (\text{Person}(x) \land \text{CanKeepSecret}(x))$

In English, the negation is "There exists a person who can keep a secret", or equivalently, "It is not true that no one can keep a secret."

d)

The proposition "there is someone in this class who does not have visited Saint-Martin" can be expressed using quantifiers as:

$\exists x\, (x \text{ is in this class} \land \neg\text{VisitedSaintMartin}(x))$

The negation of this proposition is:

$\neg(\exists x\, (x \text{ is in this class} \land \neg\text{VisitedSaintMartin}(x)))$

Using De Morgan's laws, this can be simplified as:

$(\forall x)(x \text{ is in this class} \rightarrow \text{VisitedSaintMartin}(x))$

In English, the negation is "Every person in this class has visited Saint-Martin", or equivalently, "No one in this class has not visited Saint-Martin."