Sequential Circuit Analysis

Circuit Diagram to State Diagram
State Diagram to Circuit Diagram
Counters
Application of Counters

What is sequential logic

❏ Sequential circuit has additional dimension which is time

❏ Combinational logic only depends on current input

❏ Sequential circuit output depends on previous input other than current input

❏ More powerful than combination logic

❏ Able to model condition which can't be modeled by combinational logic

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Circuit Diagram
to
State Diagram

Example 1: Draw the state diagram for the given circuit

Sequential circuit using D flip flop

Present States: **Next States:**

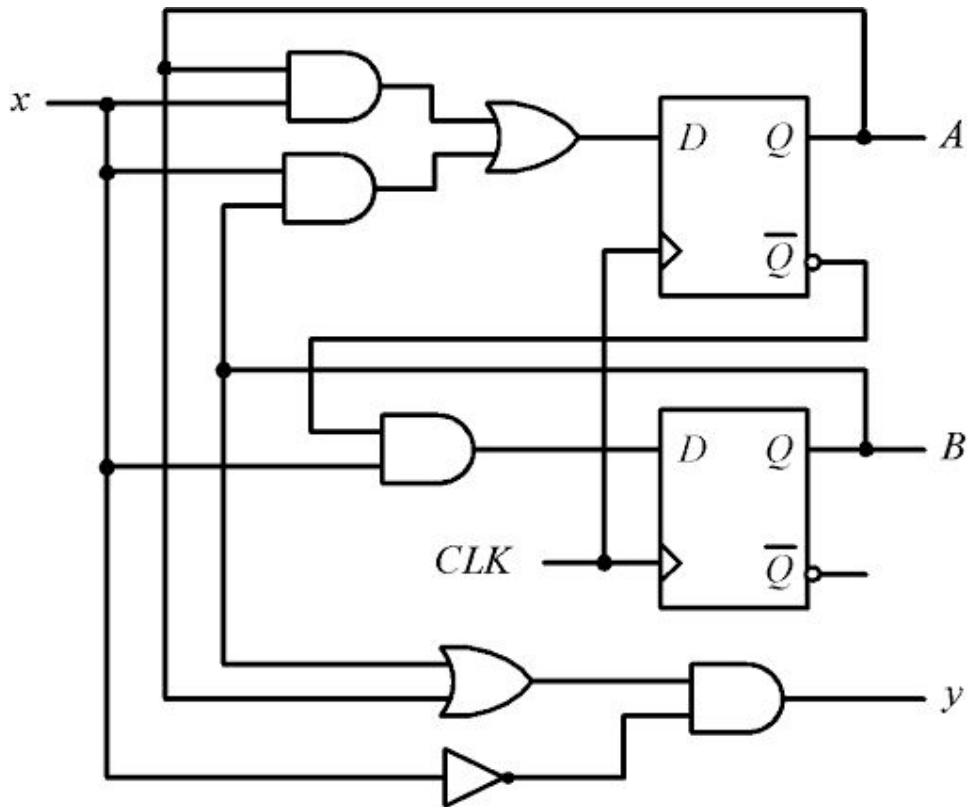A and B   A+ and B+

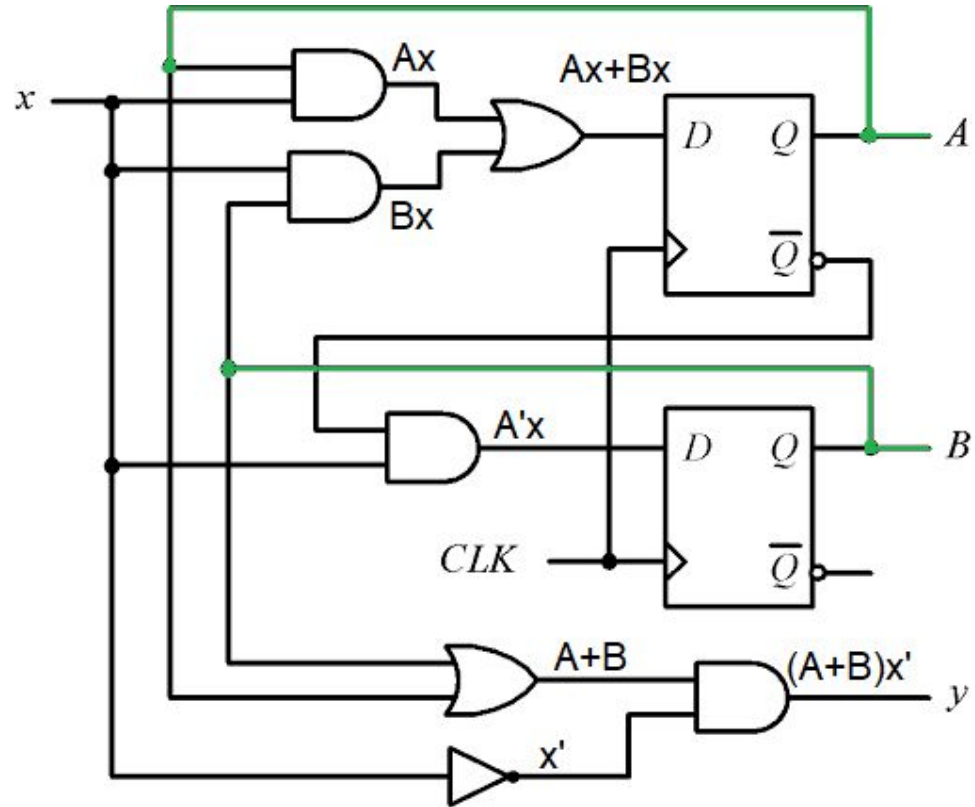**Input:**   **Output:**

DA, DB, x   y

Example 1: Draw the state diagram for the given circuit

1. Find out the input equations of the flip flop and the output equation

DA = Ax + Bx

DB = A'x

y = (A+B)x'

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Example 1: Draw the state diagram for the given circuit

2. Construct the state table according to the flip flop input and output equations ⟹

A	B	x	DA	DB	y
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0

DA = Ax + Bx

DB = A'x

y = (A+B)x'

When A=0, B=0, x=0,

DA = 0.0 + 0.0 = 0

DB = 0'.0 = 0
y = (0+0).0' = 0

When A=0, B=0, x=1,

DA = 0.1 + 0.1 = 0

DB = 0'.1 = 1
y = (0+0).1' = 0

Example 1: Draw the state diagram for the given circuit

3. Find out the next states from the flip flop input values

D Flip-flop Truth table

D	Q
0	0
1	1

∴ DA = A+
∴ DB = B+

A	B	x	DA	DB	y	A+	B+
0	0	0	0	0	0	0	0
0	0	1	0	1	0	0	1
0	1	0	0	0	1	0	0
0	1	1	1	1	0	1	1
1	0	0	0	0	1	0	0
1	0	1	1	0	0	1	0
1	1	0	0	0	1	0	0
1	1	1	1	0	0	1	0

Example 1: Draw the state diagram for the given circuit

A	B	x	A+	B+	y
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0

4. Draw the state diagram from the state table
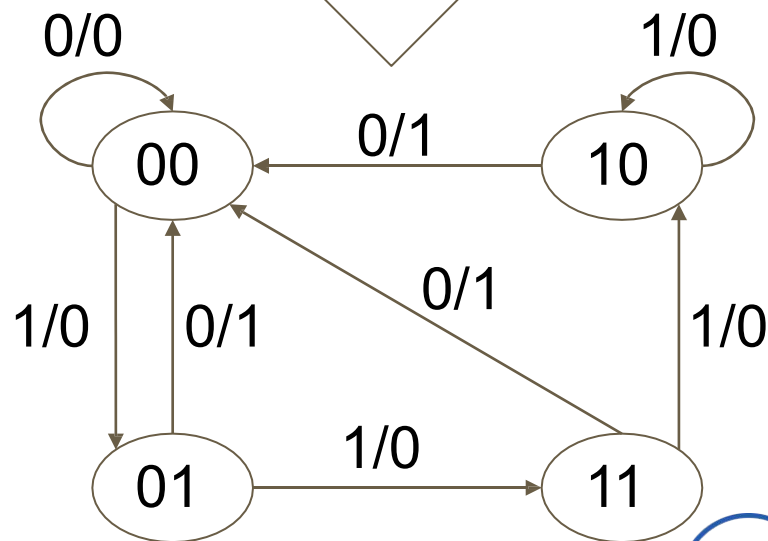
State Diagram

Example 2: Draw the state diagram for the given circuit

Sequential circuit using JK flip flop

Sequential circuit using JK flip flop

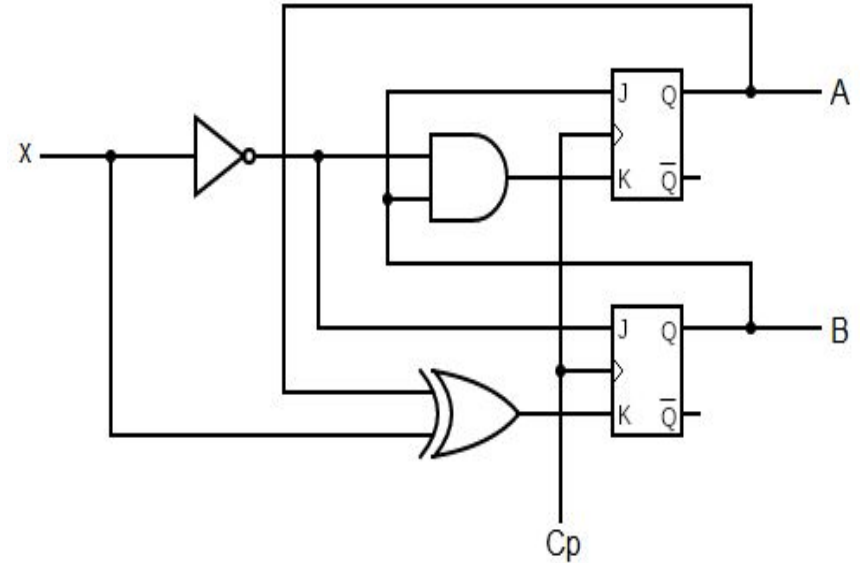**Present States:**   **Next States:**

A and B   A+ and B+

Input:

JA, KA, JB, KB, x

Example 2: Draw the state diagram for the given circuit

1. Find out the input equations of the flip flop and the output equation

JA = B
KA = Bx'
JB = x'
KB = A xor X

Example 2: Draw the state diagram for the given circuit

2. Construct the state table according to the flip flop input and output equations

JA = B
KA = Bx'
JB = x'
KB = A xor X

A	B	x	JA	KA	JB	KB
0	0	0	0	0	1	0
0	0	1	0	0	0	1
0	1	0	1	1	1	0
0	1	1	1	0	0	1
1	0	0	0	0	1	1
1	0	1	0	0	0	0
1	1	0	1	1	1	1
1	1	1	1	0	0	0
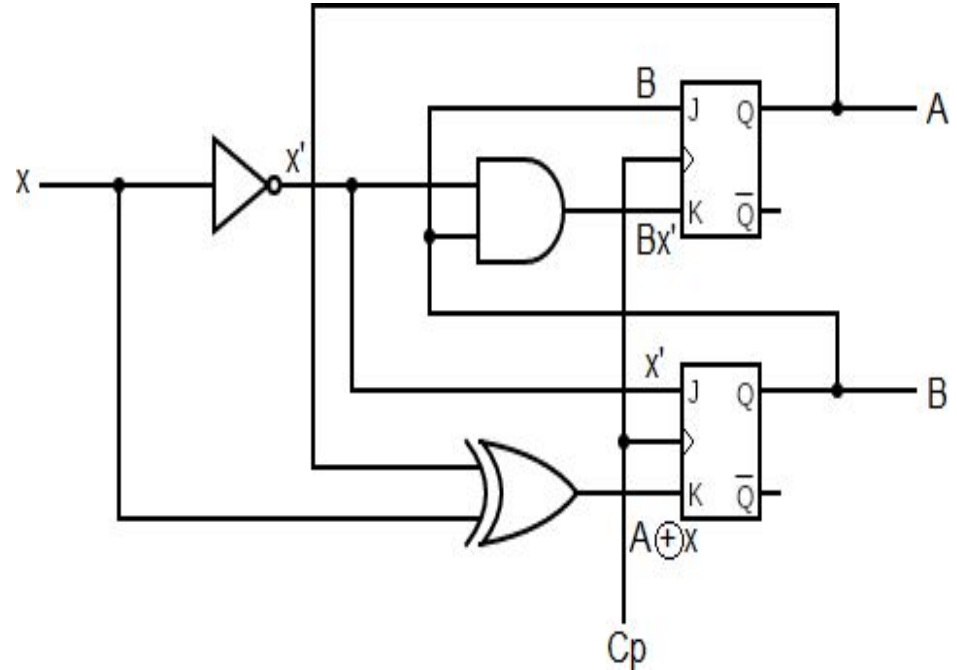
When A=0, B=0, x=0,
JA = 0
KA = 0.0' = 0
JB = 0' = 1
KB = 0 xor 0 = 0

When A=0, B=0, x=1,
JA = 0
KA = 0.1' = 0
JB = 1' = 0
KB = 0 xor 0 = 0

Example 2: Draw the state diagram for the given circuit

3. Find out the next states from the flip flop input values

JK Flip-flop Truth table

J	K	Q
0	0	Memory/ No change
0	1	0
1	0	1
1	1	Toggle

A	B	x	JA	KA	JB	KB	A+	B+
0	0	0	0	0	1	0	0	1
0	0	1	0	0	0	1	0	0
0	1	0	1	1	1	0	1	1
0	1	1	1	0	0	1	1	0
1	0	0	0	0	1	1	1	1
1	0	1	0	0	0	0	1	0
1	1	0	1	1	1	1	0	0
1	1	1	1	0	0	0	1	1

Example 2: Draw the state diagram for the given circuit

A	B	x	A+	B+
0	0	0	0	1
0	0	1	0	0
0	1	0	1	1
0	1	1	1	0
1	0	0	1	1
1	0	1	1	0
1	1	0	0	0
1	1	1	1	1

4. Draw the state diagram from the state table
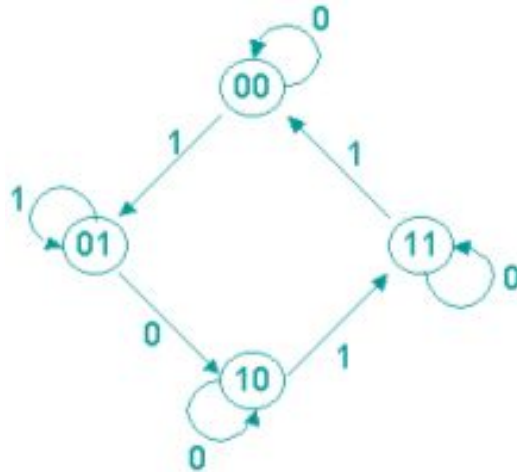

State Diagram

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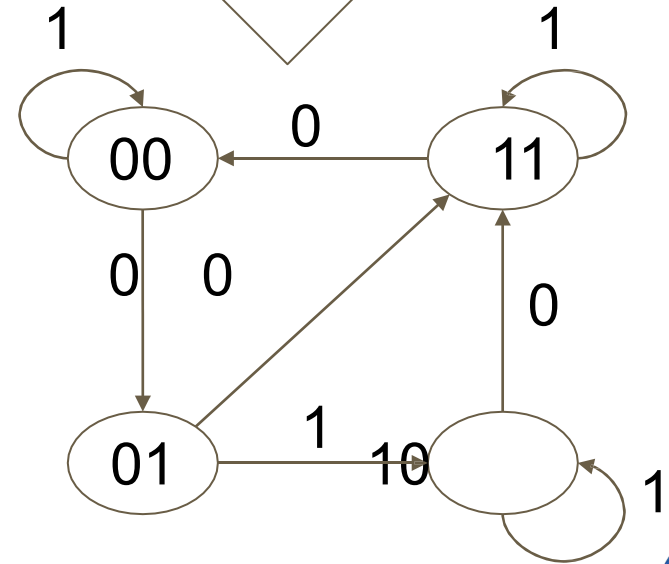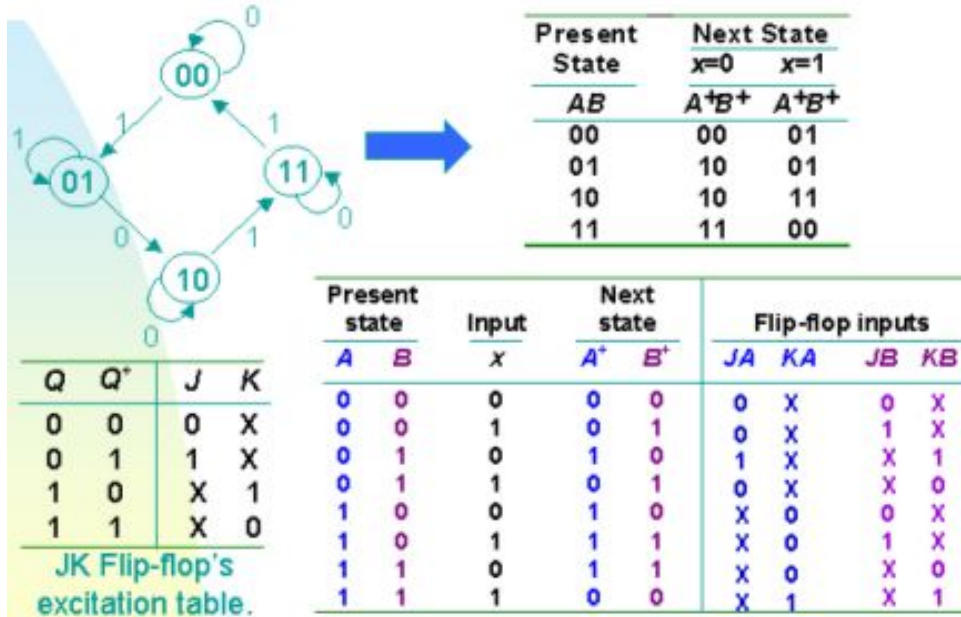
State Diagram
to
Circuit Diagram

Design: Example 1

Given state diagram as follows, get the sequential circuit using JK flip-flop

Design: Example 1

State/excitation table using JK flip-flop

Present State	Next State x=0	Next State x=1
AB	A^+B^+	A^+B^+
00	00	01
01	10	01
10	10	11
11	11	00

Q	Q⁺	J	K
0	0	0	X
0	1	1	X
1	0	X	1
1	1	X	0

JK Flip-flop's excitation table.

Present state A	Present state B	Input x	Next state A⁺	Next state B⁺	JA	KA	JB	KB
0	0	0	0	0	0	X	0	X
0	0	1	0	1	0	X	1	X
0	1	0	1	0	1	X	X	1
0	1	1	0	1	0	X	X	0
1	0	0	1	0	X	0	0	X
1	0	1	1	1	X	0	1	X
1	1	0	1	1	X	0	X	0
1	1	1	0	0	X	1	X	1

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Design: Example 1

From state table, get input flip-flop function

Present state		Input	Next state		Flip-flop inputs			
A	B	x	A⁺	B⁺	JA	KA	JB	KB
0	0	0	0	0	0	X	0	X
0	0	1	0	1	0	X	1	X
0	1	0	1	0	1	X	X	1
0	1	1	0	1	0	X	X	0
1	0	0	1	0	X	0	0	X
1	0	1	1	1	X	0	1	X
1	1	0	1	1	X	0	X	0
1	1	1	0	0	X	1	X	1

$JA = B.x'$

$KA = B.x$

$JB = x$

$KB = (A \oplus x)'$
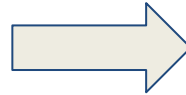
Design: Example 1

Input flip-flop function

$$JA = B.x'$$
$$KA = B.x$$

$$JB = x$$
$$KB = (A \oplus x)'$$

Logic Diagram
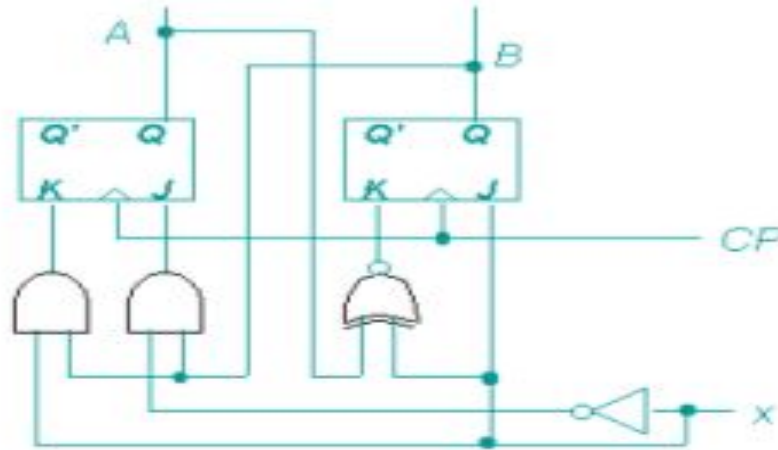
Design: Example 2

Design, using D flip-flop, circuit is based on state table below. (Exercise: How if using JK flip-flop)
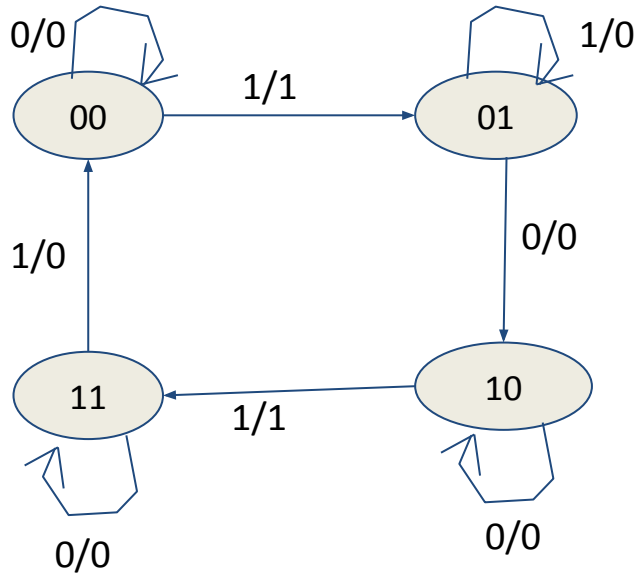
Present state		Input	Next state		Output
A	B	x	A⁺	B⁺	y
0	0	0	0	0	0
0	0	1	0	1	1
0	1	0	1	0	0
0	1	1	0	1	0
1	0	0	1	0	0
1	0	1	1	1	1
1	1	0	1	1	0
1	1	1	0	0	0

Design: Example 2

Determine input expression for flip-flop and y output

A	B	x	A+	B+	y	DA	DB
0	0	0	0	0	0	0	0
0	0	1	0	1	1	0	1
0	1	0	1	0	0	1	0
0	1	1	0	1	0	0	1
1	0	0	1	0	0	1	0
1	0	1	1	1	1	1	1
1	1	0	1	1	0	1	1
1	1	1	0	0	0	0	0

Q	Q^+	D
0	0	0
0	1	1
1	0	0
1	1	1

D Flip-flop

$DA = A.B' + B.x'$

$DB = A'.x + B'.x + A.B.x'$

$y = B'.x$
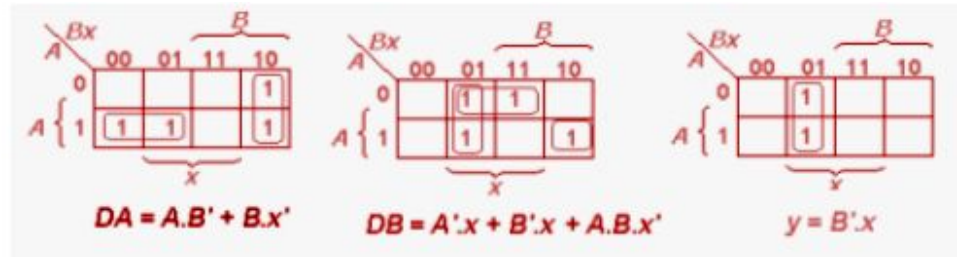
Design: Example 2

From expression built, draw logic diagram

$$DA = A.B' + B.x'$$

$$DB = A'.x + B'.x + A.B.x'$$

$$y = B'.x$$
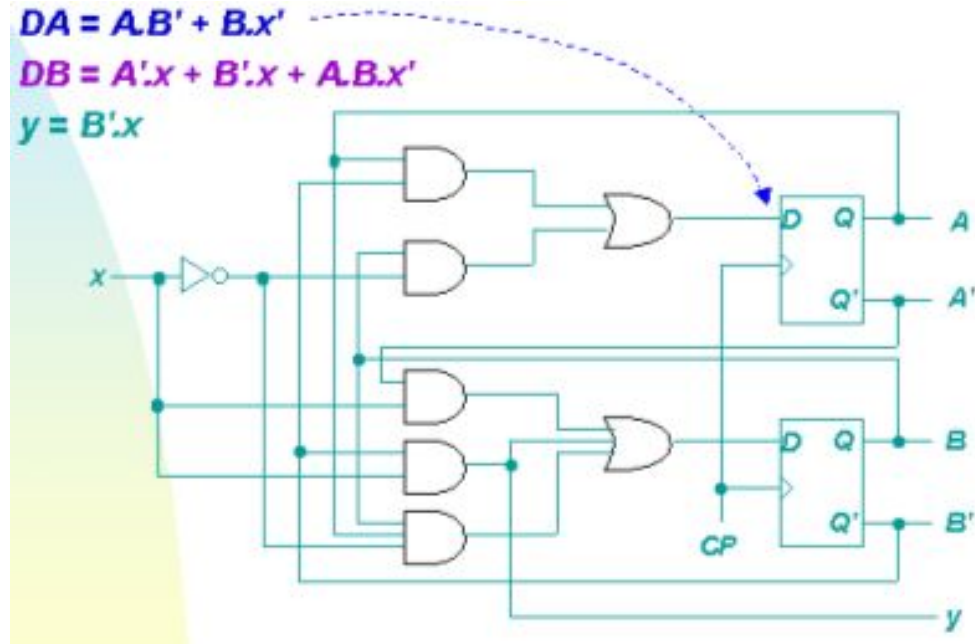
Introduction: Counters

- Counters are circuits that cycle through a specified number of states.

- Two types of counters:
  - ❖ synchronous (parallel) counters
  - ❖ asynchronous (ripple) counters

- Ripple counters allow some flip-flop outputs to be used as a source of clock for other flip-flops.

- Synchronous counters apply the same clock to all flip-flops.

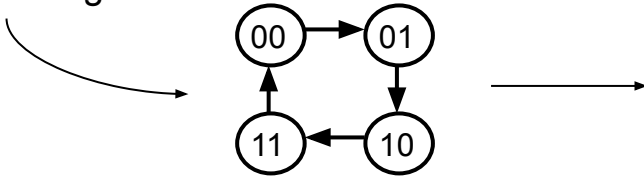- **We will cover only synchronous this time**

Synchronous (Parallel) Counters

- **Synchronous (parallel) counters**: the flip-flops are clocked at the same time by a common clock pulse.

- We can design these counters using the sequential logic design process.

- Example: 2-bit synchronous binary counter (using T flip-flops, or JK flip-flops with identical J,K inputs).

Synchronous (Parallel) Counters

Question: Design a 2-bit synchronous binary counter (using T flip-flops).

Start by drawing this

Present state		Next state		Flip-flop inputs	
A_1	A_0	A_1^+	A_0^+	TA_1	TA_0
0	0	0	1	0	1
0	1	1	0	1	1
1	0	1	1	0	1
1	1	0	0	1	1

Q_n	Q_{n+1}	T
0	0	0
0	1	1
1	0	1
1	1	0

Excitation table of T flip flop

Synchronous (Parallel) Counters

Present state		Next state		Flip-flop inputs	
A_1	A_0	A_1^+	A_0^+	TA_1	TA_0
0	0	0	1	0	1
0	1	1	0	1	1
1	0	1	1	0	1
1	1	0	0	1	1

Next, find out the equations for the flip-flop inputs.

$$TA_1 = A_1'A_0 + A_1A_0 = A_0 (A_1'+A_1) = A_0$$
$$TA_0 = 1$$

FYI, these can also be calculated using K-Maps
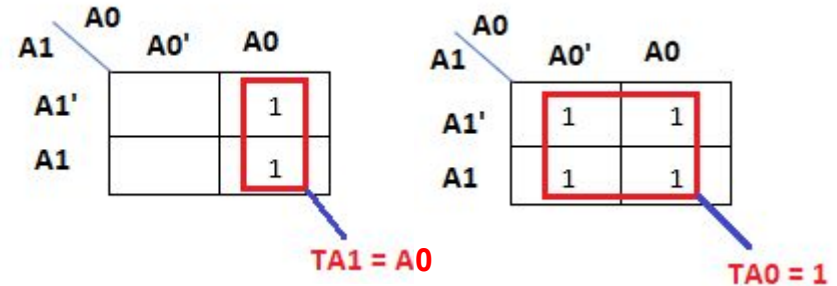
Synchronous (Parallel) Counters
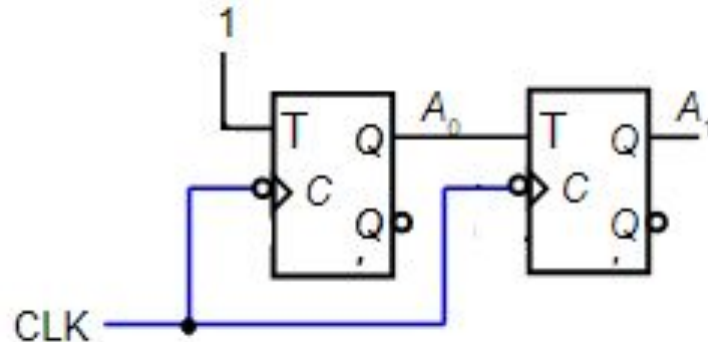
TA$_1$ = A$_1$'A$_0$ + A$_1$A$_0$ = A$_0$ (A$_1$'+A$_1$) = A$_0$
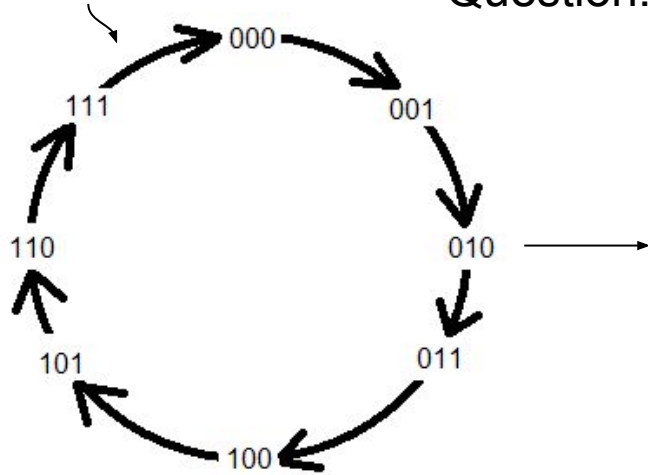**TA$_0$** = 1

Finally draw the circuit diagram

Synchronous (Parallel) Counters
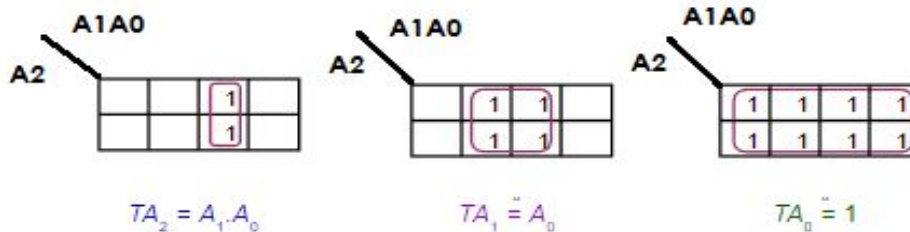
Start by drawing this

- Question: Design a 3-bit synchronous binary counter (using T).

Present state			Next state			Flip-flop inputs		
A_2	A_1	A_0	A_2^+	A_1^+	A_0^+	TA_2	TA_1	TA_0
0	0	0	0	0	1	0	0	1
0	0	1	0	1	0	0	1	1
0	1	0	0	1	1	0	0	1
0	1	1	1	0	0	1	1	1
1	0	0	1	0	1	0	0	1
1	0	1	1	1	0	0	1	1
1	1	0	1	1	1	0	0	1
1	1	1	0	0	0	1	1	1

Q_n	Q_{n+1}	T
0	0	0
0	1	1
1	0	1
1	1	0

Excitation table of T flip flop

FYI, these can also be calculated using Boolean simplifications

$TA_2 = A_1.A_0$

$TA_1 = A_0$

$TA_0 = 1$

Synchronous (Parallel) Counters

Draw the circuit diagram

$TA_2 = A_1 . A_0$ $TA_1 = A_0$ $TA_0 = 1$
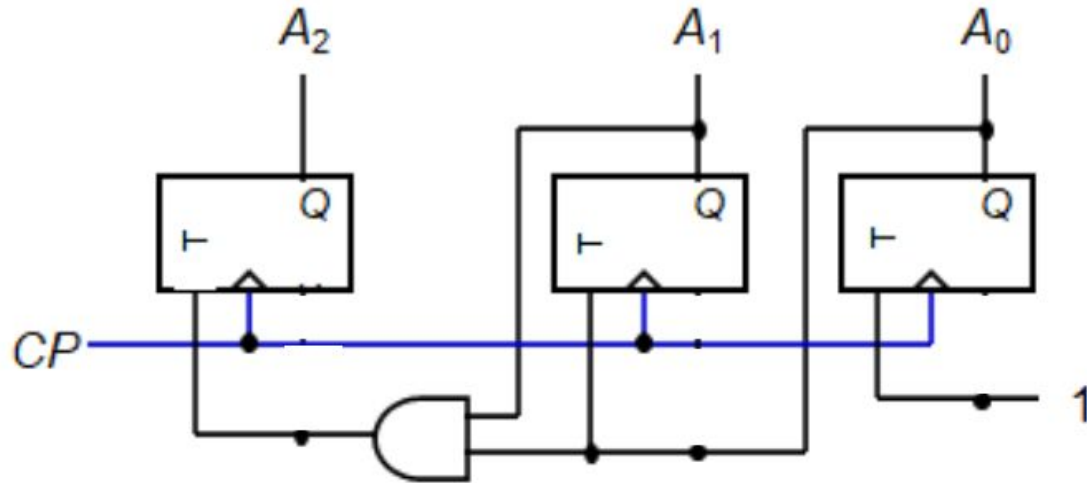
Self-Study

4 bit synchronous up counter

- Equations and diagram will look like the following:

$$TA_3 = A_2 \cdot A_1 \cdot A_0$$
$$TA_2 = A_1 \cdot A_0$$
$$TA_1 = A_0$$
$$TA_0 = 1$$
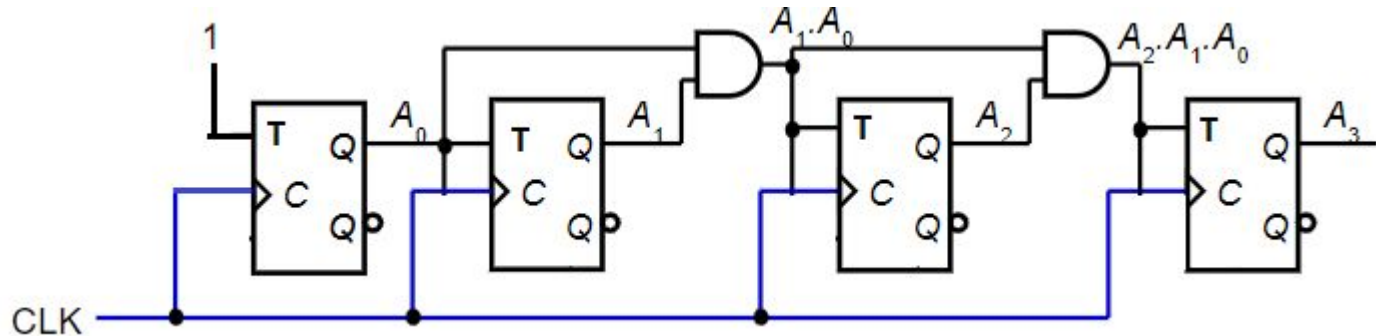
Up/Down Synchronous Counters

- **Up/down synchronous counter**: a *bidirectional* counter that is capable of counting either up or down.

- An input (control) line $Up/\overline{Down}$ (or simply Up) specifies the direction of counting.
  - ❖ $Up/\overline{Down} = 0 \rightarrow$ Count upward
  - ❖ $Up/\overline{Down} = 1 \rightarrow$ Count downward

3 bit Synchronous Up/Down Counter

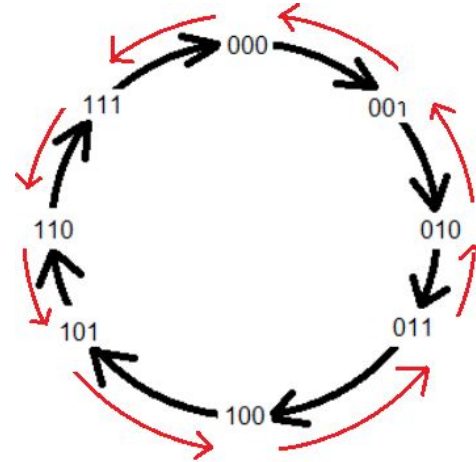| U/D	Present states			Next States			Flipflop inputs		
M	Q_2	Q_1	Q_0	Q_2+	Q_1+	Q_0+	TQ_2	TQ_1	TQ_0
0	0	0	0	0	0	1	0	0	1
0	0	0	1	0	1	0	0	1	1
0	0	1	0	0	1	1	0	0	1
0	0	1	1	1	0	0	1	1	1
0	1	0	0	1	0	1	0	0	1
0	1	0	1	1	1	0	0	1	1
0	1	1	0	1	1	1	0	0	1
0	1	1	1	0	0	0	1	1	1
1	0	0	0	1	1	1	1	1	1
1	0	0	1	0	0	0	0	0	1
1	0	1	0	0	0	1	0	1	1
1	0	1	1	0	1	0	0	0	1
1	1	0	0	0	1	1	1	1	1
1	1	0	1	1	0	0	0	0	1
1	1	1	0	1	0	1	0	1	1
1	1	1	1	1	1	0	0	0	1

Q_n	Q_{n+1}	T
0	0	0
0	1	1
1	0	1
1	1	0

Excitation table of T flip flop

3 bit Synchronous Up/Down Counter

U/D	Present states			Next States			Flipflop inputs		
M	Q_2	Q_1	Q_0	Q_2+	Q_1+	Q_0+	TQ_2	TQ_1	TQ_0
0	0	0	0	0	0	1	0	0	1
0	0	0	1	0	1	0	0	1	1
0	0	1	0	0	1	1	0	0	1
0	0	1	1	1	0	0	1	1	1
0	1	0	0	1	0	1	0	0	1
0	1	0	1	1	1	0	0	1	1
0	1	1	0	1	1	1	0	0	1
0	1	1	1	0	0	0	1	1	1
1	0	0	0	1	1	1	1	1	1
1	0	0	1	0	0	0	0	0	1
1	0	1	0	0	0	1	0	1	1
1	0	1	1	0	1	0	0	0	1
1	1	0	0	0	1	1	1	1	1
1	1	0	1	1	0	0	0	0	1
1	1	1	0	1	0	1	0	1	1
1	1	1	1	1	1	0	0	0	1

$$TQ_2 = M'Q_1 Q_0 + MQ_1'Q_0'$$

$$TQ_1 = M'Q_0 + MQ_0'$$
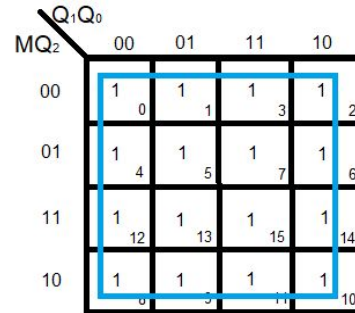
$$TQ_0 = 1$$

3 bit Up/Down Synchronous Counter

Draw the circuit diagram
(The number of present state will be equal to the number of flip-flops in the circuit)

https://www.youtube.com/watch?v=svFUEJkoeVY&list=PL
BInK6fEyqRjMH3mWf6kwqiTbT798eAOm&index=189

Practice Problem using Counters

PRACTICE PROBLEM - 1

- Implement the following counter using T FF: 1->2->3->5->7->11->13->1

Start by drawing this

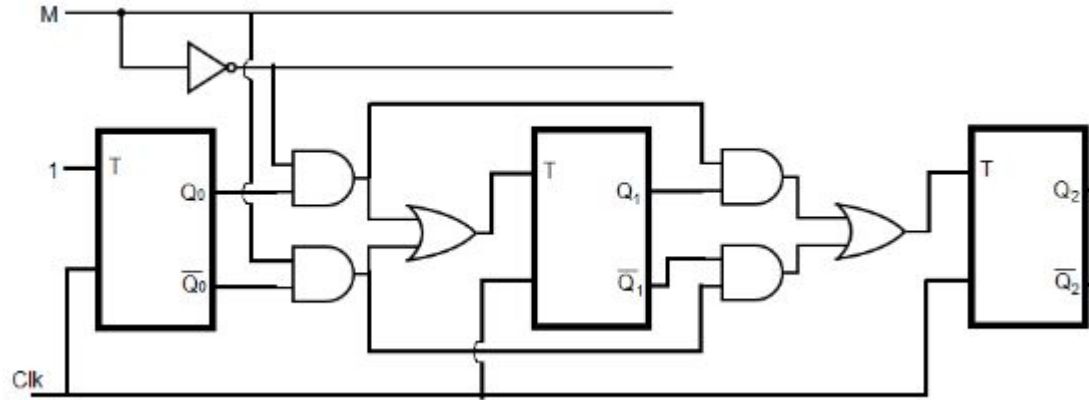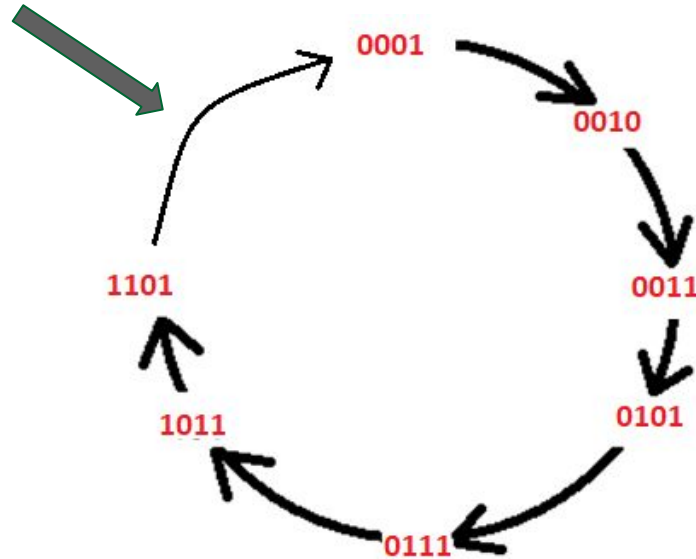| Present states				Next States				Flip-flop inputs			
A_3	A_2	A_1	A_0	A_3+	A_2+	A_1+	A_0+	TA_3	TA_2	TA_1	TA_0
0	0	0	0	0	0	0	1	0	0	0	1
0	0	0	1	0	0	1	0	0	0	1	1
0	0	1	0	0	0	1	1	0	0	0	1
0	0	1	1	0	1	0	1	0	1	1	0
0	1	0	0	0	0	0	1	0	1	0	1
0	1	0	1	0	1	1	1	0	0	1	0
0	1	1	0	0	0	0	1	0	1	1	1
0	1	1	1	1	0	1	1	1	1	0	0
1	0	0	0	0	0	0	1	1	0	0	1
1	0	0	1	0	0	0	1	1	0	0	0
1	0	1	0	0	0	0	1	1	0	1	1
1	0	1	1	1	1	0	1	0	1	1	0
1	1	0	0	0	0	0	1	1	1	0	1
1	1	0	1	0	0	0	1	1	1	0	0
1	1	1	0	0	0	0	1	1	1	1	1
1	1	1	1	0	0	0	1	1	1	1	0

Q_n	Q_{n+1}	T
0	0	0
0	1	1
1	0	1
1	1	0

Excitation table of T flip flop

For present states not mentioned in the question, next state will be the INITIAL STATE [0001 in this question]. It's important to show the states that are not mentioned in the question.Otherwise, the circuit will stop working if any of those states show up.

Present states				Next States				Flip-flop inputs			
A_3	A_2	A_1	A_0	A_3+	A_2+	A_1+	A_0+	TA_3	TA_2	TA_1	TA_0
0	0	0	0	0	0	0	1	0	0	0	1
0	0	0	1	0	0	1	0	0	0	1	1
0	0	1	0	0	0	1	1	0	0	0	1
0	0	1	1	0	1	0	1	0	1	1	0
0	1	0	0	0	0	0	1	0	1	0	1
0	1	0	1	0	1	1	1	0	0	1	0
0	1	1	0	0	0	0	1	0	1	1	1
0	1	1	1	1	0	1	1	1	1	0	0
1	0	0	0	0	0	0	1	1	0	0	1
1	0	0	1	0	0	0	1	1	0	0	0
1	0	1	0	0	0	0	1	1	0	1	1
1	0	1	1	1	1	0	1	0	1	1	0
1	1	0	0	0	0	0	1	1	1	0	1
1	1	0	1	0	0	0	1	1	1	0	0
1	1	1	0	0	0	0	1	1	1	1	1
1	1	1	1	0	0	0	1	1	1	1	0

$$TA3 = A3A0' + A3A1' + A2A1A0$$

$$TA2 = A2A0' + A3A2 + A1A0$$

Present states				Next States				Flip-flop inputs			
A_3	A_2	A_1	A_0	A_3+	A_2+	A_1+	A_0+	TA_3	TA_2	TA_1	TA_0
0	0	0	0	0	0	0	1	0	0	0	1
0	0	0	1	0	0	1	0	0	0	1	1
0	0	1	0	0	0	1	1	0	0	0	1
0	0	1	1	0	1	0	1	0	1	1	0
0	1	0	0	0	0	0	1	0	1	0	1
0	1	0	1	0	1	1	1	0	0	1	0
0	1	1	0	0	0	0	1	0	1	1	1
0	1	1	1	1	0	1	1	1	1	0	0
1	0	0	0	0	0	0	1	1	0	0	1
1	0	0	1	0	0	0	1	1	0	0	0
1	0	1	0	0	0	0	1	1	0	1	1
1	0	1	1	1	1	0	1	0	1	1	0
1	1	0	0	0	0	0	1	1	1	0	1
1	1	0	1	0	0	0	1	1	1	0	0
1	1	1	0	0	0	0	1	1	1	1	1
1	1	1	1	0	0	0	1	1	1	1	0

$TA1 = A3'A1'A0 + A3'A2'A0 + A2A1A0' + A3A1$

$TA0 = A0' + A3'A2'A1'$

$$TA3 = A3A0' + A3A1' + A2A1A0$$

$$TA2 = A2A0' + A3A2 + A1A0$$

$$TA1 = A3'A1'A0 + A3'A2'A0 + A2A1A0' + A3A1$$

$$TA0 = A0' + A3'A2'A1'$$

Draw the circuit yourself

PRACTICE PROBLEM - 2

- Implement the following counter using D FF: Green->Yellow->Red->Yellow->Green

Let's assume: Green = 00, Yellow = 01, Red = 10

Present states			Next States		Flip-flop inputs	
A	B	x	A+	B+	DA	DB
0	0	0	0	1	0	1
0	0	1	0	1	0	1
0	1	0	0	0	0	0
0	1	1	1	0	1	0
1	0	0	0	1	0	1
1	0	1	0	1	0	1
1	1	0	x	x	x	x
1	1	1	x	x	x	x

Find out the equations for the flip-flop inputs

DA = Bx

DB = B'

Q_n	Q_{n+1}	D
0	0	0
0	1	1
1	0	0
1	1	1

Excitation table of D flip flop

Special Note: You may need to use more than 1 external input depending upon the question.

Draw the circuit diagram (The number of present state will be equal to the number of flip-flops in the circuit)