Practice Sheet Final

1. (a) Derive formula for Secant method for finding root(s) of a nonlinear equation. Why would you use the Secant method instead of Newton's method for finding root(s) of a nonlinear equation?

   (b) Use Secant method to estimate the root of $f(x) = x^3 - 3x^2 + x$ with initial estimates x_{-1}=0.3 and x_0=0.35. Show your result along with the percentage errors in tabular form for the first three iterations.

2. Discuss the limitations of Newton's Method of solving a root of a nonlinear equation?

3

Using Gauss elimination method solve the below system:

$$3x_1 + 5x_2 + 7x_3 + 9x_4 = 1.4$$

$$7x_1 + 3x_2 + 11x_3 + 4x_4 = 1.8$$

$$2x_1 + 5x_2 + 3x_3 + 2x_4 = 2.7$$

$$8x_1 + 7x_2 + 7x_3 + 4x_4 = 3.4$$

4.

Find the root of the below equation using secant method with initial value $\mathbf{x_{-1}=0.3}$ and $\mathbf{x_0=0.9}$. Do your calculation for the first three iterations and show you results in a tabular form with all the percentage errors.

$$f(x) = x^2 - e^{-2x} - (x)$$

5.

Use bi-section method to find the root(s) of $f(x) = x^2 - e^{-2x} - (x)$ with $\mathbf{x_l=0}$ and $\mathbf{x_u = -0.8}$.

6

Solve the system of equations below using LU decomposition:

$$x_1 + 3x_2 + 2x_3 + 4x_4 = 1.4$$

$$2x_1 + x_2 + x_3 + 3x_4 = 1.8$$

$$2x_1 + 5x_2 + x_3 + x_4 = 2.7$$

$$3x_1 + 4x_2 + 2x_3 + 5x_4 = 3.4$$

7

Using Gauss elimination method solve the below system:

$$12x_1 + 10x_2 - 7x_3 = 15$$

$$6\,x_1 + 5x_2 + 3x_3 = 14$$

$$24x_1 - x_2 + 5x_3 = 28$$

8

Discuss 2 problems of Gaussian elimination.

9

Determine the real root of $f(x) = -26 + 85x - 91x^2 + 44x^3 - 8x^4 + x^5$ using the bisection method. Employ initial guesses of $x_l = 0.5$ and $x_u = 1.0$. Iterate the process until the approximate error falls below a stopping criterion of $\varepsilon_s = 10\%$. Note that you must show the detail calculation of the first iteration.

10

With the help of Trapezoidal rule of Integration, integrate $\int_{1}^{21} \left(1 + e^{2x}\right) dx$ using a single segment. Show, in a tabular form, the effect step size on the value of the integration considering step size=1, 2 and 4. In the table, show the values of integration and percentage true error against the value of 'n'.

11

Consider the following set of vectors $S = \{v_1, v_2, v_3\}$ in R^3 where

$$\vec{v}_1 = \begin{pmatrix} \frac{3}{5} \\ 0 \\ \frac{4}{5} \end{pmatrix}, \quad \vec{v}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad \text{and} \quad \vec{v}_3 = \begin{pmatrix} -\frac{3}{5} \\ 0 \\ \frac{4}{5} \end{pmatrix}.$$

a. Check if S is an orthonormal set.
b. Let's say the following matrix below is an orthonormal matrix

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix}.$$

Compute the value of $(A^T A)^{-1}$

12

a)
$$\begin{aligned} -4x_1 + 7x_2 - 2x_3 &= 2 \\ x_1 - 2x_2 + x_3 &= 3 \\ 2x_1 - 3x_2 + x_3 &= -4 \end{aligned}$$

a. Construct $F^{(1)}$ and $F^{(2)}$
b. Find Lower Triangular Matrix
c. Solve the System using LU Decomposition

13.

Suppose $f(x) = e^x \sin(1.5x)$.

a. Find $f'(1)$ with step size = 0.1 using Central Difference method
b. Find $f'(1)$ with step size = 0.05 using Central Difference method
c. Now Compute $D_h^{(1)}$
d. Also Find the upper bound of Truncation error with ζ {0.5,3.5}