

1. Given $\beta=2$, $m=2$, $e_{min} = -1$, $e_{max} = 1$. Using the normalized form answer the following:

a) Compute the Machine Epsilon. [3 marks]

b) Compute the minimum of $|x|$ [3 marks]

c) How many non-negative numbers can you represent using this system [1 marks]

2. Suppose a system with $\beta=2$, $m=3$. What will be the product of $x = 4/8$ and $y = 7/8$ for this system. [3 marks]

Question -

(3 marks) Consider the quadratic equation, $x^2 - 60x + 1 = 0$. Working to 6 significant figures, compute the roots of the quadratic equation and check that there is a loss of significance. Find the correct roots such that loss of significance does not occur.

Solution: The discriminant of the given quadratic equation is

$$\text{Discriminant} = \frac{-(-60) \pm \sqrt{(-60)^2 - 4 \times 1 \times 1}}{2 \times 1} = 30 \pm 29.9833 = 59.9833 \quad \text{and} \quad 0.0167000 \quad (\text{upto 6 sig. fig.})$$

But the product of the roots must be equal to 1, but here we get, $59.9833 \times 0.0167000 = 1.00172 \neq 1$. Since the product is not equal to one there is loss of significance due to the subtraction of two close numbers 30 and 29.9833.

One root is $x_1 = 59.9833$. To find the second root, we use, $x_2 = 1/x_1 = 1/(59.9833) = 0.0166713$ up to 6 sig. fig. Now, check that $x_1 + x_2 = 59.9833 + 0.0166713 = 59.99997 = 60.0000$ up to 6 sig. fig. This is the correct solution that does not have any loss of significance.