Practice Sheet Final

- 1. (a) Derive formula for Secant method for finding root(s) of a nonlinear equation. Why would you use the Secant method instead of Newton's method for finding root(s) of a nonlinear equation?
 - (b) Use Secant method to estimate the root of $f(x) = x^3 3x^2 + x$ with initial estimates $x_{-1}=0.3$ and $x_0=0.35$. Show your result along with the percentage errors in tabular form for the first three iterations.
- 2. Discuss the limitations of Newton's Method of solving a root of a nonlinear equation?

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Using Gauss elimination method solve the below system:

$$3x_{1} + 5x_{2} + 7x_{3} + 9x_{4} = 1.4$$

$$7x_{1} + 3x_{2} + 11x_{3} + 4x_{4} = 1.8$$

$$2x_{1} + 5x_{2} + 3x_{3} + 2x_{4} = 2.7$$

$$8x_{1} + 7x_{2} + 7x_{3} + 4x_{4} = 3.4$$

4.

Find the root of the below equation using secant method with initial value x_{-1} =0.3 and x_0 =0.9. Do your calculation for the first three iterations and show you results in a tabular form with all the percentage errors.

$$f(x) = x^2 - e^{-2x} - (x)$$

5.

Use bi-section method to find the root(s) of $f(x) = x^2 - e^{-2x} - (x)$ with $x_i = 0$ and $x_u = -0.8$.

Solve the system of equations below using LU decomposition:

$$x_{1} + 3x_{2} + 2x_{3} + 4x_{4} = 1.4$$

$$2x_{1} + x_{2} + x_{3} + 3x_{4} = 1.8$$

$$2x_{1} + 5x_{2} + x_{3} + x_{4} = 2.7$$

$$3x_{1} + 4x_{2} + 2x_{3} + 5x_{4} = 3.4$$

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Using Gauss elimination method solve the below system:

$$12x_{1} + 10x_{2} - 7x_{3} = 15$$

$$6x_{1} + 5x_{2} + 3x_{3} = 14$$

$$24x_{1} - x_{2} + 5x_{3} = 28$$

8

Discuss 2 problems of Gaussian elimination.

Determine the real root of $f(x) = -26 + 85x - 91x^2 + 44x^3 - 8x^4 + x^5$ using the bisection method. Employ initial guesses of $x_l = 0.5$ and $x_u = 1.0$. Iterate the process until the approximate error falls below a stopping criterion of $\varepsilon_s = 10\%$. Note that you must show the detail calculation of the first iteration.

10

With the help of Trapezoidal rule of Integration, integrate $\int_{1}^{21} (1 + e^{2x}) dx$ using a single segment. Show, in a tabular form, the effect step size on the value of the integration considering step size=1, 2 and 4. In the table, show the values of integration and percentage true error against the value of 'n'.

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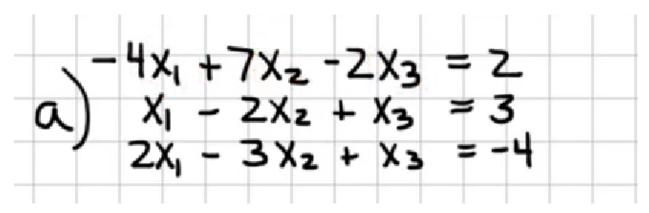
Consider the following set of vectors $S = \{v_1, v_2, v_3\}$ in R^3 where

$$ec{v}_1 = egin{pmatrix} rac{3}{5} \ 0 \ rac{4}{5} \end{pmatrix}, \quad ec{v}_2 = egin{pmatrix} 0 \ 1 \ 0 \end{pmatrix} \quad ext{and} \quad ec{v}_3 = egin{pmatrix} -rac{3}{5} \ 0 \ rac{4}{5} \end{pmatrix} \,.$$

- a. Check if S is an orthonormal set.
- b. Let's say the following matrix below is an orthonormal matrix

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix}.$$

Compute the value of $(A^T A)^{-1}$



- ^{a.} Construct $F^{(1)}$ and $F^{(2)}$
- b. Find Lower Triangular Matrix
- c. Solve the System using LU Decomposition

13.

Suppose $f(x) = e^x Sin (1.5x)$.

- a. Find f'(1) with step size = 0.1 using Central Difference method
- b. Find f'(1) with step size = 0.05 using Central Difference method
- c. Now Compute D_h⁽¹⁾
- d. Also Find the upper bound of Truncation error with ζ $\{0.5,3.5\}$