Simpson's Rule :

Newton's Cotes Formula for n=1 [Trapezium Rule].

Newton's Cotes Formula for n=2 [Simpson's Rule]

For n=1,

$$I(f) = \int_a^b P_1(x)\,dx.$$

$$P_1(x) = l_0(x) f(x_0) + l_1(x) f(x_1)$$

$$I(f) = \int_a^b l_0(x) f(x_0) + l_1(x) f(x_1)$$

$$I(f) = f(x_0) \underbrace{\int_a^b l_0(x)}_{\text{weight function}} + f(x_1) \int_a^b l_1(x)$$

$$I(f) = \delta_0 f(x_0) + \delta_1 f(x_1) \rightarrow \sum_{k=0}^{1} \delta_k f(x_k)$$

$$I(f) = \frac{b-a}{2}\left[f(a) + f(b)\right] \rightarrow \text{Trapezium Rule.}$$

For n = 2, [Simpson's Rule] $\rightarrow [a, b]$

$$I_2(f) = \sum_{k=0}^{2} \delta_k f(x_k) \qquad h = \frac{b-a}{2}$$

$x_0 = a$

$x_1 = (a+b)/2 = m.$

$x_2 = b$

$$l_0(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} = \frac{(x-m)(x-b)}{(a-m)(a-b)}$$

$$l_1(x) = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} = \frac{(x-a)(x-b)}{(m-a)(m-b)}$$

$$l_2(x) = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} = \frac{(x-a)(x-m)}{(b-a)(b-m)}$$

$$\delta_0 = \int_a^b l_0(x)\,dx$$

$$\delta_0 = \frac{1}{(a-m)(a-b)}\int_a^b (x-m)(x-b)\,dx$$

$$\delta_0 = \frac{1}{6}(b-a)$$

$$\delta_1 = \frac{2}{3}(b-a)$$

$$\delta_2 = \frac{1}{6}(b-a)$$

$$\delta_1 = \int_a^b l_1(x)\,dx$$

$$\delta_2 = \int_a^b l_2(x)\,dx$$

$I_2(f) = \delta_0 f(x_0) + \delta_1 f(x_1) + \delta_2 f(x_2)$

$I_2(f) = \frac{(b-a)}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right].$

→ Simpson's Rule.

Assignment - 06 :

b) Use Simpson's Rule since $n = 2$,

$I_2(f) = \frac{(b-a)}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$

Given interval : $[0, 2]$.

$f(0) = e^{0.5(0)} + \sin(0) = 1.$

$f(2) = e^{0.5(2)} + \sin(2) = 3.6276.$

$f\left(\frac{0+2}{2}\right) = f(1) = e^{0.5(1)} + \sin(1) = 2.4902.$

$I_2(f) = \frac{(2-0)}{6}\left[1 + 4(2.4902) + 3.6276\right]$

$I_2(f) = \frac{1}{3} \times 14.5884$

$I_2(f) = 4.8628.$