PRACTICE QUESTIONS

Question 1

Question # 1 : A function is given by $f(x) = 6e^{-3x}$. Now Answer the following:

1. [1 Mark] Calculate $f'(x)$ at $x = 0.5$ with $h = 0.32$ using the central difference formula.
2. [1 Mark] Calculate $f'(x)$ at $x = 0.5$ with $h = 0.16$ using the central difference formula.
3. [3 Marks] Now compute $D_{0.32}^{(1)}$ at $x = 0.5$ using Richardson extrapolation method.
4. [2 Marks] If the exact value of the derivative, $f'(0.5)$ is 1.3388, find the percentage error with extrapolated value found in the previous part.

Question 2

1. Consider the following data set:

x	1.1	1.2	1.3
$f(x)$	-0.57941	-0.90730	-1.2807

(a) (4 marks) Using the above data, compute $f'(1.2)$ using the central difference method.

(b) (6 marks) For in interval $[1.1, 1.3]$, compute the error bound (truncation error) if the above data is generated by the function, $f(x) = x\cos(x) - x^2\sin(x)$.

(c) (2 marks) Also compute the actual error.

Solution: Here: $h = 0.1$, $x_0 = 1.2$. Therefore, the derivative at $x = 1.2$ is

$$f'(1.2) = \left.\frac{f(x_0+h) - f(x_0-h)}{2h}\right|_{x_0=1.2, h=0.1} = \frac{-1.2807 - (-0.57941)}{2 \times 0.1} = -3.5065 \ .$$

Solution: For $\xi \in [1.1, 1.3]$, the error bound is given by

$$\begin{aligned}
\left|\frac{h^2}{6} f'''(\xi)\right| &= \frac{h^2}{6}\left|-9\cos(\xi) + 7\xi\sin\xi + \xi^2\cos\xi\right| \ , \\
&\leq \frac{(0.1)^2}{6}\Big(9|\cos(1.1)| + 7(1.3)|\sin(1.3)| + (1.3)^2|\cos(1.1)|\Big) \ , \\
&= 0.022696 \ .
\end{aligned}$$

$$\begin{aligned}
f'(1.2) &= \Big(\cos x - 3x\sin x - x^2\cos x\Big)\Big|_{x=1.2} = -3.5148 \ . \\
\therefore \text{Actual Error} &= |-3.5148 - (-3.5065)| = 0.0083 \ .
\end{aligned}$$