# FINAL EXAM TOTAL MARKS: 50 DURATION: 105 MINUTES

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There are a total of six problems. You have to solve all of them.

### Problem 1 (CO5): Pumping Lemma (5 points)

Let  $\Sigma = \{0, 1\}$ . Consider the following language.

$$L = \{ w \in \Sigma^* : w = 0^a 1^b 1^c 0^d, \text{ where } a + b = c + d \text{ and } a, b, c, d \ge 0 \}$$

Use the pumping lemma to **demonstrate** that *L* is not regular language.

## Problem 2 (CO3): Designing Context-Free Grammars (10 points)

Let  $\Sigma = \{0, 1\}$ . Consider the following languages. Recall that for a string w, |w| denotes the length of w.

$$L_1=\{w\in \Sigma^*: w \text{ is an even length palindrome}\}$$
 
$$L_2=\{w\in \Sigma^*: \text{length of } w \text{ is even}\}$$
 
$$L_3=\{x\text{11}y: x,y\in L_2, |x|=|y|\}$$
 
$$L_4=L_1\cap L_3$$

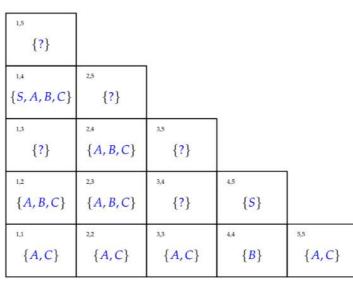
Now solve the following problems.

- (a) Give a context-free grammar for the language  $L_1$ . (3 points)
- (b) Give a context-free grammar for the language  $L_3$ . (4 points)
- (c) Give a context-free grammar for the language  $L_4$ .(3 point)

### Problem 3 (CO4): The CYK Algorithm (5 points)

**Apply** the CYK algorithm to fill up the table for the string aaaba using the following grammar. Here a and b are terminals, and the rest are variables.

$$S \rightarrow BA \mid BC$$
  
 $A \rightarrow AB \mid AC \mid$  a  
 $B \rightarrow CB \mid CC \mid$  b  
 $C \rightarrow CA \mid$  a



a a a b a

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### Problem 4 (CO3): Constructing Pushdown Automata (10 points)

Let  $\Sigma = \{0, 1\}$ . Consider the following languages.

 $L_1 = \{w \mid w \text{ starts and ends with the same character}\}$ 

 $L_2 = \{w \mid \text{the number of 0s in } w \text{ is not same as the number of 1s} \}$ 

- (a) Give the state diagram of a pushdown automaton that recognizes  $L_1$ . (4 points)
- (b) **Give** the state diagram of a pushdown automaton that recognizes  $L_2$ . (6 points)

## Problem 5 (CO3): Derivations, Parse Trees and Ambiguity (10 points)

Take a look at the grammar below and solve the following problems.

$$A \to 1A \mid 1C \mid 0B \mid 00A$$
  
 $B \to 0A \mid 1B \mid 00B$   
 $C \to 0C0 \mid 0C1 \mid 1C0 \mid 1C1 \mid \epsilon$ 

- (a) Give a leftmost derivation for the string 01011001. (3 points)
- (b) Sketch the parse tree corresponding to the derivation you gave in (a). (2 points)
- (c) **Demonstrate** that the given grammar is ambiguous by showing two more parse trees (apart from the one you already found in (b)) for the same string. (3 points)
- (d) Find a string w of length six such that w has exactly one parse tree in the grammar above. (1 point)
- (e) **Desgin** an unambiguous Context Free Grammar for the language represented by the given ambiguous grammar. (1 point)

### Problem 6 (CO4): Chomsky Normal Form (10 points)

Answer the following questions.

(a) **List** the productions that violate the conditions of the Chomsky Normal Form (CNF) in the following grammar. (5 points)

$$\begin{split} M &\rightarrow \varepsilon \mid \mathbf{x} \mid MN \\ N &\rightarrow NC \mid \mathbf{yy} \mid P \\ P &\rightarrow QQ \mid \mathbf{z} \\ Q &\rightarrow \mathbf{yQ} \mid \varepsilon \end{split}$$

(b) **Write** down the additional rules that need to be added to the following grammar if the production  $B \to \varepsilon$  is removed. (3 points)

$$S \rightarrow Aa \mid BB$$
  
 $A \rightarrow CA \mid BaB \mid \varepsilon$   
 $B \rightarrow AB \mid b \mid \varepsilon$ 

(c) Write down the additional rules that need to be added to the following grammar if all the unit productions are removed. (2 points)

$$X \rightarrow 0X1Y1 \mid Y$$
$$Y \rightarrow XY \mid YY \mid Z$$
$$Z \rightarrow 1Z \mid Z$$