CSE 331: Automata and Computability (Section 10, 11)
Spring 2025

Assignment - 2

Total Marks: 120

Deadline: 14 May 2025, 11:55 pm

You have to upload the scanned PDF of your handwritten assignment.

1. **Proof that the following languages are non-regular using the pumping lemma. [30 marks]**
   a. $L = \{w \in \{0,1\}^*, w = 0^i1^j0^k \text{ where } i + k = j \text{ and } i, j, k > 0\}$
   b. $L = \{w \in \{0,1\}^*, w = 0^i1^j0^k \text{ where } i + k > j \text{ and } i, j, k > 0\}$
   c. $L = \{w \in \{0,1\}^*, w = 0^i1^j0^k \text{ where } i + k < j \text{ and } i, j, k > 0\}$
   d. $L = \{w \in \{0,1\}^*, ww^R\}$ (w^R denotes the reverse string of w)
   e. $L = \{w \in \{0,1\}^*, ww'\}$ (w' denotes a string where every bit is flipped from w)
   f. $L = \{w \in \{0,1\}^*, w1^n \text{ where } n > 0 \text{ and } |w| = n\}$

2. **Design the Context Free Grammar (CFG) for the following languages. [30 marks]**
   a. $L = \{w \in \{0,1\}^*, \text{the length of } w \text{ is odd and the mid is } 0\}$
   b. $L = \{w \in \{0,1\}^*, w \text{ contains even number of } 0s\}$
   c. $L = \{w \in \{0,1\}^*, w \text{ starts and ends with the same symbol}\}$
   d. $L = \{w_1, w_2 \in \{0,1\}^*, w_1\#w_2 \text{ where } w_2 \text{ starts with "01" and } |w_1| = |w_2|\}$
   e. $L = \{w \in \{0,1\}^*, w \text{ contains exactly three } 1s\}$
   f. $L = \{w \in \{0,1\}^*, w = 0^i1^j0^k \text{ where } i + k < j \text{ and } i, j, k > 0\}$

3. **[10 marks]**

Take a look at the grammar below and solve the following problems.

$$S \to AX$$
$$A \to 0A \mid 1B \mid \varepsilon$$
$$B \to 0B \mid 1A$$
$$X \to 00X \mid 01X \mid 10X \mid 11X \mid 0 \mid 1$$

(a) **Give** a leftmost derivation for the string 01101010. (3 points)

(b) **Sketch** the parse tree corresponding to the derivation you gave in (a). (2 points)

(c) **Demonstrate** that the given grammar is ambiguous by showing two more parse trees (apart from the one you already found in (b)) for the same string. (4 points)

(d) **Find** a string w of length six such that w has exactly one parse tree in the grammar above. (1 point)

4. **Construct Push-Down Automata for the following languages. [30 marks]**

a. $L = \{w \in \{0,1\}^*, w = 0^i1^j0^k \text{ where } i + k = j \text{ and } i, j, k > 0\}$

b. $L = \{w \in \{0,1\}^*, w = 0^i1^j0^k \text{ where } i + k > j \text{ and } i, j, k > 0\}$

c. $L = \{w \in \{0,1\}^*, w = 0^i1^j0^k \text{ where } i + k < j \text{ and } i, j, k > 0\}$

d. $L = \{w \in \{0,1\}^*, ww^R\}$ (w^R denotes the reverse string of w)

e. $L = \{w\#x, \text{ where } x \in \{0,1\}^* \text{ and } w^R \text{ is a substring of } x\}$ (w^R denotes the reverse string of w)

f. $L = \{w \in \{0,1\}^*, w1^n \text{ where } n > 0 \text{ and } |w| = n\}$

5. Design the Turing Machines for the following languages. [20 marks]

- $L = \{w \in \{a, b\}, w \text{ is a palindrome}\}$
- Design a Turing Machine (TM) that, given an input string `w` over the alphabet `{a, b}`, writes the same string again after a delimiter symbol `#`. The TM should halt with the tape containing `w#w`, where the first occurrence of `w` is the input and the second is its copy.