

There are a total of five problems. You have to solve all of them.

Problem 1 (CO3): Designing Context-Free Grammars (10 points)

Let $\Sigma = \{0, 1\}$. Consider the following pair of languages. Recall that for a string w , $|w|$ denotes the length of w .

$$L_1 = \{w \in \Sigma^* : w \text{ contains exactly two 0s}\}$$

$$L_2 = \{x\#y : x \in L_1, y \in \Sigma^*, |x| = |y|\}$$

Now solve the following problems.

- (a) **Give** a context-free grammar for the language L_1 . (4 points)
- (b) **Write** down four seven-letter strings in L_2 . (1 point)
- (c) **Give** a context-free grammar for the language L_2 . (5 points)

Problem 2 (CO3): Derivations, Parse Trees and Ambiguity (10 points)

Take a look at the grammar below and solve the following problems.

$$S \rightarrow BA \mid Abb \mid SS$$

$$A \rightarrow a \mid \varepsilon$$

$$B \rightarrow b$$

- (a) **Give** a leftmost derivation for the string babbbba. (3 points)
- (b) **Sketch** the parse tree corresponding to the derivation you gave in (a). (2 points)
- (c) **Demonstrate** that the given grammar is ambiguous by showing two more parse tree (apart from the one you already found in (b)) for the same string. (4 points)
- (d) **Find** a string w of length five such that w has exactly one parse tree in the grammar above. (1 point)

Problem 3 (CO4): The CYK Algorithm (10 points)

Apply the CYK algorithm to determine whether the string yxxzy can be derived in the following grammar. You must show the entire CYK table. Here x, y and z are terminals, and the rest are variables.

$$S \rightarrow AB$$

$$A \rightarrow AC \mid y$$

$$B \rightarrow BD \mid EB \mid z$$

$$C \rightarrow AE \mid x$$

$$D \rightarrow y$$

$$E \rightarrow BD \mid x$$

Problem 4 (CO4): Chomsky Normal Form (10 points)

Answer the following questions.

- (a) **List** the productions that violate the conditions of the Chomsky Normal Form (CNF) in the following grammar. (4 points)

$$\begin{aligned} S &\rightarrow a \mid AB \\ A &\rightarrow SB \mid ba \mid B \\ B &\rightarrow b \mid \varepsilon \end{aligned}$$

- (b) **Write** down the additional rules that need to be added to the following grammar if the production $X \rightarrow \varepsilon$ is removed. (4 points)

$$\begin{aligned} S &\rightarrow aXbXcX \mid X \\ X &\rightarrow a \mid \varepsilon \end{aligned}$$

- (c) **Write** down the additional rules that need to be added to the following grammar if all the unit productions are removed. (2 points)

$$\begin{aligned} S &\rightarrow Aa \\ A &\rightarrow BC \mid C \\ B &\rightarrow b \\ C &\rightarrow A \mid AA \mid AaB \end{aligned}$$

Problem 5 (CO3): Constructing Pushdown Automata (10 points)

Let $\Sigma = \{a, b\}$. Note that we define $c(w, x)$ to be the count of x in the string w .

$$\begin{aligned} L_1 &= \{w \mid w \text{ is a palindrome and the length of } w \text{ is even.}\} \\ L_2 &= \{w_1\#w_2\#w_3 \mid c(w_1, b) = c(w_3, b) \text{ or } c(w_2, a) = c(w_3, a)\} \end{aligned}$$

Now solve the following problems.

- (a) **Give** the state diagram of a pushdown automaton that recognizes L_1 . (4 points)
(b) **Find** all strings $w \in L_2$ such that w starts with babbab#abba# and has a length of 15. (1 point)
(c) **Give** the state diagram of a pushdown automaton that recognizes L_2 . (5 points)

