CSE331 Section 07 & 08 Assignment 2 Solutions
Spring 2025

Question 1

a) L1 = {w ∈ {0, 1}*: 0^i1^j where i ≤ j}

Answer: Assume, for the sake of contradiction, L1 is regular. Then let p be the pumping length for L1. Now if we take the string: w = 0^p1^p ∈ L1 , then the length of w is |w| = 2p ≥ p. So w can be split into xyz such that

1. |xy| ≤ p ,
2. |y| > 0 and
3. xy^iz ∈ L1 for each i ≥ 0.

Assume, x = 0^α , y = 0^β and z = $0^{p-\alpha-\beta}1^p$

Since |xy| ≤ p, and the first p characters of w are all 0's, we can conclude that y consists of only 0's.

Then, for i = 2, $xy^2z = 0^\alpha.0^{2\beta}.0^{p-\alpha-\beta}1^p = 0^{p+\beta}1^p \notin L1$ since β can not be zero [|y| > 0].

We have excess 0's in xy^2z. Thus we get a contradiction! Hence, L1 is not a regular language.

b) L2 = {w ∈ {a, b, c}*: $a^ib^jc^{k+2}$ where i = k and i, j, k ≥ 0}

Answer: Assume, for the sake of contradiction, L2 is regular. Then let p be the pumping length for L2. Now if we take the string: w = a^pbc^{p+2} ∈ L2 , then the length of w is |w| = 2p+3 ≥ p. So w can be split into xyz such that

1. |xy| ≤ p ,
2. |y| > 0 and
3. xy^iz ∈ L2 for each i ≥ 0.

Assume, x = a^α , y = a^β and z = $a^{p-\alpha-\beta}bc^{p+2}$

Since |xy| ≤ p, and the first p characters of w are all a's, we can conclude that y consists of only a's.

Then, for i = 2, $xy^2z = a^\alpha.a^{2\beta}.a^{p-\alpha-\beta}bc^{p+2} = a^{p+\beta}bc^{p+2} \notin L2$ since β can not be zero [|y| > 0].

We have excess a's in xy^2z. Thus we get a contradiction! Hence, L2 is not a regular language.

c) L3 = {w1#w2 such that |w1| = 2.|w2| , where Σ = {0, 1} }

Answer: Assume, for the sake of contradiction, L3 is regular. Then let p be the pumping length for L3. Now if we take the string: w = $0^{2p}1^p$ ∈ L3 , then the length of w is |w| = 3p ≥ p. So w can be split into xyz such that

1. |xy| ≤ p ,
2. |y| > 0 and
3. xy^iz ∈ L3 for each i ≥ 0.

Assume, x = 0^α , y = 0^β and z = $0^{2p-\alpha-\beta}1^p$

Since |xy| ≤ p, and the first p characters of w are all 0's, we can conclude that y consists of only 0's.

Then, for i = 2, $xy^2z = 0^\alpha.0^{2\beta}.0^{2p-\alpha-\beta}1^p = 0^{2p+\beta}1^p \notin L3$ since β can not be zero [|y| > 0].

We have excess 0's in xy^2z. Thus we get a contradiction! Hence, L3 is not a regular language.

d) L4 = {w ∈ {a}*: a^{2^n} where n ≥ 0}

Answer: Assume, for the sake of contradiction, L4 is regular. Then let p be the pumping length for L4. Now if we take the string: w = a^{2^p} ∈ L4 , then the length of w is $|w| = 2^p \geq p$. So w can be split into xyz such that

1. |xy| ≤ p ,
2. |y| > 0 and
3. xy^iz ∈ L4 for each i ≥ 0.

Assume, $x = a^{\alpha}$, $y = a^{\beta}$ and $z = a^{2^p-\alpha-\beta}$

Then, for i = 2, $xy^2z = a^{\alpha}.a^{2\beta}.a^{2^p-\alpha-\beta} = a^{2^p+\beta}$.

If xy^2z ∈ L4, then $2^p + \beta$ should be a power of 2 (obviously $2^p + \beta > p$). As β can not be zero, the immediate next power of 2^p is 2^{p+1}.

So, $2^p + \beta > 2^{p+1}$

$\Rightarrow 2^p + \beta > 2.2^p$

$\Rightarrow \beta > 2^p$

But from condition 1,

$|xy| \leq p$

$\Rightarrow |y| \leq p$

$\Rightarrow \beta \leq p$

Thus we get a contradiction! Hence, L4 is not a regular language.

Question 2

A. S → 1P
P → 0P | 1P | ε
<u>Or</u>, S → S0 | S1 | 1

B. S → 0S0 | 1S1 | ε

C. S → P001P
P → 0P | 1P | ε

D. S → 0S0 | 1

E. S → PS1 | ε
P → 0 | ε
<u>Or</u>, S → 0S1 | Z
Z → Z1 | ε

F. S → 1S1 | 0P
P → 1P1 | 0

G. S → 000S11 | X0X0X
X → 0X | 1X | ε
<u>Or</u>, S → 000S11 | A0A0X
A → 1A | ε
X → 0X | 1X | ε

H. S → R1P
R → 000R | ε
P → 0P | ε

I. $0^i10^j = 0^i10^{2+3i} = 0^i10^20^{3i} = 0^i\underline{\mathbf{100}}0^{3i}$
S → 0S000 | 100

<u>Or</u>, $0^i10^j = 0^i10^{2+3i} = 0^i10^{3i}0^2 = 0^i\underline{\mathbf{1}}0^{3i}\underline{\mathbf{00}}$

S → P00

P → 0P000 | 1

J. S → P0R | Q

R → 1111R | ε

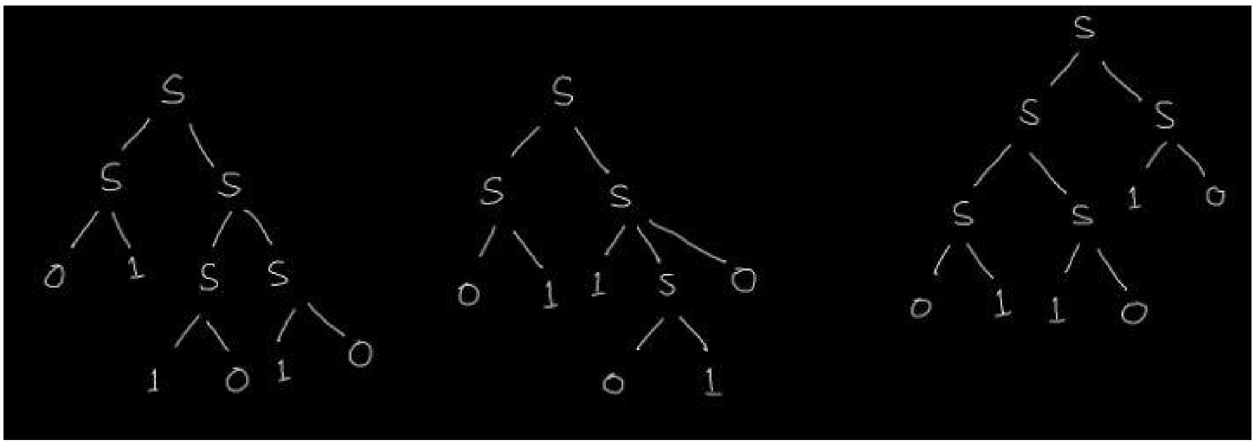
P → 1P | ε

Q → 11Q1 | 1110

Question 3

A.

a) There are three possible parse trees to derive the given string. Drawing any two of the following will be correct.



b) Correct answers include 000111, 001110, 011100, 111000, 110001, 100011. Finding any two of these will be okay. Everything else is incorrect.

B.

a) & b)

There are three different derivations of abababb.

P (odd length palindrome)	E (even length substring)
a	bababb
aba	babb
ababa	bb

$$S \Rightarrow PE \Rightarrow \mathtt{a}E \Rightarrow \mathtt{aba}E \Rightarrow \mathtt{ababa}E \Rightarrow \mathtt{abababb}E \Rightarrow \mathtt{abababb}$$

$$S \Rightarrow PE \Rightarrow \mathtt{a}P\mathtt{a}E \Rightarrow \mathtt{aba}E \Rightarrow \mathtt{ababa}E \Rightarrow \mathtt{abababb}E \Rightarrow \mathtt{abababb}$$

$$S \Rightarrow PE \Rightarrow \mathtt{a}P\mathtt{a}E \Rightarrow \mathtt{ab}P\mathtt{ba}E \Rightarrow \mathtt{ababa}E \Rightarrow \mathtt{abababb}E \Rightarrow \mathtt{abababb}$$

These are the three corresponding Parse Trees.