

Practice Questions

- (a) $L_0 = \{w : w \text{ ends with } 00\}$
- (b) $L_1 = \{w : w \text{ starts with } 11 \text{ and the substring } 11 \text{ appears only once in the string}\}$
- (c) $L_2 = \{w : w \text{ has odd number of } 0\text{'s} \text{ and ends with } 1\}$
- (d) $L_3 = \{w : w \text{ is a binary string that's decimal form is divisible by } 3\}$
- (e) $L_4 = \{w : w \text{ contains exactly two } 1\text{'s}\}$
- (f) $L_5 = \{w : w \text{ contains at least two } 0\text{'s} \text{ and at most one } 1\}$
- (g) $L_6 = \{w : \text{the second letter of } w \text{ is a } 1 \text{ and the second-to-last letter is a } 0\}$
- (h) $L_7 = \{w : \text{there exists a pair of } 1\text{'s} \text{ in } w \text{ that are separated by an even number of } 0\text{'s}\}$
- (i) $L_8 = \{w : \text{the total number of } 0\text{'s} \text{ in } w \text{ is divisible by } 3\}$
- (j) $L_0 \cap L_8$
- (k) $L_4 \cap \overline{L_5}$
- (l) $L_0 \cup L_2$
- (m) $\overline{L_0 \cup L_4}$

From Previous Mid Questions

$$\Sigma = \{0, 1, \#\}$$

$L_1 = \{w \in \Sigma^* : w \text{ does not contain } \# \text{ and the number of } 0\text{'s} \text{ in } w \text{ is not a multiple of } 3\}$

$L_2 = \{w \in \Sigma^* : \text{the substring between any two successive occurrences of } \# \text{ in } w \text{ is in } L_1\}$

- (a) If you use the “cross product” construction shown in class to obtain a DFA for $L_1 \cap L_2$, how many states will it have?

$$L_1 = \{w \in \{0, 1\}^* : w = 0^m 1^n, \text{ where } m, n \geq 0\}$$

$$L_2 = \{w \in \{0, 1\}^* : 1 \text{ does not appear at any even position in } w\}$$

- (b) If you were to use the “cross product” construction shown in class to obtain a DFA for the language $L_1 \cap L_2$, how many states would it have?

$$L_1 = \{w \in \{0, 1\}^* : w \text{ starts and ends with different letters}\}$$

$$L_2 = \{w \in \{0, 1\}^* : \text{the length of } w \text{ is at least two}\}$$

- (c) If you were to use the “cross product” construction shown in class to obtain a DFA for the language $L_1 \cap L_2$, how many states would it have?

From Practice Sheet of BUX

1. Draw a DFA for the set of binary strings that are divisible by 8 while considered as binary numbers. $\Sigma = \{0,1\}$
2. Draw a DFA for the set of strings that end with **abb**. $\Sigma = \{a, b, c\}$
3. Draw a DFA for the set of binary strings that have an even number of **0**'s or an odd number of **1**'s. $\Sigma = \{0,1\}$
4. Draw a DFA for the set of strings that have **011** as a substring and **001** as not a substring. $\Sigma = \{0,1\}$
5. Draw a DFA for the set of strings that have a length of at least **4**. $\Sigma = \{a, b\}$
6. Draw a DFA for the set of binary strings that contain at least three **1**'s. $\Sigma = \{0,1\}$
7. Draw a DFA for the set of strings that have exactly three **a**'s. $\Sigma = \{a, b, c\}$
8. Draw a DFA for the set of strings that have lengths of not more than **6**. $\Sigma = \{0,1\}$
9. Draw a DFA for the set of strings that have exactly three **1**'s and four **0**'s. $\Sigma = \{0,1,2\}$
10. Draw a DFA for the set of strings that have **three** consecutive **1**'s. $\Sigma = \{0,1\}$

From Micheal Sipsers books Exercise :

■ Introduction to the Theory of Computation, Third Edition -- Michael Sipser -- Third edition, ...

Page 83-84

- 1.4** Each of the following languages is the intersection of two simpler languages. In each part, construct DFAs for the simpler languages, then combine them using the construction discussed in footnote 3 (page 46) to give the state diagram of a DFA for the language given. In all parts, $\Sigma = \{a, b\}$.
- a. $\{w \mid w \text{ has at least three a's and at least two b's}\}$
 - ^Ab. $\{w \mid w \text{ has exactly two a's and at least two b's}\}$
 - c. $\{w \mid w \text{ has an even number of a's and one or two b's}\}$
 - ^Ad. $\{w \mid w \text{ has an even number of a's and each a is followed by at least one b}\}$
 - e. $\{w \mid w \text{ starts with an a and has at most one b}\}$
 - f. $\{w \mid w \text{ has an odd number of a's and ends with a b}\}$
 - g. $\{w \mid w \text{ has even length and an odd number of a's}\}$

1.5 Each of the following languages is the complement of a simpler language. In each part, construct a DFA for the simpler language, then use it to give the state diagram of a DFA for the language given. In all parts, $\Sigma = \{a, b\}$.

- ^A**a.** $\{w \mid w \text{ does not contain the substring } ab\}$
- ^A**b.** $\{w \mid w \text{ does not contain the substring } baba\}$
- c.** $\{w \mid w \text{ contains neither the substrings } ab \text{ nor } ba\}$
- d.** $\{w \mid w \text{ is any string not in } a^*b^*\}$
- e.** $\{w \mid w \text{ is any string not in } (ab^+)^*\}$
- f.** $\{w \mid w \text{ is any string not in } a^* \cup b^*\}$
- g.** $\{w \mid w \text{ is any string that doesn't contain exactly two } a\text{'s}\}$
- h.** $\{w \mid w \text{ is any string except } a \text{ and } b\}$

1.6 Give state diagrams of DFAs recognizing the following languages. In all parts, the alphabet is $\{0,1\}$.

- a.** $\{w \mid w \text{ begins with a } 1 \text{ and ends with a } 0\}$
- b.** $\{w \mid w \text{ contains at least three } 1\text{'s}\}$
- c.** $\{w \mid w \text{ contains the substring } 0101 \text{ (i.e., } w = x0101y \text{ for some } x \text{ and } y)\}$
- d.** $\{w \mid w \text{ has length at least } 3 \text{ and its third symbol is a } 0\}$
- e.** $\{w \mid w \text{ starts with } 0 \text{ and has odd length, or starts with } 1 \text{ and has even length}\}$
- f.** $\{w \mid w \text{ doesn't contain the substring } 110\}$
- g.** $\{w \mid \text{the length of } w \text{ is at most } 5\}$
- h.** $\{w \mid w \text{ is any string except } 11 \text{ and } 111\}$
- i.** $\{w \mid \text{every odd position of } w \text{ is a } 1\}$
- j.** $\{w \mid w \text{ contains at least two } 0\text{'s and at most one } 1\}$
- k.** $\{\epsilon, 0\}$
- l.** $\{w \mid w \text{ contains an even number of } 0\text{'s, or contains exactly two } 1\text{'s}\}$
- m.** The empty set
- n.** All strings except the empty string