

There are a total of five problems. You have to solve all of them.

Set A CFG

Problem 1 (CO3): Designing Context-Free Grammars (10 points)

- (a) **Convert** the given regular expression into a context-free grammar. (3 points)

$$a^*(ba + bb)^* + \#$$

Let $\Sigma = \{a, b, \#\}$. Consider the following languages over Σ . Recall that for a string w , $|w|$ denotes the length of w .

$$L_1 = \{a^{3i}, \text{ where } i \geq 0\}$$

$$L_2 = \{b^{2j}, \text{ where } j \geq 0\}$$

$$L_3 = \{x\#y, \text{ where } x \in L_1, y \in L_2 \text{ and } |x| = |y|\}$$

Now solve the following problems.

- (b) Your Friend writes the following context-free grammars for the language L_1 . **Choose** the correct grammar for L_1 . There could be more than one correct grammar. (3 points)

(i) $S \rightarrow ABS \mid \varepsilon$

$$A \rightarrow a \mid \varepsilon$$

$$B \rightarrow aa \mid \varepsilon$$

(ii) $S \rightarrow aaaS \mid \varepsilon$

(iii) $S \rightarrow AAAS \mid \varepsilon$

$$A \rightarrow aA \mid \varepsilon$$

(iv) $S \rightarrow aP \mid aQ \mid \varepsilon$

$$P \rightarrow aR$$

$$R \rightarrow aS$$

$$Q \rightarrow aR$$

- (c) **Write** down a string of any length in L_3 . (1 point)
- (d) **Give** a context-free grammar for the language L_3 . (3 points)

Set B CFG

Problem 1 (CO3): Designing Context-Free Grammars (10 points)

- (a) **Convert** the given regular expression into a context-free grammar. (3 points)

$$a(ba)^* + \#a^*b$$

Let $\Sigma = \{a, b, \#\}$. Consider the following languages over Σ . Recall that for a string w , $|w|$ denotes the length of w .

$$L_1 = \{a^{4i}, \text{ where } i \geq 0\}$$

$$L_2 = \{b^{2j}, \text{ where } j \geq 0\}$$

$$L_3 = \{x\#y, \text{ where } x \in L_1, y \in L_2 \text{ and } |x| = |y|\}$$

Now solve the following problems.

- (b) Your Friend writes the following context-free grammars for the language L_1 . **Choose** the correct grammar for L_1 . There could be more than one correct grammar. (3 points)

(i) $S \rightarrow ASA \mid \epsilon$

$$A \rightarrow aa$$

(ii) $S \rightarrow AAS \mid \epsilon$

$$A \rightarrow aa \mid \epsilon$$

(iii) $S \rightarrow AAAAS \mid \epsilon$

$$A \rightarrow a$$

(iv) $S \rightarrow aP \mid aQ \mid \epsilon$

$$P \rightarrow aR$$

$$R \rightarrow aQ$$

$$Q \rightarrow aS$$

- (c) **Write** down a string of any length in L_3 . (1 point)
- (d) **Give** a context-free grammar for the language L_3 . (3 points)

Set C CFG

Problem 1 (CO3): Designing Context-Free Grammars (10 points)

- (a) **Convert** the given regular expression into a context-free grammar. (3 points)

$$(ab^* + \#a)^* + b$$

Let $\Sigma = \{a, b, \#\}$. Consider the following languages over Σ . Recall that for a string w , $|w|$ denotes the length of w .

$$L_1 = \{b^{5i}, \text{ where } i \geq 0\}$$

$$L_2 = \{a^j, \text{ where } j \geq 0\}$$

$$L_3 = \{x\#y, \text{ where } x \in L_1, y \in L_2 \text{ and } |x| = |y|\}$$

Now solve the following problems.

- (b) Your Friend writes the following context-free grammars for the language L_1 . **Choose** the correct grammar for L_1 . There could be more than one correct grammar. (3 points)

(i) $S \rightarrow ASA \mid b$

$$A \rightarrow bb$$

(ii) $S \rightarrow AAAAAAS \mid \epsilon$

$$A \rightarrow bA \mid b$$

(iii) $S \rightarrow AASB \mid \epsilon$

$$A \rightarrow bb$$

$$B \rightarrow b$$

(iv) $S \rightarrow bP \mid bbbbQ \mid \epsilon$

$$P \rightarrow bbR$$

$$R \rightarrow bQ$$

$$Q \rightarrow bS$$

- (c) **Write** down a string of any length in L_3 . (1 point)
- (d) **Give** a context-free grammar for the language L_3 . (3 points)

Set A

Problem 2 (CO3): Derivations, Parse Trees and Ambiguity (10 points)

Take a look at the grammar below and solve the following problems.

$$S \rightarrow SS \mid 0S1 \mid 1S0$$

$$S \rightarrow \epsilon$$

- (a) **Give** a leftmost derivation for the string 011010. (3 points)
- (b) **Sketch** the parse tree corresponding to the derivation you gave in (a). (2 points)
- (c) **Demonstrate** that the given grammar is ambiguous by showing three more parse trees (apart from the one you already found in (b)) for the same string. (3 points)
- (d) **Find** a string w of length six such that w has exactly one parse tree in the grammar above. (1 point)
- (e) **Design** an unambiguous Context Free Grammar for the language represented by the given ambiguous grammar. (1 point)

Set B

Problem 2 (CO3): Derivations, Parse Trees and Ambiguity (10 points)

Take a look at the grammar below and solve the following problems.

$$A \rightarrow 0A1 \mid AA \mid 1A0$$

$$A \rightarrow \epsilon$$

- Give a leftmost derivation for the string 100101. (3 points)
- Sketch the parse tree corresponding to the derivation you gave in (a). (2 points)
- Demonstrate that the given grammar is ambiguous by showing three more parse trees (apart from the one you already found in (b)) for the same string. (3 points)
- Find a string w of length six such that w has exactly one parse tree in the grammar above. (1 point)
- Design an unambiguous Context Free Grammar for the language represented by the given ambiguous grammar. (1 point)

set A for CYK

Problem 3 (CO4): The CYK Algorithm (10 points)

You have recently learned the CYK algorithm. Your friend filled up the first row of the CYK table and asked you to complete the rest of the table for the string cabbcc.

1,5					
1,4	2,5				
1,3	2,4	3,5			
1,2	2,3	3,4	4,5		
1,1	2,2	3,3	4,4	5,5	
{C}	{B,C}	{S,A}	{A,B}	{A}	
c	a	b	b	c	

Apply the CYK algorithm to determine whether the string cabbcc can be derived in the following grammar. You must show the entire CYK table. Here a, b and c are terminals, and the rest are variables.

$$S \rightarrow AB \mid BC \mid a$$

$$A \rightarrow BC \mid c \mid a$$

$$B \rightarrow AB \mid CB \mid b$$

$$C \rightarrow BA \mid c$$

set B for CYK

Problem 3 (CO4): The CYK Algorithm (10 points)

You have recently learned the CYK algorithm. Your friend filled up the first row of the CYK table and asked you to complete the rest of the table for the string cbcab.

1,5				
1,4	2,5			
1,3	2,4	3,5		
1,2	2,3	3,4	4,5	
1,1	2,2	3,3	4,4	5,5
{A}	{S,B}	{B,C}	{C}	{B}
c	b	c	a	b

Apply the CYK algorithm to determine whether the string cbcab can be derived in the following grammar. You must show the entire CYK table. Here a, b and c are terminals, and the rest are variables.

$S \rightarrow AB \mid BC \mid a$
 $A \rightarrow BC \mid c \mid a$
 $B \rightarrow AB \mid CB \mid b$
 $C \rightarrow BA \mid c$

set C for CYK

Problem 3 (CO4): The CYK Algorithm (10 points)

You have recently learned the CYK algorithm. Your friend filled up the first row of the CYK table and asked you to complete the rest of the table for the string *abcbc*.

1,5				
1,4	2,5			
1,3	2,4	3,5		
1,2	2,3	3,4	4,5	
1,1	2,2	3,3	4,4	5,5
{A}	{S, A}	{C, B}	{B}	{A, B}
<i>a</i>	<i>b</i>	<i>c</i>	<i>b</i>	<i>c</i>

Apply the CYK algorithm to determine whether the string *abcbc* can be derived in the following grammar. You must show the entire CYK table. Here *a*, *b* and *c* are terminals, and the rest are variables.

$S \rightarrow AB \mid BC \mid a$
 $A \rightarrow BC \mid c \mid a$
 $B \rightarrow AB \mid CB \mid b$
 $C \rightarrow BA \mid c$

Set A for CNF

Problem 4 (CO4): Chomsky Normal Form (10 points)

Answer the following questions.

- (a) **List** the productions that violate the conditions of the Chomsky Normal Form (CNF) in the following grammar. (4 points)

$$\begin{aligned} S &\rightarrow AB \mid \varepsilon \\ A &\rightarrow AS \mid a \mid \varepsilon \\ B &\rightarrow Bb \mid AaB \mid b \end{aligned}$$

- (b) **Write** down the additional rules that need to be added to the following grammar if the production $X \rightarrow \varepsilon$ is removed. (4 points)

$$\begin{aligned} S &\rightarrow XaXbX \mid X \\ X &\rightarrow aAX \mid \varepsilon \\ A &\rightarrow AA \mid a \end{aligned}$$

- (c) **Write** down the additional rules that need to be added to the following grammar if all the unit productions are removed. (2 points)

$$\begin{aligned} S &\rightarrow AB \mid CA \\ A &\rightarrow C \mid BA \mid a \\ B &\rightarrow Aba \mid Bbb \mid Ca \\ C &\rightarrow AB \mid AC \mid A \end{aligned}$$

Set B for CNF

Problem 4 (CO4): Chomsky Normal Form (10 points)

Answer the following questions.

- (a) **List** the productions that violate the conditions of the Chomsky Normal Form (CNF) in the following grammar. (4 points)

$$\begin{aligned}S_0 &\rightarrow S \mid \varepsilon \\S &\rightarrow AB \\A &\rightarrow AS \mid aAb \\B &\rightarrow BS_0 \mid \varepsilon\end{aligned}$$

- (b) **Write** down the additional rules that need to be added to the following grammar if the production $A \rightarrow \varepsilon$ is removed. (4 points)

$$\begin{aligned}S &\rightarrow aABA \mid A \\A &\rightarrow ABAB \mid \varepsilon \\B &\rightarrow BAb \mid bBABb\end{aligned}$$

- (c) **Write** down the additional rules that need to be added to the following grammar if all the unit productions are removed. (2 points)

$$\begin{aligned}S &\rightarrow Xa \mid Yb \\X &\rightarrow Y \mid ZZ \mid aX \\Y &\rightarrow XY \mid Zb \mid X \\Z &\rightarrow ZaZ \mid ZX \mid ZXX\end{aligned}$$

Set C for CNF

Problem 4 (CO4): Chomsky Normal Form (10 points)

Answer the following questions.

- (a) List the productions that violate the conditions of the Chomsky Normal Form (CNF) in the following grammar. (4 points)

$$\begin{aligned} S &\rightarrow AA \mid AS \mid \varepsilon \\ A &\rightarrow BcD \mid BD \\ B &\rightarrow CD \mid \varepsilon \\ C &\rightarrow AD \mid B \\ D &\rightarrow AB \end{aligned}$$

- (b) Write down the additional rules that need to be added to the following grammar if the production $Y \rightarrow \varepsilon$ is removed. (4 points)

$$\begin{aligned} S &\rightarrow XYaYcYX \mid Y \\ Y &\rightarrow X \mid \varepsilon \\ X &\rightarrow XX \mid d \end{aligned}$$

- (c) Write down the additional rules that need to be added to the following grammar if all the unit productions are removed. (2 points)

$$\begin{aligned} S &\rightarrow PQ \mid aaQQ \\ P &\rightarrow RS \mid aQ \mid R \\ Q &\rightarrow Qb \mid SR \mid aP \\ R &\rightarrow QS \mid aP \mid P \end{aligned}$$

Set A for PDA

Problem 5 (CO3): Constructing Pushdown Automata (10 points)

Let $\Sigma = \{0, 1\}$. Consider the following language.

$$L_1 = \{\text{Length of } w \text{ is even}\}$$

$$L_2 = \{0^{2^n}(10)^n, \text{ where } n \geq 0\}$$

- (a) Give the state diagram of a pushdown automaton that recognizes L_1 . (4 points)
(b) Give the state diagram of a pushdown automaton that recognizes L_2 . (6 points)

Set B for PDA

Problem 5 (CO3): Constructing Pushdown Automata (10 points)

Let $\Sigma = \{0, 1\}$. Consider the following language.

$$L_1 = \{w \text{ contains at least two } 1\}$$

$$L_2 = \{(10)^n 1^{2^n}, \text{ where } n \geq 0\}$$

- (a) Give the state diagram of a pushdown automaton that recognizes L_1 . (4 points)
(b) Give the state diagram of a pushdown automaton that recognizes L_2 . (6 points)

Set C for PDA

Problem 5 (CO3): Constructing Pushdown Automata (10 points)

Let $\Sigma = \{0, 1\}$. Consider the following language.

$$L_1 = \{w \text{ contains } 01 \text{ as a substring}\}$$

$$L_2 = \{w = (01)^n 0^{3n}, \text{ where } n \geq 0\}$$

- (a) **Give** the state diagram of a pushdown automaton that recognizes L_1 . (4 points)
- (b) **Give** the state diagram of a pushdown automaton that recognizes L_2 . (6 points)

