FINAL EXAM TOTAL MARKS: 50 DURATION: 105 MINUTES

B

There are a total of six problems. You have to solve all of them.

Problem 1 (CO5): Pumping Lemma (5 points)

Let $\Sigma = \{0, 1\}$. Consider the following language.

$$L = \{w \in \Sigma^* : w = 1^a 0^b 0^c 1^d, \text{ where } a + d = b + c \text{ and } a, b, c, d \ge 0\}$$

Use the pumping lemma to **demonstrate** that *L* is not regular language.

Problem 2 (CO3): Designing Context-Free Grammars (10 points)

Let $\Sigma = \{0, 1\}$. Consider the following languages. Recall that for a string w, |w| denotes the length of w.

$$L_1 = \{w \in \Sigma^* : w \text{ is a odd length palindrome}\}$$

$$L_2 = \{w \in \Sigma^* : \text{length of } w \text{ is multiple of two}\}$$

$$L_3 = \{x000y : x, y \in L_2, |x| = |y|\}$$

$$L_4 = L_1 \cap L_3$$

Now solve the following problems.

- (a) **Give** a context-free grammar for the language L_1 . (3 points)
- (b) Give a context-free grammar for the language L₃. (4 points)
- (c) Give a context-free grammar for the language L_4 .(3 point)

Problem 3 (CO4): The CYK Algorithm (5 points)

Apply the CYK algorithm to fill up the table for the string aabaa using the following grammar. Here a and b are terminals, and the rest are variables.

$$S \rightarrow BA \mid BC$$

 $A \rightarrow AB \mid AC \mid a$
 $B \rightarrow CB \mid CC \mid b$
 $C \rightarrow CA \mid a$

1,5 { ? }				
1,4	25 {S, A}			
1,3	2,4 {S, A}	3,5 {?}		
$\{A,B,C\}$	2,3 {?}	3,4 {S}	4,5 {A,B,C}	
1,1 {A,C}	2,2 {A,C}	3,3 {B}	4,4 {A,C}	5,5 {A,C}

а

а

b

a

a

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Problem 4 (CO3): Constructing Pushdown Automata (10 points)

Let $\Sigma = \{0, 1\}$. Consider the following languages.

 $L_1 = \{w \mid w \text{ starts and ends with the different character}\}$

 $L_2 = \{w \mid \text{the number of 0s in } w \text{ is not same as the number of 1s} \}$

- (a) Give the state diagram of a pushdown automaton that recognizes L_1 . (4 points)
- (b) Give the state diagram of a pushdown automaton that recognizes L_2 . (6 points)

Problem 5 (CO3): Derivations, Parse Trees and Ambiguity (10 points)

Take a look at the grammar below and solve the following problems.

$$\begin{split} A &\to 1A \mid 0B \mid 00A \\ B &\to 1B \mid 1C \mid 0A \mid 00B \\ C &\to 0C0 \mid 0C1 \mid 1C0 \mid 1C1 \mid 0 \mid 1 \end{split}$$

- (a) Give a leftmost derivation for the string 10110010. (3 points)
- (b) Sketch the parse tree corresponding to the derivation you gave in (a). (2 points)
- (c) **Demonstrate** that the given grammar is ambiguous by showing two more parse trees (apart from the one you already found in (b)) for the same string. (3 points)
- (d) Find a string w of length six such that w has exactly one parse tree in the grammar above. (1 point)
- (e) **Desgin** an unambiguous Context Free Grammar for the language represented by the given ambiguous grammar. (1 point)

Problem 6 (CO4): Chomsky Normal Form (10 points)

Answer the following questions.

(a) List the productions that violate the conditions of the Chomsky Normal Form (CNF) in the following grammar. (5 points)

$$\begin{split} P &\rightarrow \mathtt{a} \mid PR \mid \varepsilon \\ Q &\rightarrow \mathtt{b}P \mid R \\ R &\rightarrow QQ \mid \mathtt{a}T \\ T &\rightarrow QT \mid \varepsilon \end{split}$$

(b) Write down the additional rules that need to be added to the following grammar if the production $A \to \varepsilon$ is removed. (3 points)

$$S \rightarrow AA \mid bB$$

 $A \rightarrow CB \mid aA \mid \varepsilon$
 $B \rightarrow ABA \mid b \mid \varepsilon$

(c) Write down the additional rules that need to be added to the following grammar if all the unit productions are removed. (2 points)

$$A \rightarrow 0A1B1 \mid C$$

 $B \rightarrow 1B \mid B$
 $C \rightarrow BC \mid A \mid B$