

DFA To RE

State elimination method

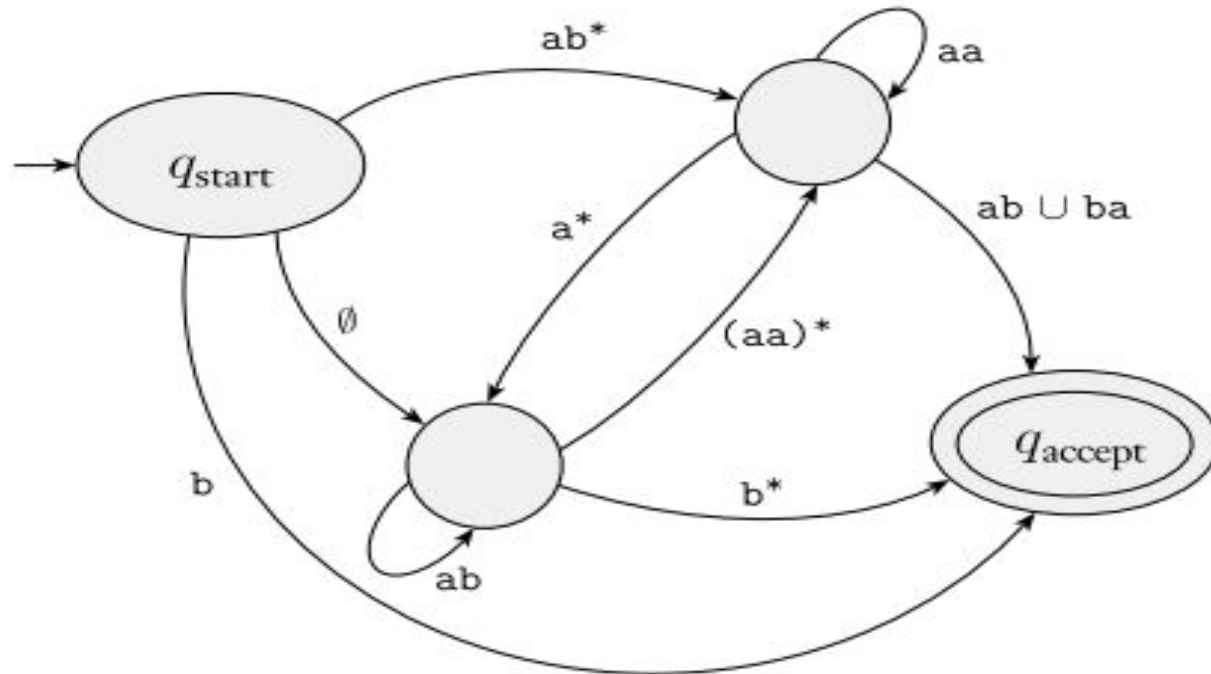
Procedure - State Elimination Method

- **Step 1 – DFA to New finite automaton GNFA**
 - **Add a new start state with epsilon transition from old start state.**
 - **Add a new final state with epsilon transition from old one.**
 - **GNFA - same as NFA**
 - **It have regular expression on transitions**
 - **Connect two states by reading a block of string**
- **Step 2 - GNFA to RE**

Rules of GNFA

- **Start state has outgoing arrow but no arrow coming**
- **Single final state ; no outgoing arrow only incoming arrows.**
- **Every state (except for start and accept) have one arrow goes from every state to every other state, and also a self loop to each state.**

Example of GNFA

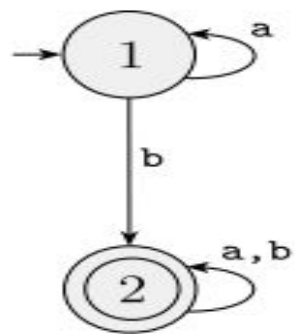


Basics

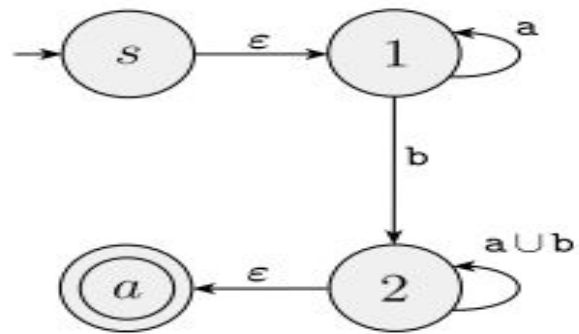
- Self- loop does not count as Inward or Outward edge
- If multiple final states are present, connect them all to the same Final state with epsilon transition
- Omit dead states at the beginning
- Sequence of elimination does not matter

Process

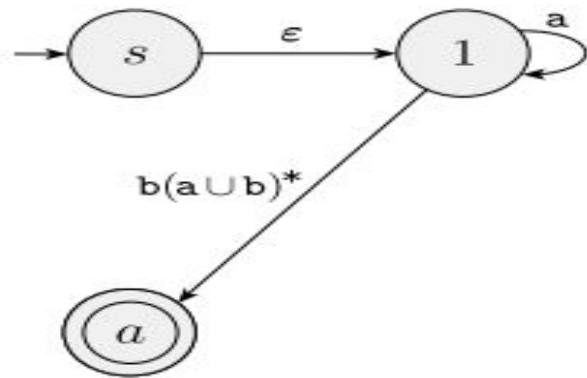
- Need to eliminate all states that are not start or final
- While elimination, we need to preserve EVERY transition
- Map every inward edge of a eliminating state to every outward edge



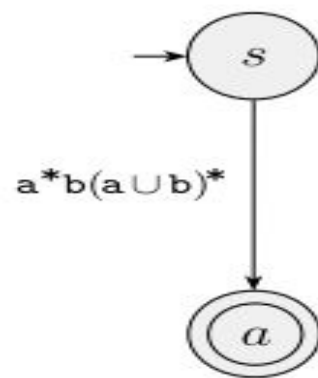
(a)



(b)

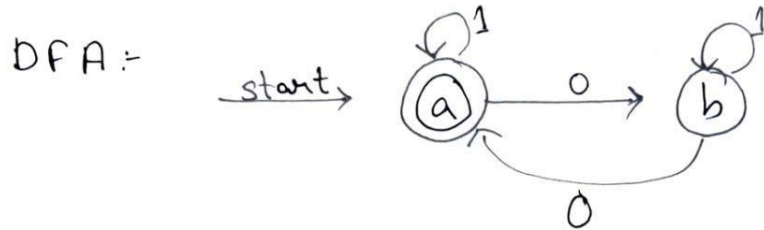


(c)



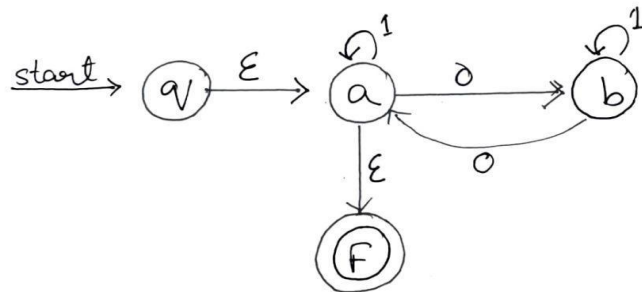
(d)

DFA that accepts the language of strings containing even number of 0s over the alphabet $\Sigma = \{0, 1\}$.

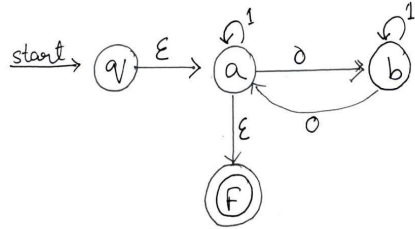


Step 1:- 1. Create new start state as 'a' has an inward edge.

2. Create new final state as 'a' has an outward edge.

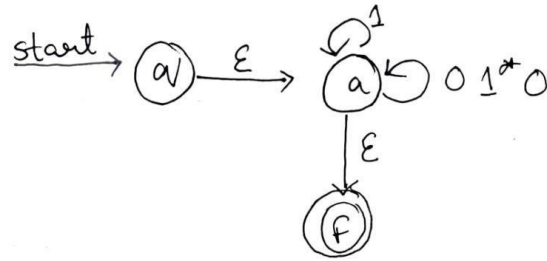


- Step 1:-
1. Create new start state as 'a' has an inward edge.
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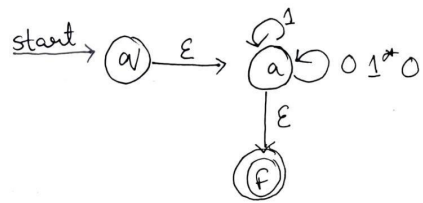
- Step 2:- Eliminate 'b' by mapping all of its inward edges to all of its outward edges.

$$\therefore a \xrightarrow{0} b \xrightarrow{1^* 0} a = 01^*0$$

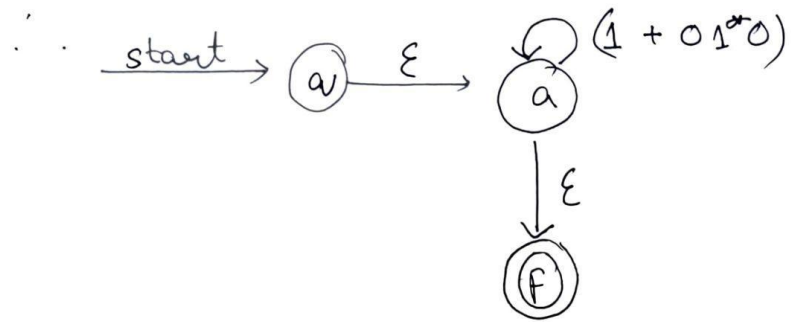


Step 2: Eliminate 'b' by mapping all of its inward edges to all of its outward edges.

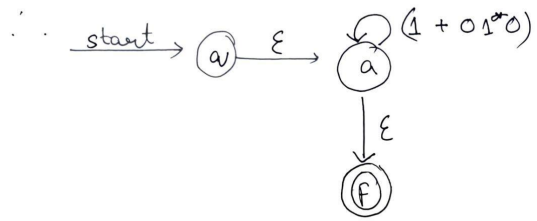
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'a' already had a self-loop. Expressions sharing the same edge/transition are connected by union.



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Step 3 \vdash Eliminate 'a' by mapping all of its inward edges to all of its outward edges.

$$\therefore a \xrightarrow{\epsilon} a \xrightarrow{(1+01^*0)^*} \epsilon \rightarrow F = (1+01^*0)^*$$

$$\therefore \xrightarrow{\text{start}} a \xrightarrow{(1+01^*0)^*} F$$

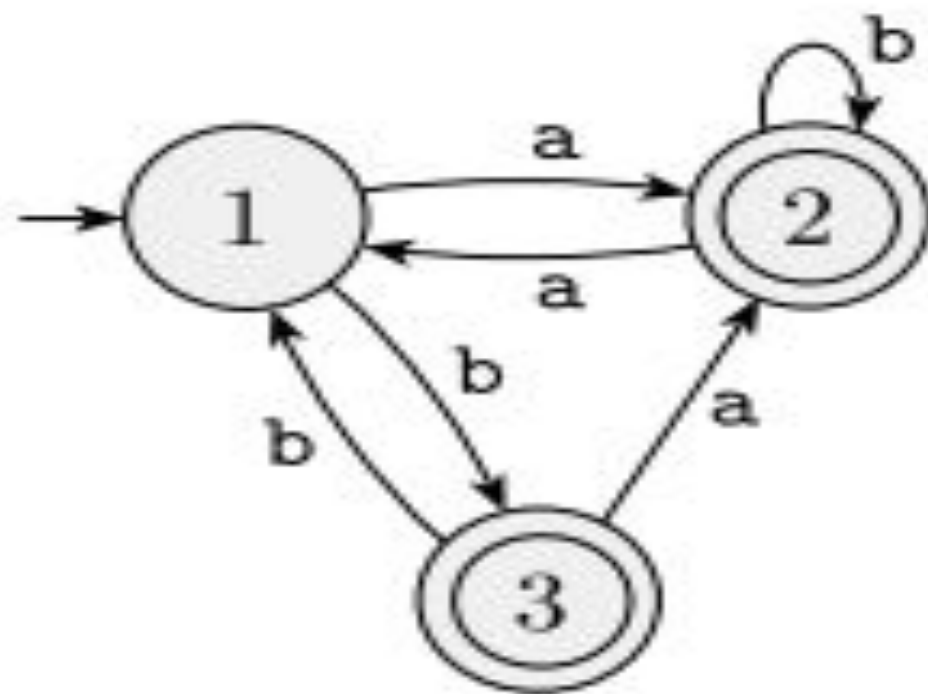
Step 3 + Eliminate 'a' by mapping all of its inward edges to all of its outward edges.

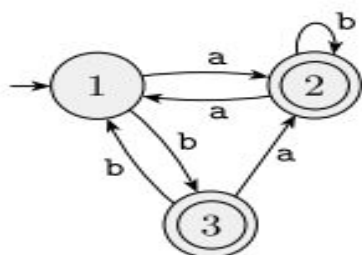
$$\therefore q \xrightarrow{\epsilon} a \xrightarrow{(1+01^*0)^*} \epsilon \rightarrow F \rightarrow (1+01^*0)^*$$

$$\therefore \text{start} \rightarrow (q) \xrightarrow{(1+01^*0)^*} (F)$$

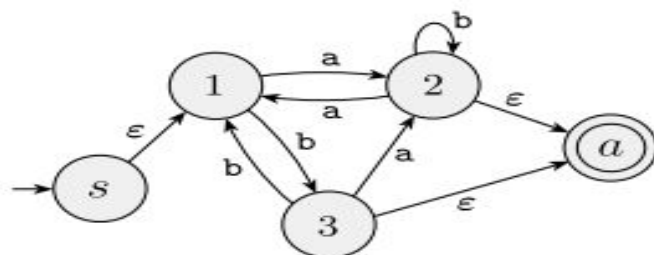
Step 4 + No other step to eliminate as only start and final states are left. \therefore RE

$$= (1+01^*0)^*$$

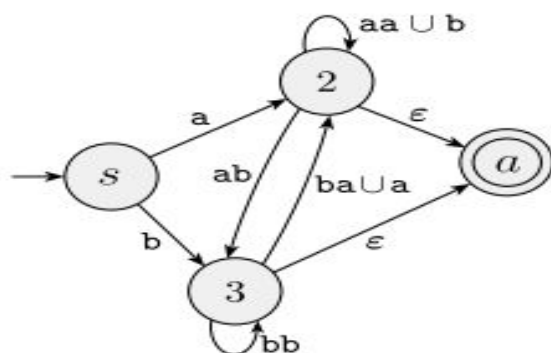




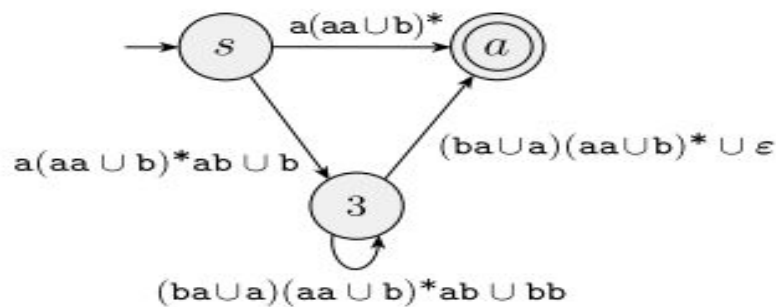
(a)



(b)



(c)



(d)



$$(a(aa \cup b)^*ab \cup b)((ba \cup a)(aa \cup b)^*ab \cup bb)^*((ba \cup a)(aa \cup b)^* \cup \epsilon) \cup a(aa \cup b)^*$$