CSE331
Automata and Computability

MIDTERM EXAM
TOTAL MARKS: 50
DURATION: 85 MINUTES

There are a total of five problems. You have to solve the first four. Problem 5 is optional.

DFA Set A

Problem 1 (CO1): DFA and Regular Languages (15 points)

We define the last two digits of your Student ID to be AB [e.g: If your Student ID is 2102895, then A = 9, B = 5]

Given, $\Sigma = \{\texttt{A}, \texttt{B}, \texttt{\#}\}$. Consider the following languages over Σ.

$$L_1 = \{w : w \text{ starts with } \texttt{A}\}$$
$$L_2 = \{w : w \text{ contains } \texttt{AB\#} \text{ as a substring}\}$$
$$L_3 = L_1 \circ L_2$$

Now solve the following problems. For questions (a)-(f), you must use your specific Σ to answer.

(a) If $\Sigma = \{\texttt{A}, \texttt{B}, \texttt{\#}\}$, then **define** Σ according to your Student ID. (1 point)

(b) **Give** the state diagram for a DFA that recognizes L_1. (3 points)

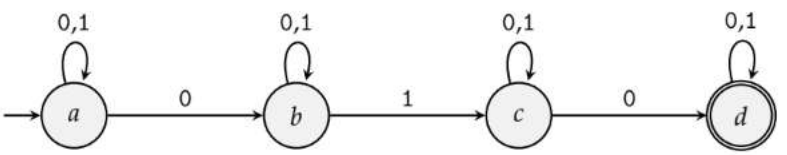
(c) **Give** the state diagram for a DFA that recognizes L_2. (3 points)

(d) **Find** all the four-letter strings in $L_1 \cap L_2$. (2 points)

(e) If you were to use the "cross product" construction shown in class to obtain a DFA for the language $L_1 \cap L_2$, how many states would it have? (1 point)

(f) **Prove** L_3 is a regular language by giving the state diagram for a DFA or an NFA that recognizes L_3. (2 points)

Now, let $\Sigma = \{0, 1\}$. Consider the following diagram of the NFA to answer the questions (g)-(h) defined for Σ.



(g) **Choose** the language recognized by this NFA? (1 point)

(i) $\{w : w \text{ has a length equal to or more than three.}\}$
(ii) $\{w : w = (010)^n, \text{n} \geq 0\}$
(iii) $\{w : w \text{ contains } 010 \text{ as a subsequence}\}$
(iv) $\{w : w \text{ contains } 010 \text{ as a substring}\}$

(h) **Select** the paths that accepts 010110 in the given NFA? There can be more than one path that accepts the string. (2 points)

(i) $\texttt{a} \to \texttt{b} \to \texttt{b} \to \texttt{b} \to \texttt{b} \to \texttt{c} \to \texttt{d}$
(ii) $\texttt{a} \to \texttt{b} \to \texttt{c} \to \texttt{d} \to \texttt{d} \to \texttt{d} \to \texttt{d}$
(iii) $\texttt{a} \to \texttt{b} \to \texttt{b} \to \texttt{b} \to \texttt{b} \to \texttt{b} \to \texttt{b}$
(iv) $\texttt{a} \to \texttt{a} \to \texttt{b} \to \texttt{b} \to \texttt{c} \to \texttt{c} \to \texttt{d}$
(v) $\texttt{a} \to \texttt{a} \to \texttt{a} \to \texttt{b} \to \texttt{c} \to \texttt{c} \to \texttt{d}$

Problem 1 (CO1): DFA and Regular Languages (15 points)

We define the last two digits of your Student ID to be AB [e.g: If your Student ID is 2102895, then A = 9, B = 5]

Given, $\Sigma = \{\texttt{A}, \texttt{B}, \texttt{\#}\}$. Consider the following languages over Σ.

$$L_1 = \{w : w \text{ ends with } \texttt{A}\}$$
$$L_2 = \{w : w \text{ contains } \texttt{B\#A} \text{ as a substring}\}$$
$$L_3 = L_1 \circ L_2$$

Now solve the following problems. For questions (a)-(f), you must use your specific Σ to answer.

(a) If $\Sigma = \{\texttt{A}, \texttt{B}, \texttt{\#}\}$, then **define** Σ according to your Student ID. (1 point)

(b) **Give** the state diagram for a DFA that recognizes L_1. (3 points)

(c) **Give** the state diagram for a DFA that recognizes L_2. (3 points)

(d) **Find** all the four-letter strings in $L_1 \cap L_2$. (2 points)

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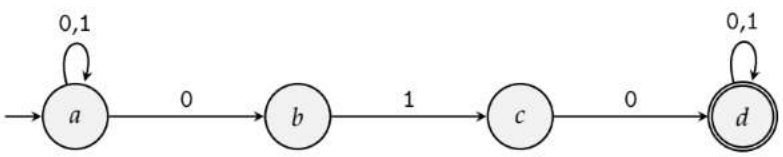

(g) **Choose** the language recognized by this NFA? (1 point)

(i) $\{w : w \text{ has a length equal to or more than three.}\}$
(ii) $\{w : w = (010)^n, \text{n} \geq 0\}$
(iii) $\{w : w \text{ ends with } 010\}$
(iv) $\{w : w \text{ contains } 010 \text{ as a substring}\}$

(h) **Select** the paths that accepts 010110 in the given NFA? There can be more than one path that accepts the string. (2 points)

(i) a → b → b → b → b → c → d
(ii) a → b → c → d → d → d → d
(iii) a → a → a → b → c → d → d
(iv) a → b → b → b → b → b → b
(v) a → a → b → b → c → c → d

Problem 1 (CO1): DFA and Regular Languages (15 points)

We define the last two digits of your Student ID to be AB [e.g: If your Student ID is 2102895, then A = 9, B = 5]

Given, $\Sigma = \{\texttt{A}, \texttt{B}, \texttt{\#}\}$. Consider the following languages over Σ.

$$L_1 = \{w : w \text{ starts with } \texttt{\#}\}$$

$$L_2 = \{w : w \text{ contains } \texttt{\#BA} \text{ as a substring}\}$$

$$L_3 = L_1 \circ L_2$$

Now solve the following problems. For questions (a)-(f), you must use your specific Σ to answer.

(a) If $\Sigma = \{\texttt{A}, \texttt{B}, \texttt{\#}\}$, then **define** Σ according to your Student ID. (1 point)

(b) **Give** the state diagram for a DFA that recognizes L_1. (3 points)

(c) **Give** the state diagram for a DFA that recognizes L_2. (3 points)

(d) **Find** all the four-letter strings in $L_1 \cap L_2$. (2 points)

(e) If you were to use the "cross product" construction shown in class to obtain a DFA for the language $L_1 \cap L_2$, how many states would it have? (1 point)

(f) **Prove** L_3 is a regular language by giving the state diagram for a DFA or an NFA that recognizes L_3. (2 points)

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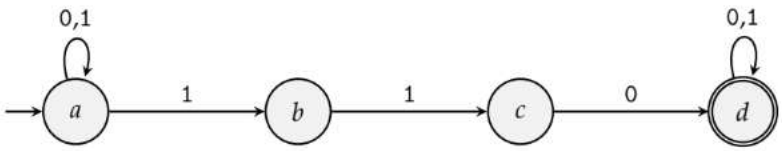


(g) **Choose** the language recognized by this NFA? (1 point)

(i) $\{w : w \text{ has a length equal to or more than three.}\}$

(ii) $\{w : w = (110)^n, \text{n} \geq 0\}$

(iii) $\{w : w \text{ contains } 110 \text{ as a substring}\}$

(iv) $\{w : w \text{ contains } 010 \text{ as a substring}\}$

(h) **Select** the paths that accepts 010110 in the given NFA? There can be more than one path that accepts the string. (2 point)

(i) $\texttt{a} \to \texttt{b} \to \texttt{b} \to \texttt{b} \to \texttt{b} \to \texttt{c} \to \texttt{d}$

(ii) $\texttt{a} \to \texttt{a} \to \texttt{a} \to \texttt{b} \to \texttt{c} \to \texttt{d} \to \texttt{d}$

(iii) $\texttt{a} \to \texttt{b} \to \texttt{c} \to \texttt{d} \to \texttt{d} \to \texttt{d} \to \texttt{d}$

(iv) $\texttt{a} \to \texttt{a} \to \texttt{a} \to \texttt{a} \to \texttt{b} \to \texttt{c} \to \texttt{d}$

(v) $\texttt{a} \to \texttt{a} \to \texttt{b} \to \texttt{b} \to \texttt{c} \to \texttt{c} \to \texttt{d}$

Problem 2 (CO1): Regular Expressions (15 points)

Let $\Sigma = \{0, 1\}$. **Give** regular expressions for each of the languages (a)-(f) over Σ.

(a) $\{w : w$ contains 11 or 101 as a substring.$\}$ (2 points)

(b) $\{w : w$ contains exactly four 1s.$\}$ (2 points)

(c) $\{w :$ The length of w is two more than multiple of five.$\}$ (2 points)

(d) $\{w : w$ consists of any combination of 01 and 110.$\}$ (2 points)

(e) $\{w : w$ doesn't end with 01$\}$ (2 points)

(f) $\{w :$ Number of 01 substring is more than number of 10 substrings in $w\}$ (2 points)

(g) You write a regular expression $0(0+1)^*1^*0^*0$. Your friends write another regular expression $01^*0^*(0+1)^*0$. Are they the same? **Write** Yes or No only. (1 point)

(h) You write a regular expression $(1+01)^*$. Your friends write another regular expression $1^*(011^*)^*$. Are they the same? **Give** justification for your answer. (2 points)

RE Set B

Problem 2 (CO1): Regular Expressions (15 points)

Let $\Sigma = \{0, 1\}$. **Give** regular expressions for each of the languages (a)-(f) over Σ.

(a) $\{w : w$ starts with 00 or 010.$\}$ (2 points)

(b) $\{w : w$ contains at least three 1s.$\}$ (2 points)

(c) $\{w :$ The length of w is three more than multiple of five.$\}$ (2 points)

(d) $\{w : w$ consists of any combination of 10 and 001.$\}$ (2 points)

(e) $\{w : w$ doesn't end with 11$\}$ (2 points)

(f) $\{w :$ Number of 01 substring is less than number of 10 substrings in $w\}$ (2 points)

(g) You write a regular expression $11^*(0+1)^*0^*1$. Your friends write another regular expression $10^*1^*(0+1)^*1$. Are they the same? **Write** Yes or No only. (1 point)

(h) You write a regular expression $(0+10)^*$. Your friends write another regular expression $0^*(100^*)^*$. Are they the same? **Give** justification for your answer. (2 points)

RE Set C

Problem 2 (CO1): Regular Expressions (15 points)

Let $\Sigma = \{0, 1\}$. **Give** regular expressions for each of the languages (a)-(f) over Σ.

(a) $\{w : w$ ends with 001 or 11.$\}$ (2 points)

(b) $\{w : w$ contains at most two 1s.$\}$ (2 points)

(c) $\{w :$ The length of w is three more than multiple of four.$\}$ (2 points)

(d) $\{w : w$ consists of any combination of 11 and 010.$\}$ (2 points)

(e) $\{w : w$ doesn't end with 00$\}$ (2 points)

(f) $\{w :$ Number of 01 substring and 10 substrings in w is unequal$\}$ (2 points)

(g) You write a regular expression $01^*(0+1)^*0^*1$. Your friends write another regular expression $00^*1^*(0+1)^*1$. Are they the same? **Write** Yes or No only. (1 point)

(h) You write a regular expression $(1+01)^*$. Your friends write another regular expression $1^*(011^*)^*$. Are they the same? **Give** justification for your answer. (2 points)

Problem 3 (CO3): Converting Regular Expressions to NFAs (10 points)

Convert the following regular expression over $\Sigma = \{a, b, c\}$ into an equivalent NFA. Note that $R_1 + R_2$ is the same as $R_1 \cup R_2$.

$$(bc)^*(a + c) + (bc^* + a)^*b$$

RE to NFA Set A

Problem 3 (CO2): Converting Regular Expressions to NFAs (10 points)

Convert the following regular expression over $\Sigma = \{0, 1, 2\}$ into an equivalent NFA. Note that $R_1 + R_2$ is the same as $R_1 \cup R_2$.

$$(0^*1 + 2)^* + 0(12)^*0$$

RE to NFA Set B

Problem 3 (CO2): Converting Regular Expressions to NFAs (10 points)

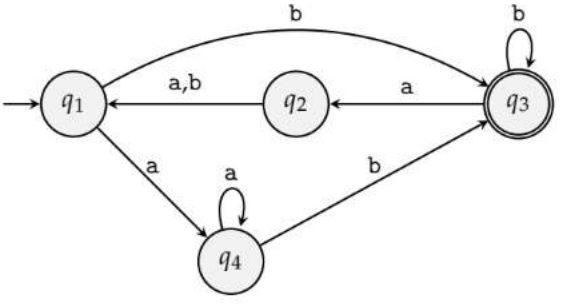
Convert the following regular expression over $\Sigma = \{0, 1, 2\}$ into an equivalent NFA. Note that $R_1 + R_2$ is the same as $R_1 \cup R_2$.

$$1^*0 + (0^*2 + (20)^*1)$$

DFA to RE Set A

Problem 4 (CO2): Converting Finite Automata to Regular Expressions (10 points)

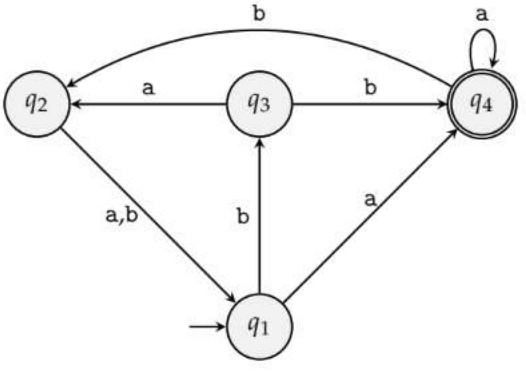
Convert the following DFA into an equivalent regular expression using the state elimination method. First eliminate q_3, then q_2, next q_4, and finally q_1. You must show work.



DFA to RE Set B

Problem 4 (CO2): Converting Finite Automata to Regular Expressions (10 points)

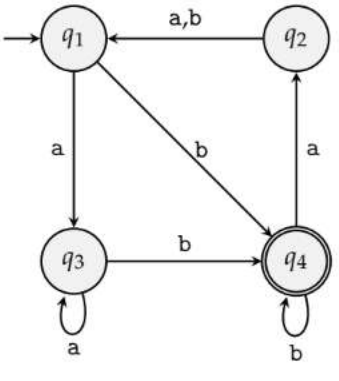
Convert the following DFA into an equivalent regular expression using the state elimination method. First eliminate q_4, then q_2, next q_3, and finally q_1. You must show work.



DFA to RE Set C

Problem 4 (CO2): Converting Finite Automata to Regular Expressions (10 points)

Convert the following DFA into an equivalent regular expression using the state elimination method. First eliminate q_4, then q_2, next q_3, and finally q_1. You must show work.



Problem 5 (Bonus): Even Odd (5 points)

Disclaimer: This is a bonus problem. Attempt it only after you are done with everything else. Even if you do not attempt it, you can get a perfect score. So, do not worry if you find it too hard!

Let $\Sigma = \{0, 1\}$.

$$L = \{w | \text{ 0s in even position of w are followed by odd numbers of 1s}\}$$

Give a five state diagram for a DFA that recognizes L.