
Your solutions need to be handwritten. After writing down your solutions, scan and compile those into a single pdf file. Then submit in the following link:

<https://forms.gle/zmeVaG8jAxVk3x3H8>

[The form will be accessible only from your G-Suite email account.]

Problem 1

Write down regular expressions for the following languages.

- a) $\{w \in \{0, 1\}^* : w \text{ does not end in } 101\}$ (2 points)
- b) $\{w \in \{a, b, c\}^* : w \text{ starts with } abba \text{ and ends in } bac\}$ (2 points)
- c) $\{w \in \{0, 1\}^* : w \text{ has length at least 3 and its third symbol is a } 0\}$ (2 points)
- d) $\{w \in \{0, 1\}^* : w \text{ contains an even number of } 0\text{'s or exactly two } 1\text{'s}\}$ (2 points)
- e) $\{w \in \{a, b\}^* : \text{the number of } a\text{'s in } w \text{ is 1 more than a multiple of } 3\}$ (2 points)

Problem 2

Describe the languages that the following regular expressions generate. (5 points)

- a) $01(1 + 0)^*$
- b) $0(0 + 1)^* + (0 + 1)^*1$
- c) $(1 + 01)^*(0 + \epsilon)$
- d) $(0 + \epsilon)(10)^*(1 + \epsilon)$
- e) $(0 + 10)^*(1 + \epsilon) + (1 + 01)^*(0 + \epsilon)$

Problem 3

Construct deterministic finite automata for the following languages.

- a) $\{w \in \{0, 1\}^* : w \text{ contains exactly two } 0\text{'s}\}$ (2 points)
- b) $\{w \in \{0, 1\}^* : w \text{ contains an even number of } 0\text{'s and an odd number of } 1\text{'s}\}$ (2 points)
- c) $\{w \in \{0, 1\}^* : \text{the length of } w \text{ is even and } w \text{ contains } 0\text{'s at all the odd positions}\}$ (3 points)
- d) $\{w \in \{0, 1\}^* : w \text{ contains } 01^m0 \text{ as a substring where } m \text{ is divisible by } 3\}$ (3 points)
- e) $\{w \in \{0, 1\}^* : w \text{ contains } 0\text{'s and } 1\text{'s in alternate}\}$ (3 points)
- f) $\{w \in \{0, 1\}^* : \text{the last two symbols of } w \text{ are the same}\}$ (3 points)

- g) $\{ w \in \{0, 1\}^* : w = 0^m 1^n \text{ where } m \text{ is even and } n \geq 0 \}$ (3 points)
- h) $\{ w \in \{0, 1\}^* : w \text{ doesn't contain } 00 \text{ and } 11 \}$ (3 points)
- i) $\{ w \in \{0, 1, 2\}^* : w \text{ when interpreted in three base number system is divisible by nine} \}$ [Try using 3 states maximum] (3 points)

j) [Bonus Question] Let $\Sigma = \{0, 1\}$. For $w \in \Sigma^*$, we denote by $h(w)$ the number of 1s in w . For example, $h(100101) = 3$.

Consider the following language over Σ . As always, $|w|$ is the length of the string w .

$$L = \{ w : |w|^2 + h(w)^2 \text{ is two more than a multiple of three} \}$$

Give the state diagram for a DFA that recognizes L .

Problem 4

Answer the following questions. Add explanations if needed. (5 points)

- a) Are regular expressions $(ba)^+(a^*b^* \mid a^*)$ and $(ba)^*ba^+b^*$ the same?
- b) Write down the shortest nonempty string generated by the expression:
 $(0+1)^* (0 (0+1) 0 + 1 (100 + 001 + 11^*)) 00$
- c) What is the shortest string generated by the regular expression
 $(00 + 01) (101)^* ((00)^* + 110)$ with exactly three 0s?
- d) Suppose, you are given a regular expression: $a(a^*b^* \mid b^+)b$. Now, is the expression $a^*(ab^+)$ equivalent to your given expression?
- e) Are regular expressions $a^*(b \mid b^* \mid b^+)$ and $a^?(a^+b^+ \mid b^* \mid \epsilon)$ the same?