

Take a look at the grammar below and solve the following problems.

$$S \rightarrow AB \mid bAA \mid SS$$

$$A \rightarrow a \mid aA$$

$$B \rightarrow b \mid \epsilon$$

- Give a leftmost derivation for the string abaaab. (3 points)
- Sketch the parse tree corresponding to the derivation you gave in (a). (2 points)
- Demonstrate that the given grammar is ambiguous by showing two more parse tree (apart from the one you already found in (b)) for the same string. (4 points)
- Find a string w of length five such that w has exactly one parse tree in the grammar above. (1 point)

a, b)

Leftmost derivation	Corresponding Parse Tree
$ \begin{aligned} S &\rightarrow SS \rightarrow SSS \rightarrow ABSS \rightarrow aBSS \rightarrow \\ &abSS \rightarrow abABS \rightarrow abaBS \rightarrow aba\epsilon S \\ &\rightarrow abaSS \rightarrow abaABS \rightarrow abaabS \rightarrow \\ &abaa\epsilon S \rightarrow abaaAB \rightarrow abaaaB \rightarrow abaaab \end{aligned} $	<pre> graph TD S1[S] --- S2[S] S1 --- S3[S] S2 --- S4[S] S2 --- S5[S] S3 --- S6[S] S3 --- S7[S] S4 --- A1[A] S4 --- B1[B] S5 --- A2[A] S5 --- B2[B] S6 --- A3[A] S6 --- B3[B] S7 --- A4[A] S7 --- B4[B] A1 --- a1[a] B1 --- b1[b] A2 --- a2[a] B2 --- e1["ε"] A3 --- a3[a] B3 --- e2["ε"] A4 --- a4[a] B4 --- b2[b] </pre>

c)

$ \begin{aligned} S &\rightarrow SS \rightarrow ABS \rightarrow aBS \rightarrow abS \rightarrow \\ abAB &\rightarrow abaAB \rightarrow abaaAB \rightarrow \\ abaaaB &\rightarrow abaaaab \end{aligned} $	<pre> graph TD S1[S] --- S2[S] S1 --- S3[S] S2 --- A1[A] S2 --- B1[B] A1 --- a1[a] B1 --- b1[b] S3 --- A2[A] S3 --- B2[B] A2 --- a2[a] A2 --- A3[A] A3 --- a3[a] A3 --- A4[A] A4 --- a4[a] B2 --- b2[b] </pre>
$ \begin{aligned} S &\rightarrow SS \rightarrow ABS \rightarrow aBS \rightarrow abS \rightarrow \\ abSS &\rightarrow abABS \rightarrow abaBS \rightarrow \\ abaES &\rightarrow abaAB \rightarrow abaaAB \rightarrow \\ abaaaB &\rightarrow abaaaab \end{aligned} $	<pre> graph TD S1[S] --- S2[S] S1 --- S3[S] S2 --- A1[A] S2 --- B1[B] A1 --- a1[a] B1 --- b1[b] S3 --- S4[S] S3 --- A2[A] S3 --- B2[B] S4 --- A3[A] S4 --- B3[B] A3 --- a3[a] B3 --- e[ε] A2 --- a2[a] A2 --- A4[A] A4 --- a4[a] B2 --- b2[b] </pre>

d) abbaa, baaab