

## CSE331: Automata and Computability

### Assignment 1

Deadline: 20/11/2024 11:59 pm

## DFA (40 points)

1.  $L_1 = \{w \mid w \in \{0, 1, 2\}^* \text{ the sum of the symbols in } w \text{ is a multiple of } 3\}$   
For example, 021201 is part of the language because the sum of all its symbols equals 6 ( $6 \bmod 3 = 0$ ); whereas, 010012 is not in the language because it sums up to 4 ( $4 \bmod 3 = 1$ ).
2.  $L_2 = \{w \mid w \in \{0, 1\}^* \text{ } w \text{ when interpreted as a binary number, is divisible by } 5\}$
3.  $L_3 = \{w \mid w \in \{0, 1\}^* \text{ } w \text{ is any string that doesn't contain exactly two a's}\}$
4.  $L_4 = \{w \mid w \in \{0, 1\}^* \text{ } w \text{ starts with a 0 and has odd length, or starts with 1 and has even length}\}$
5.  $L_5 = \{w \mid w \in \{0, 1\}^* \text{ } w \text{ is any string not in } 0^*1^*\}$
6.  $L_6 = \{w \mid w \in \{0, 1\}^* \text{ } w \text{ contains } 01^m0 \text{ as a substring where } m \text{ leaves a remainder of } 2 \text{ when divided by } 3\}$
7.  $L_7 = \{w \mid w \in \{0, 1\}^* \text{ where the set of binary strings where numbers of 0s between two successive 1s will be even}\}$
8.  $L_8 = \{w \mid w \in \{a, b\}^* \text{ each "b" is followed by at least one "a"}\}$
9.  $L_9 = \{w \mid w \in \{a, b, c\}^* \text{ } w = a^m b^n c^l \text{ for } m, n, l \geq 1\}$
10.  $L_{10} = \{w \mid w \in \{0, 1\}^* \text{ set of strings where 0's and 1's appear in alternating groups of odd numbered length.}\}$
11.  $L_{11} = \{w \mid w \in \{0, 1\}^* \text{ } w \text{ contains an equal number of occurrences of the substrings } 01 \text{ and } 10\}$
12.  $L_{12} = \{w \mid w \in \{a, b\}^* \text{ and for every } x \text{ that is a prefix of } w, \mid \#a(x) - \#b(x) \mid \leq 2\}$   
Accept strings where the difference between a's and b's in the prefix is in the range -2,...,2. For example, the language contains  $\epsilon$ , b, bba and aaba.  
Hint: bbaaaa is accepted but aaaabb is not accepted.

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13. Let  $\Sigma = \{0, 1\}$ . Consider the following languages over  $\Sigma$ .

$L_{13} = \{w : \text{every second letter of } w \text{ is } 0\}$

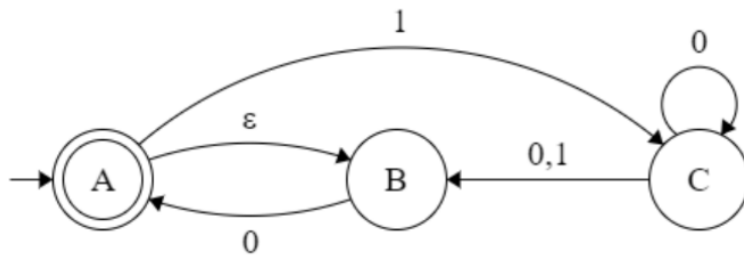
$L_{14} = \{w : \text{every third letter of } w \text{ is } 1\}$

- (a) Write down a length 5 string that is in  $L_1 \cap L_2$ .
- (b) Give the state diagram for a DFA that recognizes  $L_1 \cap L_2$ .

### NFA (9 points)

- 1.  $L_{15} = \{w \mid w \in \{0, 1\}^* \text{ } w \text{ contains } 1001 \text{ or } 11\}$  (use 5 states)
- 2.  $L_{16} = \{w \mid w \in \{0, 1\}^* \text{ } w \text{ contains a } 1 \text{ in the third position from the end}\}$
- 3.  $L_{17} = \{w \mid w \in \{0, 1\}^* \text{ } \text{length of } w \text{ is a multiple of } 2 \text{ or } 3\}$

### NFA to DFA [Subset construction] (4 points)



### Regular expression (21 points)

Give regular expressions that generate the following languages, assuming the alphabet  $\{0, 1\}$

- 1.  $L_{18} = \{w \mid w \text{ starts with } 0 \text{ and has odd length, or starts with } 1 \text{ and has even length}\}$
- 2.  $L_{19} = \{w \mid w \text{ does not contain the substring } 110\}$
- 3.  $L_{20} = \{w \mid w \text{ contains an even number of } 0\text{'s or contains exactly two } 1\text{'s}\}$
- 4.  $L_{21} = \{w \mid w \text{ contains equal number of } 01 \text{ and } 10\}$
- 5.  $L_{22} = \{w \mid \text{every third position in } w \text{ is } 1\}$
- 6.  $L_{23} = \{w \mid \text{every } 1 \text{ in } w \text{ is followed by at least two } 0\}$
- 7.  $L_{24} = L_{22} \cap L_{23}$

### RE to NFA (6 points)

- 1.  $10(01 \mid 0)^*$
- 2.  $(0 \mid 01^*0)^*01^*0$