Midterm Exam
Total marks: 40
Duration: 70 minutes

CSE331
Set 2

There are a total of four problems. You have to solve **all** of them.

Problem 1 (CO1): DFA and Regular Languages (10 points)

Let $\Sigma = \{0, 1\}$. Consider the following languages over Σ.

$$L_1 = \{w : w \text{ starts with either } 01 \text{ or } 10\}$$

$$L_2 = \{w : w \text{ does not start with } 11\}$$

$$L_3 = \{w : \text{the length of } w \text{ is at least two}\}$$

Now solve the following problems.

(a) Give the state diagram for a DFA that recognizes L_1. (3 points)

(b) Give the state diagram for a DFA that recognizes L_2. (3 points)

(c) Give the state diagram for a DFA that recognizes L_3. (2 points)

(d) Give the state diagram for a DFA that recognizes $\overline{L_1} \cap L_2 \cap L_3$ using only four states. Here $\overline{L}$ denotes the complement of the language L i.e., $\overline{L} = \Sigma^* - L$. (2 points)

Problem 2 (CO1): Regular Expressions (10 points)

Let $\Sigma = \{0, 1\}$. Consider the following pair of languages over Σ.

$$L_1 = \{w : w \text{ contains } 11 \text{ as a substring}\}$$

$$L_2 = \{w : w \text{ contains } 10 \text{ as a substring}\}$$

Now solve the following problems.

(a) Write down a regular expression for the language L_1. (2 points)

(b) Write down a regular expression for the language L_2. (2 points)

(c) Your friend wants a regular expression for the language $\overline{L_1 \cap L_2}$ where $\overline{L}$ denotes the complement of the language L i.e., $\overline{L} = \Sigma^* - L$. He wants your help. You tell him to make use of the fact $\overline{L_1 \cap L_2} = \overline{L_1} \cup \overline{L_2}$.

(i) Write down a regular expression for the language $\overline{L_1}$. (2 points)

(ii) Write down a regular expression for the language $\overline{L_2}$. (2 points)

(iii) Using the fact above, write down a regular expression for the language $\overline{L_1 \cap L_2}$. (2 points)

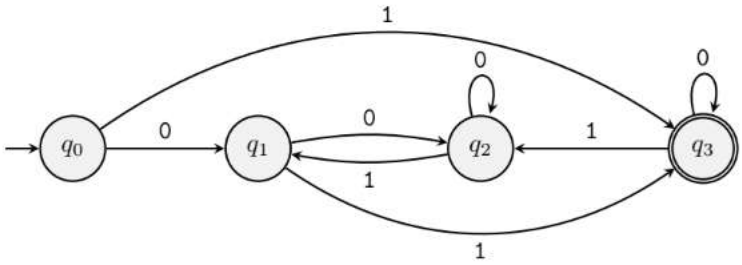
Problem 3 (CO1): Converting Regular Expressions to NFAs (10 points)

Convert the following regular expression into an equivalent NFA. Note that $R_1 + R_2$ is the same as $R_1 \cup R_2$.

$$\mathtt{c} + \left(\mathtt{ab}^* + (\mathtt{bb}^*\mathtt{c})^*\right)^*$$

Problem 4 (CO1): Converting Finite Automata to Regular Expressions (10 points)

Convert the following DFA into an equivalent regular expression using the state elimination method. First eliminate q_1, then q_2 and finally q_3. You must show work.



After you are done with the test, please indicate where you stand on the smiley face spectrum.