Final Exam
Total marks: 50
Duration: 95 minutes CSE331: Automata and Computability

There are a total of five problems. You have to solve **all** of them.

Problem 1 (CO2): Designing Context-Free Grammar (10 points)

Let $\Sigma = \{0, 1, 2\}$. Consider the following languages over Σ.

$$L_1 = \{1^i 0 2^j 1^k \mid i, j, k \geq 0, i = k\}$$
$$L_2 = \{1^i 0 2^j 1^k \mid i, j, k \geq 0, k = i + 2j\}$$

Now solve the following problems.

(a) **Give** a context-free grammar for the language L_1. (5 points)

(b) **Give** a context-free grammar for the language L_2. (5 points)

Problem 2 (CO2): Derivations, Parse Trees and Ambiguity (10 points)

Take a look at the grammar below and solve the following problems.

$$S \to \mathtt{aa}S \mid \mathtt{ab}S \mid \mathtt{ba}S \mid \mathtt{bb}S \mid X$$
$$X \to \mathtt{aa}Y \mid \mathtt{ba}Y$$
$$Y \to \mathtt{a}Y \mid \mathtt{b}Y \mid \varepsilon$$

(a) **Give** a leftmost derivation for the string `abbabbaa`. (3 points)

(b) **Sketch** the parse tree corresponding to the derivation you gave in (a). (2 points)

(c) **Demonstrate** that the given grammar is ambiguous by showing one more parse tree (apart from the one you already found in (b)) for the same string. (4 points)

(d) **Find** a string w of length nine such that w has exactly one parse tree in the grammar above. (1 point)

Problem 3 (CO2): Chomsky Normal Form (10 points)

(a) **List** the rules that violate the conditions of Chomsky Normal form in the following grammar. Here `a`, `b`, and `c` are terminals and the rest are variables.

$$A \to BC \mid \mathtt{b}B \mid \mathtt{a}$$
$$B \to \mathtt{bb} \mid C\mathtt{b} \mid \mathtt{b} \mid C$$
$$C \to \mathtt{c}$$

(b) **Write** down the additional rules that need to be added to the following grammar if the production, $B \to \varepsilon$ is removed. Here `0` and `1` are terminals and the rest are variables.

$$S \to AB \mid \mathtt{1}$$
$$A \to BAB \mid ABA \mid B \mid 11$$
$$B \to \mathtt{00} \mid \varepsilon$$

(c) **Write** down the additional rules that need to be added to the following grammar if the unit productions are removed. Here `0` and `1` are terminals and the rest are variables.

$$S \to XYX \mid YX \mid X \mid Y$$
$$Y \to XY \mid X\mathtt{0} \mid \mathtt{0}$$
$$X \to \mathtt{1} \mid Y$$

Problem 4 (CO4): The CYK Algorithm (10 points)

Apply the CYK algorithm to determine whether the string `baaab` can be derived in the following grammar. You must show the entire CYK table. Here `a` and `b` are terminals and the rest are variables.

$$
\begin{aligned}
S &\to CA \mid DB \mid \mathtt{a} \mid \mathtt{b} \\
Z &\to CA \mid DB \mid \mathtt{a} \mid \mathtt{b} \\
C &\to AZ \\
D &\to BZ \\
A &\to \mathtt{a} \\
B &\to \mathtt{b}
\end{aligned}
$$

Problem 5 (CO2): Constructing Pushdown Automata (10 points)

Let $\Sigma = \{\mathtt{0}, \mathtt{1}\}$. Consider the following pair of languages over Σ.

$$L_1 = \{w \mid \text{the length of } w \text{ is divisible by four}\}$$
$$L_2 = \{w \mid w = \mathtt{0}^{n+2}\mathtt{1}^{2n}, n \geq 0\}$$

Now solve the following problems.

(a) **Construct** a pushdown automaton that recognizes L_1. (4 points)

(b) **Construct** a pushdown automaton that recognizes L_2. (6 points)

Problem 6: (Bonus) Closed Under Intersection? (5 points)

(Note that this is a bonus problem. Attempt it only after you are done with everything else. Even if you do not attempt it, you can get a perfect score. So, do not worry if you find it too hard!)

Consider the language $L = L_1 \cap L_2$ where L_1 and L_2 were defined in Problem 5. **Construct** a pushdown automaton that recognizes L. Describe what your automaton is doing in two or three sentences.

After you are done with the test, please indicate where you stand on the smiley face spectrum.