BRAC UNIVERSITY

Kha – 224, Merul Badda, Dhaka, Bangladesh

CSE331 : Automata and Computability

Assignment 02

Summer 2024

Q1. Which of the following statements about regular expressions is true? (2 points)

1. Regular expressions can describe only Regular Languages.
2. Regular expressions can describe only Context Free Languages.
3. Regular expressions can describe both Regular Language and Context Free Languages.

Write an explanation for your answer.

Answer : Regular languages are recognized by finite automata, which have limited amount of memory. However, Context Free Languages require unbounded amount of memory, so regular expressions can't describe Context Free Languages.

Q2. Which is the correct regular expression for the language $L = \{w \in \{0,1\}^*: 0^n$, where n is even$\}$? (2 points)

1. $0\ (00)^*$
2. $(00)^*$
3. $0^*\ (00)^*\ 0^*$
4. None of the above

Write an explanation for your answer.

Answer : Since n is even, the number of 0s in the language will also be even and $(00)^*$ contains even number of 0s.

Q3. Which is the correct regular expression for the language $L = \{w \in \{0,1\}^*: 0^n1^n$, where n is even$\}$? (2 points)

1. $(00)^*\ (11)^*$

2. ((0 ∪ 1) (0 ∪ 1)) *

3. (00 ∪ 11)*

4. None of the above

Write an explanation for your answer.

Answer : Since, every option has * above it, none can guarantee that in the string the number of 0s and 1s will be same.

Q4. You are given two languages, A and B. You can construct a regular expression for A. However, it is impossible to construct a regular expression for B. Now read the options below. (2 points)

a) A is a regular language

b) B is a regular language.

c) B is a nonregular language.

d) A∪B is a regular language. Choose the best answer from below.

1. a and b

2. a and c

3. a and d

4. a, b and d

5. a, c and d

Write an explanation for your answer.

Answer : Regular expressions describe regular language. Since I can construct regular expression for A but not for B, A is a regular language, and B is a nonregular language. A∪B will be regular if and only if A and B are regular. Though, A∪B they are closed under union, but when one of the languages is nonregular, the union will be nonregular.

Q5. What does the regular expression (0 ∪ 1)* 0 (0 ∪ 1)* describe? (2 points)

1.   L = {w ∈ {0,1}*: w contains exactly one 0}

2.   L = {w ∈ {0,1}*: w contains at least three 0}

3.   L = {w ∈ {0,1}*: w contains at least one 0}

4.   L = {w ∈ {0,1}*: w contains at most one 0} Write an explanation for your answer.

Write an explanation for your answer.

answer.

Answer : In the given regular expression (0 ∪ 1)* 0 (0 ∪ 1)*, the expression (0 ∪ 1)* can generate any number of 0s or 1s or empty. However, there is a 0 in the middle that always exists, which means that w contains at least one 0.

Q6. (0 ∪ 1)* (0 (0 ∪ 1) 0 ∪ 1 (100 ∪ 001 ∪ 11*)) 00

Write down the shortest nonempty string generated by the given expression. Write down the string only. (2 points)

Write an explanation for your answer.

Answer : As we are searching for shortest nonempty strings, let (0 ∪ 1)* be empty. Then 0 (0 ∪ 1) 0 becomes either 000 or 010. Next, 1 (100 ∪ 001 ∪ 11*) simplifies to 11. So, (0 ∪ 1)* (0 (0 ∪ 1) 0 ∪ 1 (100 ∪ 001 ∪ 11*)) 00 simplifies to ((000 ∪ 11) ∪ (010 ∪ 11))00 = 1100. Therefore, the shortest nonempty string is 1100.

Q7. (00 ∪ 01) (101)* ((00)* ∪ 110)

What is the shortest string generated by the regular expression with exactly three 0s? Write down the string. (2 points)

Write an explanation for your answer.

Answer : As we are searching for shortest string with exactly three 0s, from (00 ∪ 01), we choose 01, it gives one 0. Let (101)* be empty. Next, from ((00)* ∪ 110), we choose 00 from (00)* , which provides two 0s, making a total of three 0s. Therefore, the shortest string with exactly three 0s is 0100.

Q8. Does the string 00100001011 belong to the regular expression 0 (0 ∪ 1)* 01 (0 ∪ 1)* ((0 ∪ 1)*)* 11 (0 ∪ 1)* 1? (2 points)

1. Yes
2. No

Write an explanation for your answer.

Answer : If we look at the regular expression, from the part 11 (0 ∪ 1)* 1, even if we consider (0 ∪ 1)* as empty, there will still be three 1s. However, in the string 00100001011, we see only two 1s at the end, so it doesn't fit the regular expression.

Q9. Let's, $\Sigma = \{a, b\}$. You are given a regular expression, **a (Σ* a$^+$ ε) ∪ b (Σ* b$^+$ ε)**, what is the correct language for this regex? Here, $a^+ = aa$*.

Note, for a regular language L, the regular expression will be correct if and only if it generates all the strings, $w \in L$, and doesn't generate any string, $w \notin L$. (2 points) a) w starts and ends with the same symbols

b) w contains either at least two a or at least two b

c) w contains substring ab

d) w contains an a followed by a b

e) w contains equal numbers of ab and ba Choose the best answer from below.

1. a
2. a and b
3. c and d
4. a and e
5. a, b and e

Write an explanation for your answer.

Answer : If we look at the regular expression a (Σ* a$^+$ ε) ∪ b (Σ* b$^+$ ε), we can see that if a string starts with a, it ends with a, and if it starts with b, it ends with b. Since, the string starts with a or b and ends with a+ or b+, it contains either at least two a or at least two b. Then, Σ* may not appear at all, and even if it does, it does not imply that w must contain the substring ab, an a followed by a b or equal numbers of ab and ba.

Q10. Which of the following Regular expressions are equivalent to **a(a* b* ∪ b$^+$)b** (2 points) a) a*(ab$^+$)

b) a$^+$(ab$^+$)

c) a$^+$b$^+$

d) a(aa*b*)b

e) a*(a*b$^+$ ∪ b*)b

Choose the best answer from below.

1. a and b

2. c and d

3. d and e

4. b and c

Write an explanation for your answer.

Answer : b, c are correct because they ensure strings must start with a and end with b.

Q11. (01 ∪ 1010 ∪ 1) * (00 ∪ 010)

The length of a string, w can be expressed by |w|. Find out how many distinct strings can be generated using the given regular expression such that 3 <= |w| <= 5. Write a numeric value only. (2 points)

Write an explanation for your answer.

Answer : Possible distinct strings of length between 3 and 5 are : 010, 1+00 = 100, 01+00 = 0100, 1+010 = 1010, 11+00 = 1100, 01+010 = 01010, 11+010 = 11010, 1+01+00 = 10100. The total number of distinct strings is 8.

Q12. You have a regular expression (0 ∪ 1) (0 ∪ 1) 0 0* (0 ∪1)* 1* . And another regular expression (00 ∪ 11 ∪ 10 ∪ 01) 0 1* 0* (1 ∪ 0 ∪ ε)* 0*. Are these two the same? (2 points)

1. Yes

2. No

Write an explanation for your answer.

Answer : In the part '00*' of the first regular expression, it indicates that zero or more 0s can occur. In the part '01*0*' of the second regular expression, it indicates that zero or more 0s can occur, but any number of 1s can appear between them, which is not possible in the first regular expression. Additionally, the first regular expression can end with any number of 1s, while the second regular expression can end with any number of 0s. Essentially, they generate different sets of strings.

Q13. Let's ∑ = {0,1}. What is the regular expression for the strings that have even length and start with 1? (2 points) a) 1 (0 ∪ 1)*

b) (10 ∪ 11) ((0 ∪ 1) (0 ∪ 1))*

c) (1 (0 ∪ 1))*

d) 1 (0 ∪ 1) ((0 ∪ 1) (0 ∪ 1))*

e) 1 (0 ∪ 1) (00 ∪ 01 ∪ 11 ∪ 10)* ∪ ε Select the best answer.

1. d and e

2. b and e

3. b and d

4. b, d and e

5. a, c and e

Write an explanation for your answer.

Answer : Option b starts with 10 or 11, ensuring that the string starts with 1 and has an even length. It then continues with an even number of symbols. Option d is similar to option b. However, the other options do not require the strings to start with 1 or maintain an even length.

Q14. L = {w ε {0,1}*: w doesn't contain 001}

Note, for a regular language L, the regular expression will be correct if and only if it generates all the strings, w ∈ L, and doesn't generate any string, w ∉ L. Which of the following Regular Expressions generate L:

- (1* (01)*)* (ε ∪ 0 ∪ 000*) (1 point)

  1. Correct

  2. Incorrect

- (1* (0 ∪ ε) 1*)* (1 point)

  1. Correct

  2. Incorrect

- (1 ∪ 10)* (ε ∪ 0 ∪ 000*) (1 point)

  1. Correct

  2. Incorrect

- (1 ∪ 01)* 0* (1 point)

  1. Correct

  2. Incorrect

- (1* (10)*)* 0* (1 point)

  1. Correct

  2. Incorrect

Q15. (0 ∪ ε) (10)* (1 ∪ ε)

Note, for a regular language L, the regular expression will be correct if and only if it generates all the strings, w ∈ L, and doesn't generate any string, w ∉ L.

Which of the following regular languages are generated by the given regular expression

- L = { w ∈ {0,1}*: w doesn't contain 00} (1 point)
  1. Correct
  2. Incorrect

- L = { w ∈ {0,1}*: 0 and 1 alternates in w} (1 point)
  1. Correct
  2. Incorrect

- L = { w ∈ {0,1}*: w doesn't contain either 00 or 11} (1 point)
  1. Correct
  2. Incorrect

- L = { w ∈ {0,1}*: The period of w is either 01 or 10 }
  The period of a string is the smallest prefix, when repeated one or more times, can recreate the entire string. (1 point)
  1. Correct
  2. Incorrect

All answers are not correct.