

BRAC UNIVERSITY

Kha – 224, Merul Badda, Dhaka, Bangladesh

CSE331 : Automata and Computability

Assignment 03

Summer 2024

Q1. Which of the following statements about Context Free Grammar (CFG) is true? (2 points)

1. Context Free Grammar (CFG) can describe only Context Free Languages.
2. Context Free Grammar (CFG) can describe all the Non Regular Languages and Regular Languages.
3. Context Free Grammar (CFG) can describe both Context Free Languages and Regular Language.

Write an explanation for your answer.

Q2. Consider the following languages.

A) $L = \{ w \in \{a,b,c\}^* : a^n b^n c^n, \text{ where } n \geq 0 \}$

B) $L = \{ w \in \{0,1\}^* : ww \}$

C) $L = \{ w \in \{0,1\}^* : ww^R \}$ [w^R means reverse of w . For example, if $w = 10110$, then $w^R = 01101$.]

D) $L = \{ w \in \{a,b,c\}^* : a^i b^j c^k, \text{ where } 0 \leq i \leq j \leq k \}$

Which languages are Context Free Languages?

1. Only C
2. B and C
3. C and D
4. A, B and C

Write an explanation for your answer.

Q3. Let, L is a context-free language. Then L^* will also be a context-free language.

The statement is

1. True
2. False
3. Not enough information to answer

Write an explanation for your answer.

Q4. Consider the following Context Free Grammar.

$S \rightarrow AB \mid BC$

$A \rightarrow aA \mid Aa \mid a \mid \varepsilon$

$B \rightarrow bBb \mid bb$

$C \rightarrow c \mid \varepsilon$

Choose the correct regular expression for the language covered by the given CFG

1. $a^*(bb)^* \mid (bb)^*c^*$
2. $a^+(bb)^* \mid (bb)^+c^?$
3. $a^+(bb)^+ \mid (bb)^+(c|\varepsilon)$
4. $a^*(bb)^+ \mid (bb)^+(c|\varepsilon)$
5. $a^+(bb)^* \mid (bb)^*(c|\varepsilon)$

Write an explanation for your answer.

Q5. $A \rightarrow 1A \mid 0A1 \mid 01$

Which of the following string can be generated by the given CFG.

There could be more than one correct answer, choose all of them.

1. 000111
2. 001111
3. 001011
4. 0100111
5. 1000111

Write an explanation for your answer.

Q6. You are given a Context Free Grammar.

$S \rightarrow AB$

$A \rightarrow aA \mid \varepsilon$

$B \rightarrow Bb \mid \varepsilon$

Now consider the following languages.

- a) $L = \{ w \in \{a,b\}^*: a^n b^m, \text{ where } n, m \geq 0 \}$
- b) $L = \{ w \in \{a,b\}^*: a^n b^n, \text{ where } n \geq 0 \}$
- c) $L = \{ w \in \{a,b\}^*: \text{number of } a \text{ and number of } b \text{ is unequal in } w \}$
- d) $L = \{ w \in \{a,b\}^*: \text{all } a \text{ in } w \text{ always precede } b \}$

Which of the following statements are true regarding the language generated by this grammar?

Note, for a language L , the CFG will be correct if and only if it can parse all the strings, $w \in L$, and doesn't parse any string, $w \notin L$.

1. Only A
2. Only B
3. A and C
4. A and D
5. B and D

Write an explanation for your answer.

Q7. $L = \{ w \in \{a,b,c\}^*: a^m b^n c^k \mid n, m \geq 0 \text{ \& } k=3m+2 \}$

Let's say, we have the following four CFGs labeled as (A) to (D).

CFG A:

$S \rightarrow aScc \mid Y$

$Y \rightarrow bY \mid ccc$

CFG B:

$S \rightarrow aSccc \mid Y$

$Y \rightarrow bY \mid cc$

CFG C:

$S \rightarrow Xcc$

$X \rightarrow aXccc \mid Z$

$Z \rightarrow bZ \mid \varepsilon$

CFG D:

$S \rightarrow aSccc \mid bS \mid cc$

What will be the correct CFG for the language L ?

Note, for a language L , the CFG will be correct if and only if it can parse all the strings, $w \in L$, and doesn't parse any string, $w \notin L$.

1. Only B
2. Only C
3. A and C
4. A and D

5. B and C
6. B and D

Write an explanation for your answer.

Q8. $L = \{ w \in \{a,b\}^* : w = a^i b^j \text{ where } i = 2+j, j \geq 0 \}$

Let's say, we have the following four CFGs labeled as (A) to (C).

CFG A:

$S \rightarrow aaN$

$N \rightarrow aNb \mid \epsilon$

CFG B:

$S \rightarrow aSb \mid aa$

CFG C:

$S \rightarrow aSb \mid N$

$N \rightarrow aaN \mid \epsilon$

What will be the correct CFG for the language L?

Note, for a language L, the CFG will be correct if and only if it can parse all the strings, $w \in L$, and doesn't parse any string, $w \notin L$.

1. A and C
2. A and B
3. B and C
4. A, B and C

Write an explanation for your answer.

Q9. Consider the following two Context Free Grammars. Do these two grammars (G1 and G2) represent the same language?

G1:

$S \rightarrow AB$

$A \rightarrow 1A00 \mid C$

$C \rightarrow 0C \mid 0$

$B \rightarrow 1B \mid \epsilon$

G2:

$S \rightarrow ABC$

$A \rightarrow 1A00 \mid \epsilon$

$B \rightarrow 0B \mid 0$

$C \rightarrow 1C \mid \epsilon$

1. Yes
2. No

Write an explanation for your answer.

Q10. $L = \{ w \in \{a,b,c\}^* : a^m b^n c^k \mid n = m/2 \text{ or } n = k - 2 ; \text{ where } n \geq 0, m \text{ is even \& } k \geq 2 \}$

Let's say, we have the following 4 CFGs labeled as (A) to (D).

CFG A:

$S \rightarrow XY \mid PQ$

$X \rightarrow aXbb \mid \varepsilon$

$Y \rightarrow ccY \mid c$

$P \rightarrow aP \mid b$

$Q \rightarrow bQc \mid c$

CFG B:

$S \rightarrow XY \mid PQ$

$X \rightarrow aXb \mid \varepsilon$

$Y \rightarrow cYb \mid c$

$P \rightarrow aP \mid \varepsilon$

$Q \rightarrow bQcc \mid c$

CFG C:

$S \rightarrow XY \mid PQ$

$X \rightarrow aXb \mid c$

$Y \rightarrow cYb \mid \varepsilon$

$P \rightarrow aP \mid \varepsilon$

$Q \rightarrow bQcc \mid c$

CFG D:

$S \rightarrow XY \mid PQ$

$X \rightarrow aaXb \mid \varepsilon$

$Y \rightarrow cY \mid cc$

$P \rightarrow aaP \mid \varepsilon$

$Q \rightarrow bQc \mid cc$

What will be the correct CFG for the language L?

Note, for a language L , the CFG will be correct if and only if it can parse all the strings, $w \in L$, and doesn't parse any string, $w \notin L$.

1. A
2. B
3. C
4. D

Write an explanation for your answer.

Q11. $L = \{w \in \{0,1\}^*: w_1w_2w_3: \text{ where } |w_1| = |w_2| \text{ or } |w_3| = |w_1|\}$

Let's say, we have the following 4 CFGs labeled as (A) to (D).

CFG A:

$S \rightarrow RQ \mid P$
 $P \rightarrow 0P1 \mid 1P0 \mid \varepsilon$
 $Q \rightarrow 0Q \mid 1Q \mid \varepsilon$
 $R \rightarrow 0R0 \mid 1R1 \mid 0R1 \mid 1R0 \mid Q$

CFG B:

$S \rightarrow PR \mid Q$
 $P \rightarrow 0P0 \mid 1P1 \mid \varepsilon$
 $Q \rightarrow 0Q \mid 1Q \mid \varepsilon$
 $R \rightarrow 0R0 \mid 1R1 \mid 0R1 \mid 1R0 \mid Q$

CFG C:

$S \rightarrow PQ \mid R$
 $P \rightarrow 0P0 \mid 1P1 \mid 0P1 \mid 1P0 \mid \varepsilon$
 $Q \rightarrow 0Q \mid 1Q \mid \varepsilon$
 $R \rightarrow 0R0 \mid 1R1 \mid 0R1 \mid 1R0 \mid Q$

CFG D:

$S \rightarrow PQ \mid R$
 $P \rightarrow 0P0 \mid 1P1 \mid 0P1 \mid 1P0 \mid \varepsilon$
 $Q \rightarrow 0Q \mid 1Q$
 $R \rightarrow 0R0 \mid 1R1 \mid 0R1 \mid 1R0 \mid P$

What will be the correct CFG for the language L ?

Note, for a language L , the CFG will be correct if and only if it can parse all the strings, $w \in L$, and doesn't parse any string, $w \notin L$.

1. A and C

2. B and C
3. C and D
4. A, B and C
5. A, B, C and D

Write an explanation for your answer.

Q12. $L = \{w \in \{0,1\}^* : w_1\#w_2\#w_3 : \text{where } w_3 = w_1^R\}$

Given a string w over some alphabet Σ , let w^R be its reverse. For example, if $w = 10110$, then $w^R = 01101$.

Let's say, we have the following 4 CFGs labeled as (A) to (C).

CFG A:

$S \rightarrow 1S0 \mid 0S1 \mid \#P\#$

$P \rightarrow 0P \mid 1P \mid \epsilon$

CFG B:

$S \rightarrow 0S \mid 1S \mid \#P\#$

$P \rightarrow 0P0 \mid 1P1 \mid \epsilon$

CFG C:

$S \rightarrow 0S0 \mid 1S1 \mid \#P\#$

$P \rightarrow 0P \mid 1P \mid \epsilon$

What will be the correct CFG for the language L ?

Note, for a language L , the CFG will be correct if and only if it can parse all the strings, $w \in L$, and doesn't parse any string, $w \notin L$.

1. A
2. B
3. C

Write an explanation for your answer.

Q13. $L = \{w \in \{0,1\}^* : w \in \Sigma^* \mid \text{every prefix of } w \text{ has at least as many 0's as 1's}\}$

Clarification: L states "Every prefix of w contains equal or more 0s than 1s."

Select the correct CFG for L :

A) $S \rightarrow 0S1S \mid 0S \mid \epsilon$

B) $S \rightarrow SS \mid 0S1 \mid 1 \mid \varepsilon$

C) $S \rightarrow SS \mid 0S1 \mid 0 \mid \varepsilon$

D) $S \rightarrow 0S1S \mid 1S \mid \varepsilon$

A prefix is a string consisting of several first letters of a given string, without any reorders. An empty prefix is also a valid prefix. For example, the string "abcd" has 5 prefixes: empty string, "a", "ab", "abc" and "abcd".

1. a and d
2. a and b
3. a and c
4. a, b and d
5. a, b and c
6. a, b, c and d

Write an explanation for your answer.

Q14. Let $\Sigma = \{a, b\}$ and let $L = \{a^n b^m \mid n, m \in \mathbb{N} \text{ and } n \leq m \leq 5n\}$. The CFG for L is given below.

Here is a CFG of the language

$S \rightarrow aSb? \mid \varepsilon$

$A \rightarrow b \mid \varepsilon$

What will be the '?' such that the grammar can describe the language, L correctly.

Fill out the question mark with one or more variables/terminals. Write down the missing string only.

Write an explanation for your answer.

Q15.

$E \rightarrow E + P \mid P$

$P \rightarrow P * Q \mid Q$

$Q \rightarrow \text{id}$

Is the grammar ambiguous?

1. Yes
2. No

Write an explanation for your answer.

Q16. You are given two Context Free Grammar G1 and G2.

G1:

$S \rightarrow (S)S \mid \epsilon$

G2:

$S \rightarrow (S) \mid SS \mid \epsilon$

Now, consider the string $w = ((()((()))((()()))))()$

Which of the following statements is true?

1. w can be generated using only G1
2. w can be generated using only G2
3. w can be generated using G1 and G2
4. w cannot be generated using either G1 or G2

Write an explanation for your answer.

Q17. Consider the two Context free grammars G1 and G2. Which of the following is $L(G1) \cap L(G2)$?

G1:

$S \rightarrow ASA$

$S \rightarrow 0$

$A \rightarrow 0 \mid 1$

G2:

$S \rightarrow ASA$

$S \rightarrow \epsilon$

$A \rightarrow 0 \mid 1$

1. $L = \{w \in \Sigma^* \mid w \text{ contains } 0 \text{ in the middle.}\}$
2. $L = \{w \in \Sigma^* \mid \text{length of } w \text{ is even.}\}$
3. $L = \{w \in \Sigma^* \mid w \text{ is a even length string with } 0 \text{ in the middle.}\}$
4. $L = \{w \in \Sigma^* \mid w \text{ is a odd length string with } 0 \text{ in the middle.}\}$
5. None of the above.

Write an explanation for your answer.

Q18. Consider the following Context Free Grammar, G.

G:

$P \rightarrow XP1 \mid XQ0$

$Q \rightarrow XQ1 \mid XR0$
 $R \rightarrow XRX \mid \#$
 $X \rightarrow 0 \mid 1$

Now, answer the following questions.

What is the shortest string can be parsed from the grammar, G? If there are multiple correct answer write only one of those.

Write an explanation for your answer.

Find out how many distinct strings can be generated using the given Context Free Grammar, G, such that $9 \leq |w| \leq 11$. Write the numeric value only.

Write an explanation for your answer.

Q19. Consider the following Context Free Grammar, G.

G:

$S \rightarrow aaS \mid abS \mid baS \mid bbS \mid X$
 $X \rightarrow aaY \mid baY$
 $Y \rightarrow aY \mid bY \mid \epsilon$

Write a eight length string that starts with "bb" and has only one parse tree. Write the string only.

Write an explanation for your answer.

Q20. $L = \{w \in \{a,b,c\}^* : a^i b^j c^k \mid i, j, k \geq 0 \text{ and if } i > j \text{ then } k = i - j, \text{ else } k = 0\}$

Which of the following CFGs generates L?

CFG A:

$S \rightarrow AB \mid AC$
 $A \rightarrow aA \mid \epsilon$
 $B \rightarrow bB \mid b$
 $C \rightarrow aCc \mid \epsilon$

CFG B:

$S \rightarrow AB \mid A$
 $A \rightarrow aA \mid a \mid \epsilon$
 $B \rightarrow bBc \mid b$

CFG C:

$S \rightarrow A \mid B$
 $A \rightarrow aAb \mid Ab \mid \varepsilon$
 $B \rightarrow aBc \mid aCc$
 $C \rightarrow aCb \mid \varepsilon$

CFG D:

$S \rightarrow AB \mid AC$
 $A \rightarrow aAb \mid \varepsilon$
 $B \rightarrow bB \mid \varepsilon$
 $C \rightarrow aCc \mid \varepsilon$

Note, for a language L, the CFG will be correct if and only if it can parse all the strings, $w \in L$, and doesn't parse any string, $w \notin L$.

1. A
2. B
3. C
4. D

Write an explanation for your answer.

Q21. $L = \{w \in \{a, b\}^* : \text{each } b \text{ in } w \text{ is followed by at least two } a\}$

Note, for a language L, the CFG will be correct if and only if it can parse all the strings, $w \in L$, and doesn't parse any string, $w \notin L$.

Which of the following Context Free Grammar can generate L?

$S \rightarrow XY$
 $X \rightarrow aX \mid \varepsilon$
 $Y \rightarrow ZY \mid \varepsilon$
 $Z \rightarrow PQ$
 $P \rightarrow ba$
 $Q \rightarrow aQ \mid a$

1. Correct
2. Incorrect

$P \rightarrow aP \mid bQ \mid \varepsilon$
 $Q \rightarrow aR \mid bT$
 $R \rightarrow aS \mid bT$
 $S \rightarrow aS \mid bQ \mid \varepsilon$
 $T \rightarrow aT \mid bT$

1. Correct

2. Incorrect

$$S \rightarrow AbR \mid A$$

$$A \rightarrow aA \mid \varepsilon$$

$$R \rightarrow aAabR \mid A$$

1. Correct

2. Incorrect