

### Problem 1

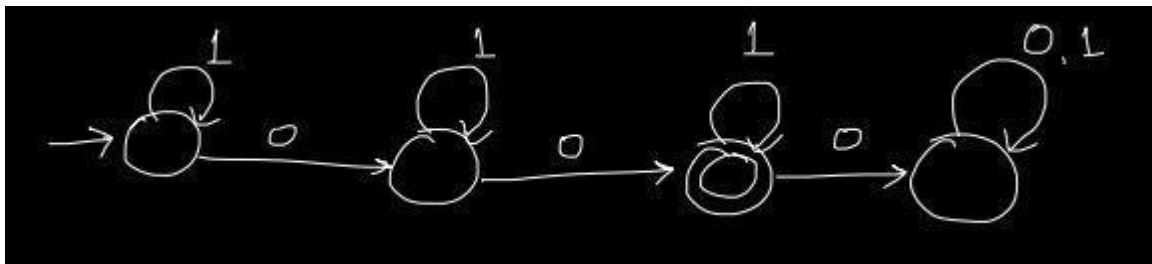
- a)  $(0 \mid 1)^* (0 \mid 11 \mid 001) \mid \epsilon \mid 1 \mid 01$
- b)  $abba (a|b|c)^* bac \mid abba$
- c)  $(0 \mid 1) (0 \mid 1) 0 (0 \mid 1)^*$
- d)  $(1^* 0 1^* 0 1^*)^* \mid 0^* 1 0^* 1 0^*$
- e)  $(b^* ab^* ab^* ab^*)^* b^* ab^*$

### Problem 2

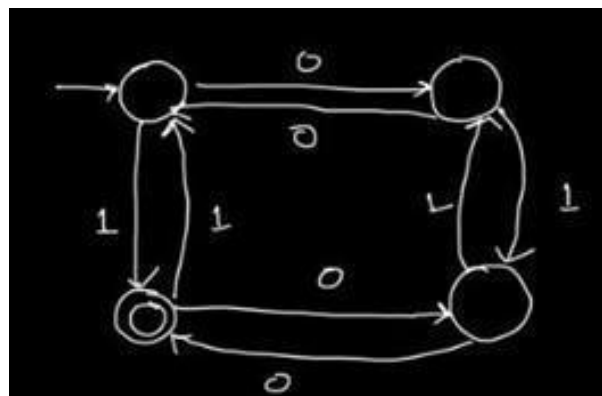
- a) Strings over  $\{0,1\}$  that start with 01
- b) Strings over  $\{0,1\}$  that start with 0 or end with 1
- c) Strings over  $\{0,1\}$  that do not contain 00 as a substring
- d) Strings over  $\{0,1\}$  that have neither consecutive 0's, nor consecutive 1's
- e) Strings over  $\{0,1\}$  that may have consecutive 0's, or consecutive 1's, but not both

### Problem 3

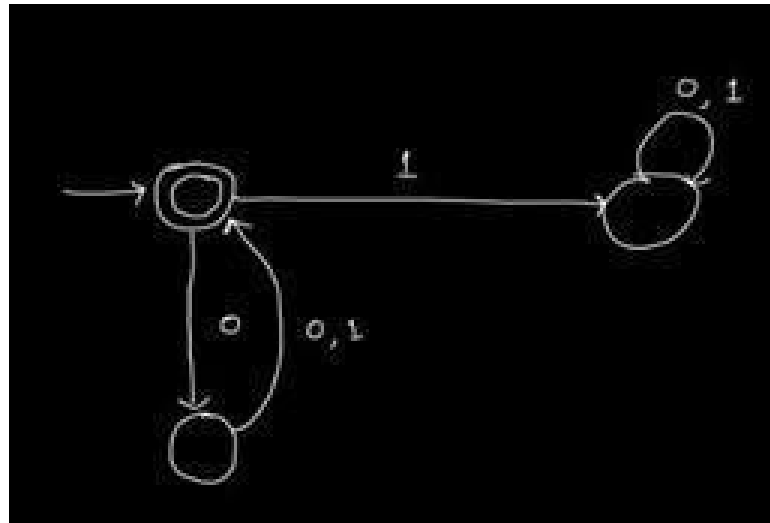
a)



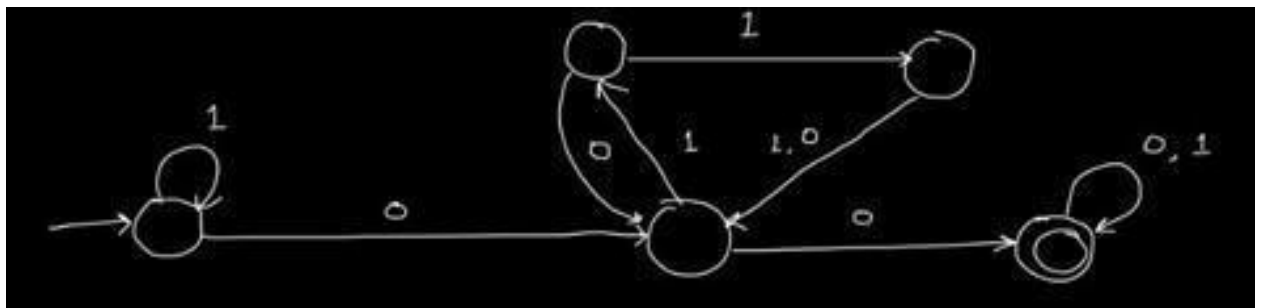
b)



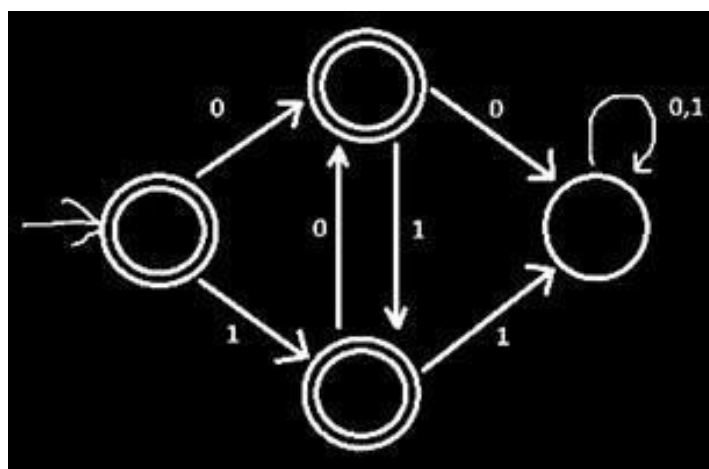
c)



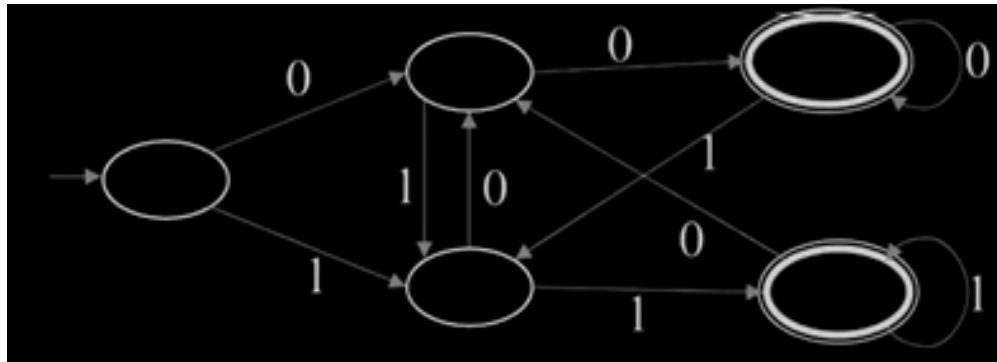
d)



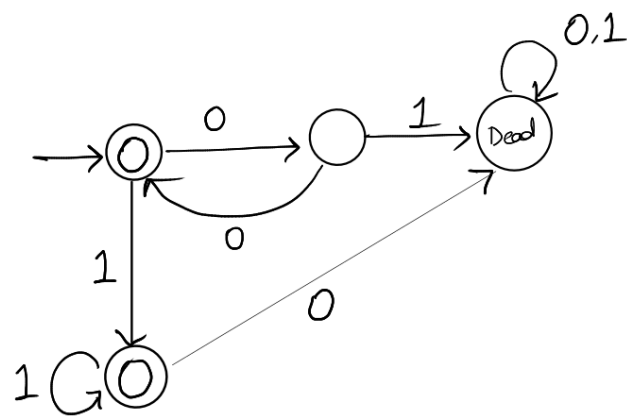
e)



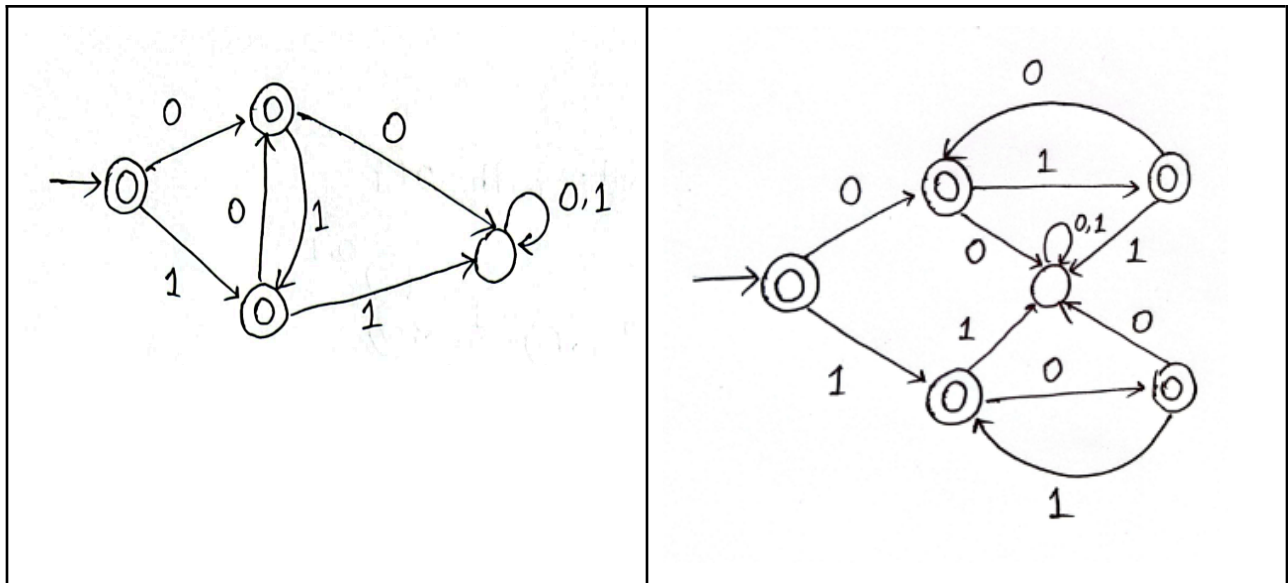
f)



g)

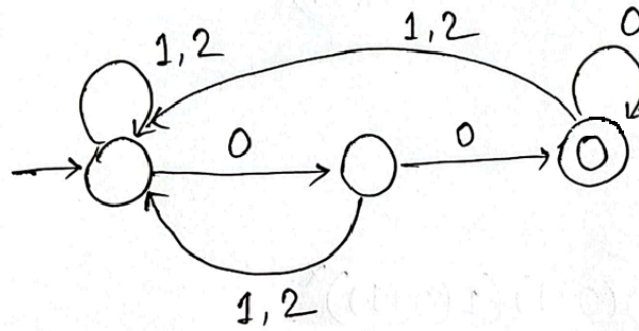


h)



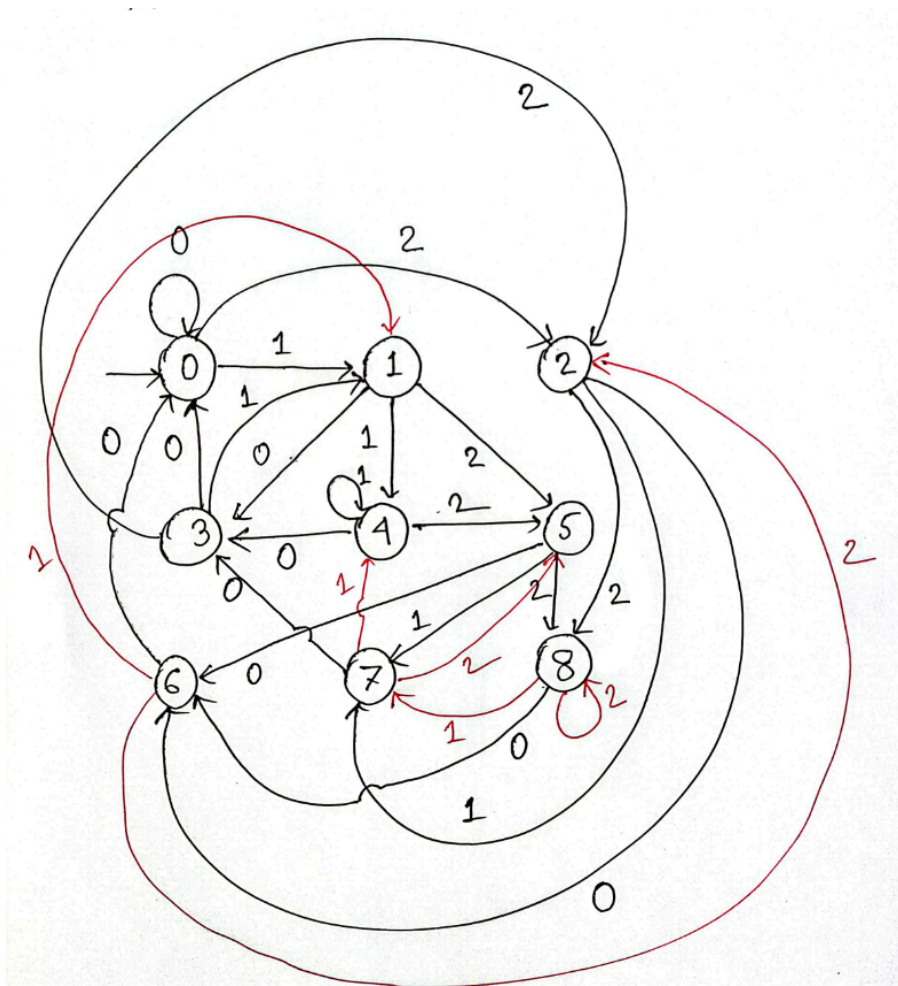
i)

In three base number system, the numbers which ends with 00, only those numbers are divisible by 9.

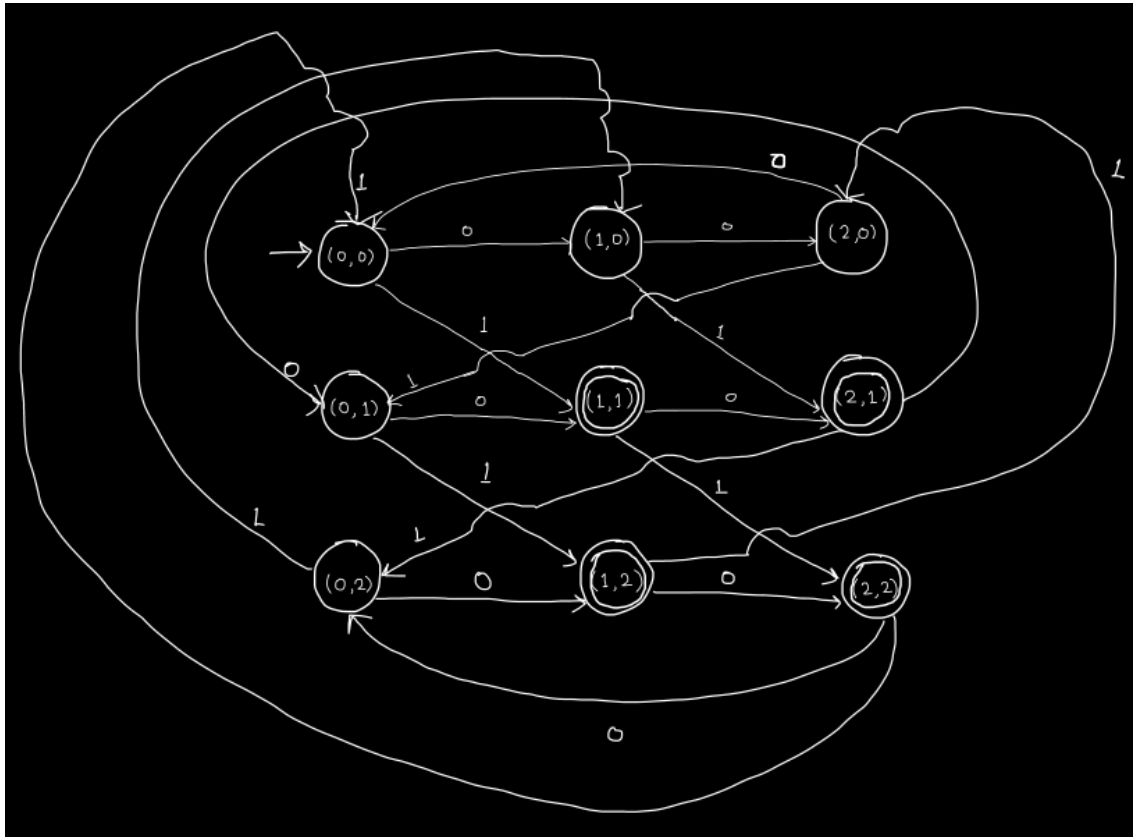


There is also a DFA with 9 states and 27 transitions - which is also correct.

The logic is - if there is a number  $x$ , in ternary representation, adding a 0 at the end will make the number  $3x$ , adding 1 at the end will make it  $3x+1$  and for 2 it will become  $3x+2$ .



j)



A correct solution is anything equivalent to the above. The DFA diagram is a bit clunky due to there being a lot of states and transitions. However, the idea is simple: the states are indexed by a pair  $(i, j)$  where  $0 \leq i, j \leq 2$ , and  $i, j$  are the length and the number of 1s seen so far modulo 3. Upon seeing a new letter  $i$  goes up by 1, while  $j$  goes up by 1 only if the new letter is a 1.

The main observation is that  $i^2 + j^2 = 2 \pmod 3$  iff  $i = 1, 2$  and  $j = 1, 2 \pmod 3$ . This gives us four accepting states.

Any DFA for this language necessarily requires 9 states. So, any DFA with fewer states is wrong.

#### **Problem 4**

- a) Yes
- b) 1100
- c) 0100
- d) Yes
- e) No