Solution: Surprise Quiz 03

Question 01: Determine the Bias from the 20-bit IEEE 754 Representation Format

Understanding the Format: The 20-bit IEEE 754 representation consists of:

- **1 bit** for the sign.
- **6 bits** for the exponent, as the bias exponent (63) fits within indices 1 to 7 and its binary representation is '111111' (6 bits).
- **13 bits** for the mantissa, as the remaining bits are allocated after the sign and exponent.

Bias Calculation: The bias is calculated as:

$$\text{Bias} = 2^{k-1} - 1 \quad \text{where } k \text{ is the number of exponent bits.}$$

Here, $k = 6$, so:

$$\text{Bias} = 2^{6-1} - 1 = 2^5 - 1 = 31.$$

Solutions:

i) Given hexadecimal: `0xFF123`, Bias Exponent: 63.

$$\text{Bias} = 31 \quad \text{(as calculated above).}$$

ii) Given hexadecimal: `0xE2143`, Bias Exponent: 49.

$$\text{Bias} = 31 \quad \text{(as calculated above).}$$

Convert $111.1011_2 \times 2^5$ to 32-bit IEEE-754 Floating Point Representation

Steps to Solve:

1. **Normalize the binary number:**

   $$111.1011_2 = 1.111011_2 \times 2^2$$

   Combine the normalization shift with the given exponent 2^5:

   $$111.1011_2 \times 2^5 = 1.111011_2 \times 2^7$$

2. **Determine the exponent with bias:** The bias for single precision is 127. Adding the actual exponent (7):

   $$\text{Exponent (E)} = 127 + 7 = 134$$

   Convert 134 to binary:

   $$134 = 10000110_2$$

3. **Extract the mantissa:** The fractional part after normalization ($1.\mathbf{111011}_2$) is:

$$\text{Mantissa} = \mathbf{11101100000000000000000}_2$$

(Padded with zeros to make 23 bits.)

4. **Combine the components:** The final 32-bit IEEE 754 representation is:

$$\text{Sign (S)} = 0 \quad \text{(Positive number)},$$

$$\text{Exponent (E)} = 10000110_2,$$

$$\text{Mantissa (M)} = 11101100000000000000000_2.$$

Thus:

$$\text{IEEE 754 Representation} = \mathbf{0\ 10000110\ 11101100000000000000000}.$$

5. **Convert to hexadecimal:** Group the bits into 4-bit chunks:

$$0100\ 0011\ 0111\ 0110\ 0000\ 0000\ 0000\ 0000$$

Convert to hexadecimal:

$$\text{Hexadecimal Representation: } 0x43760000$$