

## 5.2 Evaluation Orders for SDD's

“Dependency graphs” are a useful tool for determining an evaluation order for the attribute instances in a given parse tree. While an annotated parse tree shows the values of attributes, a dependency graph helps us determine how those values can be computed.

In this section, in addition to dependency graphs, we define two important classes of SDD's: the “S-attributed” and the more general “L-attributed” SDD's. The translations specified by these two classes fit well with the parsing methods we have studied, and most translations encountered in practice can be written to conform to the requirements of at least one of these classes.

### 5.2.1 Dependency Graphs

A *dependency graph* depicts the flow of information among the attribute instances in a particular parse tree; an edge from one attribute instance to another means that the value of the first is needed to compute the second. Edges express constraints implied by the semantic rules. In more detail:

- For each parse-tree node, say a node labeled by grammar symbol  $X$ , the dependency graph has a node for each attribute associated with  $X$ .
- Suppose that a semantic rule associated with a production  $p$  defines the value of synthesized attribute  $A.b$  in terms of the value of  $X.c$  (the rule may define  $A.b$  in terms of other attributes in addition to  $X.c$ ). Then, the dependency graph has an edge from  $X.c$  to  $A.b$ . More precisely, at every node  $N$  labeled  $A$  where production  $p$  is applied, create an edge to attribute  $b$  at  $N$ , from the attribute  $c$  at the child of  $N$  corresponding to this instance of the symbol  $X$  in the body of the production.<sup>2</sup>
- Suppose that a semantic rule associated with a production  $p$  defines the value of inherited attribute  $B.c$  in terms of the value of  $X.a$ . Then, the dependency graph has an edge from  $X.a$  to  $B.c$ . For each node  $N$  labeled  $B$  that corresponds to an occurrence of this  $B$  in the body of production  $p$ , create an edge to attribute  $c$  at  $N$  from the attribute  $a$  at the node  $M$

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<sup>2</sup>Since a node  $N$  can have several children labeled  $X$ , we again assume that subscripts distinguish among uses of the same symbol at different places in the production.

that corresponds to this occurrence of  $X$ . Note that  $M$  could be either the parent or a sibling of  $N$ .

**Example 5.4:** Consider the following production and rule:

PRODUCTION	SEMANTIC RULE
$E \rightarrow E_1 + T$	$E.val = E_1.val + T.val$

At every node  $N$  labeled  $E$ , with children corresponding to the body of this production, the synthesized attribute  $val$  at  $N$  is computed using the values of  $val$  at the two children, labeled  $E$  and  $T$ . Thus, a portion of the dependency graph for every parse tree in which this production is used looks like Fig. 5.6. As a convention, we shall show the parse tree edges as dotted lines, while the edges of the dependency graph are solid.

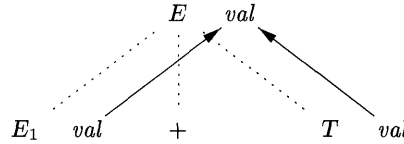


Figure 5.6:  $E.val$  is synthesized from  $E_1.val$  and  $E_2.val$

**Example 5.5:** An example of a complete dependency graph appears in Fig. 5.7. The nodes of the dependency graph, represented by the numbers 1 through 9, correspond to the attributes in the annotated parse tree in Fig. 5.5.

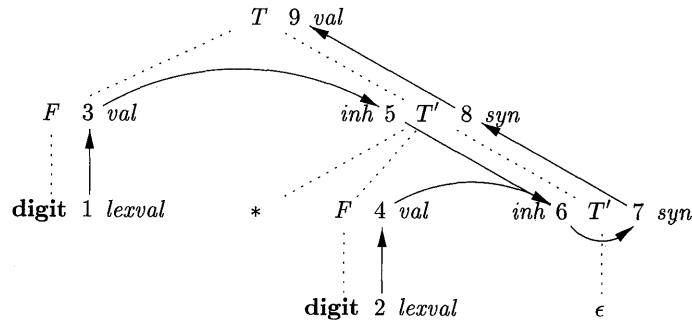


Figure 5.7: Dependency graph for the annotated parse tree of Fig. 5.5

Nodes 1 and 2 represent the attribute *lexval* associated with the two leaves labeled **digit**. Nodes 3 and 4 represent the attribute *val* associated with the two nodes labeled  $F$ . The edges to node 3 from 1 and to node 4 from 2 result

from the semantic rule that defines  $F.val$  in terms of **digit.lexval**. In fact,  $F.val$  equals **digit.lexval**, but the edge represents dependence, not equality.

Nodes 5 and 6 represent the inherited attribute  $T'.inh$  associated with each of the occurrences of nonterminal  $T'$ . The edge to 5 from 3 is due to the rule  $T'.inh = F.val$ , which defines  $T'.inh$  at the right child of the root from  $F.val$  at the left child. We see edges to 6 from node 5 for  $T'.inh$  and from node 4 for  $F.val$ , because these values are multiplied to evaluate the attribute  $inh$  at node 6.

Nodes 7 and 8 represent the synthesized attribute  $syn$  associated with the occurrences of  $T'$ . The edge to node 7 from 6 is due to the semantic rule  $T'.syn = T'.inh$  associated with production 3 in Fig. 5.4. The edge to node 8 from 7 is due to a semantic rule associated with production 2.

Finally, node 9 represents the attribute  $T.val$ . The edge to 9 from 8 is due to the semantic rule,  $T.val = T'.syn$ , associated with production 1.  $\square$

## 5.2.2 Ordering the Evaluation of Attributes

The dependency graph characterizes the possible orders in which we can evaluate the attributes at the various nodes of a parse tree. If the dependency graph has an edge from node  $M$  to node  $N$ , then the attribute corresponding to  $M$  must be evaluated before the attribute of  $N$ . Thus, the only allowable orders of evaluation are those sequences of nodes  $N_1, N_2, \dots, N_k$  such that if there is an edge of the dependency graph from  $N_i$  to  $N_j$ , then  $i < j$ . Such an ordering embeds a directed graph into a linear order, and is called a *topological sort* of the graph.

If there is any cycle in the graph, then there are no topological sorts; that is, there is no way to evaluate the SDD on this parse tree. If there are no cycles, however, then there is always at least one topological sort. To see why, since there are no cycles, we can surely find a node with no edge entering. For if there were no such node, we could proceed from predecessor to predecessor until we came back to some node we had already seen, yielding a cycle. Make this node the first in the topological order, remove it from the dependency graph, and repeat the process on the remaining nodes.

**Example 5.6:** The dependency graph of Fig. 5.7 has no cycles. One topological sort is the order in which the nodes have already been numbered: 1, 2,  $\dots$ , 9. Notice that every edge of the graph goes from a node to a higher-numbered node, so this order is surely a topological sort. There are other topological sorts as well, such as 1, 3, 5, 2, 4, 6, 7, 8, 9.  $\square$

## 5.2.3 S-Attributed Definitions

As mentioned earlier, given an SDD, it is very hard to tell whether there exist any parse trees whose dependency graphs have cycles. In practice, translations can be implemented using classes of SDD's that guarantee an evaluation order,

since they do not permit dependency graphs with cycles. Moreover, the two classes introduced in this section can be implemented efficiently in connection with top-down or bottom-up parsing.

The first class is defined as follows:

- An SDD is *S-attributed* if every attribute is synthesized.

**Example 5.7:** The SDD of Fig. 5.1 is an example of an S-attributed definition. Each attribute, *L.val*, *E.val*, *T.val*, and *F.val* is synthesized.  $\square$

When an SDD is S-attributed, we can evaluate its attributes in any bottom-up order of the nodes of the parse tree. It is often especially simple to evaluate the attributes by performing a postorder traversal of the parse tree and evaluating the attributes at a node *N* when the traversal leaves *N* for the last time. That is, we apply the function *postorder*, defined below, to the root of the parse tree (see also the box “Preorder and Postorder Traversals” in Section 2.3.4):

```

postorder(N) {
    for ( each child C of N, from the left ) postorder(C);
    evaluate the attributes associated with node N;
}

```

S-attributed definitions can be implemented during bottom-up parsing, since a bottom-up parse corresponds to a postorder traversal. Specifically, *postorder* corresponds exactly to the order in which an LR parser reduces a production body to its head. This fact will be used in Section 5.4.2 to evaluate synthesized attributes and store them on the stack during LR parsing, without creating the tree nodes explicitly.

### 5.2.4 L-Attributed Definitions

The second class of SDD's is called *L-attributed definitions*. The idea behind this class is that, between the attributes associated with a production body, dependency-graph edges can go from left to right, but not from right to left (hence “L-attributed”). More precisely, each attribute must be either

1. Synthesized, or
2. Inherited, but with the rules limited as follows. Suppose that there is a production  $A \rightarrow X_1X_2 \cdots X_n$ , and that there is an inherited attribute  $X_i.a$  computed by a rule associated with this production. Then the rule may use only:
  - (a) Inherited attributes associated with the head *A*.
  - (b) Either inherited or synthesized attributes associated with the occurrences of symbols  $X_1, X_2, \dots, X_{i-1}$  located to the left of  $X_i$ .

- (c) Inherited or synthesized attributes associated with this occurrence of  $X_i$  itself, but only in such a way that there are no cycles in a dependency graph formed by the attributes of this  $X_i$ .

**Example 5.8:** The SDD in Fig. 5.4 is L-attributed. To see why, consider the semantic rules for inherited attributes, which are repeated here for convenience:

PRODUCTION	SEMANTIC RULE
$T \rightarrow F T'$	$T'.inh = F.val$
$T' \rightarrow * F T'_1$	$T'_1.inh = T'.inh \times F.val$

The first of these rules defines the inherited attribute  $T'.inh$  using only  $F.val$ , and  $F$  appears to the left of  $T'$  in the production body, as required. The second rule defines  $T'_1.inh$  using the inherited attribute  $T'.inh$  associated with the head, and  $F.val$ , where  $F$  appears to the left of  $T'_1$  in the production body.

In each of these cases, the rules use information “from above or from the left,” as required by the class. The remaining attributes are synthesized. Hence, the SDD is L-attributed.  $\square$

**Example 5.9:** Any SDD containing the following production and rules cannot be L-attributed:

PRODUCTION	SEMANTIC RULES
$A \rightarrow B C$	$A.s = B.b;$ $B.i = f(C.c, A.s)$

The first rule,  $A.s = B.b$ , is a legitimate rule in either an S-attributed or L-attributed SDD. It defines a synthesized attribute  $A.s$  in terms of an attribute at a child (that is, a symbol within the production body).

The second rule defines an inherited attribute  $B.i$ , so the entire SDD cannot be S-attributed. Further, although the rule is legal, the SDD cannot be L-attributed, because the attribute  $C.c$  is used to help define  $B.i$ , and  $C$  is to the right of  $B$  in the production body. While attributes at siblings in a parse tree may be used in L-attributed SDD's, they must be to the left of the symbol whose attribute is being defined.  $\square$

## 5.2.5 Semantic Rules with Controlled Side Effects

In practice, translations involve side effects: a desk calculator might print a result; a code generator might enter the type of an identifier into a symbol table. With SDD's, we strike a balance between attribute grammars and translation schemes. Attribute grammars have no side effects and allow any evaluation order consistent with the dependency graph. Translation schemes impose left-to-right evaluation and allow semantic actions to contain any program fragment; translation schemes are discussed in Section 5.4.

We shall control side effects in SDD's in one of the following ways:

- Permit incidental side effects that do not constrain attribute evaluation. In other words, permit side effects when attribute evaluation based on any topological sort of the dependency graph produces a “correct” translation, where “correct” depends on the application.
- Constrain the allowable evaluation orders, so that the same translation is produced for any allowable order. The constraints can be thought of as implicit edges added to the dependency graph.

As an example of an incidental side effect, let us modify the desk calculator of Example 5.1 to print a result. Instead of the rule  $L.val = E.val$ , which saves the result in the synthesized attribute  $L.val$ , consider:

	PRODUCTION	SEMANTIC RULE
1)	$L \rightarrow E \mathbf{n}$	$print(E.val)$

Semantic rules that are executed for their side effects, such as  $print(E.val)$ , will be treated as the definitions of dummy synthesized attributes associated with the head of the production. The modified SDD produces the same translation under any topological sort, since the print statement is executed at the end, after the result is computed into  $E.val$ .

**Example 5.10:** The SDD in Fig. 5.8 takes a simple declaration  $D$  consisting of a basic type  $T$  followed by a list  $L$  of identifiers.  $T$  can be **int** or **float**. For each identifier on the list, the type is entered into the symbol-table entry for the identifier. We assume that entering the type for one identifier does not affect the symbol-table entry for any other identifier. Thus, entries can be updated in any order. This SDD does not check whether an identifier is declared more than once; it can be modified to do so.

PRODUCTION	SEMANTIC RULES
1) $D \rightarrow T L$	$L.inh = T.type$
2) $T \rightarrow \mathbf{int}$	$T.type = \text{integer}$
3) $T \rightarrow \mathbf{float}$	$T.type = \text{float}$
4) $L \rightarrow L_1, \mathbf{id}$	$L_1.inh = L.inh$ $addType(\mathbf{id}.entry, L.inh)$
5) $L \rightarrow \mathbf{id}$	$addType(\mathbf{id}.entry, L.inh)$

Figure 5.8: Syntax-directed definition for simple type declarations

Nonterminal  $D$  represents a declaration, which, from production 1, consists of a type  $T$  followed by a list  $L$  of identifiers.  $T$  has one attribute,  $T.type$ , which is the type in the declaration  $D$ . Nonterminal  $L$  also has one attribute, which we call  $inh$  to emphasize that it is an inherited attribute. The purpose of  $L.inh$



### 5.2.6 Exercises for Section 5.2

**Exercise 5.2.1:** What are all the topological sorts for the dependency graph of Fig. 5.7?

**Exercise 5.2.2:** For the SDD of Fig. 5.8, give annotated parse trees for the following expressions:

- a) `int a, b, c.`
- b) `float w, x, y, z.`

**Exercise 5.2.3:** Suppose that we have a production  $A \rightarrow BCD$ . Each of the four nonterminals  $A$ ,  $B$ ,  $C$ , and  $D$  have two attributes:  $s$  is a synthesized attribute, and  $i$  is an inherited attribute. For each of the sets of rules below, tell whether (i) the rules are consistent with an S-attributed definition (ii) the rules are consistent with an L-attributed definition, and (iii) whether the rules are consistent with any evaluation order at all?

- a)  $A.s = B.i + C.s$ .
- b)  $A.s = B.i + C.s$  and  $D.i = A.i + B.s$ .
- c)  $A.s = B.s + D.s$ .
- ! d)  $A.s = D.i$ ,  $B.i = A.s + C.s$ ,  $C.i = B.s$ , and  $D.i = B.i + C.i$ .

**! Exercise 5.2.4:** This grammar generates binary numbers with a “decimal” point:

$$\begin{aligned} S &\rightarrow L . L \mid L \\ L &\rightarrow L B \mid B \\ B &\rightarrow 0 \mid 1 \end{aligned}$$

Design an L-attributed SDD to compute  $S.val$ , the decimal-number value of an input string. For example, the translation of string `101.101` should be the decimal number 5.625. *Hint:* use an inherited attribute  $L.side$  that tells which side of the decimal point a bit is on.

**!! Exercise 5.2.5:** Design an S-attributed SDD for the grammar and translation described in Exercise 5.2.4.

**!! Exercise 5.2.6:** Implement Algorithm 3.23, which converts a regular expression into a nondeterministic finite automaton, by an L-attributed SDD on a top-down parsable grammar. Assume that there is a token `char` representing any character, and that `char.lexval` is the character it represents. You may also assume the existence of a function `new()` that returns a new state, that is, a state never before returned by this function. Use any convenient notation to specify the transitions of the NFA.