Game Playing

Today's class

- Game playing
  - State of the art and resources
  - Framework
- Game trees
  - Minimax
  - Alpha-beta pruning
  - Adding randomness

Why study games?

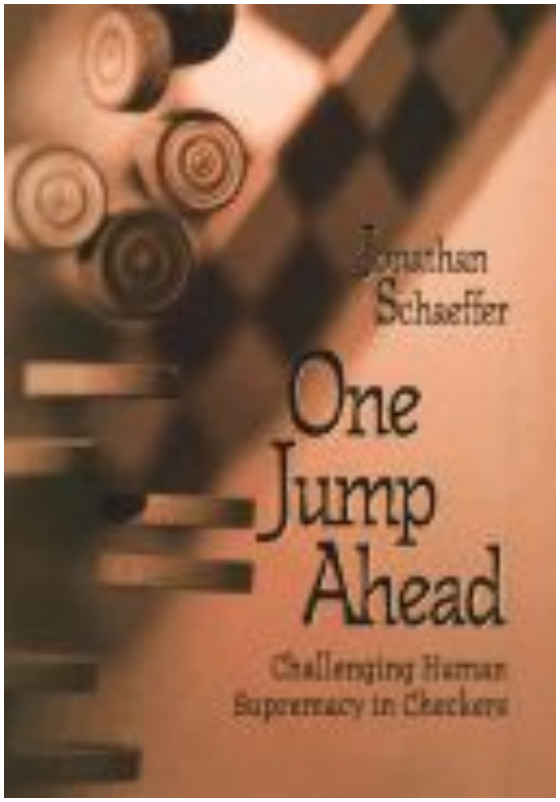
- Clear criteria for success
- Offer an opportunity to study problems involving {hostile, adversarial, competing} agents.
- Historical reasons
- Fun
- Interesting, hard problems which require minimal "initial structure"
- Games often define very large search spaces
  - chess 35^{100} nodes in search tree, 10^{40} legal states

State of the art

- How good are computer game players?
  - **Chess**:
    - Deep Blue beat Gary Kasparov in 1997
    - Garry Kasparav vs. Deep Junior (Feb 2003): tie!
    - Kasparov vs. X3D Fritz (November 2003): tie! http://www.thechessdrum.net/tournaments/Kasparov-X3DFritz/index.html
    - Deep Fritz beat world champion Vladimir Kramnik (2006)
  - **Checkers**: Chinook (an AI program with a *very large* endgame database) is the world champion and can provably never be beaten. Retired in 1995
  - **Go**: Computer players have finally reached tournament-level play
  - **Bridge**: "Expert-level" computer players exist (but no world champions yet!)
- Good places to learn more:
  - http://www.cs.ualberta.ca/~games/
  - http://www.cs.unimass.nl/icga

Chinook

- Chinook is the World Man-Machine Checkers Champion, developed by researchers at the University of Alberta.
- It earned this title by competing in human tournaments, winning the right to play for the (human) world championship, and eventually defeating the best players in the world.
- Visit http://www.cs.ualberta.ca/~chinook/ to play a version of Chinook over the Internet.
- The developers have fully analyzed the game of checkers, and can provably *never* be beaten (http://www.sciencemag.org/cgi/content/abstract/1144079v1)
- "One Jump Ahead: Challenging Human Supremacy in Checkers" Jonathan Schaeffer, University of Alberta (496 pages, Springer. $34.95, 1998).



Typical case

- 2-person game
- Players alternate moves
- **Zero-sum**: one player's loss is the other's gain
- **Perfect information**: both players have access to complete information about the state of the game. No information is hidden from either player.
- No chance (e.g., using dice) involved
- Examples: Tic-Tac-Toe, Checkers, Chess, Go, Nim, Othello
- Not: Bridge, Solitaire, Backgammon, ...

How to play a game

- A way to play such a game is to:
  - Consider all the legal moves you can make
  - Compute the new position resulting from each move
  - Evaluate each resulting position and determine which is best
  - Make that move
  - Wait for your opponent to move and repeat
- Key problems are:
  - Representing the "board"
  - Generating all legal next boards
  - Evaluating a position

Evaluation function

- **Evaluation function** or **static evaluator** is used to evaluate the "goodness" of a game position.
  - Contrast with heuristic search where the evaluation function was a non-negative estimate of the cost from the start node to a goal and passing through the given node
- The zero-sum assumption allows us to use a single evaluation function to describe the goodness of a board with respect to both players.
  - **f(n) >> 0**: position n good for me and bad for you
  - **f(n) << 0**: position n bad for me and good for you
  - **f(n) near 0**: position n is a neutral position
  - **f(n) = +infinity**: win for me
  - **f(n) = -infinity**: win for you

Evaluation function examples

- Example of an evaluation function for Tic-Tac-Toe:

  f(n) = [# of 3-lengths open for me] - [# of 3-lengths open for you]

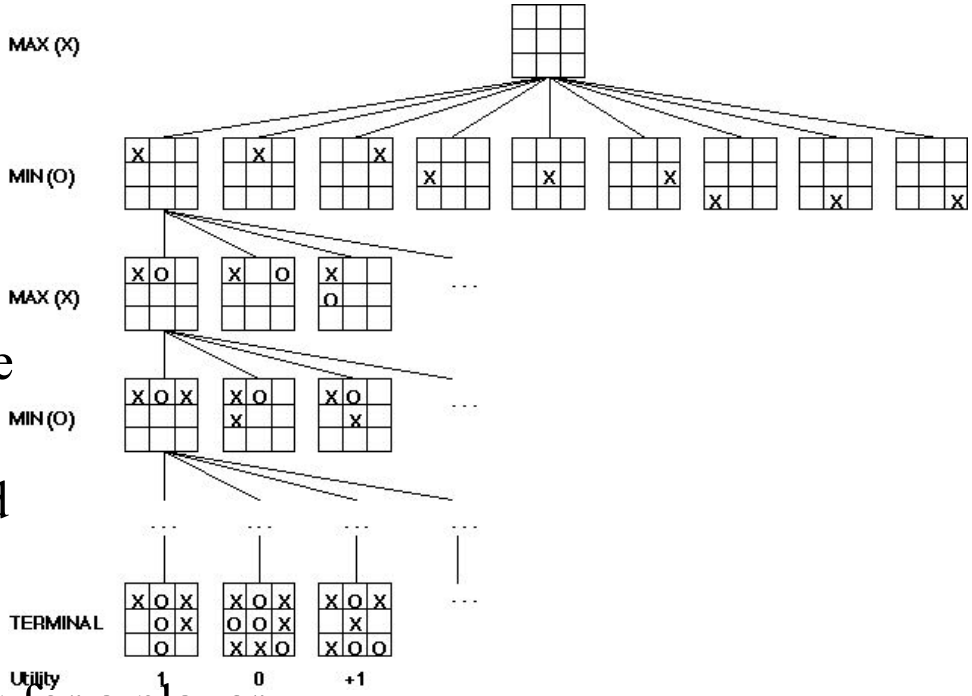
  where a 3-length is a complete row, column, or diagonal

- Alan Turing's function for chess
  - **f(n) = w(n)/b(n)** where w(n) = sum of the point value of white's pieces and b(n) = sum of black's
- Most evaluation functions are specified as a weighted sum of position features:

  $f(n) = w_1 * feat_1(n) + w_2 * feat_2(n) + ... + w_n * feat_k(n)$

- Example features for chess are piece count, piece placement, squares controlled, etc.
- Deep Blue had over 8000 features in its evaluation function

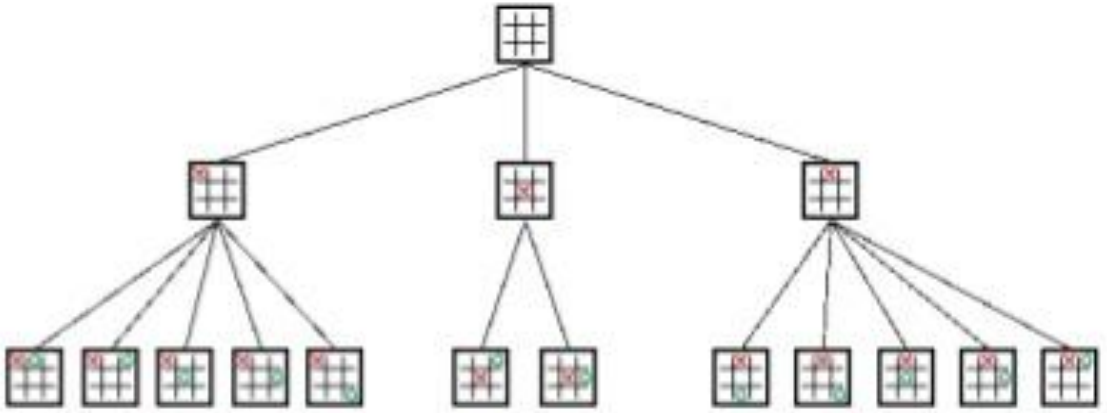
Game trees



- Problem spaces for typical games are represented as trees
- Root node represents the current board configuration; player must decide the best single move to make next
- **Static evaluator function** rates a board position. f(board) = real number with f>0 “white” (me), f<0 for black (you)
- Arcs represent the possible legal moves for a player
- If it is **my turn** to move, then the root is labeled a "**MAX**" node; otherwise it is labeled a "**MIN**" node, indicating **my opponent's turn**.
- Each level of the tree has nodes that are all MAX or all MIN; nodes at level i are of the opposite kind from those at level i+1

MinMax - Overview

- Search tree
  - *Squares* represent decision states (ie- after a move)
  - *Branches* are decisions (ie- the move)
  - Start at root
  - Nodes at end are leaf nodes
  - Ex: Tic-Tac-Toe (symmetrical positions removed)

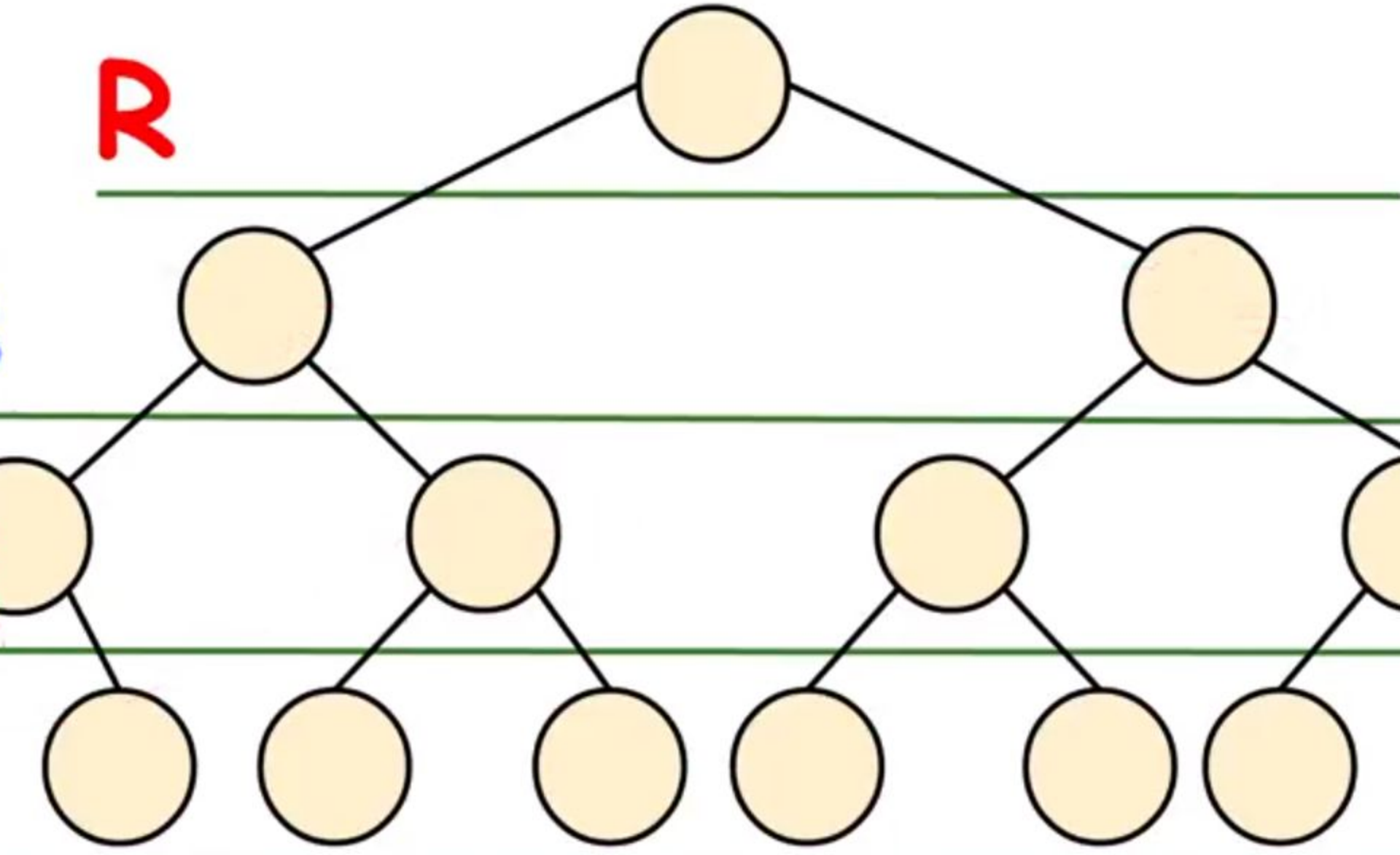


- Unlike binary trees can have any number of children
  - Depends on the game situation
- Levels usually called *plies* (a *ply* is one level)
  - Each ply is where "turn" switches to other player
- Players called *Min* and *Max* (next)

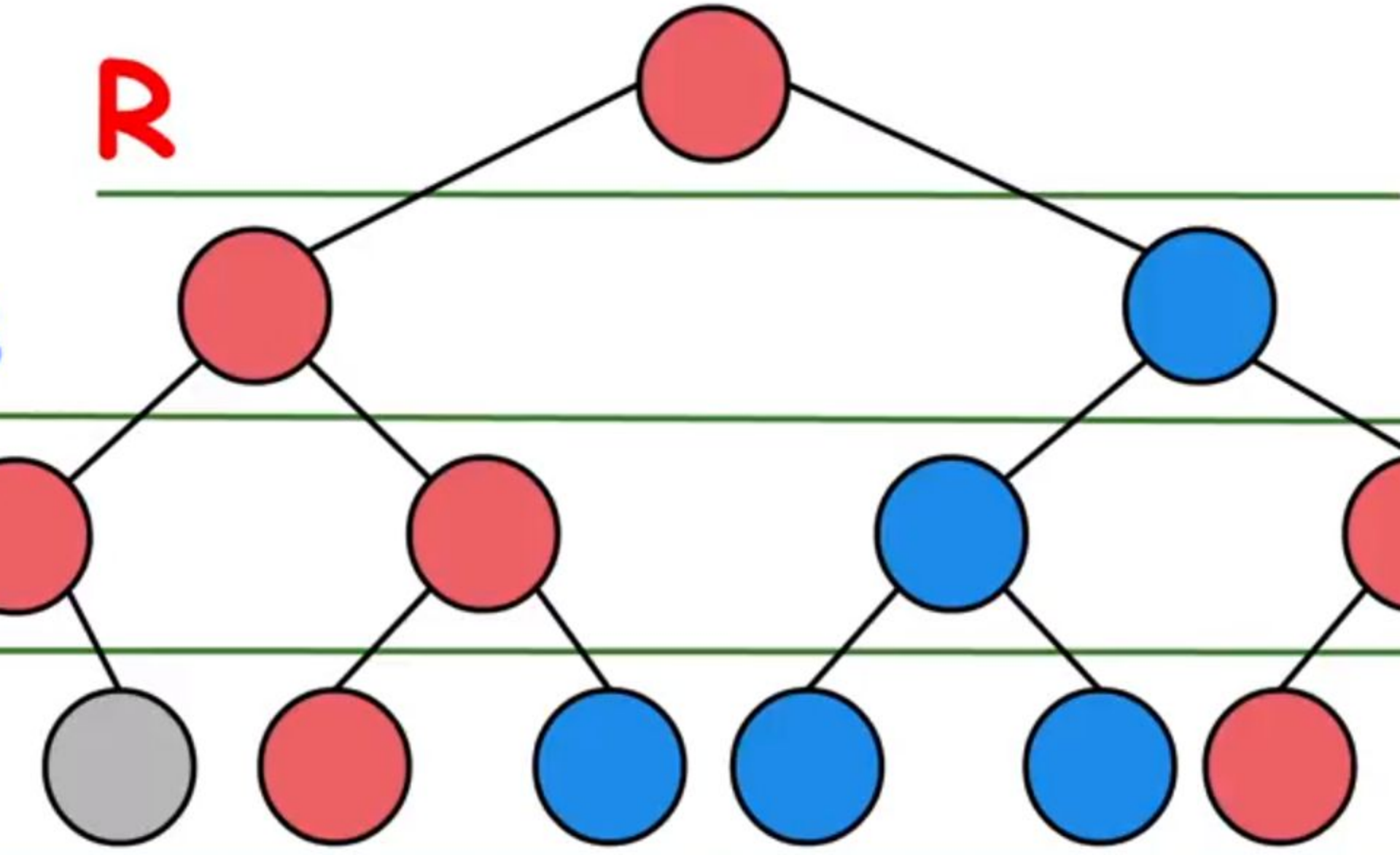
Minimax procedure

- Create start node as a MAX node with current board configuration
- Expand nodes down to some **depth** (a.k.a. **ply**) of lookahead in the game
- Apply the evaluation function at each of the leaf nodes
- "Back up" values for each of the non-leaf nodes until a value is computed for the root node
  - At MIN nodes, the backed-up value is the **minimum** of the values associated with its children.
  - At MAX nodes, the backed-up value is the **maximum** of the values associated with its children.
- Pick the operator associated with the child node whose backed-up value determined the value at the root

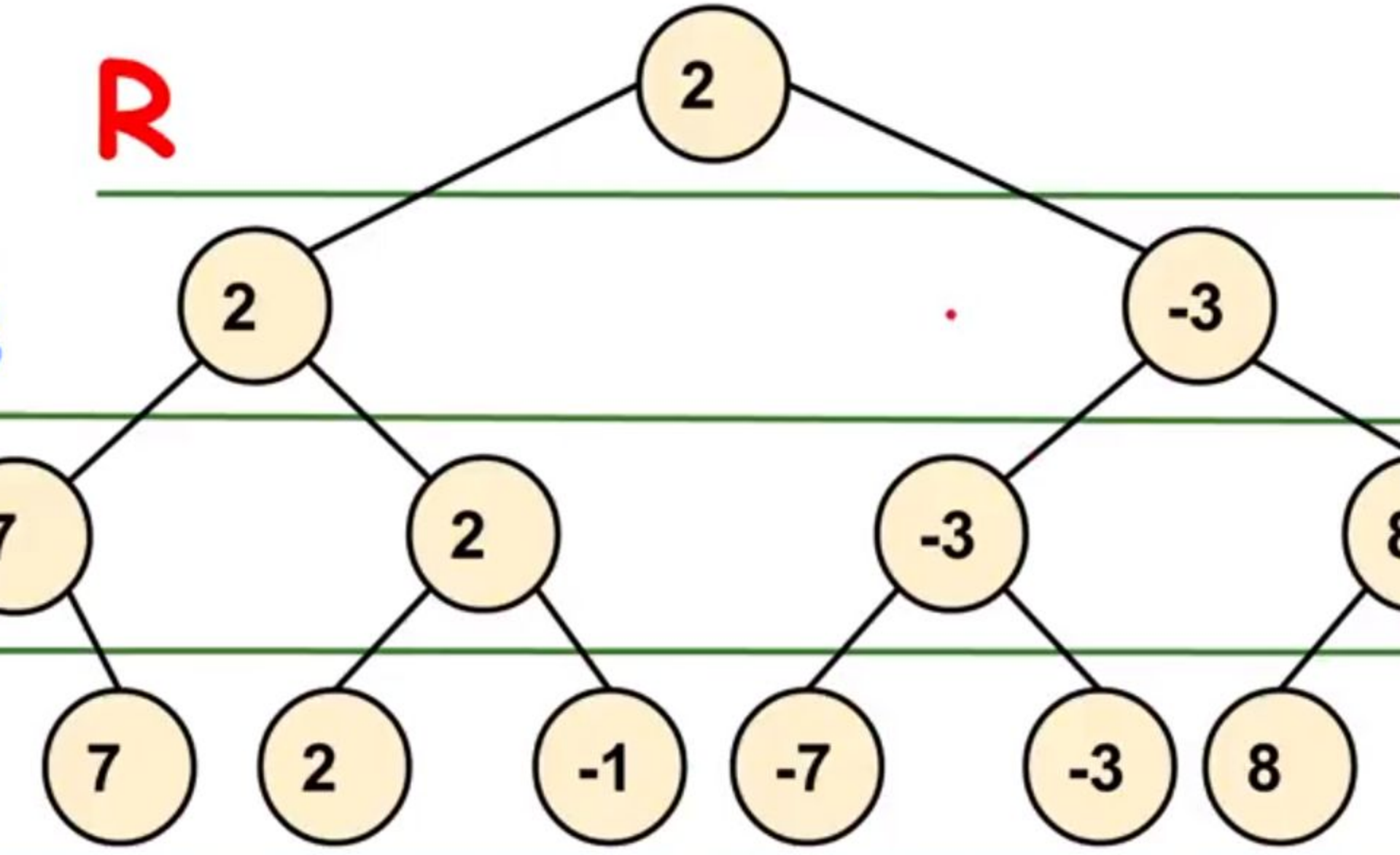
Minimax



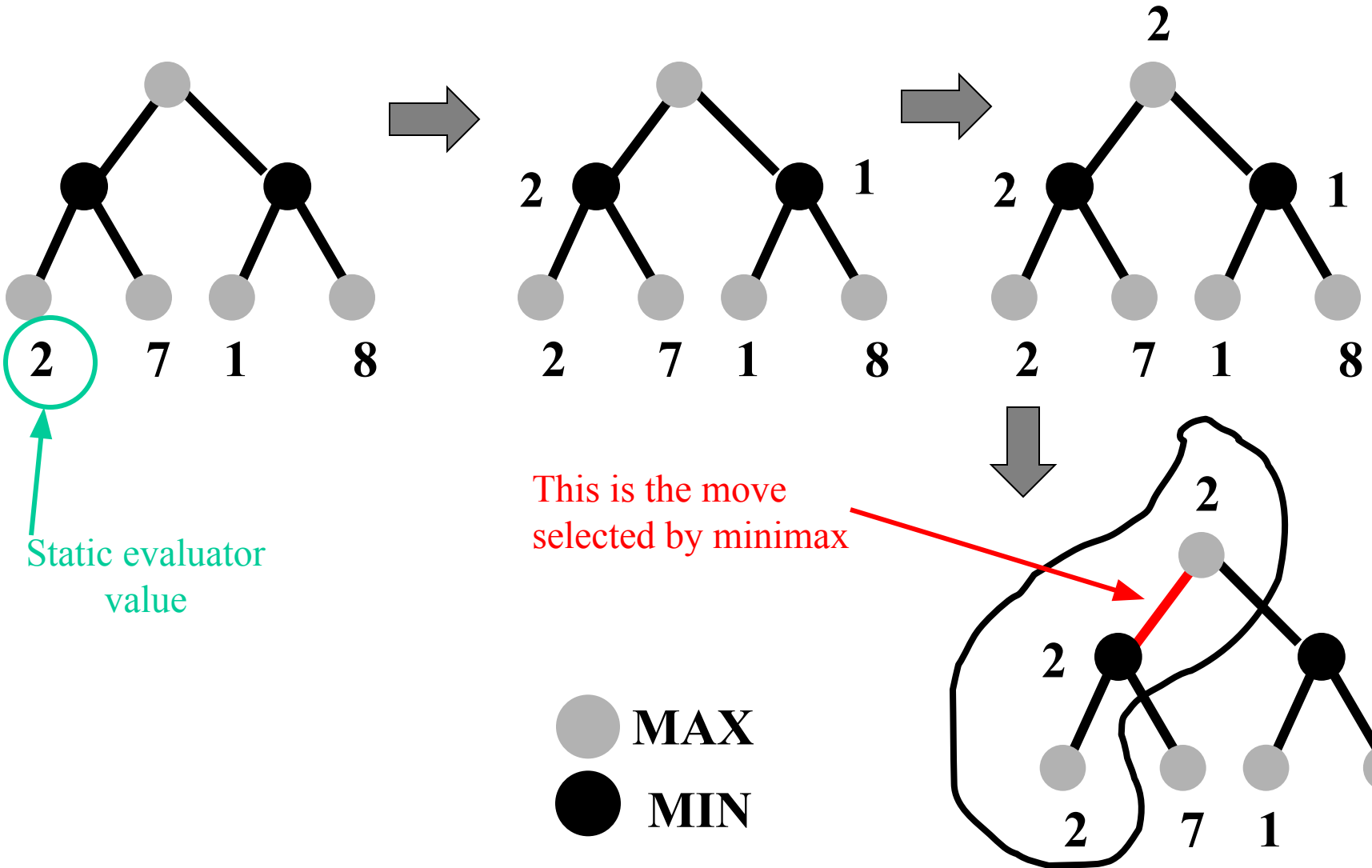
Minimax

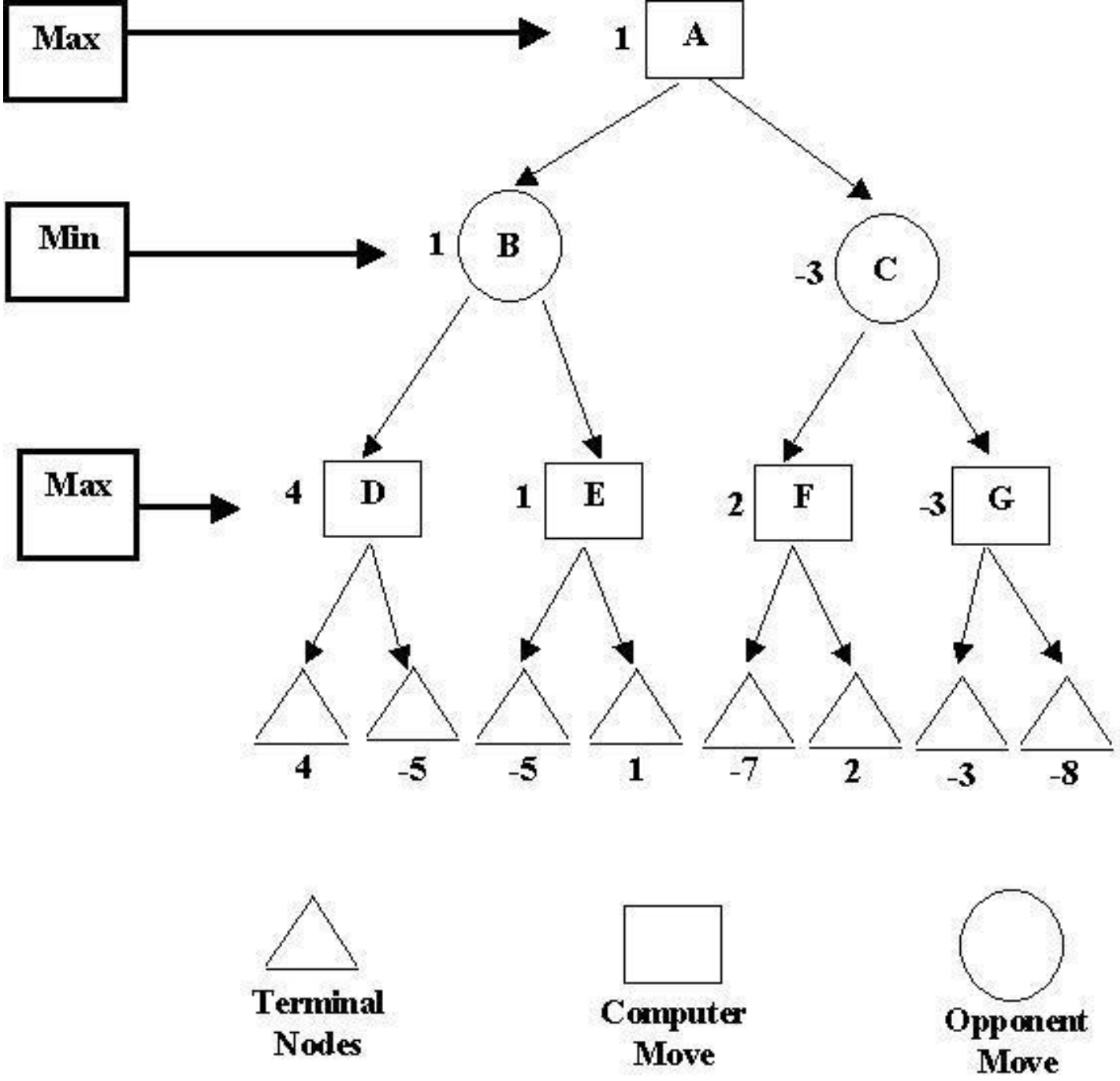


Minimax

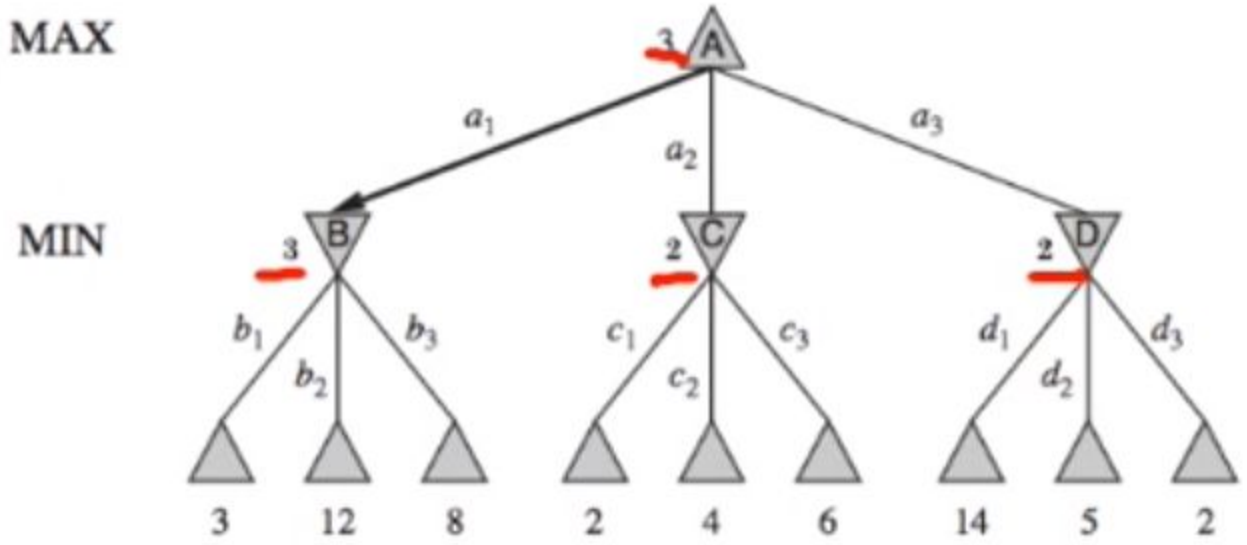


Minimax Algorithm



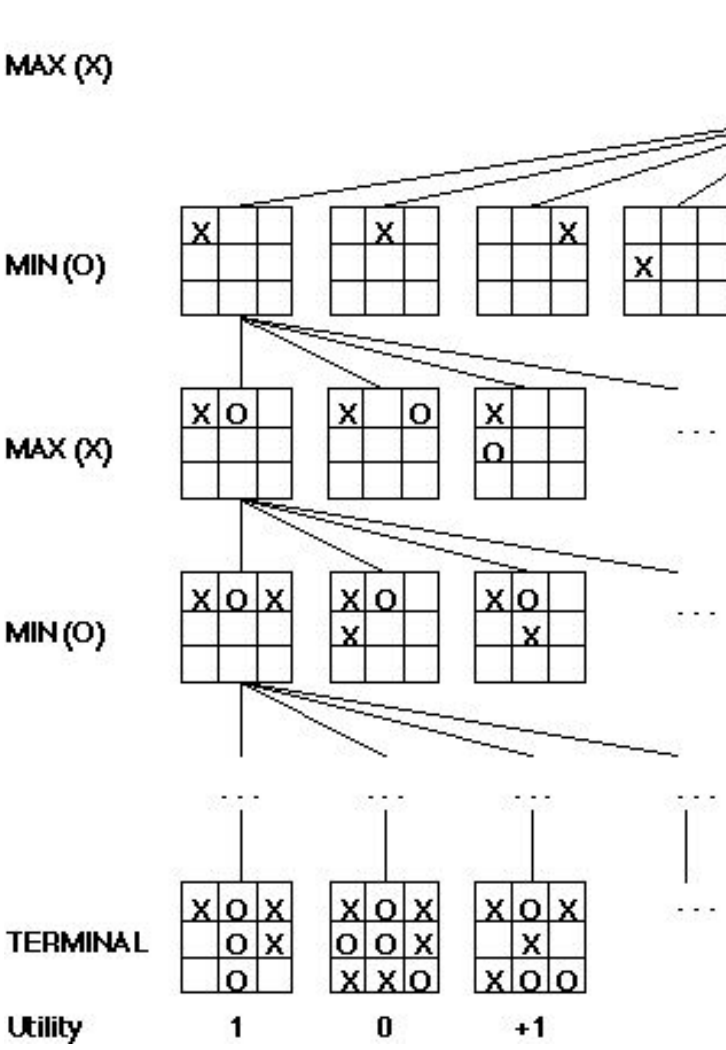

Max
1 A
Min
1 B
-3 C
Max
4 D
1 E
2 F
-3 G
4
-5
-5
1
-7
2
-3
-8
Terminal Nodes
Computer Move
Opponent Move

Minimax
Picking my best move against your best move



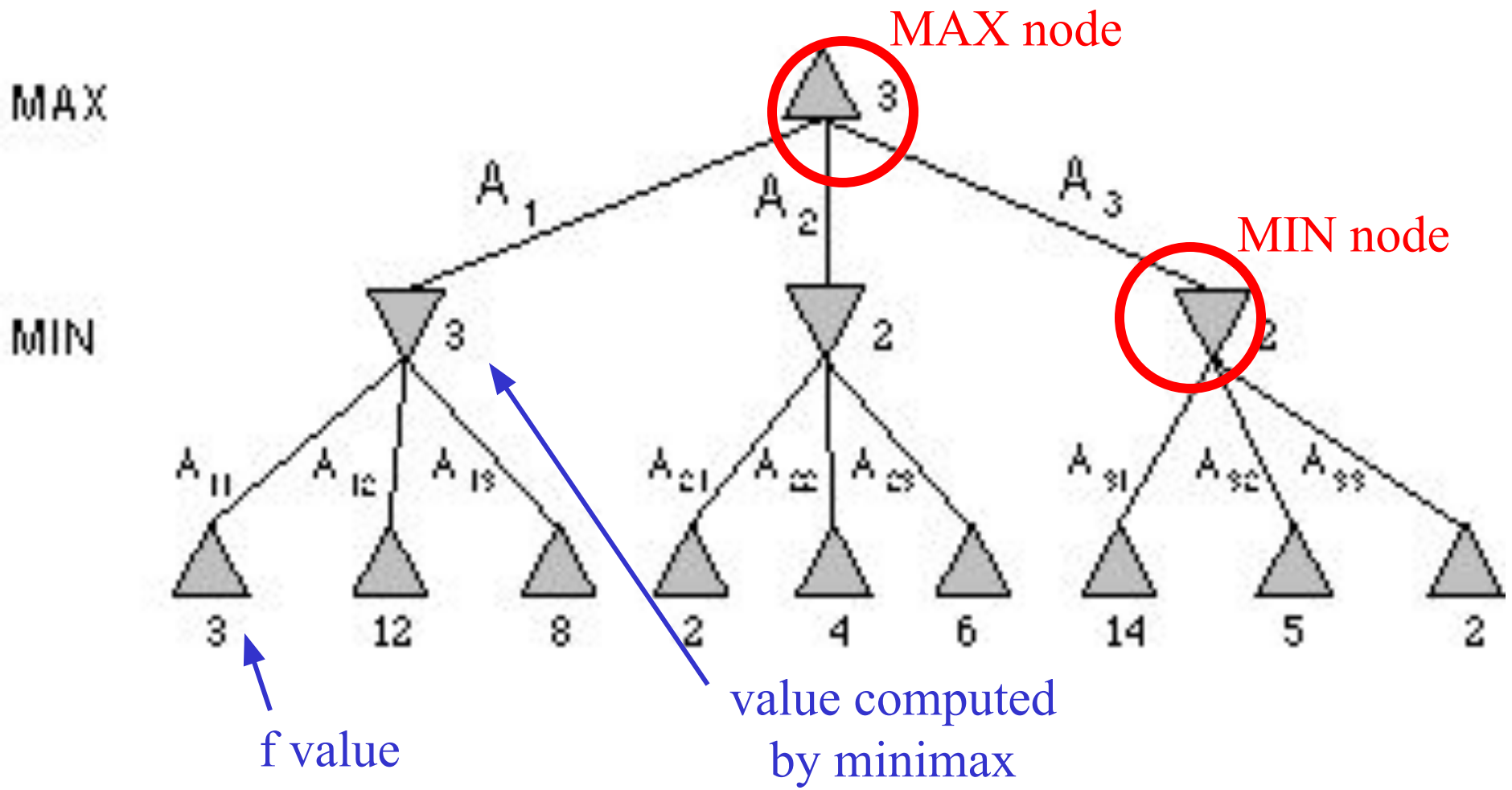
$$minimax(s) = \begin{cases} utility(s) & \text{if } terminal(s) \\ \max_{a \in action(s)} minimax(result(s,a)) & \text{if } player(s) = MAX \\ \min_{a \in action(s)} minimax(result(s,a)) & \text{if } player(s) = MIN \end{cases}$$

Partial Game Tree for Tic-Tac-Toe



- f(n) = +1 if the position is a win for X.
- f(n) = -1 if the position is a win for O.
- f(n) = 0 if the position is a draw.

Minimax Tree



Minimax
Discussion

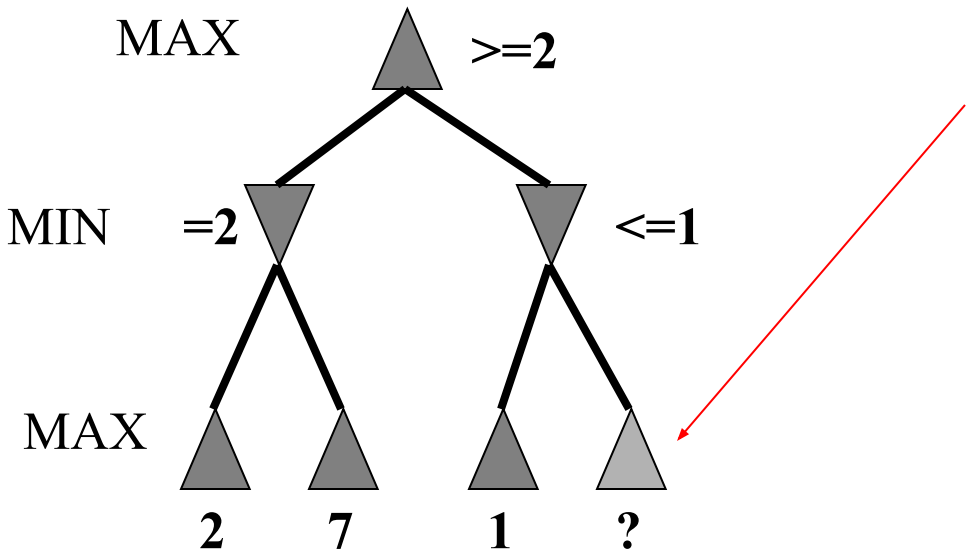
- Complete depth first exploration
- Depth m with b legal moves. $O(b^m)$
- Space complexity (memory) $O(bm)$
- Chess: $m \approx 35$; on average:$50 \leq b \leq 100$
- Impractical for most games, but basis of other algs.

Nim Game Tree

- Let us take an example of a simple game 5- stone Nim. So this game is played with two players and a pile of stones. Suppose we have 15 stones to start with. Each player removes either one or two stones from the pile and the player who removes the last stone wins the game. This is the description of the game of the 5-stone Nim where each player can remove one or two stones at a time. Let us draw the game tree of this game.

Alpha-beta pruning

- We can improve on the performance of the minimax algorithm through **alpha-beta pruning**
- Basic idea: *"If you have an idea that is surely bad, don't take the time to see how truly awful it is."* -- Pat Winston

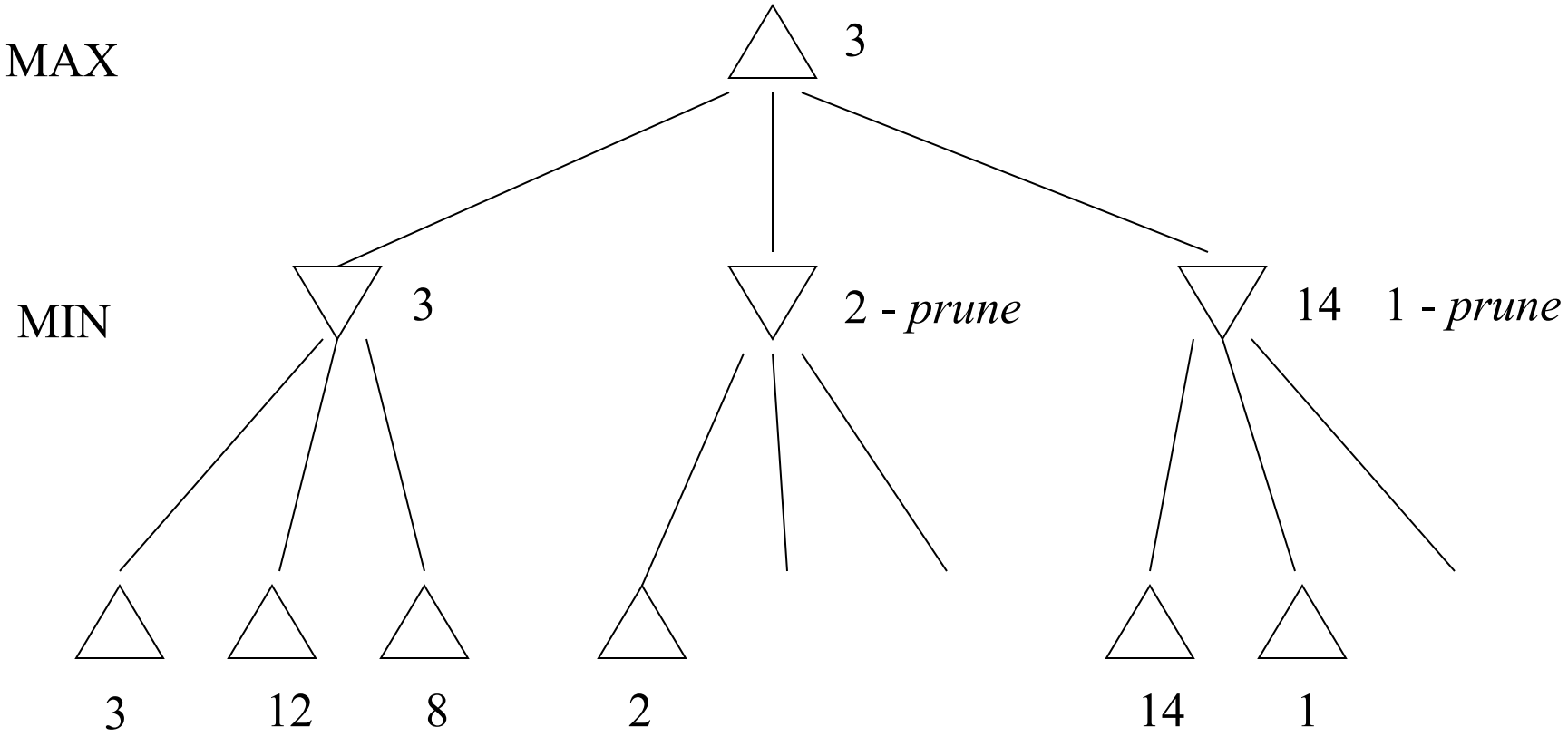


- We don't need to compute the value at this node.
- No matter what it is, it can't affect the value of the root node.

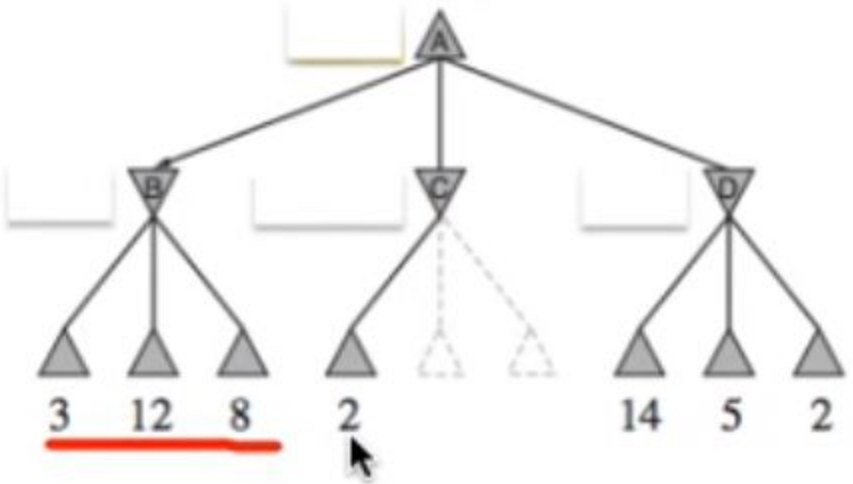
Alpha-beta pruning

- Traverse the search tree in depth-first order
- At each **MAX** node n, **alpha(n)** = maximum value found so far
- At each **MIN** node n, **beta(n)** = minimum value found so far
  - Note: The alpha values start at -infinity and only increase, while beta values start at +infinity and only decrease.
- **Beta cutoff**: Given a MAX node n, cut off the search below n (i.e., don't generate or examine any more of n's children) if alpha(n) >= beta(i) for some MIN node ancestor i of n.
- **Alpha cutoff:** stop searching below MIN node n if beta(n) <= alpha(i) for some MAX node ancestor i of n.

Alpha-beta example



Alha-Beta prunning
Intuition



Do we need to expand all nodes?

$$
\begin{aligned}
minimax(root) &= max(min(3,12,8), min(2,x,y), min(14,5,2)) \\
&= max(3, min(2,x,y), 2) \\
&= max(3,z,2) \\
&= 3
\end{aligned}
$$

Do we need z?

Alpha-beta algorithm

```
function MAX-VALUE (state, α, β)
    ;; α = best MAX so far; β = best MIN
if TERMINAL-TEST (state) then return UTILITY(state)
v := -∞
for each s in SUCCESSORS (state) do
    v := MAX (v, MIN-VALUE (s, α, β))
    if v >= β then return v
    α := MAX (α, v)
end
return v

function MIN-VALUE (state, α, β)
if TERMINAL-TEST (state) then return UTILITY(state)
v := ∞
for each s in SUCCESSORS (state) do
    v := MIN (v, MAX-VALUE (s, α, β))
    if v <= α then return v
    β := MIN (β, v)
end
return v
```

Alpha–Beta prunning

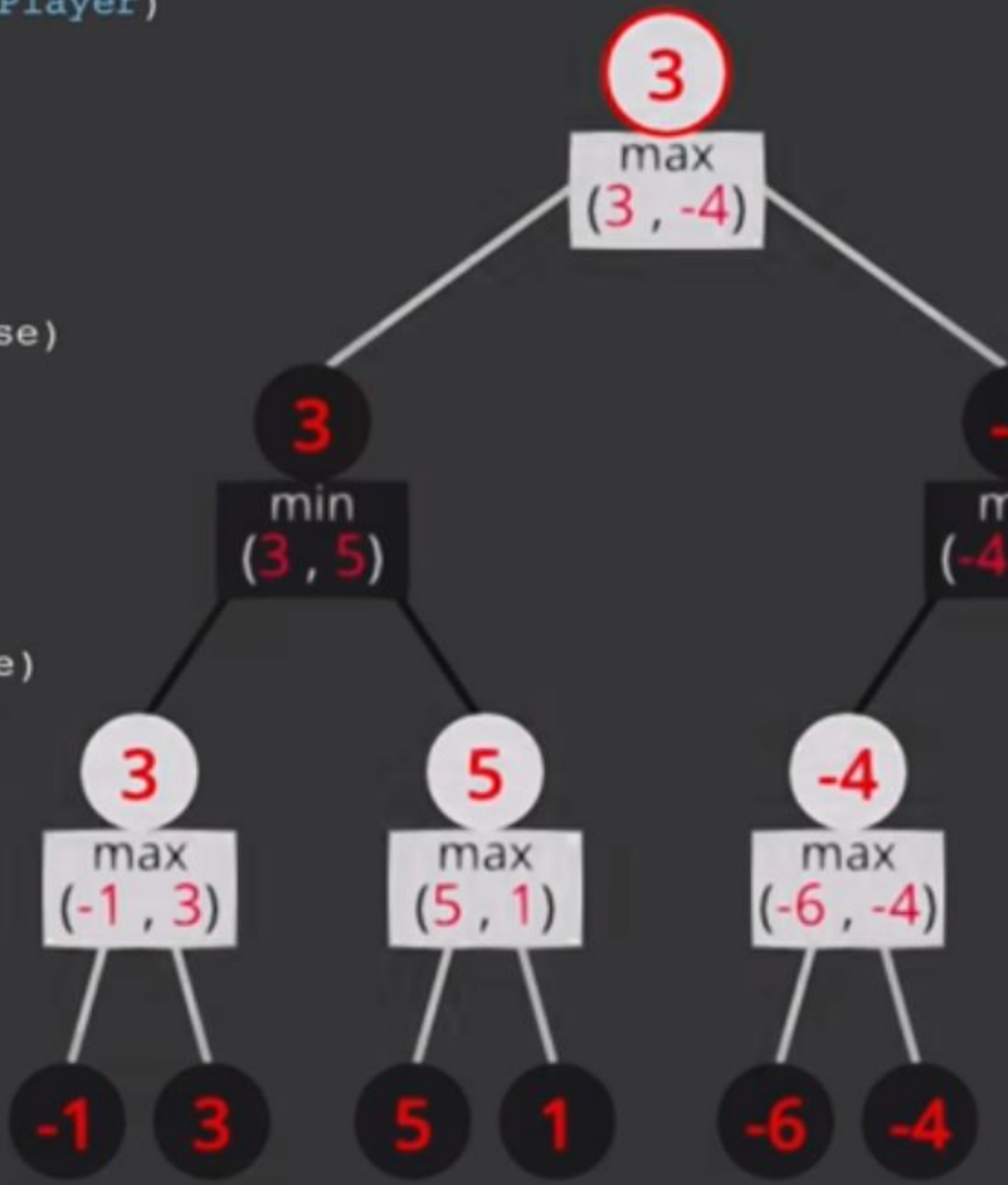
Two values:

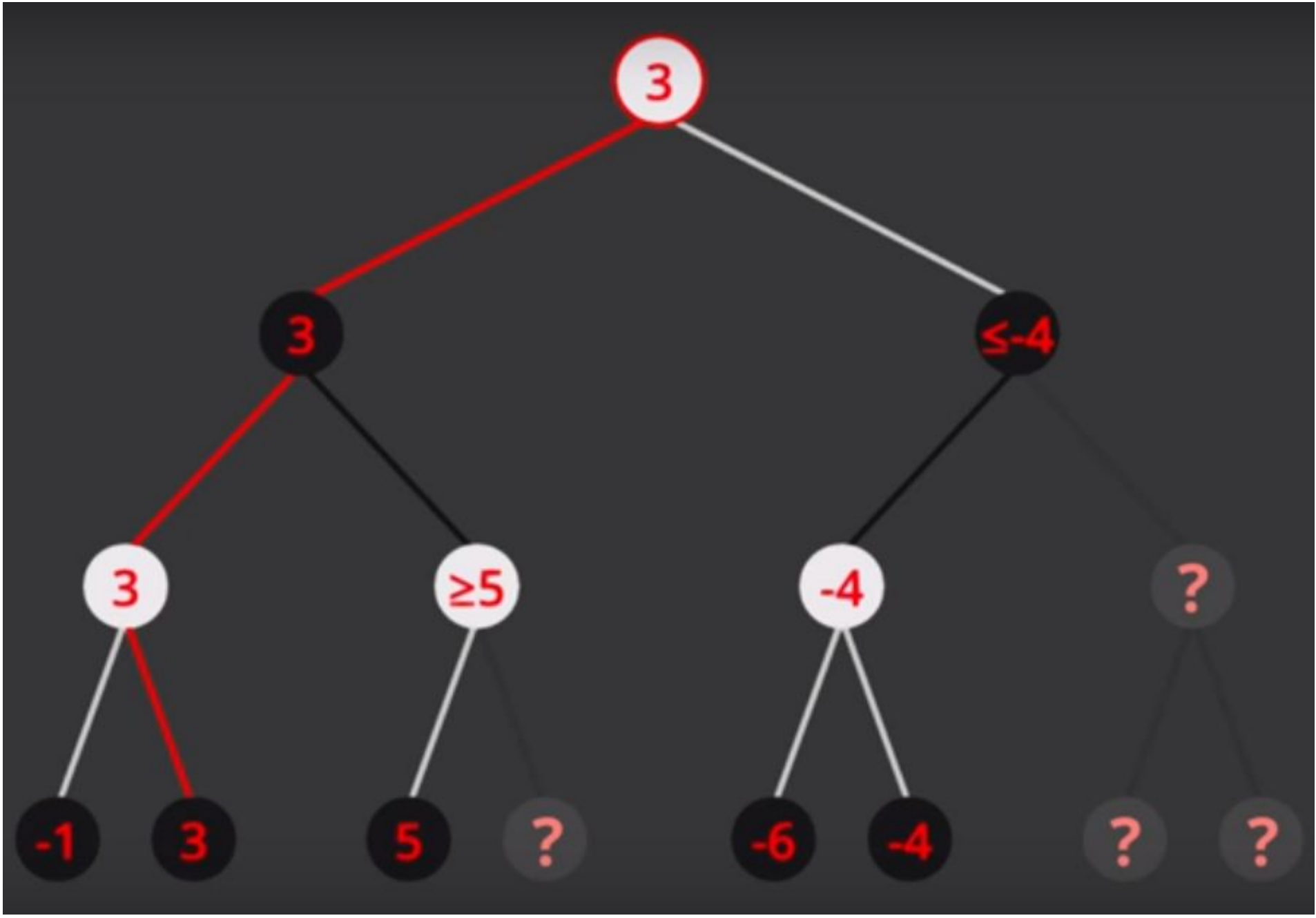
- $\alpha =$ value of best choice so far for MAX (highest-value)
- $\beta =$ value of best choice so far for MIN (lowest-value)
- Each node keeps track of its $[\alpha, \beta]$ values

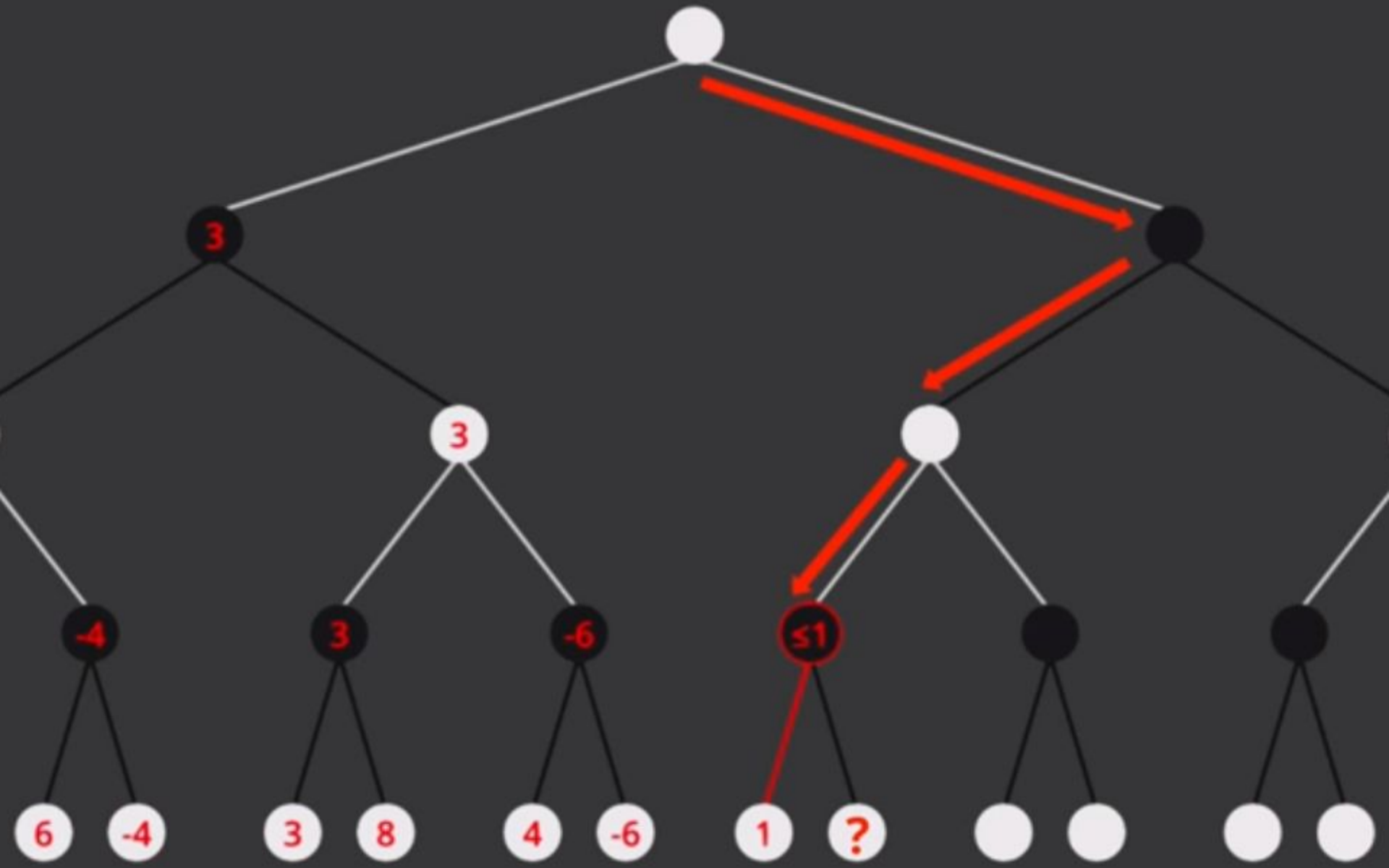
Alpha-Beta Prunning
Properties

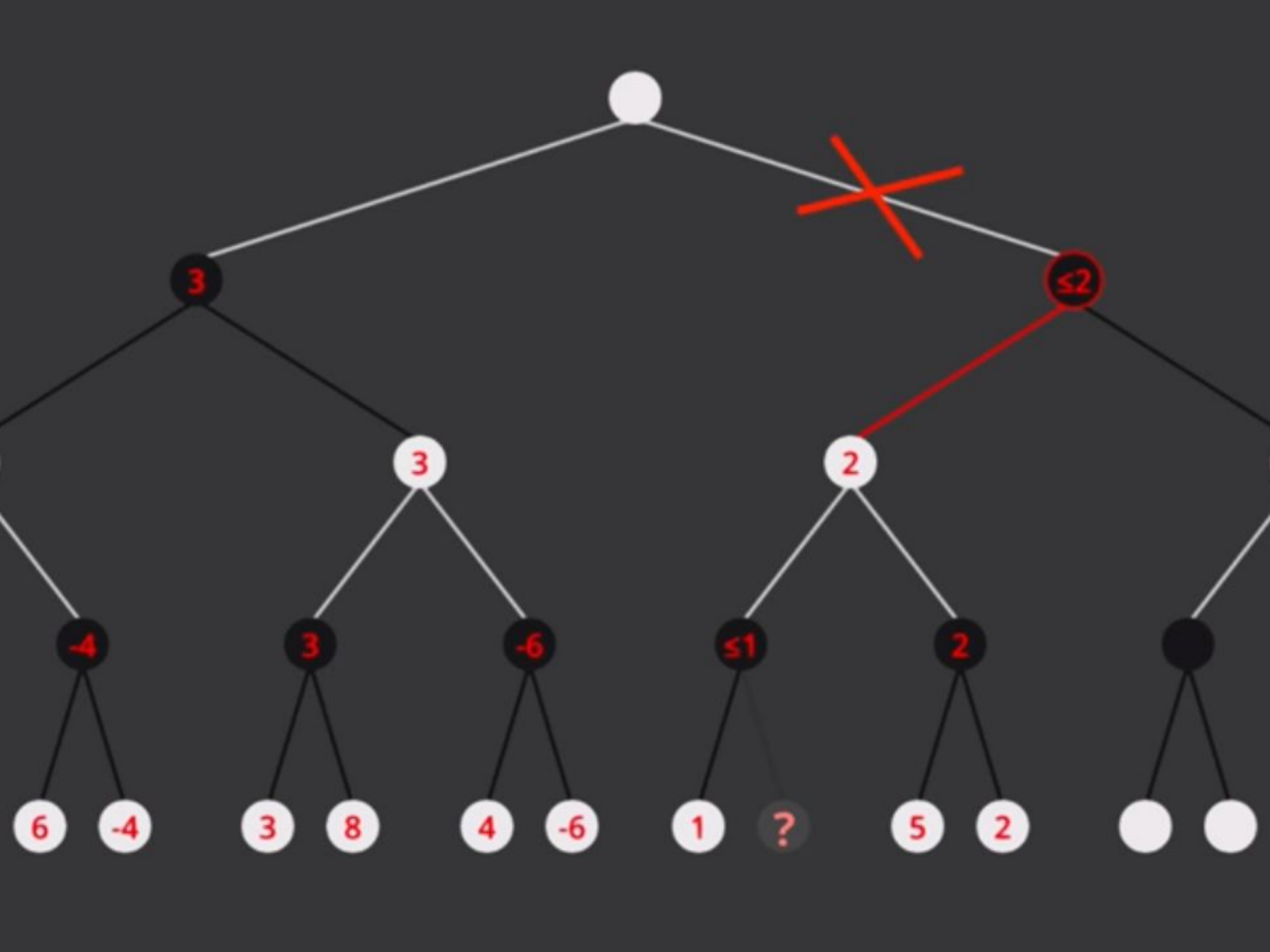
- Prunning does not affect final outcome
- Sorting moves by result improves $\alpha - \beta$ performance
- Perfect ordering: $O(b^{\frac{m}{2}})$
- An exercise on **metareasoning**

```
max(position, depth, maximizingPlayer)
= 0 or game over in position
static evaluation of position

ingPlayer
= -infinity
a child of position
= minimax(child, depth - 1, false)
al = max(maxEval, eval)
axEval


= +infinity
a child of position
= minimax(child, depth - 1, true)
al = min(minEval, eval)
inEval

ll
ntPosition, 3, true)
```



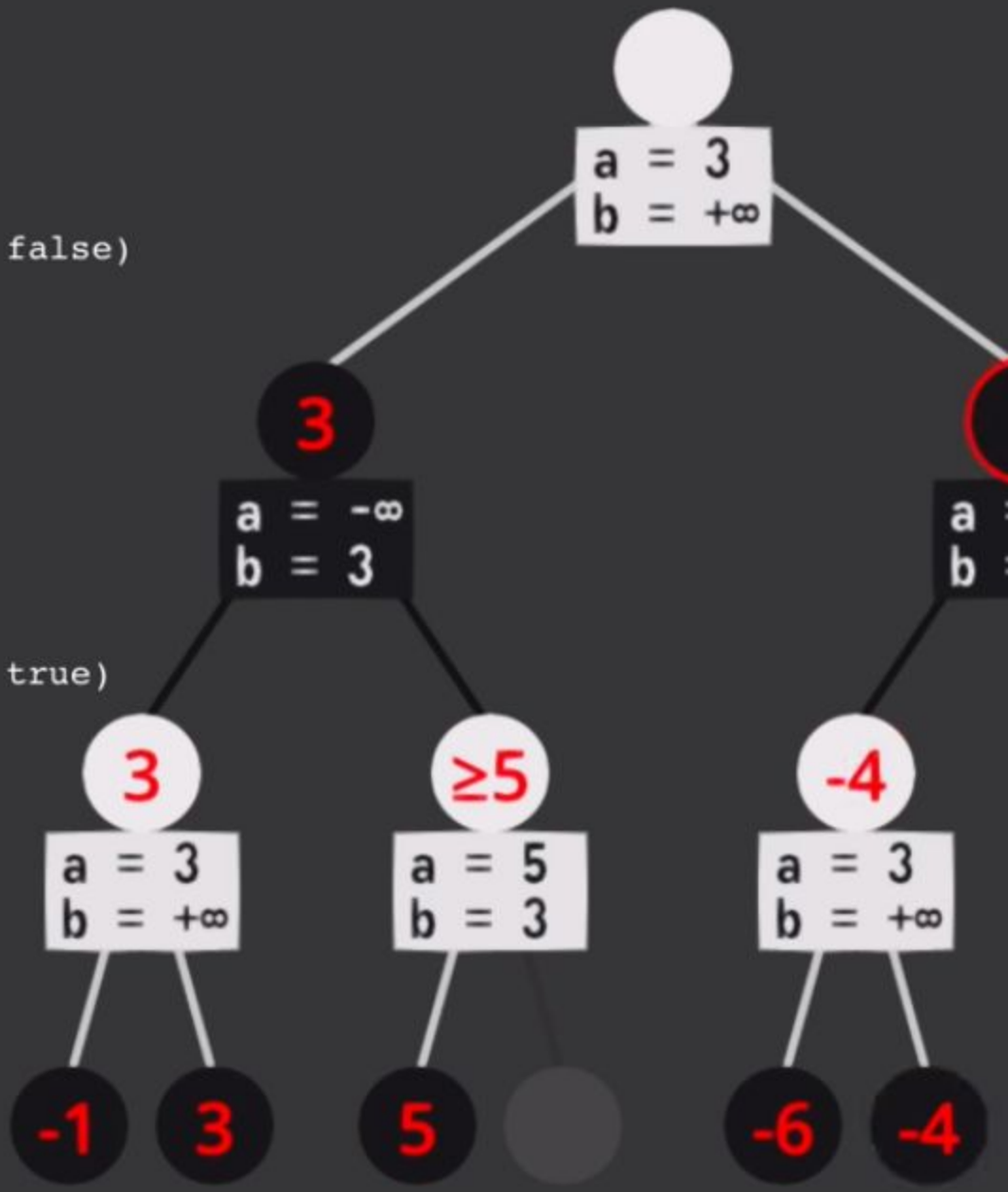

3
3
≤-4
3
≥5
-4
?
-1
3
5
?
-6
-4
?
?


3
3
-4
3
-6
≤1
6
-4
3
8
4
-6
1
?


3
≤2
3
2
-4
3
-6
≤1
2
6
-4
3
8
4
-6
1
?
5
2

```
x(position, depth, alpha, beta, maximizingPlayer)
 or game over in position
tic evaluation of position

Player
-infinity
hild of position
inimax(child, depth - 1, alpha, beta, false)
= max(maxEval, eval)
max(alpha, eval)
<= alpha

Eval


+infinity
hild of position
inimax(child, depth - 1, alpha, beta, true)
= min(minEval, eval)
in(beta, eval)
<= alpha

Eval
```

```
ll
ntPosition, 3, -∞, +∞, true)
```

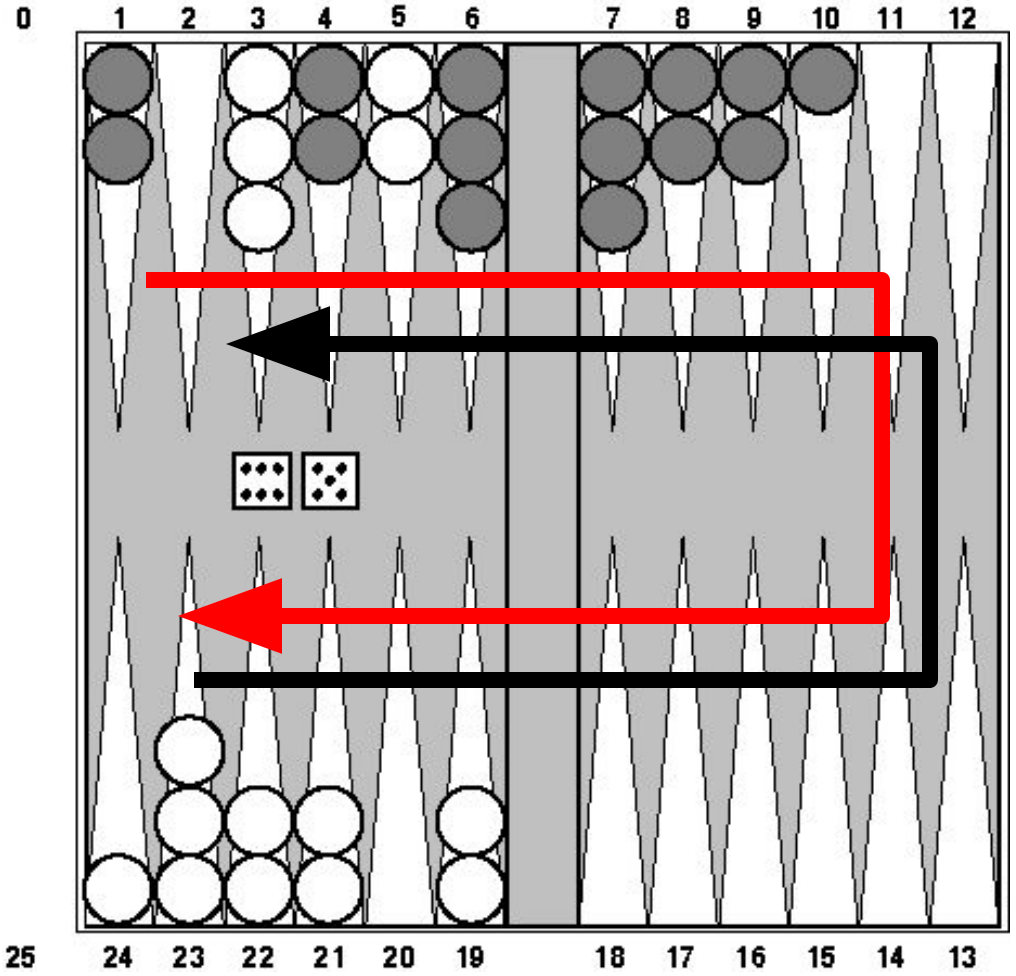


Effectiveness of alpha-beta

- Alpha-beta is guaranteed to compute the same value for the root node as computed by minimax, with less or equal computation
- **Worst case:** no pruning, examining b^d leaf nodes, where each node has b children and a d-ply search is performed
- **Best case:** examine only $(2b)^{d/2}$ leaf nodes.
  - Result is you can search twice as deep as minimax!
- **Best case** is when each player's best move is the first alternative generated
- In Deep Blue, they found empirically that alpha-beta pruning meant that the average branching factor at each node was about 6 instead of about 35!

Games of chance

- Backgammon is a two-player game with **uncertainty**.
- Players roll dice to determine what moves to make.
- White has just rolled *5 and 6* and has four legal moves:
  - 5-10, 5-11
  - 5-11, 19-24
  - 5-10, 10-16
  - 5-11, 11-16
- Such games are good for exploring decision making in adversarial problems involving skill and luck.



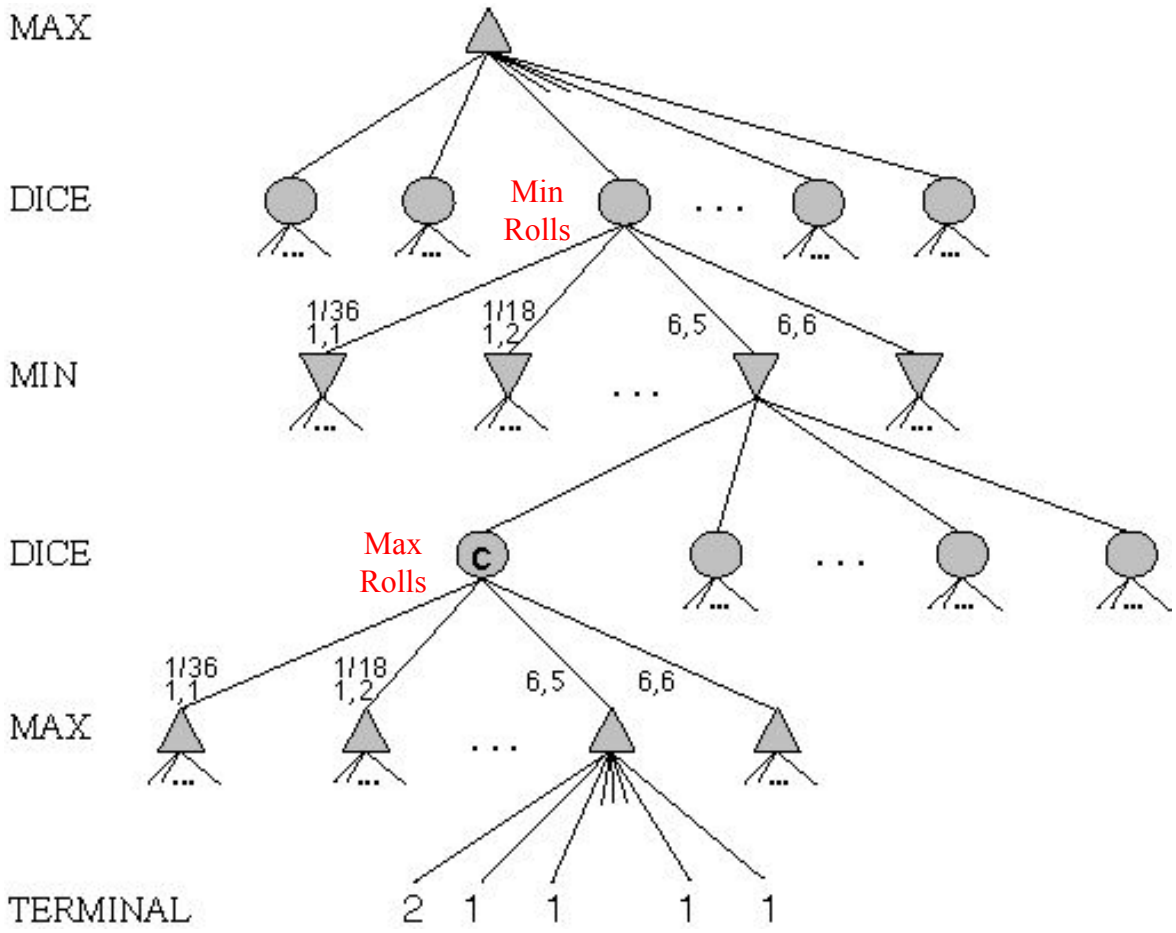
Game trees with chance nodes

- **Chance nodes** (shown as circles) represent random events
- For a random event with N outcomes, each chance node has N distinct children; a probability is associated with each
- (For 2 dice, there are 21 distinct outcomes)
- Use minimax to compute values for MAX and MIN nodes
- Use **expected values** for chance nodes
- For chance nodes over a max node, as in C:

expectimax(C) = $\sum_i (P(d_i) * \text{maxvalue}(i))$

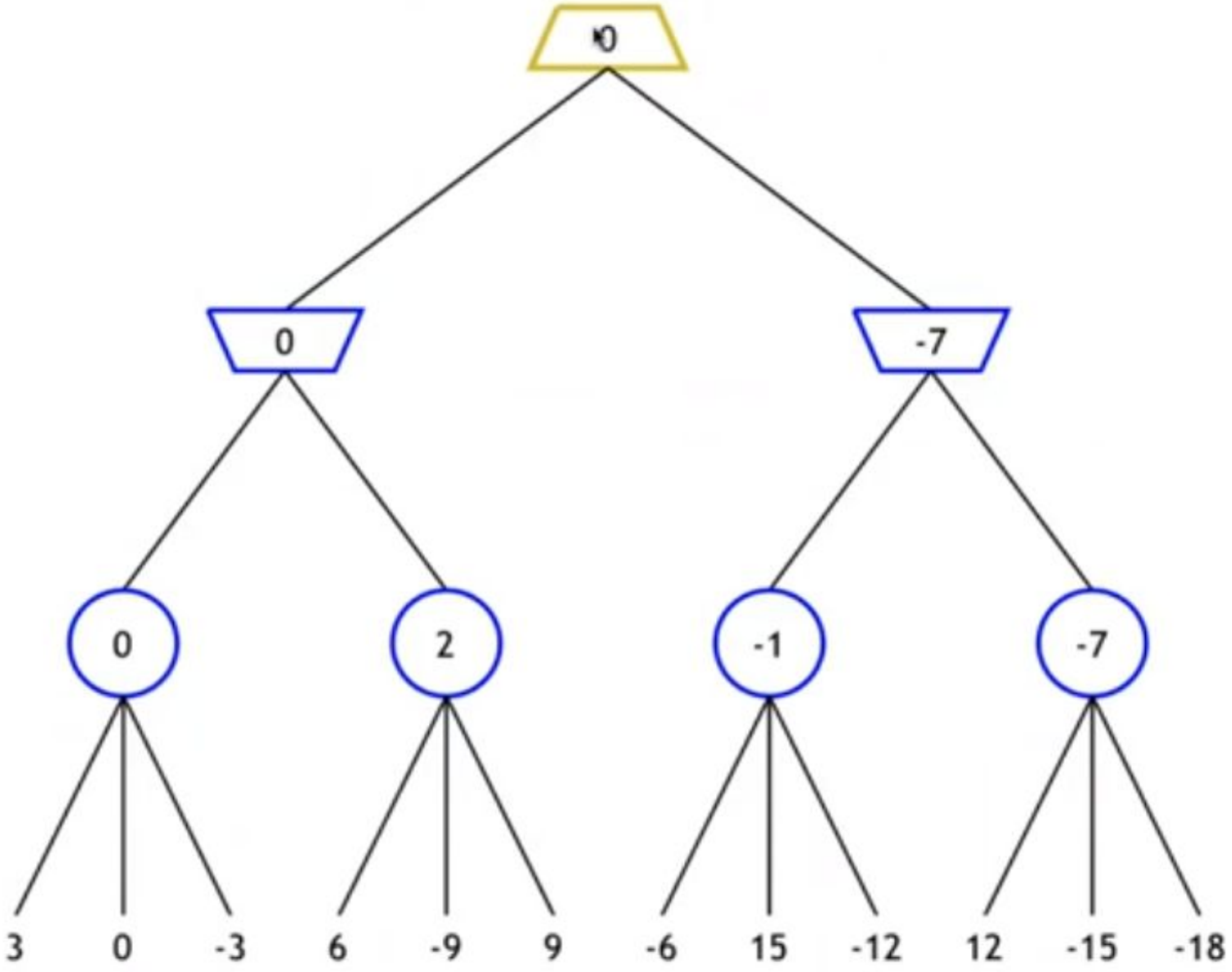
- For chance nodes over a min node:

expectimin(C) = $\sum_i (P(d_i) * \text{minvalue}(i))$

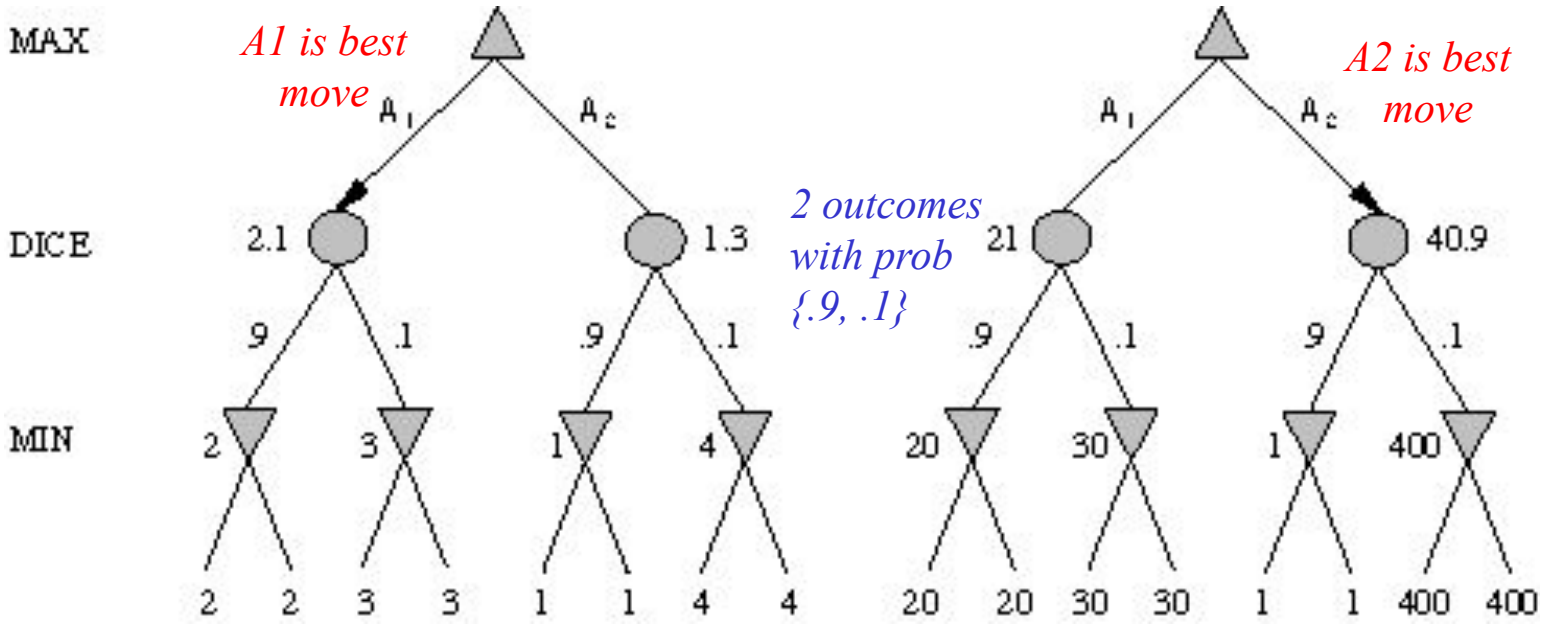


Expectiminimax

Consider the game tree shown below. The trapezoidal nodes mean the same thing as in the previous question; the circular nodes represent chance nodes in which each of the possible actions may be taken with equal probability. Assuming both players act optimally, carry out the expectiminimax search algorithm. Enter the value of each node inside the corresponding node.

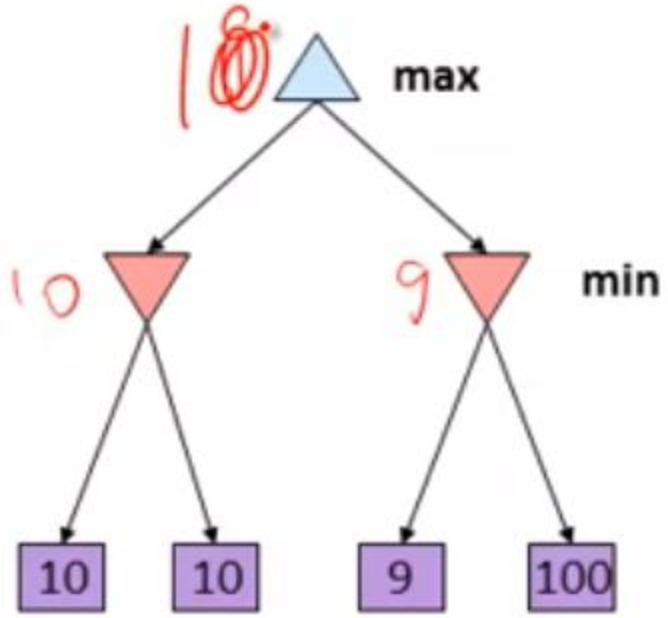


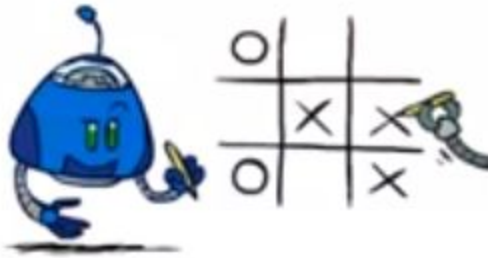
Meaning of the evaluation function



- Dealing with probabilities and expected values means we have to be careful about the "meaning" of values returned by the static evaluator.
- Note that a "relative-order preserving" change of the values would not change the decision of minimax, but could change the decision with chance nodes.
- Linear transformations are OK

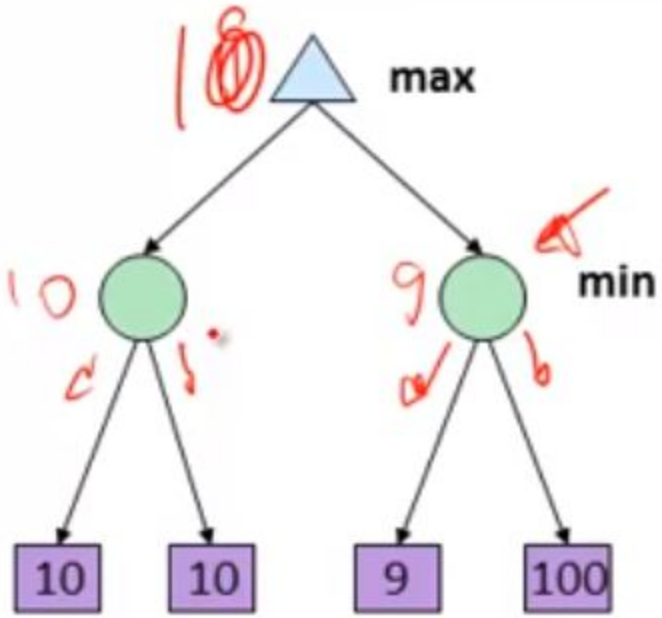
Worst-Case vs. Average Case





a: Uncertain outcomes controlled by chance, not an adversa

Worst-Case vs. Average Case



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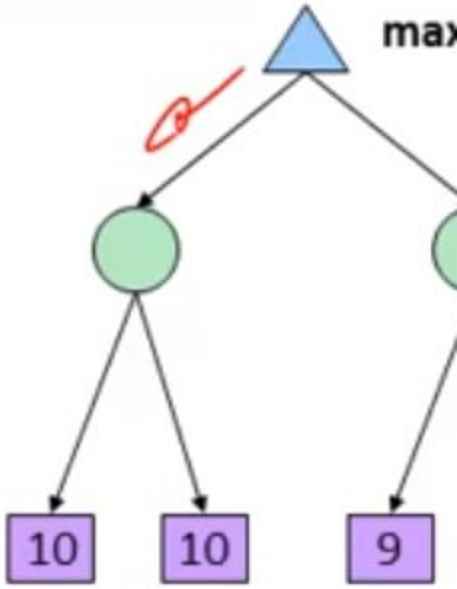
Expectimax Search

uldn't we know what the result of an action will be?
cit randomness: rolling dice
edictable opponents: the ghosts respond randomly
ns can fail: when moving a robot, wheels might slip

should now reflect average-case (expectimax)
es, not worst-case (minimax) outcomes

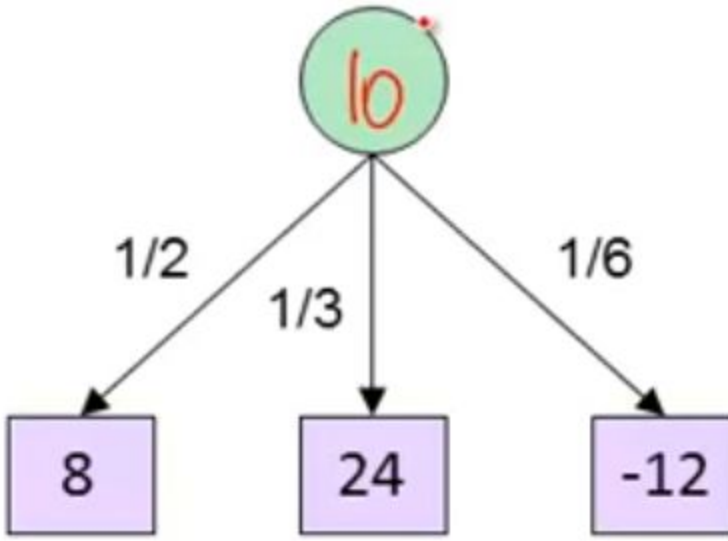
nax search: compute the average score under
play
nodes as in minimax search
ce nodes are like min nodes but the outcome is uncertain
ulate their expected utilities
ake weighted average (expectation) of children

e'll learn how to formalize the underlying uncertain-
roblems as Markov Decision Processes



Expectimax Pseudocode

```
-value(state):
ialize v = 0
each successor of state:
 p = probability(successor)
 v += p * value(successor)
urn v
```



$$\frac{1}{2} * 8 + \frac{1}{3} * 24 + \frac{1}{6} * (-12)$$

v = (1/2) (8) + (1/3) (24) + (1/6) (-12) = 10

4 + 8 − 2 = 10

Reminder: Probabilities

n variable represents an event whose outcome is unknown
ility distribution is an assignment of weights to outcomes

Traffic on freeway
om variable: T = whether there's traffic
mes: T in {none, light, heavy}
bution: P(T=none) = 0.25, P(T=light) = 0.55, P(T=heavy) = 0.20

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bilities are always non-negative
bilities over all possible outcomes sum to one

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eavy) = 0.20, P(T=heavy | Hour=8am) = 0.60
talk about methods for reasoning and updating probabilities later

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Example: Oopsy-Nim

- Starts out like Nim
- Each player in turn has to pick up either one or two objects
- Sometimes (with probability 0.25), when you try to pick up two objects, you drop them both
- Picking up a single object always works
- Whoever picks up the last object loses

- Question: Why can't we draw the entire game tree?
- Exercise: Draw the 2-ply game tree (2 moves per player)