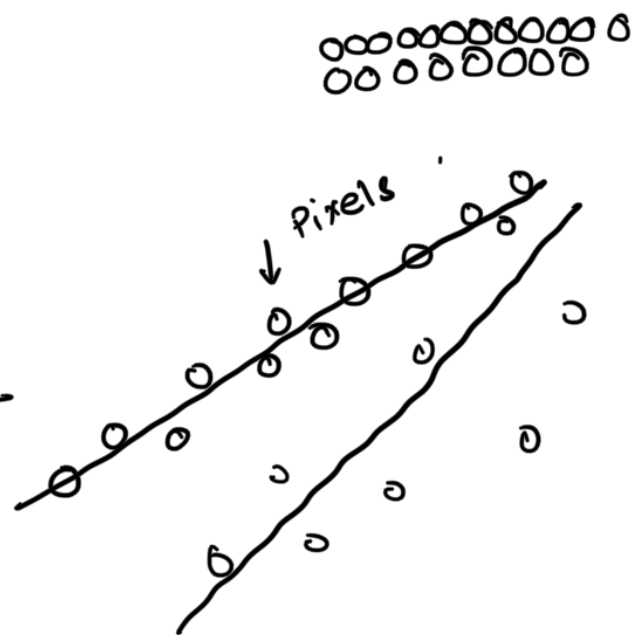
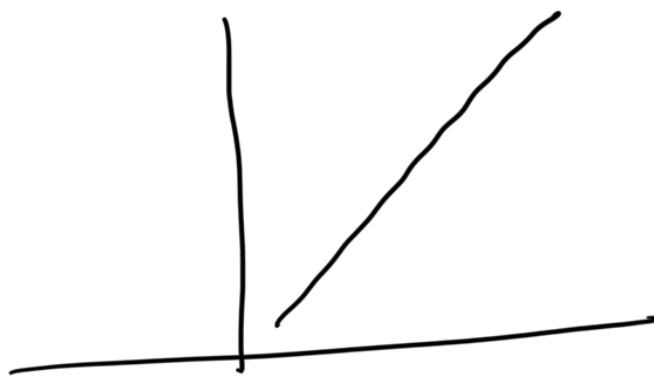


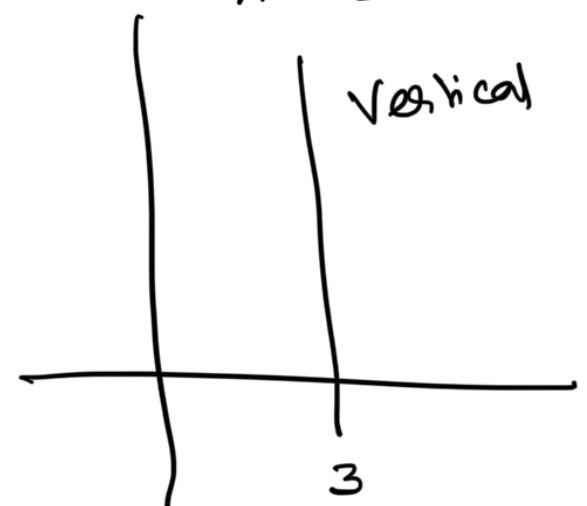
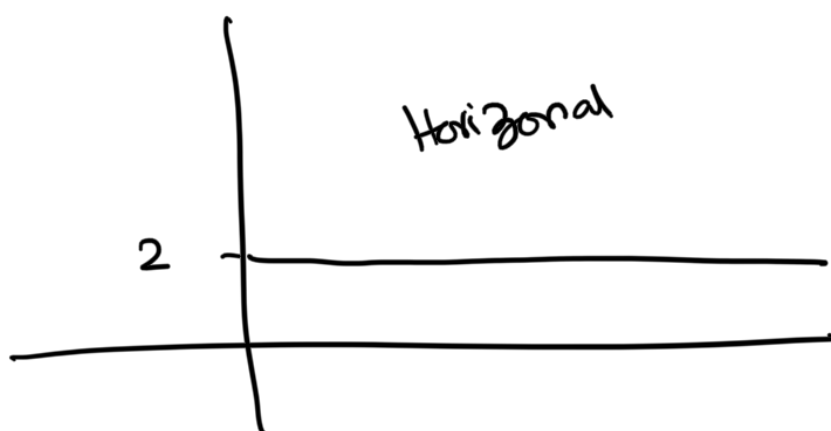
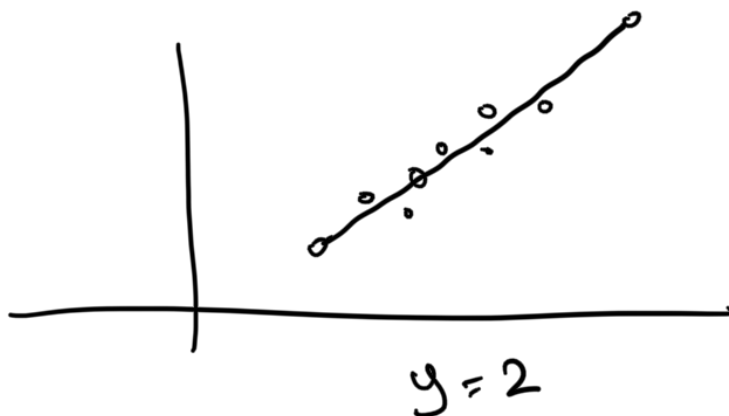
CSE423 Week 1

Line Drawing Algorithm

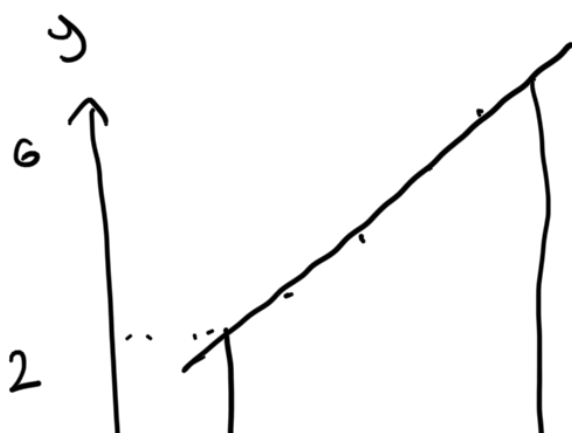
- DDA
- Midpoint
- Midpoint with eight way symmetry



Simple Approach



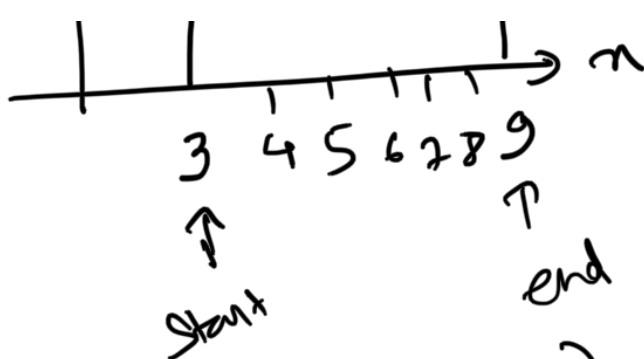
Target \rightarrow To find out all the necessary pixels to draw a line x_1, y_1 to x_2, y_2
 $(3, 2)$ to $(9, 6)$



$$y = mx + c$$

\downarrow
gradient

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$



$$m = \frac{6-2}{9-3}$$

$$= \frac{4}{6} = \frac{2}{3}$$

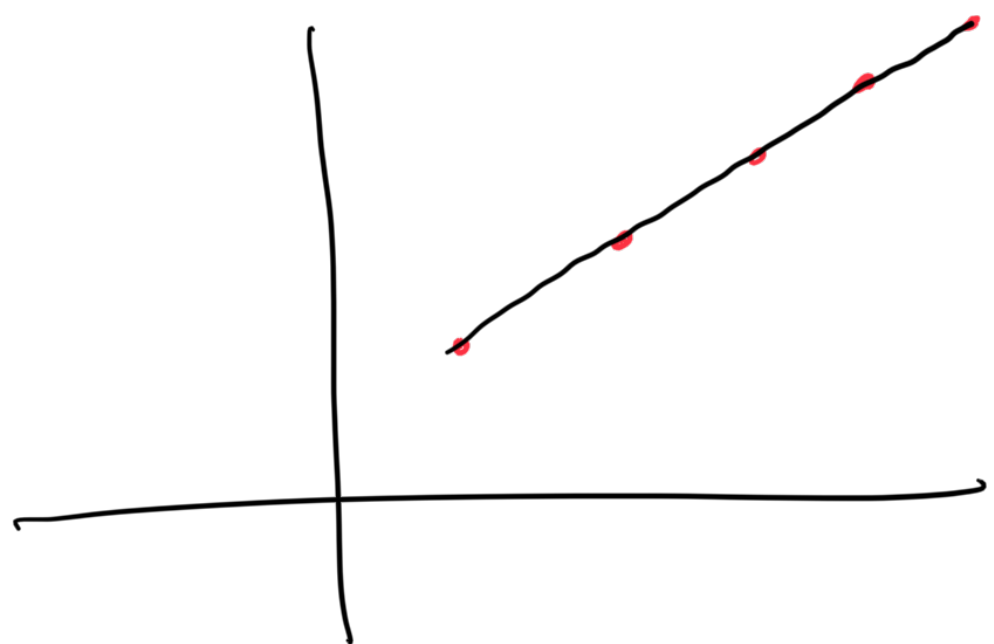
$(3, 2)$ $(1050, 100)$

$$y - y_1 = m(n - n_1)$$

$$y - 2 = \frac{2}{3}(n - 3)$$

$$y = \frac{2}{3}n - 2 + 2$$

n	y	$y = \frac{2}{3}n$	rounded	
3	$y = \frac{2}{3} \times 3 = 2$	≈ 2	2	$(3, 2)$
4	$y = \frac{2}{3} \times 4 = 2.7$	≈ 3	3	$(4, 3)$
5	$y = \frac{2}{3} \times 5 = 3.3$	≈ 3	3	$(5, 3)$
6	$y = \frac{2}{3} \times 6 = 4$	≈ 4	4	$(6, 4)$
7	$y = \frac{2}{3} \times 7 = 4.6$	≈ 5	5	$(7, 5)$
8	$y = \frac{2}{3} \times 8 = 5.3$	≈ 5	5	$(8, 5)$
9	$y = \frac{2}{3} \times 9 = 6$	≈ 6	6	$(9, 6)$



Problem!

→ Slow
→ Continuous y but here her

DDA Algorithm

↳ Digital Differential Analyzer.

if m is between $-1 < m < 1$
= $x_{k+1} = x_k + 1$ ←
 $y_{k+1} = y_k + m$

else,
 $y_{k+1} = y_k + 1$
 $x_{k+1} = x_k + 1/m$

Then, round off to the nearest pixel.

For example 1

$(2, 2)$ $(7, 5)$

$$m = \frac{5-2}{7-2} = \frac{3}{5} = 0.6$$

$-1 < m < 1$

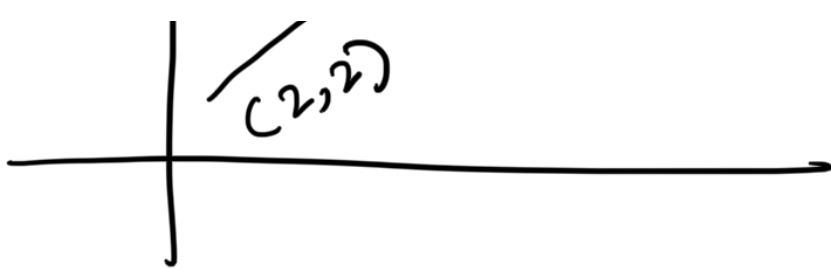
$x = 2 + 1 =$

x	y	$y(\text{round off})$	Pixel
2	2	2	(2, 2)
3	$2 + 0.6 = 2.6$	3	(3, 3)
4	$2.6 + 0.6 = 3.2$	3	(4, 3)
5	$3.2 + 0.6 = 3.8$	4	(5, 4)
6	$3.8 + 0.6 = 4.4$	4	(6, 4)
7	$4.4 + 0.6 = 5$	5	(7, 5)

End

Example 2

$(5, 7)$.



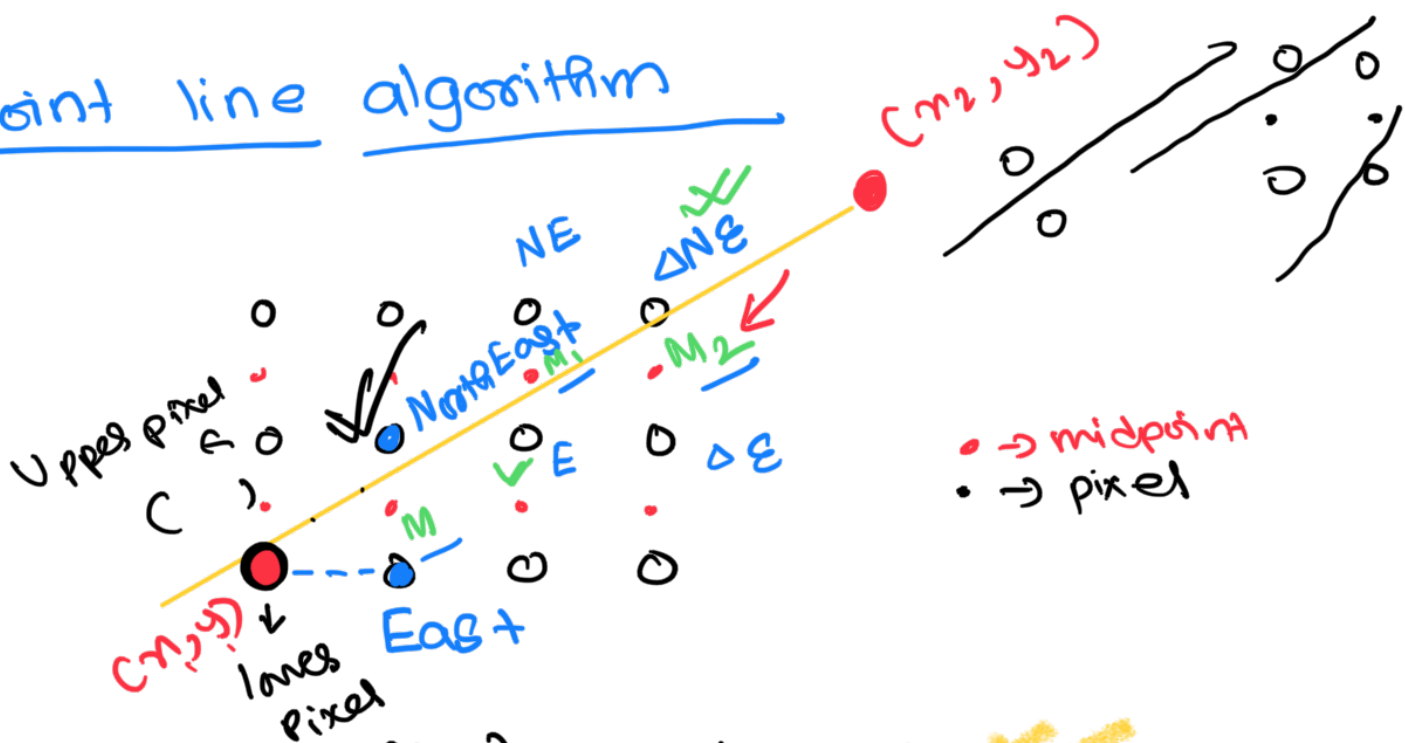
$$m = \frac{7-2}{5-2} = \frac{5}{3} = 1.7$$

y	x	x (rounded off)	pixel
2	2	2	(2,2)
3	$2 + \frac{1}{3} = 2.58$	3	(3,3)
4	$2.58 + \frac{1}{3} = 3.16$	3	(3,4)
5	$3.16 + \frac{1}{3} = 3.74$	4	(4,5)
6	$3.74 + \frac{1}{3} = 4.32$	4	(4,6)
7	$4.32 + \frac{1}{3} = 4.9$	5	(5,7)

2+1

verify →

Midpoint line algorithm



Conditions :

$$f(x, y) = ax + by + c$$

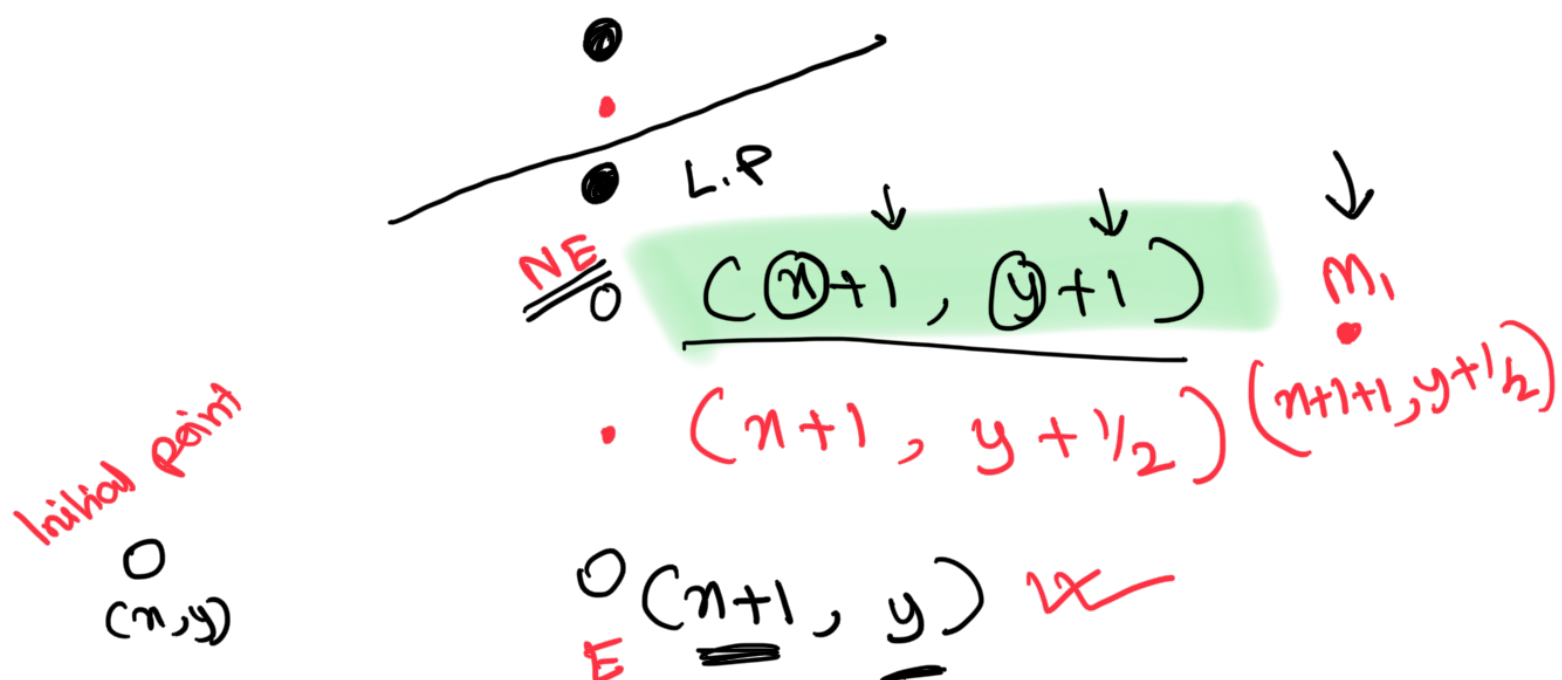
✓ $f(\text{midpoint})$
 ① -ve ✓

Midpoint's Position

Upperside of line

NE
 • $(x+1, y+1)$
 M • $(x+1, y+1/2)$
 • $(x+1, y)$
 Pixel
 lower Pixel (E)

② \checkmark +ve \checkmark | lower side of line | Upper Pixel (NE) \checkmark



$$y = mx + c \dots (1)$$

$$m = \frac{\Delta y}{\Delta x}$$

$$y = \frac{\Delta y}{\Delta x} x + c$$

We know,

$$\Delta x y - \Delta y x - \Delta x c = 0 \quad (x-1)$$

$$\rightarrow \Delta y x - \Delta x y + \Delta x c = 0$$

$$\rightarrow ax + by + c = 0$$

$x = x+1, y = y+1/2$

$$\begin{aligned} a &= \Delta y \\ b &= -\Delta x \\ c &= \Delta x \end{aligned}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$

$$f(x) / d_{old} = f(x+1, y+1/2)$$

Initial \rightarrow

$$d_{old} = a(x+1) + b(y+1/2) + c$$

Case 1:

Upper Pixel (NE)

$$(x+1+1, y+1/2+1)$$

$$f(M_1) = (n+2, y+1/2)$$

$$d_{new} = a(n+2) + b(y+3/2) + c$$

$$d_{new} - d_{old} = a(n+2) + b(y+3/2) + c - a(n+1) - b(y+1/2) - c$$

$$= \cancel{an} + 2a + \cancel{by} + 3/2b + \cancel{c} - \cancel{an} - a - \cancel{by} - 1/2b - \cancel{c}$$

$$d_{new} - d_{old} = a + b$$

$$d_{new} = d_{old} + a + b \quad \checkmark \Delta NE$$

Case 2 : Lower Pixel / E

$$f(M_1) / d_{new} = f(n+1+1, y+1/2) = f(n+2, y+1/2)$$

$$d_{new} = a(n+2) + b(y+1/2) + c$$

$$d_{new} - d_{old} = a(n+2) + b(y+1/2) + c - a(n+1) - b(y+1/2) - c$$

$$d_{new} - d_{old} = \cancel{an} + 2a + \cancel{by} + 1/2b + \cancel{c} - \cancel{an} - a - \cancel{by} - 1/2b - \cancel{c}$$

$$d_{new} = d_{old} + a \quad \Delta E$$

Initially, $m = (n+1, y+1/2)$ (n, y)

$$d_{old} = a(n+1) + b(y+1/2) + c$$

$$= an + a + by + 1/2b + c$$

$$= an + bu + c + a + 1/2b$$

$$d_{old} = 0 + a + \frac{1}{2}b$$

$$d_{old} = a + \frac{1}{2}b \quad (\text{multiply by 2})$$

we need to convert to integers from floating point

$$d_{old} = 2a + b$$

$$d_{old} = 2\Delta y - \Delta x$$

$$\begin{array}{l} a = \Delta y \\ b = -\Delta x \end{array}$$

V.P / ΔNE

$$d_{new} = d_{old} + \underbrace{2\Delta y - 2\Delta x}_{\Delta NE} \quad \checkmark$$

L.P / ΔE

$$d_{new} = d_{old} + \underbrace{2\Delta y}_{\Delta E} \quad \checkmark$$

$$\Delta NE = 2(\Delta y - \Delta x)$$

$$\Delta E = 2\Delta y$$

$$d_{old} = 2\Delta y - \Delta x$$

↓
Void draw-line(int x_0 , int y_0 , int x_1 , int y_1)

$$\text{int } \Delta x = x_1 - x_0;$$

$$\text{if } \Delta y = y_1 - y_0;$$

$$\text{int } d_{old} = 2\Delta y - \Delta x;$$

$$\text{int } dE = 2 * \Delta y;$$

$$\text{int } dNE = 2 * (\Delta y - \Delta x);$$

$$\text{int } x = x_0, \text{ int } y = y_0$$

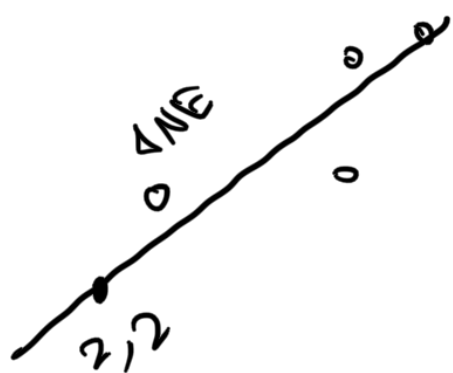
draw pixel (x, y);

while (x < x_1) {

if ($d_{old} < 0$) { ΔE

$x++$;

(2,2)



$$d_{old} = d_{old} + dE ;$$

else { ΔNE

$x++ ;$

$y++ ;$

$$d_{old} = d_{old} + dNE ;$$

}

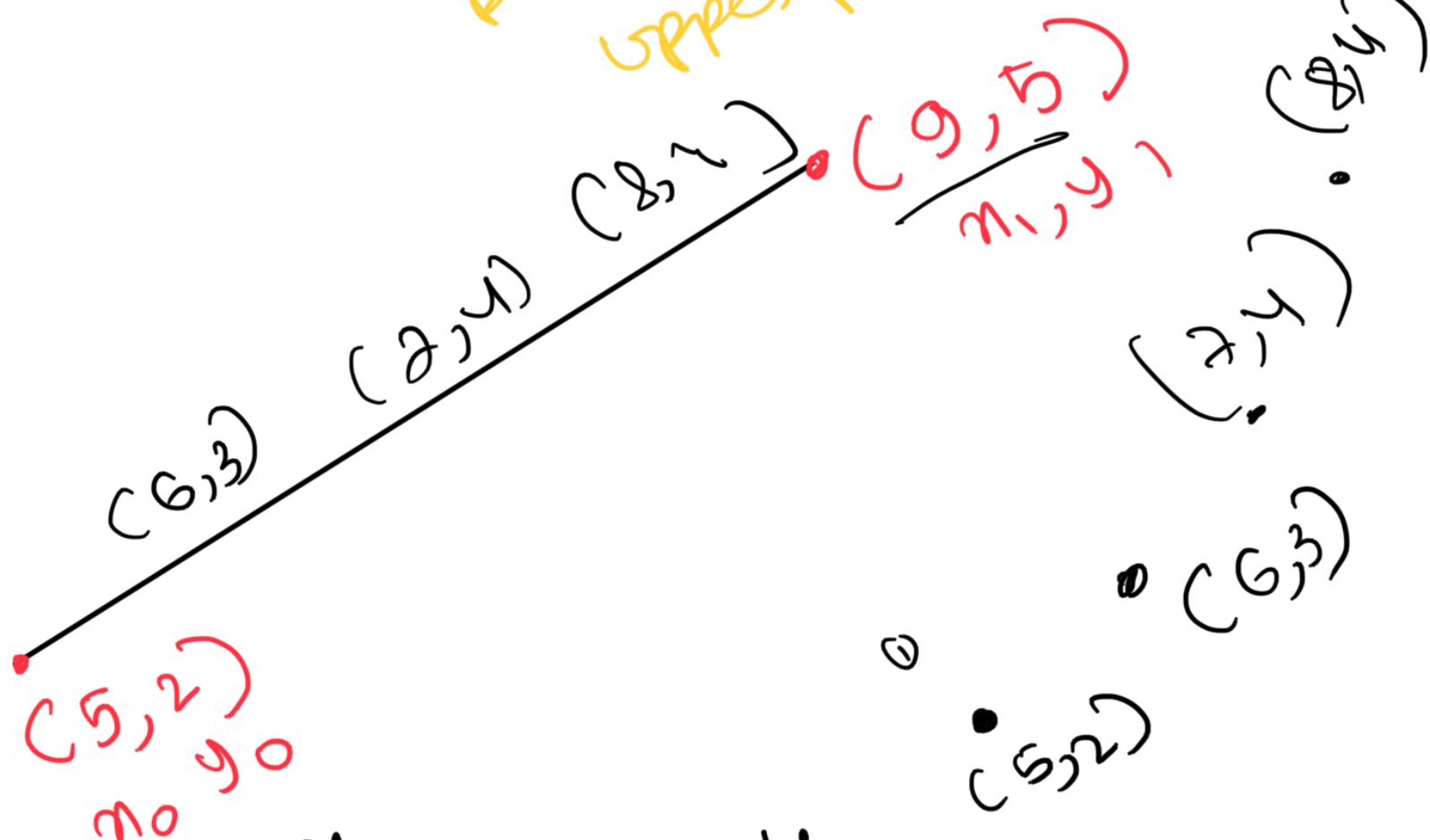
draw pixel (x, y) ;

}

}

end.

* midpoint diya
pass kare tohale
upper pixel ΔNE.



$$\Delta x = 9 - 5 = 4$$

$$\Delta y = 5 - 2 = 3$$

$$d_{old} = 2 \times 3 - 4 = 2 \quad \checkmark$$

$$dE = 2 \times 3 = 6 \leftarrow$$

$$\Delta NE = 2 \times (5-4) \\ = -2 \times E$$

$$x=5, \quad y=2$$

(5,2) Pixel

$$5 < 9$$

① ΔNE

$$x = 5+1 = 6$$

$$y = 2+1 = 3$$

$$d_{old} = 2-2 = 0$$

(6,3) Pixel.

$$6 < 9$$

②

ΔNE

$d_{old} = 0$ or $d > 0$
NE

$$x = 6+1 = 7$$

$$y = 3+1 = 4$$

$$d = 0-2 = -2 \times W$$

(7,4) Pixel.

③ ΔE $7 < 9$

$$x = 7+1 = 8$$

$$d = -2+6 = 4$$

$(8, 4)$ (Pixel)

$$8 < 9$$

④

ΔNE

$$x = 8 + 1 = 9$$

$$y = 5$$

$$d = 4 - 2 = 2$$

$(9, 5)$ pixel.