Rigid | Euclidean – Preserves distances + angles
Similitudes – " angles
Linear T. – $\alpha(p+q) = \alpha(p) + \alpha(q)$
$\alpha(ap) = a\alpha(p)$

Affine – preserves parallel lines
Projective " lines

$x' = ax + by + c$
$y' = ex + fy + d$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a & b \\ e & f \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} c \\ d \end{pmatrix}$$

Cartesian formulation

$$p' = Mp + t$$

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} a & b & c \\ e & f & d \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

Homogenous formulation

$$p' = Mp$$

Translation : $P' = P + T$
Scaling : $P' = S \cdot P$
Rotate : $P' = R \cdot P$

Non Homogenous

Composition is difficult to express ~~ion~~, since translation not expressed as a matrix multiplication.

w/ Homogenous all 3 can be expressed w/ MM by 3×3 matrices
$w = 1 \rightarrow$ affine transformations

2D = 3×3
3D = 4×4

$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix}$$

Multiplying Homogenous coordinates by an affine matrix keeps w unchanged.

$$\Rightarrow \begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Homogenous Visualization

$$\left.\begin{aligned}(0,0,1) &\rightarrow (0,0,2)\\ (7,1,1) &\rightarrow (14,2,2)\\ (4,5,1) &\rightarrow (8,10,2)\end{aligned}\right\} w=2$$

Translation - 2D

$$(4,5) \ \& \ (7,5) \xrightarrow{T} (7,1) \ \& \ (10,1)$$

$T \rightarrow$ Translation $(3,-4)$

$$x' = x + dx \qquad P = \begin{pmatrix} x \\ y \end{pmatrix} \qquad T = \begin{pmatrix} dx \\ dy \end{pmatrix}$$
$$y' = y + dy$$
$$P' = \begin{pmatrix} x' \\ y' \end{pmatrix} \qquad P' = P + T$$

Homogenous form

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & dx \\ 0 & 1 & dy \\ 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

Translation - 3D

$$x' = x + dx$$
$$y' = y + dy$$
$$z' = z + dz$$

$$\begin{matrix} 1 & 0 & 0 & dx \\ 0 & 1 & 0 & dy \\ 0 & 0 & 1 & dz \\ 0 & 0 & 0 & 1 \end{matrix} * \begin{matrix} x \\ y \\ z \\ 1 \end{matrix} = \begin{matrix} x' \\ y' \\ z' \\ 1 \end{matrix}$$

$$T(dx, dy, dz) * P = P'$$

Scaling - 2D

$$(4,5) \ \& \ (7,5) \xrightarrow{S} (2, 5/4) \ \& \ (7/2, 5/4)$$

$S \rightarrow$ Scaling $(1/2, 1/4)$

$$x' = Sx * x \qquad S * P = P'$$
$$y' = Sy * y$$

$$\begin{pmatrix} Sx & 0 \\ 0 & Sy \end{pmatrix} * \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x * Sx \\ y * Sy \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} Sx & 0 & 0 \\ 0 & Sy & 0 \\ 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

Translations + Rotations

$TR \neq RT$

R first & then T.

$$p' = TRp$$
$$= Mp$$
$$= Rp + T$$

$$M = \begin{matrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{matrix} \quad * \quad \begin{matrix} R_{11} & R_{12} & R_{13} & 0 \\ R_{21} & R_{22} & R_{23} & 0 \\ R_{31} & R_{32} & R_{33} & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$$

$$= \left(\begin{array}{ccc|c} R_{11} & R_{12} & R_{13} & T_x \\ R_{21} & R_{22} & R_{23} & T_y \\ R_{31} & R_{32} & R_{33} & T_z \\ \hline 0 & 0 & 0 & 1 \end{array} \right) = \begin{pmatrix} R & T \\ 0 & 1 \end{pmatrix}$$

$$p' = RTp$$
$$= MP = R(P+T) = RP + RT$$

Not commutative $TS \neq ST$

But associative $((CB)A)p = (C(BA))p = \underbrace{C(B(Ap))}_{\text{one point}} = q$

many points $M = CBA$
$q = Mp$

Rotation of θ about $P(h,k)$: $R_{\theta,P}$

(i) Translate $P(h,k)$ to 0.
(ii) Rotate θ wrt 0.
(iii) Translate $(0,0)$ to $P(h,k)$

$$R_{\theta,P} = T(h,k) * R_\theta * T(-h,-k)$$

(i) Translate $(0,b)$ to origin
(ii) Rotate $-\theta$
(iii) Mirror reflect about X-axis
(iv) Rotate θ
(v) Translate origin to $(0,b)$

$$M_L = T(0,b) * R(\theta) * M_x * R(-\theta) * T(0,-b)$$

Shearing

x-axis

$$SH_x = \begin{matrix} 1 & a & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$$

y-axis

$$SH_y = \begin{matrix} 1 & 0 & 0 \\ b & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$$

xy-axis

$$SH_{xy} = \begin{matrix} 1 & a & 0 \\ b & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$$

z-axis

~~$SH_{xy}(x,y)$~~ $SH_{xy}(sh_x, sh_y) * P = P'$

$$\begin{pmatrix} 1 & 0 & sh_x & 0 \\ 0 & 1 & sh_y & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x + z * sh_x \\ y + z * sh_y \\ z \\ 1 \end{pmatrix}$$

Inverse Transformation –

A^{-1} will undo anything A does.

Translation → $T^{-1}_{(dx,dy)} = T_{(-dx,-dy)}$

Rotation → $R^{-1}_{(\theta)} = R_{(-\theta)} = R^T_{(\theta)}$ { flips a matrix over its diagonal }

Scaling → $S^{-1}_{(sx,sy)} = S(1/sx, 1/sy)$

Mirror Reflection → $M^{-1}_x = M_x$

$M^{-1}_y = M_y$

Composite Transformations –

$(1,1) \xrightarrow[S]{} (2,2) \xrightarrow[T]{} (5,3)$

$TSP \neq STP$

S → Scaling (2,2)

T → Translate (3,1)

M.M. = $T(SP) = P'$

↓

$TSP = P'$

$$\begin{matrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{matrix} * \begin{matrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{matrix} = \begin{matrix} 2 & 0 & 3 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{matrix} * \begin{matrix} x \\ y \\ 1 \end{matrix} = \begin{matrix} x' \\ y' \\ 1 \end{matrix}$$

Rotation-3D

* angle of R
* axis of R

$R_{\theta,k} * P = P'$

ABOUT z AXIS-

$$\begin{pmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x\cos\theta - y\sin\theta \\ x\sin\theta + y\cos\theta \\ z \\ 1 \end{pmatrix}$$

ABOUT y AXIS-

$$\begin{pmatrix} \cos\theta & 0 & +\sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x\cos\theta + z\sin\theta \\ y \\ -x\sin\theta + z\cos\theta \\ 1 \end{pmatrix}$$

ABOUT X AXIS-

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y\cos\theta - z\sin\theta \\ y\sin\theta + z\cos\theta \\ 1 \end{pmatrix}$$

Mirror Reflection-

$(1,1) \rightarrow (1,-1)$ Reflection about X-axis $\quad x' = x \quad y' = -y$

$(1,1) \rightarrow (-1,1)$ " " Y-axis $\quad x' = -x \quad y' = y$

$$M_x = \begin{matrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{matrix} \qquad M_y = \begin{matrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$$

Scaling - 3D

$x' = s_x * x$
$y' = s_y * y$
$z' = s_z * z$

$S(s_x, s_y, s_z) * P = P'$

$$\begin{pmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix}$$

Rotation - 2D

$$V = \begin{vmatrix} r\cos\varphi \\ r\sin\varphi \end{vmatrix} \qquad V' = \begin{vmatrix} r\cos(\varphi+\theta) \\ r\sin(\varphi+\theta) \end{vmatrix}$$

Expand $(\varphi+\theta) \rightarrow$ $x' = r\cos\varphi\cos\theta - r\sin\varphi\sin\theta$
$y' = r\cos\varphi\sin\theta + r\sin\varphi\cos\theta$

$\Rightarrow$ $x' = x\cos\theta - y\sin\theta$
$y' = x\sin\theta + y\cos\theta$

$R * P = P'$

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} * \begin{pmatrix} x \\ y \end{pmatrix} = \begin{matrix} x' \\ y' \end{matrix}$$

$$\text{" " "} = \begin{pmatrix} x\cos\theta - y\sin\theta \\ x\sin\theta + y\cos\theta \end{pmatrix}$$

$\Rightarrow$

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

CSE423 – Computer Graphics – AJA

Color Model

Achromatic + Colored Light

Achromatic / Monochrome ← B/W

* different intensities of greys
* lacks hue + saturation + brightness
* measured using the quantity of light
* all RGB values are equal
  (R=G=B)
* different values for these 3 generate different shades of greys.

Visible Spectrum – 400nm to 700nm (approx.)
Blue – 380nm
Red – 780nm

A color model is a specification of a coordinate system & a subspace within that system where each color is represented by a single point.

RGB (Red, Green, Blue)
CMY (Cyan, Magenta, Yellow)
HSV (Hue, Saturation, Value)
HLS (Hue, Lightness, ~~Value~~ Saturation)

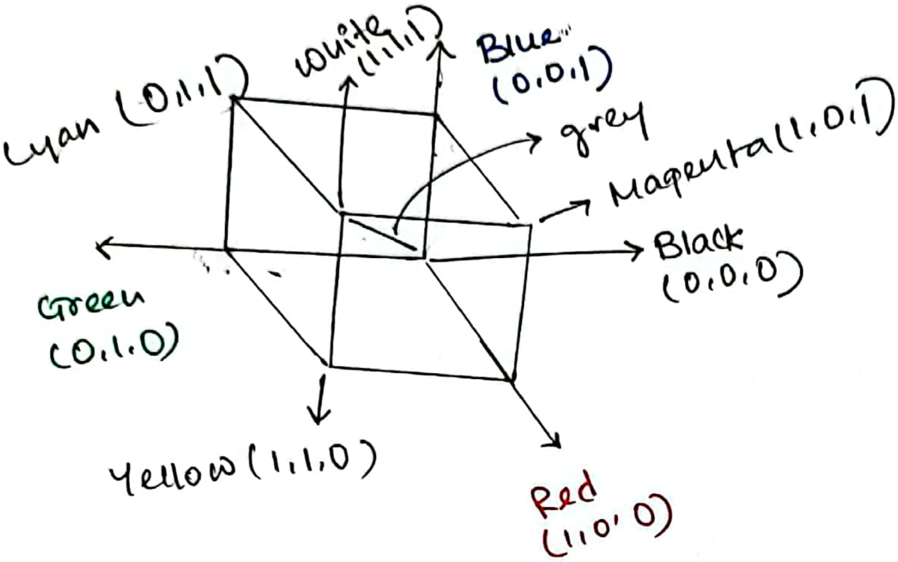
Painting – mixed colors are achieved by subtracting color method.
For computers – additive color method ve

Colors on screen are created w/ light using the additive color method. Begins w/ black & ends w/ white. As more colors are added, the result is lighter & tends to white.

Painting – opposite : Subtractive Color Method
Begins w/ white & ends w/ black.
↑ more colors, tends to black.

<u>RGB Model</u> (TVs + Computers)

* Based on Cartesian Coordinates System



Here, black → absence of any color
white → presence of all colors

* values [0,1]
* Grey scale → equal R, G & B.

* represented by a unit cube w/ axes RGB & one corner at the origin of a 3D color coordinate system (Black)

~~CYK Model~~

secondary wrt RGB

<u>CMY Model</u> (Cyan, Magenta, Yellow)

↳ primary in subtractive model + RGB → secondary
↓
adding causes colors to not be reflected + thus not be seen.
white = absence
black = presence

* Printers
* Copiers

C = Y = M → Black
Muddy Black when printed
↓
True Black, 4th color black is added = CMY<u>K</u> Model

$$\begin{pmatrix} C \\ M \\ Y \end{pmatrix} = \begin{pmatrix} 1.00 \\ 1.00 \\ 1.00 \end{pmatrix} - \begin{pmatrix} R \\ G \\ B \end{pmatrix}$$

C	M	Y	Color
0	0	0	white
1	0	0	cyan
0	1	0	Magenta
0	0	1	Yellow
1	1	0	Blue
1	0	1	Green
0	1	1	Red
1	1	1	Black

HSV Model

H(Hue) — λ of pure color observed in the signal
- distinguishes R/Y/G etc
- >400 hues → human eye

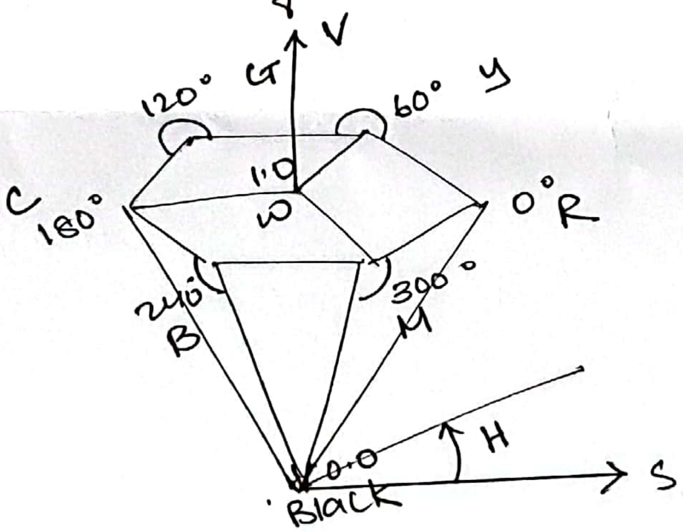
↓
dominant color seen

S(degree of dilution) — Inverse of the quantity of "white" present in the signal. Pure color — 100% sat^n
White + Gray — 0% sat^n
- how far the color is from equal intensity grey
- distinguished R from P etc.
- ≈ 20 sat^n levels are visible per hue.
- Range [0,1]

↓
%

V(Value) — Distinguishes grey levels.

* Vision + Image Processing



RGB to HSV Conversion –

Max = max (RGB)
Min = min (RGB)

V = max → Value of HSV → V

if max ≠ 0
  s = max-min/max   s → Saturation of HSV
else
  s = 0

if s = 0
  h = undefined   h → Hue of HSV
else
  Δ = max-min
  if r = max
    h = g-b/Δ
  elif g = max
    h = 2+ (b-r)/Δ
  elif b = max
    h = 4+ (r-g)/Δ

  h = h * 60
  if h < 0
    h = h+360

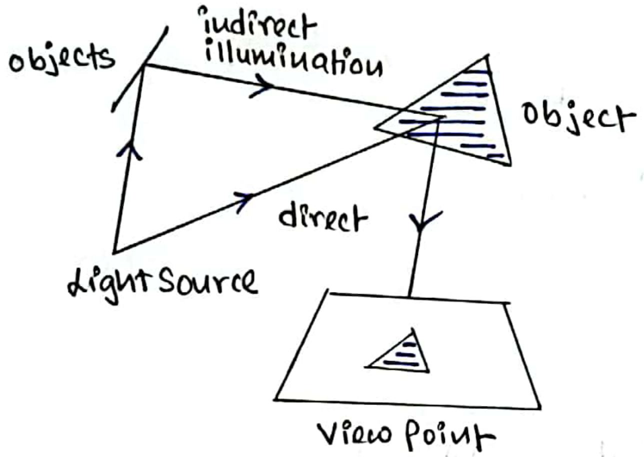
R	G	B	Hue	Saturation	Value	C	M	Y
0.25	0.30	1.00	236°	75%	100%	0.75	0.70	0.0
0.01	1.00	0.99	179.4°	99%	100%	0.99	0.00	0.01
1.00	0.11	0.01	61°	99%	100%	0.00	0.89	0.99

CSE423 – Computer Graphics – AJA
Lecture / Chapter 06 – Lighting Model

Illumination – transport of energy from light sources to surfaces + points

↙ Local ↘ global

Illumination Model – Express the factors determining a surface's color / luminous intensity at a particular 3D point.
(outgoing / reflected light)



2 components of illumination –
(I) Light sources
(II) surface properties

* Light sources are described by a luminance / intensity "I"
  - each color is described separately
  - $I = [I_R \ I_G \ I_B]$

* Types of light sources –
  (I) Ambient
  (II) Diffuse
  (III) Spot

Dark object
↓
Ambient Reflection (colour) + Diffuse Reflection (some depth) = Correct color + object → Object
+ Specular Reflection (shininess)

Ambient Light
- no identifiable source or direction
- product of multiple reflections of light from many surfaces present in the environment
- computationally inexpensive

Ambient Light

↙ Global
- independent of L's.
- Lights entire scene
- Ex. reflection of sunlight from several surfaces

↘ Local
- from addn light sources
- can be different for each light + primary color
- Ex. reflection of fluorescent lamps from several surfaces

* Ambient Reflection

- reflection of light that does not come directly from a light source
  - * In darkness, we can make out the edges of objects (light bounces off of objects)
  - * Phong - Direct Lighting Model (assume that ambient light falls equally on all objects).

$$A = I_a k_a$$

$I_a \rightarrow$ intensity of the ambient light

$k_a \rightarrow$ ambient coefficient $[0,1]$

- is set to provide right amount of ambient light for a scene.

$k \rightarrow 1$ Bright scenes

$k \rightarrow 0$ Dark scenes

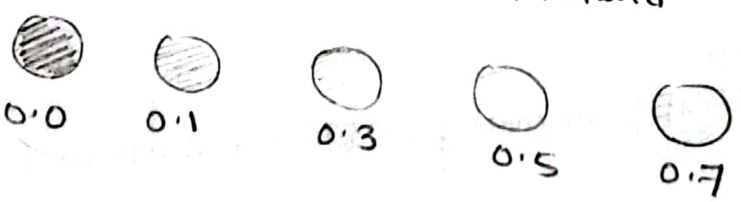
* Ambient Reflection Coefficient

* Effect of adding ambient light to the diffuse light reflected by a sphere

- Diffuse source intensity $\rightarrow$ 1.0
- " reflection coefficient $\rightarrow$ 0.4
- Ambient source intensity $\rightarrow$ 1.0
- " reflection coefficient $\rightarrow$ 0.0 0.1 0.3 0.5 0.7

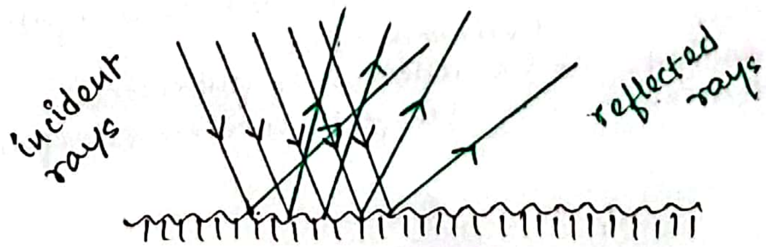
↓ ambient light ; shadows too deep + harsh

↑ " " ; picture washed out + bland

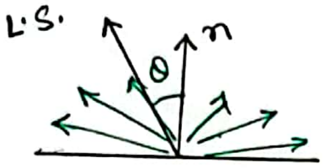


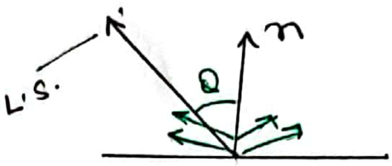
Diffuse Reflection -

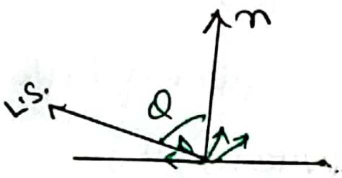
* ∥ rays from L.S.
* Reflected rays are scattered (not smooth surface)
* direction of reflection
* some are visible, some are not



Diffuse Reflection – Phong's Model







* light reflected equally in all direction
* magnitude of reflection depends on θ

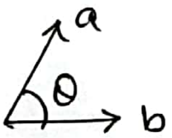
Phong's Diffuse Reflection –

$$D = I_p k_d \max[\underbrace{\cos(\theta)}, 0]$$

no light is reflected if L.S. is behind the surface

I_p → intensity of L.S.

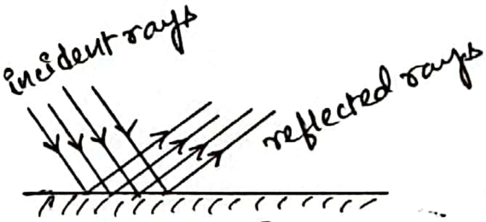
k_d → [0,1] diffuse coefficient



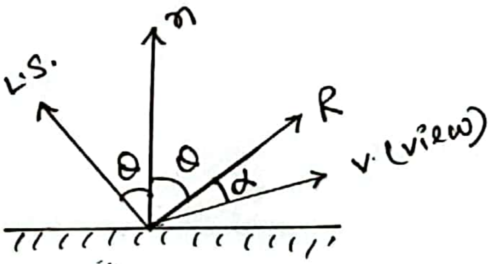
$a \cdot b = |a| \cdot |b| \cos\theta$

$\therefore \cos\theta = \underbrace{L \cdot n}_{\text{unit vectors}}$

$$D = I_p k_d \max[L \cdot n, 0] \quad \text{— (1)}$$



Specular Reflection



Phong's Specular Model

amount of light being reflection

$$S = I_p k_s \cdot \underbrace{\cos^n(\alpha)} \quad \text{— (2)}$$

k_s → [0,1] specular coefficient

n → specular exponent/shininess

α → angle betn R & V

$\therefore \cos\alpha = \underbrace{V \cdot R}_{\text{unit vectors}}$

where $\bar{R} = 2(\hat{L} \cdot \hat{n})\hat{n} - \hat{L}$ — (3)

$I = I_a k_a + I_p k_d \max(L \cdot n, 0) + I_p k_s (\max(V \cdot R, 0))^n$

Ambient light — Diffuse light — Specular light

Attenuation — loss of light energy over space

$$f_{att} = \max\left[1 - (d/r)^{\gamma}, 0\right]$$

r = radius of the L.S. sphere of influence

$$I = I_a k_a + I_p f_{att} (k_d \cdot \max(L \cdot n, 0) + k_s \cdot (\max(V \cdot R, 0))^n)$$

For multiple L.S.,

$$I = I_a k_a + \sum_{i=1}^{m} I_{p_i} f_{att} (k_d \cdot \max(L_i \cdot n, 0) + k_s (\max(V \cdot R_i, 0))^n)$$

— x —

Ex. A light source w/ intensity 5 and radius of influence 50 is located at (2,3,4) from which you are called to calculate the illumination of a point on the xy plane. The camera is set at a point (5,6,3) & the light is reflected back from (4,4) of the plane. The ambient, diffuse and specular coefficient is given at 0.2, ~~0.3~~ 0.5 and 0.4 respectively.

(a) Represent the reflected ray R in the unit vector
(b) Specular Reflection Intensity for a shininess factor of n
(c) Attenuation factor
(d) If ambient light ~~inten~~ intensity is at 2, calculate the total reflected light intensity at the given point according to Phong's model w/ the attenuation factor.

(a) $L = (2,3,4) - (4,4,0)$

$= (-2,-1,4)$

xy plane normal = z-axis $= \hat{k}$

$\hat{L} = \dfrac{-2\hat{i} - \hat{j} + 4\hat{k}}{\sqrt{21}}$

$\hat{L}\cdot\hat{n} = \dfrac{(-2\hat{i} - \hat{j} + 4\hat{k})}{\sqrt{21}} \times \hat{k}$

$= \dfrac{4}{\sqrt{21}}$

$\bar{R} = 2(\hat{L}\cdot\hat{n})\hat{n} - \hat{L}$

$= \dfrac{8}{\sqrt{21}}\hat{k} - \cancel{(-2,-1,4)} \dfrac{(-2\hat{i} - \hat{j} + 4\hat{k})}{\sqrt{21}}$

$= \dfrac{1}{\sqrt{21}}(+2\hat{i} + \hat{j} - 4\hat{k} + 8\hat{k})$

$= \dfrac{1}{\sqrt{21}}(2\hat{i} + \hat{j} + 4\hat{k})$

$\hat{i}\cdot\hat{i} = 1$

(b) $\bar{V} = (5,6,3) - (4,4,0)$

$= (1,2,3)$

$\hat{V} = \dfrac{1}{\sqrt{14}}(\hat{i} + 2\hat{j} + 3\hat{k})$

$\hat{R}\cdot\hat{V} = \dfrac{1}{\sqrt{21}}\cdot\dfrac{1}{\sqrt{14}}(2\hat{i} + 2\hat{j} + 12\hat{k})$

$= \dfrac{1}{\sqrt{294}}(16) = \dfrac{16}{\sqrt{294}}$

$I = I_p \cdot k_s \cdot \left(\dfrac{16}{\sqrt{294}}\right)^n$

$= 5 \times 0.4 \times \left(\dfrac{16}{\sqrt{294}}\right)^{10}$

(c) $d = [(2-4)^2 + (3-7)^2 + (4-0)^2]^{1/2}$

$= \sqrt{21}$

$f_{att} = \max(1 - (d/r)^2, 0)$

$= \max(1 - (\frac{\sqrt{21}}{50})^2, 0)$

$= 0.9916$

(d) $I = I_a k_a + I_p f_{att} (k_d \max(\hat{L} \cdot \hat{n}, 0) + k_s (\max(\hat{R} \cdot \hat{V}, 0))^n)$

$= 2 \times 0.2 + 5 \times 0.9916 (0.5 \times \frac{4}{\sqrt{21}} + 0.4 \times (\frac{16}{\sqrt{294}})^{10})$

$= 0.8333$

(Ans)

CSE423 – Computer Graphics – AJA
Lecture 11+12+13 – Projection

Projection – a process by which we can transform points in a coordinate system of dimension "n" (3D) into points in a coordinate system of dimension <u>less</u> than "n" (2D).

3D Model → Projection → 2D Model
I/P   O/P

Centre of Projection → C.O.P.

* Projection → onto a plane
* Projectors → straight lines

3D to 2D Projection is defined by straight projection rays (projectors) emerging from the C.O.P. passing through each point of the object & intersecting the "projection plane" to form a projection.

2 types of Projection –

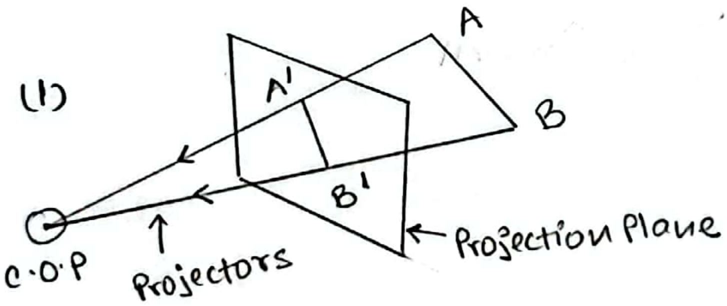
(I) Perspective (distance to C.O.P is finite)

(II) Parallel (∞)

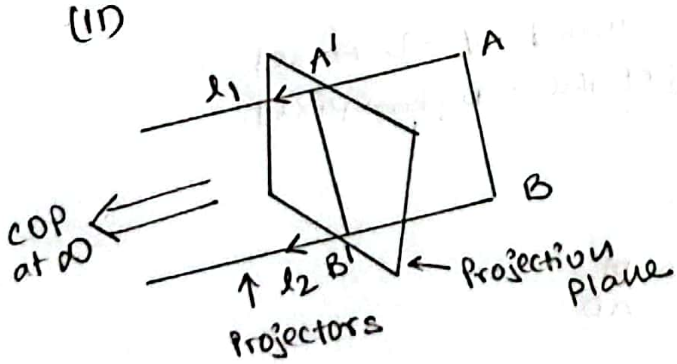
– l_1 & l_2 rays of sight

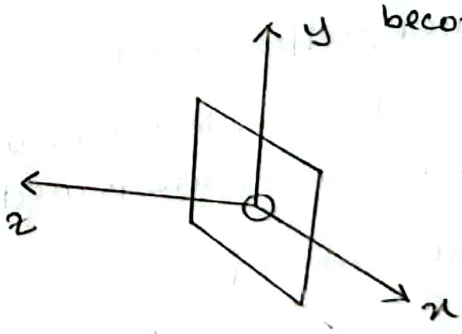
– inc. d to ∞, the rays become ||.

(I)



(II)

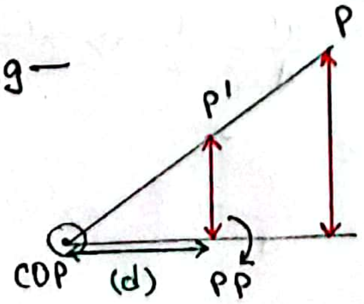




Monitor in 3D-view.

–ve z axis inside & +ve z axis outside the plane.

Simplifying –



d → distance from C.O.P to P.P.

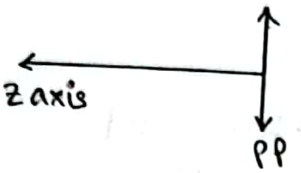
$|P'| = |P_p| \cdot |P|$

$|P'|$ → projection, $|P_p|$ → projection matrix, $|P|$ → object

Based on the orientation of PP :

(I) Orthographic Projection –

z-axis perpendicular to the projection plane



(II) Oblique Projection –

z-axis not perpendicular to the projection plane

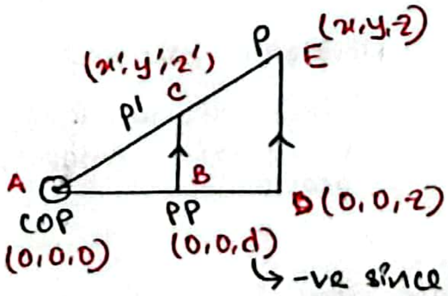
(excluded from the syllabus)

Perspective	Parallel
— human vision	— less realistic due to no foreshortening
— C.O.P. distance ↑ object size ↓ (inversely proportional)	— ∥ lines remain ∥
— angles remain intact for faces ∥ to projection plane.	— ✓ as well
— ∥ lines do not project ∥ lines	

P' → final pixel/coordinate

For an object, calculations + view are determined w/ 3D Model. And the projection plane is used to project the 2D ~~plane~~ pixel.

(*) Origin is at COP.



↳ -ve since it goes to the -ve z axis; z is also -ve.

Similar △s,

$$\frac{BC}{AB} = \frac{DE}{AD}$$

x' :

$$\frac{x'}{d} = \frac{x}{z} \Rightarrow x' = x/(z/d)$$

z' :

$$\frac{z'}{d} = \frac{z}{z} \Rightarrow z' = z/(z/d)$$

y' :

$$\frac{BC}{AB} = \frac{DE}{AD} \Rightarrow \frac{y'}{d} = \frac{y}{z}$$

$$\Rightarrow y' = y/(z/d)$$

$$\begin{bmatrix}x'\\y'\\z'\\1\end{bmatrix} = \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&1/d&0\end{bmatrix}\cdot\begin{bmatrix}x\\y\\z\\1\end{bmatrix}$$

$\rightarrow$?

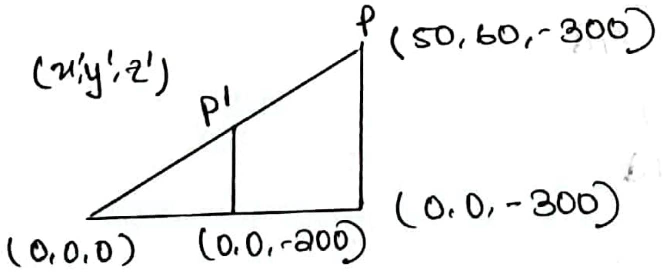
projection matrix

$$\begin{bmatrix}x'\\y'\\z'\\1\end{bmatrix} = \begin{bmatrix}x\\y\\z\\z/d\end{bmatrix} \rightarrow \begin{bmatrix}x'\\y'\\z'\\1\end{bmatrix} = \begin{bmatrix}x'/z/d\\y'/z/d\\z'/z/d\\1\end{bmatrix} ✓$$

$\hookrightarrow$ X

$\Rightarrow$ To make it match, we need all terms in z/d (divide)

Ex. origin at COP, calculate projected point coordinates for a point (50, 60, -300) where the projection plane is at a distance of 200 from the COP?



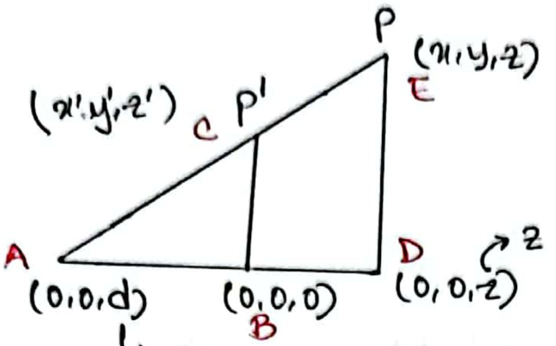
$$\begin{bmatrix}x'\\y'\\z'\\1\end{bmatrix} = \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&-1/200&0\end{bmatrix}\begin{bmatrix}50\\60\\-300\\1\end{bmatrix}$$

$$= \begin{bmatrix}50\\60\\-300\\3/2\end{bmatrix} \rightarrow \text{Divide by } 3/2$$

$$= \begin{bmatrix}33.33\\40\\-200\\1\end{bmatrix}$$

$\therefore$ P' = (3333, 40, -200)

(4) Origin is at PP.



z → -ve as it is in the -ve z-axis

(0,0,d) ↳ d is +ve as it is towards the +ve z-axis

$$y' \rightarrow \frac{BC}{AB} = \frac{DE}{AD}$$

$$\Rightarrow \frac{y'}{d} = \frac{y}{d-z} \rightarrow z = -ve \text{ so actually an add}^n.$$

$$\Rightarrow y' = \frac{y}{d-z/d} = \frac{y}{1-z/d}$$

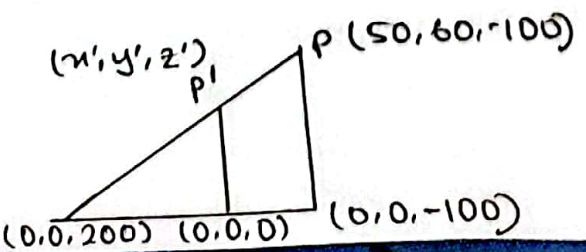
$$x' \Rightarrow x' = \frac{x}{d-z/d} = \frac{x}{1-z/d}$$

$$z' \Rightarrow z' = 0 \text{ (origin at PP)}$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1/d & 1 \end{bmatrix}}_{\text{Projection Matrix}} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 0 \\ 1-z/d \end{bmatrix} \Rightarrow \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} x/1-z/d \\ y/1-z/d \\ 0 \\ 1 \end{bmatrix}$$

Ex. for origin at PP, calculate projected point coordinates for a point (50, 60, -100) where d = 200 from COP?



$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1/200 & 1 \end{bmatrix} \cdot \begin{bmatrix} 50 \\ 60 \\ -100 \\ 1 \end{bmatrix} = \begin{bmatrix} 50 \\ 60 \\ 0 \\ 1.5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 50/1.5 \\ 60/1.5 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 33.33 \\ 40 \\ 0 \\ 1 \end{bmatrix}$$

$\therefore P'(33.33, 40, 0)$

→ one plane is fixed w/ a fixed value

(*) Orthographic Projection Matrix —

$$\underbrace{\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}}_{P_m} \Bigg\} \text{ Projection on } xy \text{ plane while } z = 0.$$

$$P_m * \begin{bmatrix} 2 \\ 4 \\ 9 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 0 \\ 1 \end{bmatrix} \rightarrow (2,4) \quad [\text{3D to 2D}]$$

(*) Sometimes z is fixed to a certain value.

$z = 1$

$$\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \underline{\underline{1}} \\ 0 & 0 & 0 & 1 \end{matrix} \rightarrow P_m * \begin{bmatrix} 2 \\ 4 \\ 9 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 0 + \underline{\underline{1}} \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ \underline{\underline{1}} \\ 1 \end{bmatrix}$$

P_m

(*) Suppose, yz plane & x is fixed at $x = 6$.

$$\begin{matrix} 0 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix} * \begin{bmatrix} x \\ y \\ z \\ w=1 \end{bmatrix} = \begin{bmatrix} 6 \\ y \\ z \\ 1 \end{bmatrix}$$

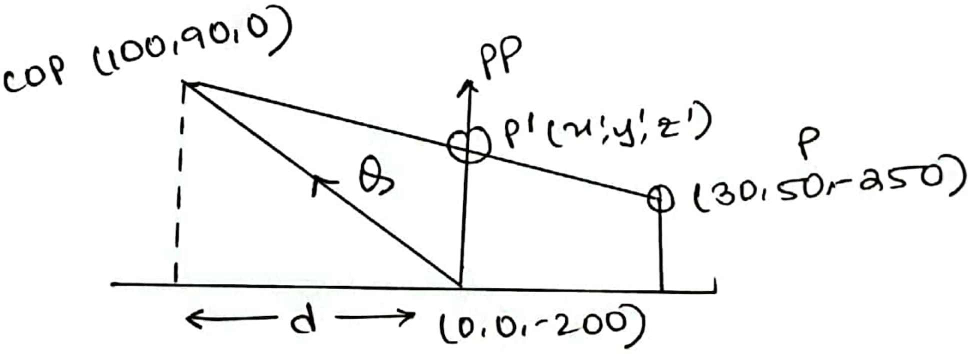
Given plane — consider their values ; ignore the others

Ex. xz plane → find for x & z ; ignore y.

General Perspective Matrix —

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\frac{Qdx}{Qdz} & zp \cdot \frac{Qdx}{Qdz} \\ 0 & 1 & -\frac{Qdy}{Qdz} & zp \cdot \frac{Qdy}{Qdz} \\ 0 & 0 & -\frac{zp}{Qdz} & zp \cdot \left(1 + \frac{zp}{Qdz}\right) \\ 0 & 0 & -1/Qdz & 1 + zp/Qdz \end{bmatrix} * \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Calculate the projected point coordinates for a point (30, 50, -250) where COP is at (100, 90, 0) & d = 200 from the COP.



$Qdx = 100$

$Qdy = 90$

$Qdz = 200$

$zp = -200$

$\therefore P' = (44, 58, -200)$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1/2 & -100 \\ 0 & 1 & -9/20 & -90 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1/200 & 0 \end{bmatrix} * \begin{bmatrix} 30 \\ 50 \\ -250 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 30+125-100 \\ 50+112.5-90 \\ -250 \\ 5/4 \end{bmatrix} = \begin{bmatrix} 55 \times 4/5 \\ 72.5 \times 4/5 \\ -250 \times 4/5 \\ 5/4 \times 4/5 \end{bmatrix} = \begin{bmatrix} 44 \\ 58 \\ -200 \\ 1 \end{bmatrix}$$