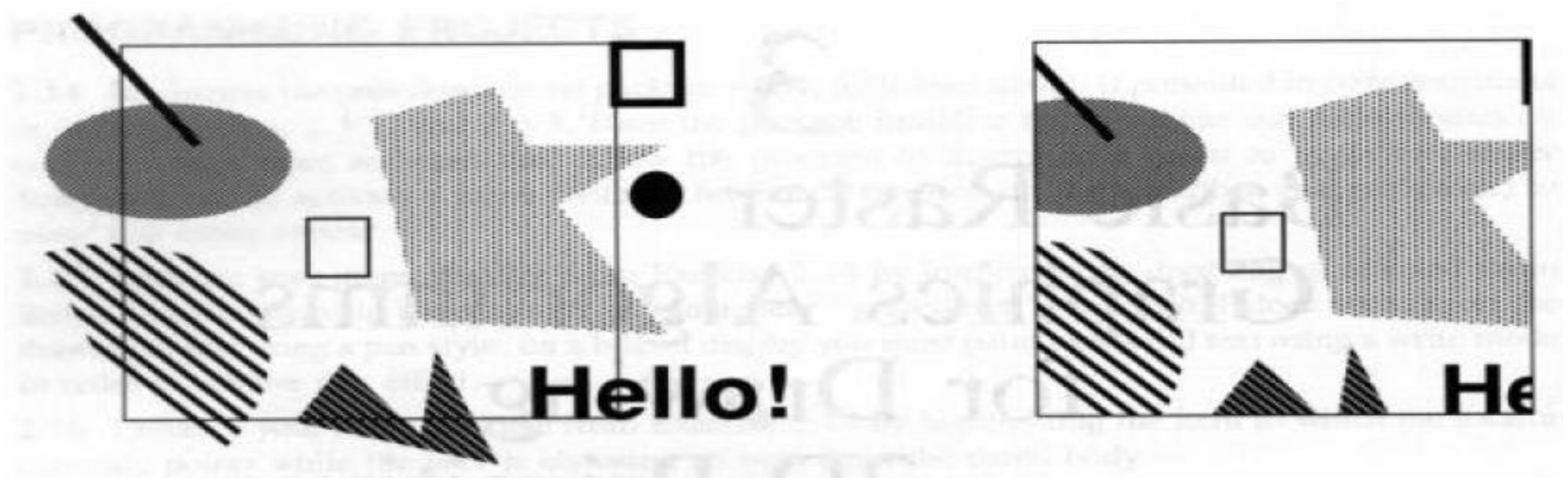
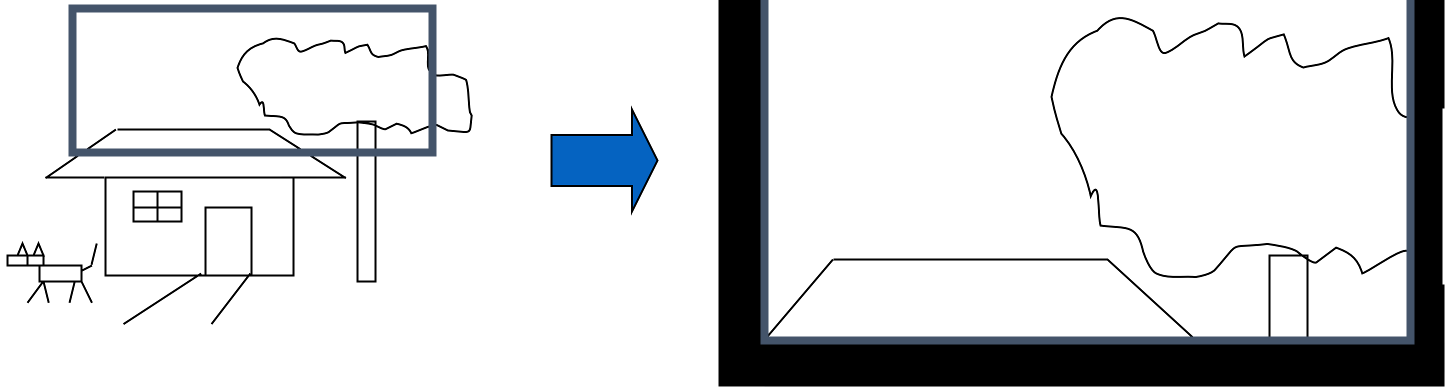
Clipping

Cohen-Sutherland Algo

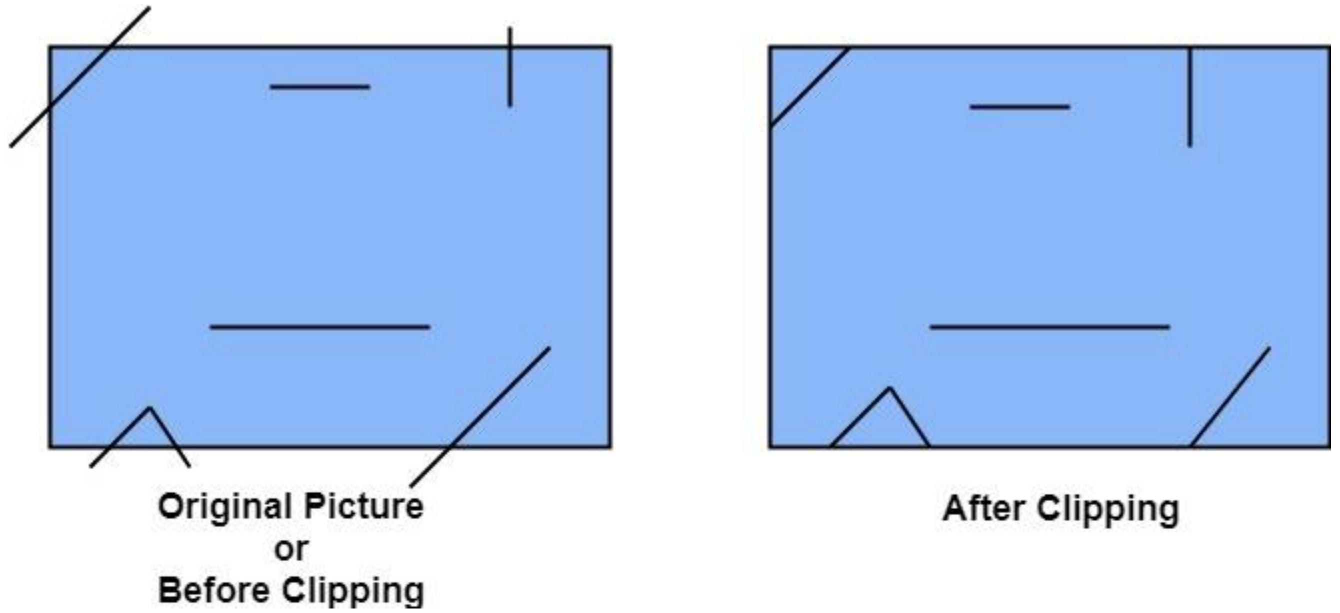
What is Clipping?



Clipped view in screen



Line Clipping



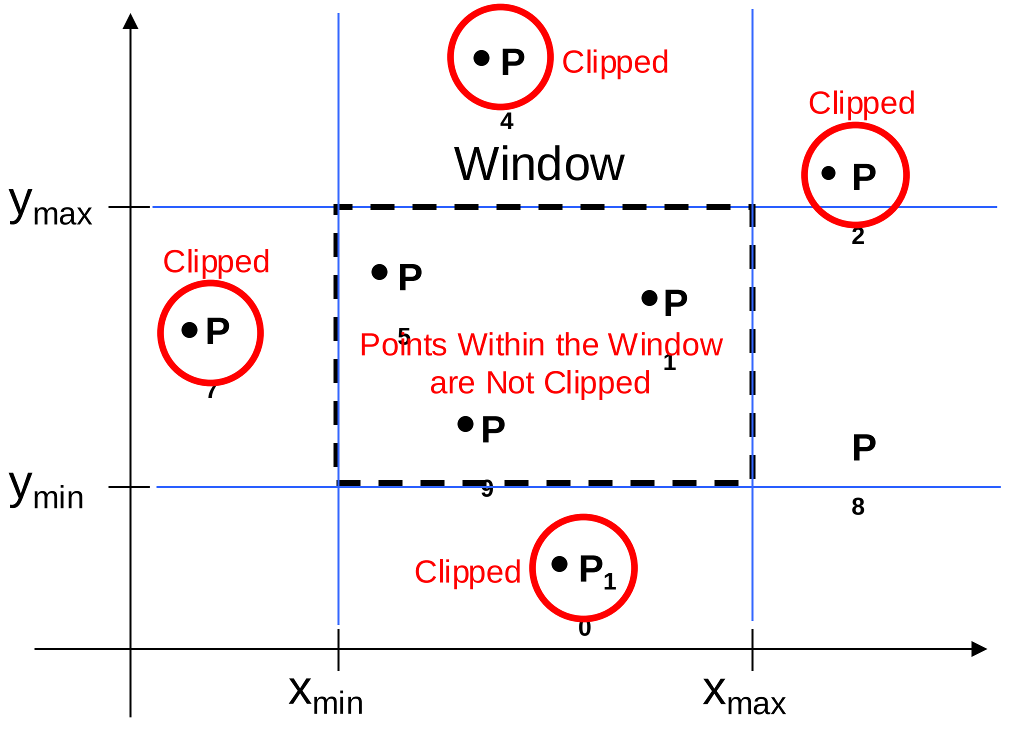
Clipping in a Raster World

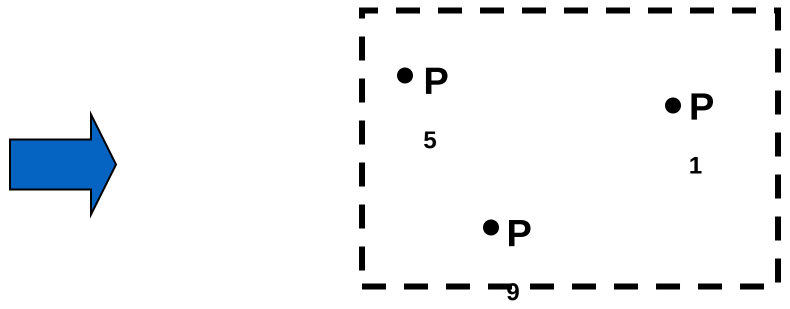
- Clipping techniques
  - **Analytical**
    - Lines, polygons etc.
    - Floating point graphics package
  - During scan conversion (scissoring)
    - Checking extrema suffices, internal points can be ignored
    - Circles, curves etc.
  - During writing a pixel
    - Outline primitive not much larger, few pixels are clipped

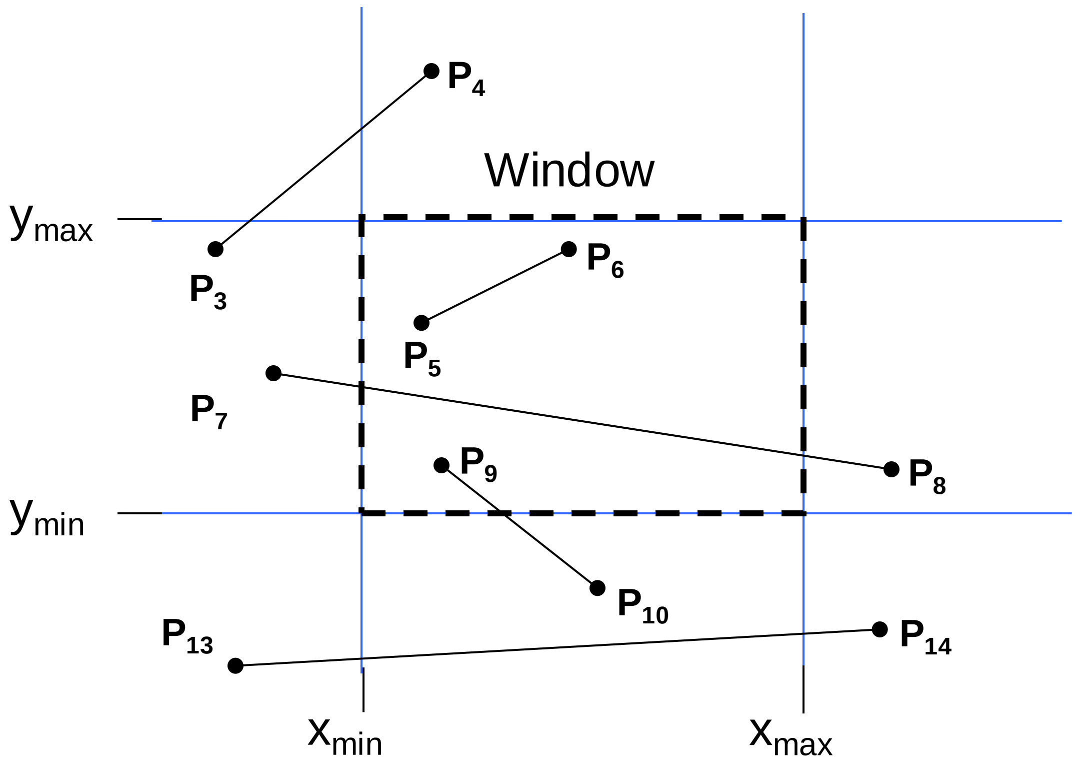
A point (x,y) is not clipped if:

$$x_{min} \leq x \leq x_{max} \text{ AND } y_{min} \leq y \leq y_{max}$$

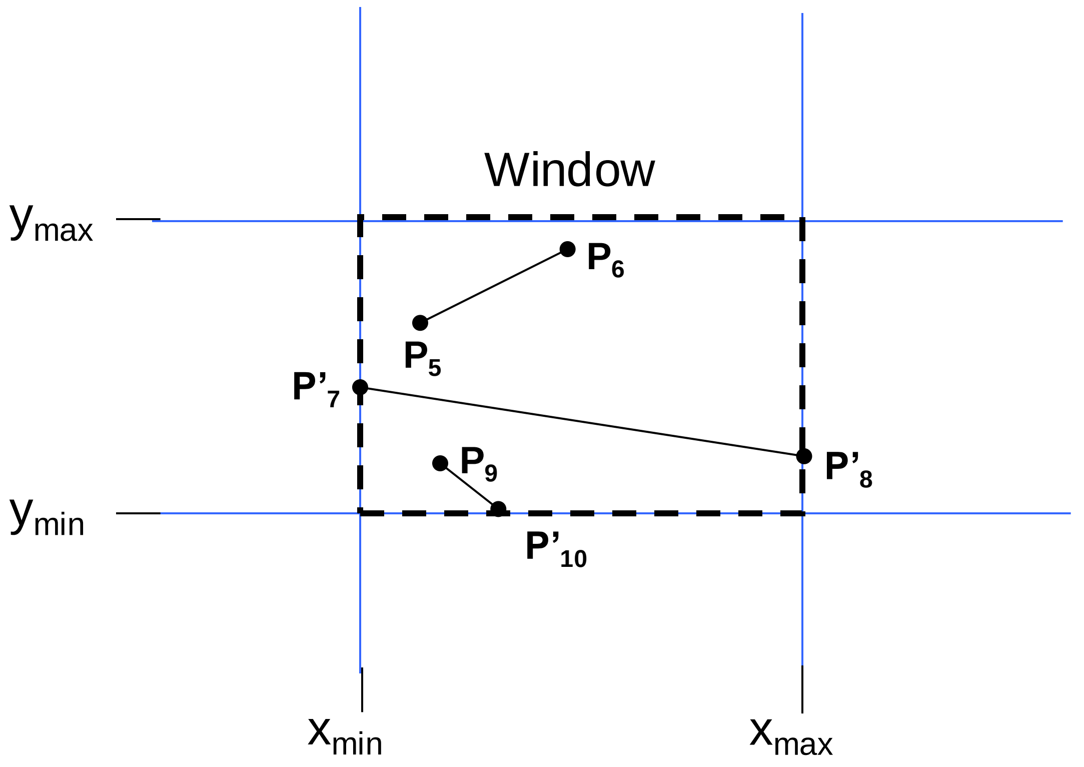
otherwise it is clipped







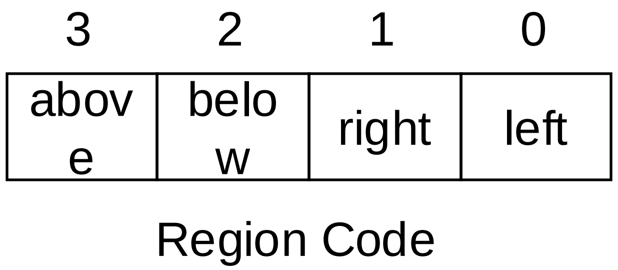
P_4
Window
y_{max}
P_3
P_6
P_5
P_7
P_9
P_8
y_{min}
P_{10}
P_{13}
P_{14}
x_{min}
x_{max}

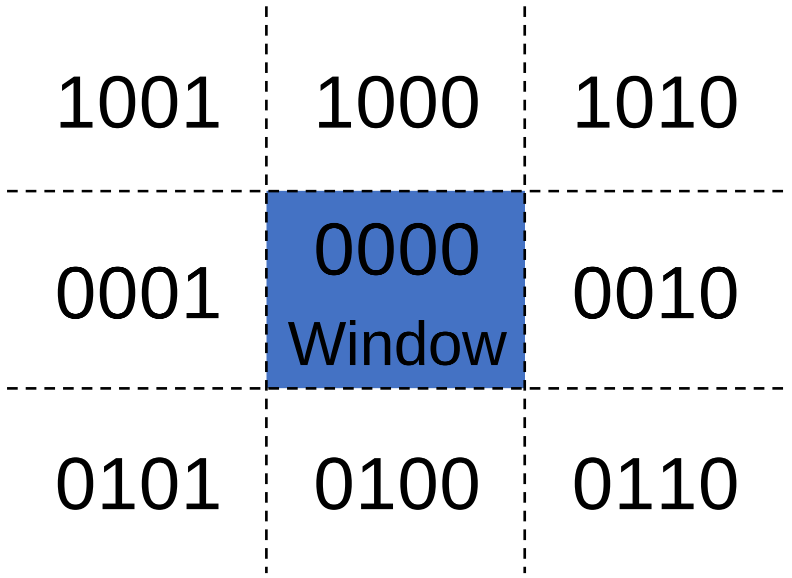


Window
y_{max}
P_6
P_5
P'_7
P_9
P'_8
y_{min}
P'_{10}
x_{min}
x_{max}

World space is divided into regions based on the window boundaries

- Each region has a unique four bit outcode
- Outcodes indicate the position of the regions with respect to the window



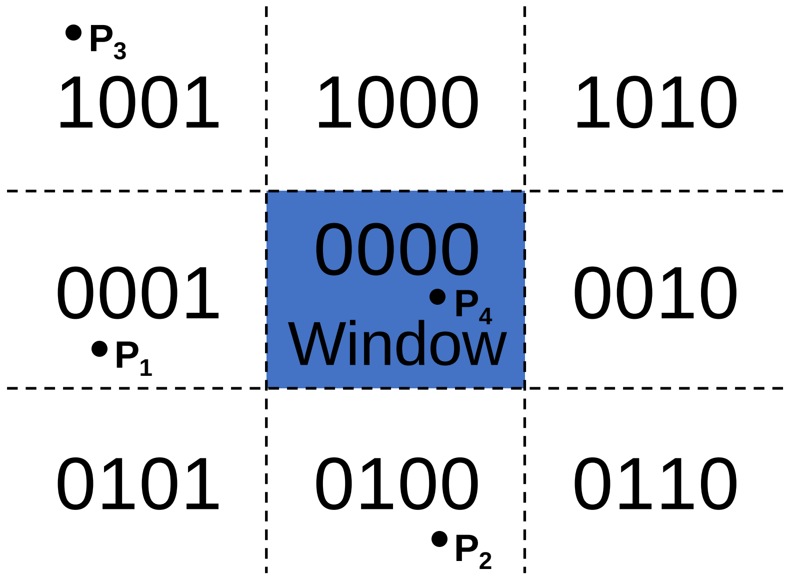


World space is divided into regions based on the window boundaries

- Each region has a unique four bit outcode
- Outcodes indicate the position of the regions with respect to the window

3	2	1	0
above	below	right	left

Region Code



```
Calculate_outcode(x,y){
    if (x<$x_{min}$) bit0 =1
    else bit0=0
    if (x>$x_{max}$) bit1=1
    else bit1=0
    if(y<$y_{min}$) bit2=1
    else bit2=0
    if (y>$y_{max}$) bit3=1
    else bit3=0
}
```

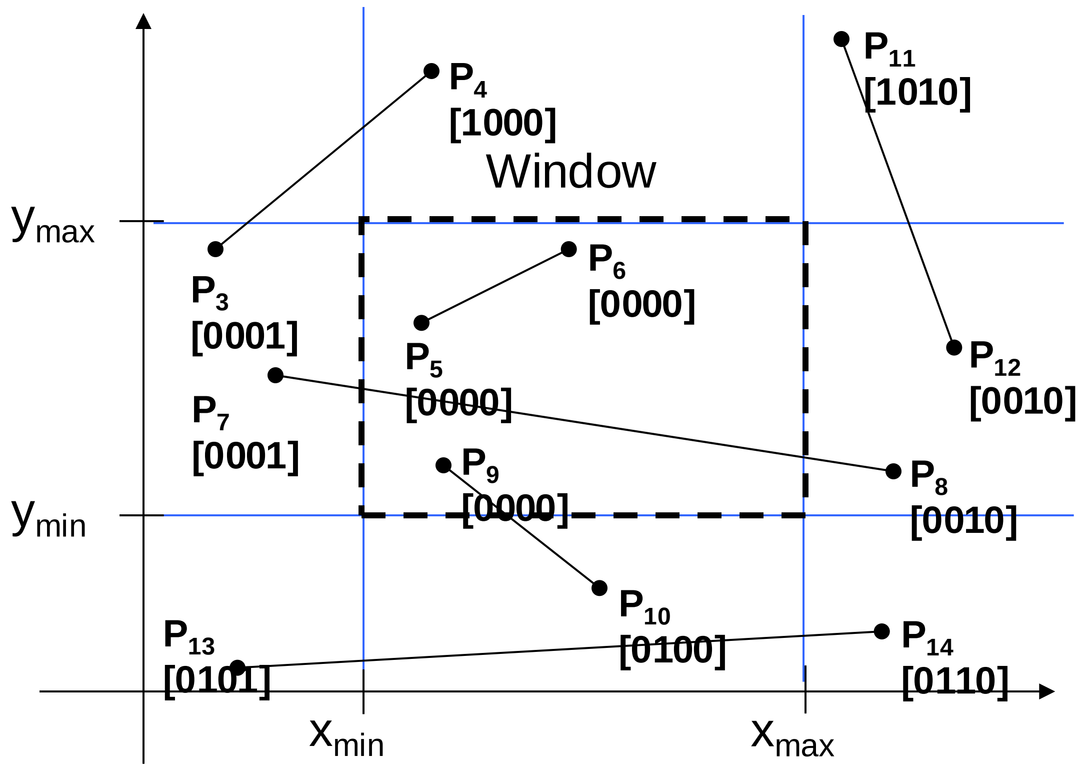
3	2	1	0
above	below	right	left

Region Code

```
Calculate_outcode(x,y){
    if ($x<x_{min}$) bit0 =1
    else bit0=0
    if ($x>x_{max}$) bit1=1
    else bit1=0
    if($y<y_{min}$) bit2=1
    else bit2=0
    if ($y>y_{max}$) bit3=1
    else bit3=0
}
```

3	2	1	0
above	below	right	left

Region Code



```
Calculate_Outcode_3D(x,y,z){
    if ($x<x_{min}$) bit0 =1
    else bit0=0
    if ($x>x_{max}$) bit1=1
    else bit1=0
    if($y<y_{min}$) bit2=1
    else bit2=0
    if ($y>y_{max}$) bit3=1
    else bit3=0
    if($z<z_{min}$) bit4=1
    else bit4=0
    if ($z>z_{max}$) bit5=1
    else bit5=0
}
```

5	4	3	2	1	0
Near	Far	Above	Below	Right	Left
Region Code					

```
cohen-Sutherland(x1, y1, x2, y2):
oc1 = calculate_outcode(x1, y1),
oc2 = calculate_outcode(x2, y2);
while(true) {
    if(oc1 == oc2 == 0000) {
        //declare completely inside
        output (x1, y1), (x2, y2) as clipped line
        break
    }
    else if((oc1 AND oc2)!=0000){ // condition to check matching bit
        //declare completely outside and clip
        break
    }
    else{
        if(oc1 != 0000){
                (x1, y1) = find intersection point of line
                    and the boundary corresponding
                    to non-zero bit of oc1
                oc1 = calculate_outcode(x1, y1)
        }
        else{
                (x2, y2) = find intersection point of line
                    and the boundary corresponding
                    to non-zero bit of oc2
                oc2 = calculate_outcode(x2, y2)
        }
        continue
}
```

```
cohen-Sutherland(x1, y1, x2, y2):
oc1 = calculate_outcode(x1, y1),
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while(true) {
    if (oc1 == oc2 == 0000) {
        //declare completely inside
        output (x1, y1), (x2, y2) as clipped line
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    else{
        if(oc1 != 0000){
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                oc1 = calculate_outcode(x1, y1)
        }
        else{
                (x2, y2) = find intersection point of line
                        and the boundary corresponding
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                oc2 = calculate_outcode(x2, y2)
        }
        continue
}
```

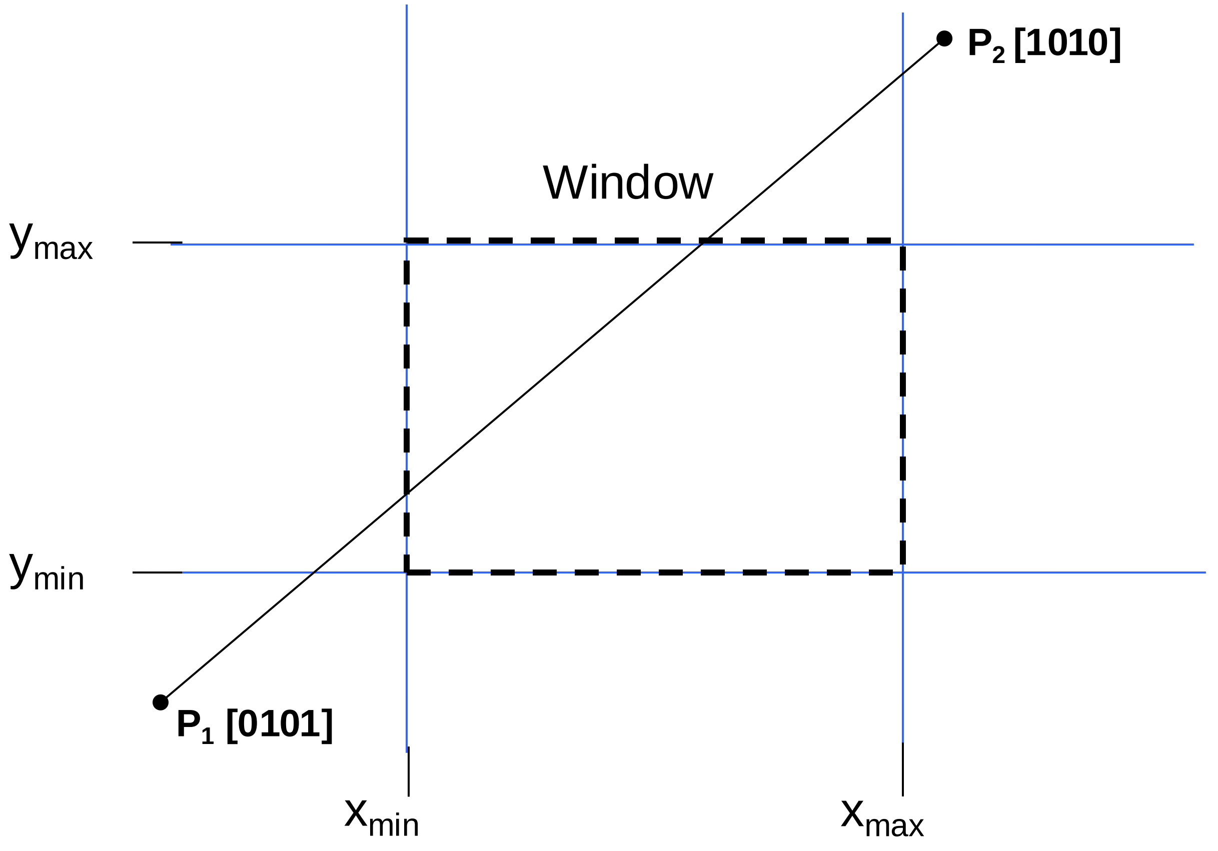

```
cohen-Sutherland(x1, y1, x2, y2):
oc1 = calculate_outcode(x1, y1),
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while(true) {
        if (oc1 == oc2 == 0000) {
                //declare completely inside
                output (x1, y1), (x2, y2) as clipped line
                break
        }
        else if ((oc1 AND oc2)!=0000) { // condition to check matching bit
                //declare completely outside and clip
                break
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                if(oc1 != 0000){
                                (x1, y1) = find intersection point of line
                                        and the boundary corresponding
                                        to non-zero bit of oc1
                                oc1 = calculate_outcode(x1, y1)
                }
                else{
                                (x2, y2) = find intersection point of line
                                        and the boundary corresponding
                                        to non-zero bit of oc2
                                oc2 = calculate_outcode(x2, y2)
                }
                continue

}
```

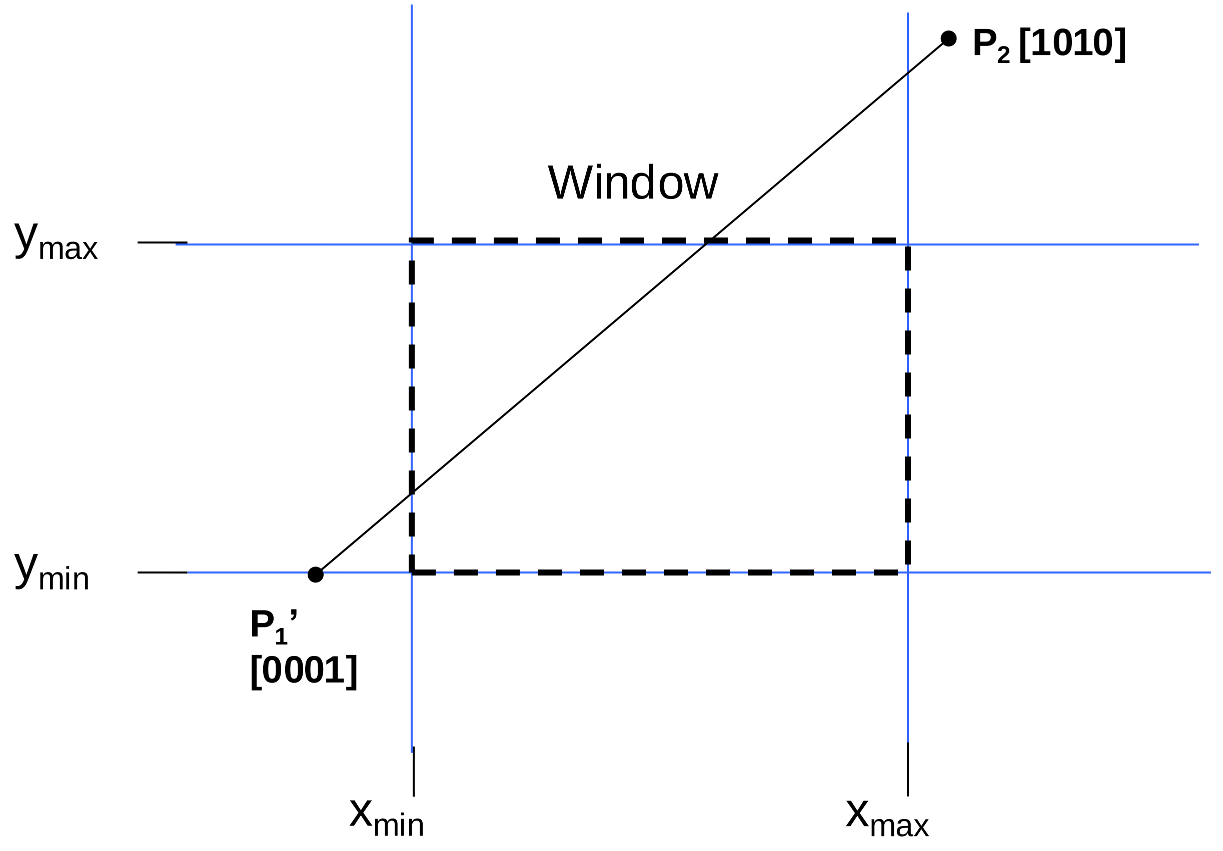

```
cohen-Sutherland(x1, y1, x2, y2):
oc1 = calculate_outcode(x1, y1),
oc2 = calculate_outcode(x2, y2);
while(true) {
    if (oc1 == oc2 == 0000) {
        //declare completely inside
        output (x1, y1), (x2, y2) as clipped line
        break
    }
    else if ((oc1 AND oc2)!=0000) { // condition to check matching bit
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    else{
        if(oc1 != 0000){
                (x1, y1) = find intersection point of line
                    and the boundary corresponding
                    to non-zero bit of oc1
                oc1 = calculate_outcode(x1, y1)
        }
        else{
                (x2, y2) = find intersection point of line
                    and the boundary corresponding
                    to non-zero bit of oc2
                oc2 = calculate_outcode(x2, y2)
        }
        continue

}
```
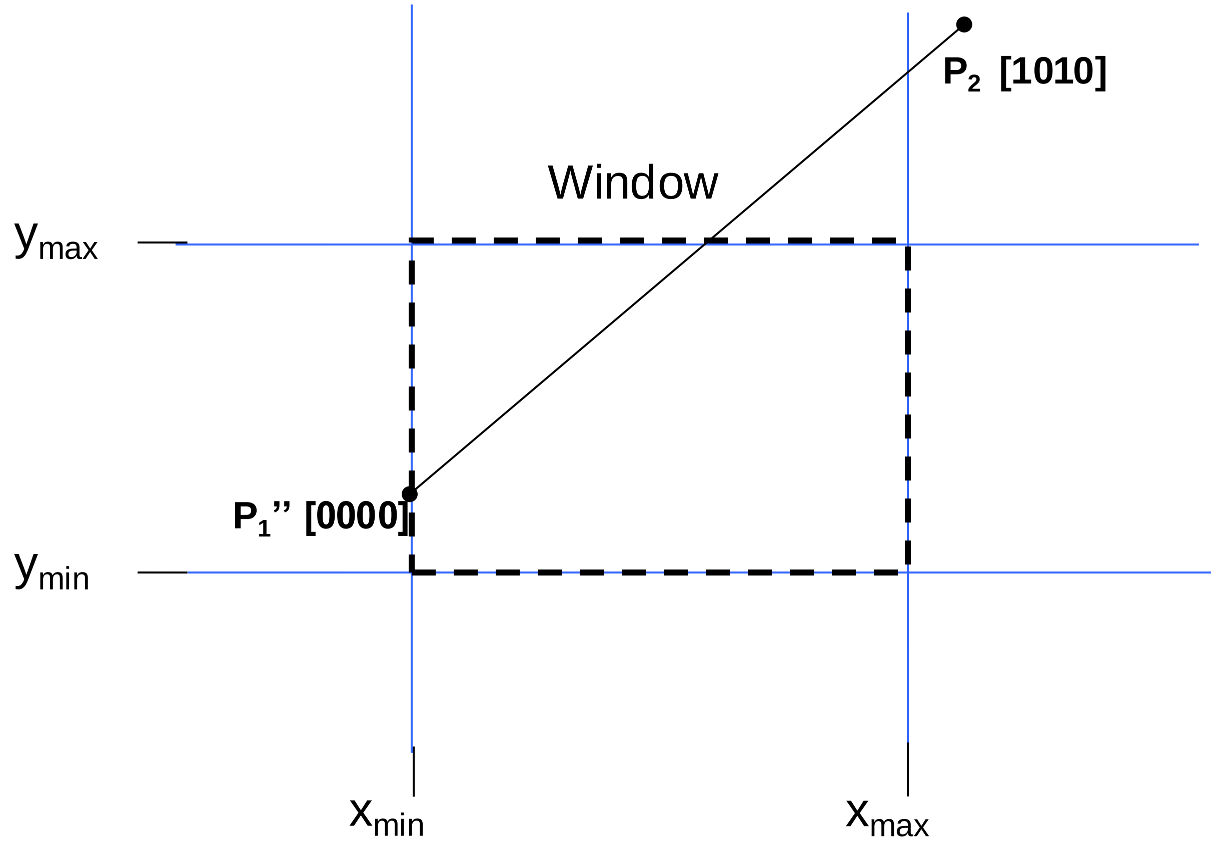
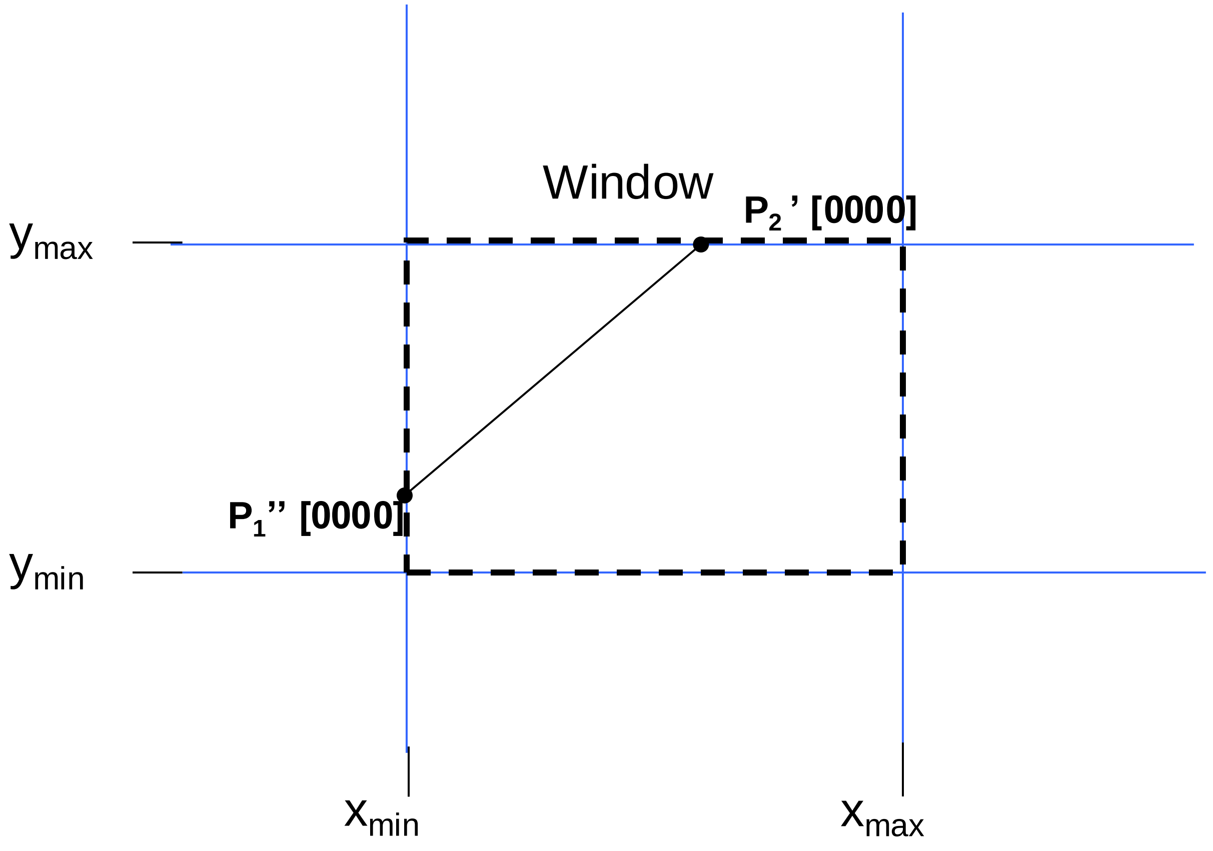
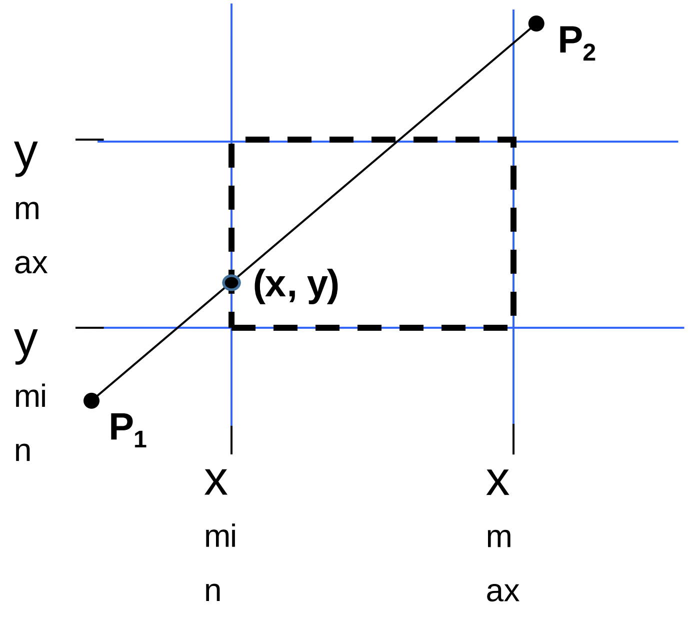



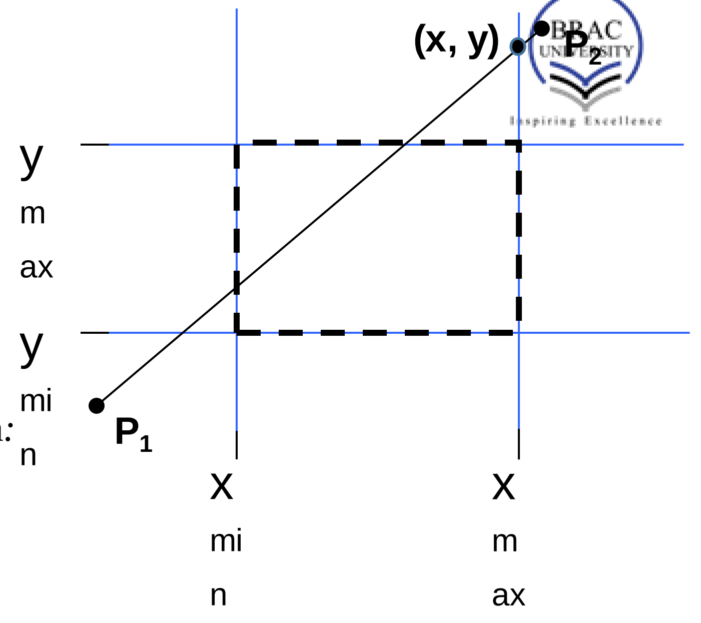
```
cohen-Sutherland(x1, y1, x2, y2):
oc1 = calculate_outcode(x1, y1),
oc2 = calculate_outcode(x2, y2);
while(true) {
        if (oc1 == oc2 == 0000) {
                //declare completely inside
                output (x1, y1), (x2, y2) as clipped line
                break
        }
        else if ((oc1 AND oc2)!=0000) { // condition to check matching bit
                //declare completely outside and clip
                break
        }
        else{
                if(oc1 != 0000){
                                (x1, y1) = find intersection point of line
                                        and the boundary corresponding
                                        to non-zero bit of oc1
                                oc1 = calculate_outcode(x1, y1)
                }
                else{
                                (x2, y2) = find intersection point of line
                                            and the boundary corresponding
                                            to non-zero bit of oc2
                                oc2 = calculate_outcode(x2, y2)
                }
                continue
}
```



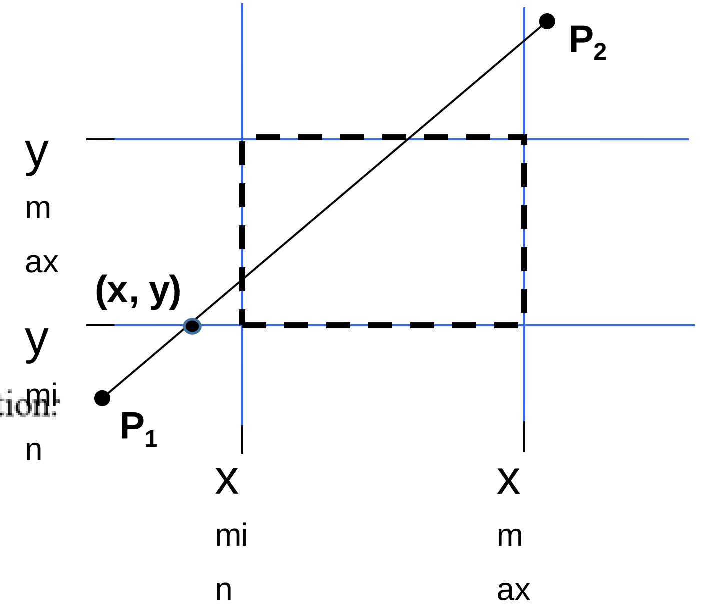
LEFT boundary intersection:
$x = x_{min}$
$y = y_1 + m\,(x_{min} - x_1)$



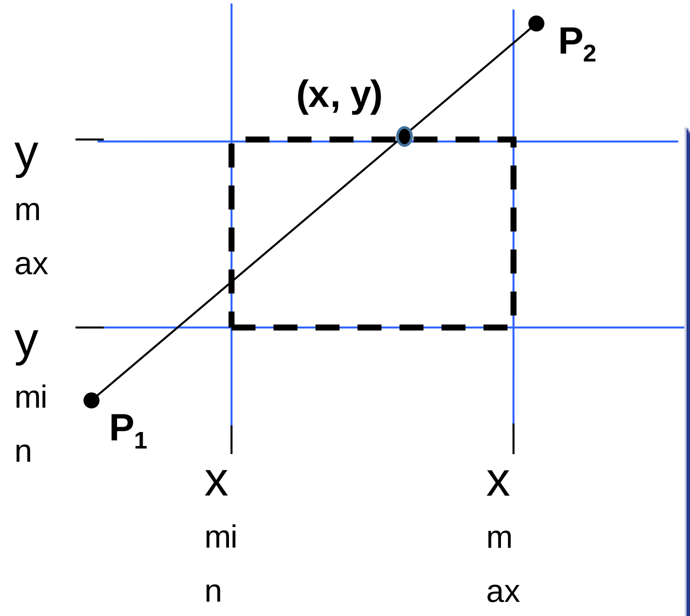
RIGHT boundary intersection:
$x = x_{max}$
$y = y_1 + m\,(x_{max} - x_1)$



BOTTOM boundary intersection:
$y = y_{min}$
$x = x_1 + \frac{1}{m}.(y_{min} - y_1)$



TOP boundary intersection:
$y = y_{max}$
$x = x_1 + \frac{1}{m}.(y_{max} - y_1)$



Determine whether the following line are accepted/rejected/partial using Cohen Sutherland line clipping algorithm.

a) Given (-250,-200) to (250,200) be the clip region.

- (i) (-100, -220) to (300, -210).
- (ii) (-250, 200) to (250, -200).

b) Given (0,0) to (300,200) be the clip region.

- (i) (50, -125) to (-100, 225).
- (ii) (-250, 200) to (250, -200).

If they are partially accepted/rejected find the line segment within the clipping window.

a)(i) boundary:

$$x_{min} = -250,\ \ x_{max} = 250,\ \ y_{min} = -200,\ \ y_{max} = 200$$

points:

$$x_1 = -100, y_1 = -220, x_2 = 300, y_2 = -210$$

Outcode calculation:

$x_{min} < x_1 < x_{max}$

so, no left or right bit

$y_1 < y_{min}$,

so, bottom bit is 1

so, outcode1 = 0100

$x_1 > x_{max}$

so, right bit

$y_2 < y_{min}$

so, bottom bit

so, outcode2 = 0110

outcode1 AND outcode2 = 0100 != 0000

so the line is completely outside.

```
cohen-Sutherland(x1, y1, x2, y2):
oc1 = calculate_outcode(x1, y1),
oc2 = calculate_outcode(x2, y2);
while(true) {
    if (oc1 == oc2 == 0000) {
        //declare completely inside
        output (x1, y1), (x2, y2) as clipped line
        break
    }
    else if ((oc1 AND oc2)!=0000) { // condition to check matching bit
        //declare completely outside and clip
        break
    }
    else{
        if(oc1 != 0000){
                (x1, y1) = find intersection point of line
                    and the boundary corresponding
                    to non-zero bit of oc1
                oc1 = calculate_outcode(x1, y1)
        }
        else{
                (x2, y2) = find intersection point of line
                    and the boundary corresponding
                    to non-zero bit of oc2
                oc2 = calculate_outcode(x2, y2)
        }
        continue

}
```

a)(ii) boundary:

$$x_{min} = -250,\ \ x_{max} = 250,\ \ y_{min} = -200,\ \ y_{max} = 200$$

points:

$$x_1 = -250, y_1 = 200, x_2 = 250, y_2 = -200$$

Outcode calculation:

$x_{min} \leq x_1 < x_{max}$

so, no left or right bit

$y_{min} < y_1 \leq y_{max}$,

so, no top or bottom bit

so, outcode1 = 0000

similarly, outcode2 = 0000

since outcode1 & outcode 2 both are 0000

so the line is completely inside.

```
cohen-Sutherland(x1, y1, x2, y2):
oc1 = calculate_outcode(x1, y1),
oc2 = calculate_outcode(x2, y2);
while(true) {
    if (oc1 == oc2 == 0000) {
        //declare completely inside
        output (x1, y1), (x2, y2) as clipped line
        break
    }
    else if ((oc1 AND oc2)!=0000) { // condition to check matching bit
        //declare completely outside and clip
        break
    }
    else{
        if(oc1 != 0000){
                (x1, y1) = find intersection point of line
                    and the boundary corresponding
                    to non-zero bit of oc1
                oc1 = calculate_outcode(x1, y1)
        }
        else{
                (x2, y2) = find intersection point of line
                    and the boundary corresponding
                    to non-zero bit of oc2
                oc2 = calculate_outcode(x2, y2)
        }
        continue
}
```

b)(i) boundary:

$$x_{min} = 0,\ \ x_{max} = 300,\ \ y_{min} = 0,\ y_{max} = 200$$

points:

$$x_1 = 50, y_1 = -125, x_2 = -100, y_2 = 225$$

Outcode calculation:

outcode1 = 0100

outcode2 = 1001

outcode1 AND outcode2 = 0000

so partially inside

outcode1 != 0000

outcode1 has bottom bit

Applying bottom intersection:

$y_1 = y_{min} = 0$

$x_1 = x_1 + \frac{1}{m}.(y_{min} - y_1) = 50 + \frac{-150}{350}.(0 + 125) =$ -3.57

outcode1 = 0001 [recalculated]

outcode2 = 1001

outcode1 AND outcode2 = 0001

so completely outside

```
cohen-Sutherland(x1, y1, x2, y2):
oc1 = calculate_outcode(x1, y1),
oc2 = calculate_outcode(x2, y2);
while(true) {
    if (oc1 == oc2 == 0000) {
        //declare completely inside
        output (x1, y1), (x2, y2) as clipped line
        break
    }
    else if ((oc1 AND oc2)!=0000) { // condition to check matching bit
        //declare completely outside and clip
        break
    }
    else{
        if(oc1 != 0000){
                (x1, y1) = find intersection point of line
                    and the boundary corresponding
                    to non-zero bit of oc1
                oc1 = calculate_outcode(x1, y1)
        }
        else{
                (x2, y2) = find intersection point of line
                    and the boundary corresponding
                    to non-zero bit of oc2
                oc2 = calculate_outcode(x2, y2)
        }
        continue
}
```

b)(ii) boundary:

$$x_{min} = 0, \quad x_{max} = 300, \quad y_{min} = 0, \quad y_{max} = 200$$

points:

$$x_1 = -250, y_1 = 200, x_2 = 250, y_2 = -200$$

Outcode calculation:

outcode1 = 0001

outcode2 = 0100

outcode1 AND outcode2 = 0000

so partially inside

outcode1 != 0000

outcode1 has left bit

applying left intersection:
$x_1 = x_{min} = 0$

$y_1 = y_1 + m.(x_{min} - x_1) = 200 + \frac{-400}{500}.(0 + 250) = 0$

outcode1 = 0000 [recalculated]

so (x1, y1) has been clipped to (0, 0)

outcode2 = 0100

so partially inside

```
cohen-Sutherland(x1, y1, x2, y2):
oc1 = calculate_outcode(x1, y1),
oc2 = calculate_outcode(x2, y2);
while(true) {
    if (oc1 == oc2 == 0000) {
        //declare completely inside
        output (x1, y1), (x2, y2) as clipped line
        break
    }
    else if ((oc1 AND oc2)!=0000) { // condition to check matching bit
        //declare completely outside and clip
        break
    }
    else{
        if(oc1 != 0000){
                (x1, y1) = find intersection point of line
                    and the boundary corresponding
                    to non-zero bit of oc1
                oc1 = calculate_outcode(x1, y1)
        }
        else{
                (x2, y2) = find intersection point of line
                    and the boundary corresponding
                    to non-zero bit of oc2
                oc2 = calculate_outcode(x2, y2)
        }
        continue
}
```

b)(ii) boundary:

$$x_{min} = 0, \;\; x_{max} = 300, \;\; y_{min} = 0, \;\; y_{max} = 200$$

points:

$$x_1 = 0, y_1 = 0, x_2 = 250, y_2 = -200$$

(continued)

outcode1 = 0000 [recalculated]

outcode2 = 0100

so partially inside

outcode1 = 0000

so going into else codeblock,

outcode 2 has bottom bit

applying bottom intersection:
$y_2 = y_{min} = 0$

$$x_2 = x_2 + \frac{1}{m}.(y_{min} - y_2) = 250 + \frac{250}{-200}.(0 + 200) = 0$$

[Note: m has been recalculated, you can skip recalculation too]

outcode2 = 0000

outcode1=0000

so completely inside

The clipped segment is between (0, 0) to (0, 0) which is just a single point.

```
cohen-Sutherland(x1, y1, x2, y2):
oc1 = calculate_outcode(x1, y1),
oc2 = calculate_outcode(x2, y2);
while(true) {
    if (oc1 == oc2 == 0000) {
        //declare completely inside
        output (x1, y1), (x2, y2) as clipped line
        break
    }
    else if ((oc1 AND oc2)!=0000) { // condition to check matching bit
        //declare completely outside and clip
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        if(oc1 != 0000){
                (x1, y1) = find intersection point of line
                    and the boundary corresponding
                    to non-zero bit of oc1
                oc1 = calculate_outcode(x1, y1)
        }
        else{
                (x2, y2) = find intersection point of line
                    and the boundary corresponding
                    to non-zero bit of oc2
                oc2 = calculate_outcode(x2, y2)
        }
        continue
}
```

Cohen-Sutherland Line Clipping Algorithm

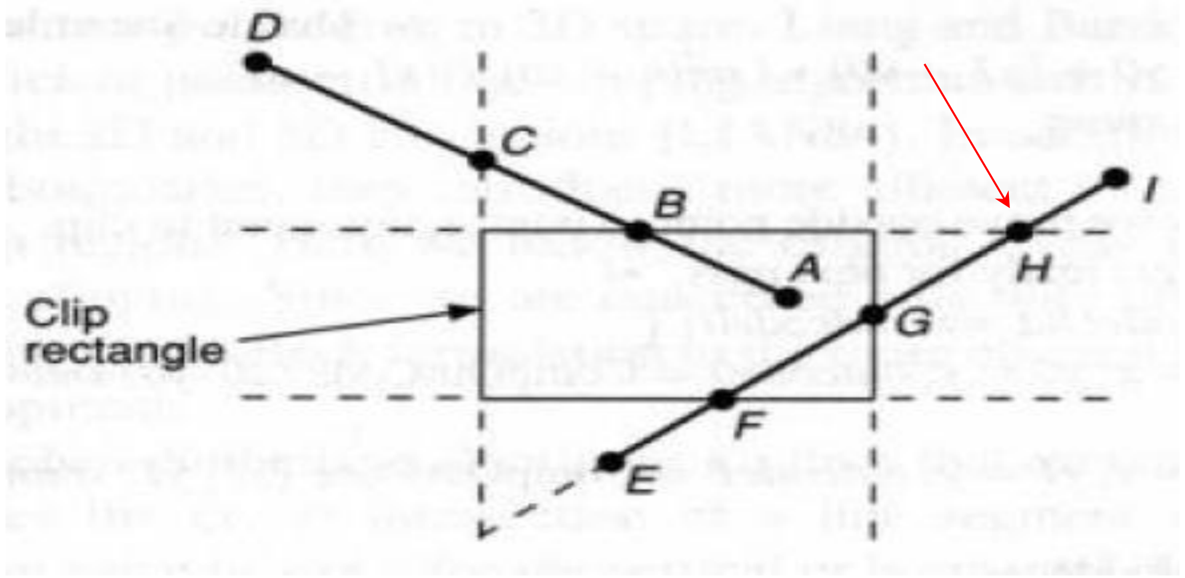
Works well for two cases:

1. Very large clip region
2. Very small clip region

- Why?

[For many trivial accept

and many trivial reject]



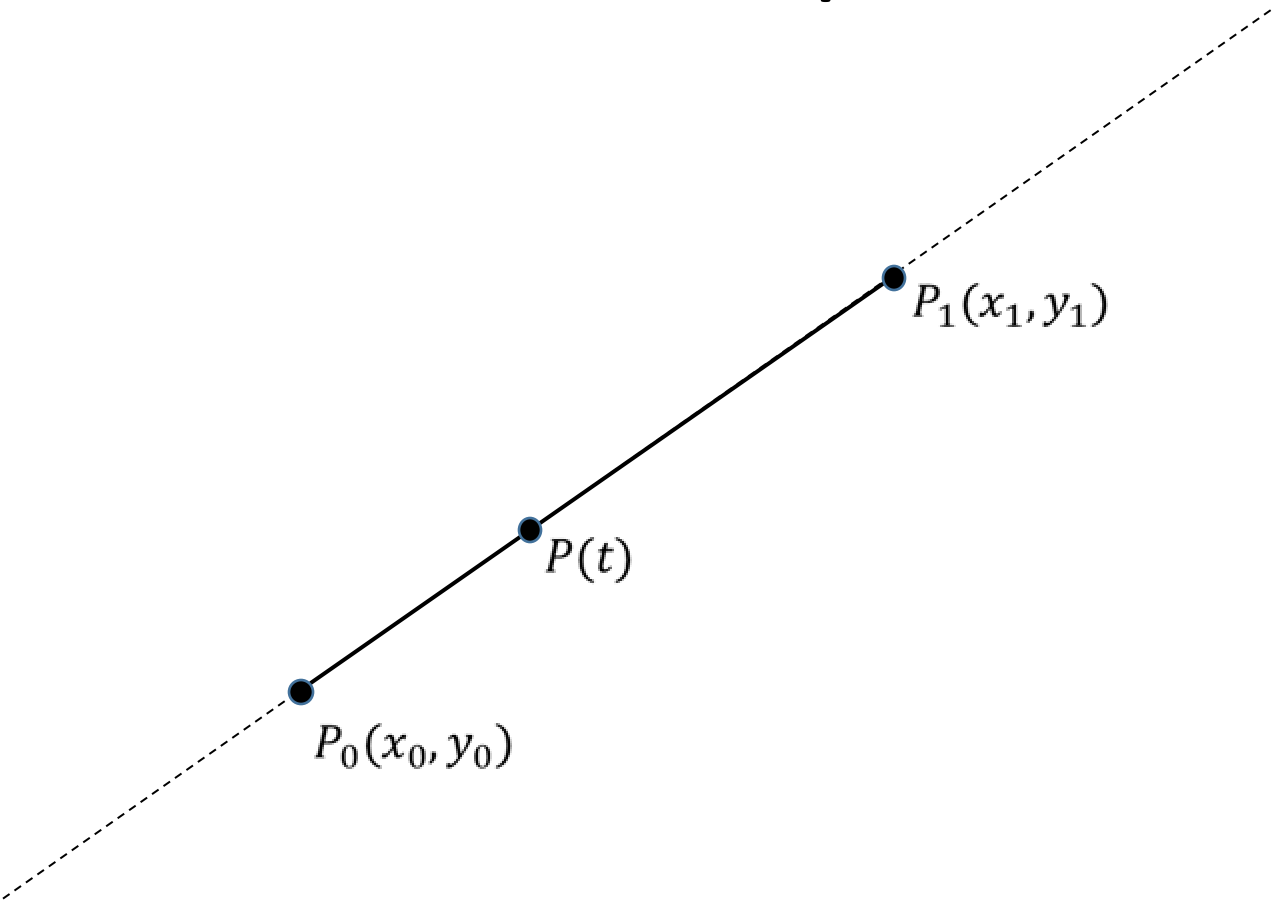
<u>Where is the problem?</u>

- Unnecessary clipping is done
- Different clipping order may take less iterations to finish

Clipping

Cyrus-Beck Algo

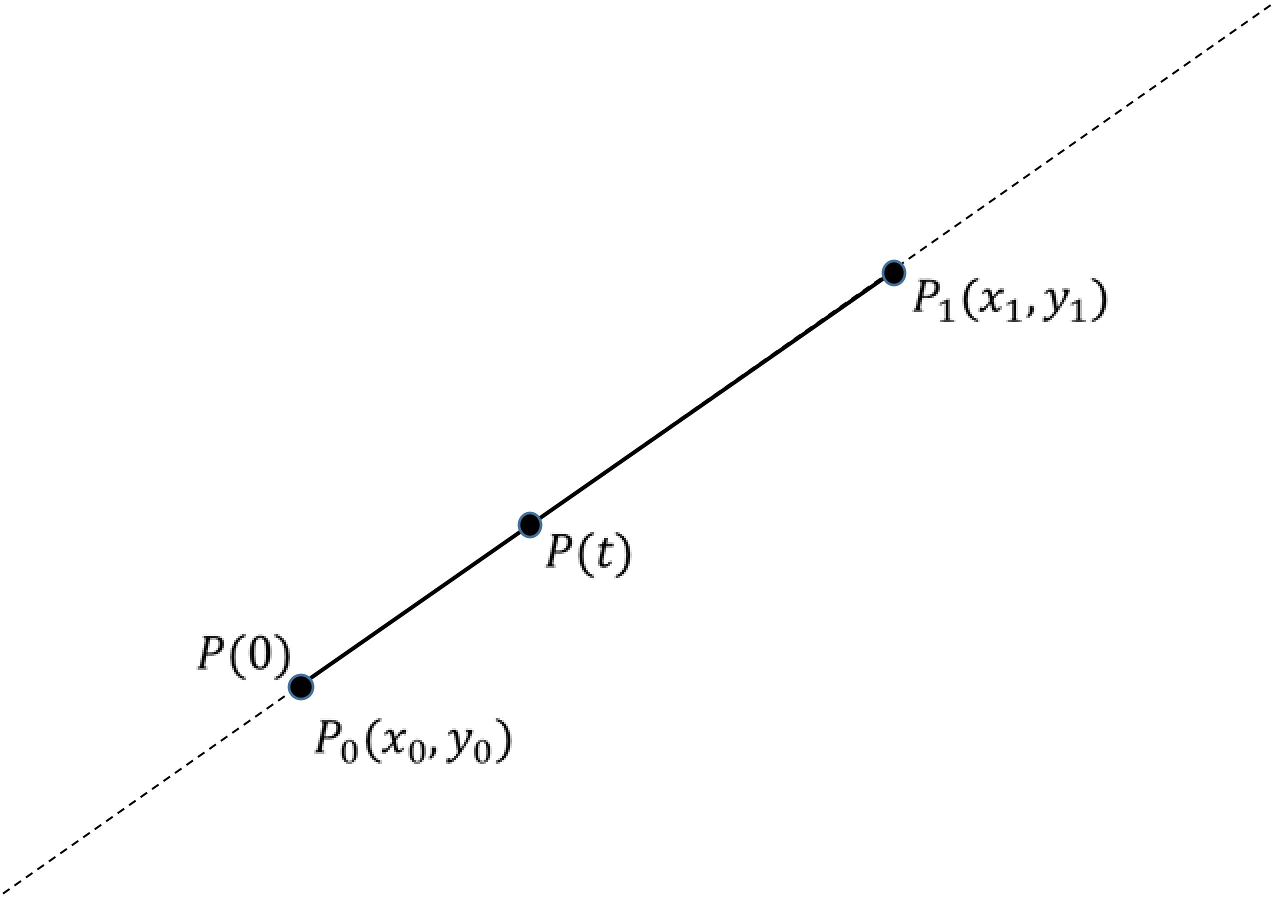
Parametric equation of Line



Parametric equation,

$$\begin{aligned} P(t) &= P_0 + t.(P_1 - P_0) \\ &= (x_0, y_0) + t(x_1 - x_0, y_1 - y_0) \\ &= (x_0 + t(x_1 - x_0), y_0 + t(y_1 - y_0)) \end{aligned}$$

Parametric equation of Line

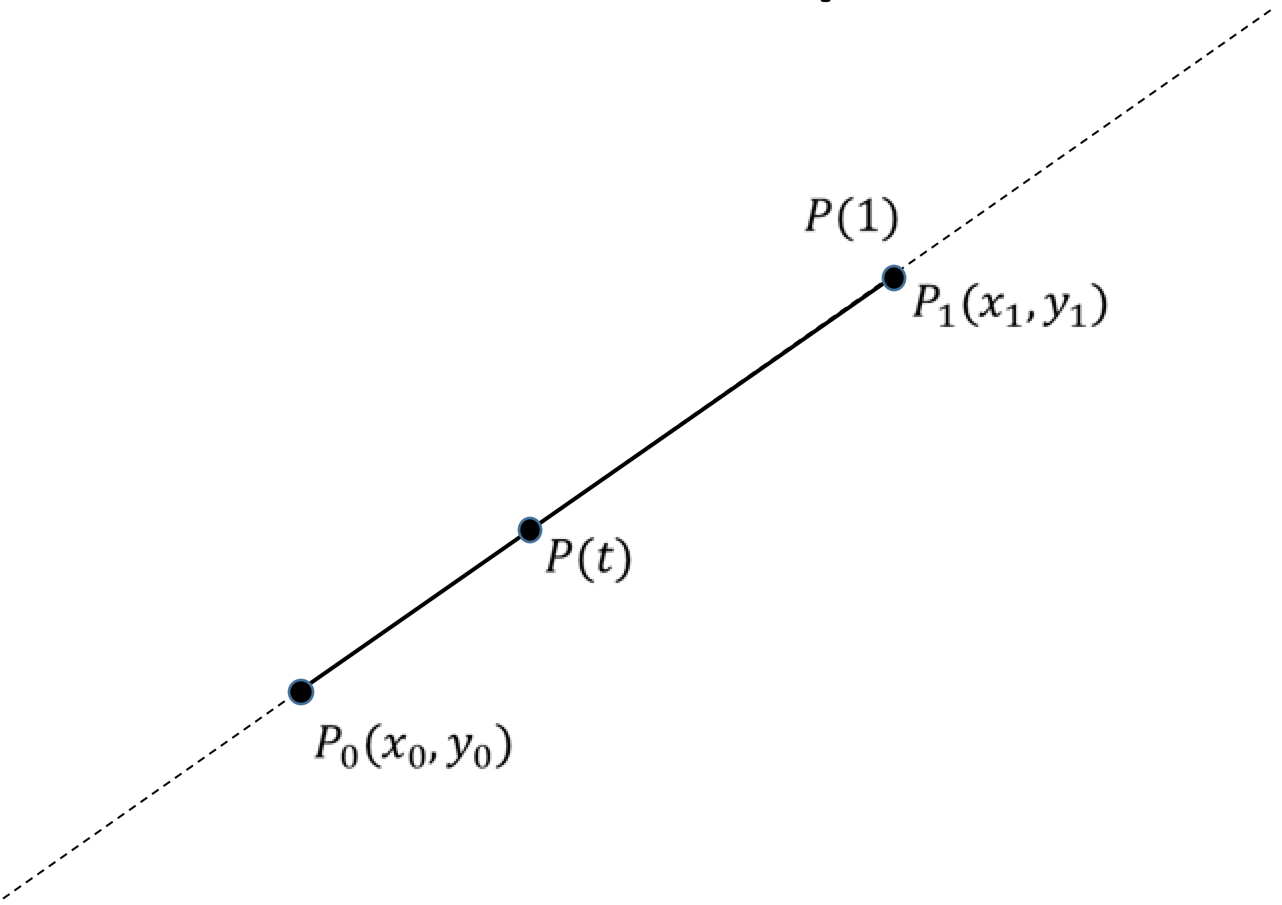


Parametric equation,

$$\begin{aligned} P(t) &= P_0 + t.(P_1 - P_0) \\ &= (x_0, y_0) + t(x_1 - x_0, y_1 - y_0) \\ &= (x_0 + t(x_1 - x_0), y_0 + t(y_1 - y_0)) \end{aligned}$$

$$\begin{aligned} P(0) &= (x_0, y_0) + 0.(x_1 - x_0, y_1 - y_0) \\ &= (x_0, y_0) \end{aligned}$$

Parametric equation of Line

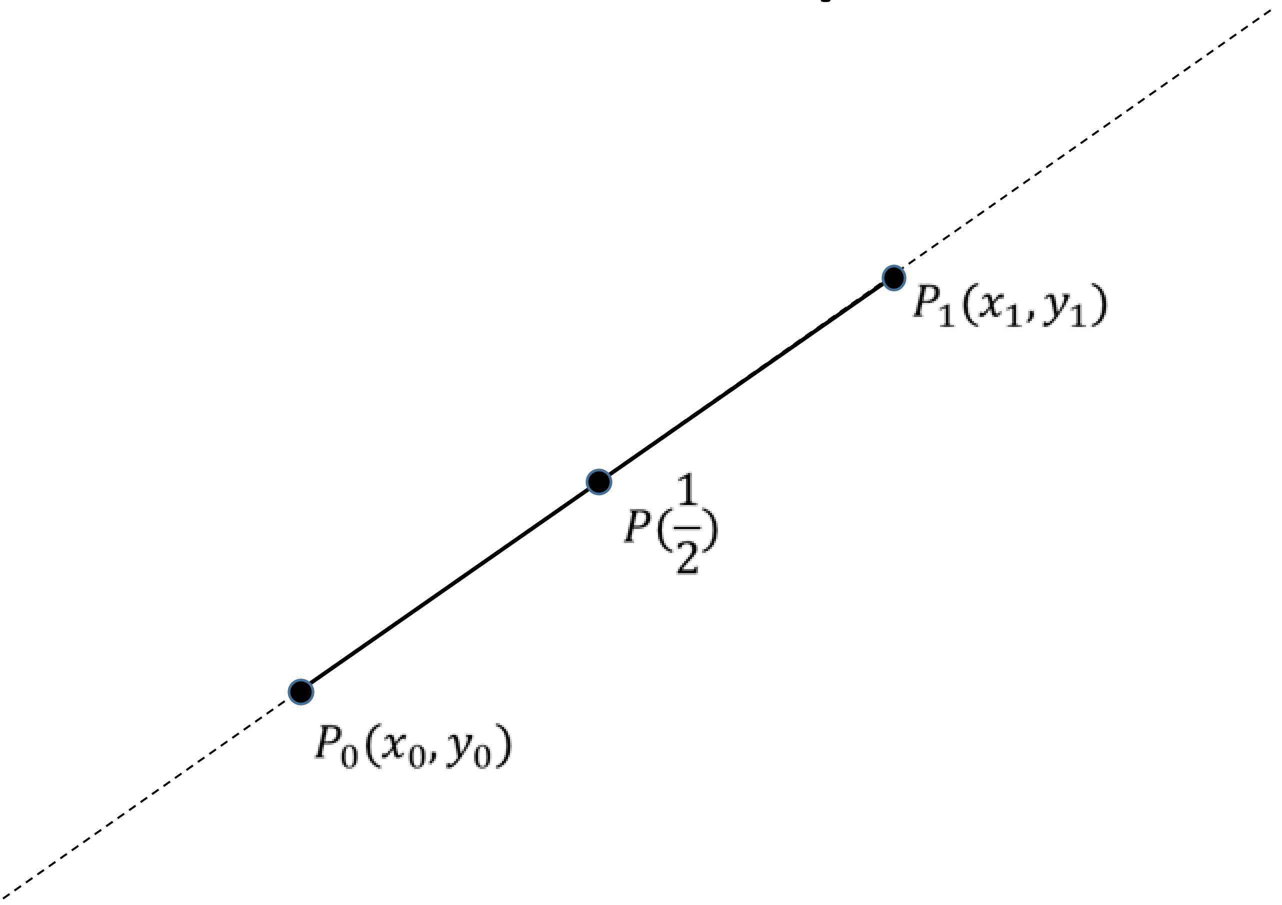


Parametric equation,

$$\begin{aligned} P(t) &= P_0 + t.(P_1 - P_0) \\ &= (x_0, y_0) + t(x_1 - x_0, y_1 - y_0) \\ &= (x_0 + t(x_1 - x_0), y_0 + t(y_1 - y_0)) \end{aligned}$$

$$\begin{aligned} P(1) &= (x_0, y_0) + 1.(x_1 - x_0, y_1 - y_0) \\ &= (x_0, y_0) + (x_1 - x_0, y_1 - y_0) \\ &= (x_0 + x_1 - x_0, y_0 + y_1 - y_0) \\ &= (x_1, y_1) \end{aligned}$$

Parametric equation of Line

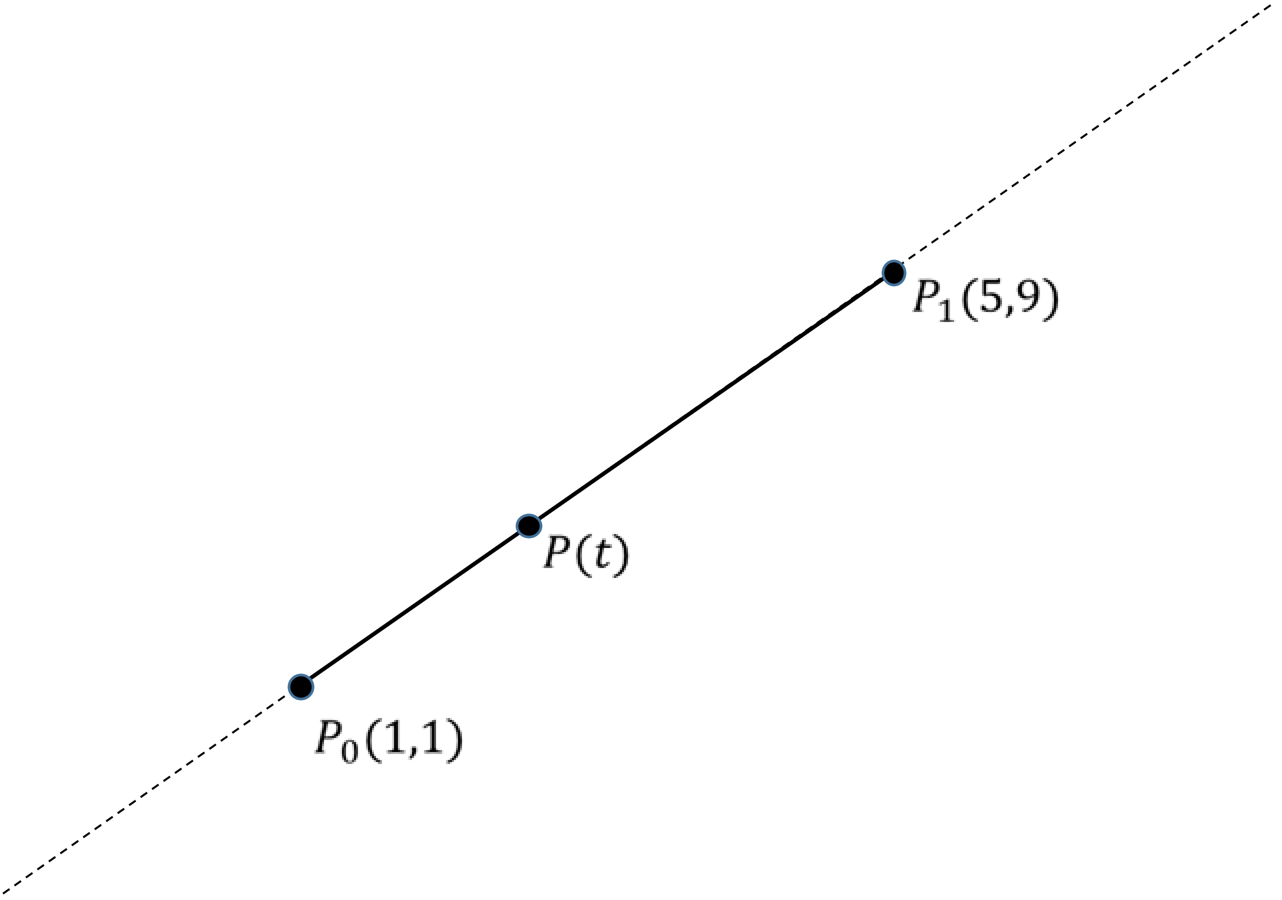


Parametric equation,

$$P(t) = P_0 + t.(P_1 - P_0)$$
$$= (x_0, y_0) + t(x_1 - x_0, y_1 - y_0)$$
$$= (x_0 + t(x_1 - x_0), y_0 + t(y_1 - y_0))$$

$$P\left(\frac{1}{2}\right) = (x_0, y_0) + \frac{1}{2}.(x_1 - x_0, y_1 - y_0)$$
$$= \left(\frac{x_0 + x_1}{2}, \frac{y_0 + y_1}{2}\right)$$

Parametric equation of Line



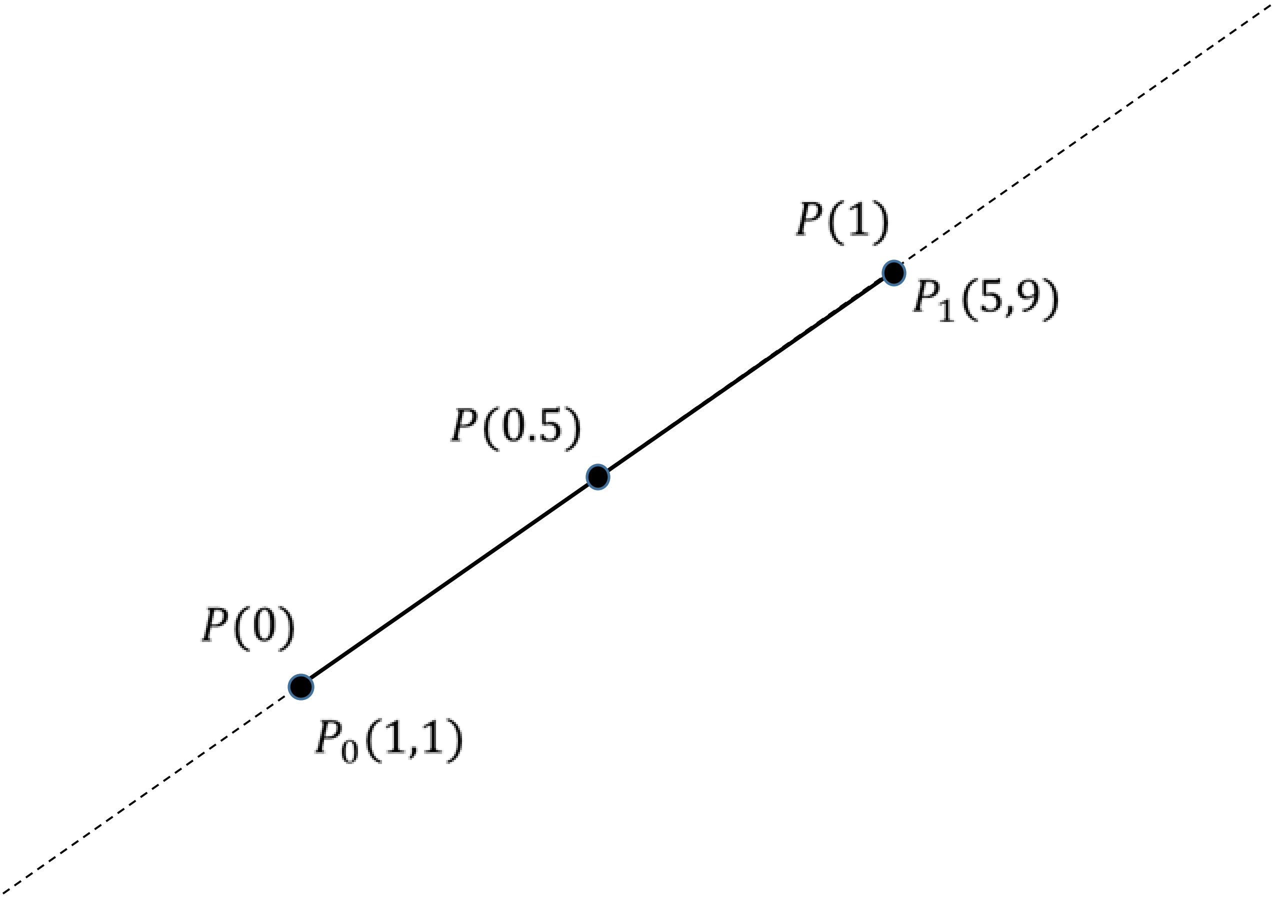
Suppose, endpoints of a line are (1, 1) and (5, 9)

Parametric equation,

$$\begin{aligned} P(t) &= P_0 + t.(P_1 - P_0) \\ &= (x_0, y_0) + t(x_1 - x_0, y_1 - y_0) \\ &= (1,1) + t(5-1, 9-1) \\ &= (1,1) + t(4,8) \\ &= (1+4t, 1+8t) \end{aligned}$$

$$P(0) = (1,1)$$
$$P(1) = (5,9)$$

Parametric equation of Line



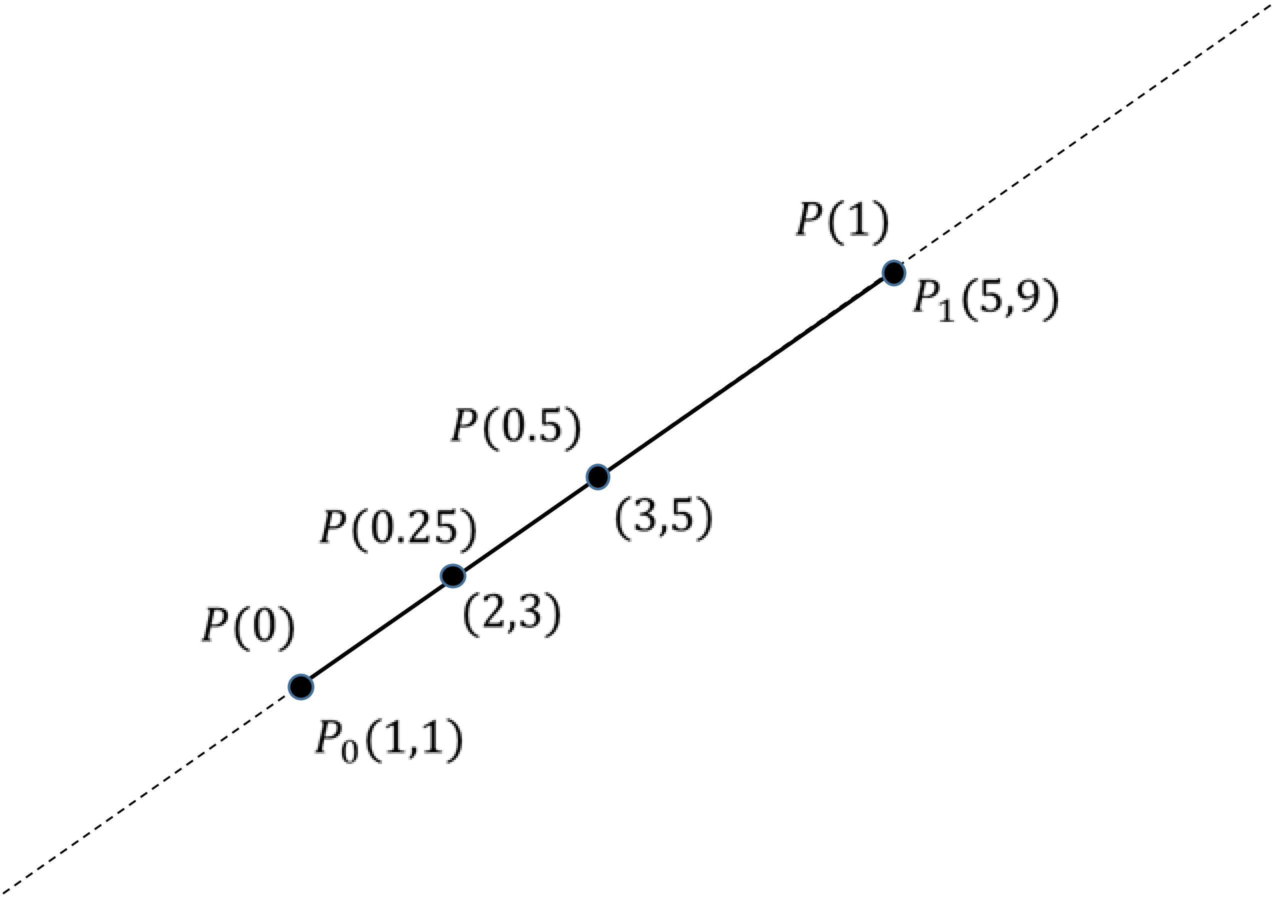
Suppose, endpoints of a line are (1, 1) and (5, 9)

Parametric equation,

$$\begin{aligned}P(t) &= P_0 + t.(P_1 - P_0)\\ &= (x_0, y_0) + t(x_1 - x_0, y_1 - y_0)\\ &= (1,1) + t(5-1, 9-1)\\ &= (1,1) + t(4,8)\\ &= (1+4t, 1+8t)\end{aligned}$$

$$P(0) = (1,1)$$
$$P(1) = (5,9)$$
$$P(0.5) = (1 + 4 \times 0.5, 1 + 8 \times 0.5) = (3,5)$$

Parametric equation of Line



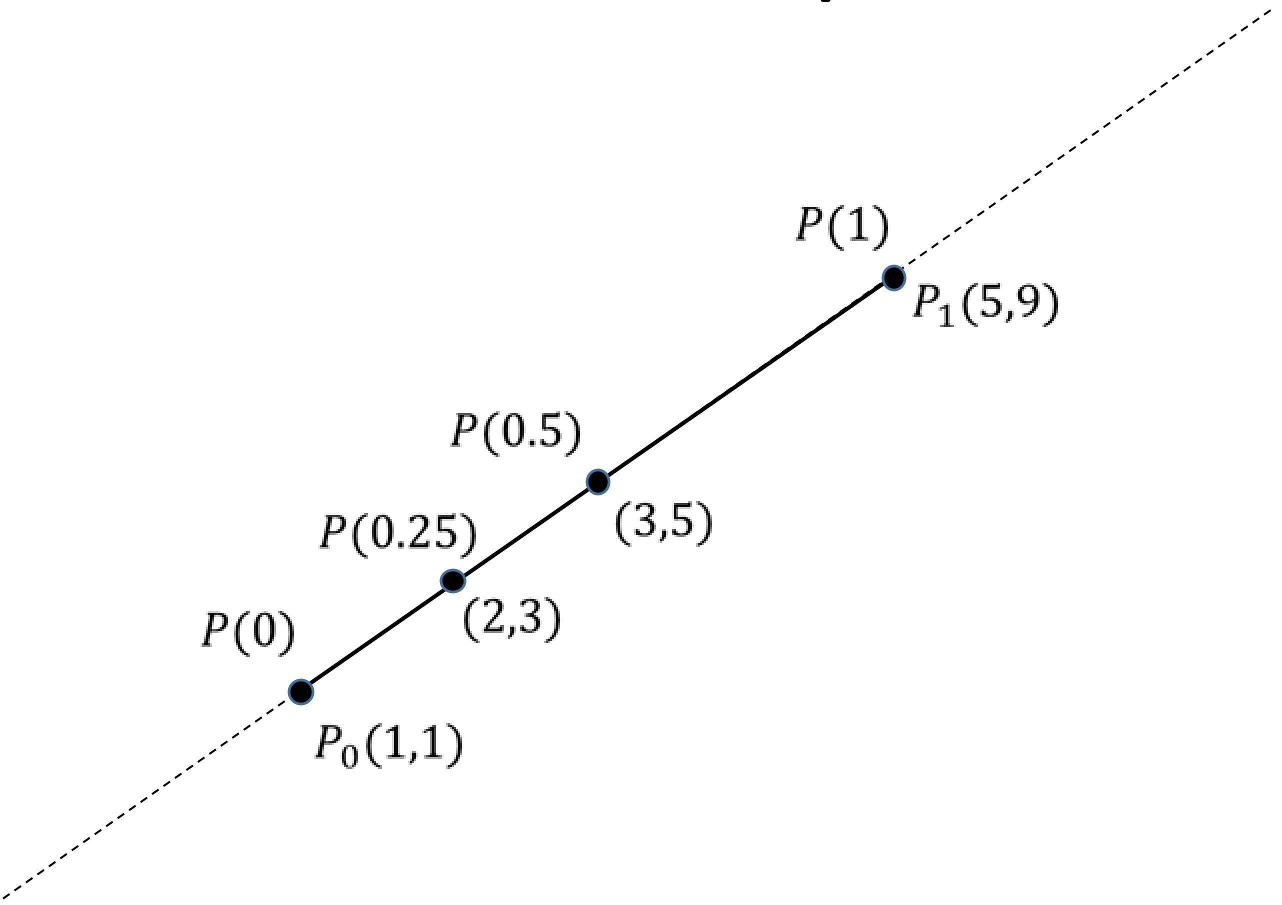
Suppose, endpoints of a line are (1, 1) and (5, 9)

Parametric equation,

$$\begin{aligned}P(t) &= P_0 + t.(P_1 - P_0)\\ &= (x_0, y_0) + t(x_1 - x_0, y_1 - y_0)\\ &= (1,1) + t(5-1, 9-1)\\ &= (1,1) + t(4,8)\\ &= (1+4t, 1+8t)\end{aligned}$$

$$P(0) = (1,1)$$
$$P(1) = (5,9)$$
$$P(0.5) = (1 + 4 \times 0.5, 1 + 8 \times 0.5) = (3,5)$$
$$P(0.25) = (1 + 4 \times 0.25, 1 + 8 \times 0.25) = (2,3)$$

Parametric equation of Line



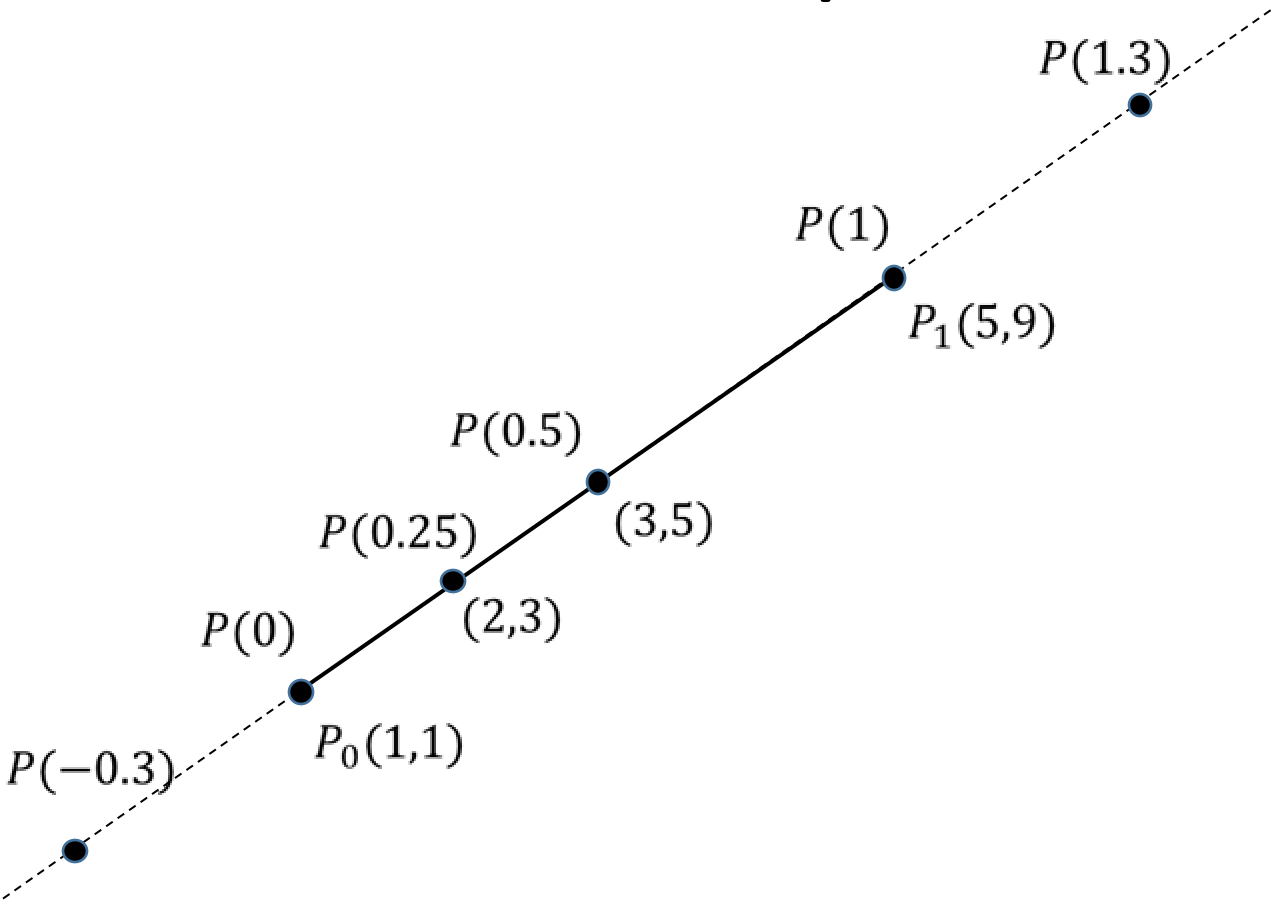
Suppose, endpoints of a line are (1, 1) and (5, 9)

Parametric equation,

$$\begin{aligned} P(t) &= P_0 + t.(P_1 - P_0) \\ &= (x_0, y_0) + t(x_1 - x_0, y_1 - y_0) \\ &= (1,1) + t(5-1, 9-1) \\ &= (1,1) + t(4,8) \\ &= (1+4t, 1+8t) \end{aligned}$$

So all points between $0 \le t \le 1$ are inside the line segment

Parametric equation of Line

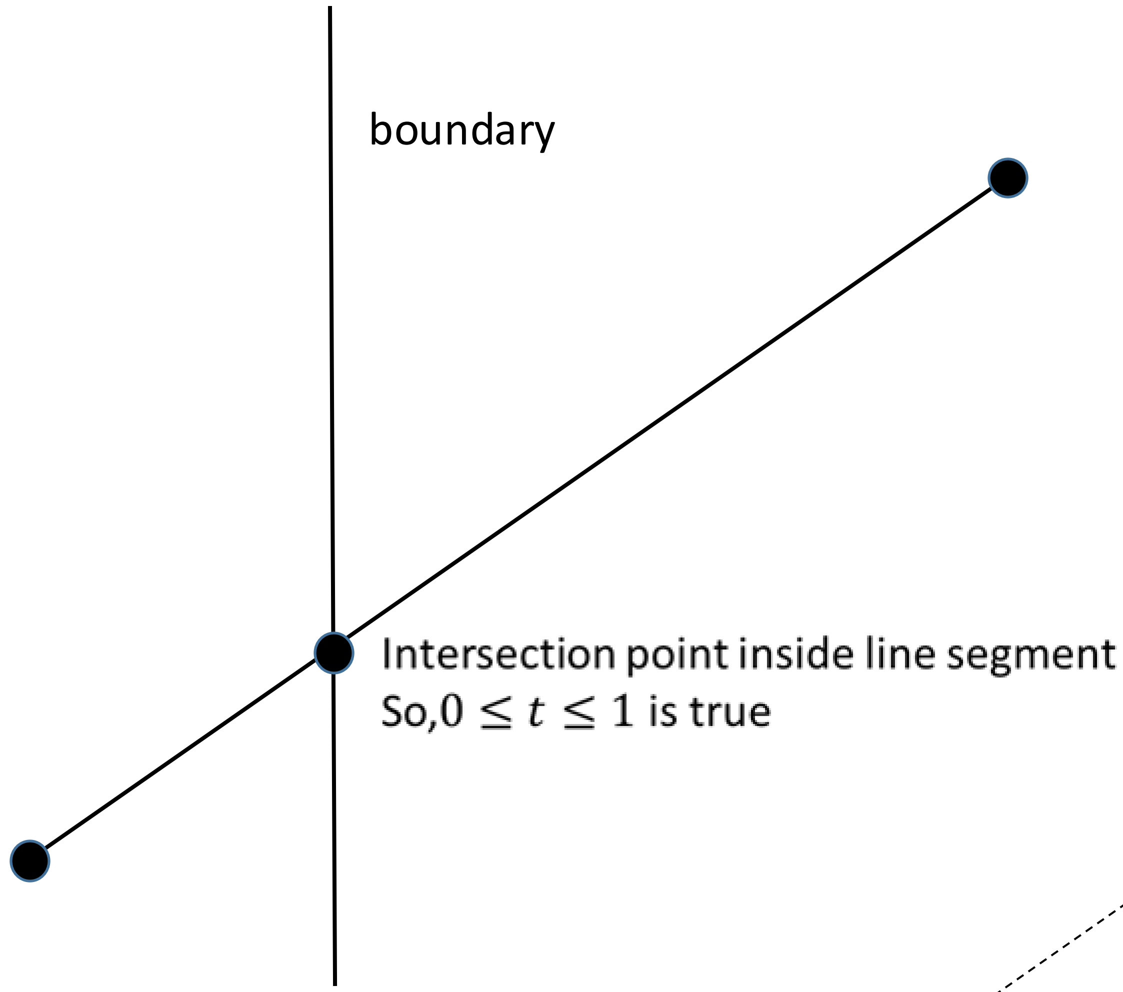


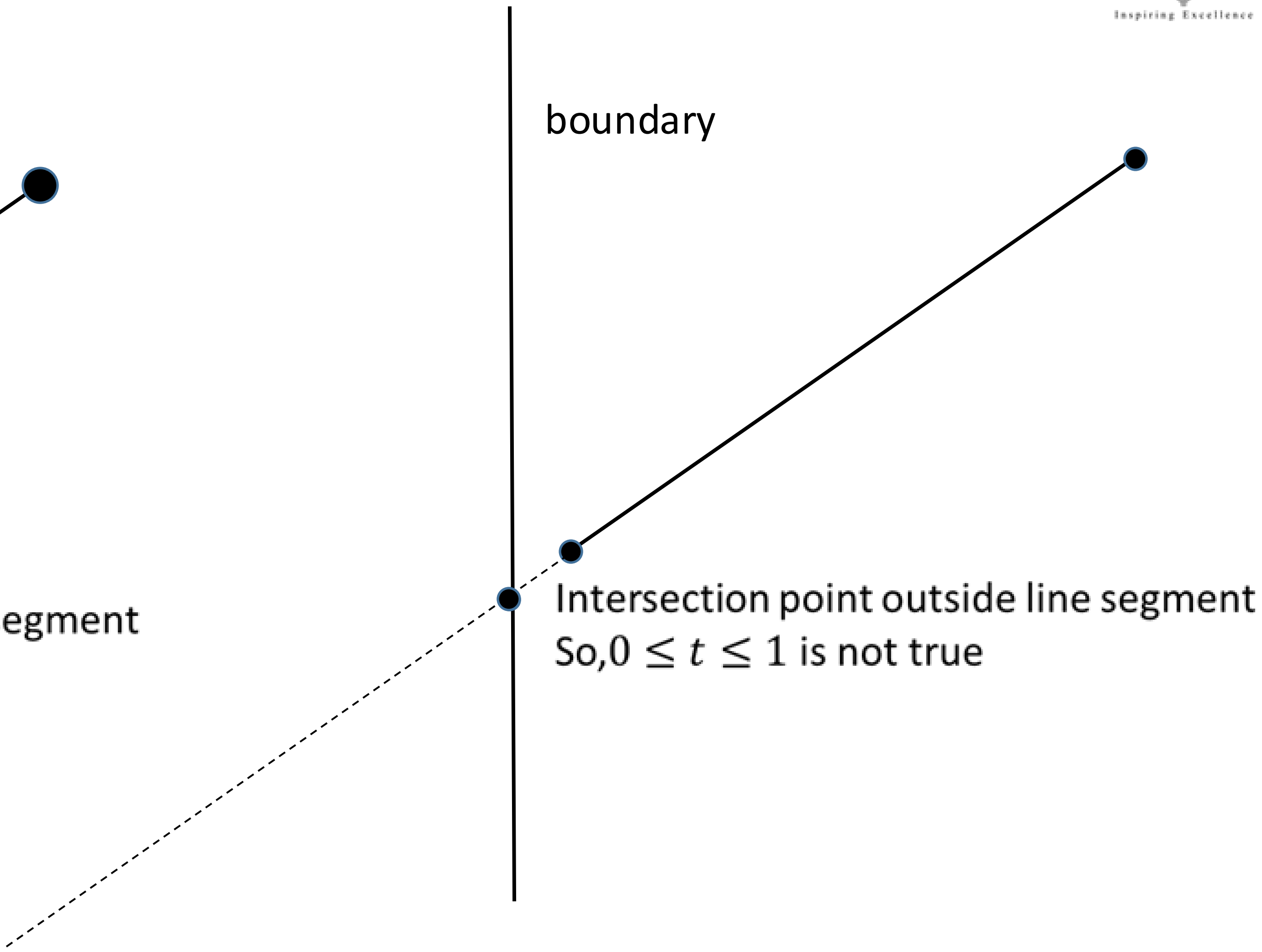
Suppose, endpoints of a line are (1, 1) and (5, 9)

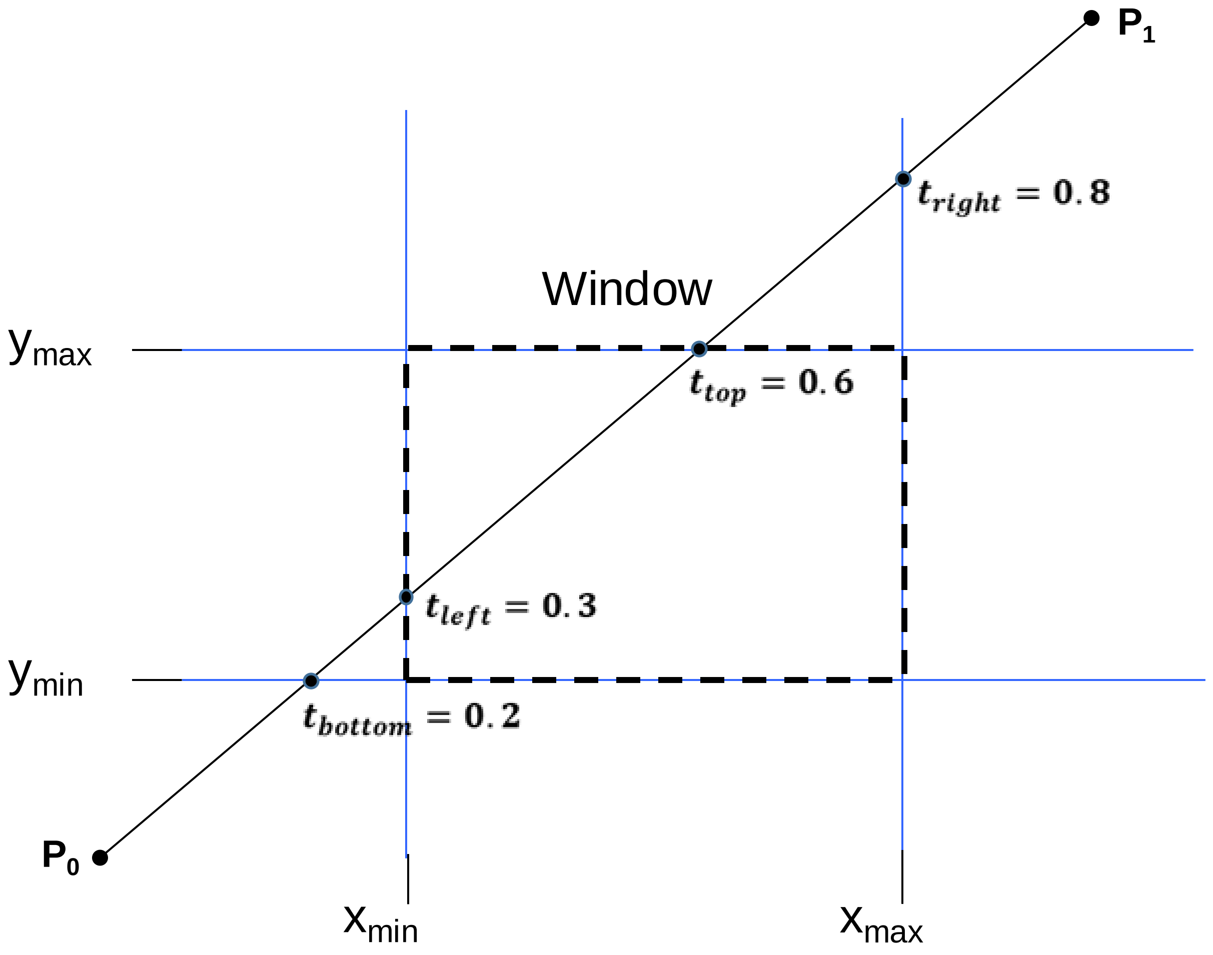
Parametric equation,

$$\begin{aligned} P(t) &= P_0 + t.(P_1 - P_0) \\ &= (x_0, y_0) + t(x_1 - x_0, y_1 - y_0) \\ &= (1,1) + t(5-1, 9-1) \\ &= (1,1) + t(4,8) \\ &= (1+4t, 1+8t) \end{aligned}$$

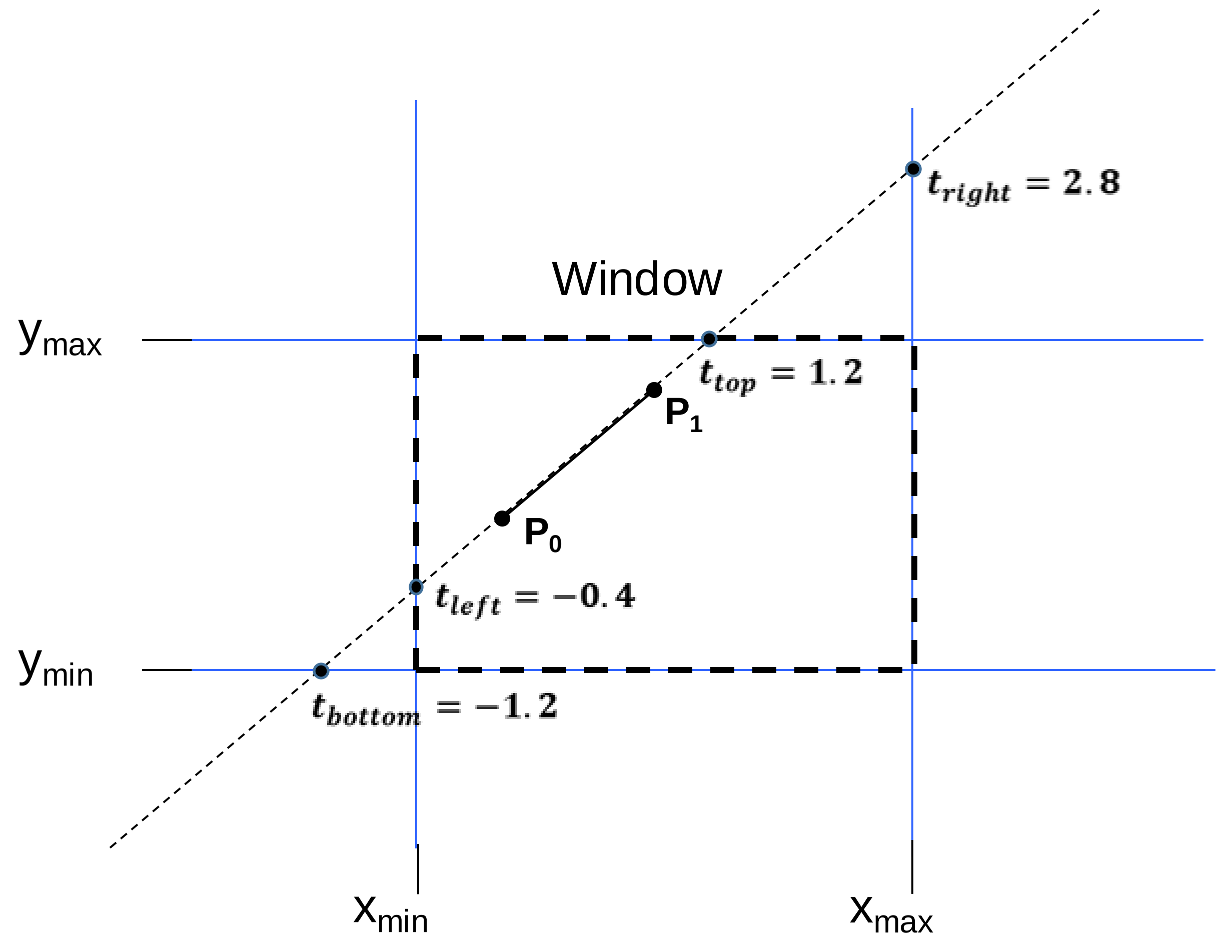
P(-0.3) and P(1.3) are outside the line segment


boundary
Intersection point inside line segment
So,$0 \leq t \leq 1$ is true


boundary
Intersection point outside line segment
So,$0 \leq t \leq 1$ is not true

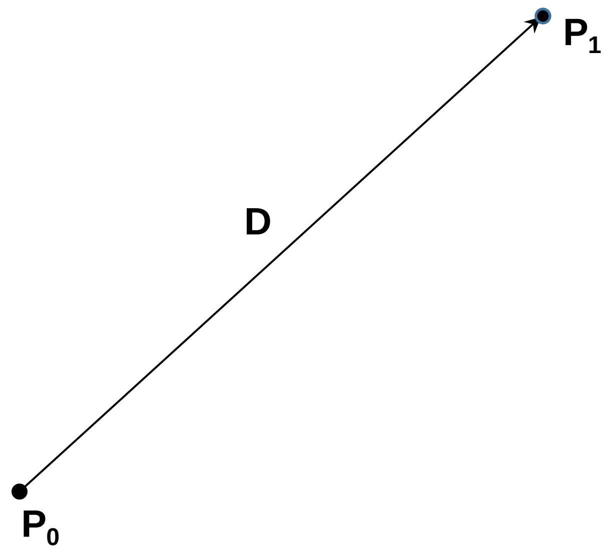


Both endpoints outside
All intersections have t between 0 and 1



Both endpoints inside
All intersections have t outside 0 to 1

Vector

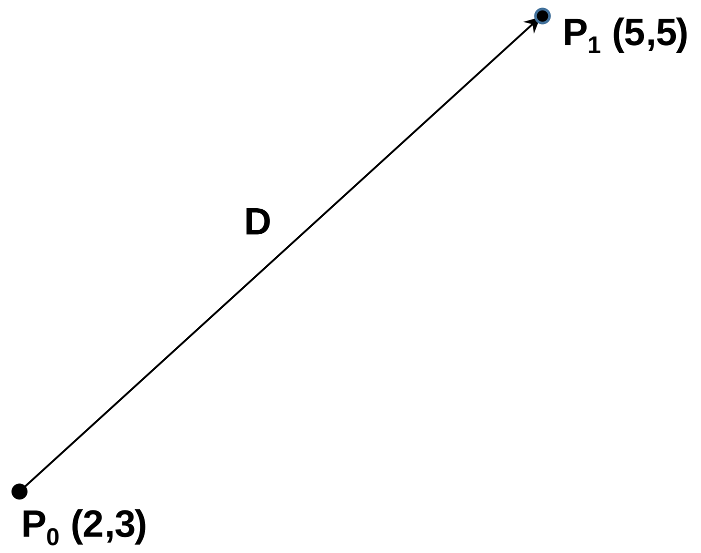


D is a vector from P_0 to P_1

$D = P_1 - P_0$

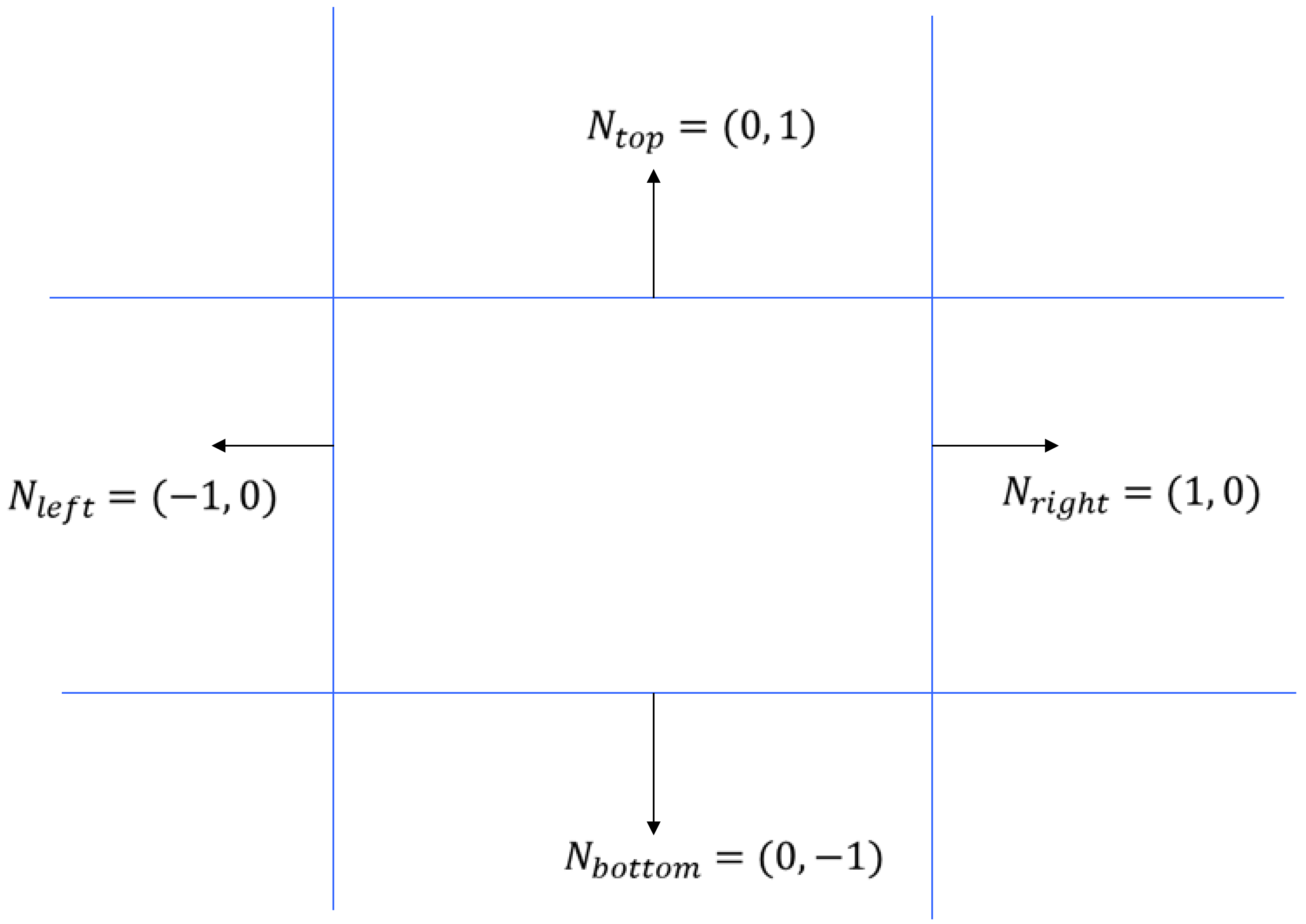
$= (x_1 - x_0, y_1 - y_0)$

Vector



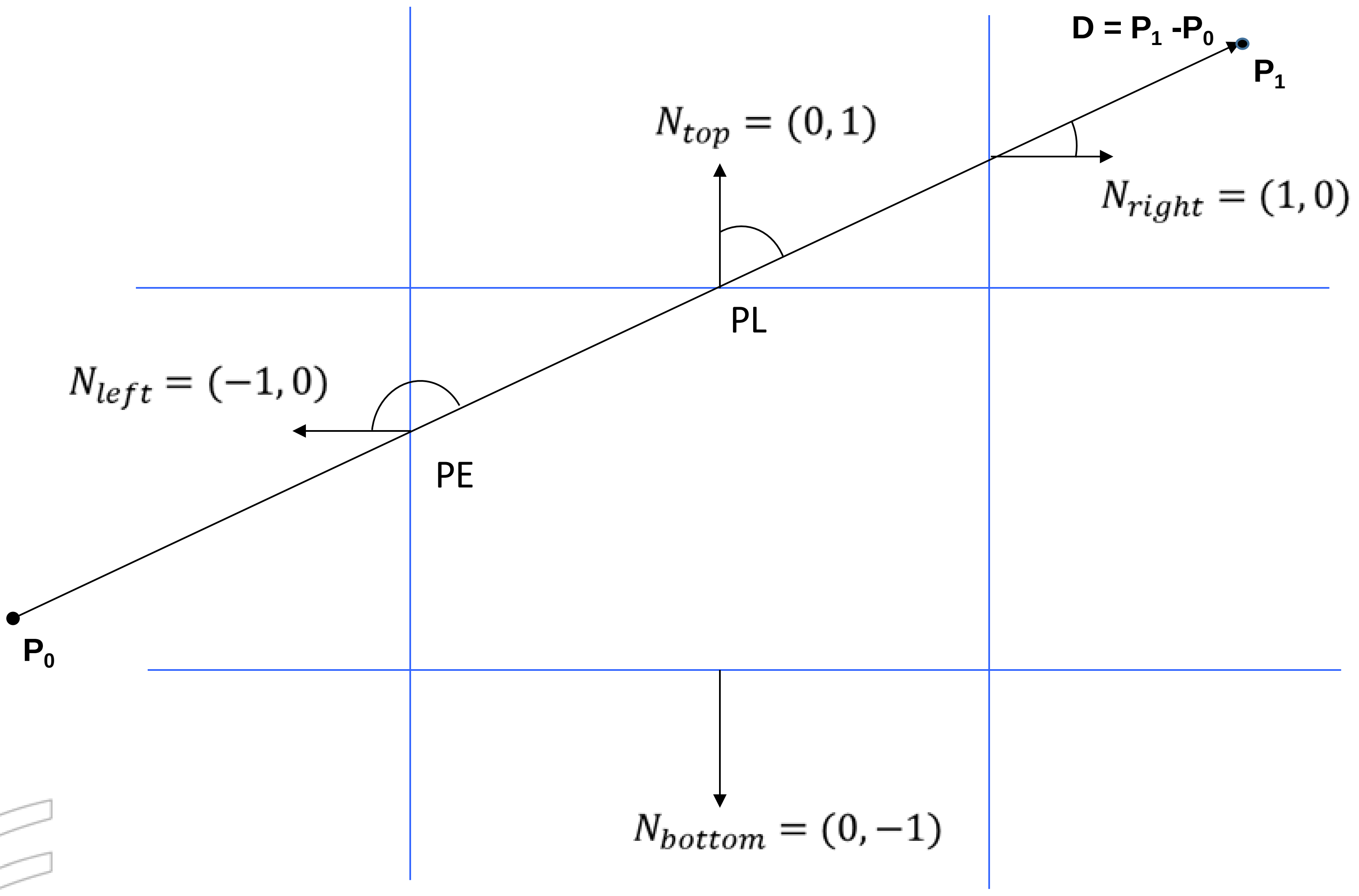
$$
\begin{aligned}
D &= P_1 - P_0 \\
&= (x_1 - x_0, y_1 - y_0) \\
&= (5 - 2, 5 - 3) \\
&= (3, 2) \\
&\cong 3i + 2j
\end{aligned}
$$

Normal vectors to boundary



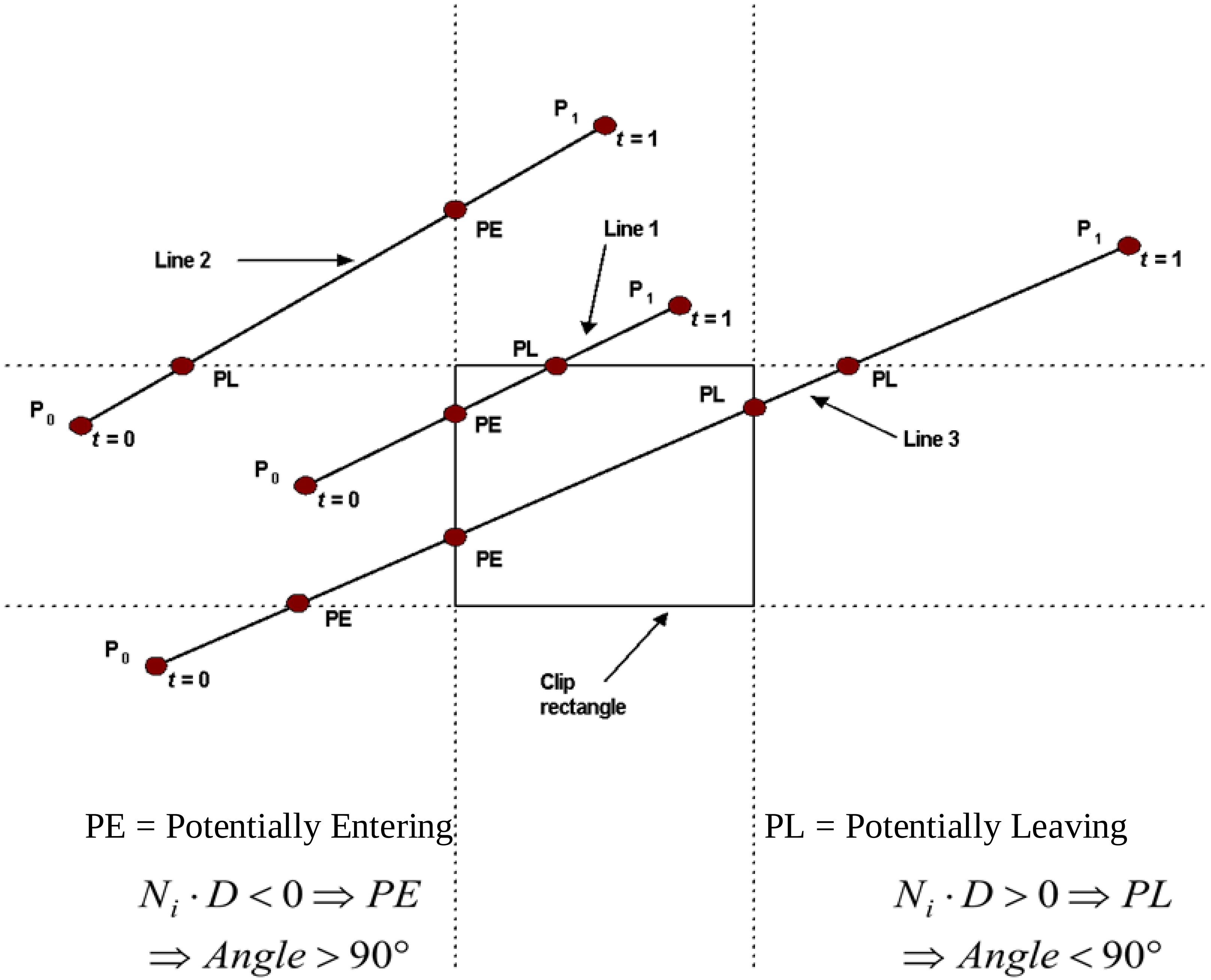
Each N_i is a perpendicular unit vector to each of the boundaries: left, right, top, bottom. N_i points away from the clip window

Normal vectors to boundary



When angle between N_i and D is more than 90 degree-
Or, $N_i \cdot D < 0$
The intersection point is PE
(Potentially entering)

When angle between N_i and D is less than 90 degree-
Or, $N_i \cdot D > 0$
The intersection point is PL
(Potentially leaving)



PE = Potentially Entering

$$N_i \cdot D < 0 \Rightarrow PE$$
$$\Rightarrow Angle > 90°$$

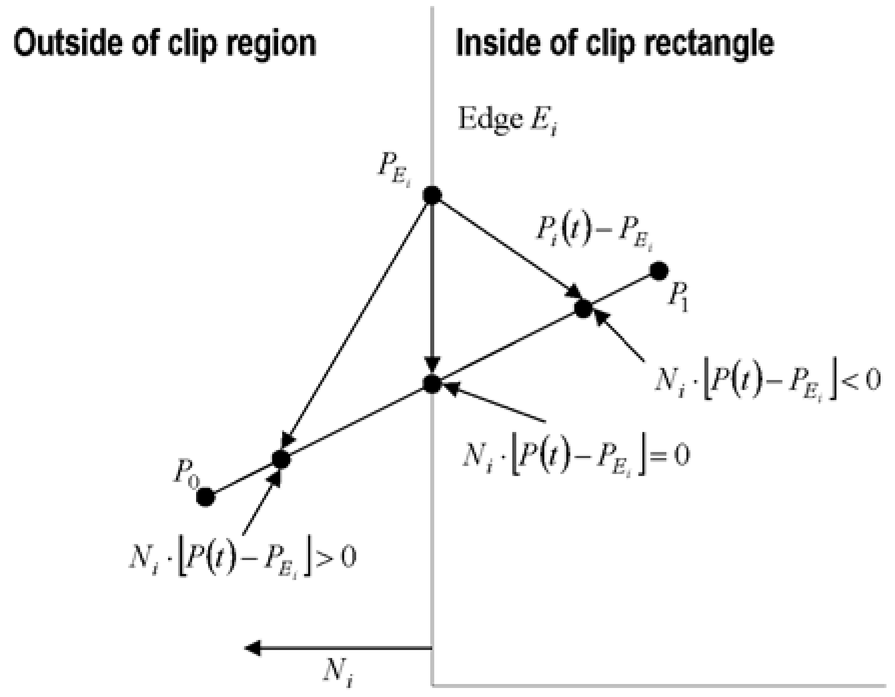
PL = Potentially Leaving

$$N_i \cdot D > 0 \Rightarrow PL$$
$$\Rightarrow Angle < 90°$$

Cyrus-Beck algo

- Calculate t values of intersection points with each 4 boundaries
- Classify intersection points whether PE/PL
- Select the PE with highest t and the PL with the lowest t
- Using parametric line eqn. find the clipped points

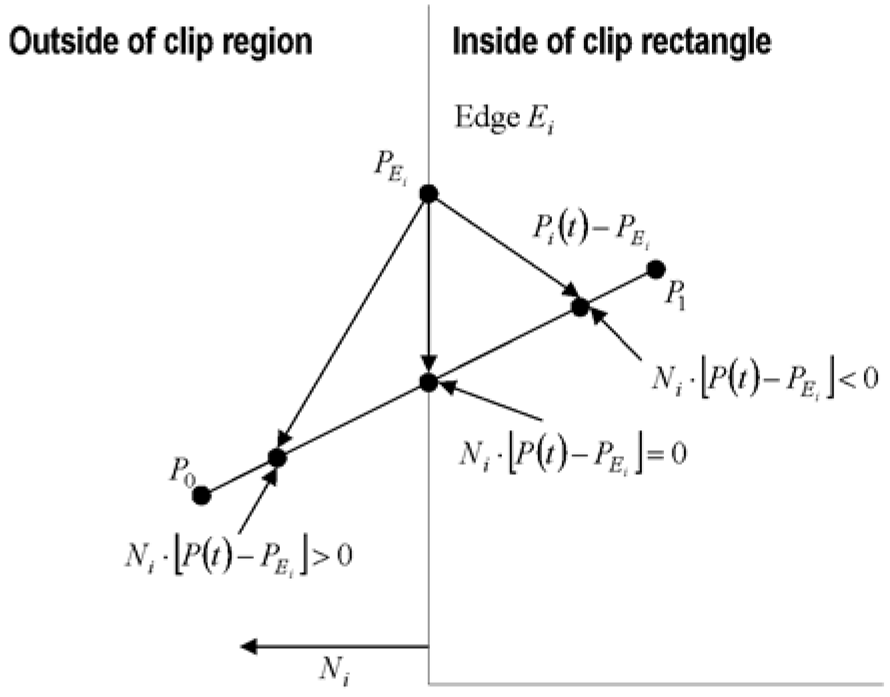
The Cyrus-Beck Algorithm



$$Line\ P_0P_1 : P(t) = P_0 + (P_1 - P_0)t$$

$$N_i \cdot [P(t) - P_{E_i}] = 0$$
$$\Rightarrow N_i \cdot [P_0 + (P_1 - P_0)t - P_{E_i}] = 0$$
$$\Rightarrow N_i \cdot [P_0 + (P_1 - P_0)t - P_{E_i}] = 0$$
$$\Rightarrow t = \frac{N_i \cdot [P_0 - P_{E_i}]}{-N_i \cdot (P_1 - P_0)}$$
$$\Rightarrow t = \frac{N_i \cdot [P_0 - P_{E_i}]}{-N_i \cdot D},\ D = P_1 - P_0$$

The Cyrus-Beck Algorithm



For left boundary,

$$t_{left} = \frac{N_i.[P_0 - P_E]}{-N_i.[P_1 - P_0]}$$
$$= \frac{(-1,0).[(x_0,y_0) - (x_{min},y)]}{-(-1,0).[(x_1,y_1) - (x_0,y_0)]}$$
$$= \frac{(-1,0).(x_0 - x_{min}, y_0 - y)}{-(-1,0).(x_1 - x_0, y_1 - y_0)}$$
$$= \frac{-1 \times (x_0 - x_{min}) + 0 \times (y_0 - y)}{-(-1 \times (x_1 - x_0) + 0 \times (y_1 - y_0))}$$
$$= \frac{-(x_0 - x_{min})}{(x_1 - x_0)}$$

Similarly,

$$t_{right} = \frac{-(x_0 - x_{max})}{(x_1 - x_0)}$$
$$t_{top} = \frac{-(y_0 - y_{max})}{(y_1 - y_0)}$$
$$t_{bottom} = \frac{-(y_0 - y_{min})}{(y_1 - y_0)}$$

The Cyrus-Beck Algorithm

```
Precalculate $N_i$ and $P_{Ei}$ for each edge
for (each line segment to be clipped) {
   if ($P_1 == P_0$)
            line is degenerated, so clip as a point;
   else {
            $t_E = 0$;  $t_L = 1$;
            for (each candidate intersection with a clip edge) {
                        if ($N_i \bullet D \;!= 0$) {   /* Ignore edges parallel to line */
                                    calculate t;
                                    use sign of  $N_i \bullet D$ to categorize as PE or PL;
                                    if (PE) $t_E = \max(t_E ,\ t)$;
                                    if (PL) $t_L = \min(t_L ,\ t)$;
                        }
            }
            if ($t_E > t_L$) return NULL;
            else return $P(t_E)$ and $P(t_L)$ as true clip intersection;
   }
}
```

a) Calculate the value of t of the lines given below for all edges and specify whether they are entering or leaving t. [Given (0,0) to (300,200) be the clip region.]

- (i) (50, -125) to (-100, 225).
- (ii) (-250, 200) to (250, -200).
- (iii) (-250, 200) to (150, 100)

Also, find the line segment within the clipping window a)(i) boundary:

$$x_{min} = 0,\ \ x_{max} = 300,\ \ y_{min} = 0,\ \ y_{max} = 200$$

points:

$$x_0 = 50, y_0 = -125, x_1 = -100, y_1 = 225$$

$$D = (x_1 - x_0, y_1 - y_0) = (-150, 350)$$

Initially, $t_E = 0, t_L = 1$

$$t_{left} = \frac{-(x_0 - x_{min})}{(x_1 - x_0)}$$

$$t_{right} = \frac{-(x_0 - x_{max})}{(x_1 - x_0)}$$

$$t_{top} = \frac{-(y_0 - y_{max})}{(y_1 - y_0)}$$

$$t_{bottom} = \frac{-(y_0 - y_{min})}{(y_1 - y_0)}$$

Boundary	N_i	$N_i.D$	t	PE/PL	t_E	t_L
Left	(-1,0)	150	$\frac{-(50-0)}{-100-50} = 0.33$	PL	0	0.33
Right	(1,0)	-150	$\frac{-(50-300)}{-100-50} = -1.67$	PE	0	0.33
Bottom	(0, -1)	-350	$\frac{-(-125-0)}{225-(-125)} = 0.357$	PE	0.357	0.33
Top	(0, 1)	350	$\frac{-(-125-200)}{225-(-125)} = 0.93$	PL	0.357	0.33

Since $t_E > t_L$, *line segment is outside clip window*

Algorithm

```
Precalculate N_i and P_Ei for each edge
for (each line segment to be clipped) {
    if (P_1 == P_0)
            line is degenerated, so clip as a point;
    else {
            t_E = 0;  t_L = 1;
            for (each candidate intersection with a clip edge) {
                    if (N_i • D != 0) {   /* Ignore edges parallel to line */
                            calculate t;
                            use sign of  N_i • D to categorize as PE or PL;
                            if (PE) t_E = max(t_E ,  t);
                            if (PL) t_L = min(t_L ,  t);
                    }
            }
            if (t_E > t_L) return NULL;
            else return P(t_E) and P(t_L) as true clip intersection;
    }
}
```

a)(ii) boundary:

$$x_{min} = 0, \;\; x_{max} = 300, \;\; y_{min} = 0, \;\; y_{max} = 200$$

points:

$$x_0 = -250, y_0 = 200, x_1 = 250, y_1 = -200$$
$$D = (x_1 - x_0, y_1 - y_0) = (500, -400)$$

Initially, $t_E = 0, t_L = 1$

Boundary	N_i	$N_i.D$	t	PE/PL	t_E	t_L
Left	(-1,0)	-500	$\frac{-(-250-0)}{500} = 0.5$	PE	0.5	1
Right	(1,0)	500	$\frac{-(-250-300)}{500} = 1.1$	PL	0.5	1
Bottom	(0, -1)	400	$\frac{-(200-0)}{-400} = 0.5$	PL	0.5	0.5
Top	(0, 1)	-400	$\frac{-(200-200)}{-400} = 0$	PE	0.5	0.5

P(0.5) and P(0.5) (same point) are the true clip intersection

$$P(0.5) = (x_0, y_0) + 0.5 \times D$$
$$= (-250, 200) + 0.5 \times (500, -400)$$
$$= (0,0)$$

$$t_{left} = \frac{-(x_0 - x_{min})}{(x_1 - x_0)}$$
$$t_{right} = \frac{-(x_0 - x_{max})}{(x_1 - x_0)}$$
$$t_{top} = \frac{-(y_0 - y_{max})}{(y_1 - y_0)}$$
$$t_{bottom} = \frac{-(y_0 - y_{min})}{(y_1 - y_0)}$$

Algorithm

```
Precalculate $N_i$ and $P_{Ei}$ for each edge
for (each line segment to be clipped) {
    if ($P_1 == P_0$)
            line is degenerated, so clip as a point;
    else {
            $t_E = 0$;  $t_L = 1$;
            for (each candidate intersection with a clip edge) {
                    if ($N_i \bullet D$ != 0) {   /* Ignore edges parallel to line */
                            calculate t;
                            use sign of $N_i \bullet D$ to categorize as PE or PL;
                            if (PE) $t_E = \max(t_E, t)$;
                            if (PL) $t_L = \min(t_L, t)$;
                    }
            }
            if ($t_E > t_L$) return NULL;
            else return $P(t_E)$ and $P(t_L)$ as true clip intersection;
    }
}
```

a)(iii) boundary:

$$x_{min} = 0, \ \ x_{max} = 300, \ \ y_{min} = 0, \ y_{max} = 200$$

points:

$$x_0 = -250, y_0 = 200, x_1 = 150, y_1 = 100$$

$$D = (x_1 - x_0, y_1 - y_0) = (400, -100)$$

Initially, $t_E = 0, t_L = 1$

$$t_{left} = \frac{-(x_0 - x_{min})}{(x_1 - x_0)}$$

$$t_{right} = \frac{-(x_0 - x_{max})}{(x_1 - x_0)}$$

$$t_{top} = \frac{-(y_0 - y_{max})}{(y_1 - y_0)}$$

$$t_{bottom} = \frac{-(y_0 - y_{min})}{(y_1 - y_0)}$$

Boundary	N_i	$N_i.D$	t	PE/PL	t_E	t_L
Left	(-1,0)	-400	$\frac{-(-250-0)}{400} = 0.625$	PE	0.625	1
Right	(1,0)	400	$\frac{-(-250-300)}{400} = 1.375$	PL	0.625	1
Bottom	(0, -1)	100	$\frac{-(200-0)}{-100} = 2$	PL	0.625	1
Top	(0, 1)	-100	$\frac{-(200-200)}{-100} = 0$	PE	0.625	1

P(0.625) and P(1) are the true clip intersection

$$P(0.625) = (x_0, y_0) + 0.625 \times D$$
$$= (-250, 200) + 0.625 \times (400, -100)$$
$$= (0, 137.5)$$
$$P(1) = (150, 100)$$

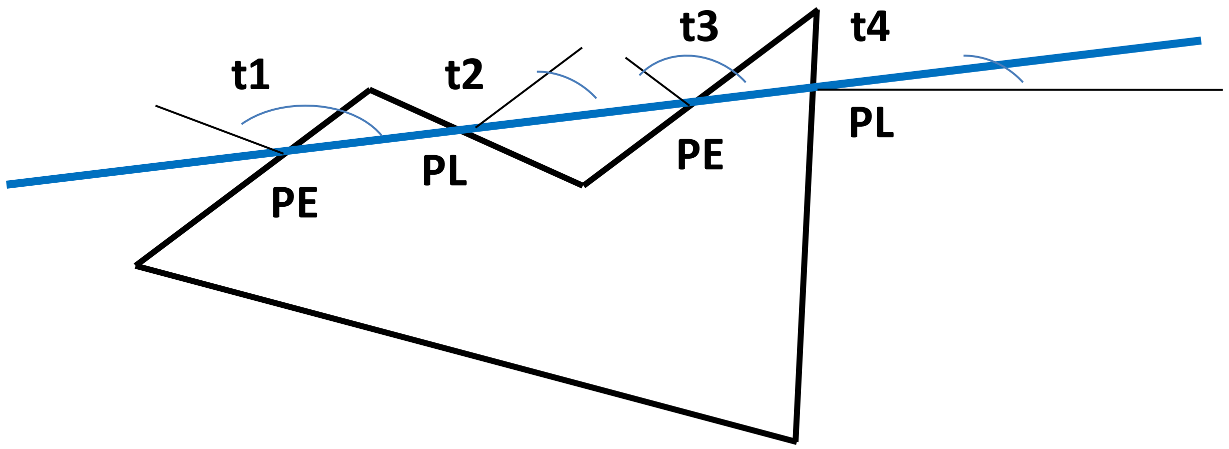
(0, 137.5) and (150, 100) are the endpoints of the clipped line

Algorithm

```
Precalculate $N_i$ and $P_{Ei}$ for each edge
for (each line segment to be clipped) {
    if ($P_1 == P_0$)
            line is degenerated, so clip as a point;
    else {
            $t_E = 0$;  $t_L = 1$;
            for (each candidate intersection with a clip edge) {
                    if ($N_i \bullet D$ != 0) {   /* Ignore edges parallel to line */
                            calculate t;
                            use sign of $N_i \bullet D$ to categorize as PE or PL;
                            if (PE) $t_E = \max(t_E, t)$;
                            if (PL) $t_L = \min(t_L, t)$;
                    }
            }
            if ($t_E > t_L$) return NULL;
            else return $P(t_E)$ and $P(t_L)$ as true clip intersection;
    }
}
```

Cyrus-Beck Parametric Line Clipping Algorithm

- Advantage
  - Works with polygons too (not only with clip rectangles)
  - Works in 3D scenario (polyhedrons)
- Problem
  - Does not work with **concave** polygon clip region



$t_E = t_3$
$t_L = t_2$
$t_E > t_L$
So the whole line is discarded though some segments should be displayed

Clipping

Sutherland-Hodgman Polygon Clipping

Sutherland-Hodgman Polygon Clipping

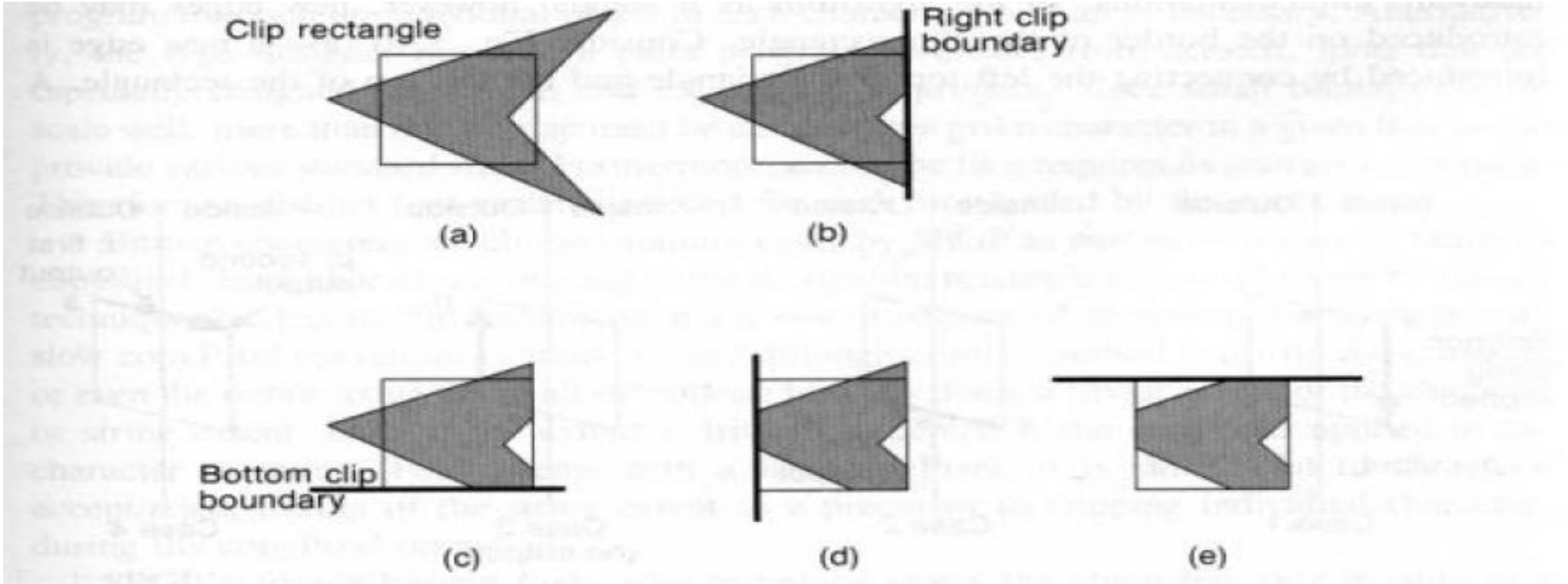


Fig. 3.47 Polygon clipping, edge by edge. (a) Before clipping. (b) Clip on right. (c) Clip on bottom. (d) Clip on left. (e) Clip on top; polygon is fully clipped.

Sutherland-Hodgman Polygon Clipping

- Divide and Conquer Strategy
  - Clip against a single infinite clip edge and get new vertices
  - Repeat for next clip edge
- Adding Vertices to Output List

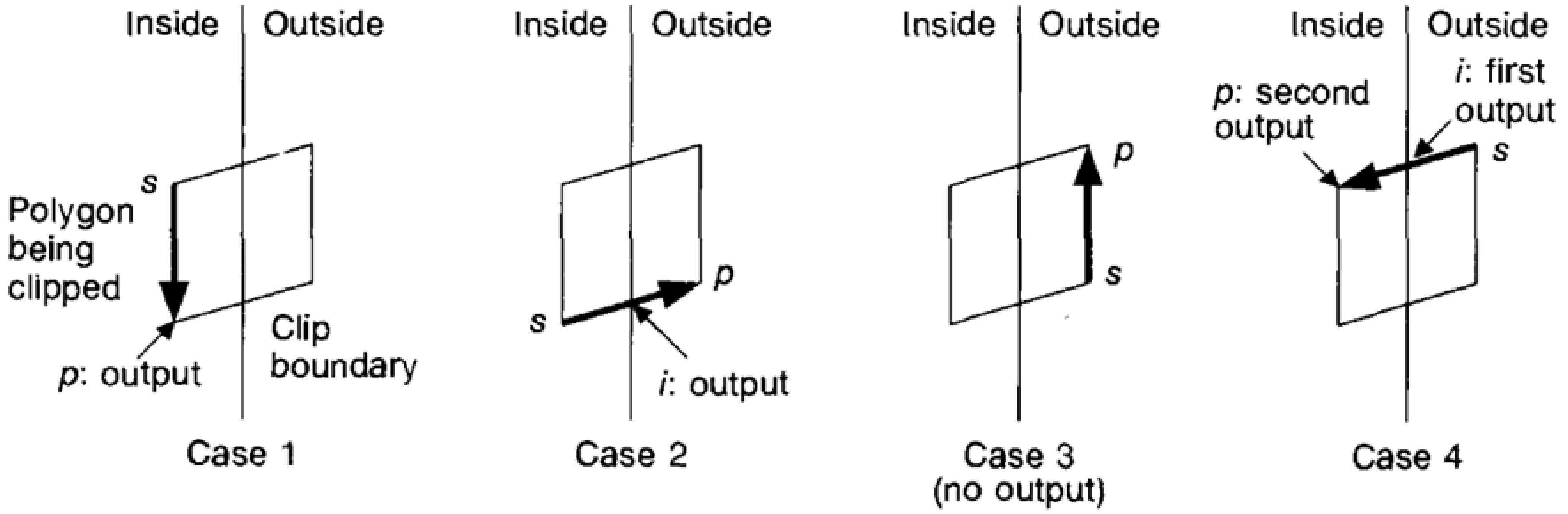


Fig. 3.48 Four cases of polygon clipping.

Sutherland-Hodgman Polygon Clipping

Input:

1. Polygon described by an input of list of vertices: $v_1, v_2, \ldots, v_n$
2. Convex clip region C

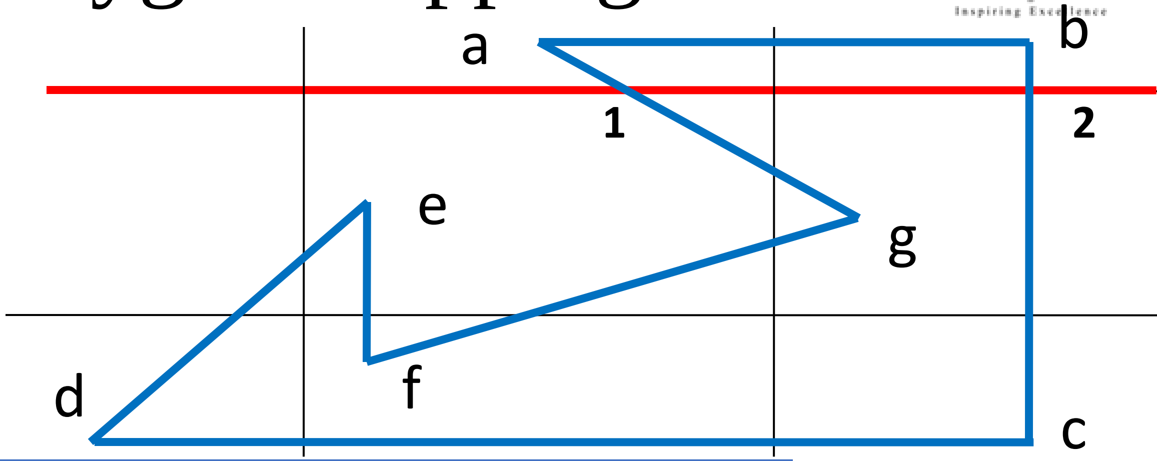
Algorithm:

```
Inputlist : $v_1, v_2, \ldots, v_n$

For each clip edge e in E do
    S □ $v_n$
    P □ $v_1$
    j □ 1
    While (j<n)
        do, if both S & P inside the clip region,
            Add p to output list
        else if S inside & P outside, then
            Find intersection point i
            Add i to output list
        else if S outside and P inside, then
            find intersection point i
            add i to output list
            add P to output list
        else if S and P both outside
            do nothing
        S □ $v_j$
        P □ $v_{j+1}$
        i ++
        inputlist □ current output list
```

Sutherland-Hodgman Polygon Clipping

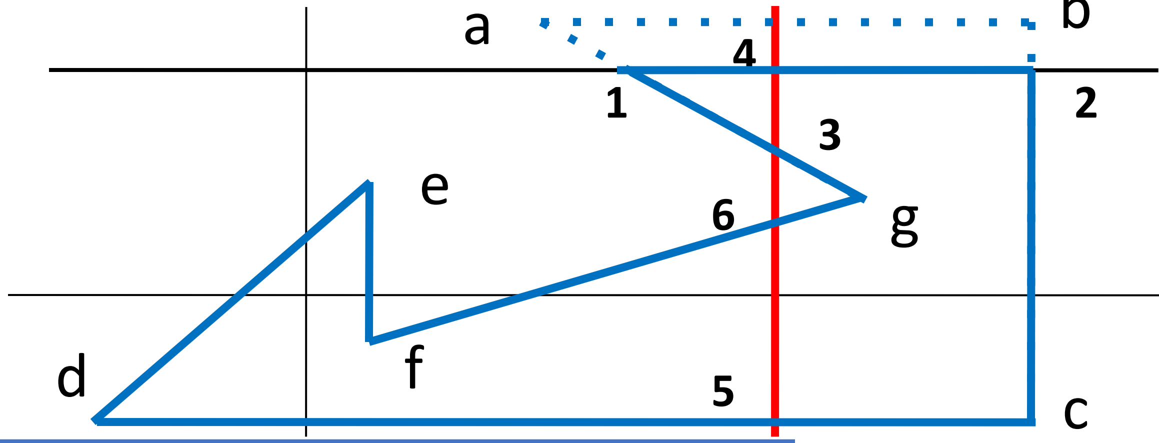
- a, b, c, d, e, f, g
- S = g, P = a
- Output: 1, 2, c, d, e, f, g



SP	Intersection	Output	Comments
g, a	1	1	g inside, a outside
a, b	-	-	Both outside
b, c	2	2,c	b outside, c inside
c, d	-	d	Both inside
d, e	-	e	Both inside
e, f	-	f	Both inside
f, g	-	g	Both inside

Sutherland-Hodgman Polygon Clipping

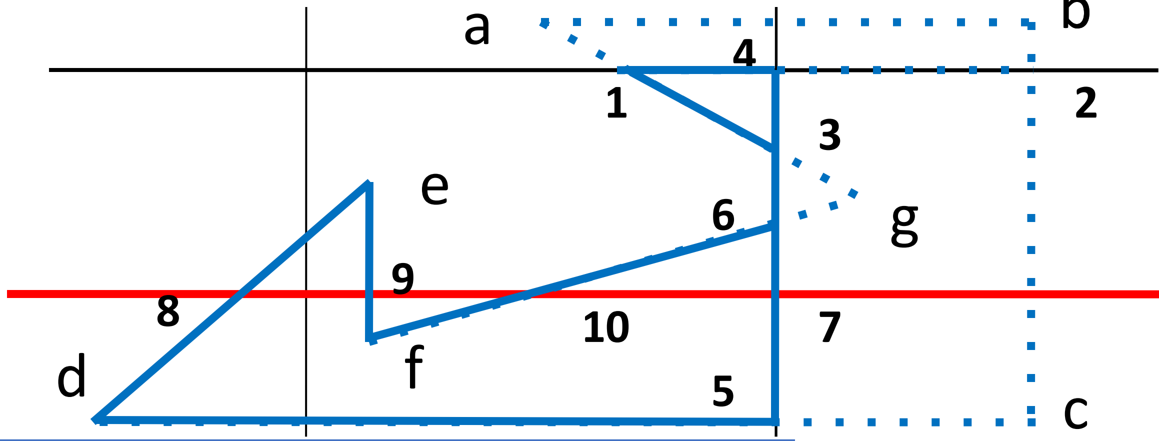
- Output of previous iteration
  1, 2, c, d, e, f, g
- S = g, P = 1
- Output: 3, 1, 4, 5, d, e, f, 6



SP	Intersection	Output	Comments
g,1	3	3,1	g outside, 1 inside
1, 2	4	4	1 inside, 2 outside
2, c	-	-	Both outside
c, d	5	5,d	d inside, c outside
d, e	-	e	Both inside
e, f	-	f	Both inside
f, g	6	6	f inside, g outside

Sutherland-Hodgman Polygon Clipping

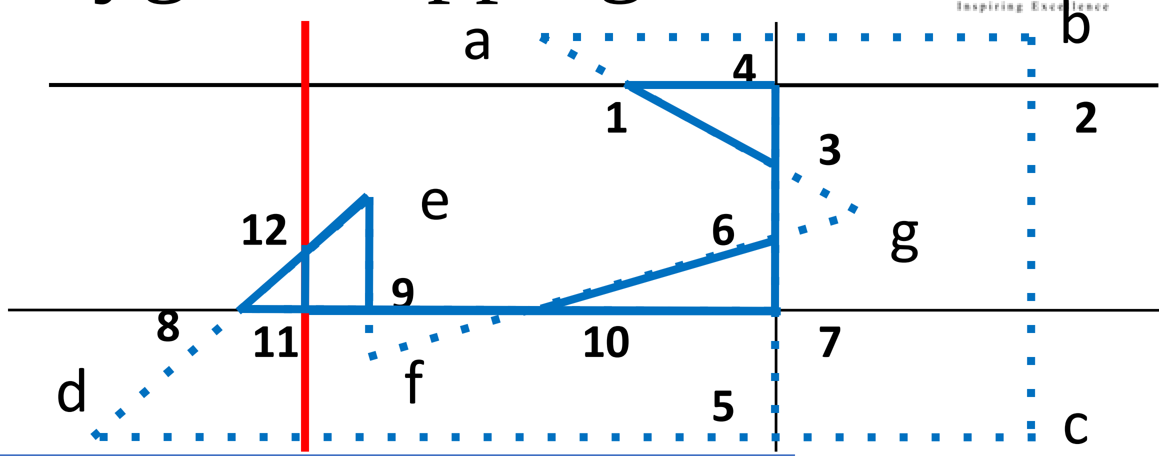
- Output of previous iteration
  3, 1, 4, 5, d, e, f, 6
- S = 6, P = 3
- Output: 3, 1, 4, 7, 8, e, 9, 10, 6



SP	Intersection	Output	Comments
6, 3	-	3	Both inside
3, 1	-	1	Both inside
1, 4	-	4	Both inside
4, 5	7	7	4 inside, 5 outside
5, d	-	-	Both outside
d, e	8	8, e	e inside, d outside
e, f	9	9	e inside, f outside
f, 6	10	10, 6	6 inside, f outside

Sutherland-Hodgman Polygon Clipping

- Output of previous iteration
  3, 1, 4, 7, 8, e, 9, 10, 6
- S = 6, P = 3
- Output: 3, 1, 4, 7, 11, 12, e,
  9, 10, 6



SP	Intersection	Output	Comments
6, 3	-	3	Both inside
3, 1	-	1	Both inside
1, 4	-	4	Both inside
4, 7	-	7	Both inside
7, 8	11	11	7 inside, 8 outside
8, e	12	12, e	e inside, 8 outside
e, 9	-	9	Both inside
9, 10	-	10	Both inside
10, 6	-	6	Both inside

Clipping Circles (and Ellipses)

- Analytical
  - Intersect circle’s extent (square of size of circle’s diameter) with clip rectangle
  - Run the algorithm of polygon clipping
    - No intersect : trivial reject
    - Intersect : divide into quadrants (and later octants if needed) and repeat
  - Compute intersection by solving equations
- During Scan Conversion
  - When circle is relatively small or scan conversion is fast
  - After extent checking scissor on a pixel by pixel basis

➢ Similar Approach for Ellipses!

Reference:

Computer Graphics: Principles and Practice: John F. Hughes, James D. Foley, Andries van Dam, Steven K. Feiner (2nd Edition)

Chapter: 3.12, 3.14