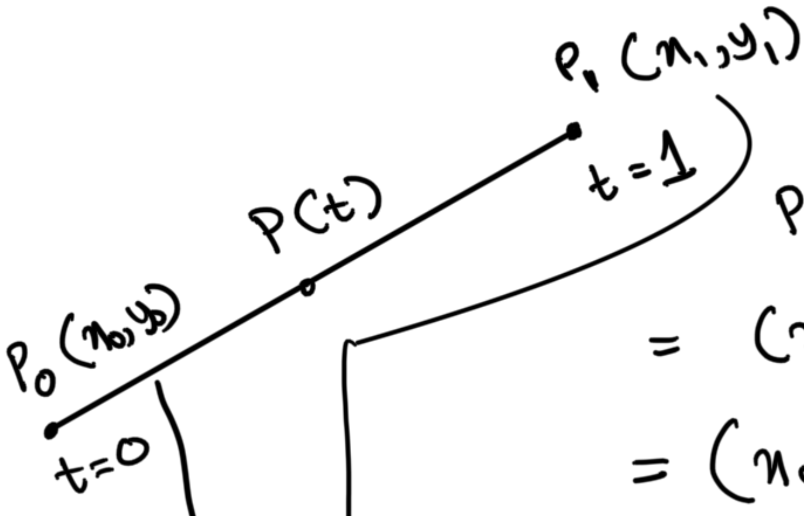
Clipping
Cyrus Beck



Parametric equation of a line $(x_1,y_1)\ (x_0, y_1)$

$$P(t) = P_0 + t(P_1 - P_0)$$
$$= (x_0, y_0) + t(x_1 - x_0, y_1 - y_0)$$
$$= (x_0 + t(x_1 - x_0)), (y_0 + t(y_1 - y_0))$$

if $t = 0$

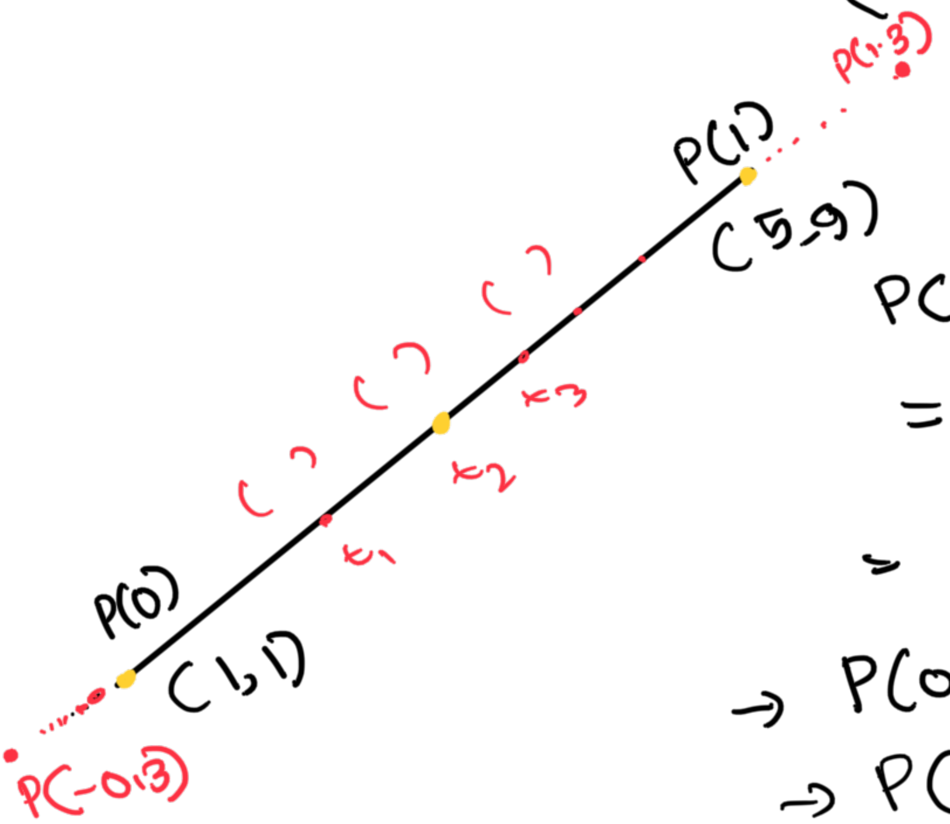
$$P(0) = (x_0 + 0\cancel{(x_1 - x_0)}), (y_0 + 0\cancel{(y_1 - y_0)})$$
$$= (x_0, y_0)$$

if $t = 1$

$$P(1) = (\cancel{x_0} + x_1 - \cancel{x_0}), (\cancel{y_0} + y_1 - \cancel{y_0})$$
$$= (x_1, y_1)$$

$$P(\tfrac{1}{2}) = \left(\frac{x_0 + x_1}{2}, \frac{y_0 + y_1}{2}\right)$$

midpoint of a line.



$$P(t) = P_0 + t(P_1 - P_0)$$
$$= (1, 1) + t(5-1, 9-1)$$
$$= (\overset{x}{1 + 4t}, \overset{y}{1 + 8t})$$

$\rightarrow P(0) = (1, 1)$ st.p

$\rightarrow P(1) = (4+1, 1+8)$

$= (5, 9)$ end.p.

$\rightarrow P(0.5) = (1 + 4\times0.5, 1 + 8\times0.5)$

$= (3, 5)$

$P(1.3) = (1 + 4\times1.3, 1 + 8\times1.3)$

$= (6.2, 11.4)$

$P(-0.3) = (1 + 4\times-0.3, 1 + 8\times-0.3)$

$= (-0.2, -1.4)$

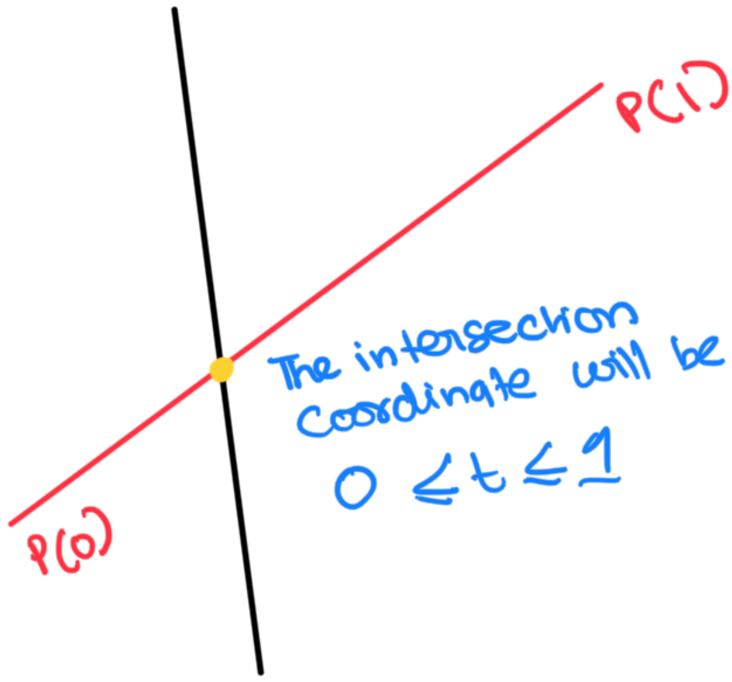
Condition

$\therefore$ $0 < t < 1$ points lie inside the line.

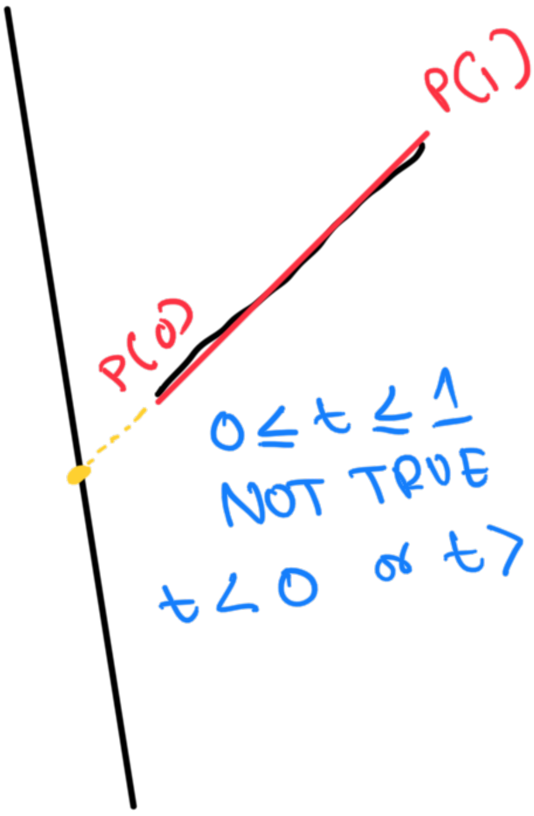
we consider the coordinates inside the line segment

$t < 0$ or $t > 1$ the coordinate will be outside the line segment.

Boundary



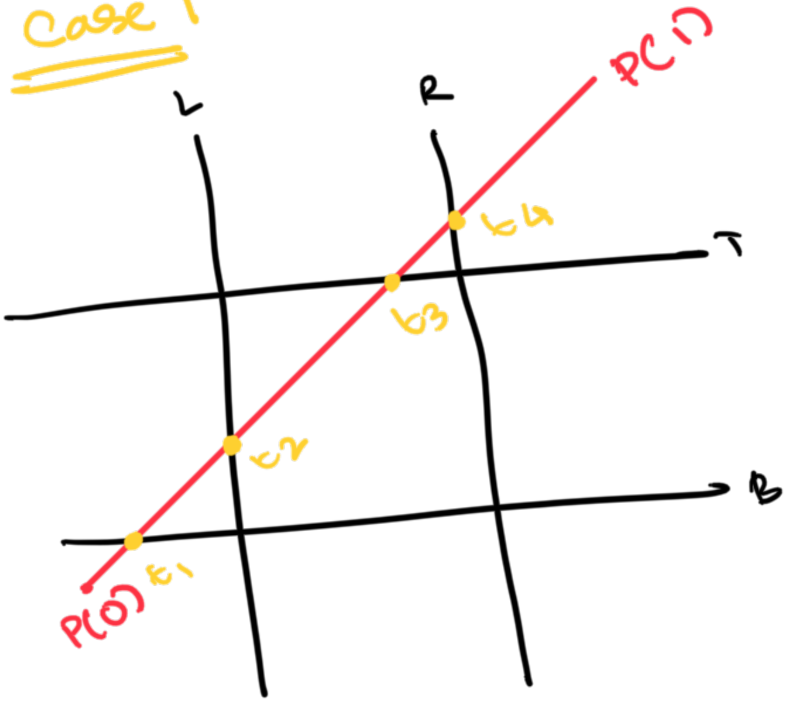
The intersection coordinate will be $0 \leq t \leq 1$



$0 \leq t \leq 1$ NOT TRUE

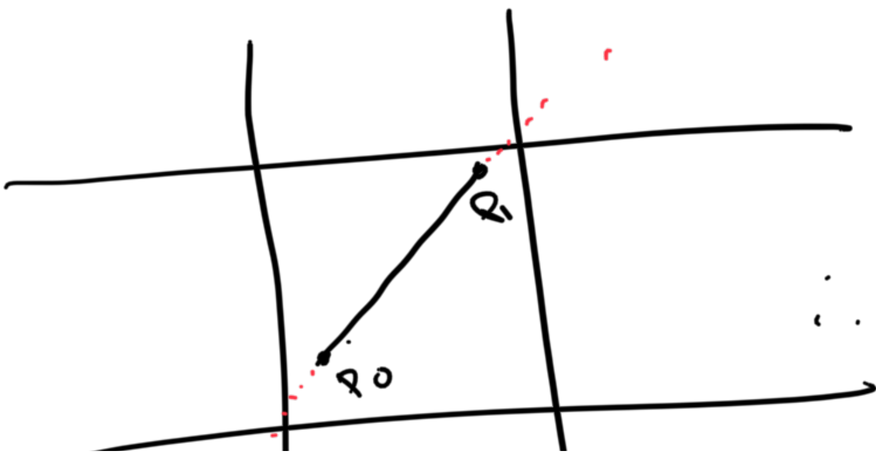
$t < 0$ or $t > 1$

Case 1



This line is partially inside $t_1 \ldots t_4$ all will be within $0 \ldots 1$.

Case 2

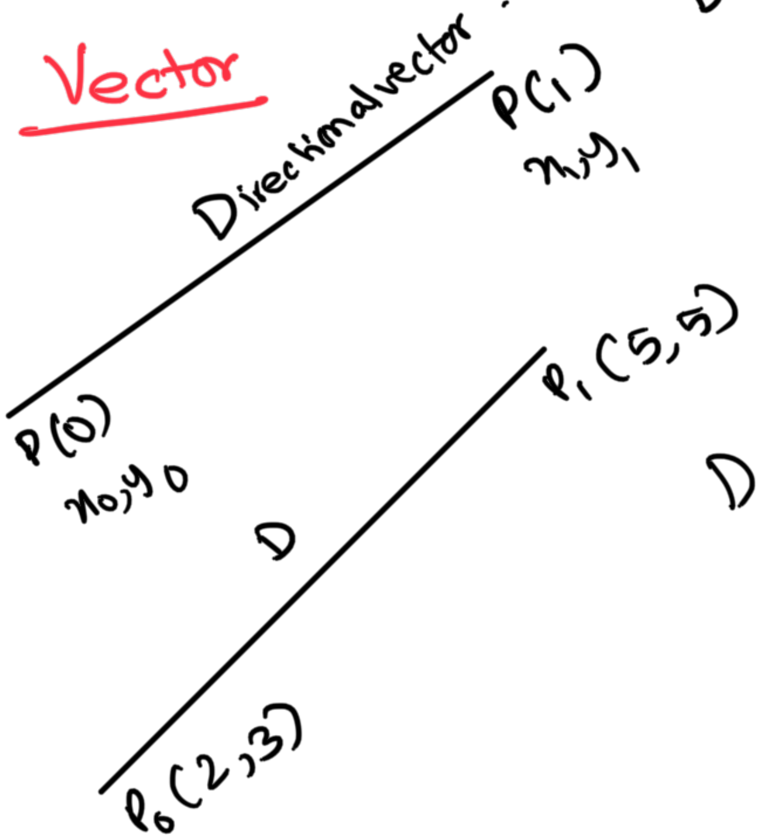


This line is completely inside

$t < 0$ or $t > 1$.

$\therefore$ t is a useful indicator for line clipping.

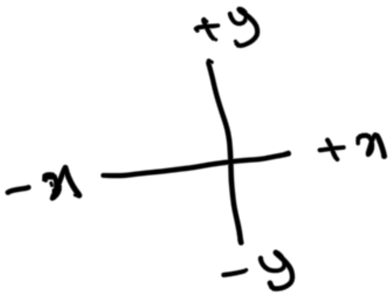
Vector



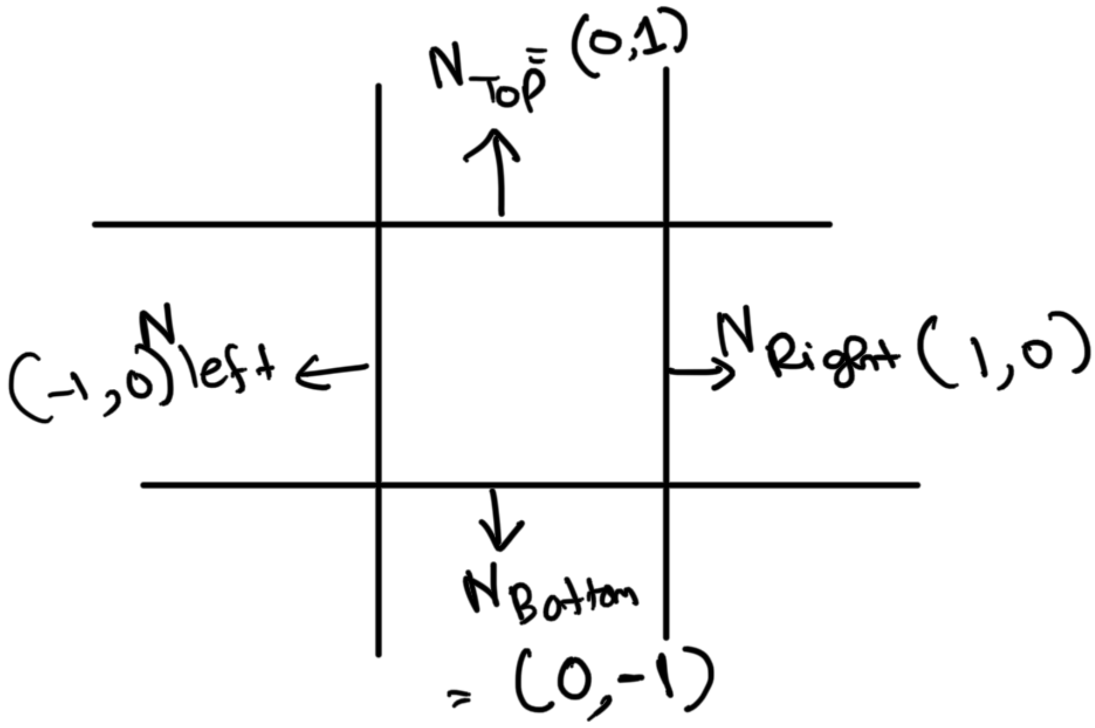
D is a vector from P_0 to P_1

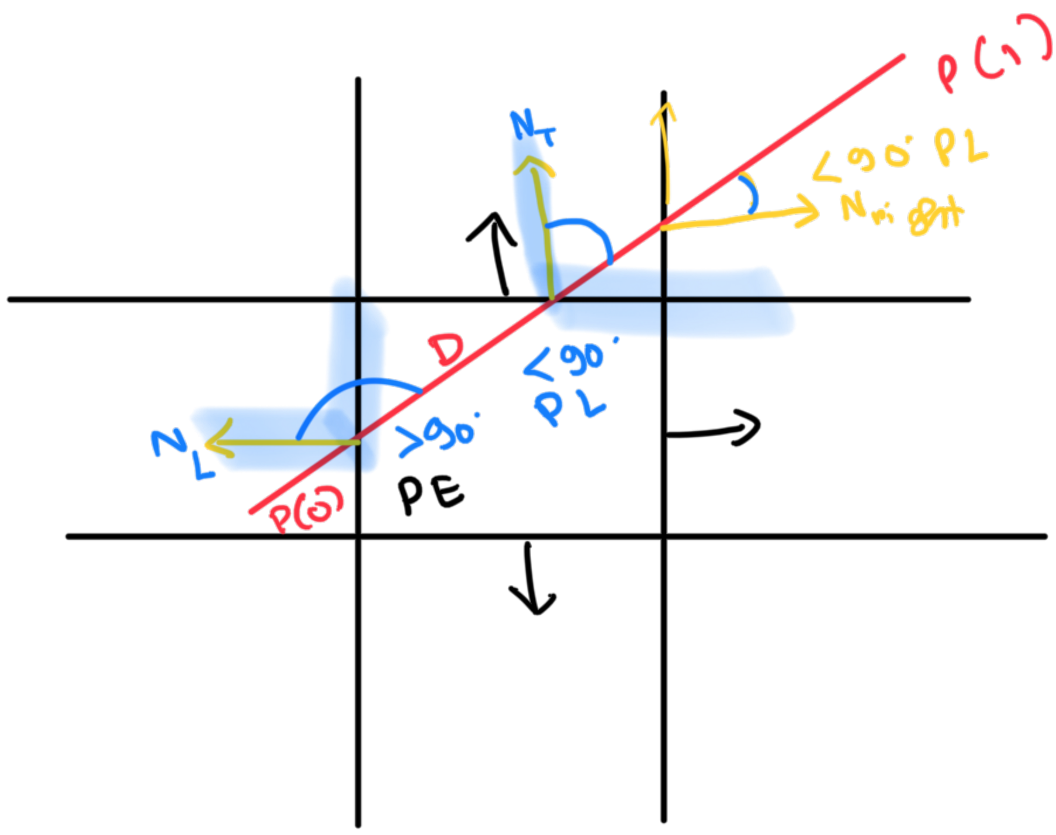
$$D = P_1 - P_0$$
$$= (x_1 - x_0, \; y_1 - y_0)$$

$$D = (5-2, \; 5-3)$$
$$= (3, 2)$$
$$= 3i + 2j$$



Normal vector to boundary





$N_i \cdot D < 0$
PE
Potentially Entering

$N_i \cdot D > 0$
PL
Potentially leaving

Cyrus Beck Algorithm

Cyrus Bec

1) Calculate t values of intersection points with each boundaries

2) Classify intersection points whether it is PE / PL
select PE with highest t / t_E and P_L with lowest value of t / t_L

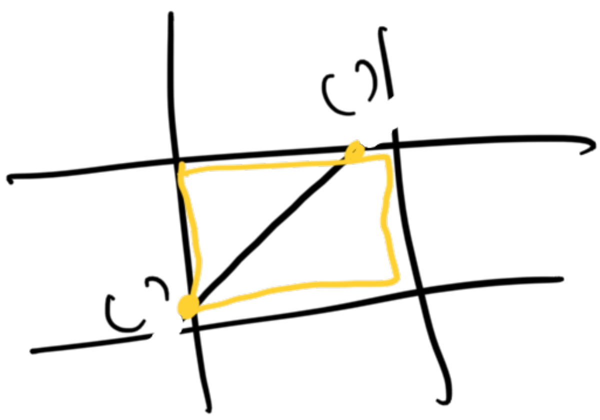
$\rightarrow N_i \cdot D < 0$ PE $\rightarrow t_E$ (max)

$\rightarrow N_i \cdot D > 0$ P_L $\rightarrow t_L$ (min)

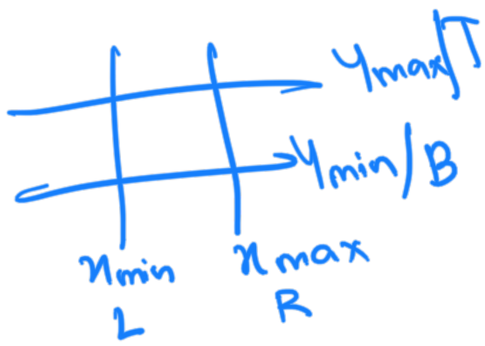
if $t_E > t_L$ line segment outside clip window

$t_E \leq t_L$ this line is in clip intersection.

$$P(t) = (x_0, y_0) + t \times D$$
$$= (x_0, y_0) + t\,(P_1 - P_0).$$



$$t = \frac{N_i\,[P_0 - P_{E_i}]}{-N_i \cdot D}$$

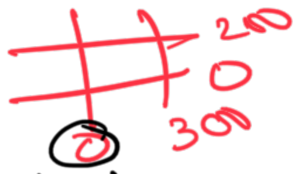


	Boundary	N	$(P_0 - P_E)N$	$(P_1 - P_0)N_i$ / $D \cdot N_i$	t
y	Top	$(0, 1)$	$y_0 - y_{max}$	$y_1 - y_0$	$\frac{y_{max} - y_0}{y_1 - y_0}$
y	Bottom	$(0, -1)$	$-(y_0 - y_{min})$	$-(y_1 - y_0)$	$\frac{y_{min} - y_0}{y_1 - y_0}$
x	Right	$(1, 0)$	$x_0 - x_{max}$	$(x_1 - x_0)$	$\frac{x_{max} - x_0}{x_1 - x_0}$
x	Left	$(-1, 0)$	$-(x_0 - x_{min})$	$-(x_1 - x_0)$	$\frac{x_{min} - x_0}{x_1 - x_0}$

Initially, $t_E = 0$, $t_L = 1$

Boundary	N_i	$N_i \cdot D$	t	PE / PL	max t_E	min t_L
Top	(0,1)	< 0	t=	PE	t_E	
Bottom	(0,−1)	> 0	t=	PL		
Right	(1,0)		t=			
left	(−1,0)		t=			

→ If $t_E > t_L$ line segment outside clip window
else, it in clip intersection.



Calculate the value of t of the line given below for all edges & specify whether they are entering or leaving t. [Given (0,0) to (300,200) be the clip region.

x_{min}, y_{min} x_{max}, y_{max}

(iii) (−250, 200) to (150, 100)

Boundary:

$x_{min} = 0$, $x_{max} = 300$, $y_{min} = 0$, $y_{max} = 200$

Points:

$x_0 = -250$, $y_0 = 200$, $x_1 = 150$, $y_1 = 100$

$$D = P_1 - P_0$$
$$= ((x_1 - x_0), (y_1 - y_0))$$
$$= (150 - (-250)), (100 - 200)$$
$$= (400, -100)$$

$t_E = 0$, $t_L = 1$

Boundary	N_i	$N_i \cdot D$	t	PE/PL	max t_E	min t_L
x_{min} left	(−1,0)	400(−1)+0 −100 = −400 <0	$\frac{x_{min} - x_0}{x_1 - x_0} = \frac{0-(-250)}{150-(-250)}$ 0.625	PE $N_i \cdot D < 0$	0.625	1
Right	(1,0)	400	$\frac{x_{max} - x_0}{x_1 - x_0} = \frac{300-(-250)}{150-(-250)}$ =1.375	$N_i \cdot D > 0$ PL	0.625	1
			$y_{min} - y_0$			

y_{min}	Bottom	$(0,-1)$	$-1(-100) = 100$	$\frac{y_{min}-y_0}{y_1-y_0}$ $\frac{0-200}{100-200} = 2$	P_L	0.625	1
y_{max}	Top	$(0,1)$	-100	$\frac{y_{max}-y_0}{y_1-y_0}$ $\frac{200-200}{100-200} = 0$	P_E	0.625	1

$t_E = 0.625$ $\quad t_E < t_1$

$t_L = 1$

It is clip intersection

$P(t) = (x_0, y_0) + t \times D$

$P(0.625) = (-250, 200) + 0.625(400, -100)$

$= (0, 137.5)$

$P(1) = (150, 100)$

$\therefore$ $(0, 137.5)$ and $(150, 100)$ are endpoints of clip line

Ans