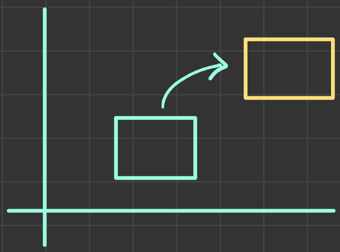
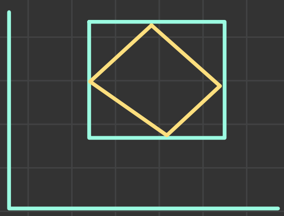
Transformation

Reposition & Resize of object

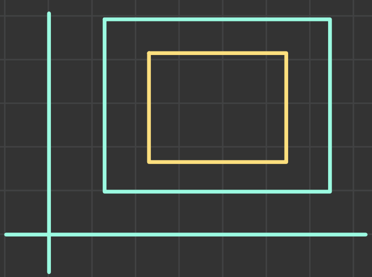
① Translation



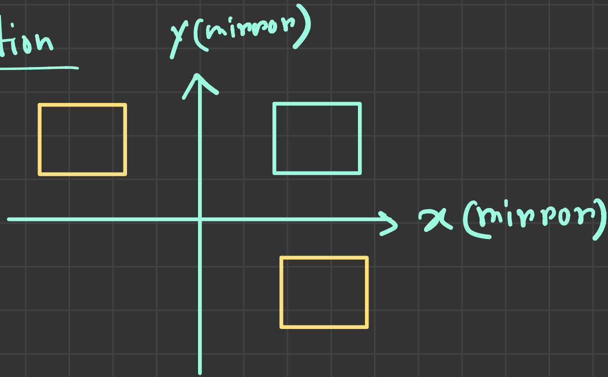
② Rotation :



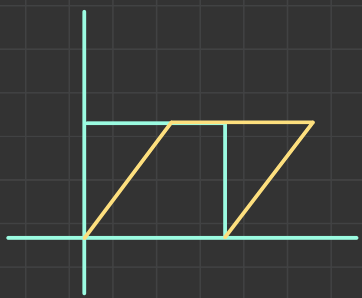
③ Scaling :



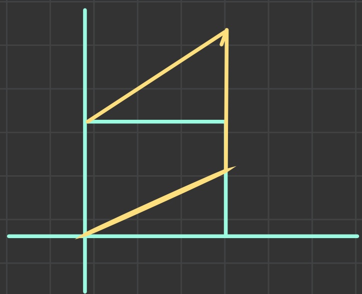
④ Reflection



⑤ Sheering

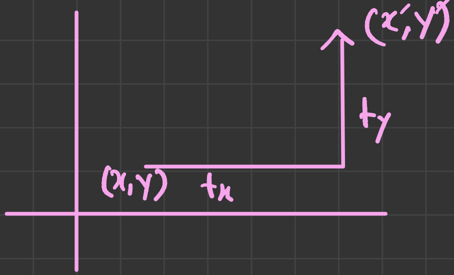


x axis same



Y axis same

① Translation (2D)



$x' = x + t_x$

$y' = y + t_y$

⊛ Cartisan represention :

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

→ extra problem

$$= \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

$$= \begin{bmatrix} x + t_x \\ y + t_y \end{bmatrix}$$

⊛ Homogenous Representation :

2D ⟶ 3x3

dummy variable

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

dummy

$$= \begin{bmatrix} x + t_x \\ x + t_y \\ 1 \end{bmatrix}$$

Translation (3D) $\longrightarrow$ (4x4)

$$x' = x + t_x$$
$$y' = y + t_y$$
$$z' = z + t_z$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

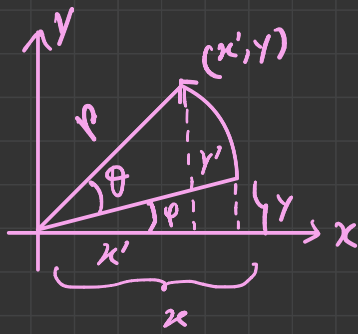
$$= \begin{bmatrix} x + t_x \\ y + t_y \\ y + t_z \\ 1 \end{bmatrix}$$

(*) Translate point (3, 2, 5). 6 step in x axis, −5 in y axis and 7 in z axis

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 3 \\ 2 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} 9 \\ -3 \\ 12 \\ 1 \end{bmatrix}$$

$\therefore (9, -3, 12)$

Rotation (z based)



$$\cos\varphi = \frac{x}{r} \Rightarrow x = r\cos\varphi$$

$$\sin\varphi = \frac{y}{r} \Rightarrow y = r\sin\varphi$$

$$\cos(\theta+\varphi) = \frac{x'}{r} \Rightarrow x' = r\cos(\theta+\varphi)$$

$$\sin(\theta+\varphi) = \frac{y'}{r} \Rightarrow y' = r\sin(\theta+\varphi)$$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$

$$x' = r\cos(\theta+\varphi) = r\cos\theta\cos\varphi - r\sin\theta\sin\varphi = x\cos\theta - y\sin\theta$$

$$y' = r\sin(\theta+\varphi) = r\sin\theta\cos\varphi + r\cos\theta\sin\varphi = x\sin\theta + y\cos\theta$$

$z = z'$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} x\cos\theta - y\sin\theta \\ x\sin\theta + y\sin\theta \\ z \\ 1 \end{bmatrix}$$

* Rotate point (9, 2, 5) at angle 45° (anti clock) with respect to origin?

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ & 0 & 0 \\ \sin 45^\circ & \cos 45^\circ & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 9 \\ 2 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} 4.95 \\ 7.78 \\ 5 \\ 1 \end{bmatrix}$$

$= (4.95, 7.78, 5)$

For clockwise, $\theta = -45^\circ$

(*) Rotate point (6,2) at an angle 90° (anti) with respect to point (2,2) ?

(i) Shift $(2,2) \longrightarrow (0,0)$ $\rightarrow T(-2,-2)$

(ii) Rotation (90°) / $\theta \rightarrow R(90°)$

(iii) Shift $(0,0) \longrightarrow (2,2)$ $\rightarrow T(2,2)$

$$\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

Output = $T(2,2) \rightarrow R(90) * T(-2,-2) * \text{Input}$

[Cant change the order]

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} \cos 90° & -\sin 90° & 0 \\ \sin 90° & \cos 90° & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 6 \\ 2 \\ 1 \end{bmatrix}$$

A = Alarm

$\neg A$ =

B = Bulgary

$\neg B$ =

	A	$\neg A$
B	$A \cap B$ 0.9	$\neg A \wedge B$ 0.6
$\neg B$	$A \cap \neg B$ 0.8	$\neg A \cap \neg B$ 0.7

$P(A) = A \cap B + A \cap \neg B$

$P(B) = A \cap B + \neg A \wedge B$

$P(A \wedge B) = 0.9$