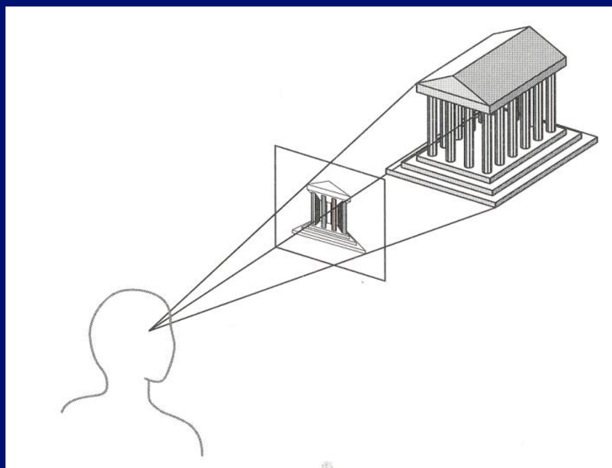
CSE 409: Computer Graphics

Camera Transformations and Projection



Angel: Interactive Computer Graphics

Acknowledgements: parts of information and pictures has been collected from the University of Virginia.

Projection

In general, projections transform points in a coordinate system of <u>dimension n</u> into points in a coordinate system of <u>dimension less than n</u>.

We shall limit ourselves to the <u>projection from 3D to 2D</u>.

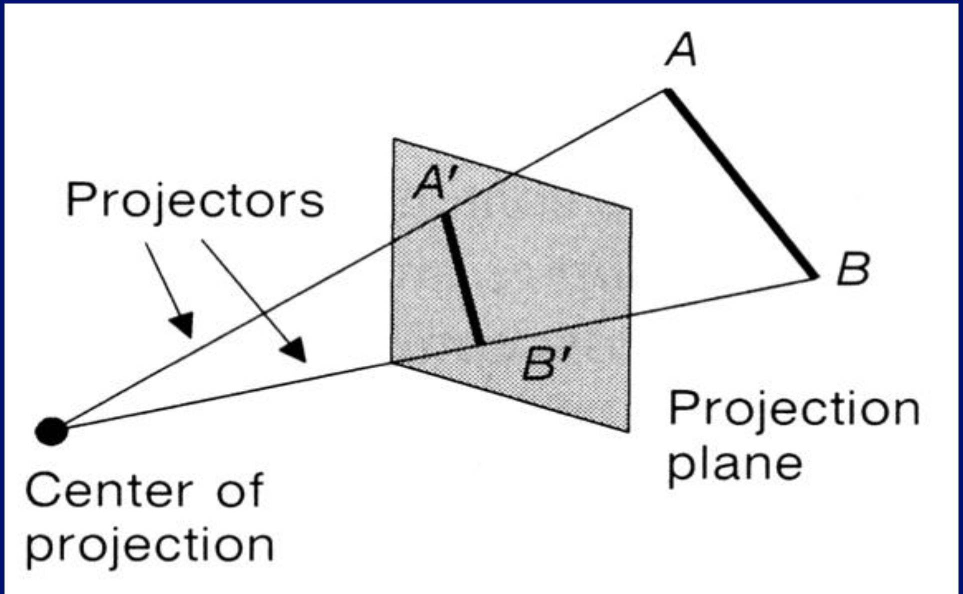
We will deal with planar geometric projections where:

- The <u>projection is onto a plane</u> rather than a curved surface
- The <u>projectors are straight lines</u> rather than curves

Projection

- key terms…

  - *Projection* from 3D to 2D is defined by straight *projection rays* (***projectors***) emanating from the '***center of projection***', passing through each point of the object, and intersecting the '***projection plane***' to form a projection.



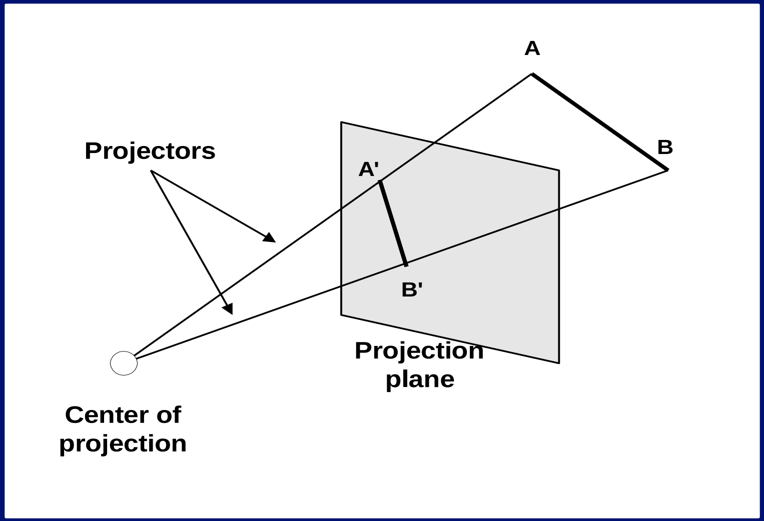
Planer Geometric Projection

2 types of projections

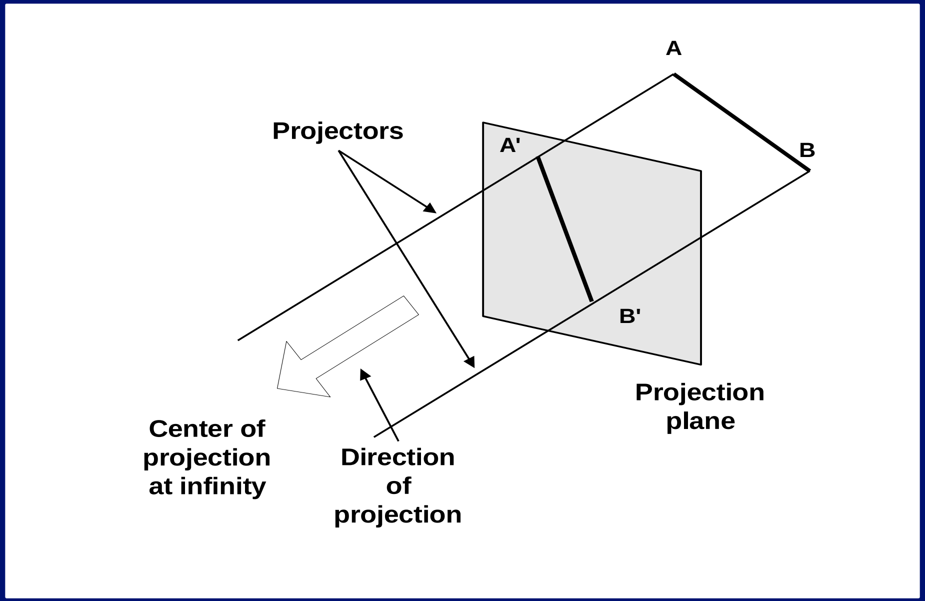
- *perspective* and *parallel*.

Key factor is the <u>center of projection</u>.

- if distance to center of projection is finite : perspective
- if infinite : parallel



Perspective projection

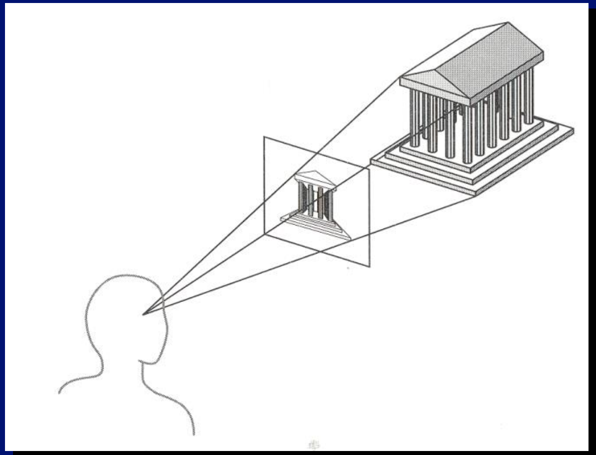


Parallel projection

Perspective v Parallel

Perspective:

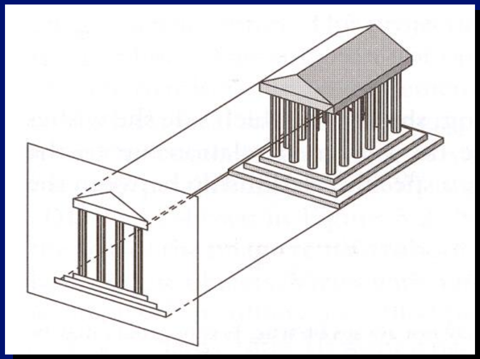
- visual effect is similar to human visual system...
- has 'perspective foreshortening'
  - size of object varies inversely with distance from the center of projection.
- Parallel lines do not in general project to parallel lines
- angles only remain intact for faces parallel to projection plane.



Parallel

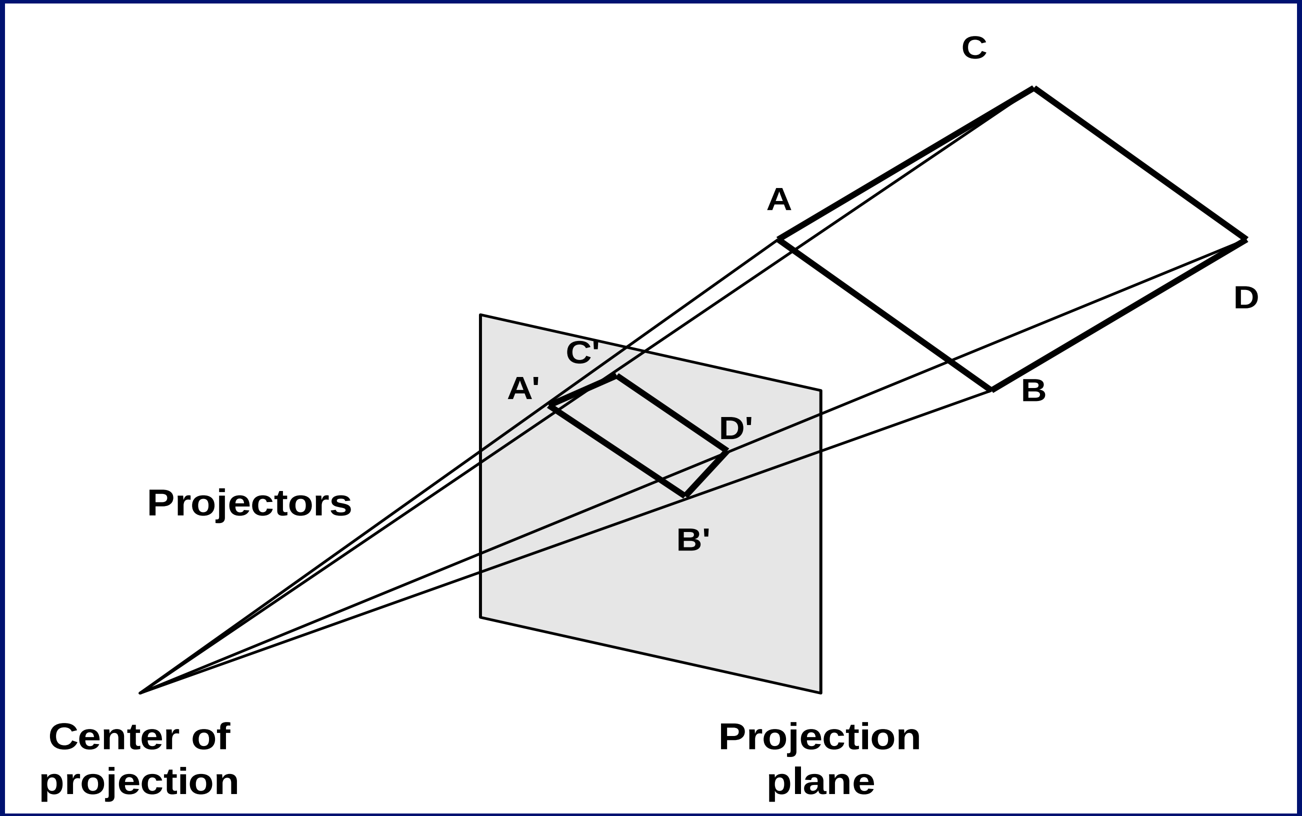
Parallel:

- less realistic view because of no foreshortening
- however, parallel lines remain parallel.
- angles only remain intact for faces parallel to projection plane.



Perspective projection- anomalies

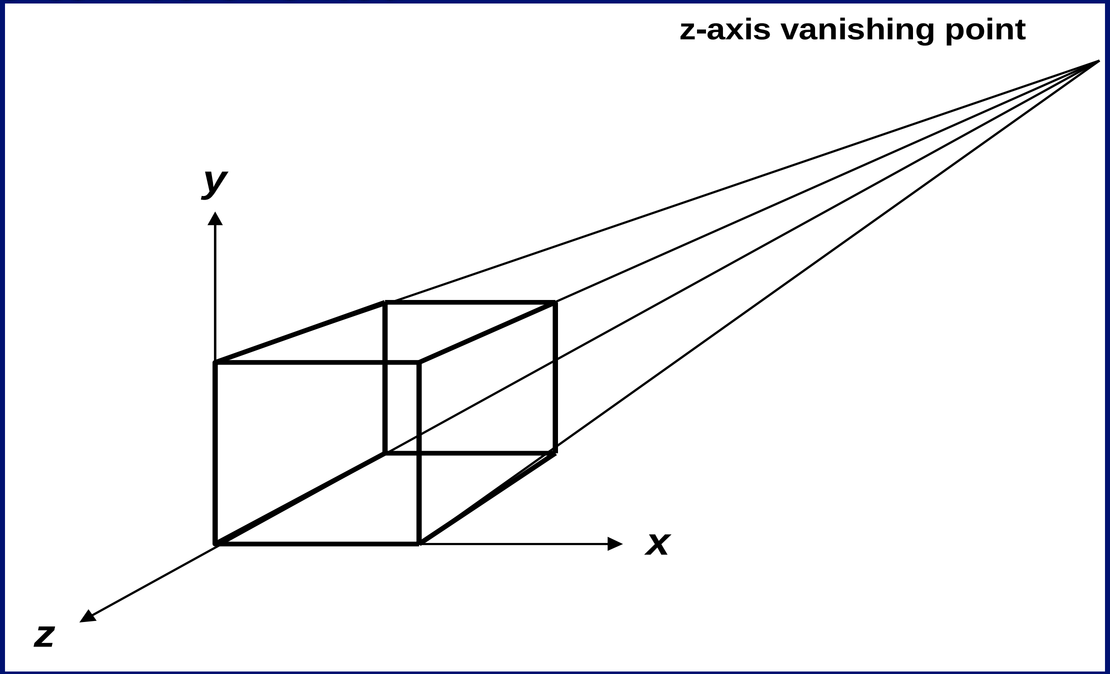
***Perspective foreshortening** The farther an object is from COP the smaller it appears*



Perspective projection- anomalies

Vanishing Points:* *Any set of parallel lines not parallel to the view plane appear to meet at some point.

- There are an infinite number of these, 1 for each of the infinite amount of directions line can be oriented



Vanishing point

Vanishing Point

Vanishing Point

If a set of lines are parallel to one of the three axes, the vanishing point is called an axis vanishing point (Principal Vanishing Point).

- There are at most 3 such points, corresponding to the number of axes cut by the projection plane

One-point:

- One principle axis cut by projection plane
- One axis vanishing point

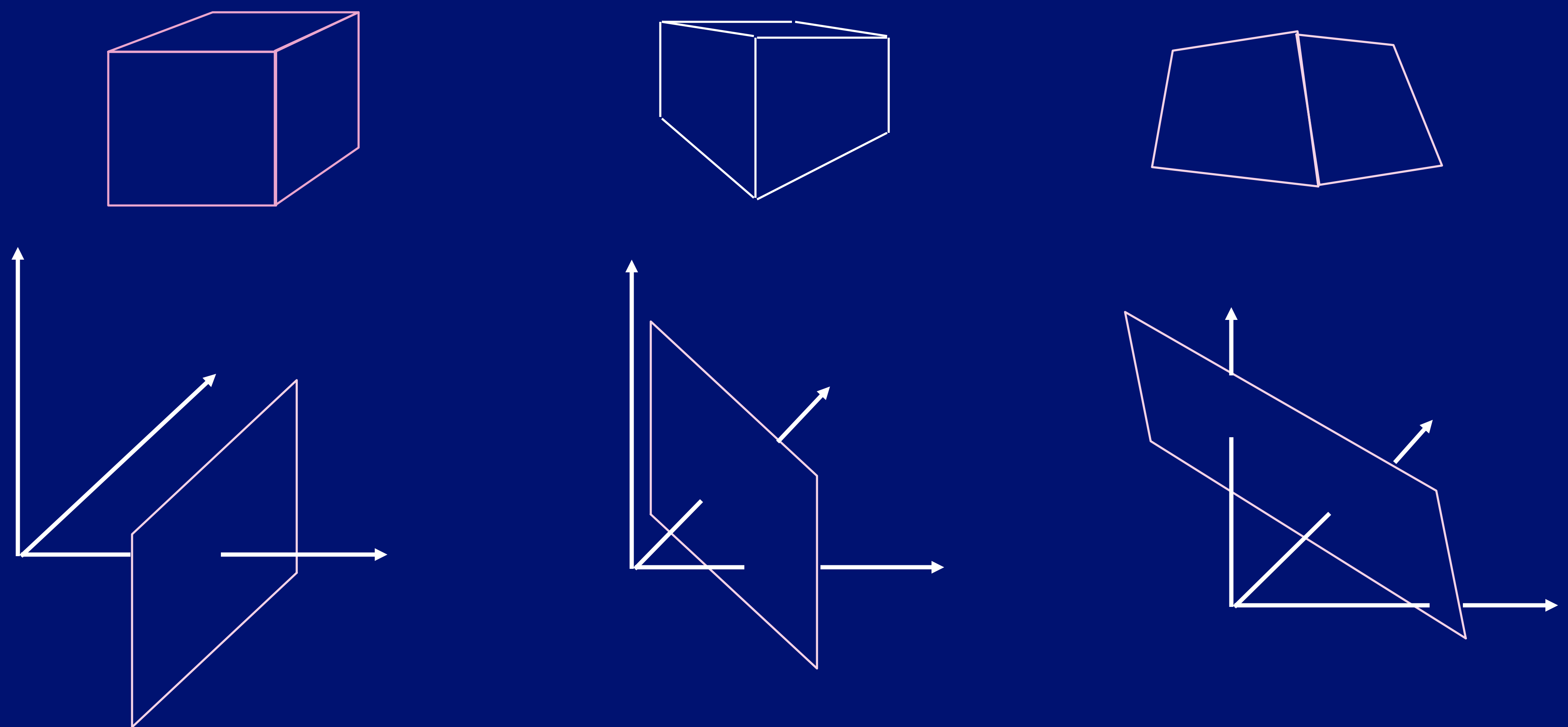
Two-point:

- Two principle axes cut by projection plane
- Two axis vanishing points

Three-point:

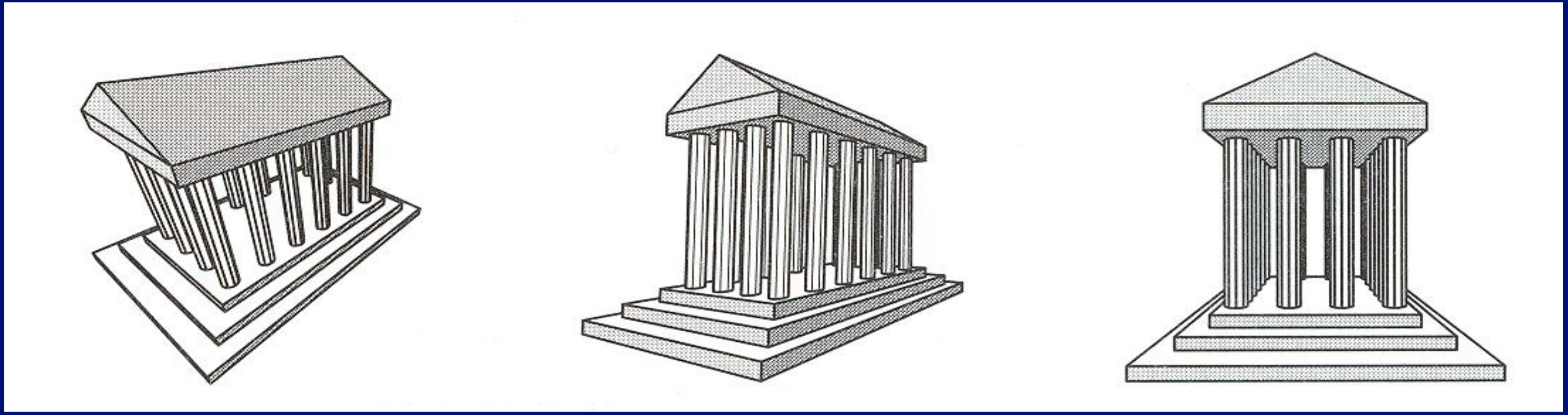
- Three principle axes cut by projection plane
- Three axis vanishing points

Vanishing Point



Perspective Projection

How many vanishing points?



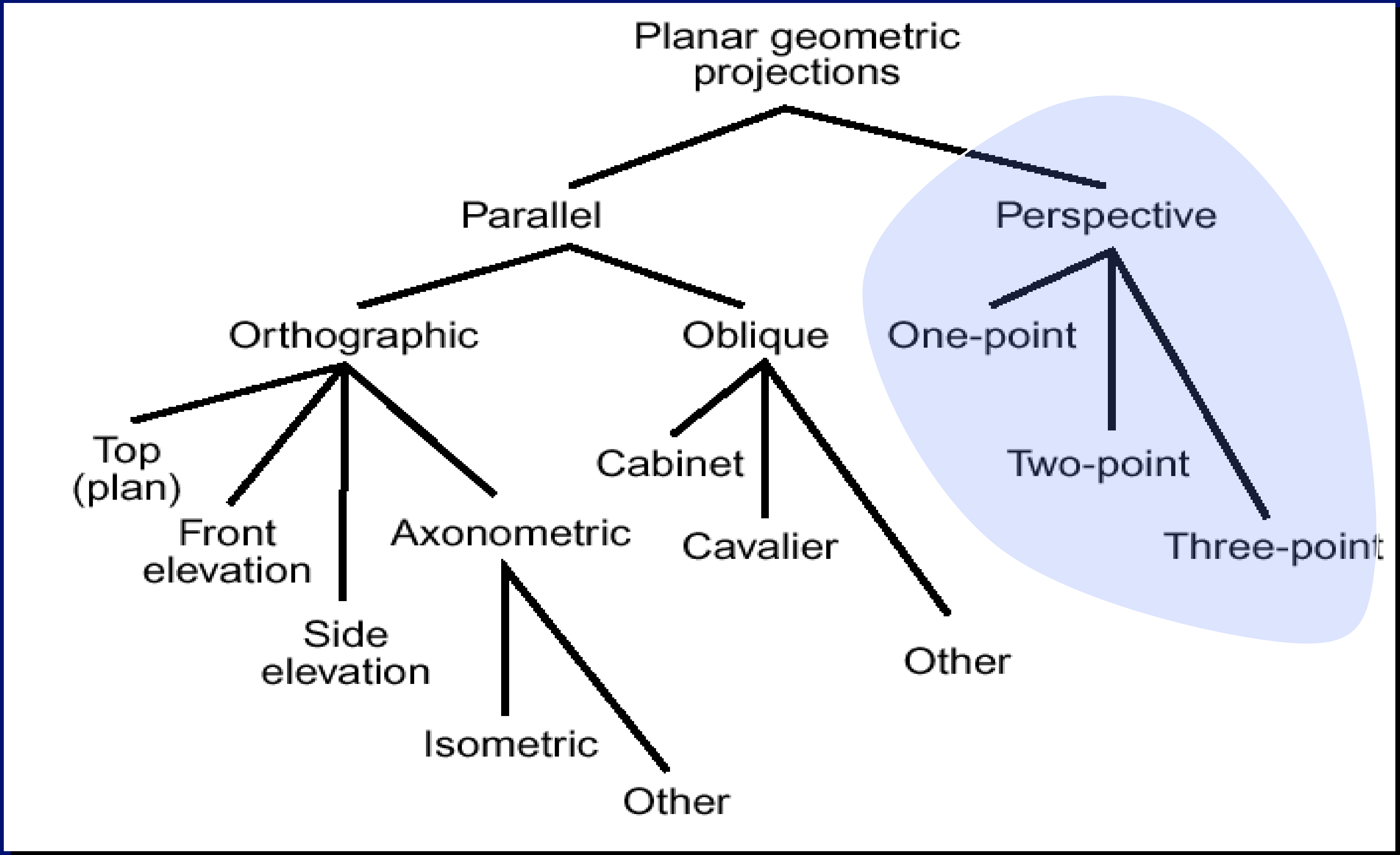
3-Point Perspective

2-Point Perspective

1-Point Perspective

Angel Figure 5.10

Taxonomy of Projection XM



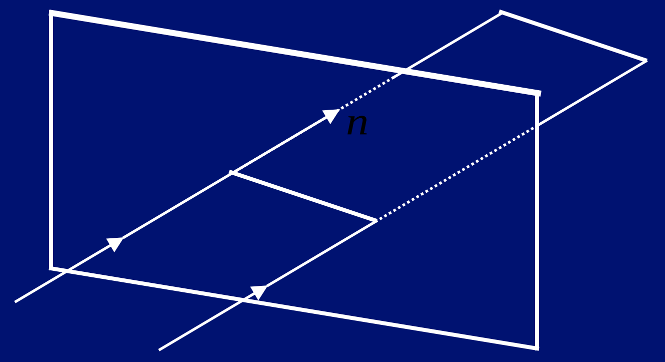
Parallel projection

2 principle types:

- *orthographic* and *oblique*.

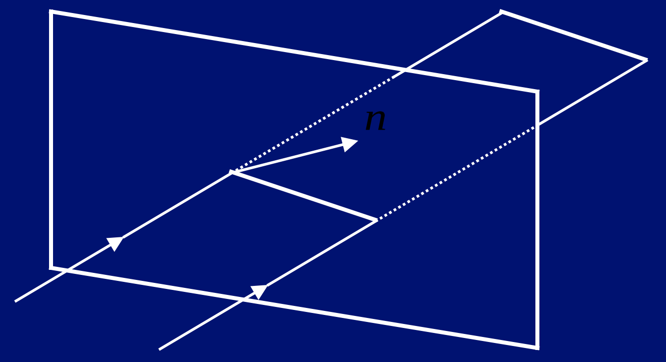
Orthographic :

- direction of projection = normal to the projection plane.



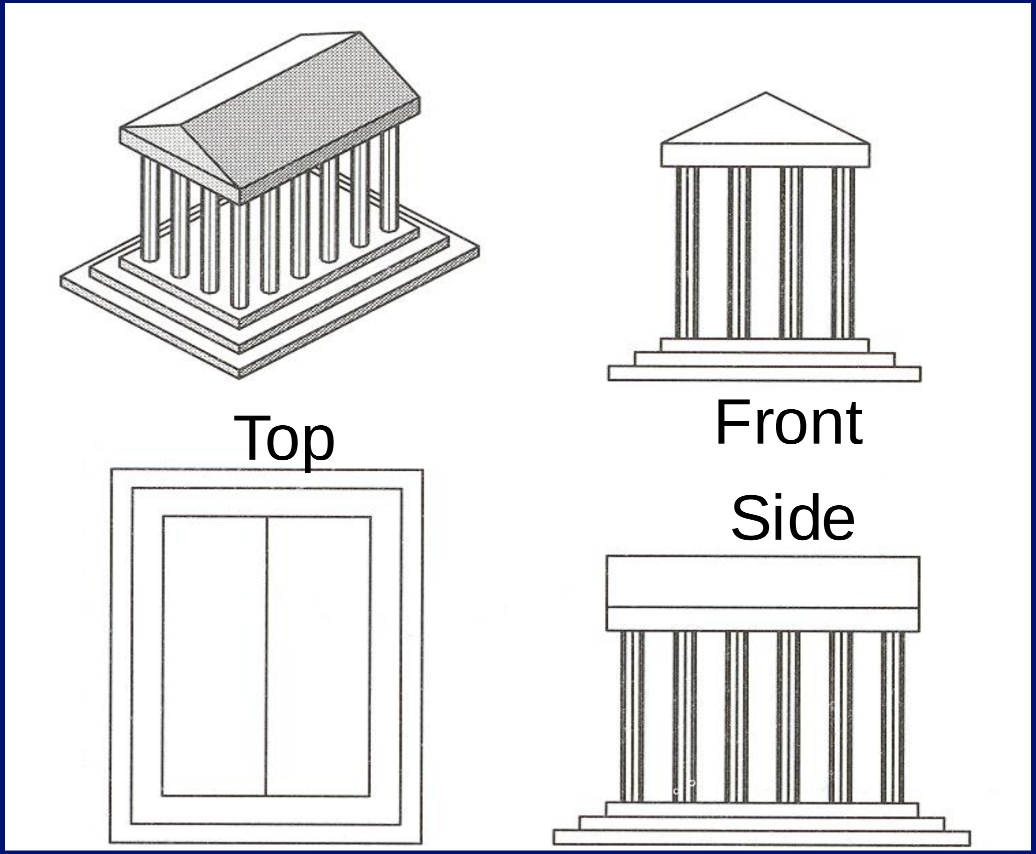
Oblique :

- direction of projection != normal to the projection plane.



Orthographic Projections

DOP perpendicular to view plane



Axonometric Orthogonal Projection

Projection plane are not normal to the principle axis and therefore show several faces of an object at once.

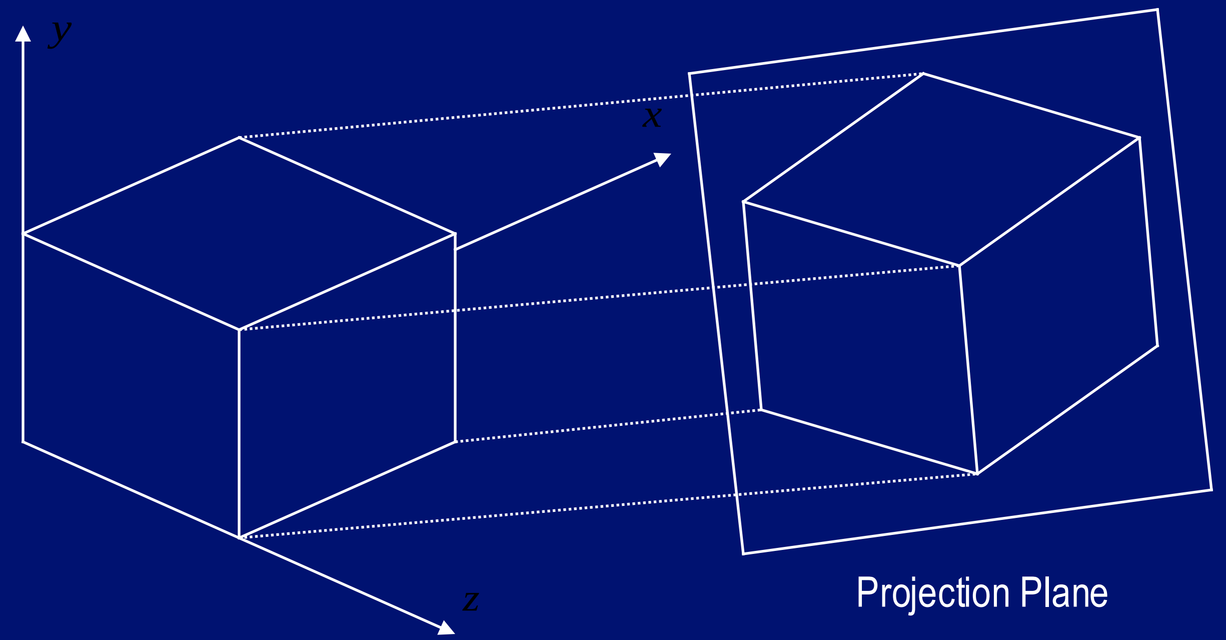
(XM) Isometric Axonometric Orthogonal Projection: Projection plane normal makes equal angles with each principle axis. If the projection plane normal is (d_x, d_y, d_z) then

$| d_x | = | d_y | = | d_z |$. There are 8 directions to satisfy this condition.

Axonometric vs Perspective

Axonometric projection shows several faces of an object at once like perspective projection.

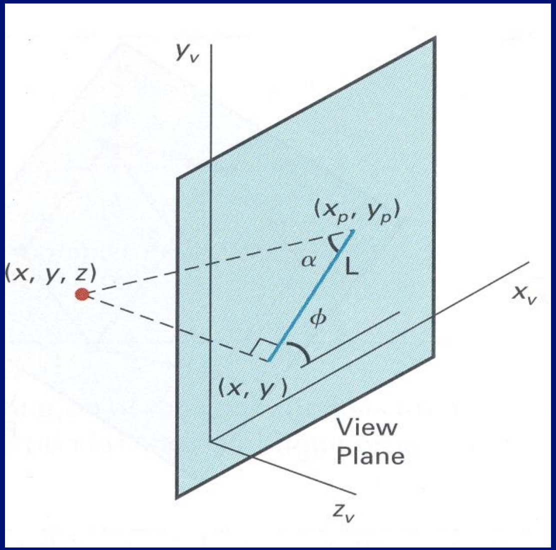
But the foreshortening is uniform rather than being related to the distance from the COP.

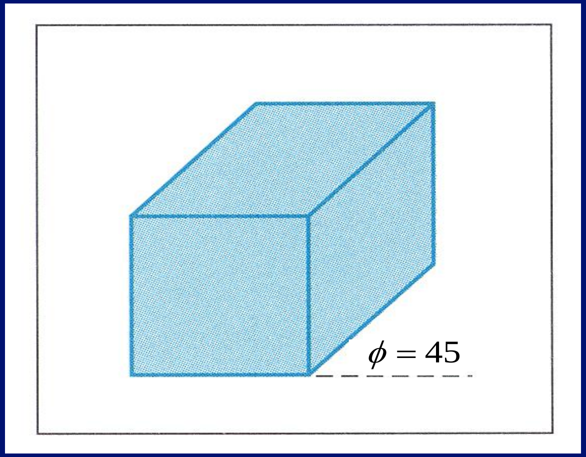


Isometric proj

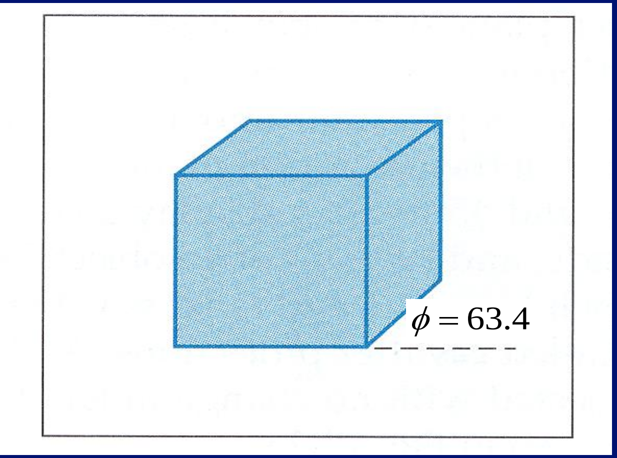
Oblique Projections

DOP not perpendicular to view plane





Cavalier
(DOP $\alpha = 45^o$)
$\tan(\alpha) = 1$



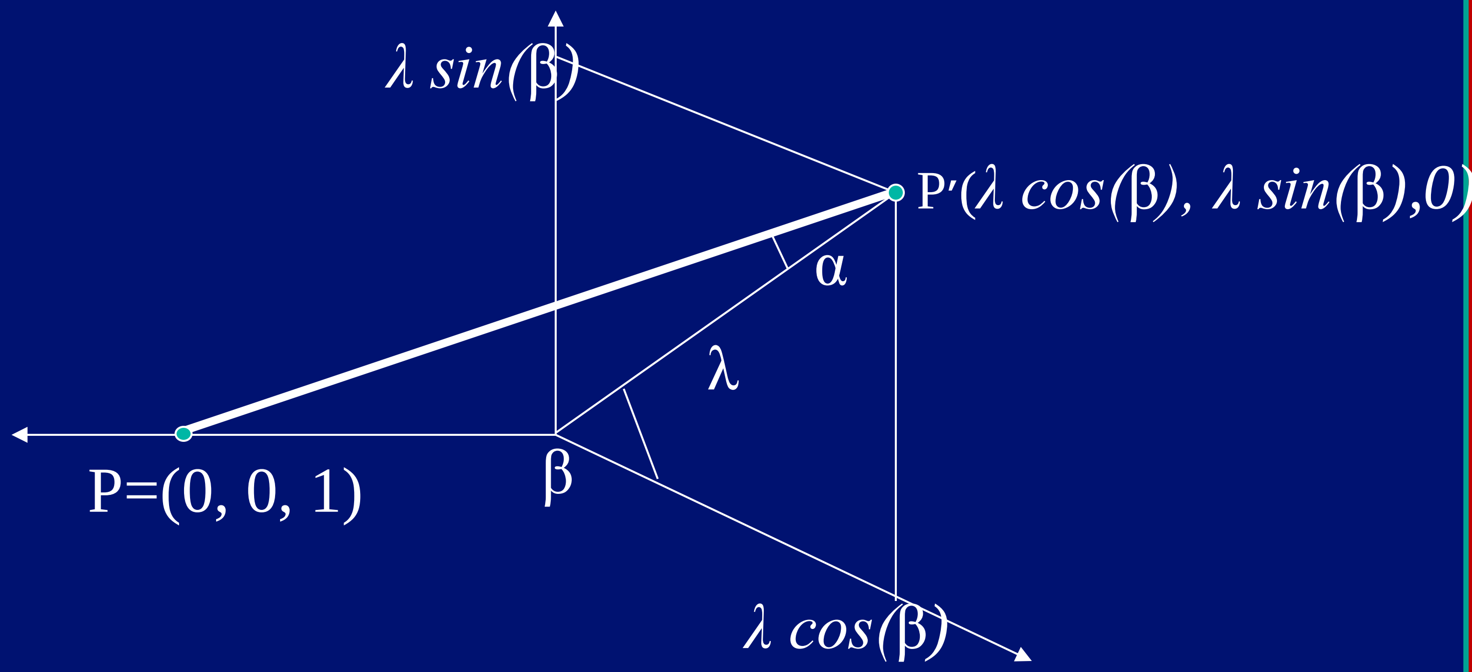
Cabinet
(DOP $\alpha = 63.4^o$)
$\tan(\alpha) = 2$

H&B

Oblique parallel projection

Cavalier, cabinet and orthogonal projections can all be specified in terms of (α, β) or (λ ,β) since

- $\tan(\alpha) = 1/\lambda$



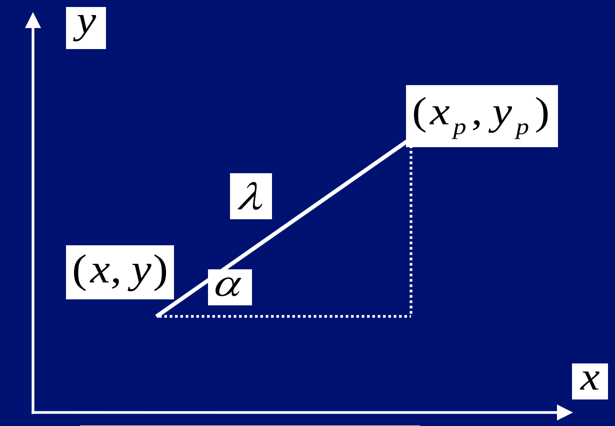
Oblique parallel projection

λ=1	α= 45	Cavalier projection	β= 0 - 360
λ=0.5	α = 63.4	Cabinet projection	β= 0 – 360
λ=0	α = 90	Orthogonal projection	β= 0 – 360

Oblique parallel projection

PP' = (λ cos(α), λ sin(α),-1) = DOP

Proj(P) = (λ cos(α), λ sin(α),0)



$$\therefore\ x_p = x + \lambda\cos\alpha$$
$$y_p = y + \lambda\sin\alpha$$

Generally

- multiply by z and allow for (non-zero) x and y

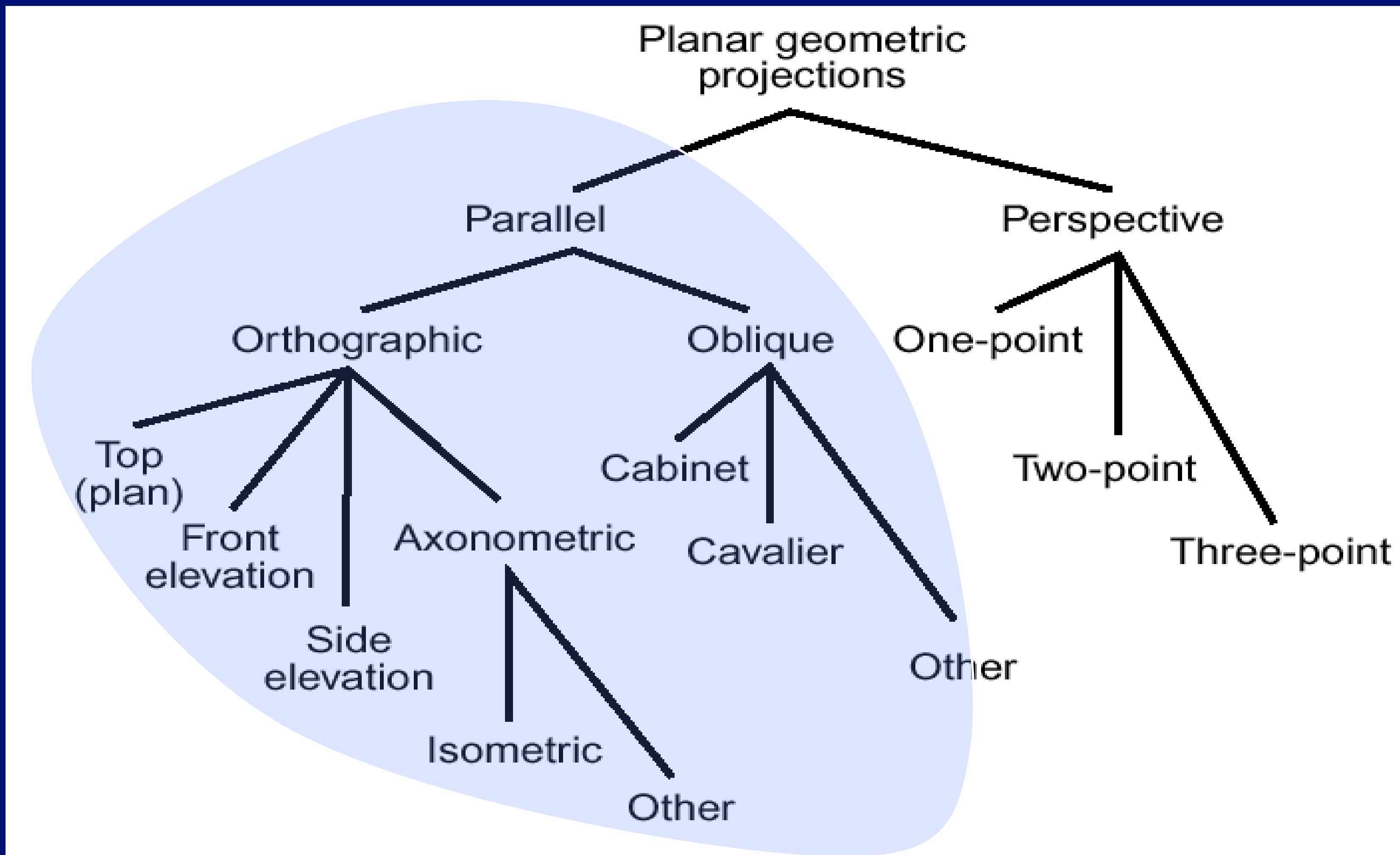
$$x' = x + z\,\lambda\cos\alpha$$

$$y' = y + z\,\lambda\sin\alpha$$

$$\begin{pmatrix} x' \\ y' \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & \lambda\cos\alpha & 0 \\ 0 & 1 & \lambda\sin\alpha & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

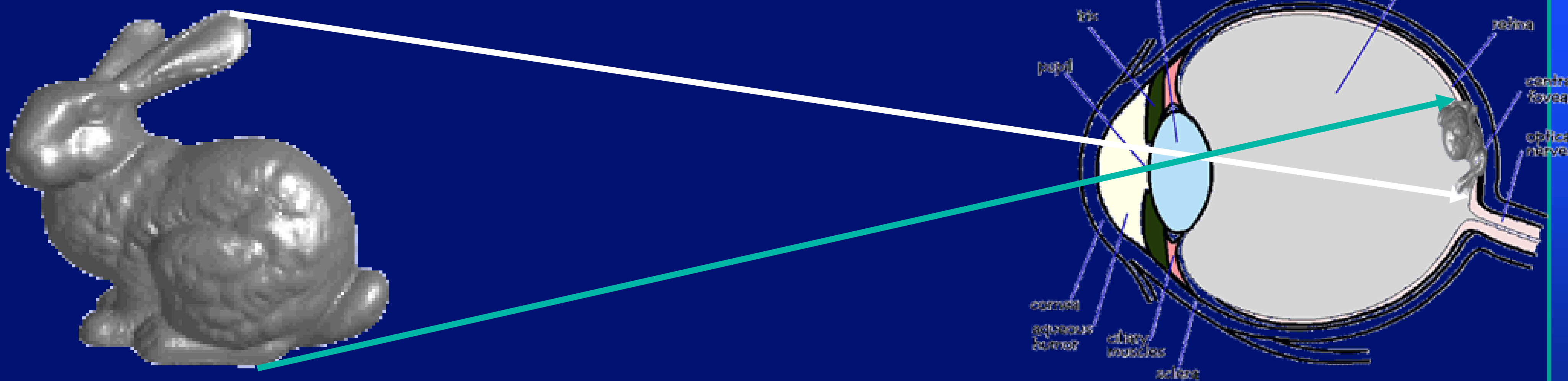
Replace α by β for this slide

Taxonomy of Projection



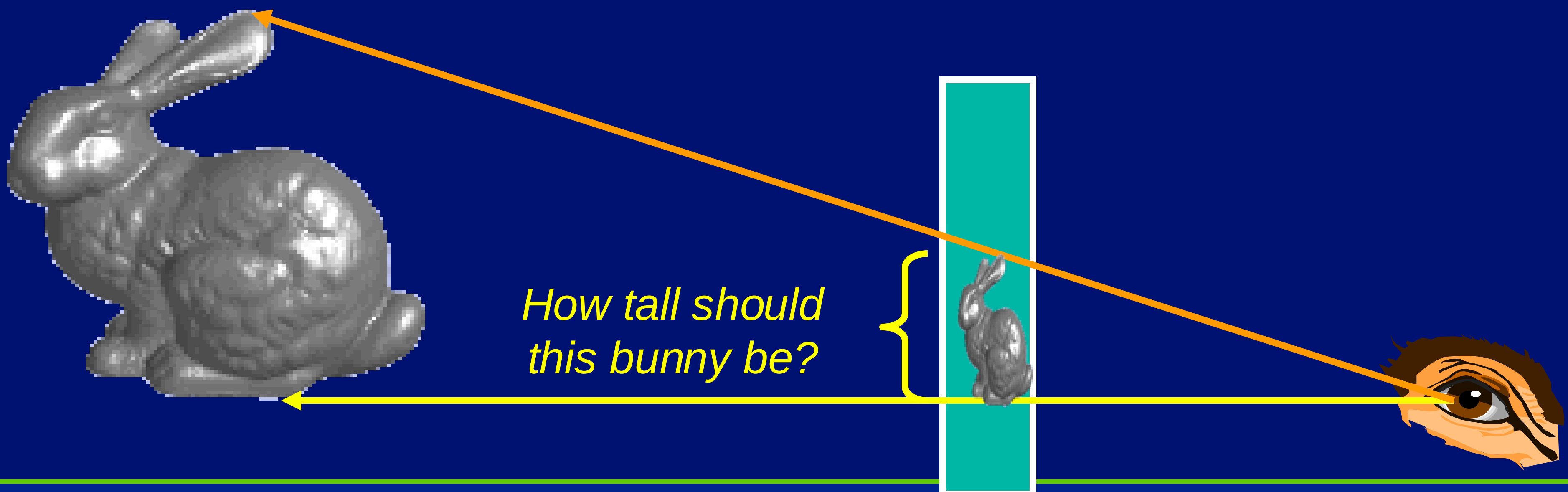
Perspective Projection

- ***In the real world, objects exhibit*** **perspective foreshortening*****: distant objects appear smaller***
- ***The basic situation:***



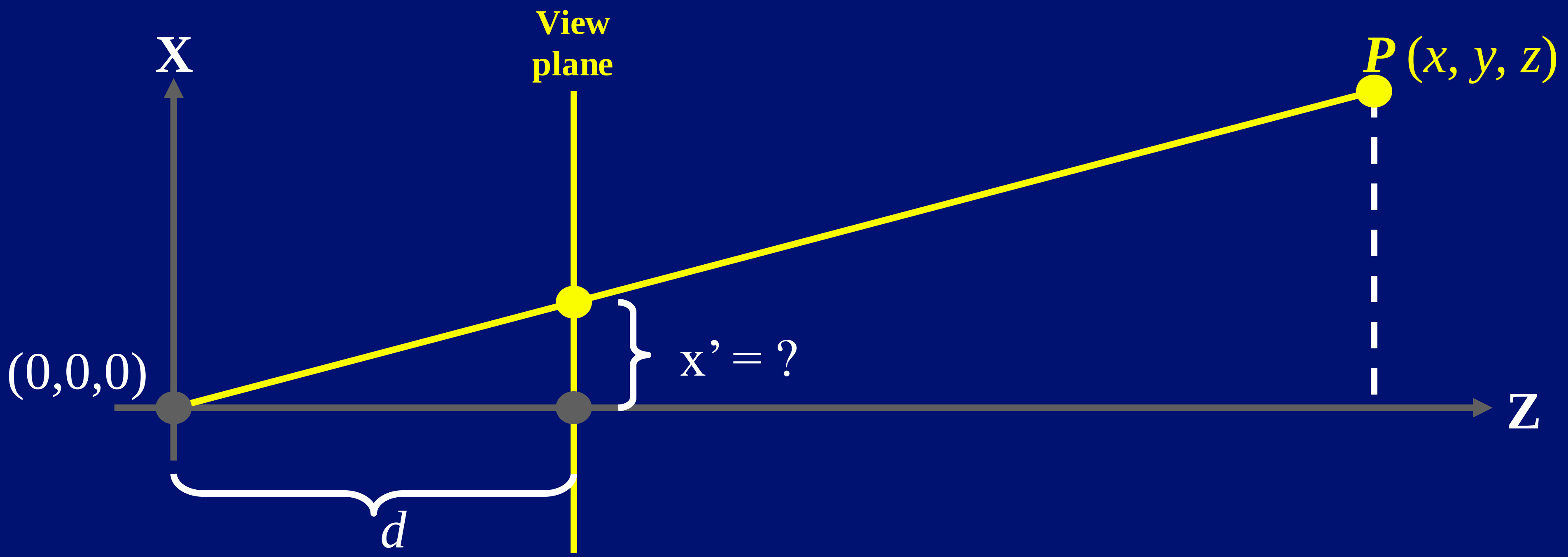
Perspective Projection

When we do 3-D graphics, we think of the screen as a 2-D window onto the 3-D world:



Perspective Projection

The geometry of the situation is that of **similar triangles.** ***View from above:***



What is x' ?

Perspective Projection

Desired result for a point $[x, y, z, 1]^T$ projected onto the view plane:

$$\frac{x'}{d} = \frac{x}{z}, \quad \frac{y'}{d} = \frac{y}{z}$$

$$x' = \frac{d \cdot x}{z} = \frac{x}{z/d}, \quad y' = \frac{d \cdot y}{z} = \frac{y}{z/d}, \quad z = d$$

What could a matrix look like to do this?

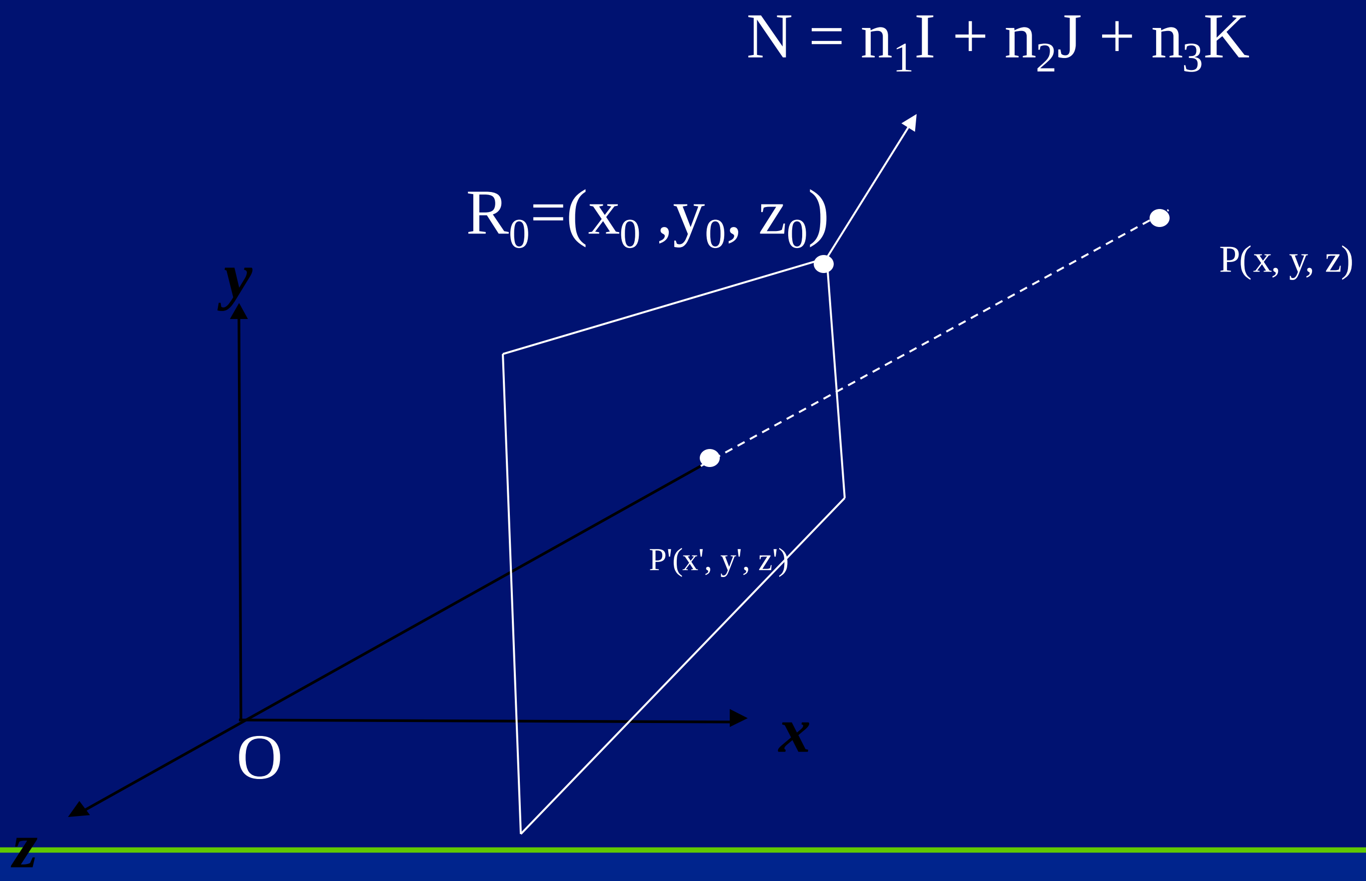
A Perspective Projection Matrix

Answer:

$$M_{perspective} = \begin{bmatrix} d & 0 & 0 & 0 \\ 0 & d & 0 & 0 \\ 0 & 0 & d & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Generalized Projection

- ***Using the origin as the center of projection, derive the perspective transformation onto the plane passing through the point $R_0(x_0, y_0, z_0)$ and having the normal vector $N = n_1I + n_2J + n_3K$.***



Generalized Projection

$$P'O = \alpha\, PO$$

$$x' = \alpha x,\ y' = \alpha y,\ z' = \alpha z$$

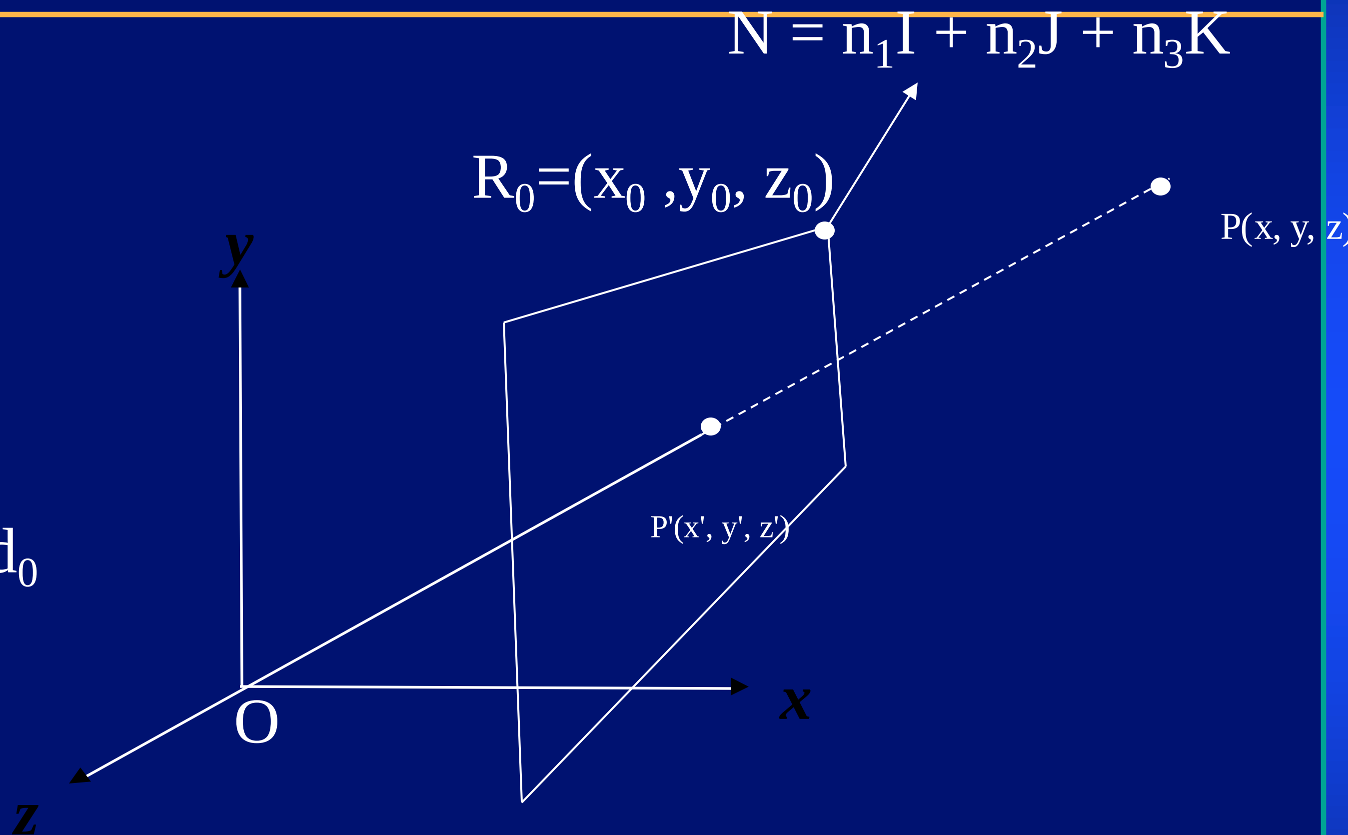
$$N.\ R_0P' = 0$$

$$n_1x' + n_2y' + n_3z'$$

$$= n_1x_0 + n_2y_0 + n_3z_0 = d_0$$

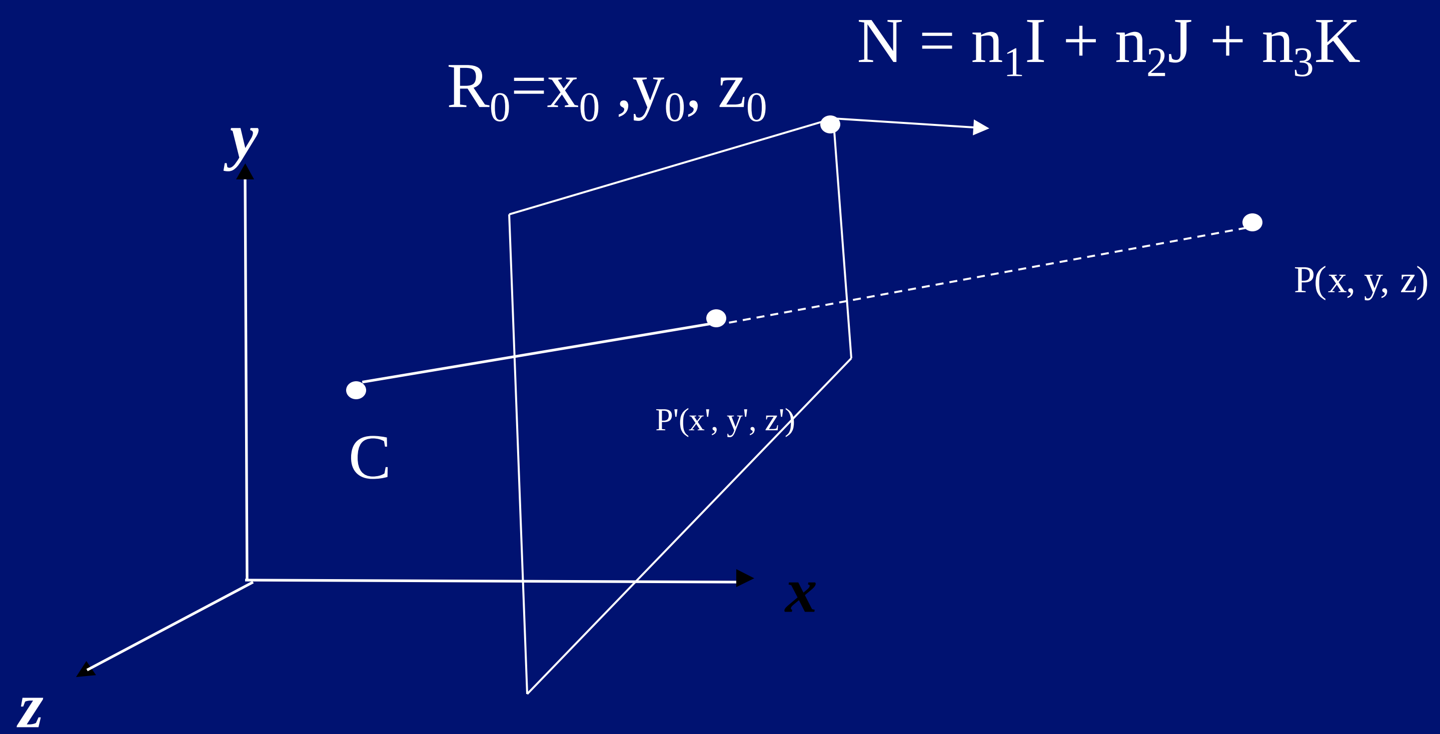
$$\alpha = \frac{d_0}{n_1x + n_2y + n_3z}$$

$$Per = \begin{pmatrix} d_0 & 0 & 0 & 0 \\ 0 & d_0 & 0 & 0 \\ 0 & 0 & d_0 & 0 \\ n_1 & n_2 & n_3 & 0 \end{pmatrix}$$



Generalized Projection

- ***Derive the general perspective transformation onto a plane with reference point R_0 and normal vector N and using C(a,b,c) as the center of projection.***



Generalized Projection

$$P'C = \alpha\, PC$$

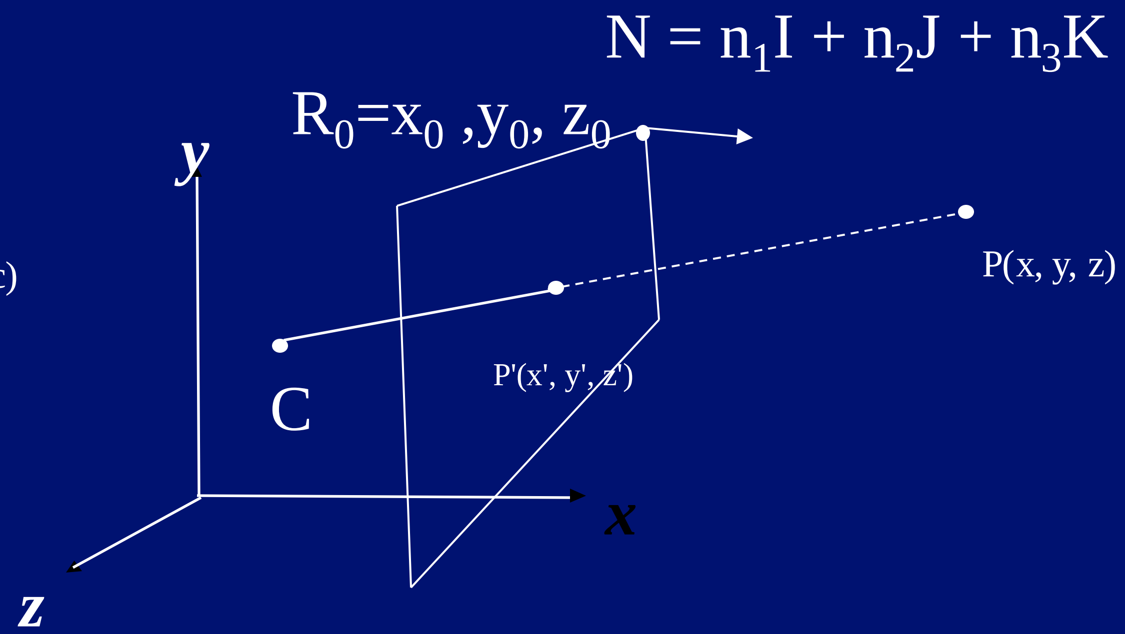
$$x' = \alpha(x-a) + a$$

$$n_1x' + n_2y' + n_3z' = d_0$$

$$\alpha = \frac{d}{n_1(x-a)+n_2(y-b)+n_3(z-c)}$$

$$d = (n_1x_0 + n_2y_0 + n_3z_0) - (n_1a + n_2b + n_3c)$$

$$= d_0 - d_1$$



Generalized Projection

- *Follow the steps –*
  - Translate so that C lies at the origin
  - Perform projection as the previous problem
  - Translate back

$$\begin{pmatrix} d + an_1 & an_2 & an_3 & -ad_0 \\ bn_1 & d + bn_2 & bn_3 & -bd_0 \\ cn_1 & cn_2 & d + cn_3 & -cd_0 \\ n_1 & n_2 & n_3 & -d_1 \end{pmatrix}$$

Viewing in OpenGL

OpenGL has multiple matrix stacks - *transformation functions* right-multiply *the top of the stack*

Two most important stacks: `GL_MODELVIEW` and `GL_PROJECTION`

Points get multiplied by the modelview matrix first, and then the projection matrix

`GL_MODELVIEW`*: Object->Camera*

`GL_PROJECTION`*: Camera->Screen*

glViewport(0,0,w,h): Screen->Device

OpenGL Example

```
void SetUpViewing()
{
        // The viewport isn’t a matrix, it’s just state...
        glViewport( 0, 0, window_width, window_height );

        // Set up camera->screen transformation first
        glMatrixMode( GL_PROJECTION );
        glLoadIdentity();
        gluPerspective( 60, 1, 1, 1000 ); // fov, aspect, near, far


        // Set up the model->camera transformation
        glMatrixMode( GL_MODELVIEW );
        gluLookAt( 3, 3, 2,    // eye point
                   0, 0, 0,    // look at point
                   0, 0, 1 ); // up vector
        glRotatef( theta, 0, 0, 1 ); // rotate the model
        glScalef( zoom, zoom, zoom ); // scale the model
}
```

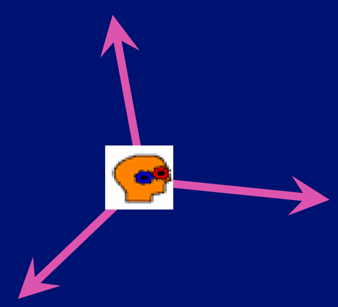
Graphics Pipeline Revisited

3D Geometric Primitives

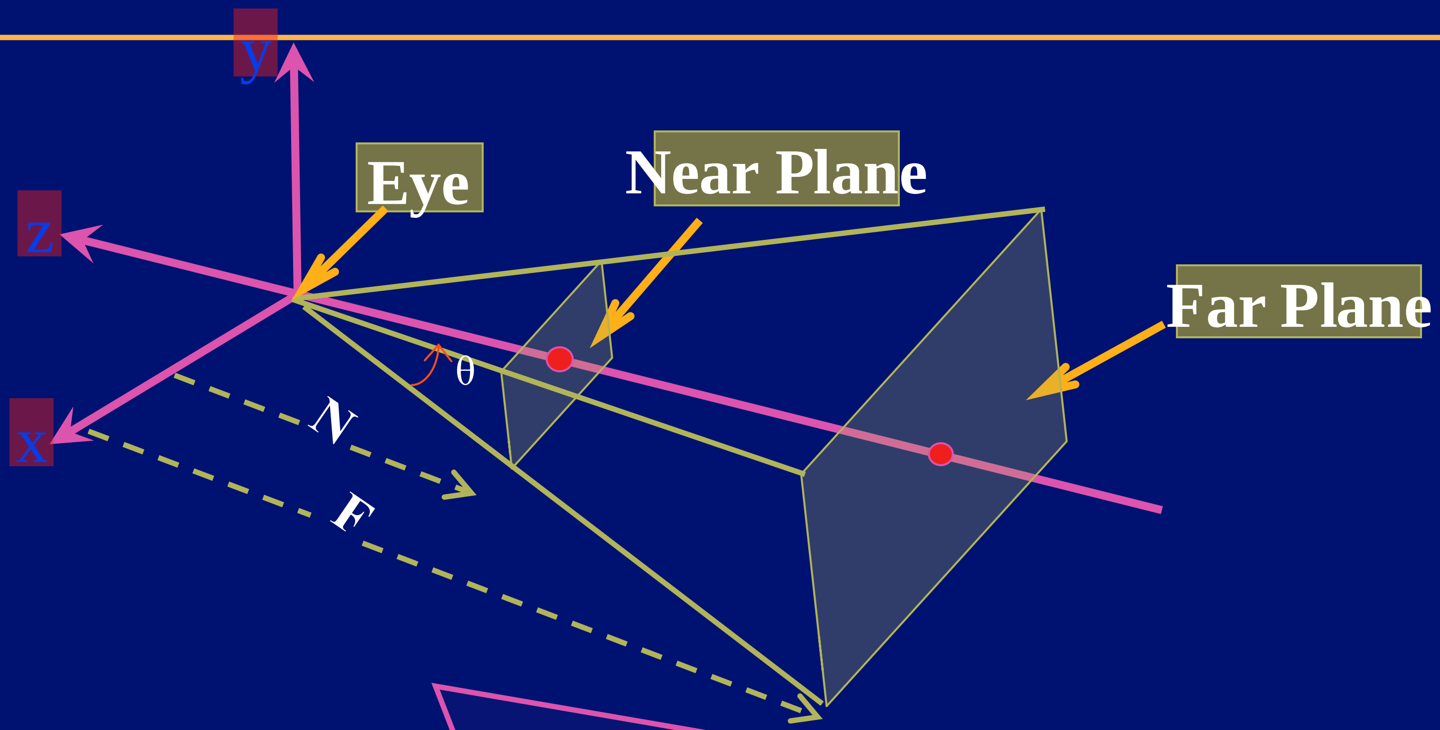
Modeling Transformation → Viewing Transformation → Projection Transformation → Clipping → Lighting → Scan Conversion

Image

```
glMatrixMode( GL_MODELVIEW );
glLoadIdentity();
gluLookAt ( eyex, eyey, eyez,
            lookx, looky, lookz,
            upx, upy, upz );
```

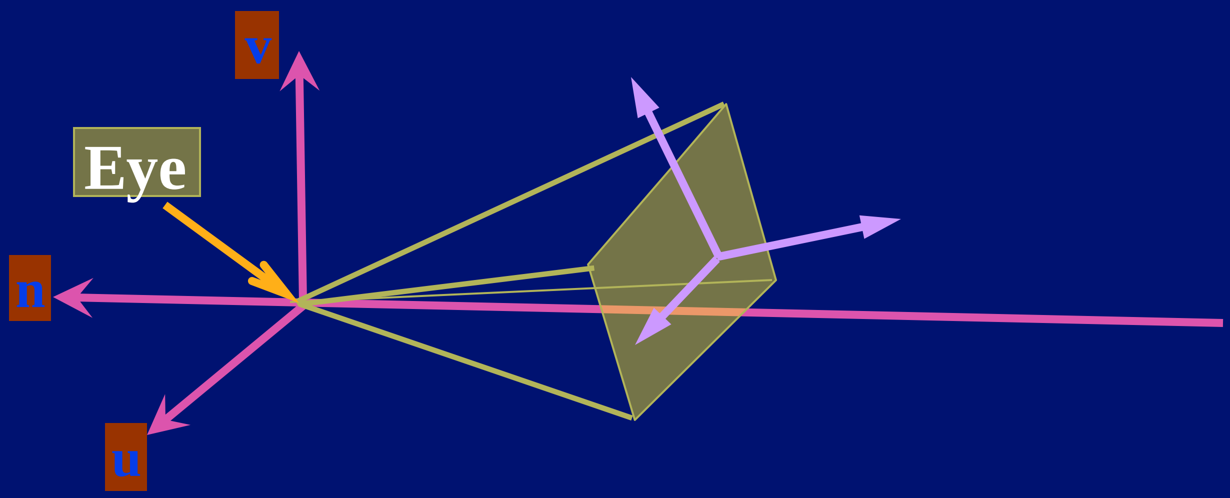


Default Camera Position

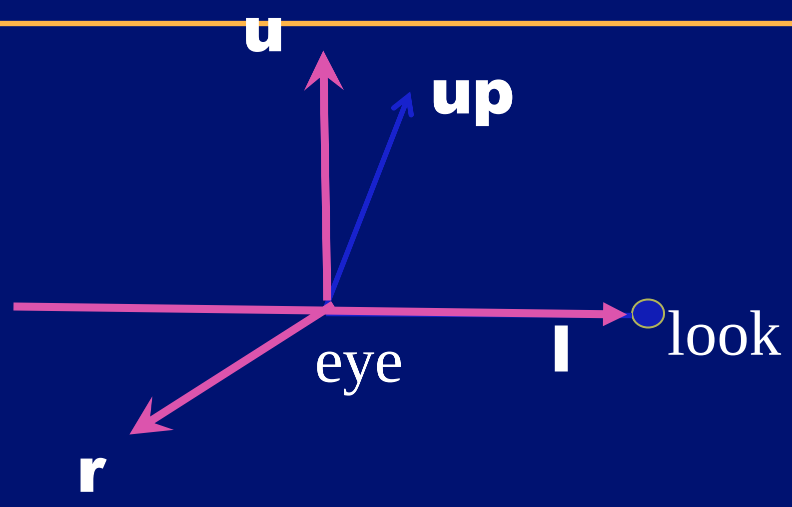


glMatrixMode(GL_PROJECTION);
glLoadIdentity();
gluPerspective(viewAngle,
aspectRatio, N, F);

General Camera Position



General Camera Position



$\mathbf{l}$ = look - eye

$\mathbf{r} = \mathbf{l} \times \mathbf{up}$

$\mathbf{u} = \mathbf{r} \times \mathbf{l}$

- ***To transform world coordinate into camera's coordinate, transform***
  - *eye* into the origin
  - **r** into the vector **i**
  - **u** into the vector **j**
  - **-l** into the vector **k**

So what is the matrix for viewing transform?

A 3D Scene

Notice the presence of the camera, the projection plane, and the world coordinate axes

Viewing transformations define how to acquire the image on the projection plane

Viewing Transformations

Create a camera-centered view

Camera is at origin

Camera is looking along negative z-axis

Camera's 'up' is aligned with y-axis

2 Basic Steps

Align the two coordinate frames by rotation

2 Basic Steps

Translate to align origins

Creating Camera Coordinate Space

Specify a point where the camera is located in world space, the eye point

Specify a point in world space that we wish to become the center of view, the lookat point

Specify a vector in world space that we wish to point up in camera image, the up vector

Intuitive camera movement

Constructing Viewing Transformation, V

Create a vector from eye-point to lookat-point

$$\begin{bmatrix} l_x \\ l_y \\ l_z \end{bmatrix} = \begin{bmatrix} lookat_x \\ lookat_y \\ lookat_z \end{bmatrix} - \begin{bmatrix} eye_x \\ eye_y \\ eye_z \end{bmatrix}$$

Normalize the vector

$$\hat{l} = \frac{\bar{l}}{\sqrt{l_x^2 + l_y^2 + l_z^2}}$$

Desired rotation matrix should map this vector to $[0, 0, -1]^T$ Why?

$$\begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} = \mathbf{V}\hat{l}$$

Constructing Viewing Transformation, V

Construct another important vector from the cross product of the lookat-vector and the vup-vector

$$\bar{r} = \bar{l} \times \overline{up}$$

This vector, when normalized, should align with $[1, 0, 0]^T$ Why?

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \mathbf{V} \frac{\bar{r}}{\sqrt{r_x^2 + r_y^2 + r_z^2}}$$

Constructing Viewing Transformation, V

One more vector to define...

$$\overline{u} = \overline{r} \times \overline{l}$$

This vector, when normalized, should align with [0, 1, 0]^T

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \mathbf{V} \frac{\overline{u}}{\sqrt{u_x^2 + u_y^2 + u_z^2}}$$

Now let's compose the results

Compositing Vectors to Form V

We know the three world axis vectors (x, y, z)

We know the three camera axis vectors (r, u, l)

Viewing transformation, V, must convert from world to camera coordinate systems

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \mathbf{V}\begin{bmatrix} \hat{r} & \hat{u} & -\hat{l} \end{bmatrix}$$

Compositing Vectors to Form V

Remember

- Each camera axis vector is unit length.
- Each camera axis vector is perpendicular to others

Camera matrix is orthogonal and normalized

- Orthonormal

Therefore, $M^{-1} = M^T$

Compositing Vectors to Form V

Therefore, rotation component of viewing transformation is just transpose of computed vectors

$$\mathbf{V}_{rotate} = \begin{bmatrix} \hat{r} \\ \hat{u} \\ -\hat{l} \end{bmatrix}$$

Compositing Vectors to Form V

Translation component too

$$\begin{bmatrix} \hat{r} \\ \hat{u} \\ -\hat{l} \end{bmatrix} \begin{bmatrix} x - eye_x \\ y - eye_y \\ z - eye_z \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$$

Multiply it through

$$\begin{bmatrix} \hat{r} \\ \hat{u} \\ -\hat{l} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} - \begin{bmatrix} \hat{r} \\ \hat{u} \\ -\hat{l} \end{bmatrix} \begin{bmatrix} eye_x \\ eye_y \\ eye_z \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$$

Final Viewing Transformation, V

To transform vertices, use this matrix:

$$\begin{bmatrix} & \hat{r} & & -\hat{r}\cdot\overline{eye} \\ & \hat{u} & & -\hat{u}\cdot\overline{eye} \\ & -\hat{l} & & \hat{l}\cdot\overline{eye} \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$$

And you get this:

Ref.

- *Chapter 7, Schaum's Outline of Computer Graphics (2nd Edition) by Zhigang Xiang, Roy A. Plastock*