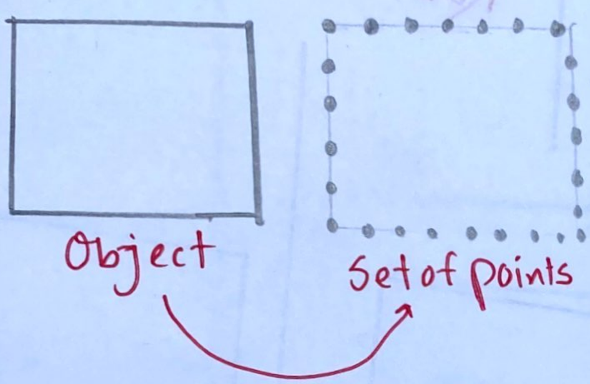
Transformation

Transformations are operations applied to geometrical description of an object to change its position, Orientation or size.

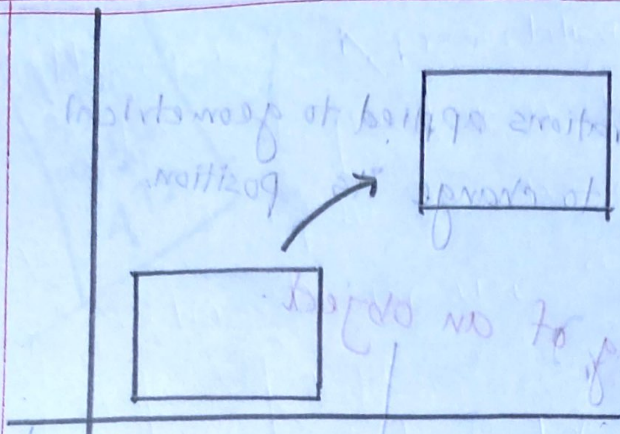
Reposition and resizing of an object.



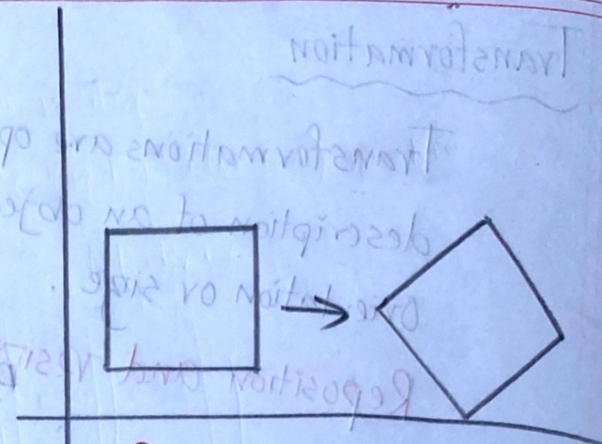
Object is nothing but set of points.

5 type of Transformation

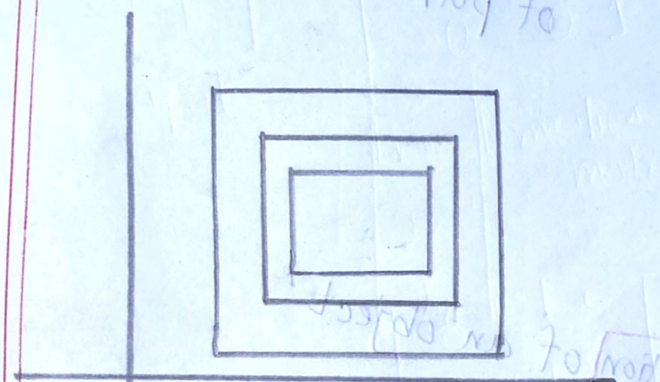
1. Translation : Changing Position of an object
2. Rotation : Changing angle of an object
3. Scaling : Shrink or resize it into bigger or smaller size by keeping the main object unchanged.
4. Reflection : We make reflection by considering any axis or a line a mirror.
5. Sharing : We put force to change shape from any side.



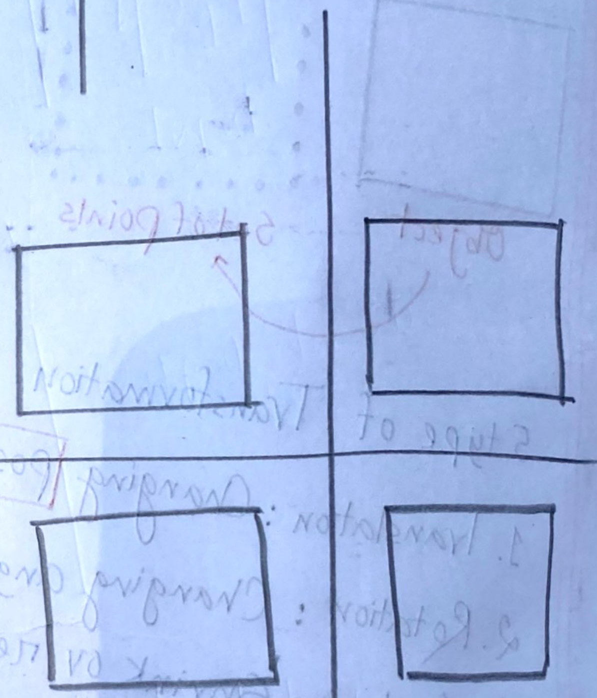
Translation.



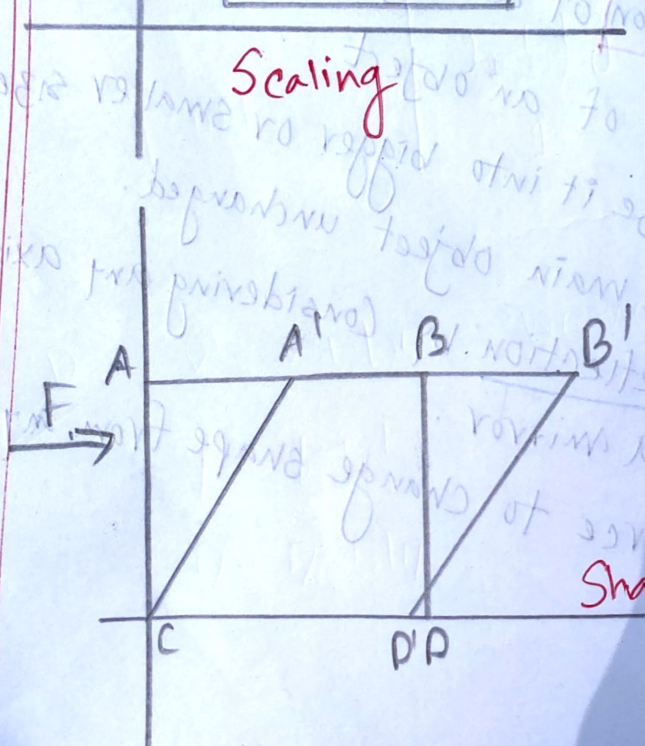
Rotation



Scaling



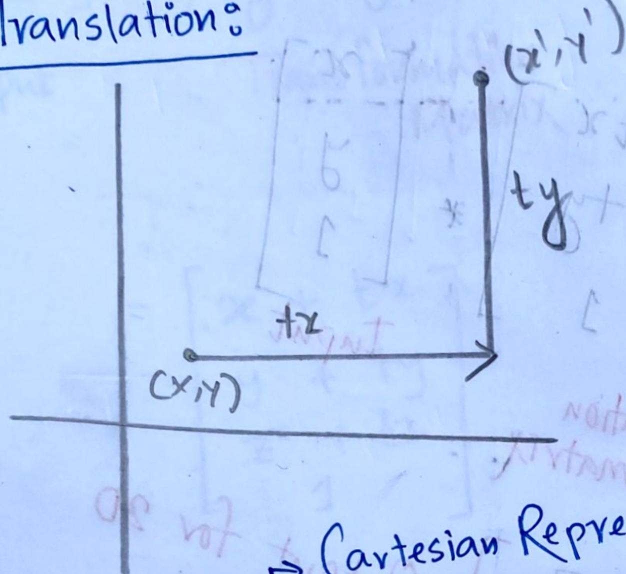
Reflection



Sharing

$$\begin{bmatrix} \text{Output} \\ \text{Matrix} \end{bmatrix} = \begin{bmatrix} \text{Transformation} \\ \text{Matrix} \end{bmatrix} \times \begin{bmatrix} \text{Input} \\ \text{Matrix} \end{bmatrix}$$

Translation:



$$x' = x + t_x$$
$$y' = y + t_y$$

Cartesian Representation

$$\underbrace{\begin{bmatrix} x' \\ y' \end{bmatrix}}_{\text{Output}} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{\text{Transformation Matrix}} * \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{\text{Input}} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

$$= \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix} = \begin{bmatrix} x + t_x \\ y + t_y \end{bmatrix}$$

Output

Homogeneous Representation:

For n-dimension
we use $(n+1)*(n+1)$ Matrix.

2D

So, for 2D - we use $3*3$ Matrix

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Output — Transformation matrix — Input

$$= \begin{bmatrix} x + tx \\ y + ty \\ 1 \end{bmatrix}$$

Output for 2D

3D

$x' = x + tx$ — we need $(4*4)$ Matrix

$y' = y + ty$

$z' = z + tz$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & tx \\ 0 & 1 & 0 & ty \\ 0 & 0 & 1 & tz \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Output Transformation Matrix Input

$$= \begin{bmatrix} x + tx \\ y + ty \\ z + tz \\ 1 \end{bmatrix}$$

Note:

Transformation Matrix for Translation:

$$\begin{bmatrix} 1 & 0 & 0 & \cdots & tx \\ 0 & 1 & 0 & \cdots & ty \\ 0 & 0 & 1 & \cdots & tz \\ \vdots & \vdots & \vdots & \ddots & \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}$$

Example

Translate point (3,2,5), 6 step in x axis. (-5) step in y-axis, 7 step in z-axis.

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 3 \\ 2 \\ 5 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 \\ -3 \\ 12 \\ 1 \end{bmatrix}$$

$\therefore$ point $(9, -3, 12)$

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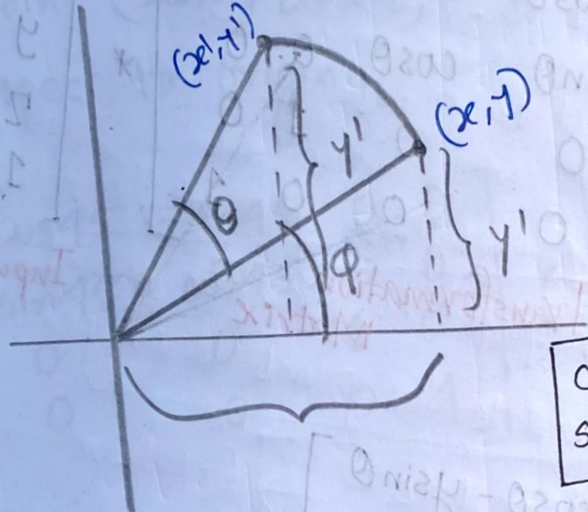
Rotation:

z based rotation $\rightarrow z' = z$



We rotate it θ amount anti-clockwise.

$\cos(A+B) = \cos A \cos B - \sin A \sin B$
$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$$\cos\phi = \frac{x}{r} \Rightarrow x = r\cos\phi$$

$$\sin\phi = \frac{y}{r} \Rightarrow y = r\sin\phi$$

$$\cos(\theta+\phi) = \frac{x'}{r} \Rightarrow x' = r\cos(\theta+\phi) = r\cos\theta\cos\phi - r\sin\theta\sin\phi$$
$$= x\cos\theta - y\sin\theta$$

$$\sin(\theta+\phi) = \frac{y'}{r} \Rightarrow y' = r\sin(\theta+\phi) = r\sin\theta\cos\phi + r\cos\theta\sin\phi$$
$$= x\sin\theta + y\sin\theta$$

$x' = x\cos\theta - y\sin\theta$
$y' = x\sin\theta + y\sin\theta$
$z' = z$

as it is being rotated z based so z is unchanged.

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Output　　Transformation Matrix　　Input

$$= \begin{bmatrix} x\cos\theta - y\sin\theta \\ x\sin\theta + y\cos\theta \\ z \\ 1 \end{bmatrix}$$

Example

Rotate point (9,2,5) at angle 45° (anti-clockwise) with respect to origin.

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos 45 & -\sin 45 & 0 & 0 \\ \sin 45 & \cos 45 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 9 \\ 2 \\ 5 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4.95 \\ 7.78 \\ 5 \\ 1 \end{bmatrix}$$

Answer: (4.95, 7.78, 5)

P.T.O

Example

Rotate Point (6,2) at an angle 90° anti-clockwise with respect to point (2,2)?

Note: When we are given a point to ~~Rat~~ rotate with respect to another point, we need to translate the point to origin.

Sequence: Translate to origin → Rotation → Translate to origin

$$\boxed{\text{Output} = T(2,2) * R(90) * T(-2,-2) * \text{Input}}$$

$$= \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} \cos 90 & -\sin 90 & 0 \\ \sin 90 & \cos 90 & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 6 \\ 2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 6 \\ 1 \end{bmatrix}$$

∴ Rotation Point (2,6)

Note:

- Output for Rotation with respect to another point:

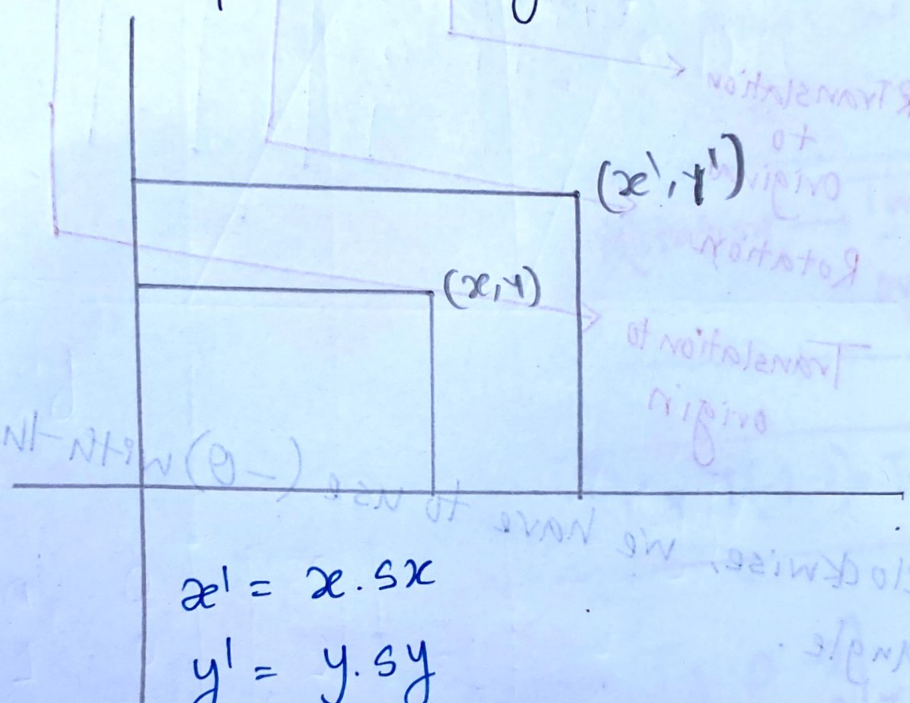
$$\text{Output} = \underbrace{T(x,y)}_{\text{Translation to Original}} * \underbrace{R(\theta)}_{\text{Rotation}} * \underbrace{T(x,y)}_{\text{Translation to origin}} * \text{Input}$$

- For clockwise, we have to use $(-\theta)$ with the given angle.

Scaling:

Cases:

Scale sx units in x with respect to origin.
Scale sy units in y with respect to origin.



$x' = x . sx$

$y' = y . sy$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} sx & 0 & 0 \\ 0 & sy & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x . sx \\ y . sy \\ 1 \end{bmatrix}$$

Scaling Matrix

Example

Scale point (3,5) $\left\{\begin{array}{l}5 \text{ units in } x\text{-axis} \\ 8 \text{ units in } y\text{-axis}\end{array}\right\}$ with respect to origin

Soln:

$$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} 15 \\ 40 \\ 1 \end{bmatrix}$$

Ans: (15, 40)

Example

Scale point (3,5) $\left\{\begin{array}{l}5 \text{ units} \longrightarrow x \\ 8 \text{ units} \longrightarrow y\end{array}\right\}$ with respect to (8,6)

Soln: Output = T(8,6) * Scale(5,8) * T(-8,-6) * Input

$$= \begin{bmatrix} 1 & 0 & 8 \\ 0 & 1 & 6 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 5 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & -8 \\ 0 & 1 & -6 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 3 \\ 5 \\ 1 \end{bmatrix}$$

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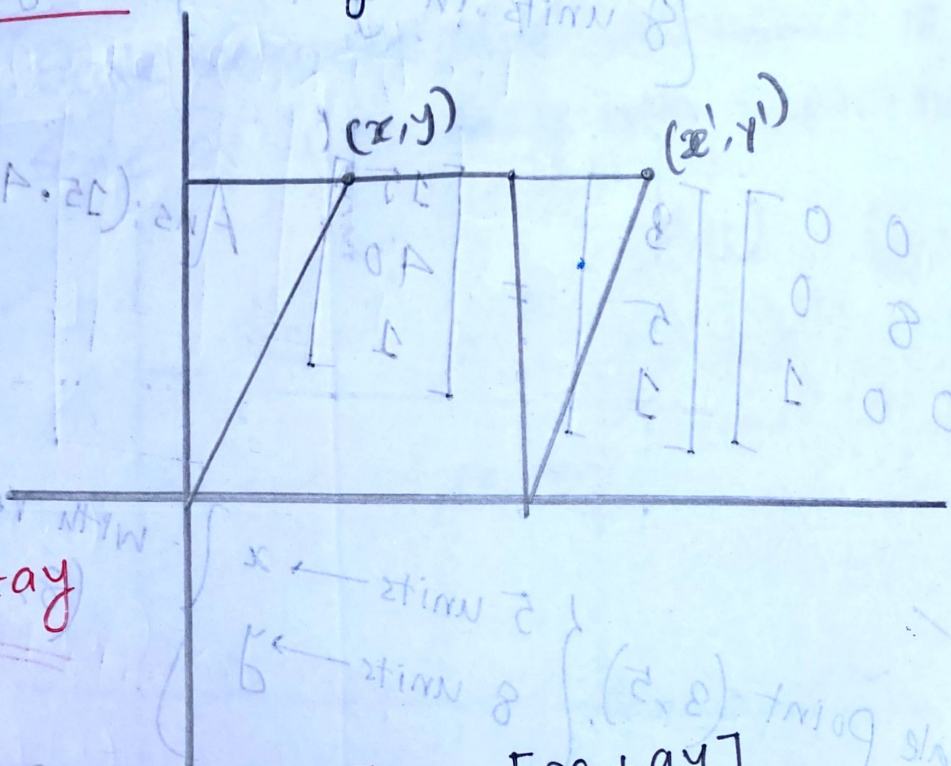
☐ Shearing:-

Case 01: Shearing with x-axis (a amount)



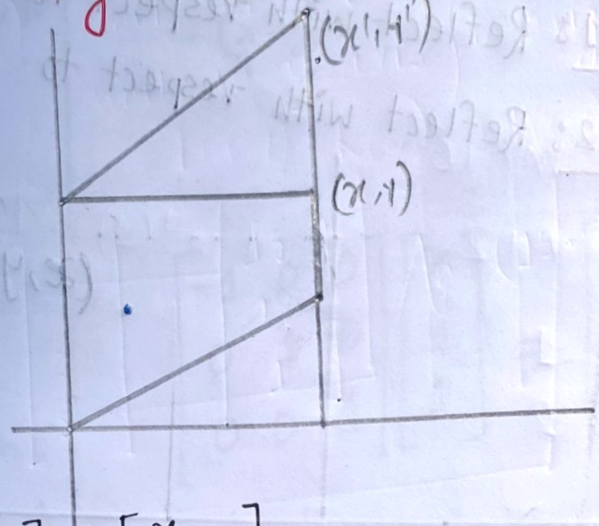
$x' = x + ay$

$y' = y$

$$\begin{bmatrix} 1 & a & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x + ay \\ y \\ 1 \end{bmatrix}$$

Case 02:

Shearing with y-axis. (b amount)



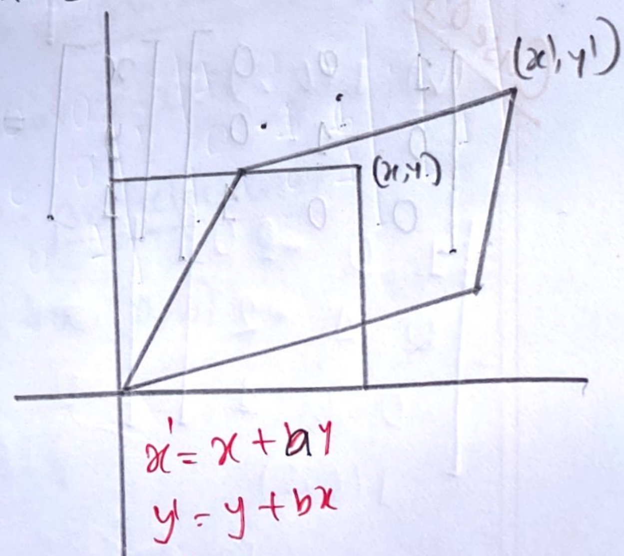
$x' = x$

$y' = y + bx$

$$\begin{bmatrix} 1 & 0 & 0 \\ b & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y+bx \\ 1 \end{bmatrix}$$

Case 03:

Shearing with both x and y axis.



$$\begin{bmatrix} 1 & a & 0 \\ b & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x+ay \\ y+bx \\ 1 \end{bmatrix}$$

$x' = x + ay$

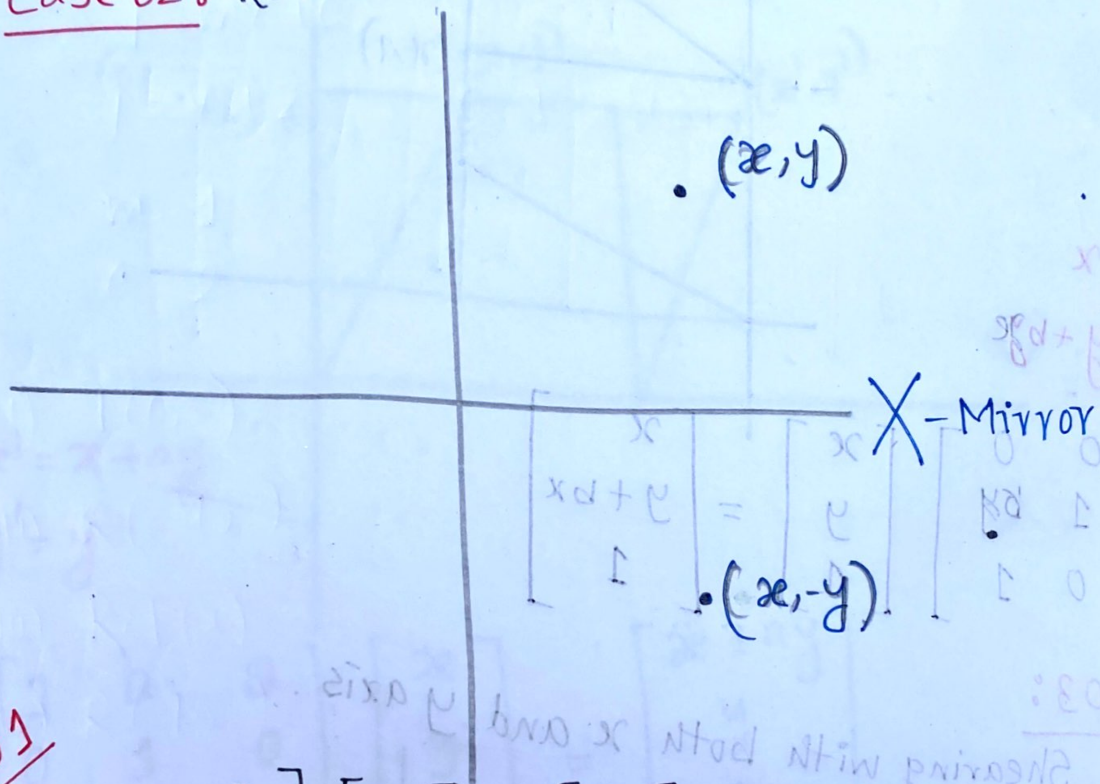
$y' = y + bx$

$z' = z$

Reflection:

Case 01: Reflect with respect to x axis.

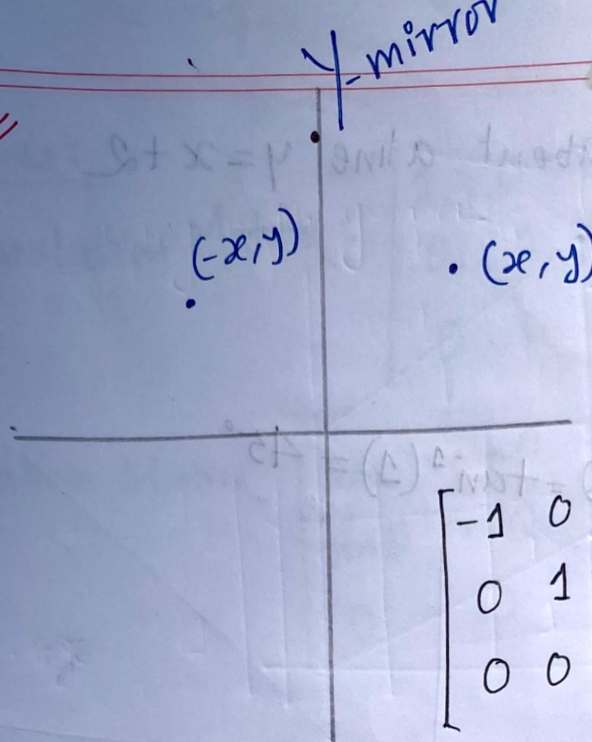
Case 02: Reflect with respect to y-axis.



Case 01

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ -y \\ 1 \end{bmatrix}$$

Case:02



$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ -y \\ 1 \end{bmatrix}$$

Case 03: Reflection with respect to a line.

Step 01: Translate (a,b) 0,b) to origin.

Step 02: Rotate- θ- degrees clockwise

Step 03: Mirror reflect about x-axis.

Step 04: Rotate θ- degree. anti-clockwise

Step 05: Translate origin to (0,b)

* Reflect (10,5) about a line $y = x + 2$

Soln:

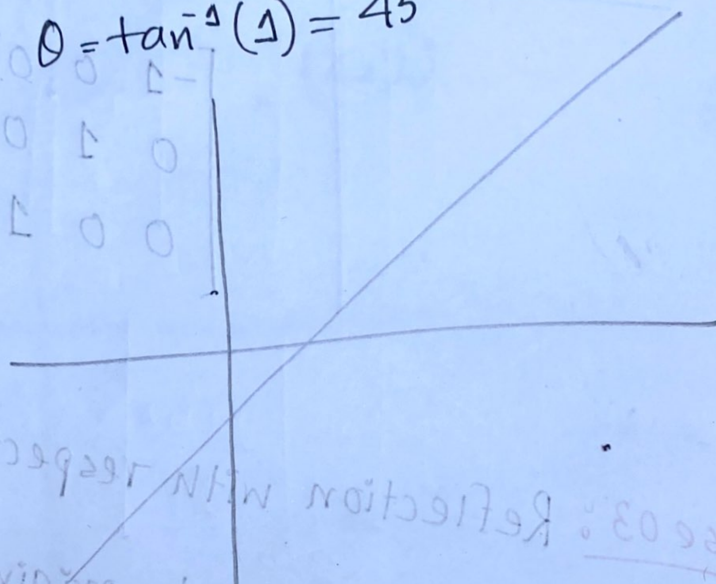
$y = mx + c$

$y = x + 2$

$m = 1$

$c = 2$

$\theta = \tan^{-1}(1) = 45^\circ$



(i) $T(0,-2)$

(ii) $R(-45^\circ)$

(iii) $Ref(x)$

(iv) $R(45^\circ)$

(v) $T(0,2)$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}}_{T(0,2)} * \underbrace{\begin{bmatrix} \cos 45 & -\sin 45 & 0 \\ \sin 45 & \cos 45 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{R(\theta)} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{Ref(x)} \underbrace{\begin{bmatrix} \cos(-45) & -\sin(-45) & 0 \\ \sin(-45) & \cos(-45) & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{Ro(-45^\circ)} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}}_{T(0,-2)}$$

$$= \begin{bmatrix} 3 \\ 12 \\ 1 \end{bmatrix} \Rightarrow (3, 12)$$

Formulas:

i) Translation Matrix:

$$\begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix}$$

ii) Rotation Matrix

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

iii) Scaling

$$\begin{bmatrix} sx & 0 & 0 \\ 0 & sy & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

iv) Shearing

$$\begin{bmatrix} 1 & a & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
x-axis

$$\begin{bmatrix} 1 & 0 & 0 \\ b & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
y-axis

$$\begin{bmatrix} 1 & a & 0 \\ b & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
both axis

vi) Reflection:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
x-axis

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
y-axis

$T(0, b)$
$R_o(\theta)$
Ref
$R_o(\theta)$
$T(0, -b)$

Color Models in Computer Graphics

Color is the aspect of things that is caused by differentiating qualities of light being reflected or emitted by them.

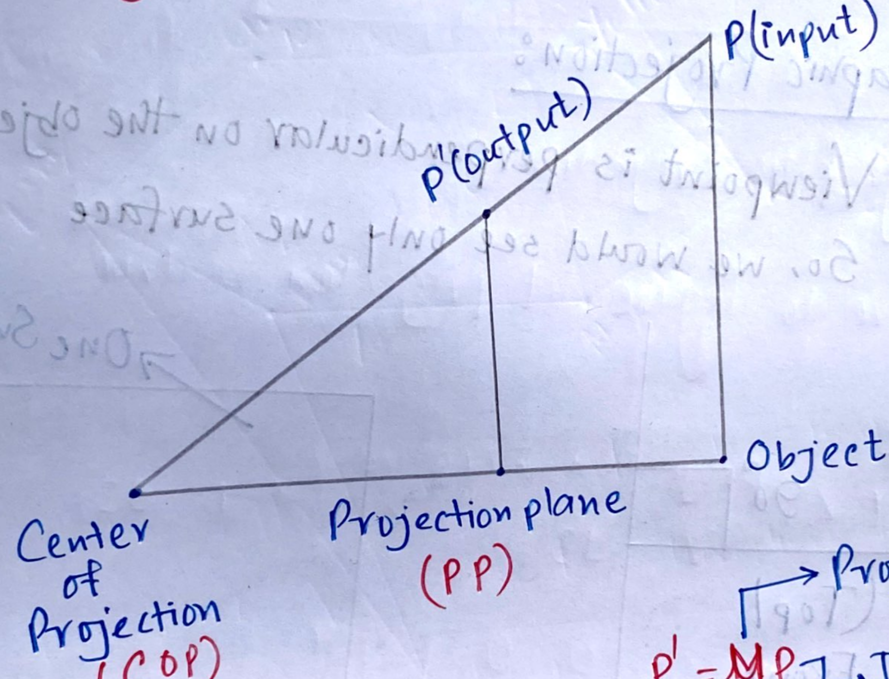
Ques: Write the differences between Additive and Substractive color model.

Additive Color Model	Substractive Color Model.
RGB, HSV, HSL	CMY, CMYK
Generates color by adding multiple colors	Generates color by substracting different colors.
Active Display → TV, Mobile, PC	Device that deposit color → printer. copiers.
No data, → Black	No data, White
Increase Brightness	Decrease Brightness.

Projection

Projection is a process where a 3D model converted to a 2D model for viewing on a plane.

3D model → $\boxed{\text{projection}}$ → 2D model output.



$$P' = M P$$

P' → Output Matrix
M → Projection
P → Input

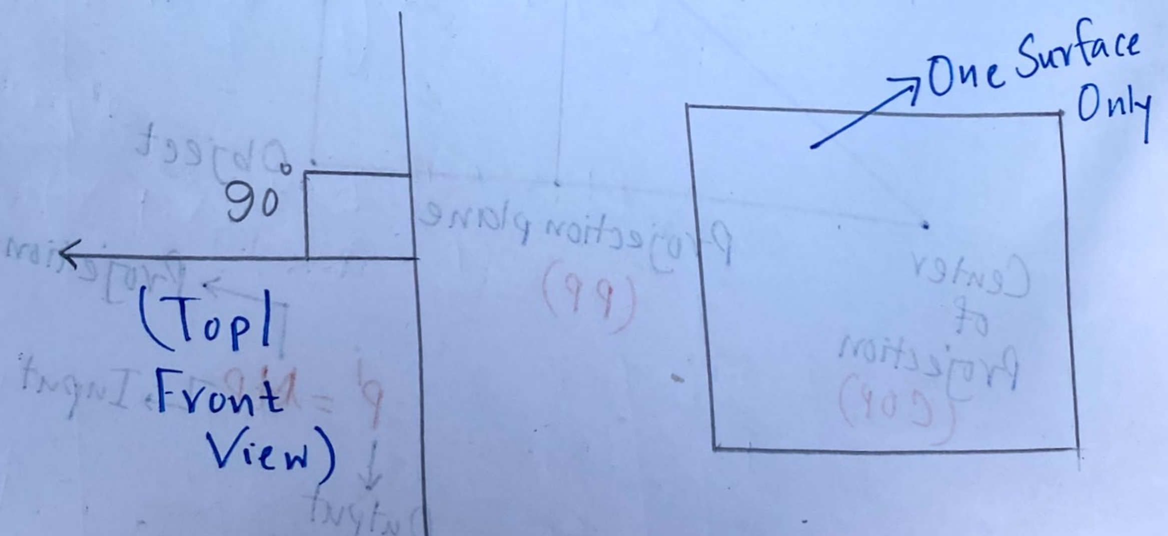
Projection

- Orthographic Projection
- Oblique Projection

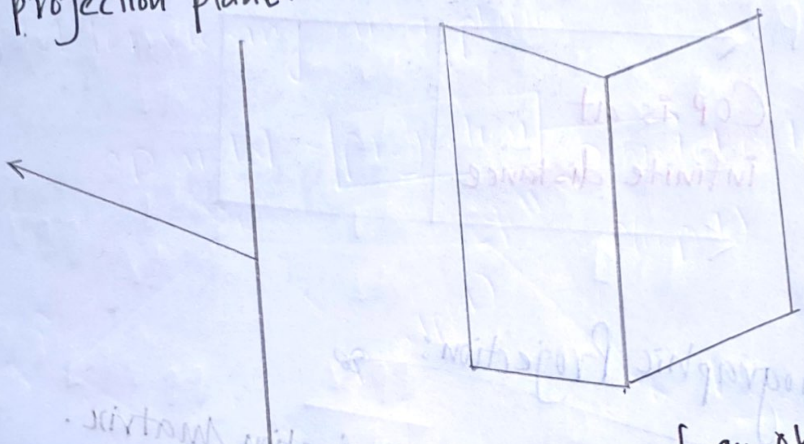
1) Orthographic Projection:

Viewpoint is perpendicular on the object.

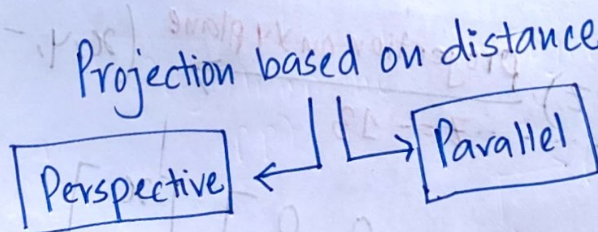
So, we would see only one surface



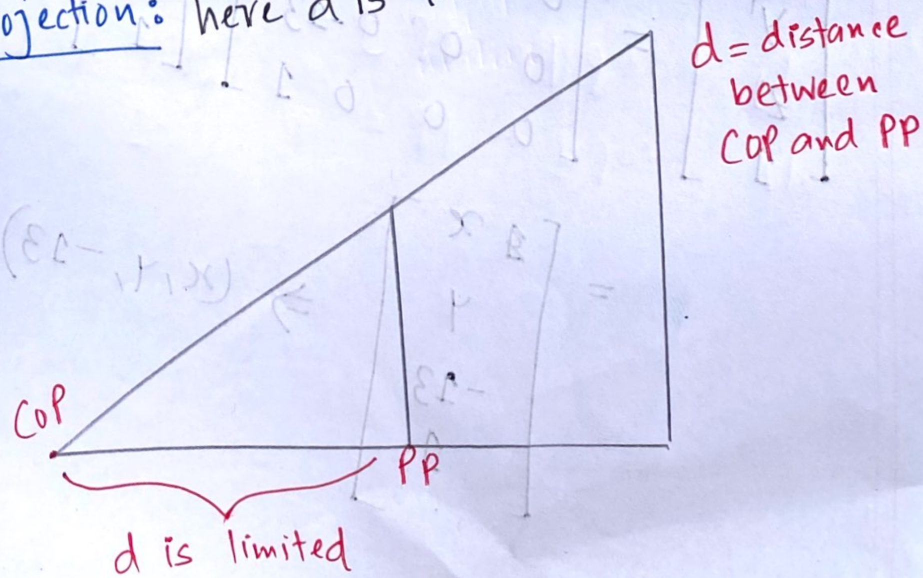
2) **Oblique Projection:** An oblique projection is a parallel projection in which the lines of sight are not perpendicular to the projection plane.



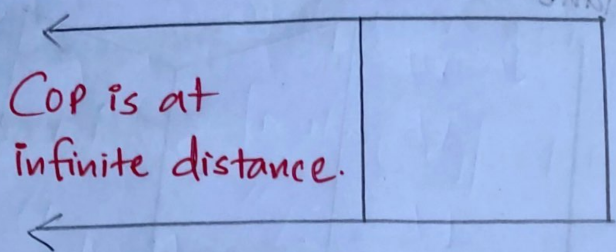
So ~~where~~ here we can see the Corner view of an object.



i) **Perspective Projection:** here d is limited.



ii) Parallel Projection: d is infinite. Example: Telescope



Orthographic Projection:

we have to figure out projection matrix.

Example

$$(x, y, z) \xrightarrow[z = -13]{\text{projection on xy plane}} (x, y, -13)$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -13 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} x \\ y \\ -13 \\ 1 \end{bmatrix} \Rightarrow (x, y, -13)$$

Formula:

$(x, y, z) \xrightarrow[z = \alpha]{\text{projection on xy plane}}$
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \alpha \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$(x, y, z) \xrightarrow[x = \alpha]{\text{projection on yz plane}}$
$$\begin{bmatrix} 0 & 0 & 0 & \alpha \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$(x, y, z) \xrightarrow[y = \alpha]{\text{projection on zx plane}}$
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

General Projection Matrix

for xy plane

$p' = Mp$

$$M = \begin{bmatrix} 1 & 0 & -\frac{dx}{dz} & \left(\frac{dx}{dz}\right) pp.z \\ 0 & 1 & -\frac{dy}{dz} & \left(\frac{dy}{dz}\right) pp.z \\ 0 & 0 & -\frac{pp.z}{dz} & pp.z\left(1 + \frac{pp.z}{dz}\right) \\ 0 & 0 & -\frac{1}{dz} & 1 + \frac{pp.z}{dz} \end{bmatrix}$$

$dx = cop.x - pp.x$

$dy = cop.y - pp.y$

$dz = cop.z - pp.z$

cop.x = center of projection at x axis

pp.x = projection plane at x coordinate.

Given COP (50, 40, 100) and PP (0, 0, −200) Find out the projected point for the given input (30, 50, −250)

$dx = COP_x - PP_x = 50 - 0 = 50$

$dy = COP_y - PP_y = 40 - 0 = 40$

$dz = COP_z - PP_z = 100 - (-200) = 300$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\frac{50}{300} & \frac{50}{300}(-200) \\ 0 & 1 & -\frac{40}{300} & \frac{40}{300}(-200) \\ 0 & 0 & -\frac{(-200)}{300} & -200\left(1 + \frac{-200}{300}\right) \\ 0 & 0 & \frac{-1}{300} & 1 + \frac{-200}{300} \end{bmatrix} \begin{bmatrix} 30 \\ 50 \\ -250 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 115/3 \\ 170/3 \\ -700/3 \\ \boxed{1.167} \end{bmatrix} \xrightarrow{\text{Normalize}} \begin{bmatrix} 32.85 \\ 48.5 \\ -200 \\ 1 \end{bmatrix}$$

As this result is not in 1, So we have to normalize it

Case 01

Find out the matrix where ~~COP~~
COP is at origin and Projection Plane
is at d distance from COP. (xy-plane)

COP $(0, 0, 0)$
PP $(0, 0, d)$

$$\boxed{\begin{aligned} dx &= 0 \\ dy &= 0 \\ dz &= 0 - d = -d \end{aligned}}$$

Projection Matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cancel{-}1 & \cancel{-d}\,0 \\ 0 & 0 & \cancel{-}\frac{1}{d} & 0 \end{bmatrix}$$

$$d\left(1 + \frac{d}{-d}\right) = d \times (1 - 1) = 0$$

$$1 + \frac{-d}{d} = 1 - 1 = 0$$

Case 02 :

Find out the matrix where $\boxed{\text{PP is at origin}}$ and $\boxed{\text{COP is at d- distance}}$ from PP

COP (0, 0, d)

PP (0, 0, 0)

$d_x = 0$

$d_y = 0$

$d_z = d - 0 = d$

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{d} & 1 \end{bmatrix}$$

Lighting / Illumination / Reflection Model

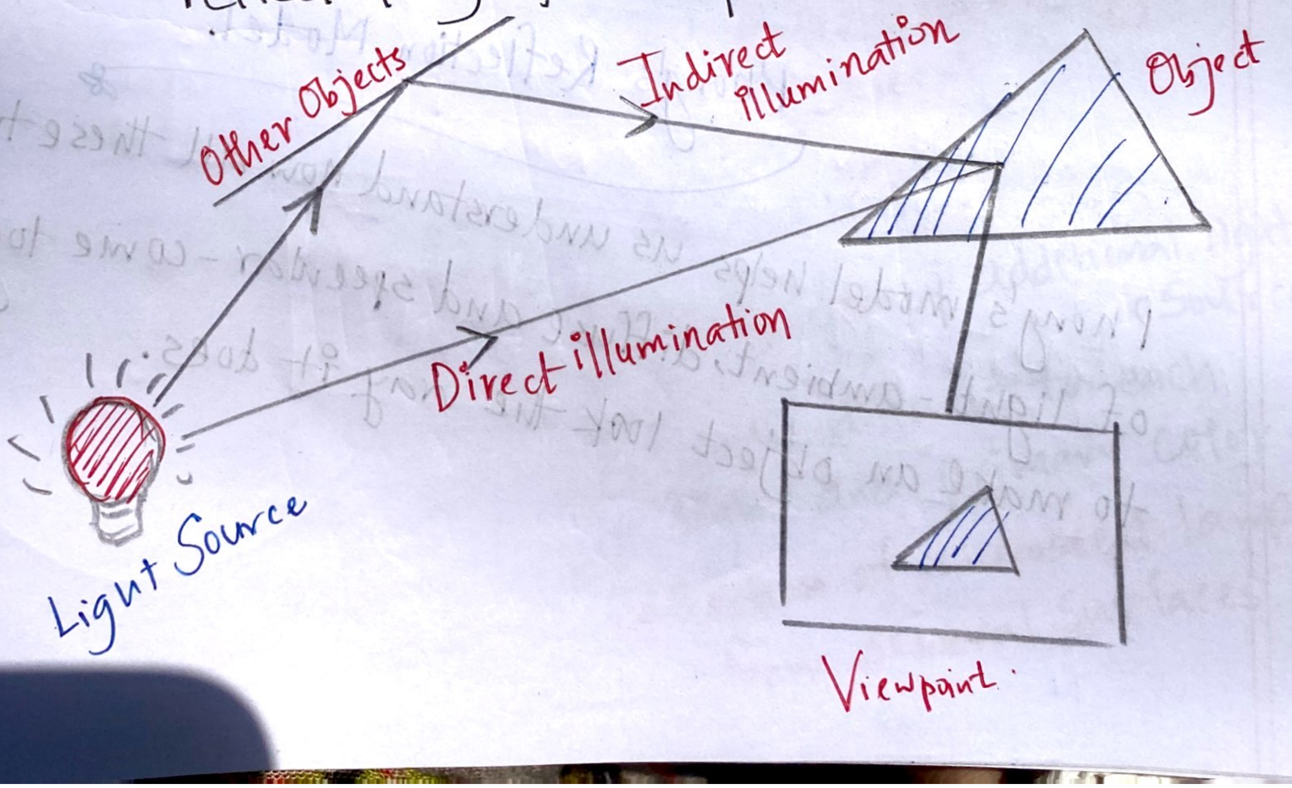
Illumination:

The transport of energy from light Sources to surfaces & points.

- → Local illumination
- → Global illumination.

Lighting model / Illumination model:

Express the factors determining a surface's Color or luminous intensity (outgoing or reflected light) at a particular point (3D point)



Components of Illumination:

1. Light source
2. Surface Properties.

I = luminance/Intensity. Each color is described separately.

$I = I_r I_g I_b$.

Type of Light Source:

1. Ambient Light. [Environment Light]
2. Diffuse Light [Main Light]
3. Spot Light. [Viewpoint - Object shines when light is upon it]

Phong's Reflection Model.

Phong's model helps us understand how all these types of light - ambient, diffuse and specular - come together to make an object look the way it does.

$$I = I_a + I_d + I_s$$

Final Intensity | Ambient | Diffuse | Specular.

Ambient Light

- No identifiable Source or direction
- Product of multiple reflections of light from the many surface present in the environment
- Categories:-
  1) Global Ambient Light
     - Independent of Light Source
     - Lights entire scene.

     Reflection of Light from Several

  2) Local Ambient Light:
     - Contributed by additional light Source
     - Can be different for each light and primary color.

     Reflection of fluroscent lamps from several Surfaces.

Ambient Reflection

** the reflection of light that doesn't Come directly from a light Source.

** As phong is a direct lighting model, we assume that ambient light falls ~~under~~ equally on all objects.

$$\boxed{A = I_a k_a}$$

here, I_a = Intensity of the ambient light

k_a = Ambient co-efficient

* Ambient Reflection Co-efficient:-

$k_a \in [0, 1]$

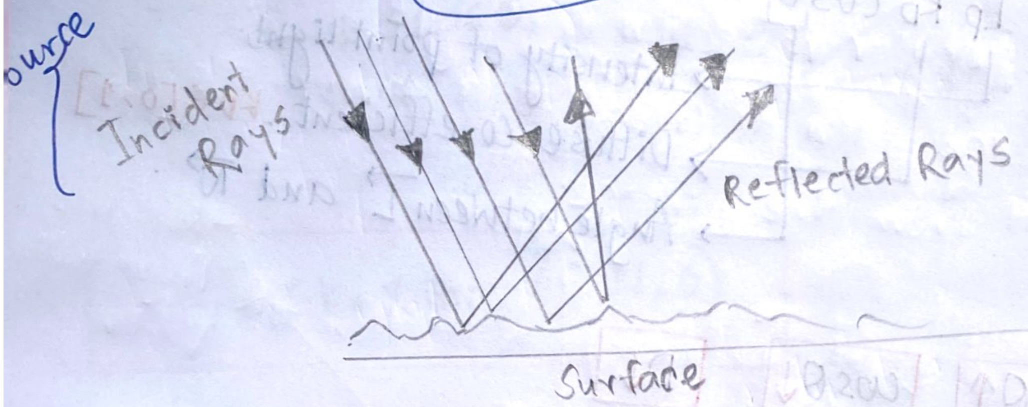
0 → Nighttime scene

1 → Bright scene.

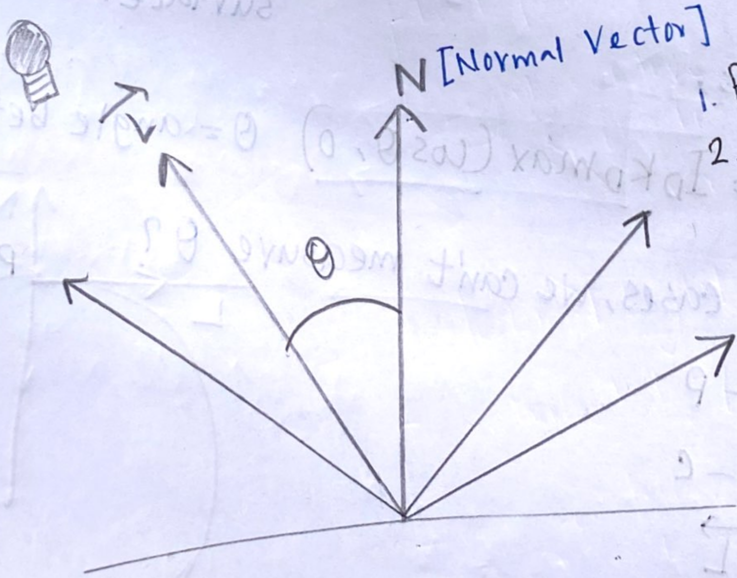
Moving from left to right the ambient.

Ambient light ↓↓	Shadows too deep and harsh.
Ambient Light ↑↑	Picture looks washed out and blend.

Diffuse Reflection



* Parallel rays are from light source.
* Reflected rays are scattered in many angles.
* Diffuse reflection is calculated based on the structure of the surface itself.



1. Point light source.
2. Light reflected equally in all direction.
3. Magnitude of Reflection depends on θ

Direction of L reversed.

Formula:

$D = I_P \, k_D \cos\theta$

- I_P → Intensity of point light.
- k_D → Diffuse co-efficient. $k_D \in [0,1]$
- θ → Angle between $\vec{L}$ and $\vec{N}$.

here, $\boxed{\theta \uparrow}$ $\boxed{\cos\theta \downarrow}$ $\boxed{D \downarrow}$

$\theta = 90^\circ \quad \cos\theta = 0 . \; D = 0$

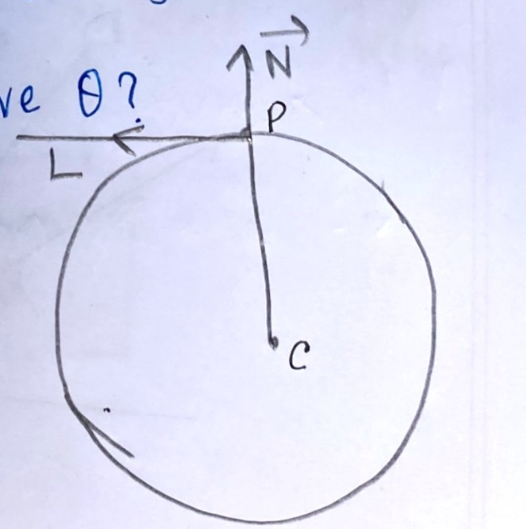
So, $\boxed{\theta > 90^\circ .}\boxed{\cos\theta < 0 . \therefore \cos\theta = -ve}\boxed{D = -ve}$

We need to solve this as no light is reflected if the light source is behind the surface.

To solve this,

$D = I_D k_D \max(\cos\theta, 0)$ θ = angle between L and N

** For the cases, we can't measure θ?



$\vec{L} = L - P$

$\vec{N} = P - C$

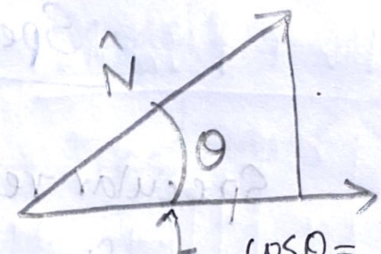
$\hat{L} = \dfrac{\vec{L}}{|\vec{L}|}$

$\hat{N} = \dfrac{\vec{N}}{|\vec{N}|}$

$$\hat{L}\cdot\hat{N} = |\hat{L}||\hat{N}|\cos\theta$$

$$= 1 \cdot 1 \cos\theta$$

$$= \hat{L}\cdot\hat{N}$$



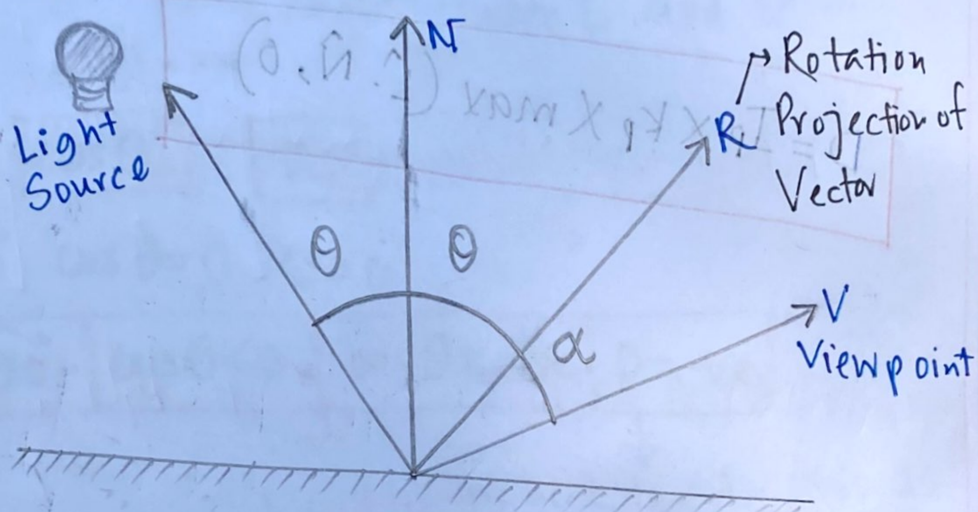
$\cos\theta = \hat{L}\cdot\hat{N}$

$$\boxed{D = I_P \times K_P \times \max(\hat{L}\cdot\hat{N}, 0)}$$

Specular Reflection

Shineness - Depends on Viewpoint.

Specular reflection is a type of surface reflectance often described as a mirror-like reflection of light from the surface.



Formula:

$$S = I_p K_s (\cos\alpha)^n$$

n → specular exp / shineness factor

I_p → Intensity of point Light

K_s → Specular Co-efficient.

α → Angle between R and V

if $n\uparrow$ $(\cos\alpha)^n \downarrow$ $S\downarrow$

Modified Formula:

$$S = I_p k_s (\hat{R}\cdot\hat{V})^n$$

$$= I_p k_s \{\max(\hat{R}\cdot\hat{V}, 0)\}^n$$

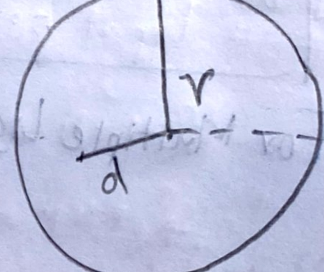
Final Formula for Phong's Reflection Model:

$$I = I_a k_a + I_p k_s \max(\hat{L}\cdot\hat{N}, 0) + I_p k_s \{ \max(\hat{R}\cdot\hat{V}$$

Attenuation

Attenuation is the loss of light energy ov space.

* Attenuation is account for by the variable f_{att} a applied to diffuse and specular components.
* Follows inversesquare law.



$$f_{att} = \frac{1}{d^2}$$ d = distance of object.

As it removes too much light,

Phong's model uses,

$$f_{att} = 1 - \left(\frac{d}{r}\right)^2$$

if point d is outside the radius,

$$d > r,$$

$$\Rightarrow \frac{d}{r} > 1 \quad \therefore \left(\frac{d}{r}\right)^2 > 1$$

Final Formula:

$$\boxed{f_{att} = Max\left(1 - \left(\frac{d}{r}\right)^{r}, 0\right)}$$

Updated Formula for Phong's Reflection Model:

$$\boxed{I = I_a k_a + I_p f_{att}\left(K_d max(\hat{L} \cdot \hat{N}, 0), K_s \left(max(\hat{V} \cdot \hat{R}, 0)\right)^n\right)}$$

or Multiple Light Sources.

$$= I_a k_a + \sum_{i=1}^{m} I_{p_i} f_{att}\left(K_d max(\hat{L}_i \cdot n, 0) + K_s\left(max(\hat{V} \cdot \hat{R}_i, 0)\right)^n\right)$$

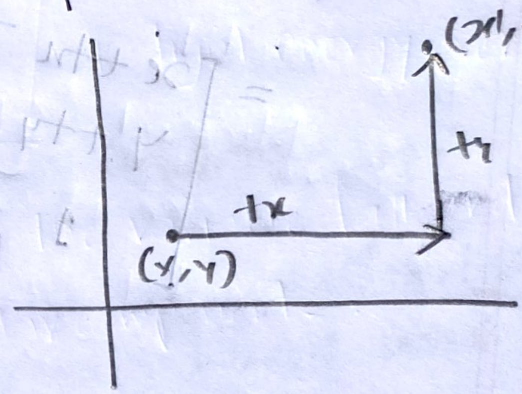
For each light source

I_p ~~is~~, L, R is multiple

Answer to the ques no. 01

1(a) Homogeneous co-ordinates is better fit for use in translation instead of Cartesian co-ordinates. Initially, translation is expressed as:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} tx \\ ty \end{bmatrix}$$



$$= \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} tx \\ ty \end{bmatrix}$$

$$= \begin{bmatrix} x + tx \\ y + ty \end{bmatrix}$$

Composition is difficult to express since translation is not expressed as a matrix multiplication. But homogeneous Co-ordinates allow all three to be expressed homogeneously using multiplication by 3X3 matrices and W is 1 for affine transformation in graphics.

P.T.O

Suppose, for 2D, we use a 3 * 3 matrix.

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} x + tx \\ y + ty \\ 1 \end{bmatrix}$$

1(b) Input points = (5, 6)

Rotation Angle 60° ~~A~~ Clockwise.

Rotation with respect to (6, 6)

$\therefore$ Output = $T(6,6) * R(-60^\circ) * T(-6,-6) * \text{Input}$

$$= \begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 6 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(-60) & -\sin(-60^\circ) & 0 \\ \sin(-60) & \cos(-60) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -6 \\ 0 & 1 & -6 \\ 6 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 6 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.5 & 0.866 & -2.196 \\ -0.866 & 0.5 & 8.196 \\ 0 & 0 & 1 \end{bmatrix}$$

1(c)

Given line,

$Y = \sqrt{3}x + 3$

$m = \sqrt{3}$ $\quad \theta = \tan^{-1}(\sqrt{3}) = 60^\circ$

$c = 3$

Output:

$T(0,-3) * R(-60^\circ) * Ref(x) * R(60^\circ) * T(0,3) * \text{Input}$

$$= \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}}_{T(0,3)} * \begin{bmatrix} \cos 60^\circ & -\sin 60^\circ & 0 \\ \sin 60^\circ & \cos 60^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(-60^\circ) & -\sin(-60^\circ) & 0 \\ \sin(-60^\circ) & \cos(60^\circ) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 5 \\ 6 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.098 \\ 8.8901 \\ 1 \end{bmatrix} \Rightarrow (0.098, 8.8321)$$

Color Model

3(a) Leonardo Da Vinci used different colors to complete his picture. So, for the final picture in this scenario, he used RGB model.

Michael Angelo snapped a picture on his phone to see it on his screen. For this, RGB model was used as well.

3(b) Algorithm to Convert RGB to HSV:

Step 01: Find Min and Max value from RGB Values.

$$Max = max(R, G, B)$$
$$Min = min(R, G, B)$$
$$l = Max - Min$$

Step 02: $Saturation = \frac{l}{Max}$

$$Value = Max$$

Scanned with CamScanner

Hue:

if R == Max:

$$H = \left(\frac{G-B}{l}\right) * 60^{\circ} \quad \left[\begin{array}{l} \text{if } H < 0^{\circ}, \\ H = H + 360^{\circ} \end{array}\right]$$

if G == Max:

$$H = \left(\frac{B-R}{l}\right) * 60^{\circ} + 120^{\circ}$$

if B == Max:

$$H = \left(\frac{R-G}{l}\right) * 60^{\circ} + 240^{\circ}$$

3(c)

$C = 0.4$

$M = 0.2$

$Y = 0.3$

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 1-0.4 \\ 1-0.2 \\ 1-0.3 \end{bmatrix} = \begin{bmatrix} 0.6 \\ 0.8 \\ 0.7 \end{bmatrix}$$

Max = G = 0.8

Min = R = 0.6.

$l = (0.8 - 0.6) = 0.2$

Scanned with CamScanner

$$\text{Saturation} = \frac{l}{\text{Max}} = \frac{0.2}{0.8} = 0.25$$

$$\text{Value} = \text{Max} = 0.8$$

Hue:

here, $G = max$.

$$\therefore \text{Hue} = \left(\frac{0.7 - 0.6}{0.2}\right) * 60^\circ + 120^\circ$$

$$= 150^\circ$$

Projection:

3(a)

Parallel Projection	Perspective Projection
Parallel Projection represents object in a different way.	Represents in 3D way.
No different effects.	Objets that are far appear smaller, Objets that are near appear bigger
distance of objet from COP is infinite	distance of object from COP is finite.
~~Lines of~~ Gives accurate projection of object.	Cannot give accurate projection.
Cannot give Realistic View	Realistic view of object

3(b) Mr. Joy Could not take Jaguar's full picture because of ~~parallel~~ perspective projection. As the jaguar was nearer, it filled up more of frames and as there was no proper distance between the jaguar and the camera so, the full picture couldnot be clicked.

3(c) · $PP(0,0,0)$

$COP(0,0,175)$

Given points $(35, 60, -300)$

$dx = COP.x - PP\,x$
$= 0$

$dy = COP_y - PP_y$
$= 0$

$dz = COP_z - PP_z$
$= 175$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{175} & \cancel{0}1 \end{bmatrix} \begin{bmatrix} 35 \\ 60 \\ -300 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 35 \\ 60 \\ 0 \\ -5/7 \end{bmatrix} \xrightarrow[\text{divide by } -5/7]{\text{Normalize}} \begin{bmatrix} -49 \\ -84 \\ 0 \\ 1 \end{bmatrix}$$

$\therefore$ Output $= (-49, -84, 0)$

Lighting Model

1(a)

Phong's model should follow the theoretical inverse square law for calculating attenuation. But it actually removes too much light. So, rather,

$f_{att} = \text{Max}\left(1 - \left(\frac{d}{r}\right)^2, 0\right)$ is used, here d represents the distance from the light source to the point illuminated. R is the maximum range of the ~~fall~~ light falls to zero.

1(b)

$\hat{E} = 2(\hat{L} \cdot \hat{N})\hat{N} - \hat{L}$

$\hat{L} = (3, 4, 6)$

$\hat{L} = \frac{3, 4, 6}{\sqrt{3^2 + 4^2 + 6^2}} =$

Scanned with CamScanner

2(b)

$$\vec{L} = (3,4,6) - (5,7,0)$$
$$= (-2,-3,6)$$

$$\hat{L} = \frac{-2,-3,6}{\sqrt{2^2+3^2+6^2}} = \left(-\frac{2}{7}, -\frac{3}{7}, \frac{6}{7}\right)$$

$$\hat{L}\cdot\hat{N} = \left(-\frac{2}{7}, -\frac{3}{7}, \frac{6}{7}\right) \cdot (0,0,1) \quad [xy\text{-plane}]$$

$$= \left(\frac{6}{7}\right)$$

$$\boxed{\cancel{\vec{E} = 2(\hat{L}\cdot\hat{N})\hat{N} - \left(-\frac{2}{7}, -\frac{3}{7}, \frac{6}{7}\right)}}$$

$$2(\hat{L}\cdot\hat{N})\hat{N} - \hat{L}$$

$$\vec{E} = 2(\hat{L}\cdot\hat{N})\hat{N} - \hat{L}$$
$$= 2\left(\frac{6}{7}\right)\cdot\hat{k} - \left(-\frac{2}{7}, -\frac{3}{7}, 6/7\right)$$
$$= \frac{12}{7}\hat{k} - \left(-\frac{2}{7}, -\frac{3}{7}, 6/7\right)$$
$$= \left(-\frac{2}{7}\hat{i}, -\frac{3}{7}\hat{j}, \frac{6}{7}\hat{k}\right)$$
$$= \left(-\frac{2}{7}\right.$$

1(c)

$$I = I_a k_a + I_p F_{att}\left(K_d \max(\hat{L}\cdot\hat{N}, 0) + K_s\left(\max(\hat{V}\cdot\hat{R}, 0)\right)^n\right)$$

1(d)

Shineness factor = 8, Ambient light intensity = 2

Ambient co-efficient = 0.3

Diffuse co-efficient = 0.1

Specular co-efficient = 0.5

Light source radius = 25

$$f_{att} = \max\left(1 - \left(\frac{d}{r}\right)^2, 0\right)$$

$$= \max\left(1 - \left(\frac{d}{25}\right)^2, 0\right)$$

$$d = \sqrt{(3_0 - 5)^2 + (4_0 - 7)^2 + (6-0)^2}$$

$$= 7$$

$$f_{att} = \max\left(1 - \left(\frac{7}{25}\right)^2, 0\right)$$

$$= \max(0.9216, 0) = 0.9216$$

$$\vec{V} = (6,4,1) - (5,7,0)$$

$$= (1,-3,1) \quad \therefore \hat{V} = \frac{1,-3,1}{\sqrt{1^2+3^2+1^2}} = \left(\frac{1}{\sqrt{11}}, \frac{-3}{\sqrt{11}}, \frac{1}{\sqrt{11}}\right)$$

$$\hat{R} = \hat{E} = \left(-\frac{2}{7}, -\frac{3}{7}, \frac{6}{7}\right)$$

$$\hat{V} \cdot \hat{R} = \left(\frac{1}{\sqrt{11}}, \frac{-3}{\sqrt{11}}, \frac{1}{\sqrt{11}}\right) \cdot \left(-\frac{2}{7}, \frac{-3}{7}, \frac{6}{7}\right)$$

$$= \left(\frac{-2\sqrt{11}}{77} + \frac{9\sqrt{11}}{77} + \frac{6\sqrt{11}}{77}\right)$$

$$= \frac{13\sqrt{11}}{77}$$

$$I = I_a K_a + F_{att} I_p K_p \max(\hat{L} \cdot \hat{N}, 0) + F_{att} I_p \{K_s \max(\hat{R} \cdot \hat{V})^n\}$$

$$= (2 \times 0.3) + (0.9216 \times 6 \times 0.1 \times 6/7) + \left(0.9216 \times 6 \times 0.5 \left(\frac{13\sqrt{11}}{77}\right)^8\right)$$

$$= 0.6 + 0.4739 + 0.\,80267$$

$$= 1.1006$$