

Practice Problem (Midpoint line + 8-way Symmetry)

i) $(-5, -100) \rightarrow (95, -35)$

$$dx = 95 - (-5) = 100$$

$$dy = -35 - (-100) = 65$$

$$|dx| > |dy|$$

$$dx > 0, dy > 0$$

$$\boxed{\text{Zone} = 0}$$

[Already Zone 0 so no need to convert]

$$d_{\text{init}} = 2dy - dx = 2 \times 65 - 100 = 30$$

$$\Delta d_{\text{NE}} = 2dy - 2dx = 2 \times 65 - 2 \times 100 = -70$$

$$\Delta d_{\text{E}} = 2dy = 2 \times 65 = 130$$

x	y	d	E/NE	d update
-5	-100	30	NE	$d = d + \Delta d_{\text{NE}} = 30 - 70 = -40$
-4	-99	-40	E	$d = d + \Delta d_{\text{E}} = -40 + 130 = 90$
-3	-99	90	NE	$90 + (-70) = 20$
-2	-98	20	NE	$20 - 70 = -50$
-1	-97	-50	E	$-50 + 130 = 80$
0	-97	80	NE	$80 - 70 = 10$
1	-96	10	NE	$10 - 70 = -60$
2	-95	-60	E	$-60 + 130 = 70$
3	-95	70	NE	$70 - 70 = 0$
4	-94	0	E	$0 + 130 = 130$

$$(i) (-5, 100) \rightarrow (20, 135)$$

$$dx = 20 - (-5) = 25$$

$$dy = 135 - 100 = 35$$

$$|dy| > |dx|$$

$$dx > 0, dy > 0$$

$$\therefore \text{Zone} = 1$$

Convert to Zone 0:

$$x = Y, y = X$$

$$(100, -5) \text{ and } (135, 20)$$

$$dx = 135 - 100 = 35, dy = 20 - (-5) = 25$$

$$d_{init} = 2dy - dx = 2 \times 25 - 35 = 15$$

$$\Delta d_{NE} = 2dy - 2dx = 2 \times 25 - 2 \times 35 = -20$$

$$\Delta d_E = 2dy = 2 \times 25 = 50$$

Original Zone

x	y	d	E/NE	d_{update}	$x = Y$	$y = X$
100	-5	15	NE	$15 + (-20) = -5$	-5	100
101	-4	-5	E	$-5 + 50 = 45$	-4	101
102	-4	45	NE	$45 - 20 = 25$	-4	102
103	-3	25	NE	$25 - 20 = 5$	-3	103
104	-2	5	NE	$5 - 20 = -15$	-2	104
105	-1	-15	E	$-15 + 50 = 35$	-1	105
106	-1	35	NE	$35 - 20 = 15$	-1	106
107	0	15	NE	$15 - 20 = -5$	0	107
108	1	-5	E	$-5 + 50 = 45$	1	108
109	1	45	NE	$45 - 20 = 25$	1	109

iii) (25, 25) and (-12, 48)

$$dx = -12 - 25 = -37$$

$$dy = 48 - 25 = 23$$

$$|dx| > |dy|$$

$$dx < 0, dy > 0$$

Zone = 3

Convert Zone 3 to Zone 0:

$$x = -X, y = Y$$

(-25, 25) and (12, 48)

$$dx = 12 - (-25) = 37, dy = 48 - 25 = 23$$

$$d_{init} = 2dy - dx = 2 \times 23 - 37 = 9$$

$$\Delta d_{NE} = 2dy - 2dx = 2 \times 23 - 2 \times 37 = -28$$

$$\Delta d_E = 2dy = 2 \times 23 = 46$$

Original Zone

x	y	d	E/NE	d update	x = -X	y = Y
-25	25	9	NE	9 - 28 = -19	25	25
-24	26	-19	E	-19 + 46 = 27	24	26
-23	26	27	NE	27 - 28 = -1	23	26
-22	27	-1	E	-1 + 46 = 45	22	27
-21	27	45	NE	45 - 28 = 17	21	27
-20	28	17	NE	17 - 28 = -11	20	28
-19	29	-11	E	-11 + 46 = 35	19	29
-18	29	35	NE	35 - 28 = 7	18	29
-17	30	7	NE	7 - 28 = -21	17	30
-16	31	-21	E	-21 + 46 = 25	16	31

(iv) (5, 52) and (-18, 89)

$$dx = -18 - 5 = -23$$

$$dy = 89 - 52 = 37$$

$$|dy| > |dx|$$

$$dx < 0, dy > 0$$

$$\therefore \text{Zone} = 2$$

Convert Zone 2 to Zone 0:

$$x = Y, y = -X$$

(52, -5) and (89, 18)

$$dx = 89 - 52 = 37, dy = 18 - (-5) = 23$$

$$d_{init} = 2dy - dx = 2 \times 23 - 37 = 9$$

$$4d_{NE} = 2dy - 2dx = 2 \times 23 - 2 \times 37 = -28$$

$$4d_E = 2dy = 2 \times 23 = 46$$

Original Zone

x	y	d	E/NE	d_{update}	$x = -Y$	$y = X$
52	-5	9	NE	$9 - 28 = -19$	5	52
53	-4	-19	E	$-19 + 46 = 27$	4	53
54	-4	27	NE	$27 + (-28) = -1$	4	54
55	-3	-1	E	$-1 + 46 = 45$	3	55
56	-3	45	NE	$45 + (-28) = 17$	3	56
57	-2	17	NE	$17 + (-28) = -11$	2	57
58	-1	-11	E	$-11 + 46 = 35$	1	58
59	-1	35	NE	$35 - 28 = 7$	1	59
60	0	7	NE	$7 - 28 = -21$	0	60
61	1	-21	E	$-21 + 46 = 22$	-1	61

✓ (11, 9) and (34, -9)

$$dx = 34 - 11 = 23$$

$$dy = -9 - 9 = -18$$

$$|dx| > |dy|$$

$$dx > 0, dy < 0$$

$$\therefore \boxed{\text{Zone} = 7}$$

Convert Zone 7 to Zone 0:

$$x = X, y = -Y$$

(11, -9) and (34, 9)

$$dx = 34 - 11 = 23, dy = 9 - (-9) = 18$$

$$d_{init} = 2dy - dx = 2 \times 18 - 23 = 13$$

$$\Delta d_{NE} = 2dy - 2dx = 2 \times 18 - 2 \times 23 = -10$$

$$\Delta d_E = 2dy = 2 \times 18 = 36$$

Original Zone

x	y	d	NE/E	d update	$x = X$	$y = -Y$
11	-9	13	NE	$13 + (-10) = 3$	11	9
12	-8	3	NE	$3 - 10 = -7$	12	8
13	-7	-7	E	$-7 + 36 = 29$	13	7
14	-7	29	NE	$29 - 10 = 19$	14	7
15	-6	19	NE	$19 - 10 = 9$	15	6
16	-5	9	NE	$9 - 10 = -1$	16	5
17	-4	-1	E	$-1 + 36 = 35$	17	4
18	-4	35	NE	$35 - 10 = 25$	18	4
19	-3	25	NE	$25 - 10 = 15$	19	3
20	-2	15	NE	$15 - 10 = 5$	20	2

(vi) $(-49, -29)$ and $(-107, -68)$

$$dx = -107 + 49 = -58$$

$$dy = -68 + 29 = -39$$

$$|dx| > |dy|$$

$$dx < 0, dy < 0$$

$$\therefore \text{Zone} = 4$$

Convert Zone 4 to Zone 0:

$$x = -X, y = -Y$$

$(49, 29)$ and $(107, 68)$

$$dx = 107 - 49 = 58, dy = 68 - 29 = 39$$

$$d_{init} = 2dy - dx = 2 \times 39 - 58 = 20$$

$$\Delta d_{NE} = 2dy - 2dx = 2 \times 39 - 2 \times 58 = -38$$

$$\Delta d_E = 2dy = 2 \times 39 = 78$$

Original Zone

x	y	d	E/NE	d update	$x = -X$	$y = -Y$
49	29	20	NE	$20 - 38 = -18$	-49	-29
50	30	-18	E	$-18 + 78 = 60$	-50	-30
51	30	60	NE	$60 - 38 = 22$	-51	-30
52	31	22	NE	$22 - 38 = -16$	-52	-31
53	32	-16	E	$-16 + 78 = 62$	-53	-32
54	32	62	NE	$62 - 38 = 24$	-54	-32
55	33	24	NE	$24 - 38 = -14$	-55	-33
56	34	-14	E	$-14 + 78 = 64$	-56	-34
57	34	64	NE	$64 - 38 = 26$	-57	-34
58	35	26	NE	$26 - 38 = -12$	-58	-35

Vii) $(-6, 9)$ and $(34, -25)$

$$dy = -25 - 9 = -34 \quad |dx| > |dy|$$

$$dx = 34 + 6 = 40 \quad dx > 0, dy < 0$$

Zone = 7

Convert Zone 7 to Zone 0°

$$x = X, y = -Y$$

$(-6, -9)$ and $(34, 25)$

$$dx = 34 + 6 = 40, dy = 25 + 9 = 34$$

$$d_{init} = 2dy - dx = 2 \times 34 - 40 = 28$$

$$d_{NE} = 2dy - 2dx = 2 \times 34 - 2 \times 40 = -12$$

$$d_E = 2dy = 2 \times 34 = 68$$

Original Zone

x	y	d	E/NE	d update	$x = X$	$y = -Y$
-6	-9	28	NE	$28 - 12 = 16$	-6	9
-5	-8	16	NE	$16 - 12 = 4$	-5	8
-4	-7	4	NE	$4 - 12 = -8$	-4	7
-3	-6	-8	E	$-8 + 68 = 60$	-3	6
-2	-6	60	NE	$60 - 12 = 48$	-2	6
-1	-5	48	NE	$48 - 12 = 36$	-1	5
0	-4	36	NE	$36 - 12 = 24$	0	4
1	-3	24	NE	$24 - 12 = 12$	1	3
2	-2	12	NE	$12 - 12 = 0$	2	2
3	-1	0	E	$0 + 68 = 68$	3	1

viii) (15, 0) and (-26, -42)

$$dx = -26 - 15 = -41$$

$$dy = -42 - 0 = -42$$

$$|dy| > |dx|$$

$$dy < 0, dx < 0$$

$$\therefore \boxed{\text{Zone} = 5}$$

Convert Zone 5 to Zone 0

$$x = -Y, y = -X$$

(0, -15) and (42, 26)

$$dx = 42 - 0 = 42, dy = 26 + 15 = 41$$

$$d_{init} = 2dy - dx = 2 \times 41 - 42 = 40$$

$$d_{NE} = 2dy - 2dx = 2 \times 41 - 2 \times 42 = -2$$

$$d_E = 2dy = 2 \times 41 = 82$$

Original Zone

x	y	d	E/NE	d update	x = -Y	y = -X
0	-15	40	NE	40 - 2 = 38	15	0
1	-14	38	NE	38 - 2 = 36	14	-1
2	-13	36	NE	36 - 2 = 34	13	-2
3	-12	34	NE	34 - 2 = 32	12	-3
4	-11	32	NE	32 - 2 = 30	11	-4
5	-10	30	NE	30 - 2 = 28	10	-5
6	-9	28	NE	28 - 2 = 26	9	-6
7	-8	26	NE	26 - 2 = 24	8	-7
8	-7	24	NE	24 - 2 = 22	7	-8
9	-6	22	NE	22 - 2 = 20	6	-9