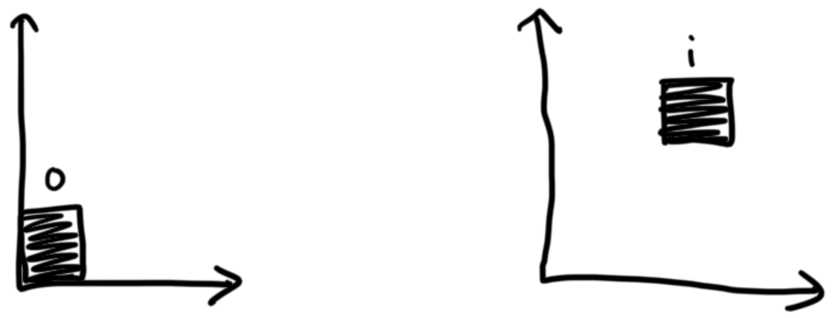
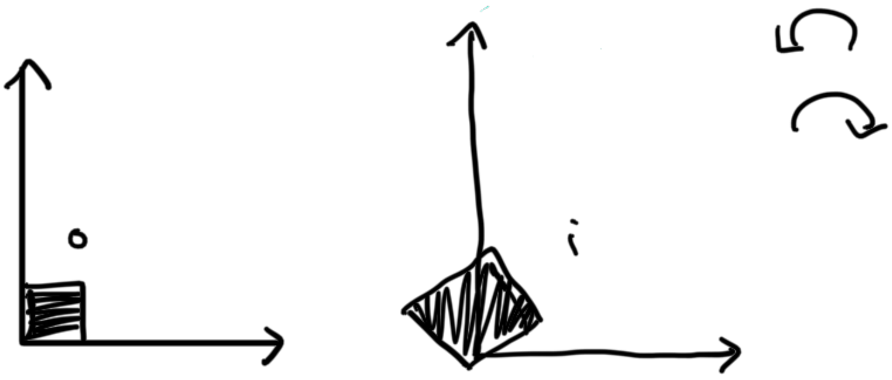
Linear Transformation

Translation



move to a new position.

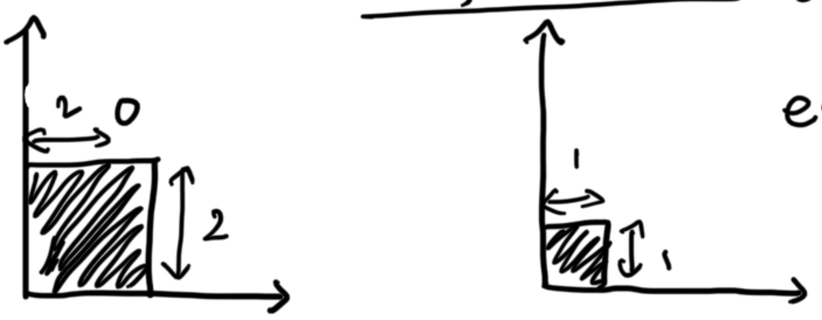
Rotation



rotate through an angle

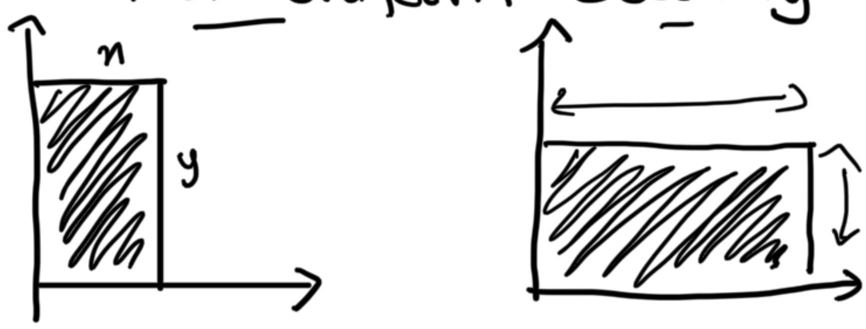
Scaling:

Uniform Scaling



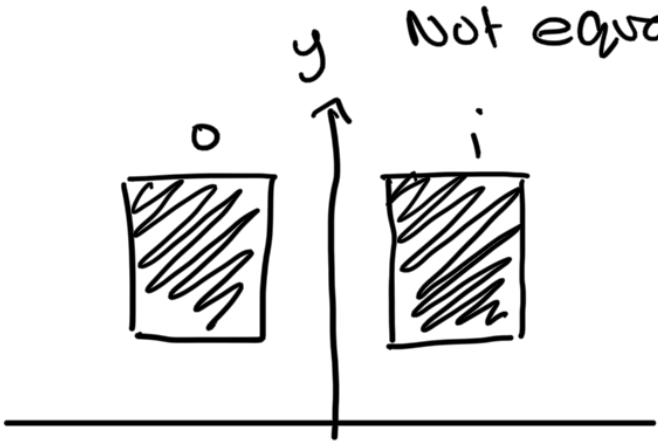
equal change in x & y

Non-uniform Scaling



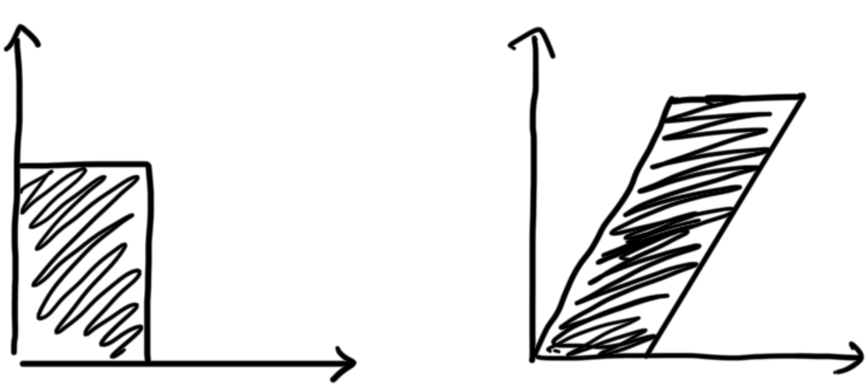
Not equal change in x & y.

Reflection:



mirror image

Shearing



Keeping centre fixed shifting x-axis & y-axis

Linear Transformation

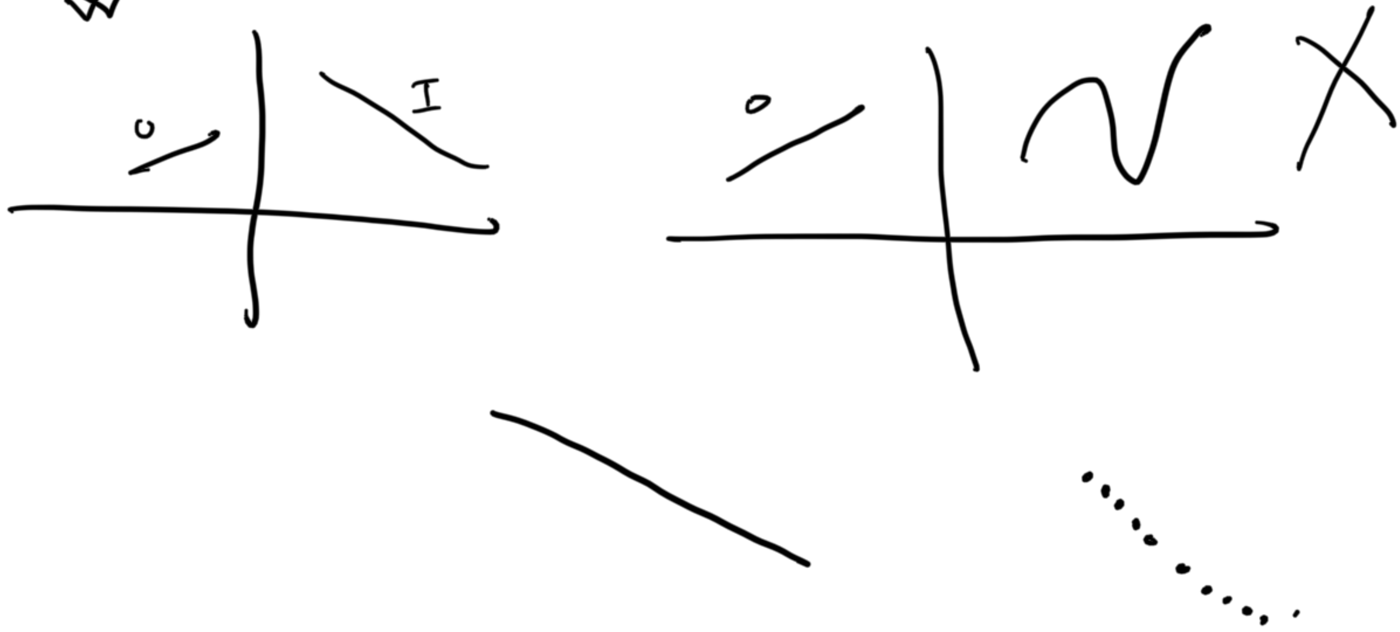
Algebraically:

- mapping between two vector spaces that preserves the operation of vector addition & scalar multiplication.

Geometrically:

- keeps origin vector unchanged & keeps straight line straight.

$$T(\vec{0}) = \vec{0}$$



If T is a linear transformation such that $T: \mathbb{R}^n \to \mathbb{R}^m$, T can be defined by matrix multiplication.

$$\boxed{T(\vec{u}) = A\vec{u}}$$

object

$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

Set.

Vector Space.

$a+b = c$

integer numbers. $\mathbb{Z}$ mult., add, $2 \times 2 = 4$

SVS, not division,

$2+2=4$

$2-2=0$

R^2 R^3

functions — polynomials are vector.

Linear Combination

- it combines two operation
  - addition & scalar multiplication.

$$\begin{bmatrix} 3 \\ -2.5 \end{bmatrix} = 3\begin{bmatrix} 1 \\ 0 \end{bmatrix} + (-2.5)\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Matrix multiplication is also a linear combination

$$\begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix}$$

$$= \begin{bmatrix} 2a + 3b \\ 1a - 4b \end{bmatrix}$$

$$= \left[a\begin{bmatrix} 2 \\ 1 \end{bmatrix} + b\begin{bmatrix} 3 \\ -4 \end{bmatrix} \right]$$

$$= \begin{bmatrix} 2a + 3b \\ 1a - 4b \end{bmatrix}$$

$T : V \to X$ as linear transformation.

1) $T(u+v) = T(u) + T(v)$

2) $T(cu) = cT(u)$

or,

$$\boxed{T(cu+dv) = cT(u) + dT(v)}$$

$$T(v) + T(u) = T(u+v)$$

result = $T\big((\ ) + (\ )\big)$

x^2

$x+4$

$4i+2j$

$4(T(i)) + 2(T(j))$

$$\boxed{T = A\vec{u}}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} 0.71 & -0.71 \\ 0.71 & 0.71 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 45^\circ \text{ rotation} \\ 45 \text{ rotation} \end{pmatrix}$$

$$\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \text{Horizontally Scale.}$$

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \text{Shear.}$$

L.H.S

$$T(cu+dv) = cT(u) + dT(v)$$

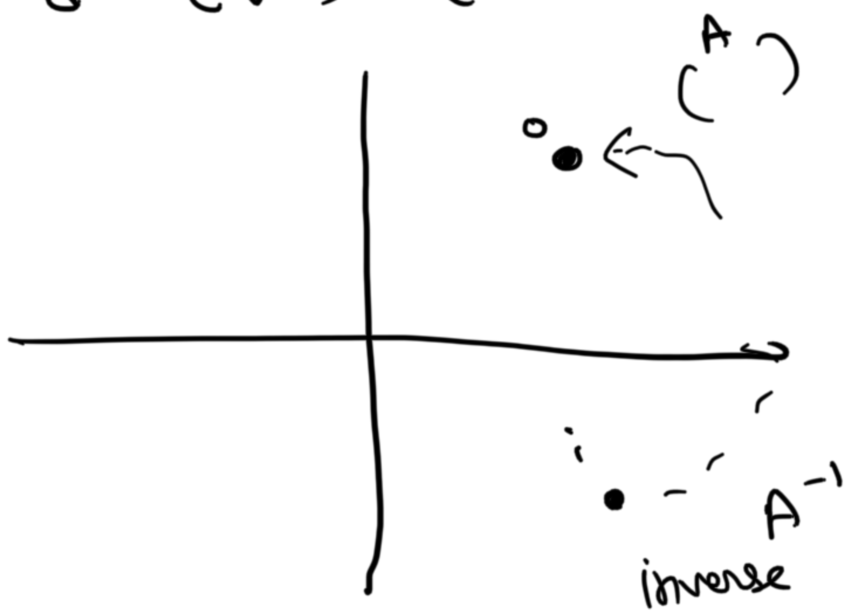
$$T(c\vec{u} + d\vec{v})$$

$$= A.(c\vec{u} + d\vec{v})$$

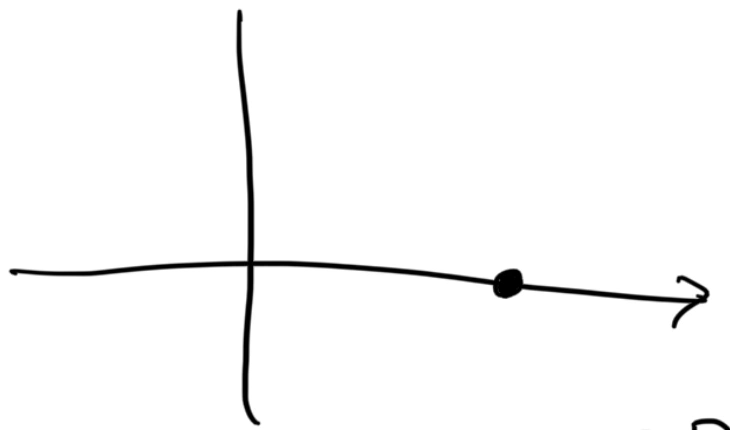
$$= \boxed{A}\cdot c\vec{u} + \boxed{A}\cdot d\vec{v} \qquad \Big| \; T(\vec{u}) = A\vec{u}, \quad T(\vec{v}) = A\vec{v}$$

$$= c\cdot \boxed{A\vec{u}} + d\cdot \boxed{A\vec{v}}$$

$$= cT(\vec{u}) + dT(\vec{v})$$ (Proved)



$$\begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2x \\ 0 \end{pmatrix}$$



$$\overset{3D}{\begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \end{bmatrix}}\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \overset{2D}{\begin{bmatrix} 2 \\ 4 \end{bmatrix}}$$

$$R^3 \rightarrow R^2$$

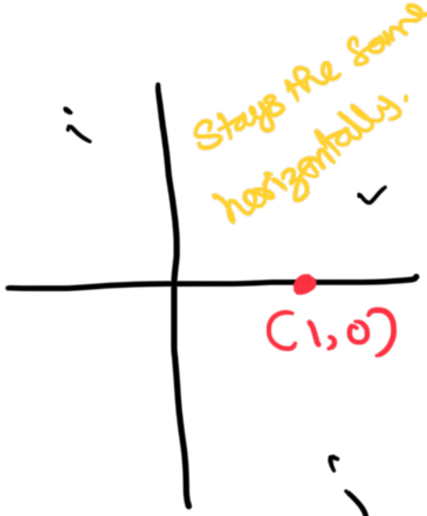
$$\begin{bmatrix} 2 & 0 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

① We don't know which transformation hasbeen applied by seeing the matrix.

② $\begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \rightarrow \begin{bmatrix} x \text{ rotate } 45^\circ \\ y \text{ " " } \end{bmatrix}$ ✓

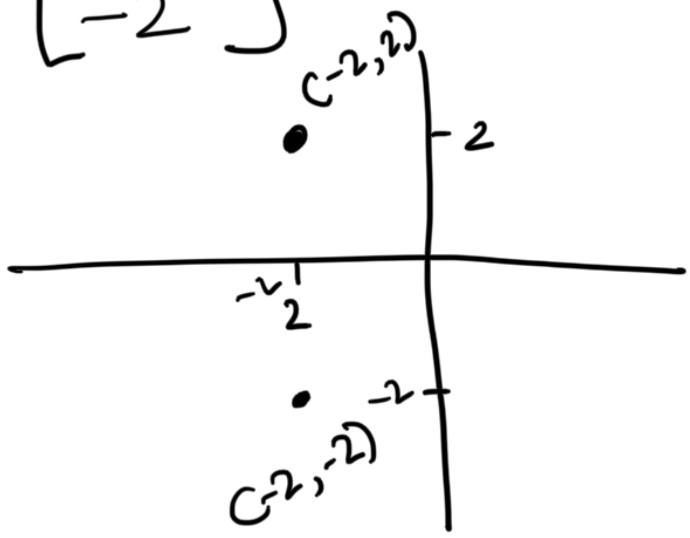
which matrix we used for 45° rotation is unknown.

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}_i = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$



$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}_j = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -2 \\ 2 \end{bmatrix}_{i,j} = \begin{bmatrix} -2 \\ -2 \end{bmatrix}$$



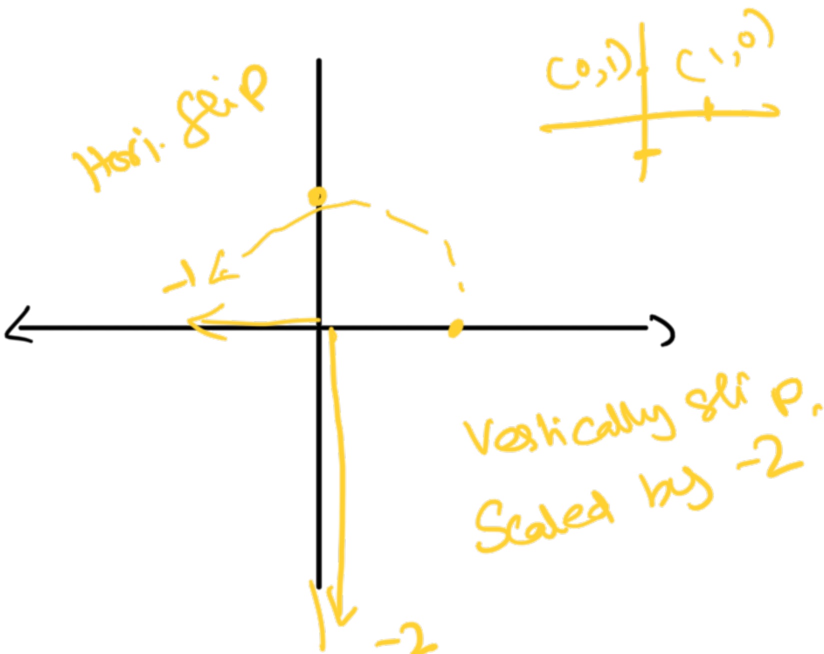
$$\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}_i = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}_j = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

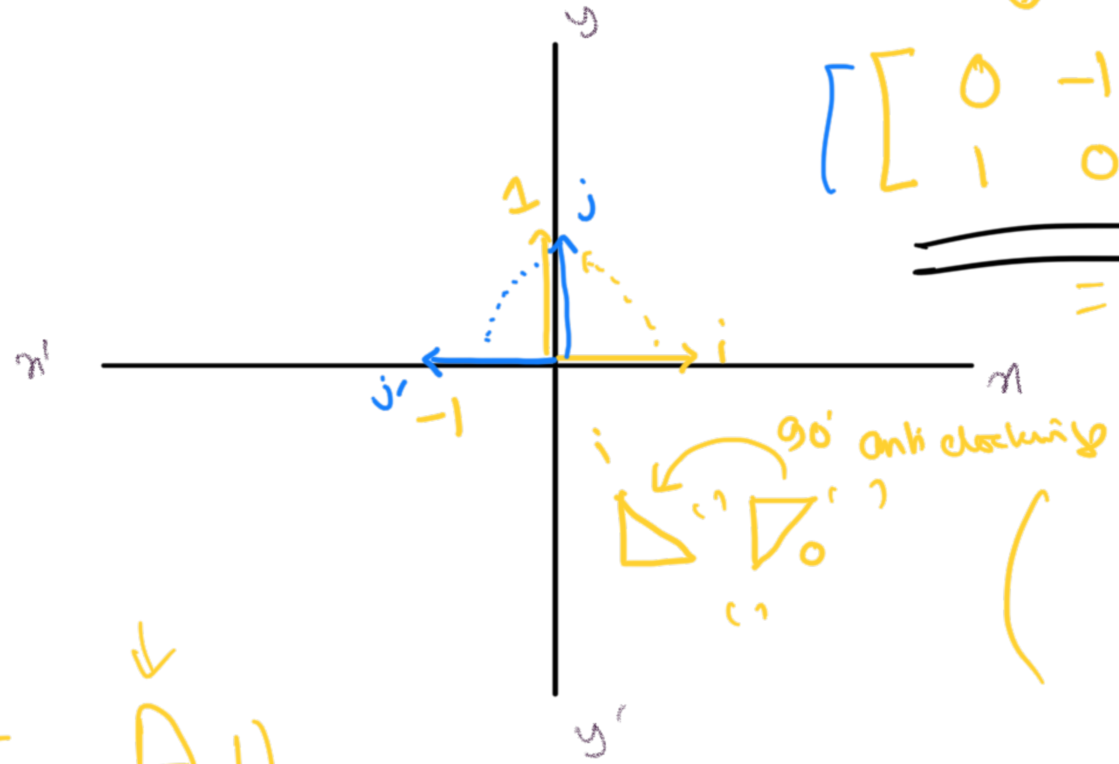
$$\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}\begin{bmatrix} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$$



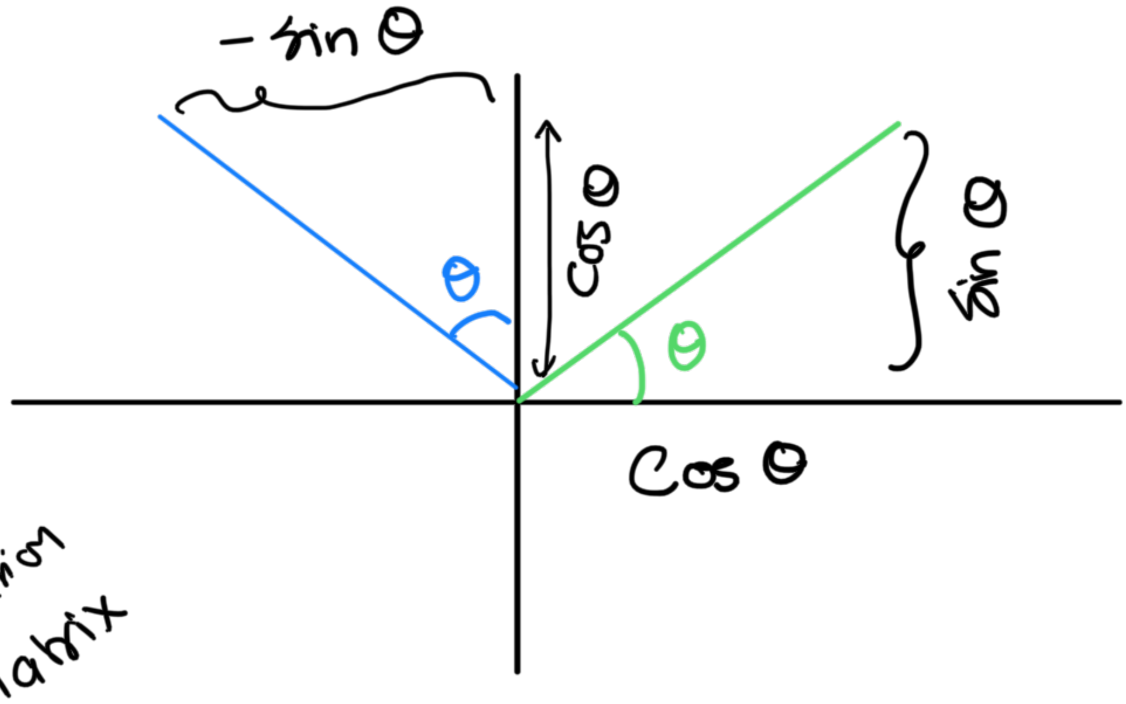
Rotation 90° anticlockwise.

$$\left[\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\right]\begin{bmatrix} \\ \end{bmatrix}$$



$T = AU$

Generic Rotation.



Rotation Matrix

$$\begin{pmatrix} \cos\theta & -\sin\theta \end{pmatrix}$$

90°

$$= \begin{pmatrix} \sin\theta & \cos\theta \end{pmatrix}$$

A .

$$\begin{pmatrix} \cos 90^\circ & -\sin 90^\circ \\ \sin 90^\circ & \cos 90^\circ \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$\underset{A}{(\quad)}\ \underset{\vec{v}}{(\quad)} = \underset{T(\vec{v})}{(\quad)}$$

$\overrightarrow{3D}$ Rotation

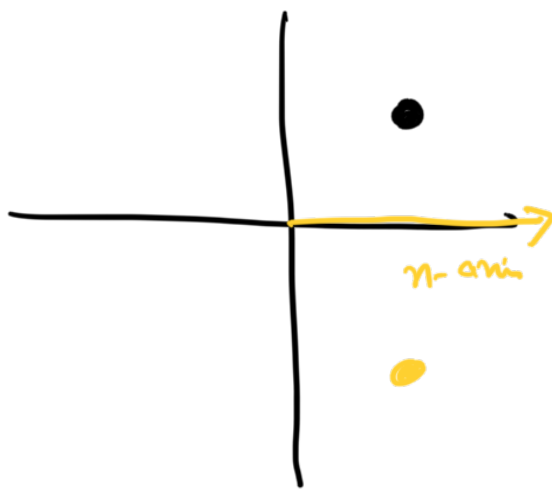
$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

$$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

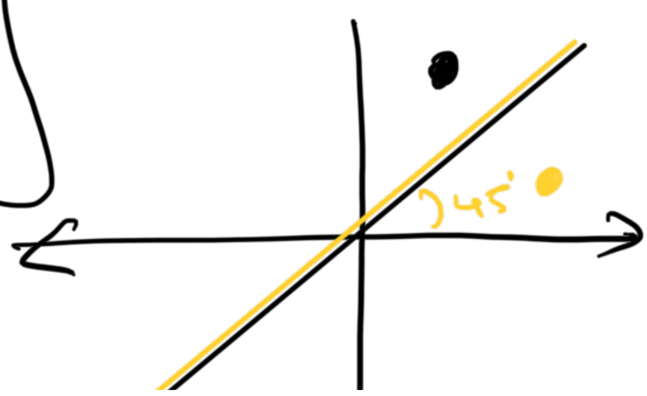
$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Reflection

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

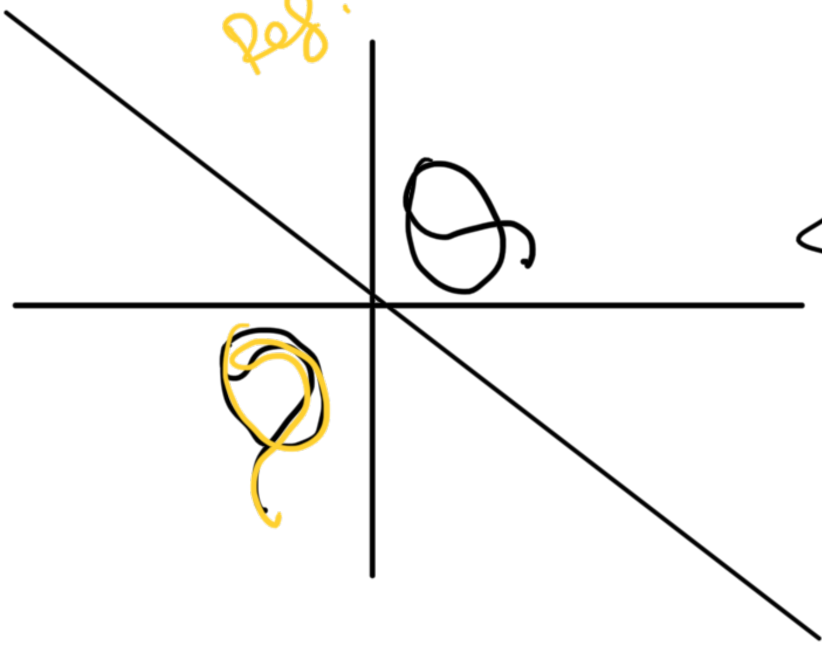


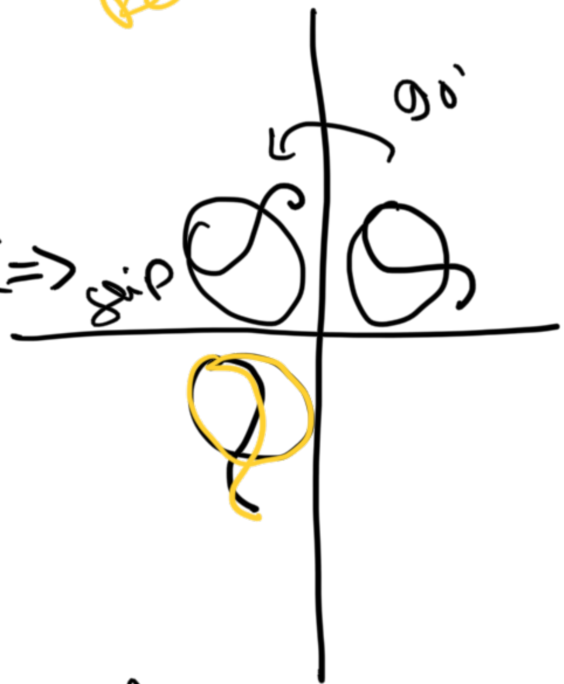
$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$



$$\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$$

Rotational

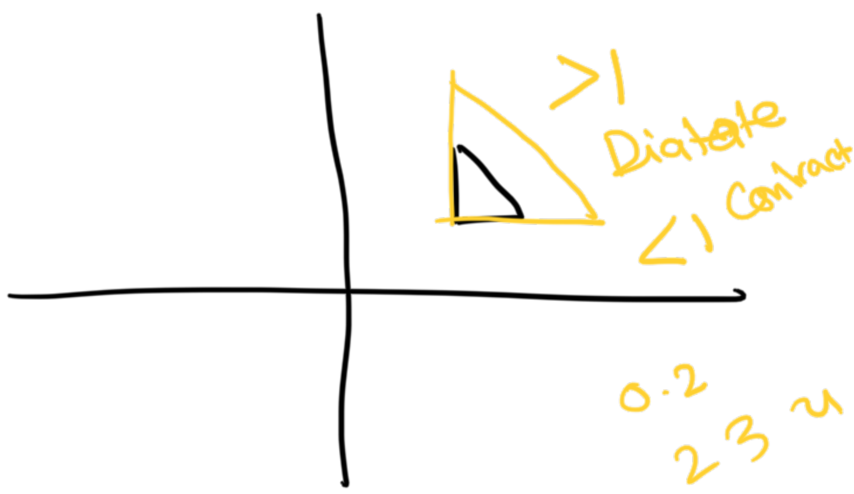


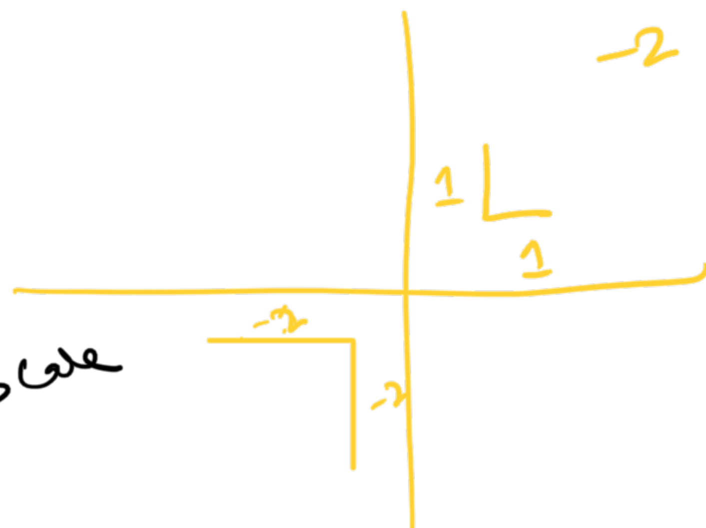


Scaling

Scale factor = 1

$$2\begin{pmatrix} 0 \\ \ \end{pmatrix} = \begin{pmatrix} i \\ \ \end{pmatrix}$$



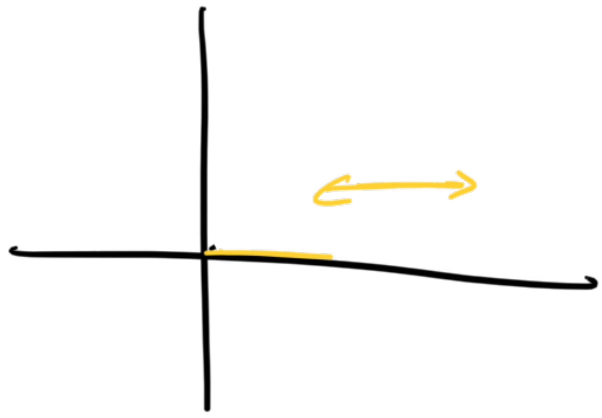


$$\begin{array}{c} \begin{array}{cc} x & \ \end{array} \\ \begin{bmatrix} a & 0 \\ 0 & 1 \end{bmatrix} \end{array}$$ x-scale

$$\begin{array}{cc} x & \\ \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} & y \end{array} \qquad \begin{array}{c} \begin{bmatrix} 1 & 0 \\ 0 & \overset{y}{b} \end{bmatrix} \end{array} \text{ y-scale}$$

x & y uniformly scaled.

Shear.



x $\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$

y $\begin{bmatrix} 1 & 0 \\ b & 1 \end{bmatrix}$

x, y $\begin{bmatrix} 1 & a \\ b & 1 \end{bmatrix}$