# Transformation Part 1

## What is Spatial Transformation?

It is similar to function.
In function we give an input and get some output.
Similarly, Spatial Transformation takes some point from space and maps it in a different place.
∴ We can say Transformation means by applying some mathematical operations and rules to a graphics and changing its position, size, shape and orientation, etc.

Mapping a point to another point

$T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$   3D to 2D transformation.

## Why do we need Transformation?

→ To move/deform/manipulate objects in space.

Our models are made of polygons
polygons are made of edges/vertices
edge/vertices are made of points.
∴ If we move a point we can move the entire object from one place to another.

→ To move the viewpoint/camera of a

scene.

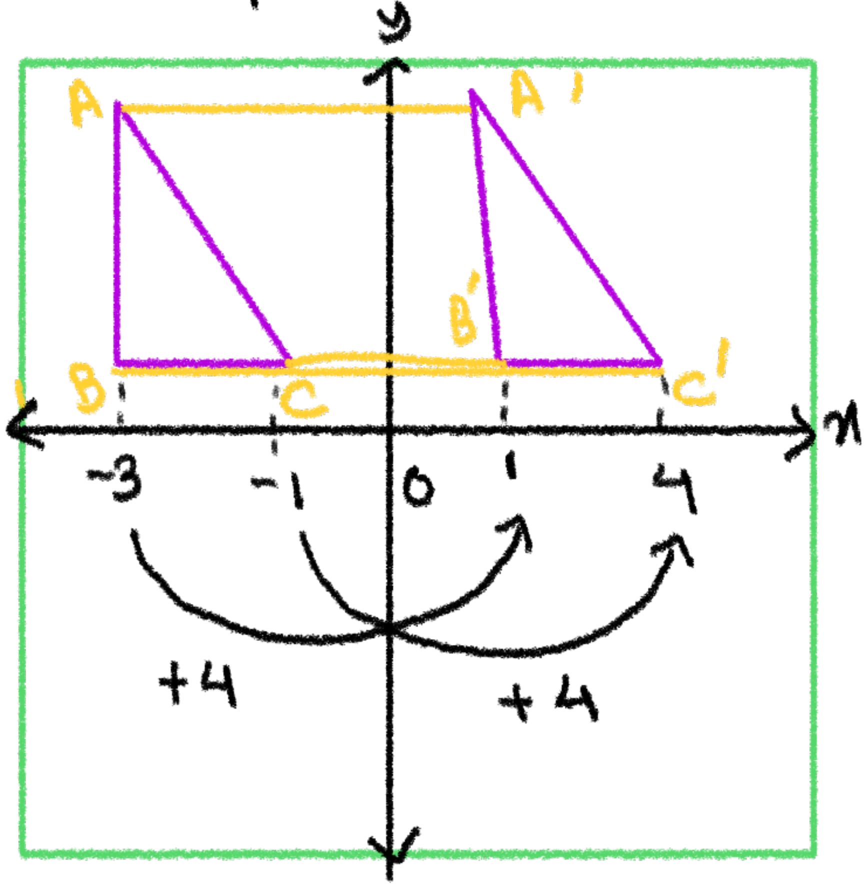
→ To map a 2D texture to a 3D object.

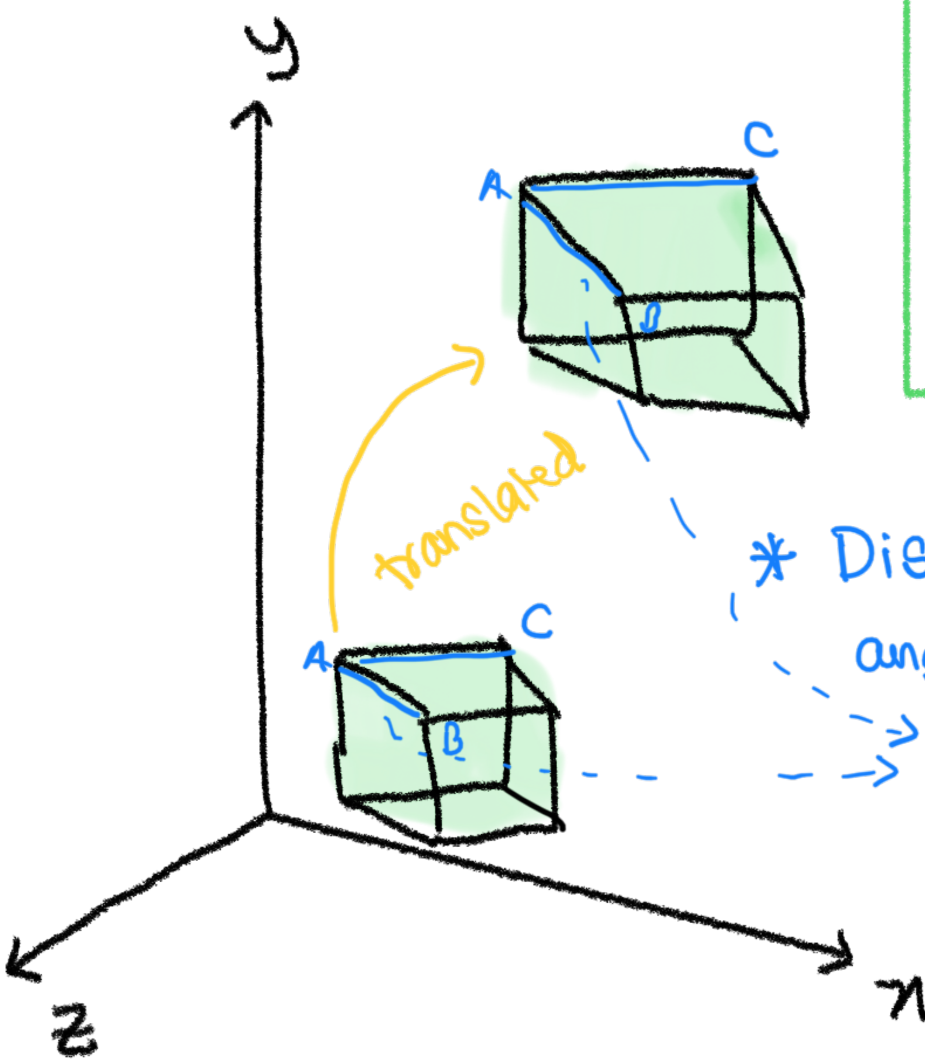
→ To project 3D object on a 2D screen.

## Translation

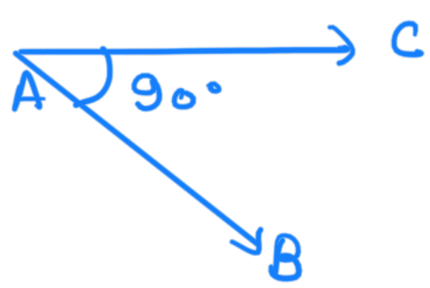
Adding an offset to the point.

Moving every point to a certain direction by a certain amount.




* Distance between points and angle betⁿ vectors are preserved.



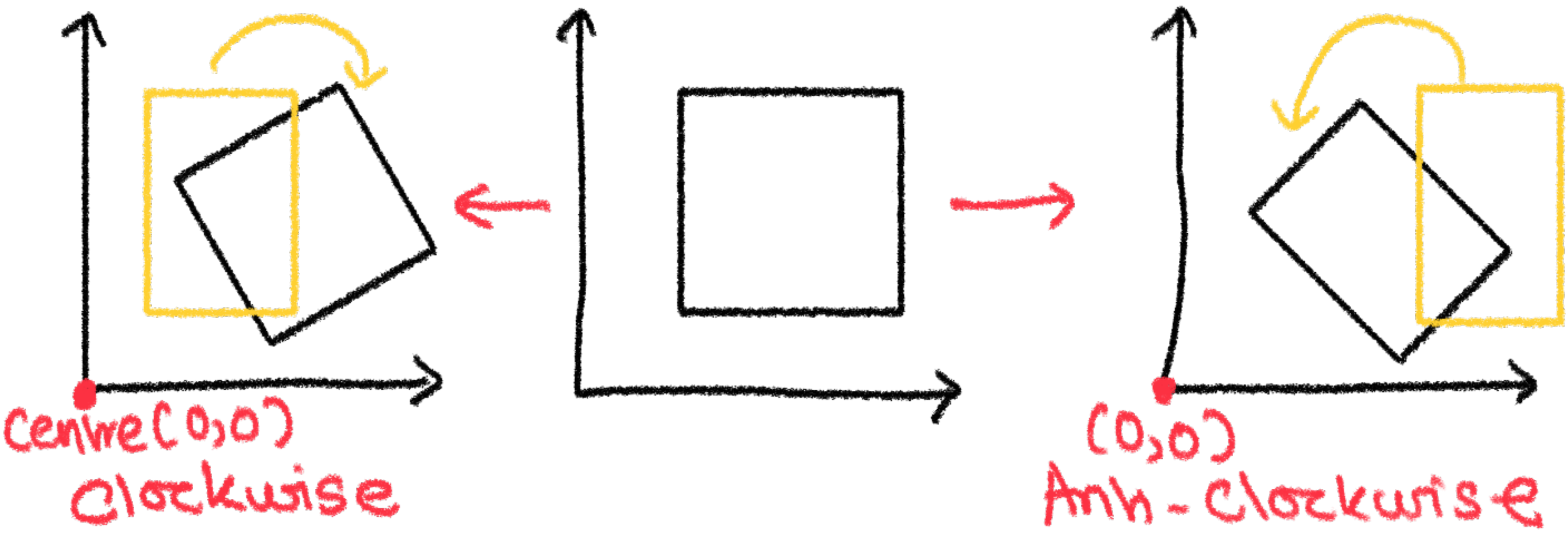

* Ratio between sides will also be preserved.

$$\frac{AB}{AC}$$

## Rotation

→ 3D objects can rotate through x-axis, y-axis and z-axis.


→ 2D objects can rotate through a point called centre (x,y) either clockwise or anti-clockwise.



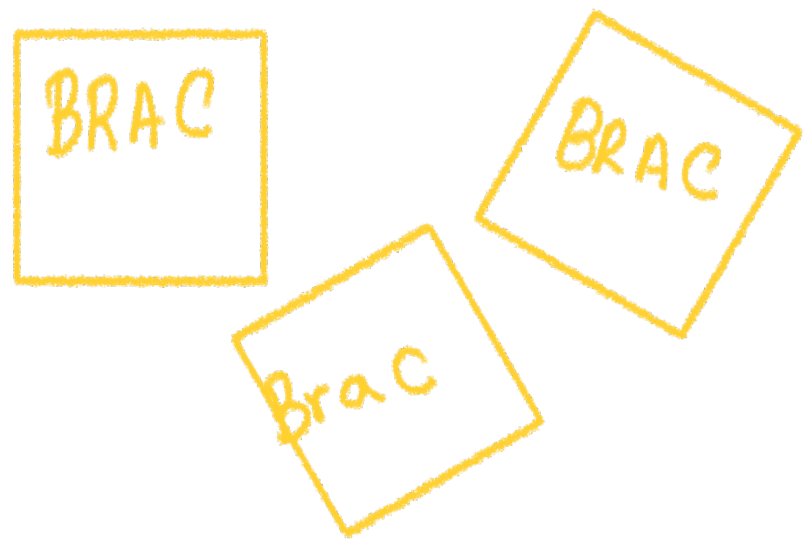

* The followings are preserved:

→ Distance between two points

→ Distance to origin (for rotation around one axes that is origin is the centre of axis)

→ Angle between two vectors

→ Orientation

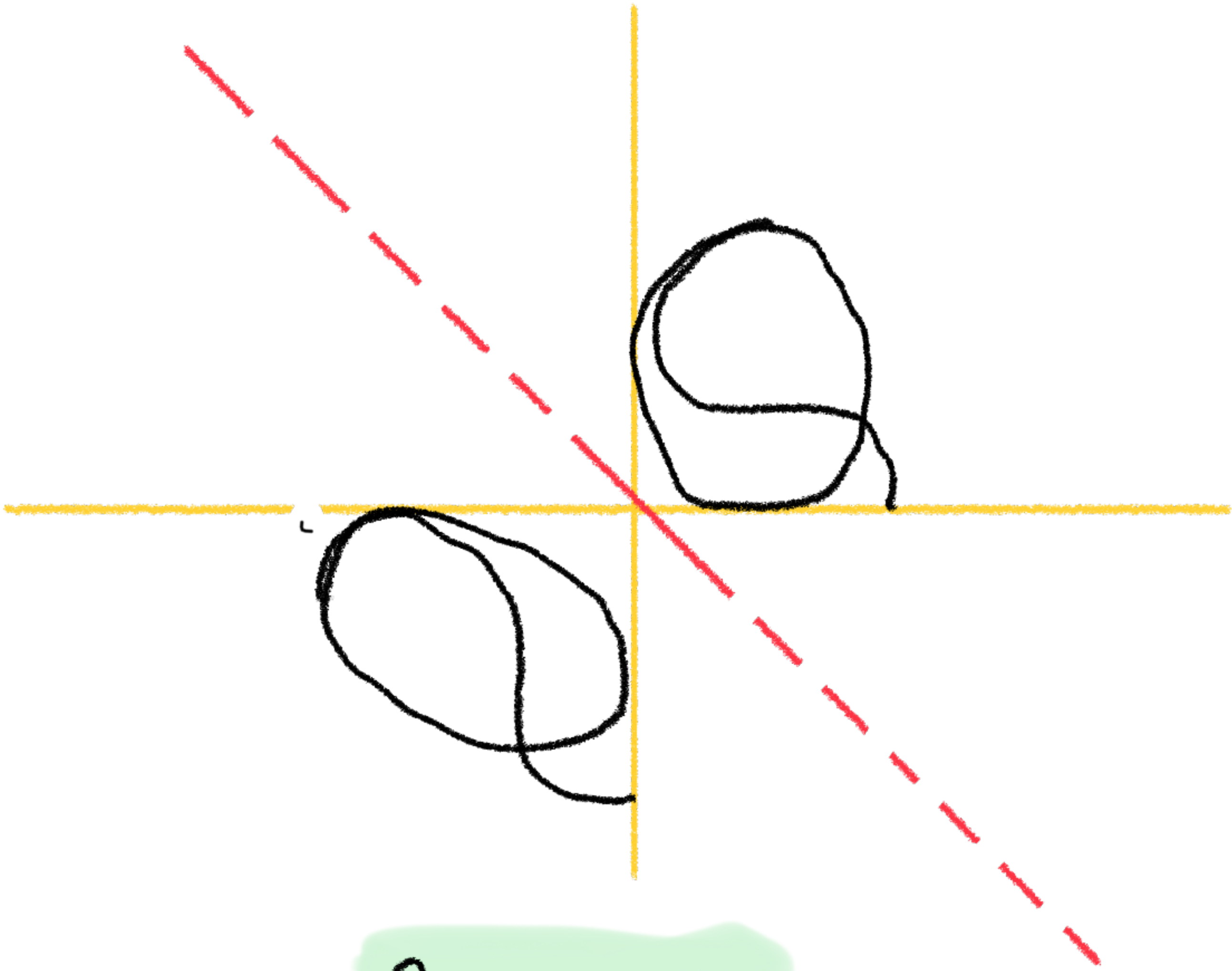



## Reflection

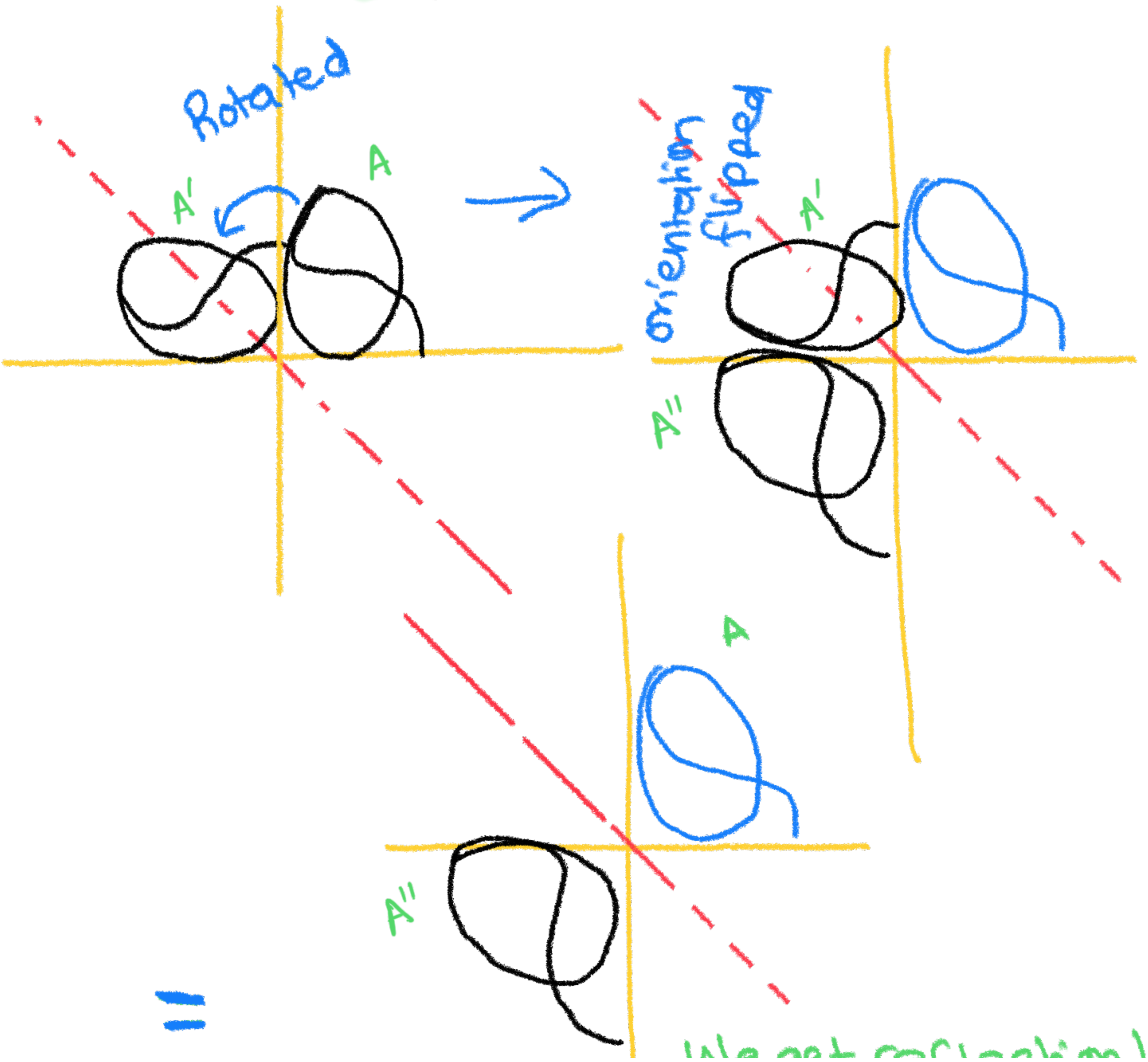
→ it is the mirror image

→ preserves everything as rotation except orientation.

∵ We can say reflection is rotation with orientation flipped!



Reflection

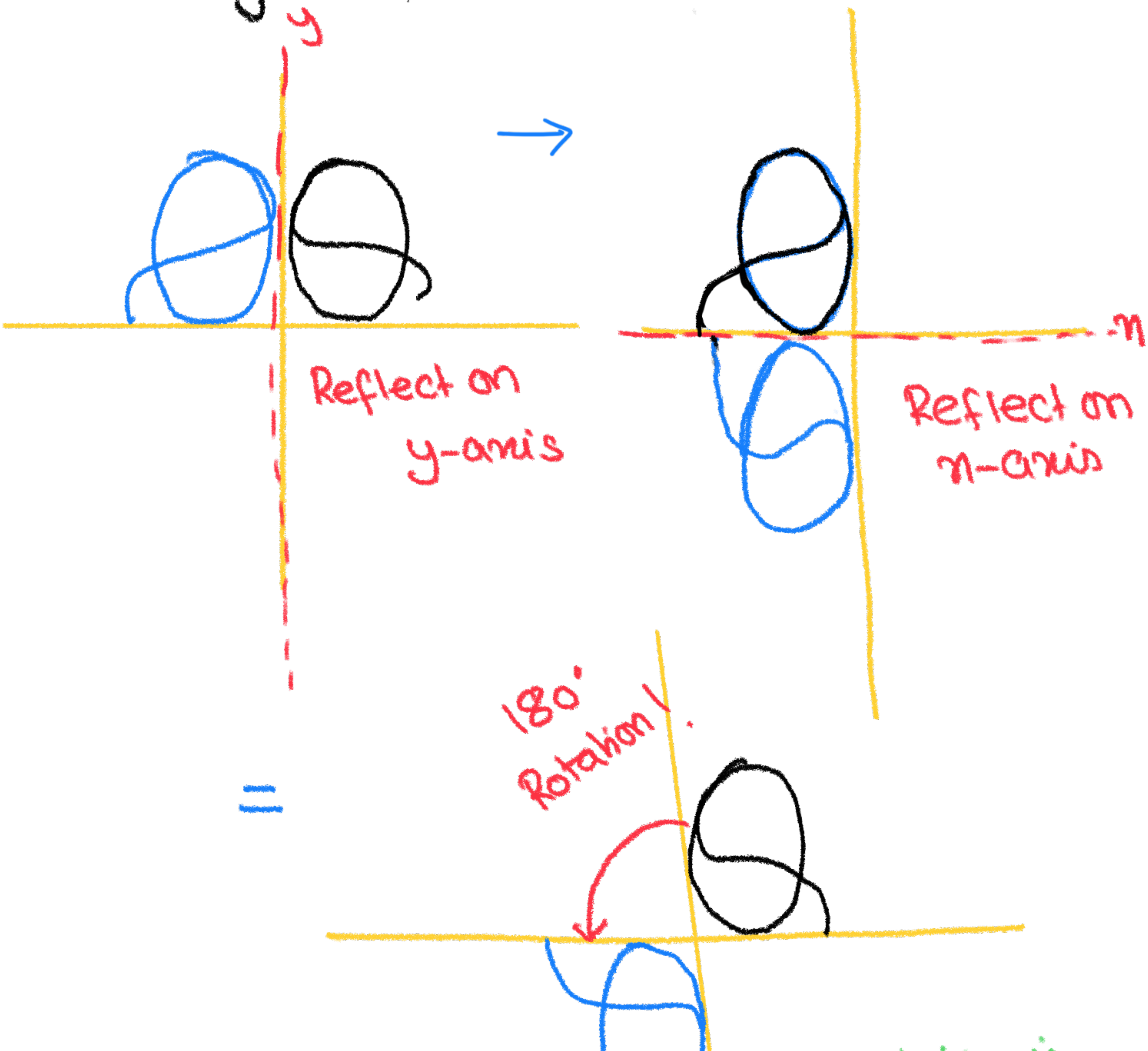


we get reflection!

If we reflect in an object n time (n is $2^n$) we will get rotation.

For ex:

if you reflect twice →




orientation is preserved, no mirror image.

## Scaling

→ increasing or decreasing size of an object using a scale factor

# Uniform Scaling

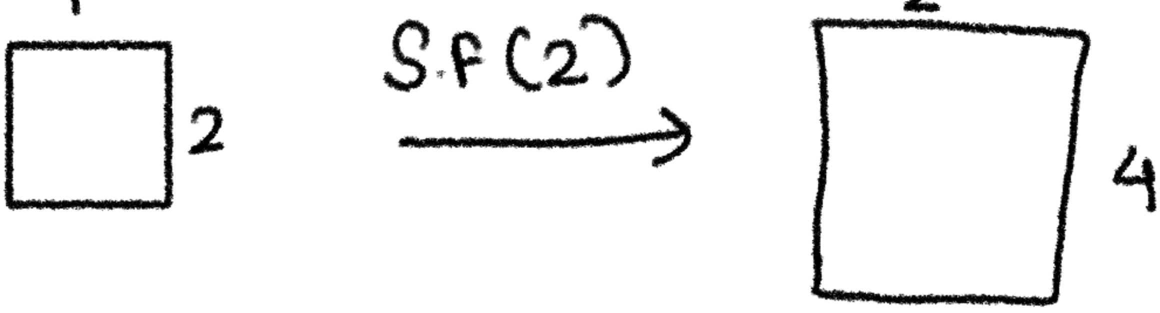
Conditions:

If scale factor = 1 [No change]

" Scalefactor > 1 [Dialation / increase]

" Scalefactor < 1 [contraction / decrease]

It is uniform as all sides of the object increases / decreases equally.




→ Preserves ratios, angles but not distances.

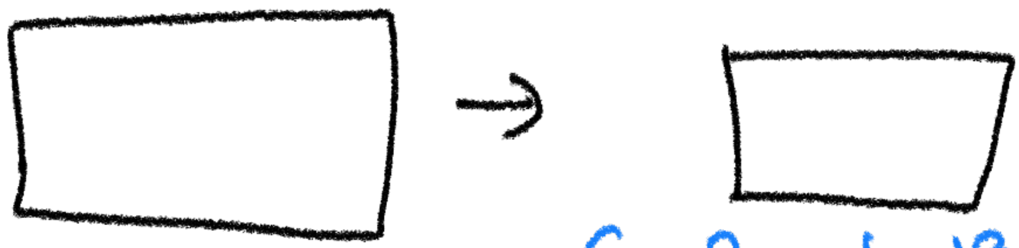
# Non-uniform Scaling

Conditions:

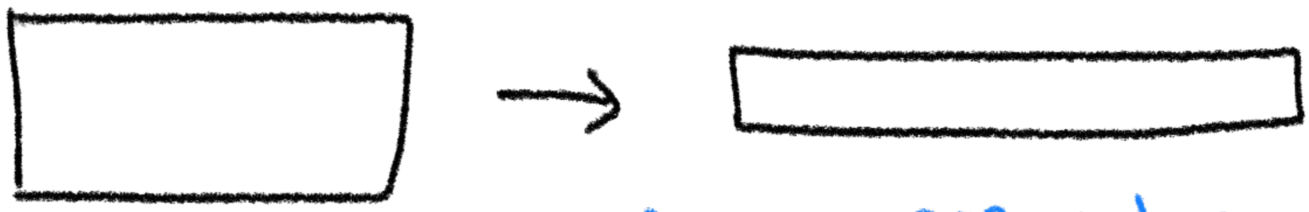
($x$ increases / decreases, $y$ remains same)
$x>1$ $x<1$

($x$ remains same, $y$ increases / decreases)
$y>1$ $y<1$

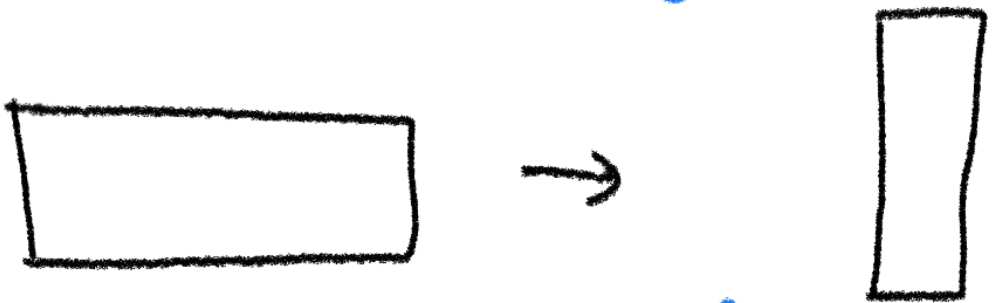
(x & y both increases)



(x & y both decreases)



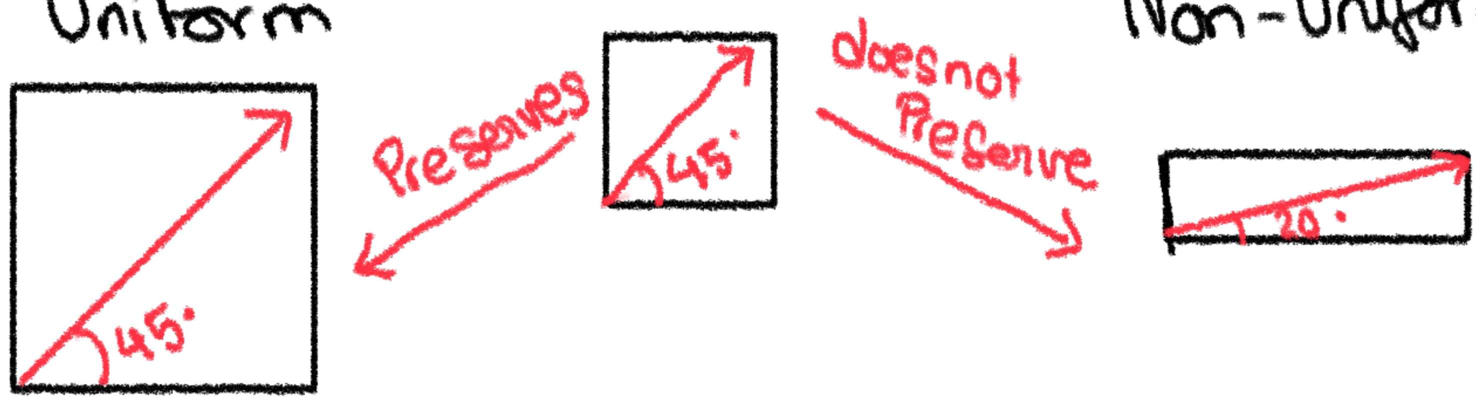
(x increases, y decreases)



(x decreases, y increase)

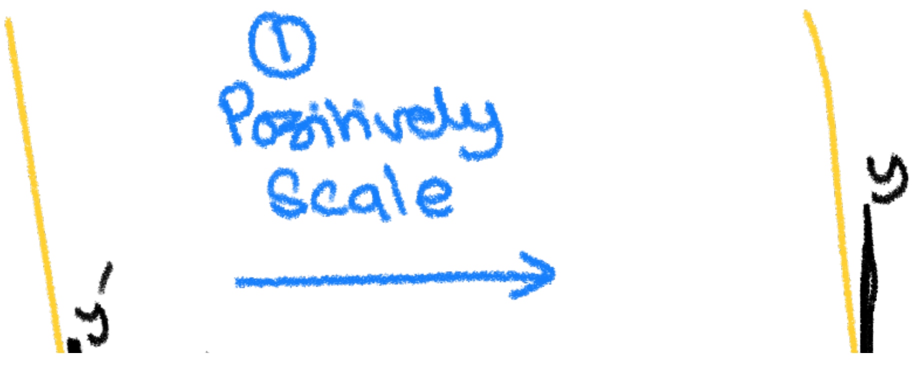
* Non-uniform doesnot preserve angle.

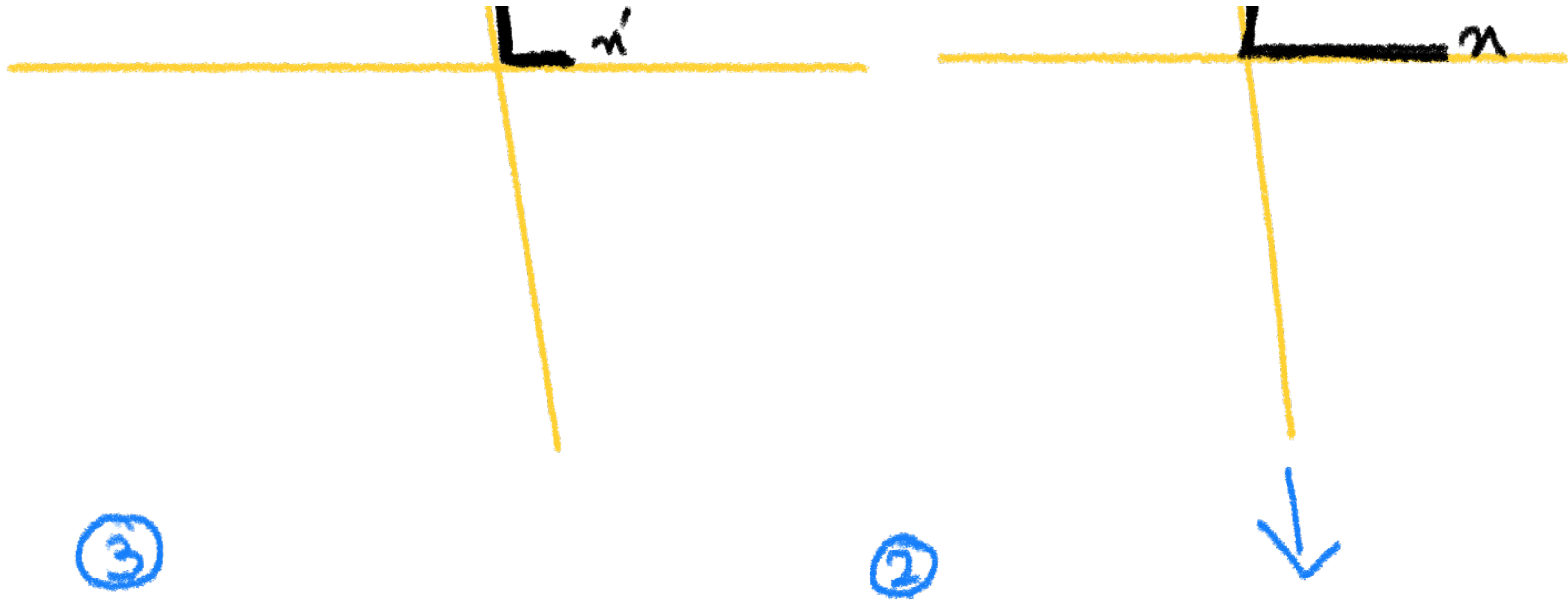
Uniform                                                                 Non-Uniform




## Scaling with a negative Factor

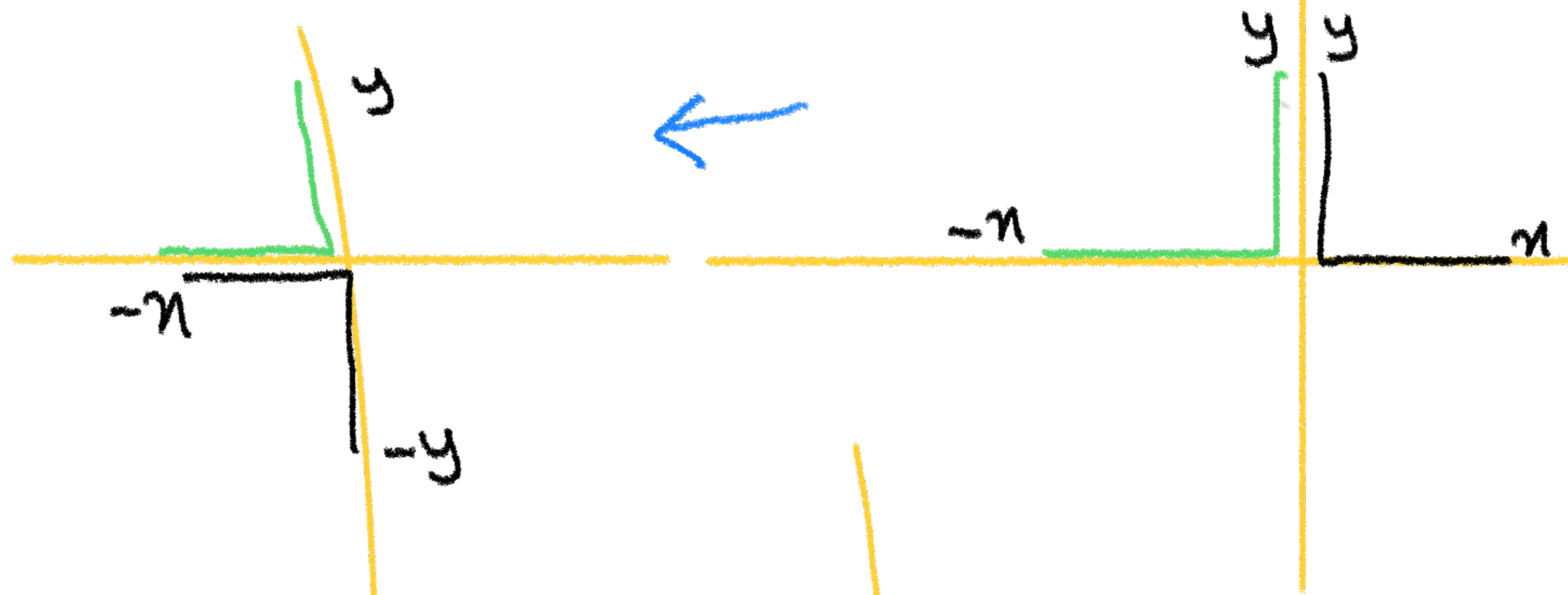
→ Incase of uniform if scale factor is negative $(-x, -y)$ we can positively scale first and reflect twice.



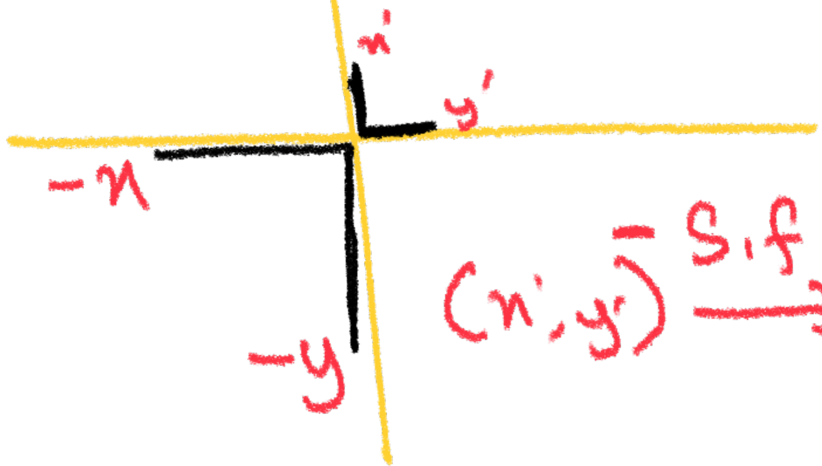



③ Reflected at $x$-axis for $-y$

② Reflected at $y$-axis for $-x$




∴ Orientation is preserved as not mirror image!




$(x', y') \xrightarrow{S.f} (-x, -y)$

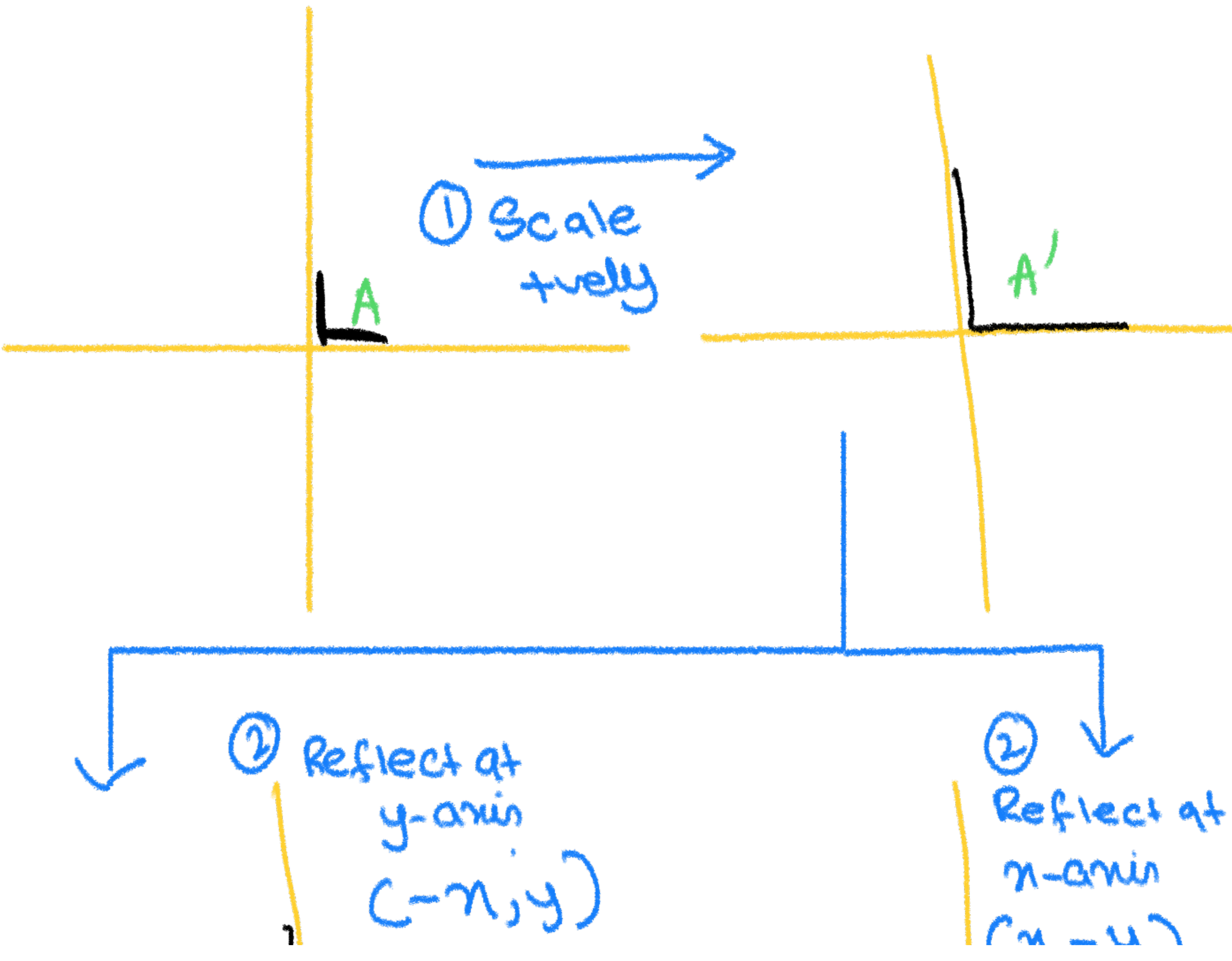
* As we reflect twice we can also say this as scaling followed by rotation / scaling followed by twice reflection

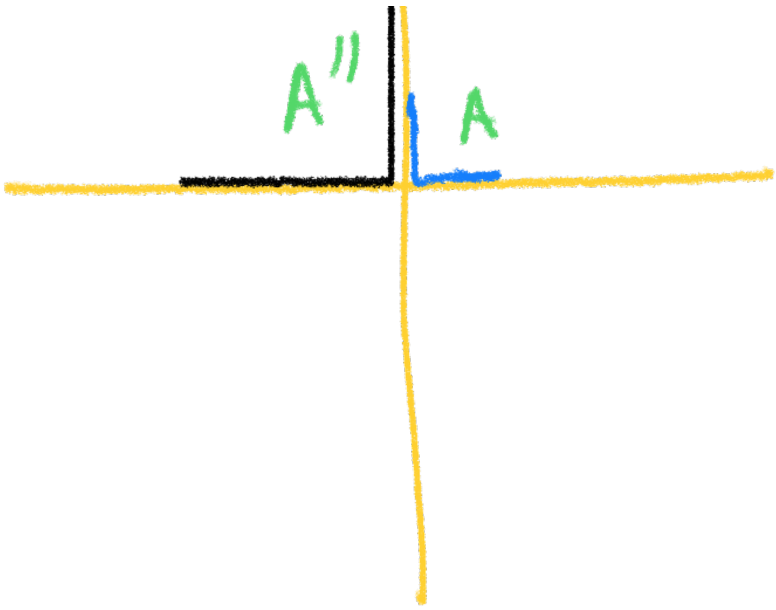
* For 3D as we reflect 3 times (odd number) $(-x, -y, -z)$ orientation is not preserved.

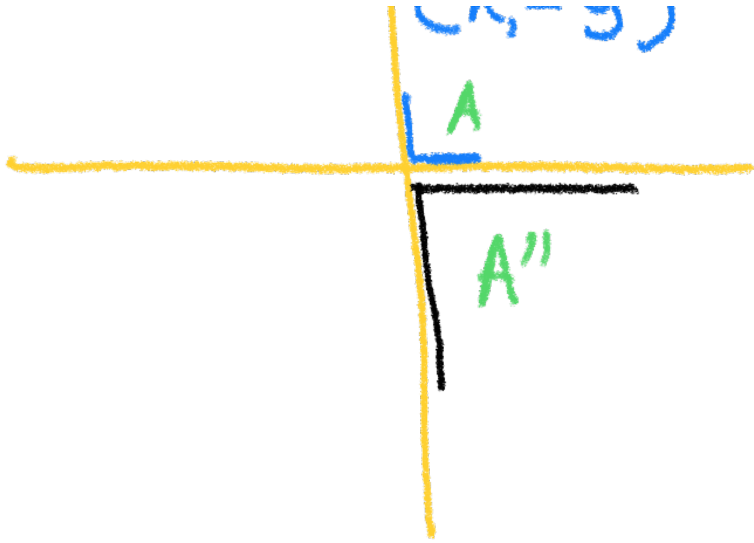

∴ we can say
orientation preservation depends on
no. of times we reflected!

→ Incase of non-uniform scaling
we reflect once depending on whether
$x$ or $y$ is negative.

If $x$ is -ve → Reflect y-axis
If $y$ is -ve → Reflect x-axis





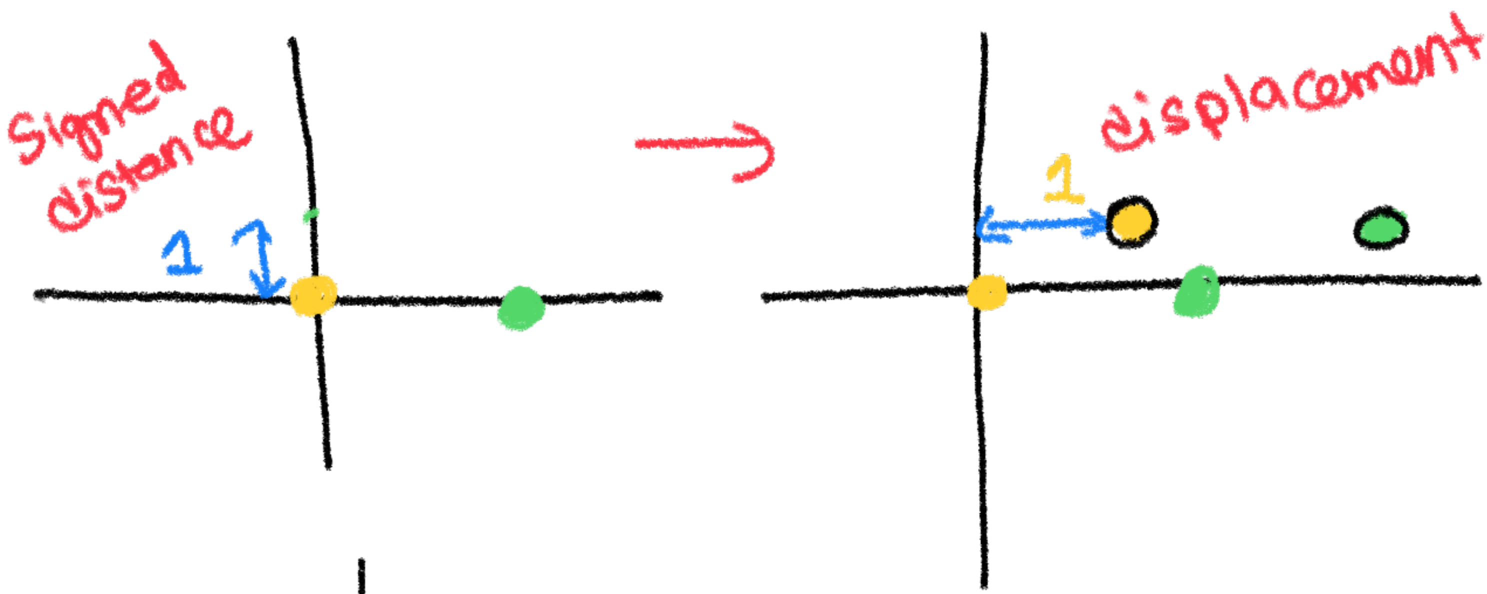



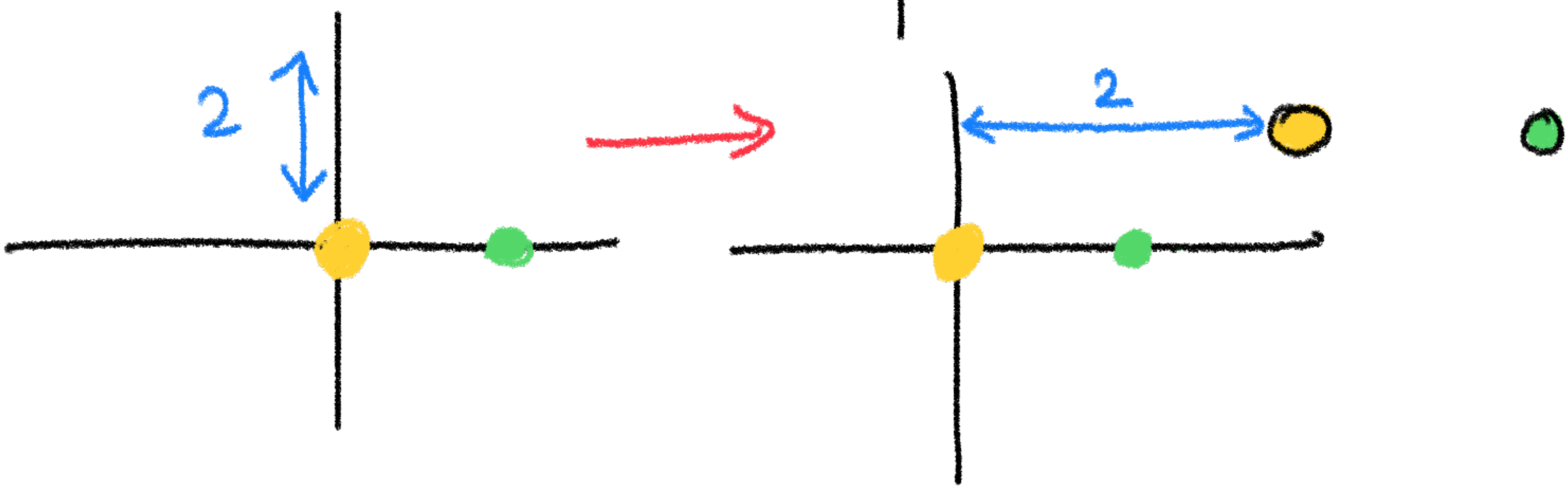

## Shear

The amount of translating each point in a Shape.

The amount of displacement is proportionate to its Signed distance

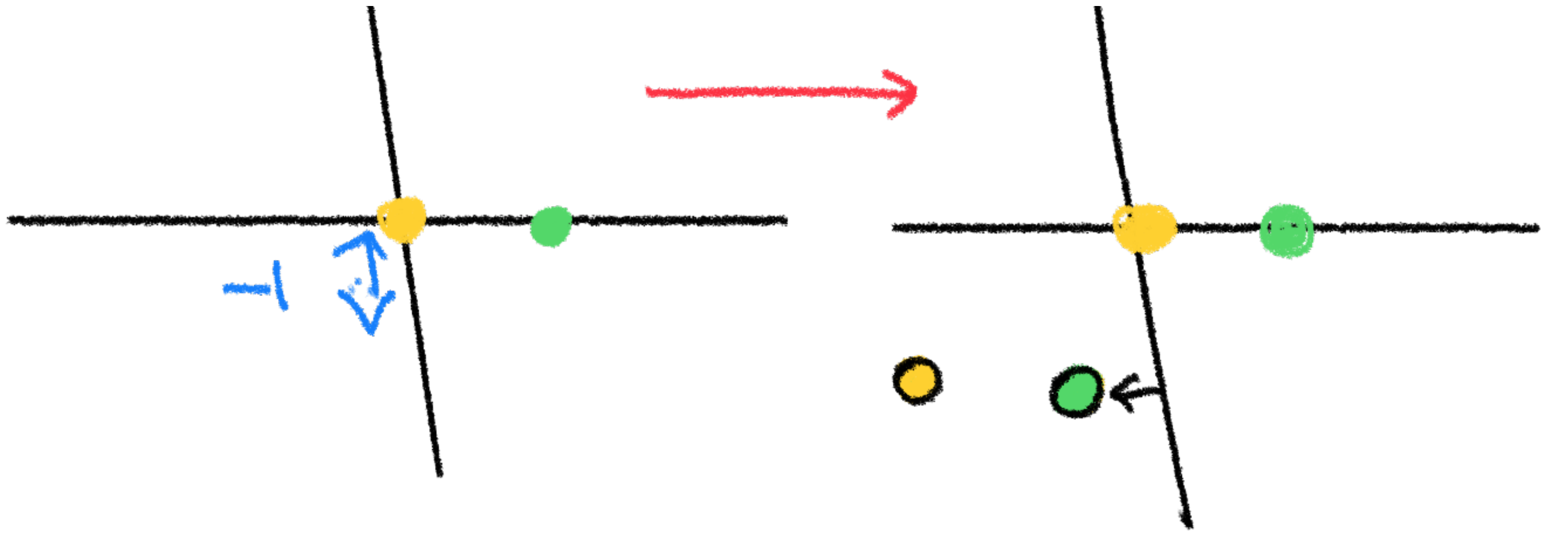
For ex: +1 updwards




Double distance = Double Displacement.

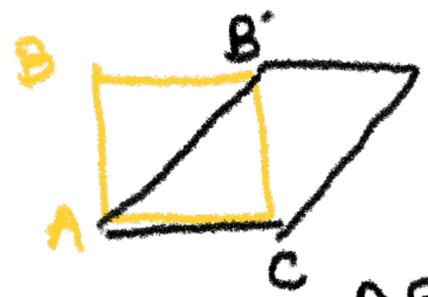
ex2: −1 downwards




* Preserves volume but not

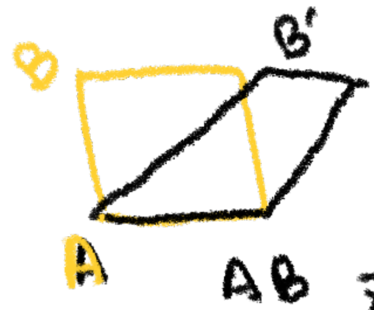
angles


ratios



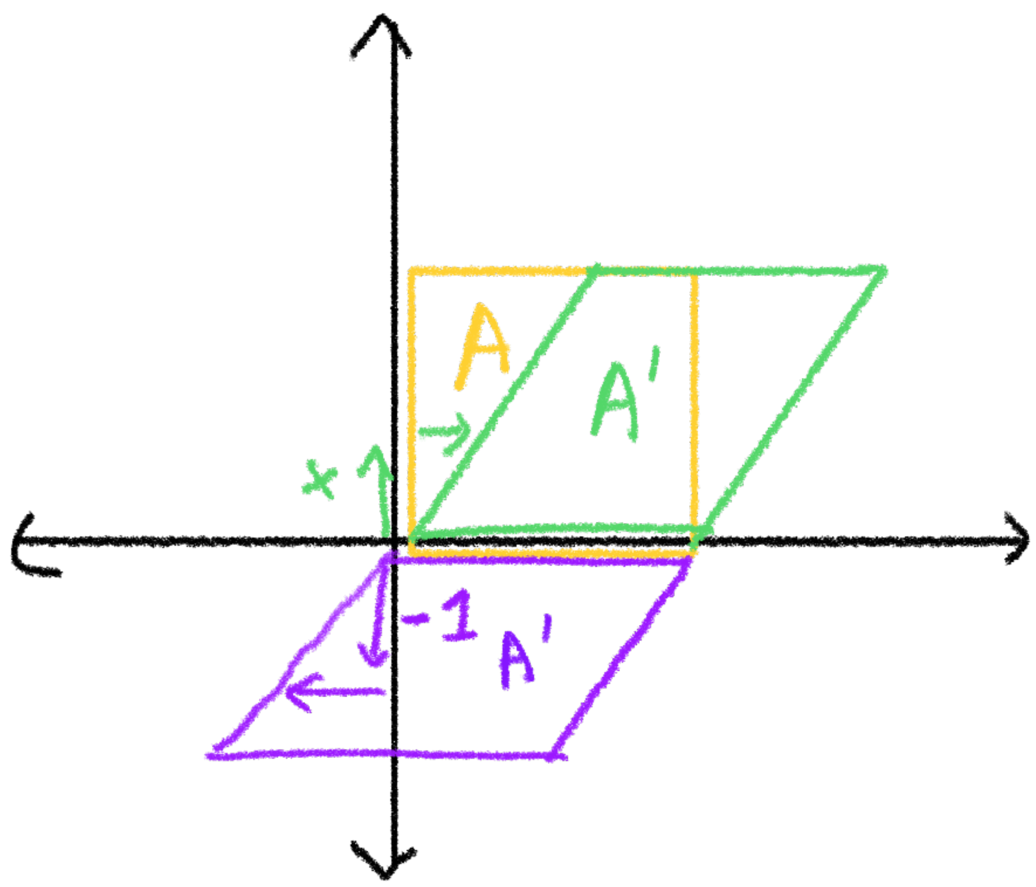

$$\frac{AB}{AC} \neq \frac{AB'}{AC}$$

distance
between two points.

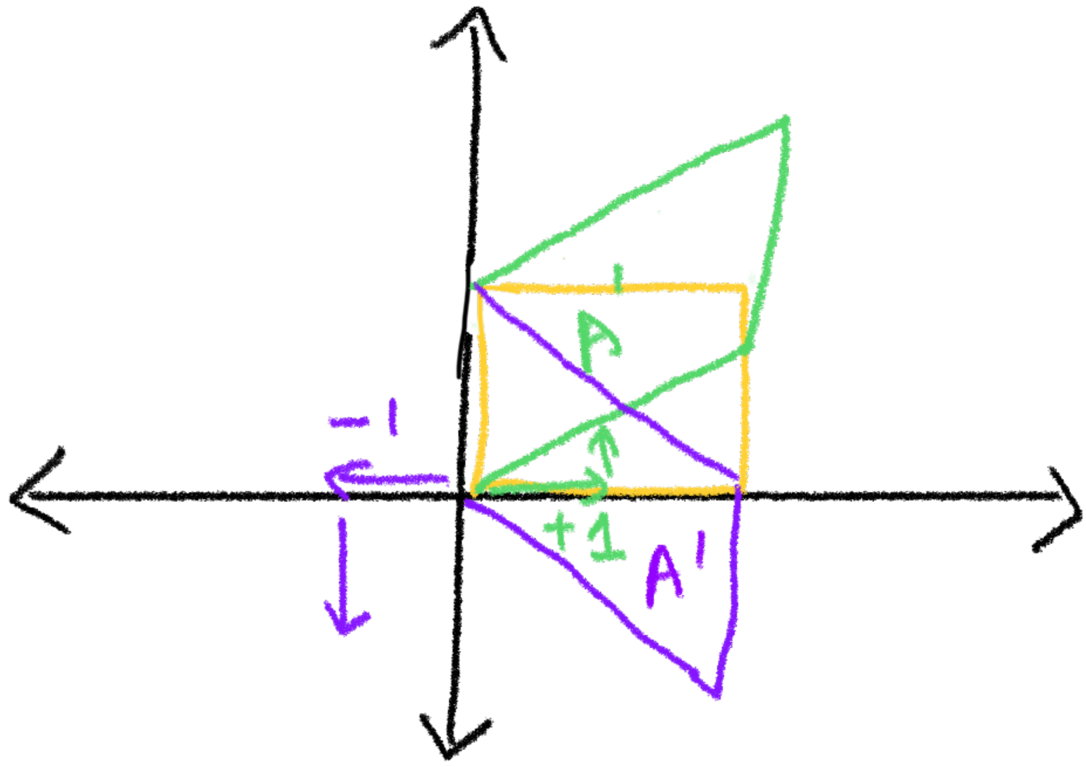



$AB \neq AB'$

## x-axis shear



# Y-axis Shear




## Summary

**Rigid / Eucledian transformation**

The combination of Translation, Rotation and Reflection
doesnot change shape & size
Angle, Ratio, Length is preserved.

**Similarity transformation.**

The combination of Translation, Rotation, Reflection & Uniform Scaling
doesnot change shape. Size can change.
Angle, Ratio preserved.

**Linear transformation.**

The combination of Shear, Rotation, Reflection & U/N.U Scaling doesnot change shape. Origin doesn't move.

Ratio preserved.

$T(\vec{u}) = \vec{v}$

$\therefore A\vec{u} = \vec{v}$

↓

matrix for different transformations.