CSE 423, Fall '21

Solution hints for Week 7 pop quiz questions

Linear Transformation 3

Q1

We want to apply a linear transformation on a rectangular object. The determinant of the transformation matrix is -17. That means, after the transformation the area of the object will be:

- ◯ 17 times smaller
- ◯ 17 times larger ✔
- ◯ same as before

If, determinant of the transformation matrix is x, area of the object will be |x| times larger after transformation. Reference from the lecture of week 7: https://youtu.be/xyRXkE1Soxw?list=PLMZnvBHNk2GWK8i1lcxx4exUXjtK8QSgK&t=930

Q2

You have learned how to perform scaling relative to the origin. However, it is possible to perform a scaling transformation that is relative to a point that is not the origin. Consider a scaling transformation relative to the point $(14, 12)$ This transformation scales everything by a factor of 5 along the x-axis and by factor of 13 along the y-axis, while $(14, 12)$ stays at where it is.The entire transformation can be expressed as a matrix M, which can be calculated as the multiplication $T_1\left(S_1\left(T_2\right)\right)$ where T_1 and T_2 are translation matrices using homogeneous coordinates and S_1 is a scaling matrix using homogeneous coordinates. If

$$M = \begin{bmatrix} M_{1,1} & M_{1,2} & M_{1,3} \\ M_{2,1} & M_{2,2} & M_{2,3} \\ M_{3,1} & M_{3,2} & M_{3,3} \end{bmatrix},$$

then answer the following questions.

What is the value of $M_{1,1}$, $M_{2,3}$ and $M_{3,2}$?

Let's assume, we are scaling relative to the point (x, y). Scaling factors along X-axis and Y-axis are α and β.

Matrix to translate to origin using homogenous coordinates, $T_2 = \begin{vmatrix} 1 & 0 & -x \\ 0 & 1 & -y \\ 0 & 0 & 1 \end{vmatrix}$

Matrix for scaling, $S_1 = \begin{vmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{vmatrix}$

Matrix to translate back, $T_1 = \begin{vmatrix} 1 & 0 & x \\ 0 & 1 & y \\ 0 & 0 & 1 \end{vmatrix}$

So, $T_1(S_1(T_2)) = \begin{vmatrix} 1 & 0 & x \\ 0 & 1 & y \\ 0 & 0 & 1 \end{vmatrix} . \begin{vmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{vmatrix} . \begin{vmatrix} 1 & 0 & -x \\ 0 & 1 & -y \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} \alpha & 0 & x - \alpha x \\ 0 & \beta & y - \beta y \\ 0 & 0 & 1 \end{vmatrix}$

Q3

Suppose, we apply a shear transformation on some 2D object along the x-axis, where our shear factor is 8. If $(38, 43)$ and $(63, 52)$ are two points within the 2D object, and if the area of the object is A, then,

Which of the following properties will be preserved after the shear transformation? (Choose all that apply)

- [ ] Lines within the 2D object will remain lines after the transformation. ✔
- [ ] The area of the 2D object, A, will change by a factor of 8.
- [ ] The area of the 2D object, A, will remain unchanged. ✔
- [ ] The distance between $(38, 43)$ and $(63, 52)$ will change after the transformation. ✔
- [ ] The distance between $(38, 43)$ and $(63, 52)$ along the y axis will change after the transformation.

All shear transformations leave areas unchanged (the determinant is 1). Lines remain lines after the transformation. If we apply the transformation along the x-axis, the x-coordinates change for all points (except on the x-axis itself), so their distances change. But distance across the y-axis doesn't.

Lighting Model

Q1

- [ ] Ambient ✔
- [ ] Diffuse
- [ ] Specular
- [ ] None of the above

Ambient reflection is applied irrespective of the position of light source. Diffuse, and Specular reflection will not be applicable here as the point on the object’s surface and light source are on opposite side to each other.

Q2

Why do we use 1 - $(d/r)^2$ as a replacement of $1/d^2$ while calculating the factor of attenuation? [Check all that apply]

- ☐ Because $1/d^2$ is not the real-life factor of attenuation, as physics tells us.
- ☐ Because using $1/d^2$ leads to a rapid decrease in attenuation over short distances. ✔
- ☐ Because when a point is very close to the light source, $1/d^2$ can lead to arithmetic overflow. ✔
- ☐ Because we want to limit the factor of attenuation between 0 and 1.

The equation $1 - (d/r)^2$ can set the upper bound of the factor of attenuation to 1 (when $d \rightarrow 0$). But it cannot limit the lower bound to 0 (when $d > r$). We need an extra condition for that.

Q3

Let's assume, Point on the surface P is (p, q); light source is at (a, b).
Then, L = (a, b) – (p, q) or (a-p, b-q).
Vector to the surface, n = (0, 1) [j-hat].

According to the derivation using vector projection, $R = 2(L.\underline{n})n - L$. Here, $\underline{n}$ is the unit vector of n. But, as n is already a unit vector, we can use as it is. If it wasn't, we would need to calculate a unit vector from n.

So, $L.\underline{n}$ = (a-p, b-q).(0, 1) = b-q [a scalar number]
Then, R = (0, 2b-2q) – (a-p, b-q) = (p-a, b-q).