Practice Problem for Transformation 2D

① $\theta = 45^\circ$ (counterclock wise)

$A(15,-5)$, $B(25,35)$

$$\text{midpoint} = \left(\frac{15+25}{2}, \frac{-5+35}{2}\right)$$
$$= (20, 15)$$

Translate $(-20,-15) \rightarrow$ Rotate $(45) \rightarrow$ Translate $(20,15)$

$$\text{Translation}(dx, dy) = \begin{bmatrix} 1 & 0 & dx \\ 0 & 1 & dy \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{Rotation}(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P' = T(20,15) \times R_{(45)} \times T_{(-20,-15)} \times P$$

Composite Matrix, $M = T_{(20,15)} \times R_{(45)} \times T_{(-20,-15)}$

$$= \begin{bmatrix} 1 & 0 & 20 \\ 0 & 1 & 15 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 45 & -\sin 45 & 0 \\ \sin 45 & \cos 45 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -20 \\ 0 & 1 & -15 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 20 \\ 0 & 1 & 15 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -20 \\ 0 & 1 & -15 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}}+0+0 & -\frac{1}{\sqrt{2}}+0+0 & 0+0+20 \\ 0+\frac{1}{\sqrt{2}}+0 & 0+\frac{1}{\sqrt{2}}+0 & 0+0+15 \\ 0+0+0 & 0+0+0 & 0+0+1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -20 \\ 0 & 1 & -15 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 20 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 15 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -20 \\ 0 & 1 & -15 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}}+0+0 & 0-\frac{1}{\sqrt{2}}+0 & -10\sqrt{2}+\frac{15}{\sqrt{2}}+20 \\ \frac{1}{\sqrt{2}}+0+0 & 0+\frac{1}{\sqrt{2}}+0 & -10\sqrt{2}-\frac{15}{\sqrt{2}}+15 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{40-5\sqrt{2}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{30-35\sqrt{2}}{2} \\ 0 & 0 & 1 \end{bmatrix}$$

Composite Matrix, $M = T_{(20,15)} \times R_{(45)} \times T_{(-20,-15)}$

$$= \begin{bmatrix} 1 & 0 & 20 \\ 0 & 1 & 15 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 45 & -\sin 45 & 0 \\ \sin 45 & \cos 45 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -20 \\ 0 & 1 & -15 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 20 \\ 0 & 1 & 15 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -20 \\ 0 & 1 & -15 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}}+0+0 & -\frac{1}{\sqrt{2}}+0+0 & 0+0+20 \\ 0+\frac{1}{\sqrt{2}}+0 & 0+\frac{1}{\sqrt{2}}+0 & 0+0+15 \\ 0+0+0 & 0+0+0 & 0+0+1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -20 \\ 0 & 1 & -15 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 20 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 15 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -20 \\ 0 & 1 & -15 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}}+0+0 & 0-\frac{1}{\sqrt{2}}+0 & -10\sqrt{2}+\frac{15}{\sqrt{2}}+20 \\ \frac{1}{\sqrt{2}}+0+0 & 0+\frac{1}{\sqrt{2}}+0 & -10\sqrt{2}-\frac{15}{\sqrt{2}}+15 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{40-5\sqrt{2}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{30-35\sqrt{2}}{2} \\ 0 & 0 & 1 \end{bmatrix}$$

Here, $P = \begin{bmatrix} 15 \\ -5 \\ 1 \end{bmatrix}$

$$\therefore P' = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{40-5\sqrt{2}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{30-35\sqrt{2}}{2} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 15 \\ -5 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{15}{\sqrt{2}} + \frac{5}{\sqrt{2}} + \frac{40-5\sqrt{2}}{2} \\ \frac{15}{\sqrt{2}} - \frac{5}{\sqrt{2}} + \frac{30-35\sqrt{2}}{2} \\ 0 + 0 + 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{40+15\sqrt{2}}{2} \\ \frac{30-25\sqrt{2}}{2} \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 30.6066 \\ -2.6776 \\ 1 \end{bmatrix}$$

$\therefore$ new coordinates $(30.6066, -2.677)$

When $P = \begin{bmatrix} 25 \\ 35 \\ 1 \end{bmatrix}$

$$P' = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{40-5\sqrt{2}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{30-35\sqrt{2}}{2} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 25 \\ 35 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{25}{\sqrt{2}} - \frac{35}{\sqrt{2}} + \frac{40-5\sqrt{2}}{2} \\ \frac{25}{\sqrt{2}} + \frac{35}{\sqrt{2}} + \frac{30-35\sqrt{2}}{2} \\ 0 + 0 + 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{40-15\sqrt{2}}{2} \\ \frac{30+25\sqrt{2}}{2} \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9.393 \\ 32.678 \\ 1 \end{bmatrix}$$

$\therefore$ new coordinates $= (9.393, 32.678)$

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(2) (i) $A(4,48)$, $B(43,0)$

$\therefore$ midpoint $\left(\frac{4+43}{2}, \frac{48+0}{2}\right) = (23.5, 24)$

Scaling 2.5 times in both axis.

Composite Matrix, $M = T_{(23.5, 24)} \times S_{(2.5, 2.5)} \times T_{(-23.5, -24)}$

$$= \begin{bmatrix} 1 & 0 & 23.5 \\ 0 & 1 & 24 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2.5 & 0 & 0 \\ 0 & 2.5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -23.5 \\ 0 & 1 & -24 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2.5+0+0 & 0+0+0 & 0+0+23.5 \\ 0+0+0 & 0+2.5+0 & 0+0+24 \\ 0+0+0 & 0+0+0 & 0+0+1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -23.5 \\ 0 & 1 & -24 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2.5 & 0 & 23.5 \\ 0 & 2.5 & 24 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -23.5 \\ 0 & 1 & -24 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2.5+0+0 & 0+0+0 & -58.75+0+23.5 \\ 0+0+0 & 0+2.5+0 & 0-60+24 \\ 0+0+0 & 0+0+0 & 0+0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 2.5 & 0 & -35.25 \\ 0 & 2.5 & -36 \\ 0 & 0 & 1 \end{bmatrix}$$

When, $P = \begin{bmatrix} 4 \\ 48 \\ 1 \end{bmatrix}$

$$P' = \begin{bmatrix} 2.5 & 0 & -35.25 \\ 0 & 2.5 & -36 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 48 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -25.25 \\ 84 \\ 1 \end{bmatrix}$$

$\therefore$ New coordinates $(-25.25, 84)$

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When, $p = \begin{bmatrix} 43 \\ 0 \\ 1 \end{bmatrix}$

$$p' = \begin{bmatrix} 2.5 & 0 & -35.25 \\ 0 & 2.5 & -36 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 43 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 72.25 \\ -36 \\ 1 \end{bmatrix}$$

$\therefore$ New coordinates $(72.25, -36)$

(3) $\theta = 45^\circ$ [counter clockwise]

Rotation with respect to $(-10, 0)$ point.

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$$M_{composite} = \cancel{T_{(0,0)}} \times T_{(-10,0)} \times R_{(45)} \times T_{(10,0)}$$

$$= \begin{bmatrix} 1 & 0 & -10 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 45 & -\sin 45 & 0 \\ \sin 45 & \cos 45 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & -10 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & -10+5\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 0 & 1 \end{bmatrix}$$

Here, $P' = MP$

$$\Rightarrow \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & -10+5\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & \cancel{1/\sqrt{2}}\ 5\sqrt{2} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

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$$M^{-1} = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & -10+5\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} & -5\sqrt{2} \\ 0 & 0 & 1 \end{bmatrix}$$

$$M^{-1}P' = M^{-1}MP$$

$$\Rightarrow P = M^{-1}P'$$

$$= \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & -10+5\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} & -5\sqrt{2} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -2.9289 \\ -7.071 \\ 1 \end{bmatrix}$$

$\therefore\ A(-2.9289, -7.071)$

(4) A(5,5)

$y = \sqrt{3} + x$

$y = x + \sqrt{3}$ $\quad \therefore m = 1,\ b = \sqrt{3}$

$\theta = \tan^{-1}(1)$

$= 45°$

$$M_{Composite} = T_{(0,\sqrt{3})} \times R_{(45)} \times Refl_{(x\,axis)} \times R_{(-45)} \times T_{(0,-\sqrt{3})}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \sqrt{3} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 45 & -\sin 45 & 0 \\ \sin 45 & \cos 45 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(-45) & -\sin(-45) & 0 \\ \sin(-45) & \cos(-45) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -\sqrt{3} \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \sqrt{3} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & -\frac{1}{\sqrt{2}} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & \frac{1}{\sqrt{2}} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -\sqrt{3} \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} & 1/\sqrt{2} & 0 \\ 1/\sqrt{2} & -1/\sqrt{2} & \sqrt{3} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -\sqrt{3} \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & -\sqrt{3} \\ 1 & 0 & \sqrt{3} \\ 0 & 0 & 1 \end{bmatrix}$$

When, $P = \begin{bmatrix} 5 \\ 5 \\ 1 \end{bmatrix}$

$$P' = \begin{bmatrix} 0 & 1 & -\sqrt{3} \\ 1 & 0 & \sqrt{3} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 5 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3.2679 \\ 6.732 \\ 1 \end{bmatrix}.$$

$\therefore$ New coordinate $(3.2679, 6.732)$

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When B $(-5,-5)$

$$P = \begin{bmatrix} -5 \\ -5 \\ 1 \end{bmatrix}$$

$$P' = \begin{bmatrix} 0 & 1 & -\sqrt{3} \\ 1 & 0 & \sqrt{3} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -5 \\ -5 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -6.732 \\ -3.2679 \\ 1 \end{bmatrix}$$

$\therefore$ new coordinate $(-6.732, -3.2679)$

When C $(0,0)$,

$$P = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\therefore P' = \begin{bmatrix} 0 & 1 & -\sqrt{3} \\ 1 & 0 & \sqrt{3} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -\sqrt{3} \\ \sqrt{3} \\ 1 \end{bmatrix}$$

$\therefore$ new coordinate $(-\sqrt{3}, \sqrt{3})$