Cyrus-Beck Clipping

2(a) $t_{left} = \dfrac{-(x_0 - x_{min})}{(x_1 - x_0)}$

Initially,

$t_E = 0,$

$t_L = 1$

$t_{right} = \dfrac{-(x_0 - x_{max})}{(x_1 - x_0)}$

$t_{top} = \dfrac{-(y_0 - y_{max})}{(y_1 - y_0)}$

$t_{bottom} = \dfrac{-(y_0 - y_{min})}{(y_1 - y_0)}$

For left boundary, $N_i = (-1, 0)$
right boundary, $N_i = (1, 0)$
bottom boundary, $N_i = (0, -1)$
top boundary, $N_i = (0, 1)$

$D = (x_1 - x_0, y_1 - y_0)$

$N_i \cdot D < 0$ হলে PE (Potentially Entering)
$N_i \cdot D > 0$ হলে PL (Potentially leaving)

2 b(i) $x_{min} = -200$, $y_{min} = -100$

$x_{max} = 200$, $y_{max} = 100$

$(-100, -100)$ to $(200, 0)$
$x_0 \; y_0 \quad x_1 \; y_1$

$D = (x_1 - x_0, y_1 - y_0) = (300, 100)$

Initially, $t_E = 0$, $t_L = 1$

Boundary	N_i	$N_i \cdot D$	t	PE/PL	t_E	t_L
Left	$(-1,0)$	$(-1\times300)+(0\times100) = -300$	-0.33	PE	0	1
Right	$(1,0)$	$(1\times300)+(0\times100) = 300$	1	PL	0	1
Bottom	$(0,-1)$	$(0\times300)+(-1\times100) = -100$	0	PE	0	1
Top	$(0,1)$	$(0\times300)+(1\times100) = 100$	2	PL	0	1

$$t_{left} = \frac{-(x_0 - x_{min})}{x_1 - x_0} = \frac{-(-100-(-200))}{(200-(-100))} = \frac{-100}{300} = -0.33$$

$$t_{right} = \frac{-(x_0 - x_{max})}{(x_1 - x_0)} = \frac{-(-100-200)}{200+100} = 1$$

$$t_{bottom} = \frac{-(y_0 - y_{min})}{(y_1 - y_0)} = \frac{-(-100+100)}{0+100} = 0$$

$$t_{top} = \frac{-(y_0 - y_{max})}{(y_1 - y_0)} = \frac{-(-100-100)}{0+100} = 2$$

$$\therefore P(0) = (x_0, y_0) + t \times D$$
$$= (-100, -100) + 0 \times (300, 100)$$
$$= (-100, -100) + (0, 0)$$
$$= (-100+0, -100+0)$$
$$= (-100, -100)$$

$$P(1) = (x_0, y_0) + t \times D$$
$$= (-100, -100) + 1 \times (300, 100)$$
$$= (-100, -100) + (300, 100)$$
$$= (-100+300, -100+100)$$
$$= (200, 0)$$

$(-100, -100)$ and $(200, 0)$ are the end points of the clipped line.

Scanned with CamScanner

2b (ii) $x_{min} = -200, \quad y_{min} = -100$

$x_{max} = 200, \quad y_{max} = 100$

$(50, -105)$ to $(150, -200)$
$x_0 \quad y_0 \qquad x_1 \quad y_1$

$D = (x_1 - x_0, y_1 - y_0) = (150 - 50, -200 + 105) = (100, -95)$

Initially $t_E = 0, \; t_L = 1$

$$t_{left} = \frac{-(x_0 - x_{min})}{(x_1 - x_0)} = \frac{-(50+200)}{150-50} = -2.5$$

$$t_{right} = \frac{-(x_0 - x_{max})}{(x_1 - x_0)} = \frac{-(50-200)}{150-50} = 1.5$$

$$t_{bottom} = \frac{-(y_0 - y_{min})}{(y_1 - y_0)} = \frac{-(-105+100)}{-200+105} = -0.05$$

$$t_{top} = \frac{-(y_0 - y_{max})}{(y_1 - y_0)} = \frac{-(-105-100)}{-200+105} = -2.16$$

Dot product

Boundary	N_i	$N_i \cdot D$	t	PE/PL	t_E	t_L
Left	$(-1,0)$	$-1\cdot100+0\cdot-95$ $= -100$	-2.5	PE	0	1
Right	$(1,0)$	$1\cdot100+0\cdot-95$ $= 100$	1.5	PL	0	1
Bottom	$(0,-1)$	$0\cdot100+(-1\cdot-95)$ $= 95$	-0.05	PL	0	-0.05
Top	$(0,1)$	$0\cdot100+1\cdot-95$ $= -95$	-2.16	PE	0	-0.05

$t_E > t_L \longrightarrow$ NULL (line is completely outside)

2b(iii) $x_{min} = -200$, $y_{min} = -100$

$x_{max} = 200$, $y_{max} = 100$

$(\underset{x_0}{-160}, \underset{y_0}{140})$ to $(\underset{x_1}{-240}, \underset{y_1}{80})$

$D = (x_1 - x_0, y_1 - y_0) = (-240+160, 80-140) = (-80, -60)$

Initially $t_E = 0$, $t_L = 1$

$$t_{left} = \frac{-(x_0 - x_{min})}{x_1 - x_0} = \frac{-(-160+200)}{-240+160} = 0.5$$

$$t_{right} = \frac{-(x_0 - x_{max})}{x_1 - x_0} = \frac{-(-160-200)}{-240+160} = -4.5$$

$$t_{bottom} = \frac{-(y_0 - y_{min})}{(y_1 - y_0)} = \frac{-(140+100)}{80-140} = 4$$

$$t_{top} = \frac{-(y_0 - y_{max})}{y_1 - y_0} = \frac{-(140-100)}{80-140} = 0.67$$

Boundary	N_i	$N_i \cdot D$	t	PE/PL	t_E	t_L
Left	(-1,0)	80	0.5	PL	0	0.5
Right	(1,0)	-80	-4.5	PE	0	0.5
Bottom	(0,-1)	60	4	PL	0	0.5
Top	(0,1)	-60	0.67	PE	0.67	0.5

$$P(0.67) = (x_0, y_0) + t \times D$$
$$= (-160, 140) + (0.67 \times (-80, -60))$$
$$= (-160, 140) + (-53.6, -40.2)$$
$$= (-213.6, 99.8)$$

Here $t_E > t_L \rightarrow$ the line is completely outside.

2b (iv) $x_{min} = -200$, $y_{min} = -100$

$x_{max} = 200$, $y_{max} = 100$

$(-300, 120)$ to $(250, -110)$

$x_0 \quad y_0 \qquad x_1 \quad y_1$

$$D = (x_1 - x_0, y_1 - y_0) = (250+300, -110-120) = (550, -230)$$

Initially $t_E = 0$, $t_L = 1$

$$t_{left} = \frac{-(x_0 - x_{min})}{(x_1 - x_0)} = \frac{-(-300+200)}{250+300} = \frac{2}{11} = 0.1818181818$$

$$t_{right} = \frac{-(x_0 - x_{max})}{(x_1 - x_0)} = \frac{-(-300-200)}{250+300} = 0.9090909091$$

$$t_{bottom} = \frac{-(y_0 - y_{min})}{y_1 - y_0} = \frac{-(120+100)}{-110-120} = 0.9565217391$$

$$t_{top} = \frac{-(y_0 - y_{max})}{(y_1 - y_0)} = \frac{-(120-100)}{-110-120} = 0.08695652174$$

Boundary	N_i	$N_i \cdot D$	t	PE/PL	t_E	t_L
left	(-1,0)	-550	0.1818181818	PE	0.181818	1
right	(1,0)	550	0.9090909091	PL	0.181818	0.9090
bottom	(0,-1)	230	0.9565217391	PL	0.181818	0.9090
top	(0,1)	-230	0.08695652174	PE	0.181818	0.9090

$$P(0.1818181818) = (-300,120) + t \times (550,-230)$$
$$= (-300,120) + (100,-41.81818182)$$
$$= (-200, 78.18181818)$$

$$P(0.9090909091) = (-300,120) + t\,(550,-230)$$
$$= (-300,120) + (500,-209.0909091)$$
$$= (200, -89.09090909)$$

(-200, 78.18181818) and (200, -89.09090909) are the true clip intersection.

2c ① $x_{min} = 0, \quad y_{min} = 0$

$x_{max} = 300, \quad y_{max} = 200$

$(100, 0)$ to $(400, 100)$
$x_0 \; y_0 \quad\quad x_1 \; y_1$

$D = (x_1 - x_0, y_1 - y_0) = (400 - 100, 100 - 0) = (300, 100)$

Initially, $t_E = 0, \; t_L = 1$

$$t_{left} = \frac{-(x_0 - x_{min})}{(x_1 - x_0)} = \frac{-(100-0)}{400-100} = -\frac{1}{3} = -0.333$$

$$t_{right} = \frac{-(x_0 - x_{max})}{(x_1 - x_0)} = \frac{-(100-300)}{400-100} = \frac{2}{3} = 0.666667$$

$$t_{top} = \frac{-(y_0 - y_{max})}{(y_1 - y_0)} = \frac{-(0-200)}{(100-0)} = 2$$

$$t_{bottom} = \frac{-(y_0 - y_{min})}{(y_1 - y_0)} = \frac{-(0-0)}{100-0} = 0$$

Boundary	N_i	$N_i \cdot D$	t	PE/PL	t_E	t_L
Left	(-1,0)	-300	-0.3333	PE	0	1
Right	(1,0)	300	0.666667	PL	0	0.66667
Bottom	(0,-1)	-100	0	PE	0	0.66667
Top	(0,1)	100	2	PL	0	0.66667

$$P(0) = (x_0, y_0) + 0 \times (300, 100)$$
$$= (100, 0) + (0, 0)$$
$$= (100, 0)$$

$$P(0.66667) = (100, 0) + 0.66667\,(300, 100)$$
$$= (100, 0) + (200, 66.66667)$$
$$= (300, 66.66667)$$

(100, 0) and (300, 66.66667) are the true clip intersection.

2c(ii) $x_{min}=0$, $y_{min}=0$

$x_{max}=300$, $y_{max}=200$

$(250,-5)$ to $(350,-100)$

$x_0 \quad y_0 \quad\quad x_1 \quad y_1$

$D = (x_1-x_0, y_1-y_0) = (350-250, -100+5) = (100,-95)$

Initially, $t_E=0$, $t_L=1$

$$t_{left} = \frac{-(x_0-x_{min})}{x_1-x_0} = \frac{-(250-0)}{350-250} = -2.5$$

$$t_{right} = \frac{-(x_0-x_{max})}{x_1-x_0} = \frac{-(250-300)}{350-250} = 0.5$$

$$t_{bottom} = \frac{-(y_0-y_{min})}{(y_1-y_0)} = \frac{-(-5-0)}{-100+5} = -0.052631$$

$$t_{top} = \frac{-(y_0-y_{max})}{(y_1-y_0)} = \frac{-(-5-200)}{(-100+5)} = -2.15789$$

Boundary	N_i	$N_i \cdot D$	t	PE/PL	t_E	t_L
Left	$(-1,0)$	-100	-2.5	PE	0	1
Right	$(1,0)$	100	0.5	PL	0	0.5
Bottom	$(0,-1)$	95	-0.052631	PL	0	-0.052631
Top	$(0,1)$	-95	-2.15789	PE	0	-0.052631

$t_E > t_L \longrightarrow$ (line is completely outside)

$$P(0) = (250, -5)$$

$$P(-0.052631) = (x_0, y_0) + \left((-0.052631) \times (100, -95)\right)$$

$$= (250, -5) + (-5.2631, 5)$$

$$= (244.7368, 0)$$

$(250, -5)$ and $(244.7368, 0)$ are the true clip intersection.

2(iii) $x_{min} = 0$, $y_{min} = 0$

$x_{max} = 300$, $y_{max} = 200$

$(40, 240)$ to $(-40, 180)$

$x_0 \;\; y_0 \qquad x_1 \;\; y_1$

$D = (x_1 - x_0, y_1 - y_0) = (-40 - 40, 180 - 240)$
$= (-80, -60)$

Initially, $t_E = 0$, $t_L = 1$

$$t_{left} = \frac{-(x_0 - x_{min})}{(x_1 - x_0)} = \frac{-(40 - 0)}{(-40 - 40)} = 0.5$$

$$t_{right} = \frac{-(x_0 - x_{max})}{(x_1 - x_0)} = \frac{-(40 - 300)}{-40 - 40} = -3.25$$

$$t_{bottom} = \frac{-(y_0 - y_{min})}{(y_1 - y_0)} = \frac{-(240 - 0)}{180 - 240} = 4$$

$$t_{top} = \frac{-(y_0 - y_{max})}{(y_1 - y_0)} = \frac{-(240 - 200)}{180 - 240} = \frac{2}{3} = 0.6667$$

Boundary	N_i	$N_i \cdot D$	t	PE/PL	t_E	t_L
Left	$(-1,0)$	80	0.5	PL	0	0.5
Right	$(1,0)$	−80	−3.25	PE	0	0.5
Bottom	$(0,-1)$	60	4	PL	0	0.5
Top	$(0,1)$	−60	0.6667	PE	0.6667	0.5

$t_E > t_L \rightarrow$ (line is completely outside)

2c (iv) $x_{min} = 0, \quad y_{min} = 0$

$x_{max} = 300, \quad y_{max} = 200$

$(-100, 220)$ to $(450, -10)$
$\quad x_0 \quad y_0 \qquad\quad x_1 \quad y_1$

$D = (x_1 - x_0, y_1 - y_0) = (450+100, -10-220) = (550, -230)$

Initially $t_E = 0, \; t_L = 1$

$$t_{left} = \frac{-(x_0 - x_{min})}{x_1 - x_0} = \frac{-(-100-0)}{450+100} = 0.181$$

$$t_{right} = \frac{-(x_0 - x_{max})}{x_1 - x_0} = \frac{-(-100-300)}{450+100} = 0.727$$

$$t_{bottom} = \frac{-(y_0 - y_{min})}{(y_1 - y_0)} = \frac{-(220-0)}{(-10-220)} = 0.957$$

$$t_{top} = \frac{-(y_0 - y_{max})}{y_1 - y_0} = \frac{-(220-200)}{-10-220} = 0.0869$$

Boundary	N_i	$N_i \cdot D$	t	PE/PL	t_E	t_L
Left	$(-1,0)$	-550	0.181	PE	0.181	1
Right	$(1,0)$	550	0.727	PL	0.181	0.727
Bottom	$(0,-1)$	230	0.957	PL	0.181	0.727
Top	$(0,1)$	-230	0.0869	PE	0.181	0.727

$$P(0.181) = (-100, 220) + 0.181 \times (550, -230)$$
$$= (-100, 220) + (100, -41.63)$$
$$= (0, 178.37)$$

$$P(0.727) = (-100, 220) + 0.727\,(550, -230)$$
$$= (-100, 220) + (400, -167.27272727)$$
$$= (300, 52.727)$$

$(0, 178.37)$ and $(300, 52.727)$ are the true clip intersec