Parameter Update 2

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Assume that the target class is at node 5. At first start with the output nodes. Define loss function and differentiate loss function with respect to all the output nodes.

$$Loss = -\log P_5$$
$$\frac{dloss}{dP_5} = -\frac{1}{P_5}$$
$$\frac{dLoss}{dY_5} = \frac{dLoss}{dP_5}.\frac{dP_5}{dY_5} = -\frac{1}{P_5}.P_5(1-P_5) = P_5 - 1$$
$$\frac{dLoss}{dY_6} = \frac{dLoss}{dP_5}.\frac{dP_5}{dY_6} = -\frac{1}{P_5}.(-P_5.P_6) = P_6$$
$$\frac{dLoss}{dY_7} = \frac{dLoss}{dP_5}.\frac{dP_5}{dY_7} = -\frac{1}{P_5}.(-P_5.P_7) = P_7$$

Write the expressions for output nodes

$$Y_5 = W_{35}Y_3 + W_{45}Y_4 + b_5$$
$$Y_6 = W_{36}Y_3 + W_{46}Y_4 + b_6$$
$$Y_7 = W_{37}Y_3 + W_{47}Y_4 + b_7$$

Now you can differentiate loss function with respect to all the output layer parameters $W_{35}, W_{45}, W_{36}, W_{46}, W_{37}, W_{47}, b_5, b_6, b_7$ using chain rule

$$\frac{dLoss}{dW_{35}} = \frac{dloss}{dY_5}.\frac{dY_5}{dW_{35}} = (P_5 - 1).Y_3$$
$$\frac{dLoss}{dW_{45}} = \frac{dloss}{dY_5}.\frac{dY_5}{dW_{45}} = (P_5 - 1).Y_4$$
$$\frac{dLoss}{dW_{36}} = \frac{dloss}{dY_5}.\frac{dY_5}{dW_{36}} = P_6.Y_3$$
$$\frac{dLoss}{dW_{46}} = \frac{dloss}{dY_6}.\frac{dY_6}{dW_{46}} = P_6.Y_4$$
$$\frac{dLoss}{dW_{37}} = \frac{dloss}{dY_7}.\frac{dY_7}{dW_{37}} = P_7.Y_3$$
$$\frac{dLoss}{dW_{47}} = \frac{dloss}{dY_7}.\frac{dY_7}{dW_{47}} = P_7.Y_4$$
$$\frac{dLoss}{db_5} = \frac{dloss}{dY_5}.\frac{dY_5}{db_5} = P_5 - 1$$
$$\frac{dLoss}{db_6} = \frac{dloss}{dY_6}.\frac{dY_6}{db_6} = P_6$$
$$\frac{dLoss}{db_7} = \frac{dloss}{dY_7}.\frac{dY_7}{db_7} = P_7$$

Now using gradient descent approach the update equations for all the output layer parameters are:

$$W_{35} = W_{35} - \eta.Y_3.(P_5 - 1)$$
$$W_{45} = W_{45} - \eta.Y_4.(P_5 - 1)$$

$$W_{36} = W_{36} - \eta.Y_3.P_6$$
$$W_{46} = W_{46} - \eta.Y_4.P_6$$

$$W_{37} = W_{37} - \eta.Y_3.P_7$$
$$W_{47} = W_{47} - \eta.Y_4.P_7$$

$$b_5 = b_5 - \eta.(P_5 - 1)$$
$$b_6 = b_6 - \eta.P_6$$
$$b_7 = b_7 - \eta.P_7$$

Now consider the hidden nodes 3 and 4. Write the expressions for Y_3 and Y_4.

$$Y_3 = Relu\{W_{13}.X_1 + W_{23}.X_2 + b_3\}$$
$$Y_4 = Relu\{W_{14}.X_1 + W_{24}.X_2 + b_4\}$$

Now differentiate the Loss function with respect to hidden outputs Y_3 and Y_4. Both nodes 3 and 4 have parents 5, 6 and 7. Following the chain rule

$$\frac{dLoss}{dY_3} = \frac{dloss}{dY_5}.\frac{dY_5}{dY_3} + \frac{dloss}{dY_6}.\frac{dY_6}{dY_3} + \frac{dloss}{dY_7}.\frac{dY_7}{dY_3} = (P_5 - 1).W_{35} + P_6.W_{36} + P_7.W_{37}$$

$$\frac{dLoss}{dY_4} = \frac{dloss}{dY_5}.\frac{dY_5}{dY_4} + \frac{dloss}{dY_6}.\frac{dY_6}{dY_4} + \frac{dloss}{dY_7}.\frac{dY_7}{dY_4} = (P_5 - 1).W_{45} + P_6.W_{46} + P_7.W_{47}$$

Now we can differentiate Loss function with respect to all the hidden layer parameters W_{13}, W_{23}, W_{14}, W_{24}, b_3 and b_4

$$\frac{dLoss}{dW_{13}} = \frac{dLoss}{dY_3}.\frac{dY_3}{dW_{13}} = \{(P_5-1).W_{35}+P_6.W_{36}+P_7.W_{37}\}.Relu^{'}(W_{13}.X_1+W_{23}.X_2+b_3).X_1$$

$$\frac{dLoss}{dW_{23}} = \frac{dLoss}{dY_3}.\frac{dY_3}{dW_{23}} = \{(P_5-1)).W_{35}+P_6.W_{36}+P_7.W_{37}\}.Relu^{'}(W_{13}.X_1+W_{23}.X_2+b_3).X_2$$

$$\frac{dLoss}{dW_{14}} = \frac{dLoss}{dY_4}.\frac{dY_4}{dW_{14}} = \{(P_5-1).W_{45}+P_6.W_{46}+P_7.W_{47}\}.Relu^{'}(W_{14}.X_1+W_{24}.X_2+b_4).X_1$$

$$\frac{dLoss}{dW_{24}} = \frac{dLoss}{dY_4}.\frac{dY_4}{dW_{24}} = \{(P_5-1).W_{45}+P_6.W_{46}+P_7.W_{47}\}.Relu^{'}(W_{14}.X_1+W_{24}.X_2+b_4).X_2$$

$$\frac{dLoss}{db_3} = \frac{dLoss}{dY_3}.\frac{dY_3}{db_3} = \{(P_5-1).W_{35}+P_6.W_{36}+P_7.W_{37}\}.Relu^{'}(W_{13}.X_1+W_{23}.X_2+b_3)$$

$$\frac{dLoss}{db_4} = \frac{dLoss}{dY_4}.\frac{dY_4}{db_4} = \{(P_5-1).W_{45}+P_6.W_{46}+P_7).W_{47}\}.Relu^{'}(W_{14}.X_1+W_{24}.X_2+b_4)$$

Therefore the updated equations for all the hidden layer parameters are as follows:

$$W_{13} = W_{13}+\eta.((P_5-1).W_{35}+P_6.W_{36}+P_7.W_{37}\}.Relu^{'}(W_{13}.X_1+W_{23}.X_2+b_3).X_1$$

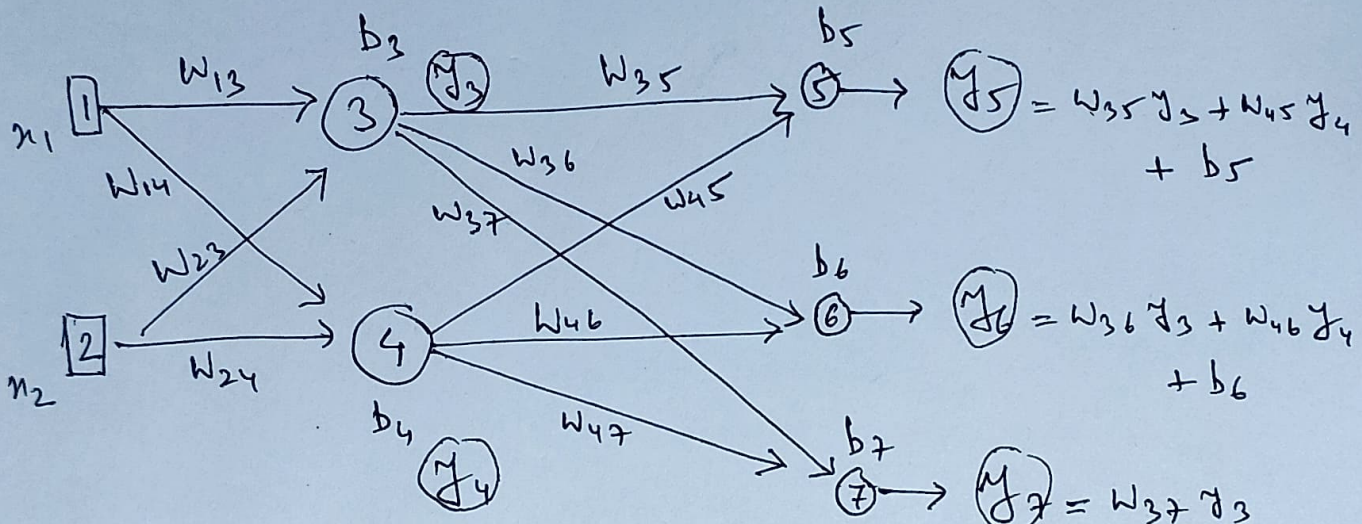
$$W_{23} = W_{23}+\eta.((P_5-1).W_{35}+P_6.W_{36}+P_7.W_{37}).Relu^{'}(W_{13}.X_1+W_{23}.X_2+b_3).X_2$$

$$W_{14} = W_{14}+\eta.((P_5-1).W_{45}+P_6.W_{46}+P_7.W_{47}).Relu^{'}(W_{14}.X_1+W_{24}.X_2+b_4).X_1$$

$$W_{24} = dW_{24}+\eta.((P_5-1).W_{45}+P_6.W_{46}+P_7.W_{47}).Relu^{'}(W_{14}.X_1+W_{24}.X_2+b_4).X_2$$

$$b_3 = b_3 + \eta.((P_5 - 1).W_{35} + P_6.W_{36} + P_7.W_{37}).Relu^{'}(W_{13}.X_1 + W_{23}.X_2 + b_3)$$

$$b_4 = b_4 + \eta.((P_5 - 1).W_{45} + P_6.W_{46} + P_7.W_{47}).Relu^{'}(W_{14}.X_1 + W_{24}.X_2 + b_4)$$



$$y_3 = \text{Relu}(W_{13} x_1 + W_{23} x_2 + b_3)$$

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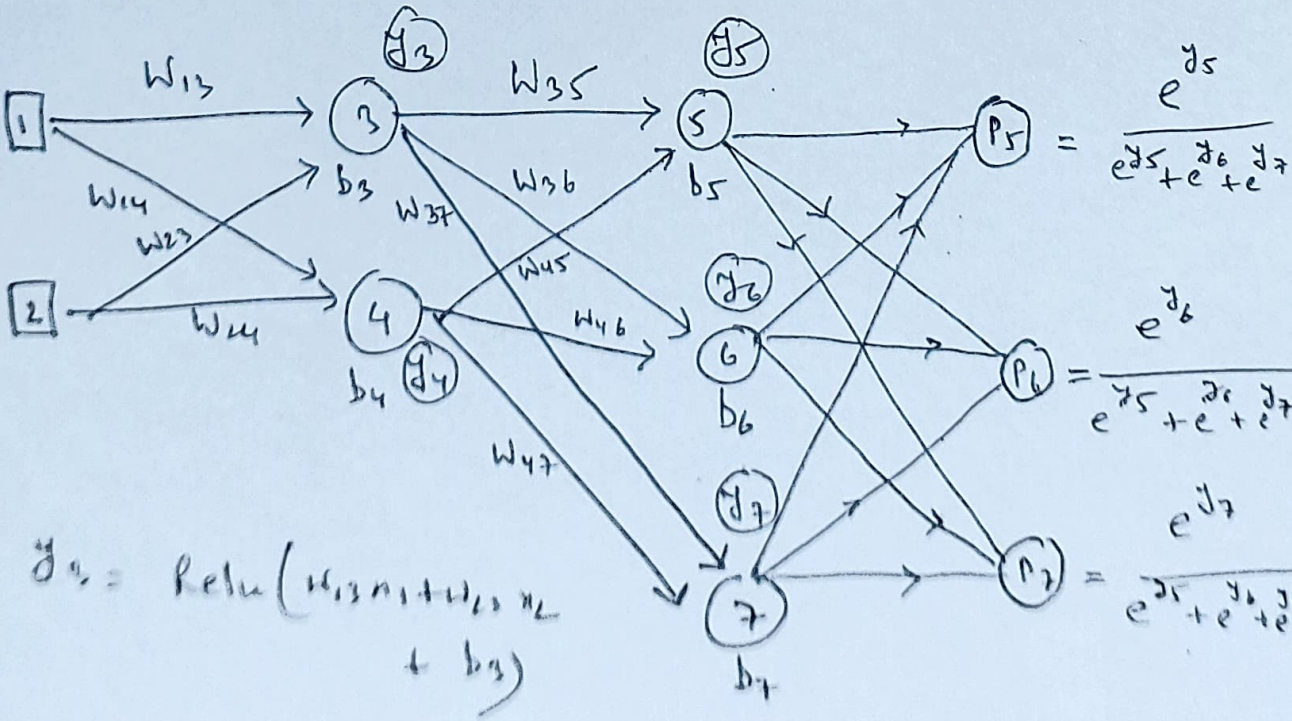

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