CSE428: Image Processing

Lecture 4: Point Processing - Part 2

June 11, 2022

Histogram
Definition

- For each gray level, **<u>how many</u>** pixels have that exact value

$$h_r(r_k) = n_k = |\{I(x,y)|I(x,y) = r_k\}| \quad r_k = 0, 1, 2, \cdots, 255$$

n_k is the number of pixels with with intensity r_k

- Normalized histogram: for each gray level, **<u>what percentage</u>** of pixels have that exact value. I.e., “unnormalized” histogram divided by the total number of pixels

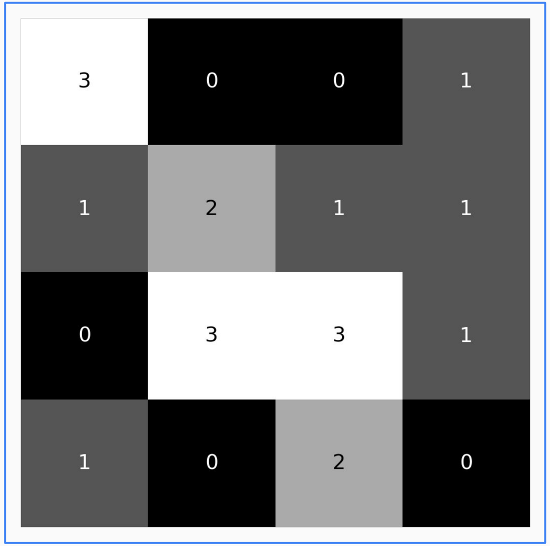
$$f_r(r_k) = \frac{n_k}{N_T}$$

n_k is the number of pixels with with intensity r_k and N_T is the total number of pixels

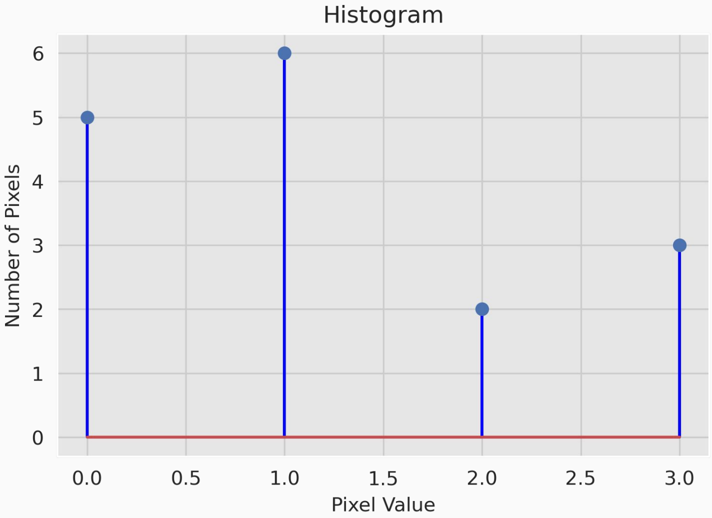
- Normalized histogram is also called the probability density function (PDF) - why?

Histogram
Example

A (4 x 4) image with 2 bit or 4 quantization levels



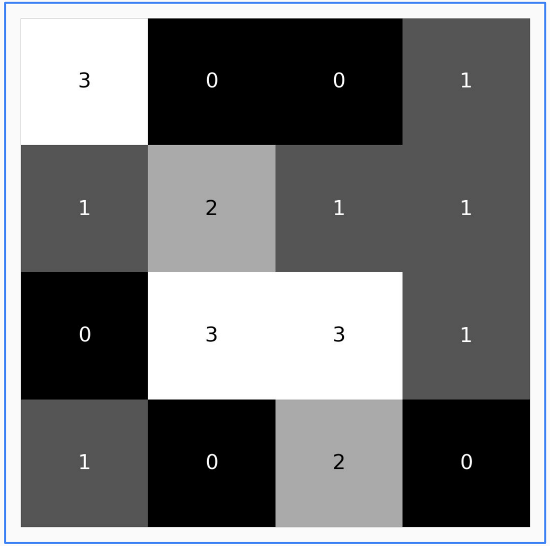
r_k	n_k
0	5
1	6
2	2
3	3



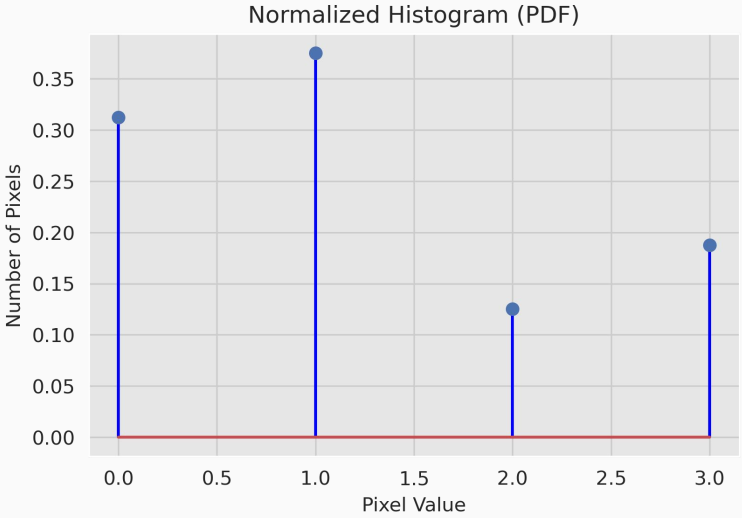
Histogram

Example

A (4 x 4) image with 2 bit or 4 quantization levels



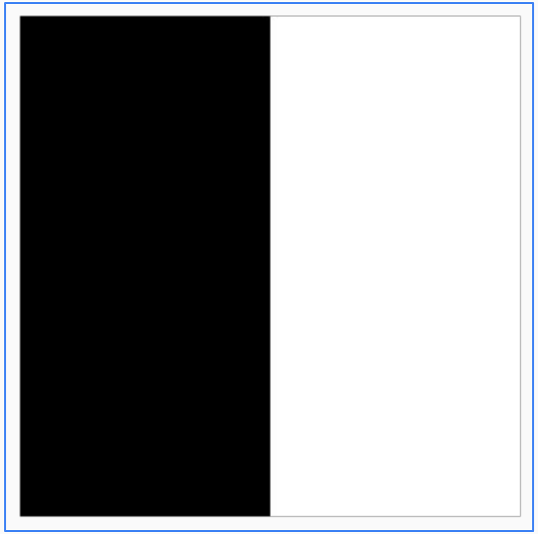
r_k	n_k	$f_r(r_k)$
0	5	0.31
1	6	0.38
2	2	0.12
3	3	0.19

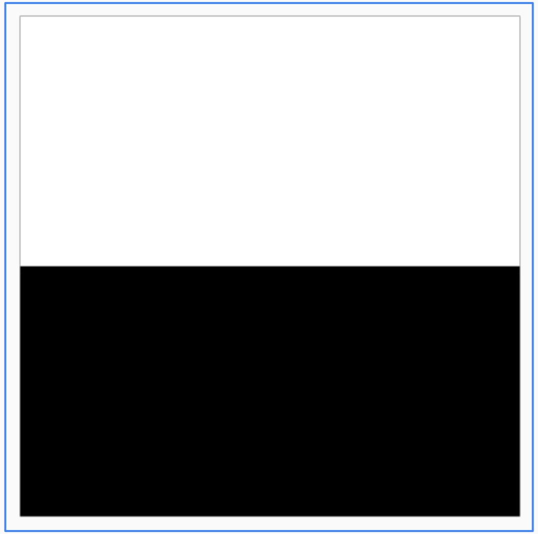


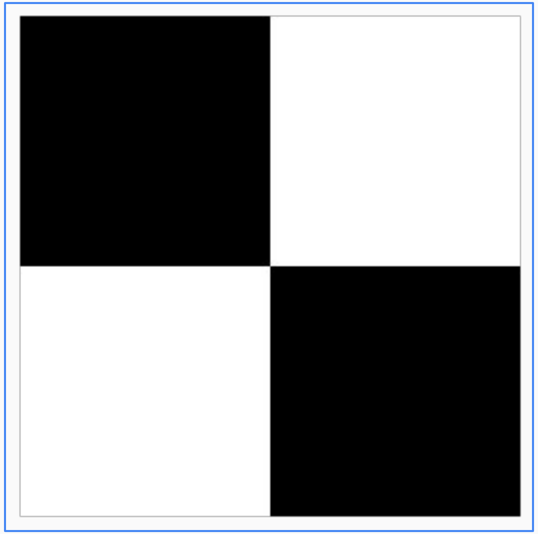
Histogram

Histograms are not unique!

All of these images have the exact same histogram (equal number of black and white)



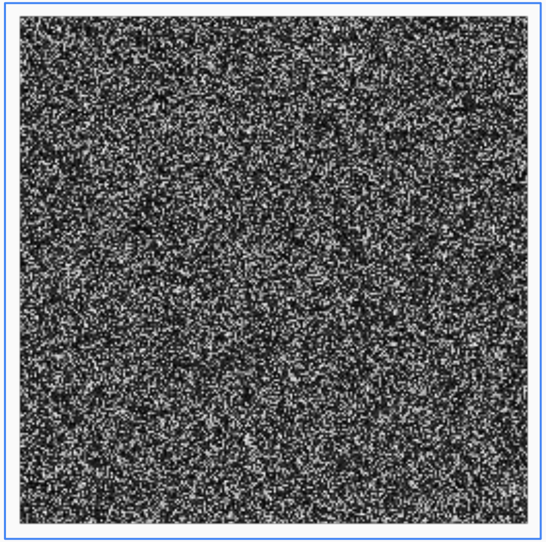


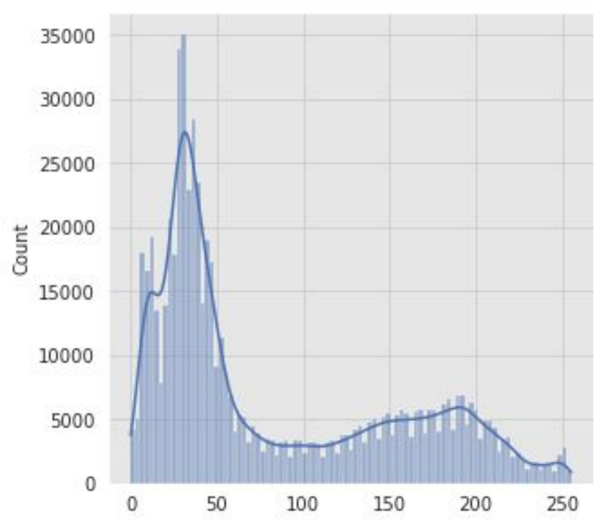


Histogram

Histograms are not unique!

The second image is a pixel permutation of the first image, hence same histogram





Cumulative Histogram
Definition

- For a given gray level, **how many** pixels have values ≤ that value.

$$H_r(r) = n_k = |\{I(x,y)|I(x,y) \leq r\}| = \int_{-\infty}^{r} h_r(x)dx \quad r \in \mathbb{R}$$

n_k is the number of pixels with with intensity ≤ r.

- Integration of histogram = cumulative histogram.

- Normalized cumulative histogram: for each gray level, **what percentage** of pixels have values ≤ that value. Also now as cumulative density function (CDF).

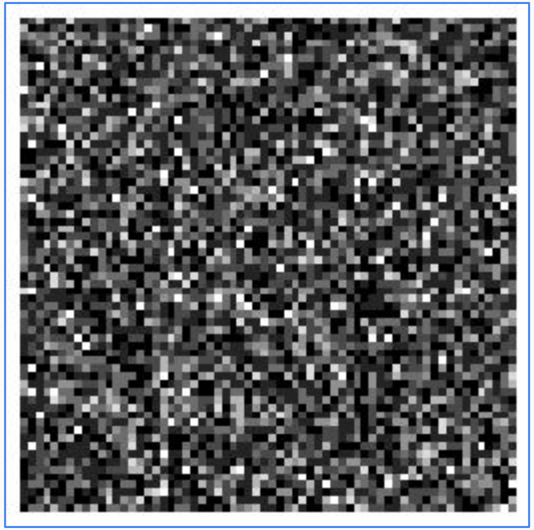
$$F_r(r) = \frac{n_k}{N_T} = \int_{-\infty}^{r} f_r(x)dx$$

- Interpretation of CDF(r) = probability that a randomly chosen pixel will have value ≤ r

Cumulative Histogram

Example

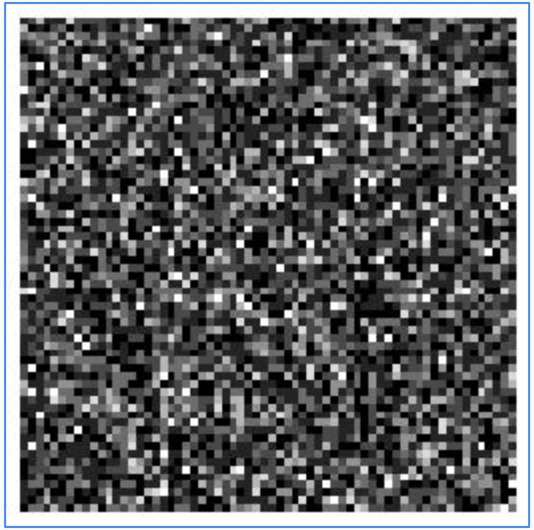
A random (64 x 64) image with 3 bit or 8 quantization levels



r_k	$h(r_k)$
0	790
1	1023
2	850
3	656
4	329
5	245
6	122
7	81

Cumulative Histogram
Example

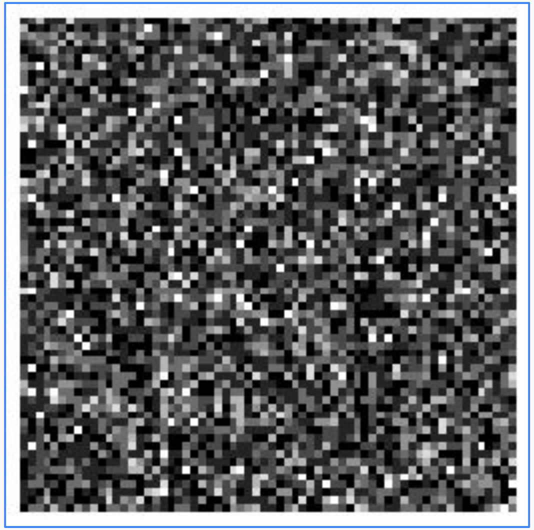
A random (64 x 64) image with 3 bit or 8 quantization levels



r_k	$h(r_k)$	$f(r_k)$ [PDF]
0	790	0.19 (=790/4096)
1	1023	0.25 (=1023/4096)
2	850	
3	656	
4	329	
5	245	
6	122	
7	81	

Cumulative Histogram
Example

A random (64 x 64) image with 3 bit or 8 quantization levels

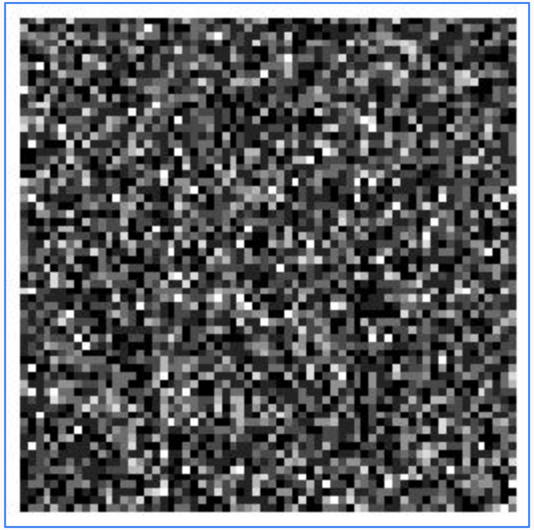


r_k	$h(r_k)$	$f(r_k)$ [PDF]
0	790	0.19
1	1023	0.25
2	850	0.21
3	656	0.16
4	329	0.08
5	245	0.06
6	122	0.03
7	81	0.02

Cumulative Histogram

Example

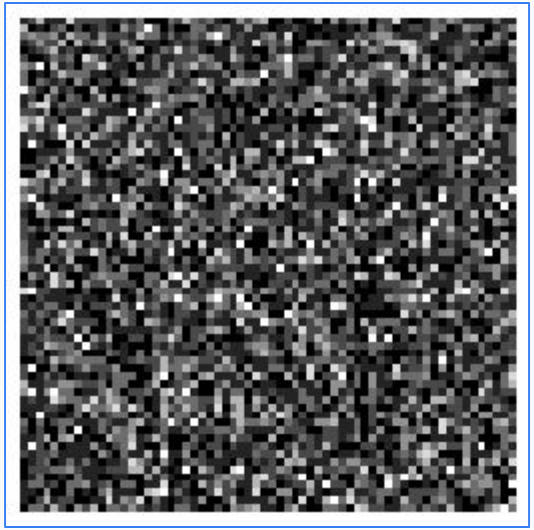
A random (64 x 64) image with 3 bit or 8 quantization levels



r_k	$h(r_k)$	$f(r_k)$ [PDF]	$H(r_k)$
0	790	0.19	790
1	1023	0.25	
2	850	0.21	
3	656	0.16	
4	329	0.08	
5	245	0.06	
6	122	0.03	
7	81	0.02	

Cumulative Histogram
Example

A random (64 x 64) image with 3 bit or 8 quantization levels

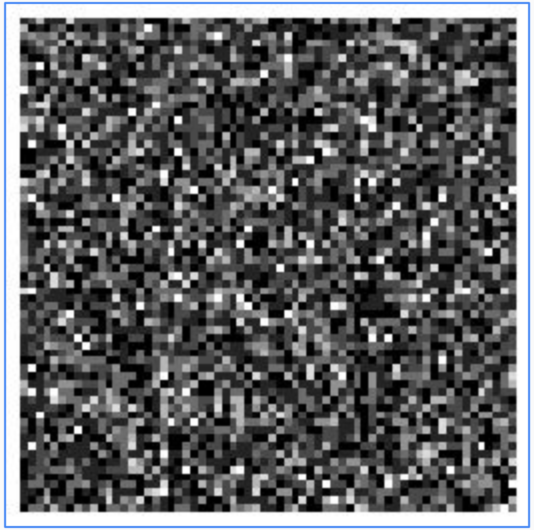


r_k	$h(r_k)$	$f(r_k)$ [PDF]	$H(r_k)$
0	790	0.19	790
1	1023	0.25	1813 (=790+1023)
2	850	0.21	
3	656	0.16	
4	329	0.08	
5	245	0.06	
6	122	0.03	
7	81	0.02	

Cumulative Histogram

Example

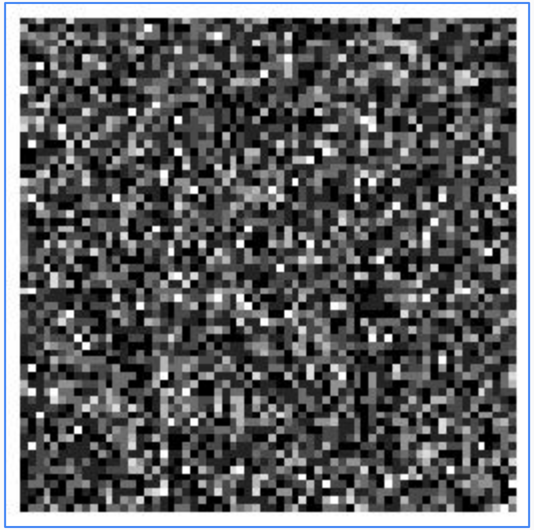
A random (64 x 64) image with 3 bit or 8 quantization levels



r_k	$h(r_k)$	$f(r_k)$ [PDF]	$H(r_k)$
0	790	0.19	790
1	1023	0.25	1813
2	850	0.21	2663 = (1813+850)
3	656	0.16	
4	329	0.08	
5	245	0.06	
6	122	0.03	
7	81	0.02	

Cumulative Histogram
Example

A random (64 x 64) image with 3 bit or 8 quantization levels

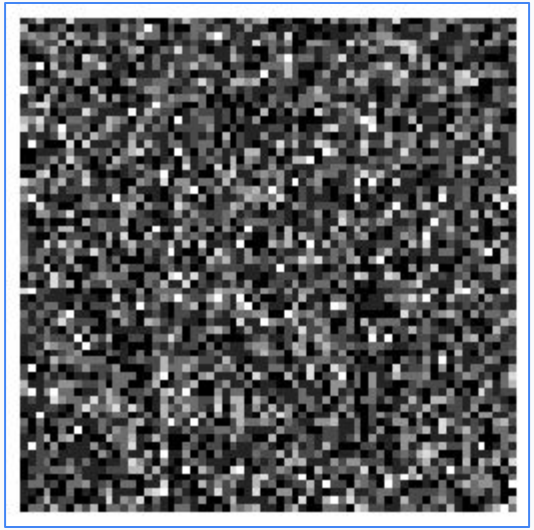


r_k	$h(r_k)$	$f(r_k)$ [PDF]	$H(r_k)$
0	790	0.19	790
1	1023	0.25	1813
2	850	0.21	2663
3	656	0.16	3319
4	329	0.08	3648
5	245	0.06	3893
6	122	0.03	4015
7	81	0.02	4096

Cumulative Histogram

Example

A random (64 x 64) image with 3 bit or 8 quantization levels

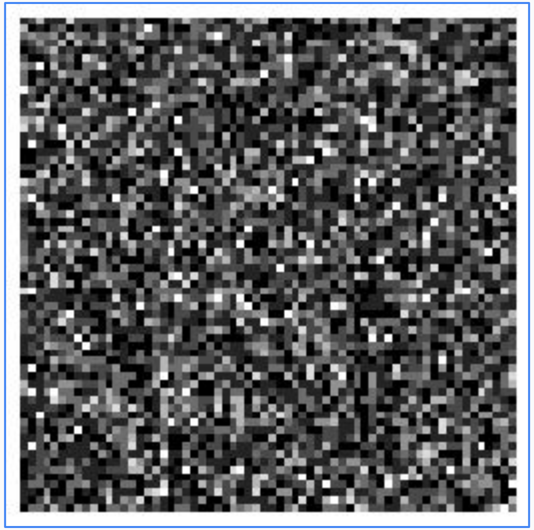


r_k	$h(r_k)$	$f(r_k)$ [PDF]	$H(r_k)$	$F(r_k)$ [CDF]
0	790	0.19	790	0.19
1	1023	0.25	1813	0.44
2	850	0.21	2663	0.65
3	656	0.16	3319	0.81
4	329	0.08	3648	0.89
5	245	0.06	3893	0.95
6	122	0.03	4015	0.98
7	81	0.02	4096	1.00

Cumulative Histogram

Example

A random (64 x 64) image with 3 bit or 8 quantization levels

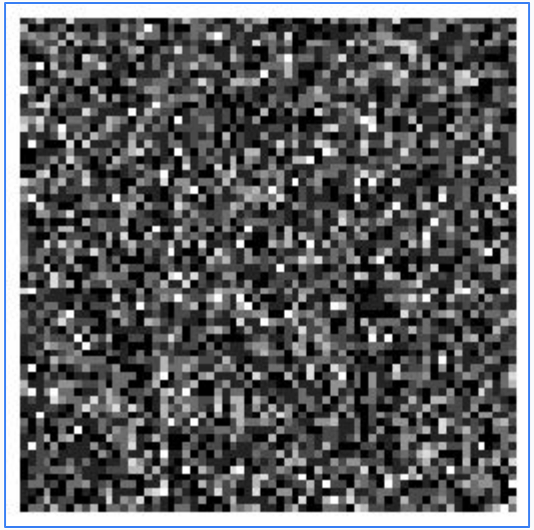


r_k	$h(r_k)$	$f(r_k)$ [PDF]	$H(r_k)$	$F(r_k)$ [CDF]
0	790	0.19	790	0.19
1	1023	0.25	1813 (=790+1023)	0.44
2	850	0.21	2663	0.65
3	656	0.16	3319	0.81
4	329	0.08	3648	0.89
5	245	0.06	3893	0.95
6	122	0.03	4015	0.98
7	81	0.02	4096	1.00

Cumulative Histogram

Example

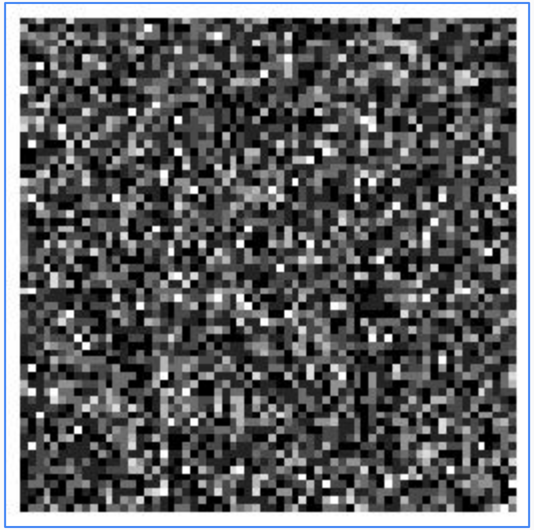
A random (64 x 64) image with 3 bit or 8 quantization levels

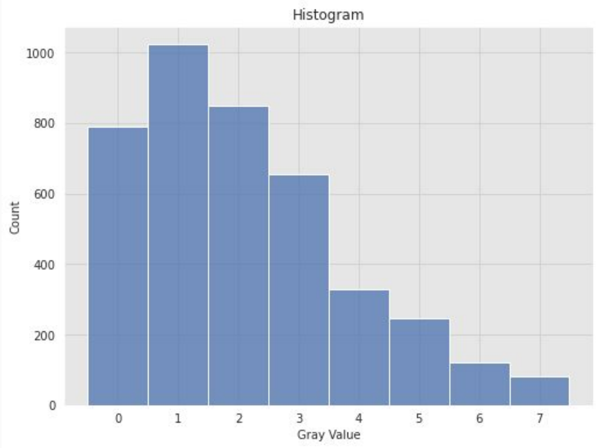


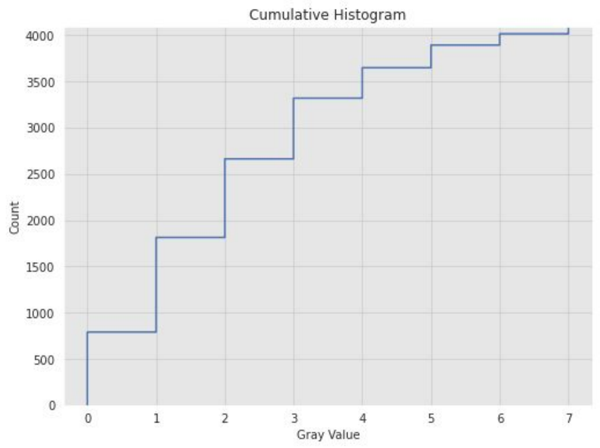
r_k	$h(r_k)$	$f(r_k)$ [PDF]	$H(r_k)$	$F(r_k)$ [CDF]
0	790	0.19	790	0.19
1	1023	0.25	1813 (=790+1023)	0.44
2	850	0.21	2663 (=1813+850)	0.65
3	656	0.16	3319	0.81
4	329	0.08	3648	0.89
5	245	0.06	3893	0.95
6	122	0.03	4015	0.98
7	81	0.02	4096	1.00

Cumulative Histogram
Example

A random (64 x 64) image with 3 bit or 8 quantization levels



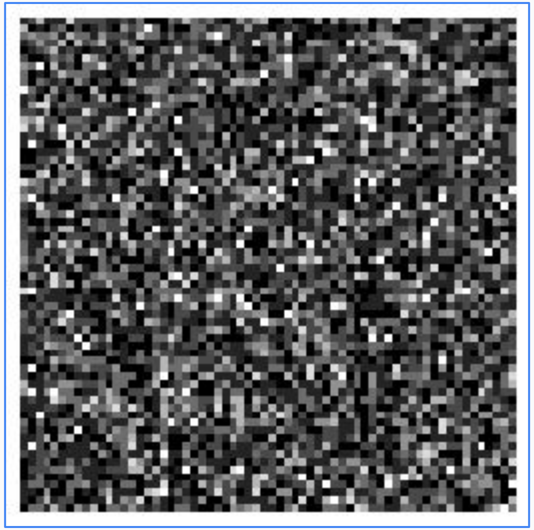


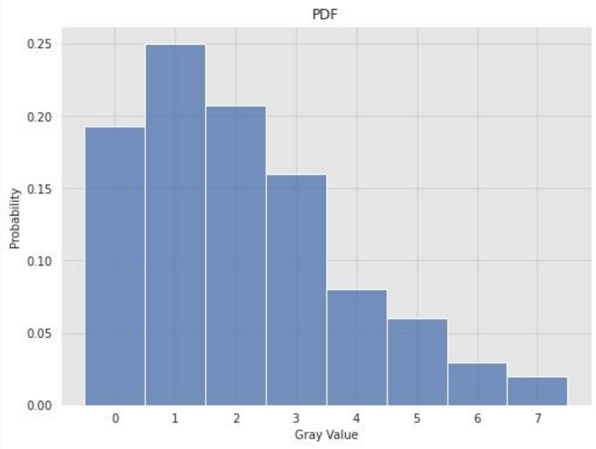


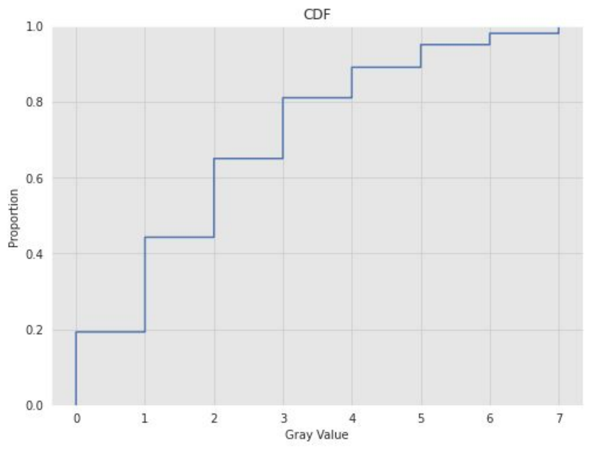
Cumulative Histogram

Example

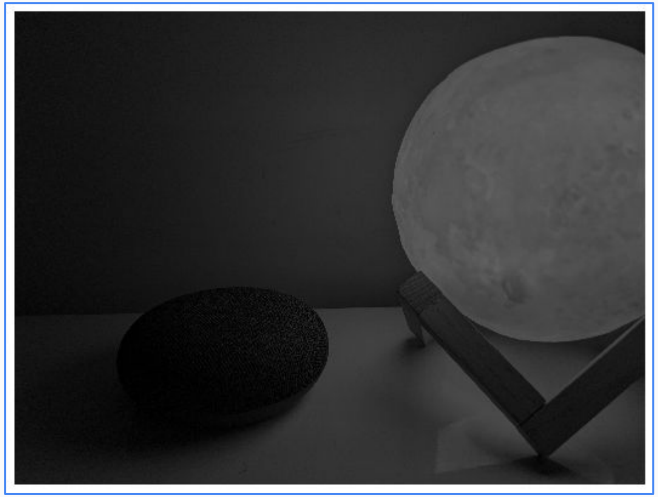
A random (64 x 64) image with 3 bit or 8 quantization levels

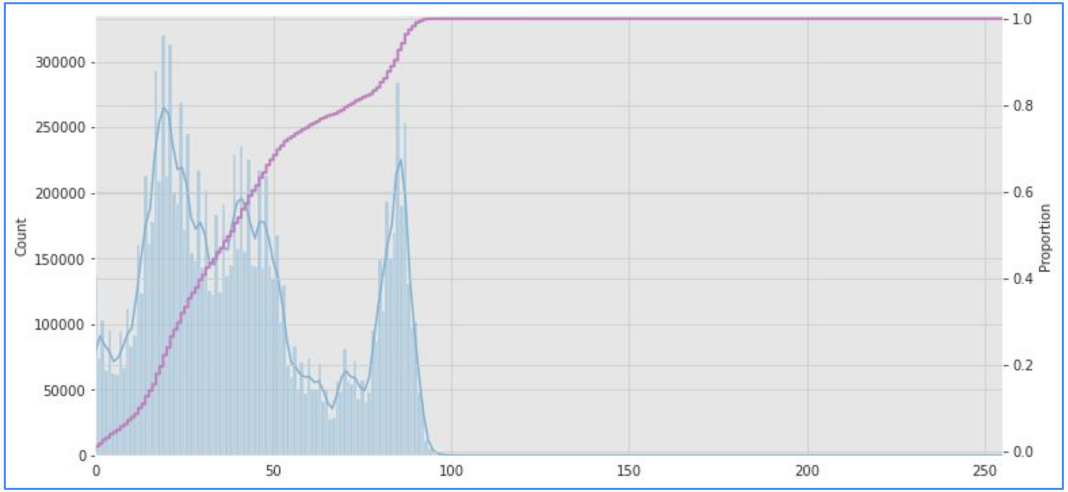






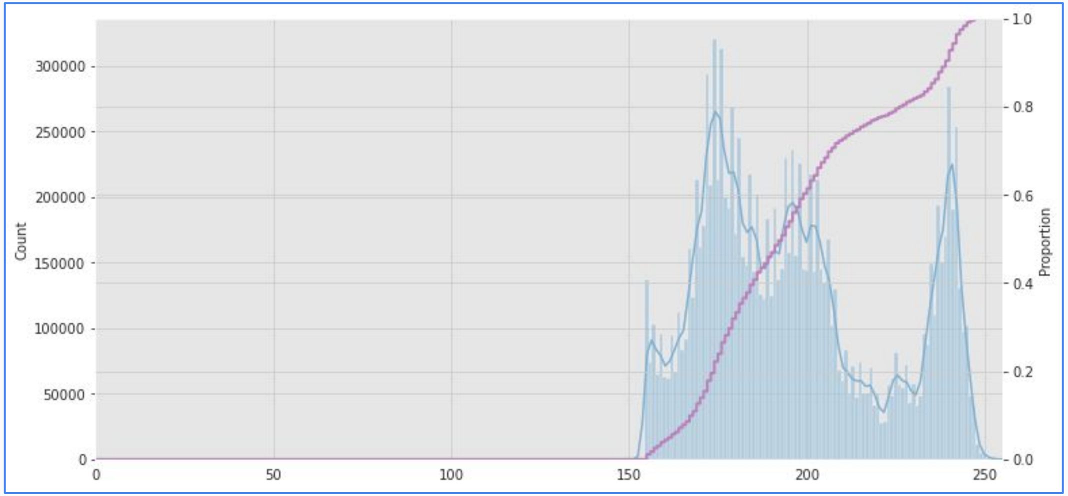
Histogram
More Example





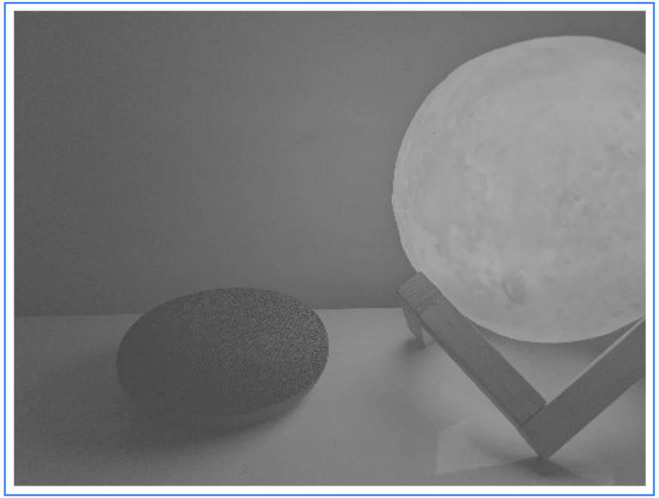
- Histogram of a dark image.
- The histogram is concentrated in the low-range gray values

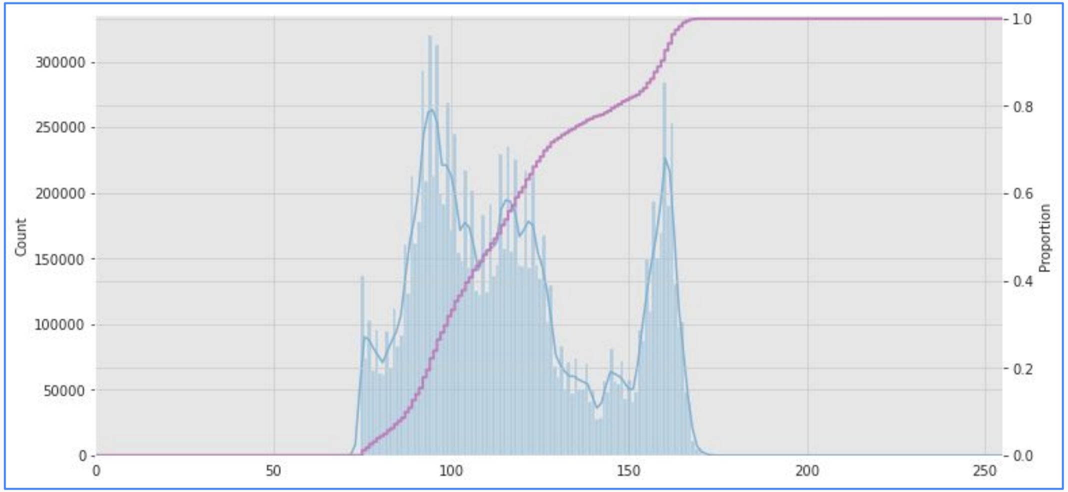
Histogram
More Example



- Histogram of a bright image.
- The histogram is concentrated in the high-range gray values

Histogram
More Example

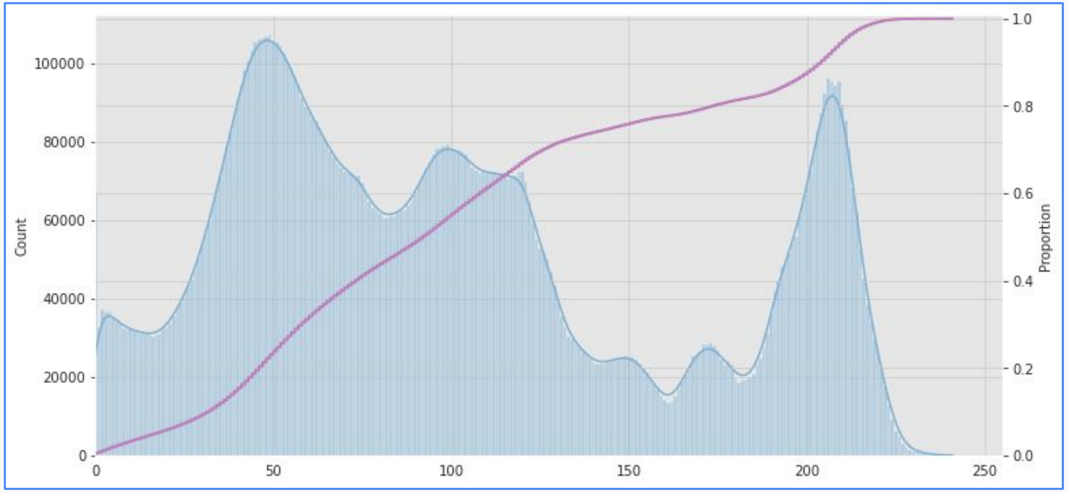




- Histogram of a low contrast image
- The histogram is concentrated in the mid-range gray values
- The image looks “washed-out”

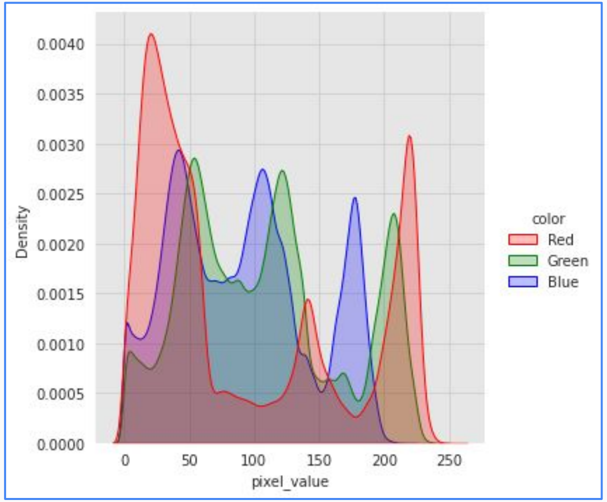
Histogram

More Example



- Histogram of a high contrast image
- The histogram is (somewhat) uniformly distributed
- Contrast can be further increased by “flattening” the histogram

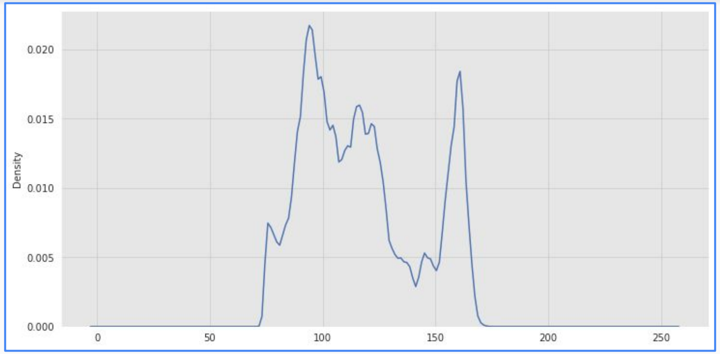
Histogram
More Example

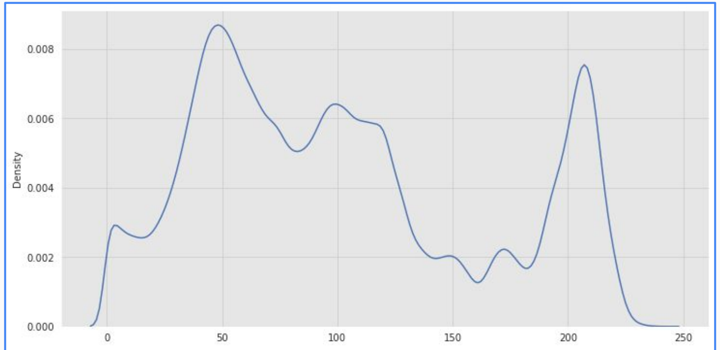


- Histograms of a color image
- The distribution of the three color channels are calculated separately

Contrast Stretching

Histogram Point of View

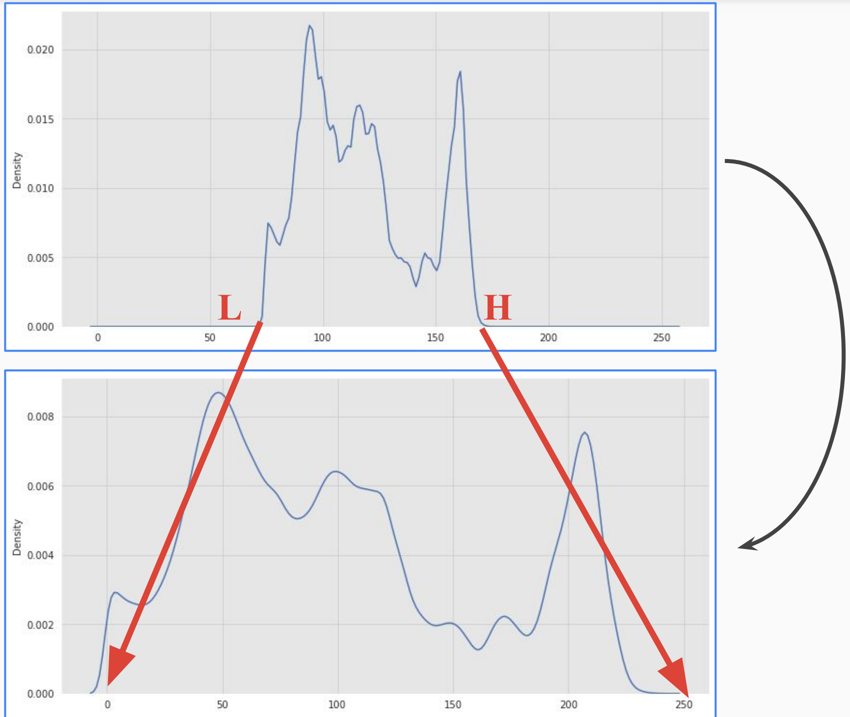




$s = T(r)$?

Contrast Stretching
Histogram Point of View

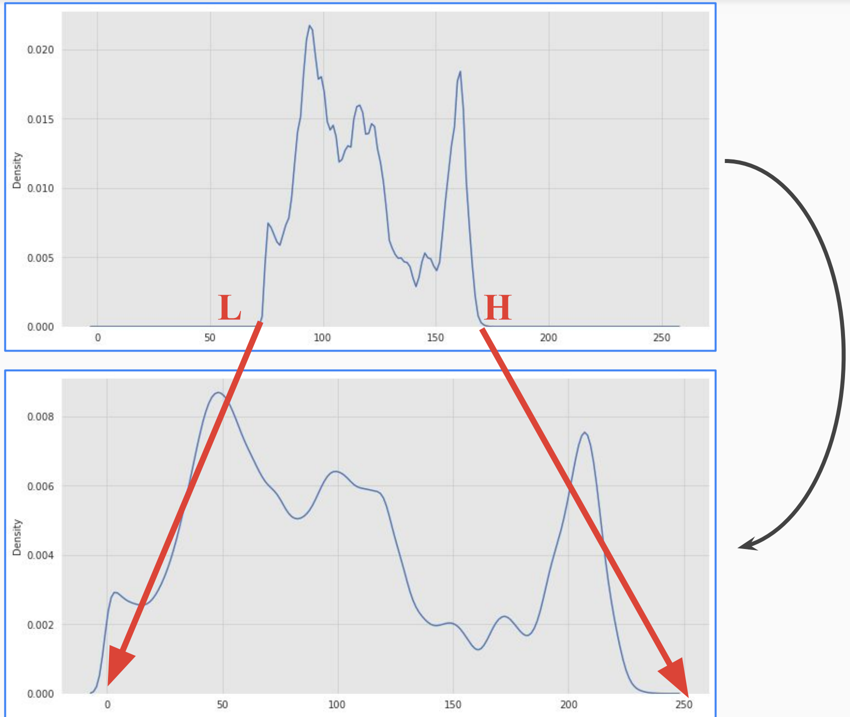
$L \rightarrow 0 \qquad H \rightarrow 255$



s = T(r)?

Contrast Stretching

Histogram Point of View



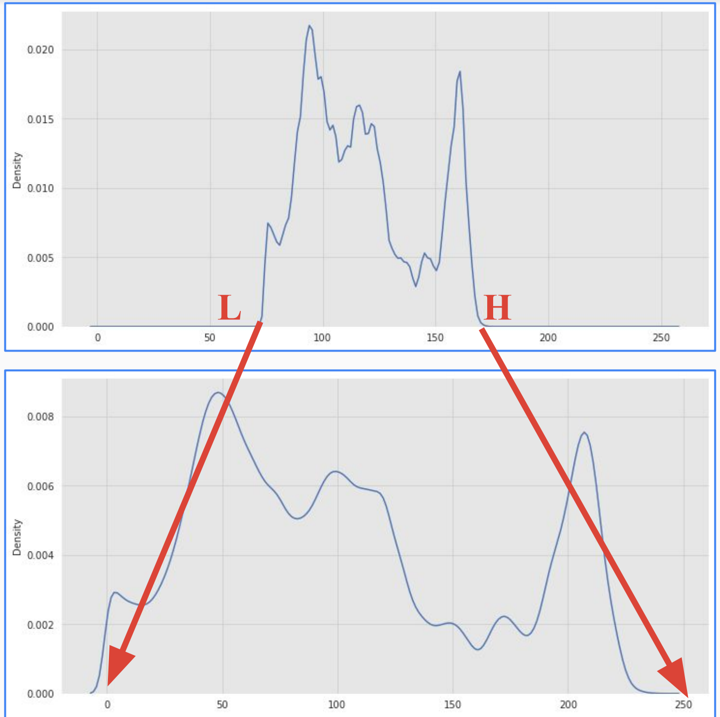
s = T(r)?

$$L \rightarrow 0 \qquad H \rightarrow 255$$

$$s = (r - L) \times \frac{255 - 0}{H - L}$$

Contrast Stretching

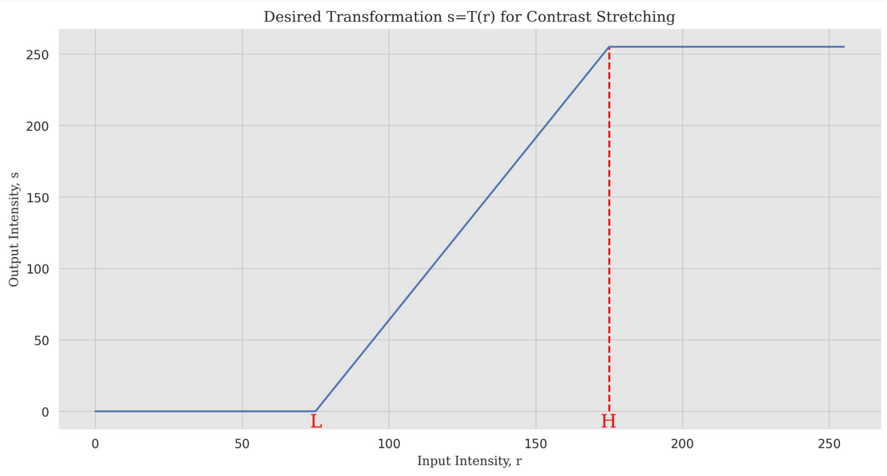
Histogram Point of View



s = T(r)?

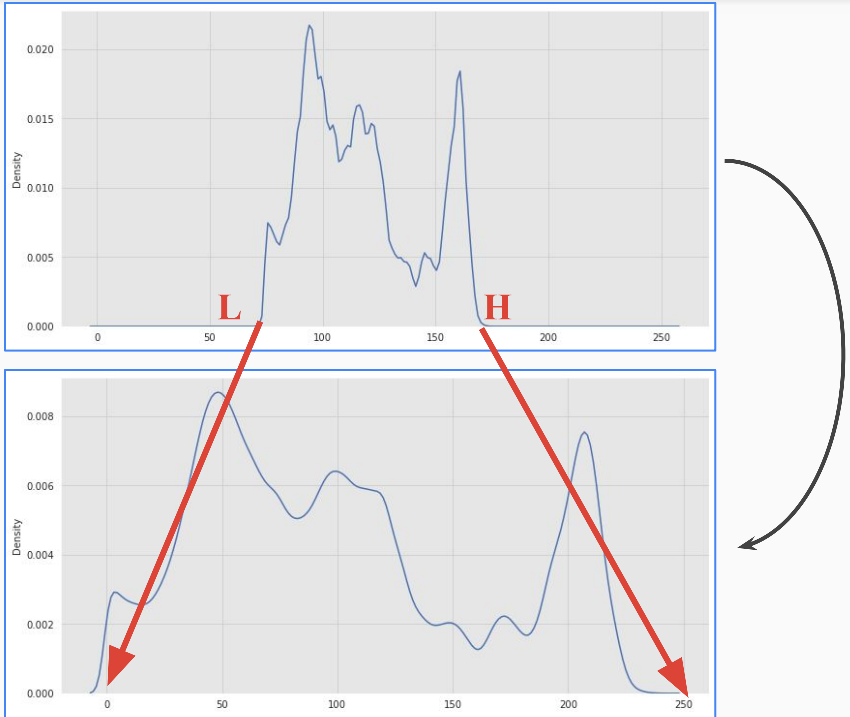
$$L \rightarrow 0 \qquad H \rightarrow 255$$

$$s = (r - L) \times \frac{255 - 0}{H - L}$$

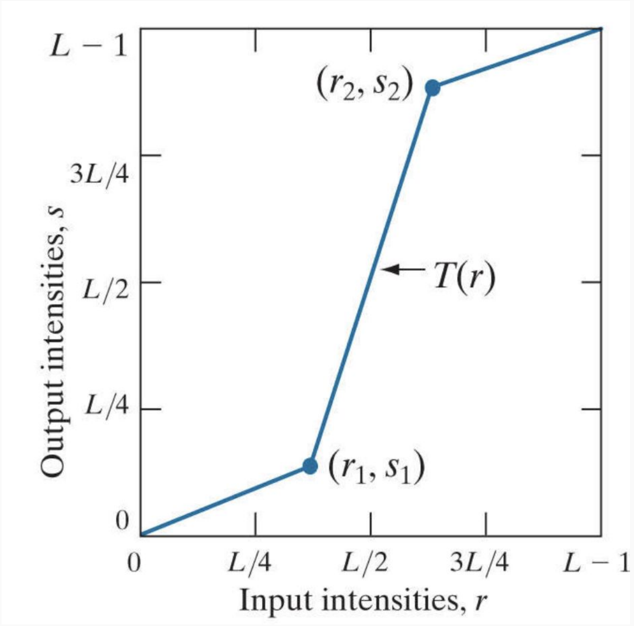


Contrast Stretching

Histogram Point of View



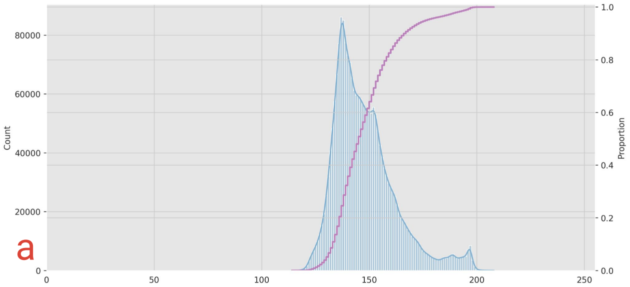
s = T(r)?

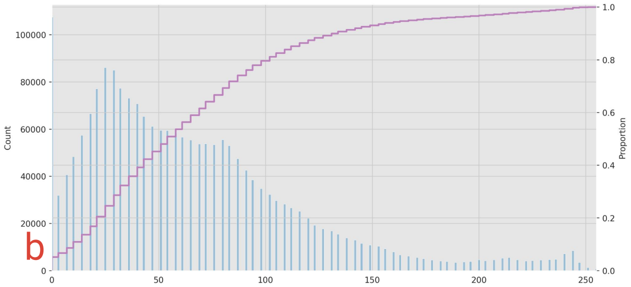


By Rafael C. Gonzalez & Richard E. Woods, 2018, *Digital Image Processing*, 4th Edition

Contrast Stretching

Histogram Point of View



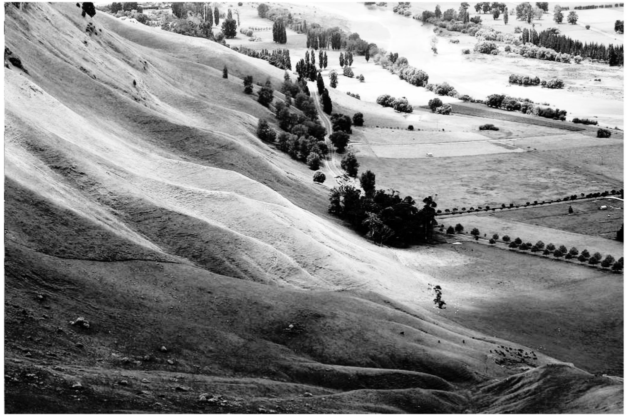


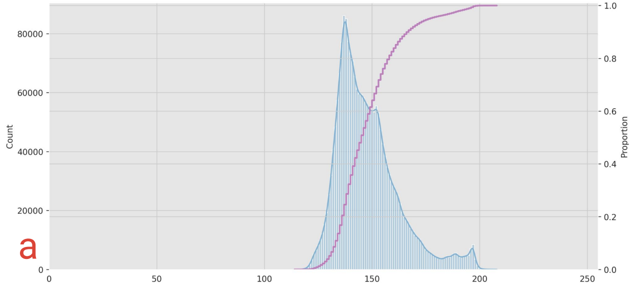
Can we do better?

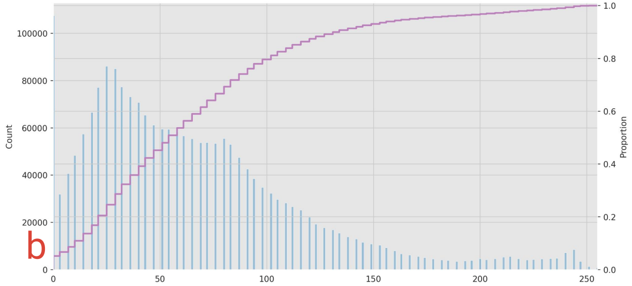
Can we do better? - YES!

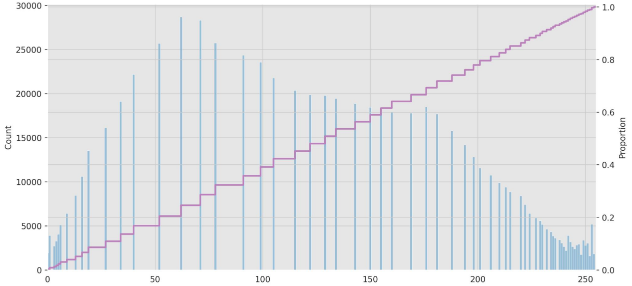
Histogram Equalization

Demo







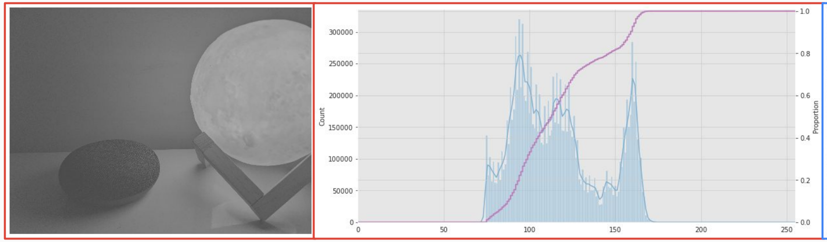


Histogram Equalization

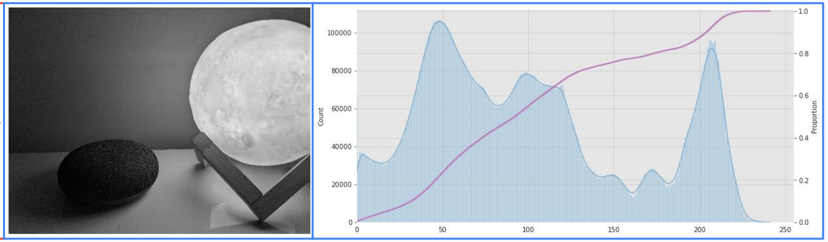
Motivation - Entropy

$$\text{Entropy}(I) = \sum_{r_k} -f(r_k)\log f(r_k)$$

- Entropy specifies the uncertainty of image pixels
- Uncertain events have more information than certain events
- Entropy is a measure of dispersion of histograms
- Uniform distribution - dispersed, high entropy, high contrast (desirable)



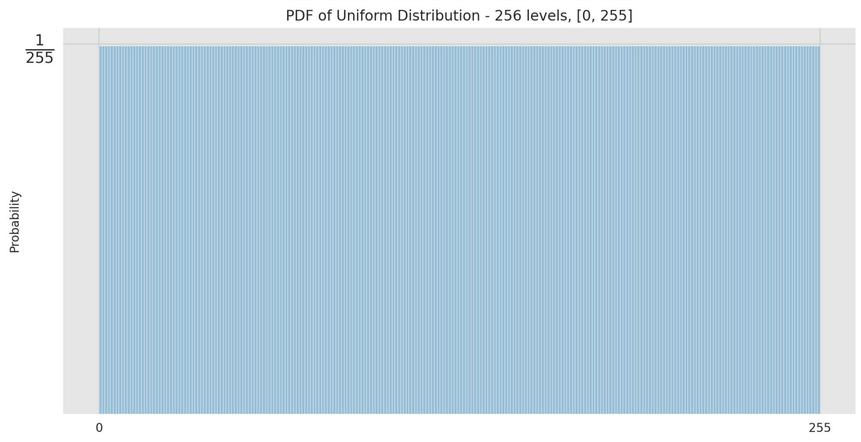
Entropy = 6.34

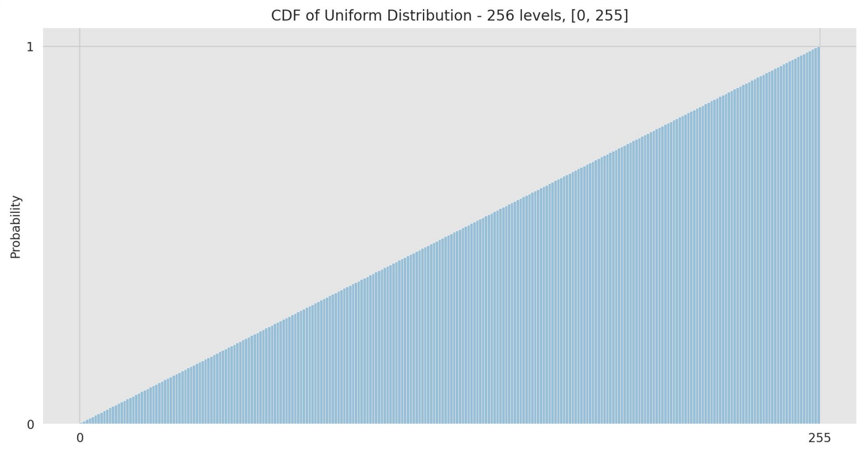


Entropy = 7.64

Histogram Equalization

Motivation - Uniform Distribution



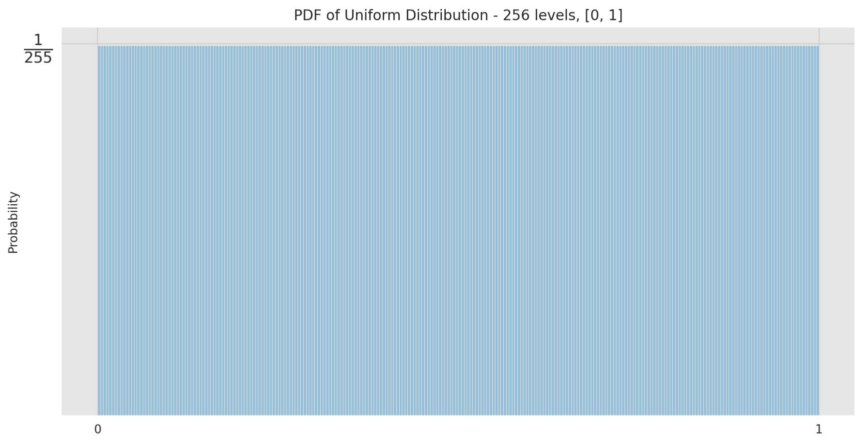


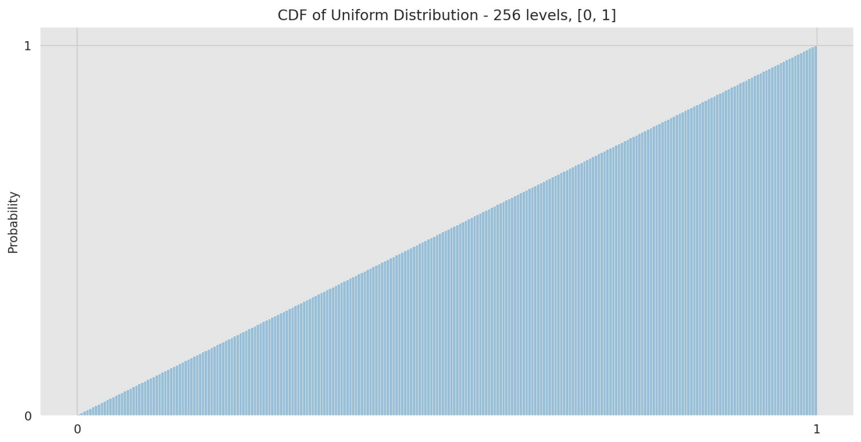
PDF is flat, $f(r_k) = \dfrac{1}{L-1}$, where L is the quantization level

CDF if linear beteween $[\text{min}, \text{max}]$, slope $= \dfrac{1}{L-1}$

Histogram Equalization

Motivation - Uniform Distribution



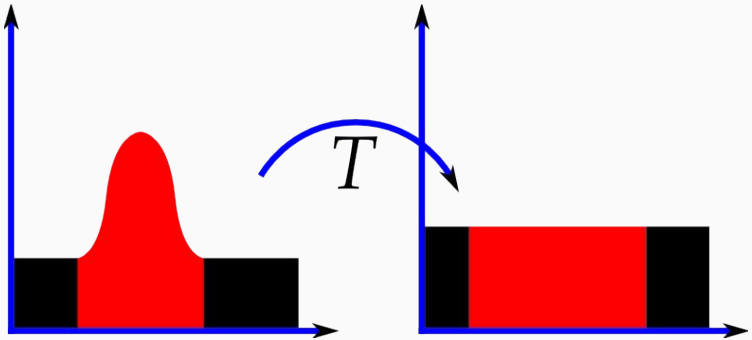


PDF is flat, $f(r_k) = \dfrac{1}{L-1}$, where L is the quantization level

CDF if linear beteween $[\text{min}, \text{max}]$, slope $= 1$

Histogram Equalization

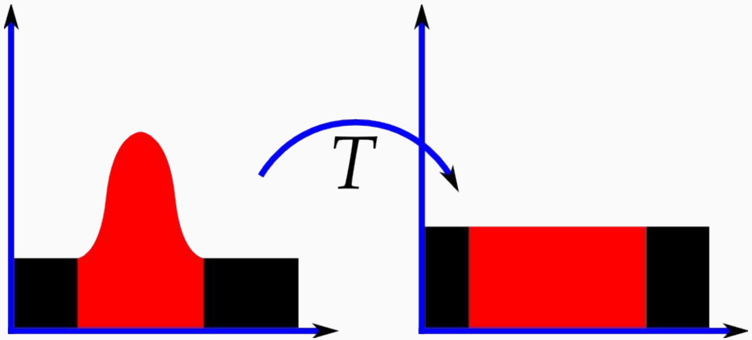
Basic Idea for Histogram Equalization



By Zefram - Own work, Public Domain, https://commons.wikimedia.org/w/index.php?curid=668605

Goal: Find a map s = T(r) such that the resulting PDF is flat, i.e., the resulting CDF is linear

Histogram Equalization
Basic Idea for Histogram Equalization



By Zefram - Own work, Public Domain, https://commons.wikimedia.org/w/index.php?curid=668605

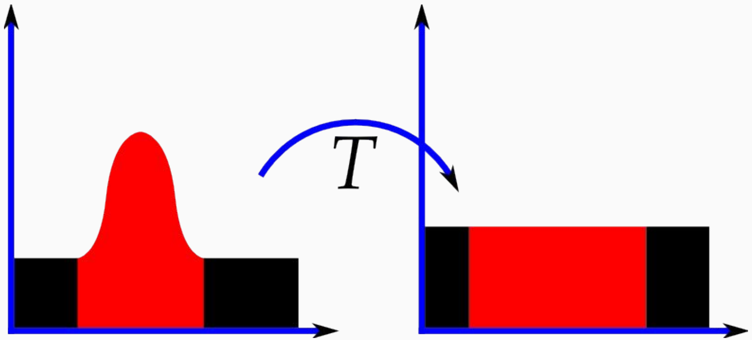
Goal: Find a map s = T(r) such that the resulting PDF is flat, i.e., the resulting CDF is linear

Observation (Assuming normalized gray values)

- For an ideal uniform distribution, y% pixels have values less than y [see the CDF]
- **Reverse**: A pixel with y% pixels having smaller values than itself should have a value y

Histogram Equalization

Basic Idea for Histogram Equalization



By Zefram - Own work, Public Domain, https://commons.wikimedia.org/w/index.php?curid=668605

Goal: Find a map s = T(r) such that the resulting PDF is flat, i.e., the resulting CDF is linear

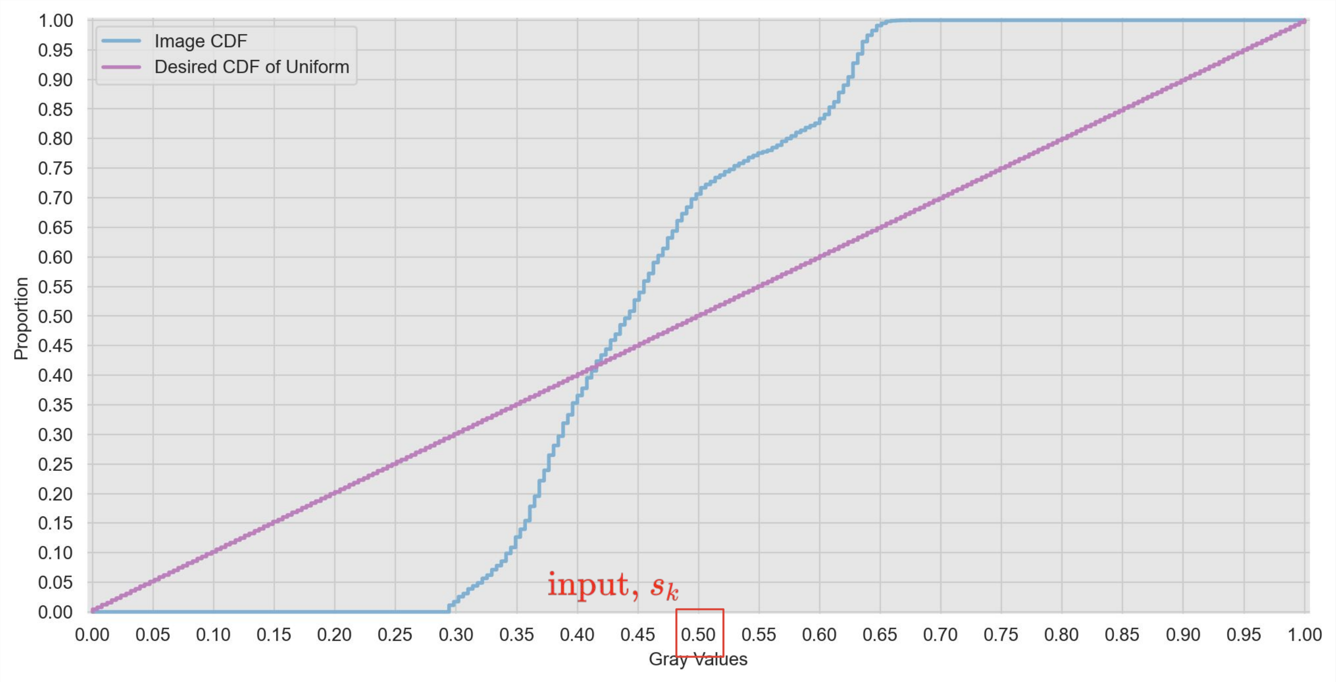
Observation (Assuming normalized gray values)

- For an ideal uniform distribution, y% pixels have values less than y [see the CDF]
- **Reverse**: A pixel with y% pixels having smaller values than itself should have a value y

Idea (Assuming normalized gray values [0, 1])

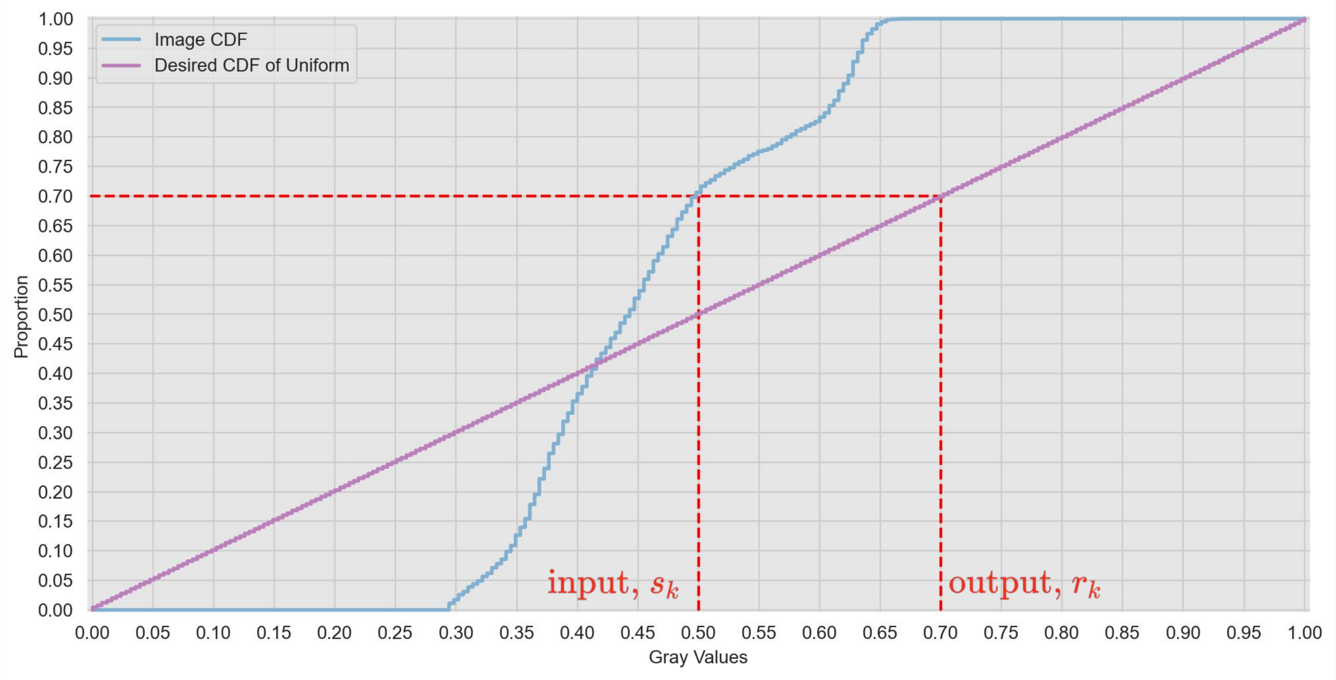
- For a given gray value x, we know the percentage of pixels with values ≤ x (CDF)
- Hence, map x to its CDF value to get a flat histogram/linear CDF.

Histogram Equalization
Basic Idea for Histogram Equalization



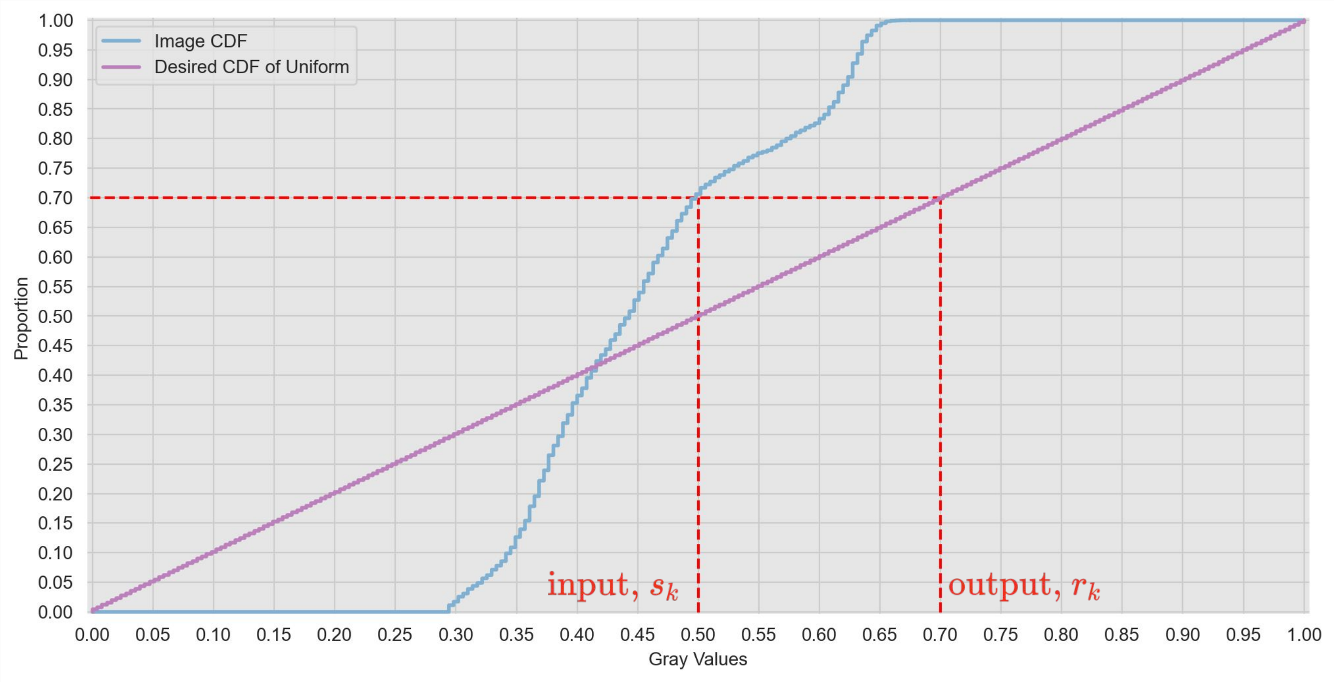
Histogram Equalization

Basic Idea for Histogram Equalization



Histogram Equalization

Basic Idea for Histogram Equalization



Hence,

$$s_k = T(r_k) = \mathrm{CDF}_r(r_k) = F_r(r_k)$$

For L quntization levels,

$$s_k = T(r_k) = \mathrm{round}((L-1)F_r(r_k))$$

Histogram Equalization

Numerical Example

A random (64 x 64) image with 3 bit or 8 quantization levels $(L = 8)$

r_k	$h(r_k)$	$f(r_k)$ [PDF]	$H(r_k)$	$F(r_k)$ [CDF]
0	790	0.19	790	0.19
1	1023	0.25	1813	0.44
2	850	0.21	2663	0.65
3	656	0.16	3319	0.81
4	329	0.08	3648	0.89
5	245	0.06	3893	0.95
6	122	0.03	4015	0.98
7	81	0.02	4096	1.00

Histogram Equalization

Numerical Example

A random (64 x 64) image with 3 bit or 8 quantization levels ($L = 8$)

r_k	$h(r_k)$	$f(r_k)$ [PDF]	$H(r_k)$	$F(r_k)$ [CDF]	$s_k = round(7xF(r_k))$
0	790	0.19	790	0.19	
1	1023	0.25	1813	0.44	
2	850	0.21	2663	0.65	
3	656	0.16	3319	0.81	
4	329	0.08	3648	0.89	
5	245	0.06	3893	0.95	
6	122	0.03	4015	0.98	
7	81	0.02	4096	1.00	

Histogram Equalization
Numerical Example

A random (64 x 64) image with 3 bit or 8 quantization levels ($L = 8$)

r_k	$h(r_k)$	$f(r_k)$ [PDF]	$H(r_k)$	$F(r_k)$ [CDF]	$s_k = \text{round}(7\text{x}F(r_k))$
0	790	0.19	790	0.19	=round(7 x 0.19)
1	1023	0.25	1813	0.44	
2	850	0.21	2663	0.65	
3	656	0.16	3319	0.81	
4	329	0.08	3648	0.89	
5	245	0.06	3893	0.95	
6	122	0.03	4015	0.98	
7	81	0.02	4096	1.00	

Histogram Equalization

Numerical Example

A random (64 x 64) image with 3 bit or 8 quantization levels ($L = 8$)

r_k	$h(r_k)$	$f(r_k)$ [PDF]	$H(r_k)$	$F(r_k)$ [CDF]	$s_k = round(7xF(r_k))$
0	790	0.19	790	0.19	=round(1.33)
1	1023	0.25	1813	0.44	
2	850	0.21	2663	0.65	
3	656	0.16	3319	0.81	
4	329	0.08	3648	0.89	
5	245	0.06	3893	0.95	
6	122	0.03	4015	0.98	
7	81	0.02	4096	1.00	

Histogram Equalization
Numerical Example

A random (64 x 64) image with 3 bit or 8 quantization levels ($L = 8$)

r_k	$h(r_k)$	$f(r_k)$ [PDF]	$H(r_k)$	$F(r_k)$ [CDF]	$s_k = \text{round}(7\text{x}F(r_k))$
0	790	0.19	790	0.19	1
1	1023	0.25	1813	0.44	
2	850	0.21	2663	0.65	
3	656	0.16	3319	0.81	
4	329	0.08	3648	0.89	
5	245	0.06	3893	0.95	
6	122	0.03	4015	0.98	
7	81	0.02	4096	1.00	

Histogram Equalization

Numerical Example

A random (64 x 64) image with 3 bit or 8 quantization levels ($L = 8$)

r_k	$h(r_k)$	$f(r_k)$ [PDF]	$H(r_k)$	$F(r_k)$ [CDF]	$s_k = round(7xF(r_k))$
0	790	0.19	790	0.19	1
1	1023	0.25	1813	0.44	3
2	850	0.21	2663	0.65	5
3	656	0.16	3319	0.81	6
4	329	0.08	3648	0.89	6
5	245	0.06	3893	0.95	7
6	122	0.03	4015	0.98	7
7	81	0.02	4096	1.00	7

Histogram Equalization

Numerical Example

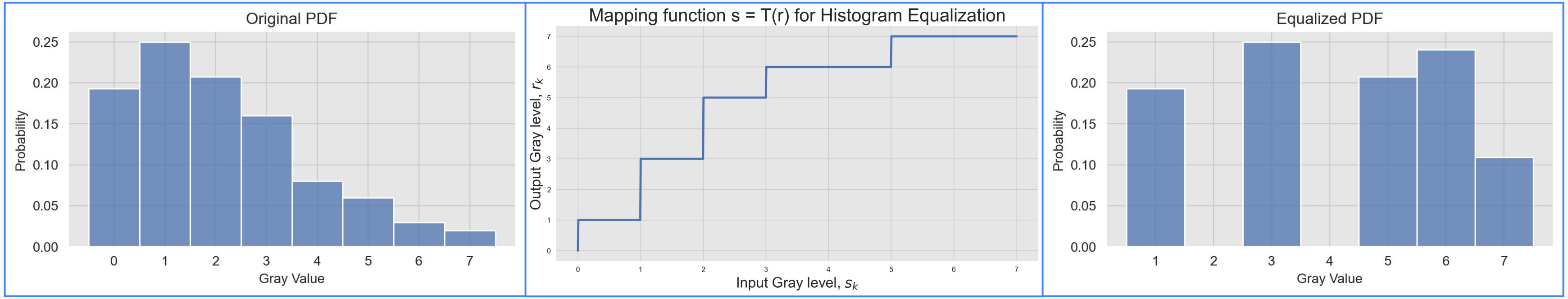
A random (64 x 64) image with 3 bit or 8 quantization levels ($L = 8$)

r_k	$h(r_k)$	$f(r_k)$ [PDF]	$H(r_k)$	$F(r_k)$ [CDF]	$s_k = round(7xF(r_k))$
0	790	0.19	790	0.19	1
1	1023	0.25	1813	0.44	3
2	850	0.21	2663	0.65	5
3	656	0.16	3319	0.81	6
4	329	0.08	3648	0.89	6
5	245	0.06	3893	0.95	7
6	122	0.03	4015	0.98	7
7	81	0.02	4096	1.00	7

Histogram Equalization

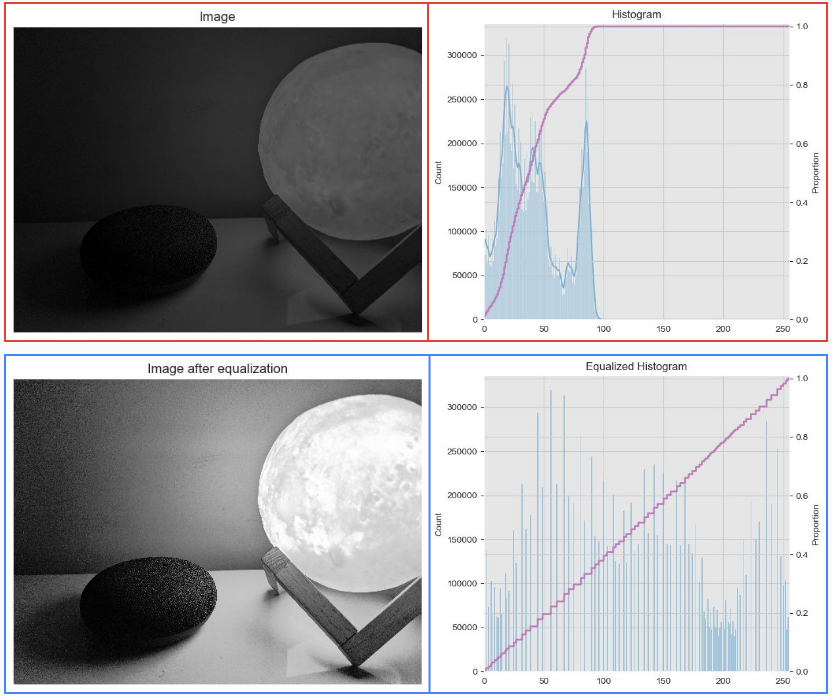
Numerical Example

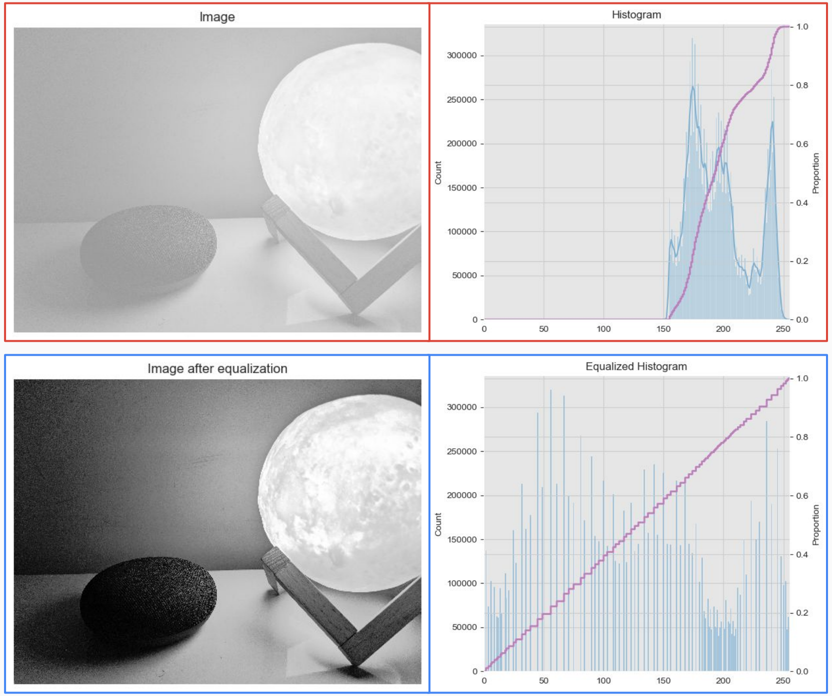
A random (64 x 64) image with 3 bit or 8 quantization levels ($L = 8$)



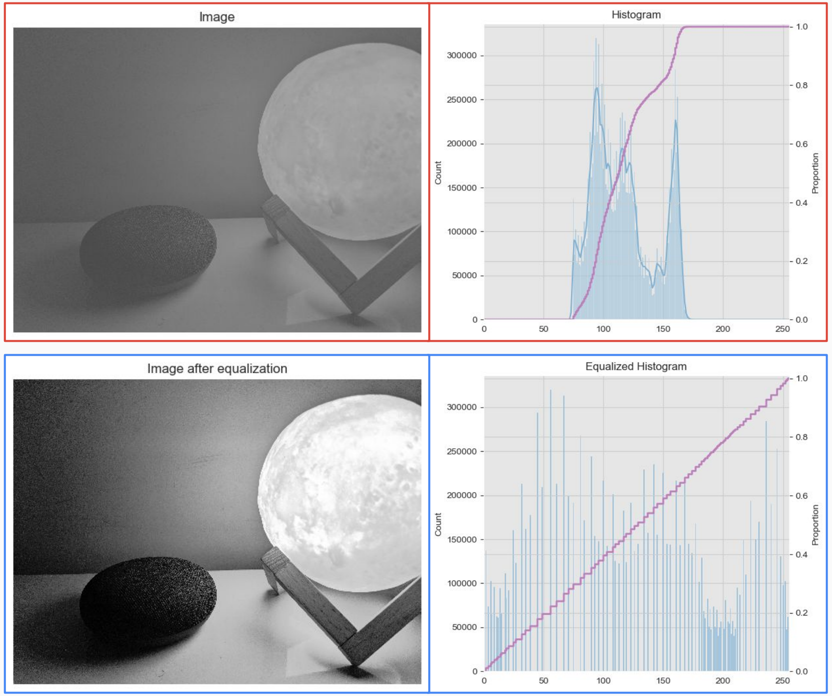
Histogram Equalization

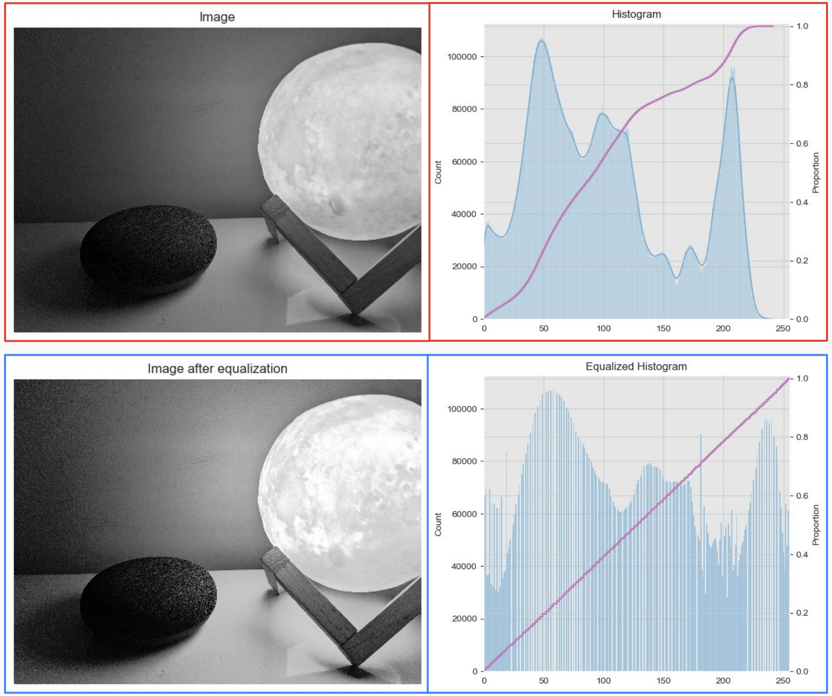
Examples





Histogram Equalization
Examples





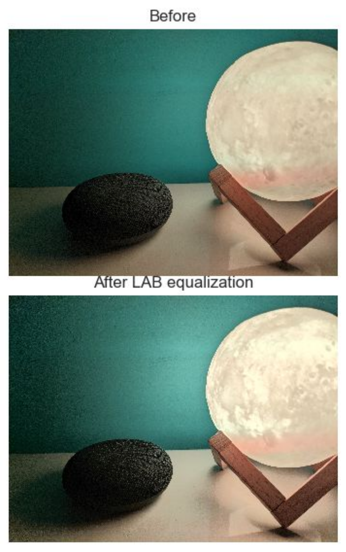
Histogram Equalization
Example

Wrong method - channel wise equalization

Histogram Equalization
Example

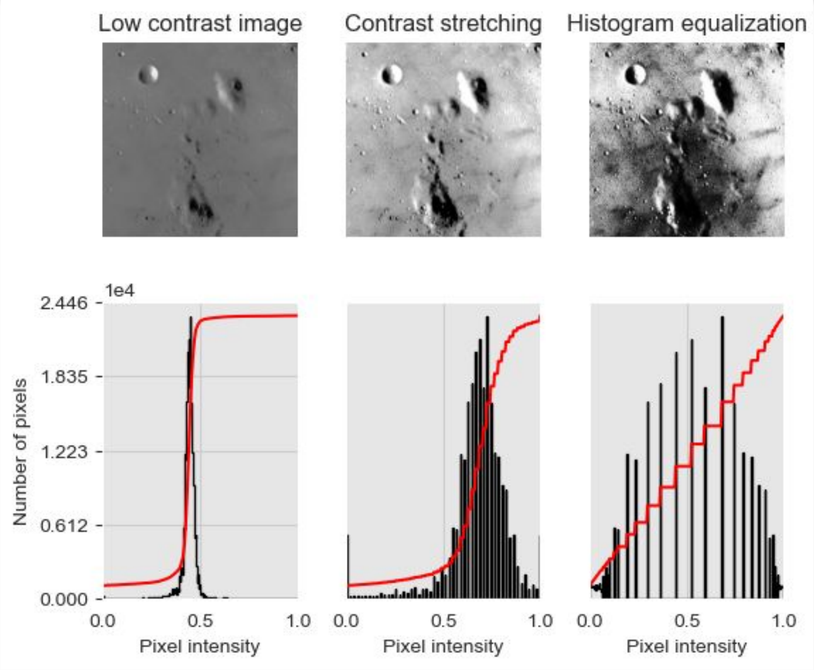
Wrong method - channel wise equalization

Correct method - RGB to LAB, equalize L, LAB to RGB



https://en.wikipedia.org/wiki/CIELAB_color_space

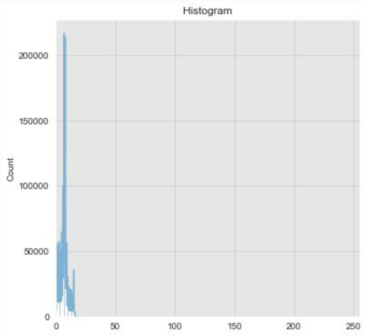
Failure of Histogram Equalization

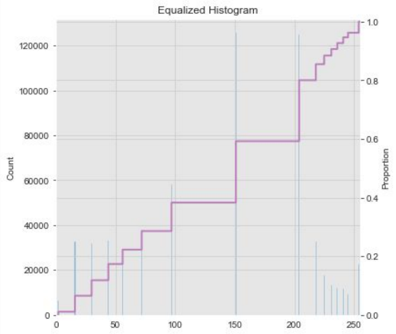


- Details in low (or high) intensity value lost
- Noise (grains) amplified

Reproduced from https://scikit-image.org/docs/stable/auto_examples/color_exposure/plot_equalize.html?highlight=histogram%20equalization

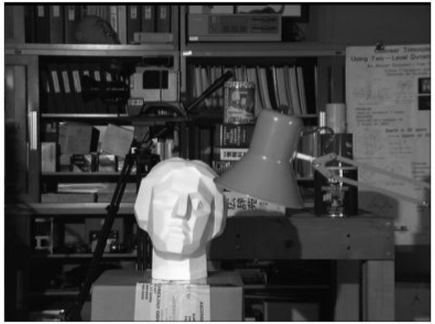
Failure of Histogram Equalization

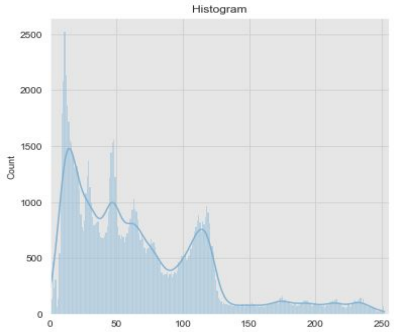


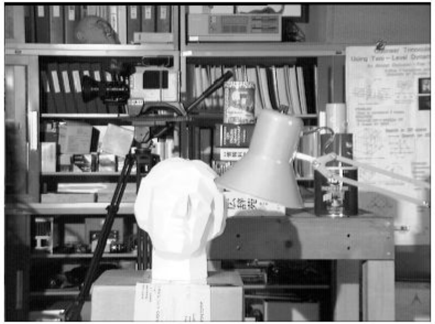


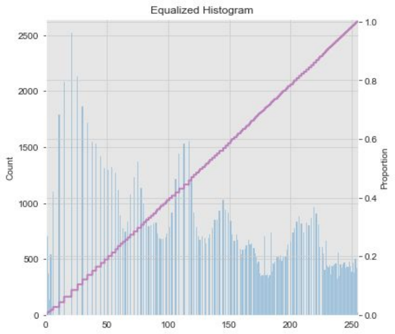
- Does not perform well for images with large area in low intensity region
- “Equalized” histogram does not have linear CDF

Failure of Histogram Equalization



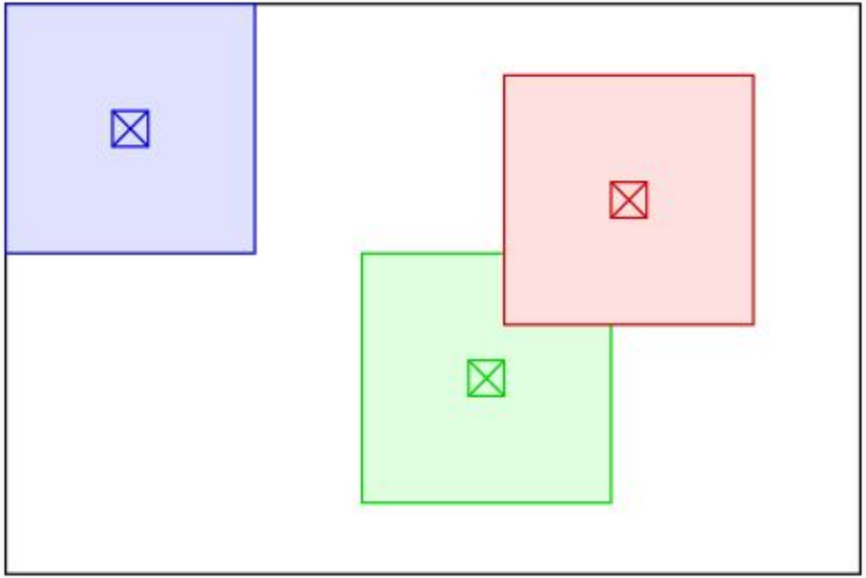






- Flat histogram (linear CDF) not always good
- Notice - Background much clear, but face details lost

Adaptive Histogram Equalization (AHE)
Idea



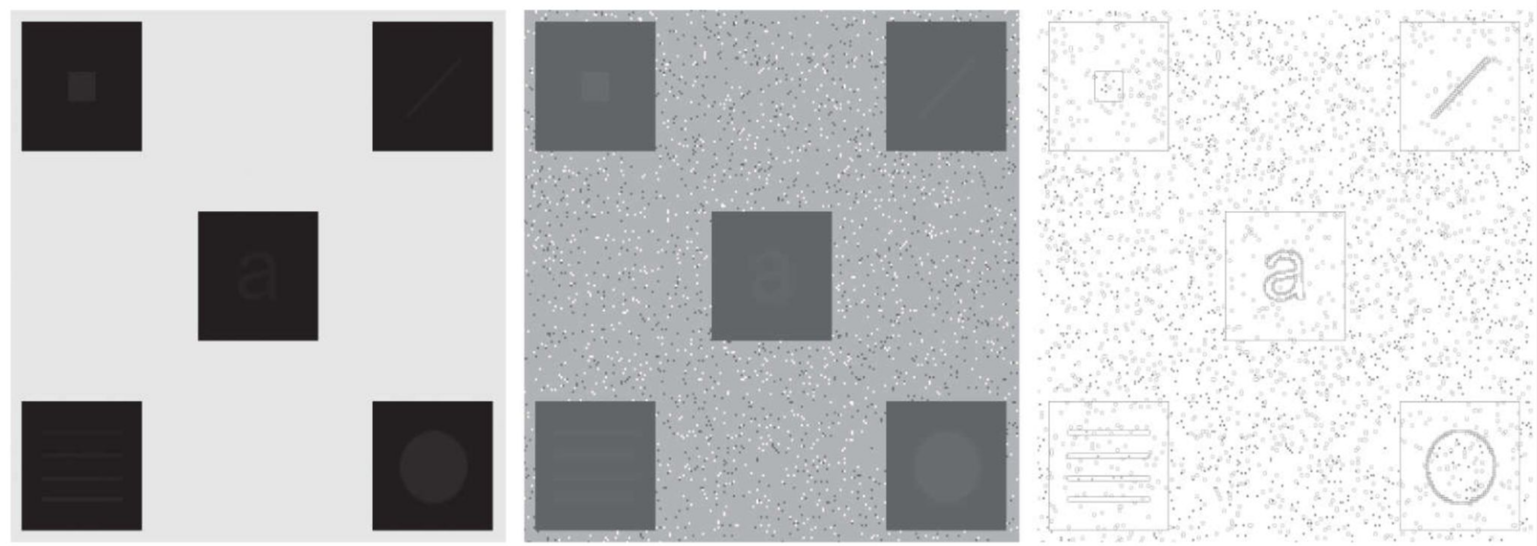
By Vswitchs - Own work, CC0, https://commons.wikimedia.org/w/index.php?curid=17467187

- If image contains regions that are significantly lighter (or darker), the contrast in those regions will not be sufficiently enhanced.
- Apply histogram equalization on a local neighbor

Adaptive Histogram Equalization (AHE)

Examples

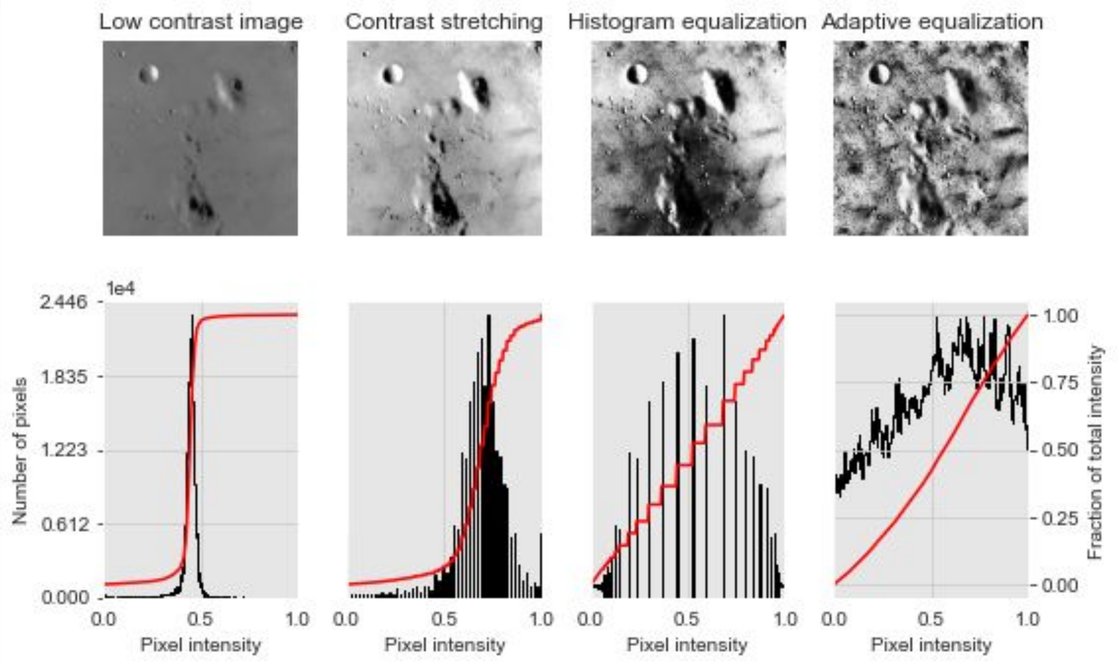
By Rafael C. Gonzalez & Richard E. Woods, 2018, *Digital Image Processing*, 4th Edition



Original Image | Global Histogram Equalization | Local (3x3) Histogram Equalization

Adaptive Histogram Equalization (AHE)

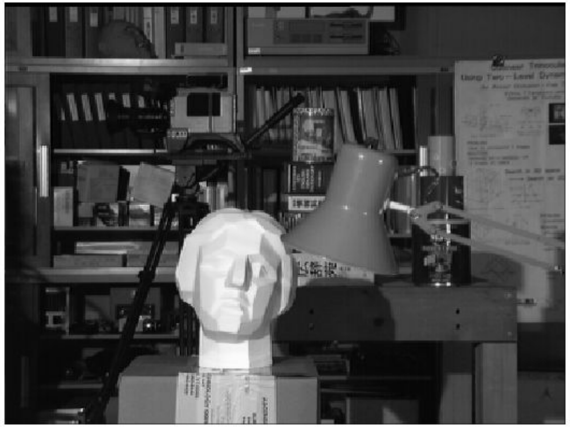
Examples



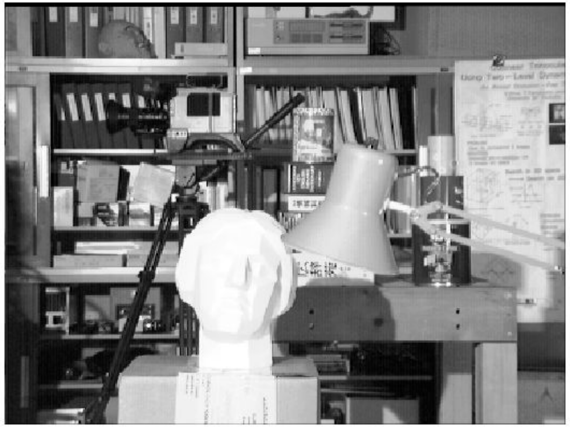
Reproduced from https://scikit-image.org/docs/stable/auto_examples/color_exposure/plot_equalize.html?highlight=histogram%20equalization

Adaptive Histogram Equalization (AHE)

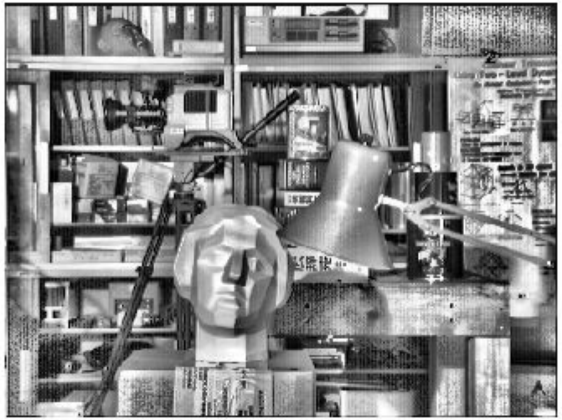
Examples



Original Image



Global Histogram Equalization



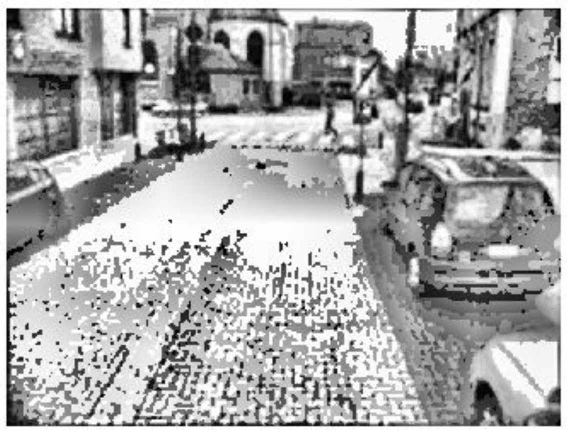
Local (36x36) Histogram Equalization

Adaptive Histogram Equalization (AHE)

Failure Example

Original Image

Global Histogram Equalization

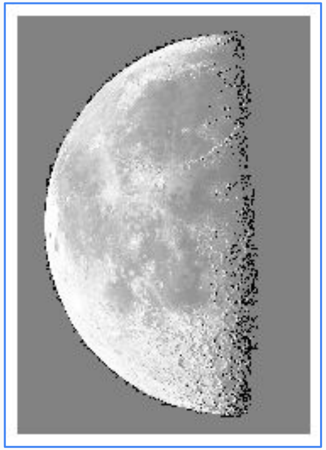


Local (100x100) Histogram Equalization

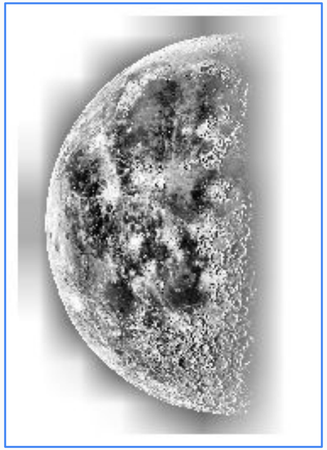
Adaptive Histogram Equalization (AHE)

Failure Example

Original Image



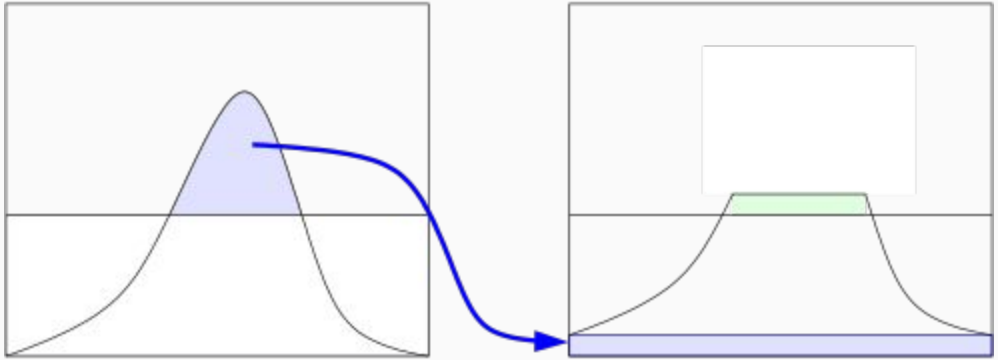
Global Histogram Equalization



Local (100x100) Histogram Equalization

Contrast Limited Adaptive Histogram Equalization (CLAHE)

- Both global and AHE tends to amplify the contrast in near-constant region
- Result - noise amplification
- Solution: Contrast Limited AHE (CLAHE), where the contrast amplification is limited



By Vswitchs - Own work, CC0, https://commons.wikimedia.org/w/index.php?curid=17500777

Contrast Limited Adaptive Histogram Equalization (CLAHE)

Original Image

Global Histogram Equalization

Local (100x100) Histogram Equalization

Clip limit = 1% of kernel size

Contrast Limited Adaptive Histogram Equalization (CLAHE)

Original Image

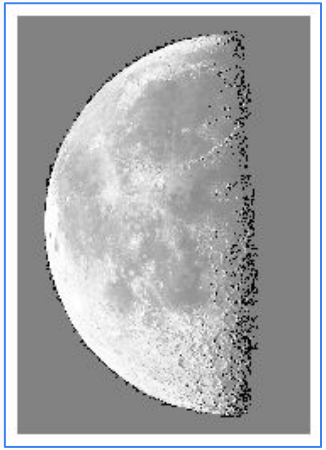
Global Histogram Equalization

Local (100x100) Histogram Equalization

Clip limit = 1% of kernel size

Contrast Limited Adaptive Histogram Equalization (CLAHE)

Original Image

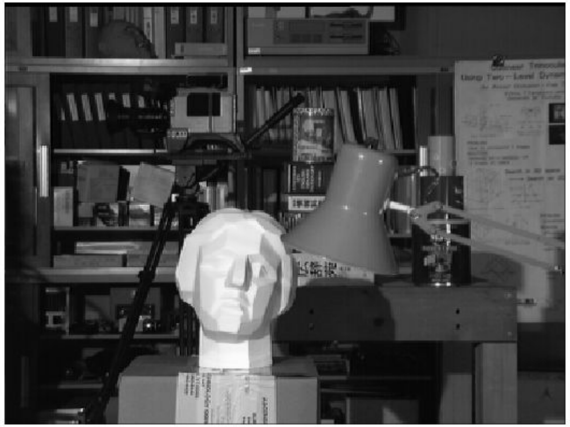


Global Histogram Equalization

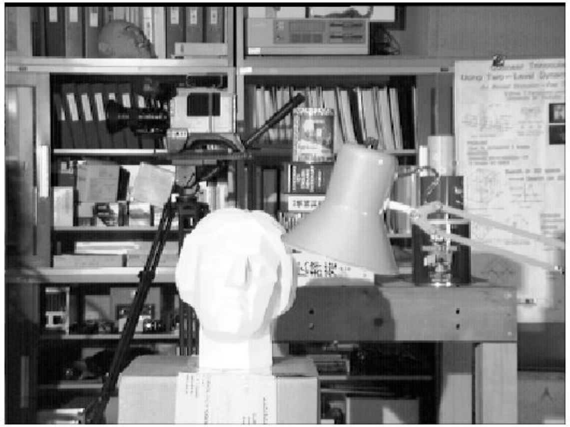
Local (100x100) Histogram Equalization

Clip limit = 5% of kernel size

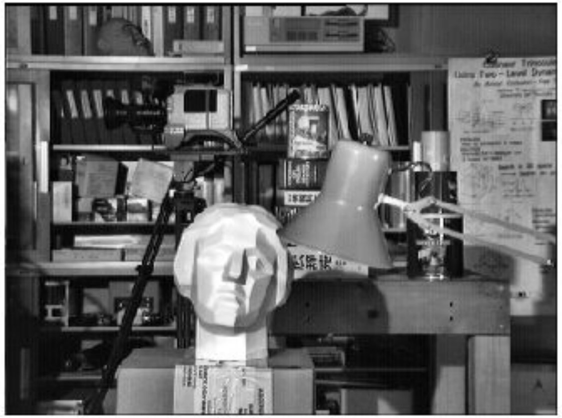
Contrast Limited Adaptive Histogram Equalization (CLAHE)



Original Image



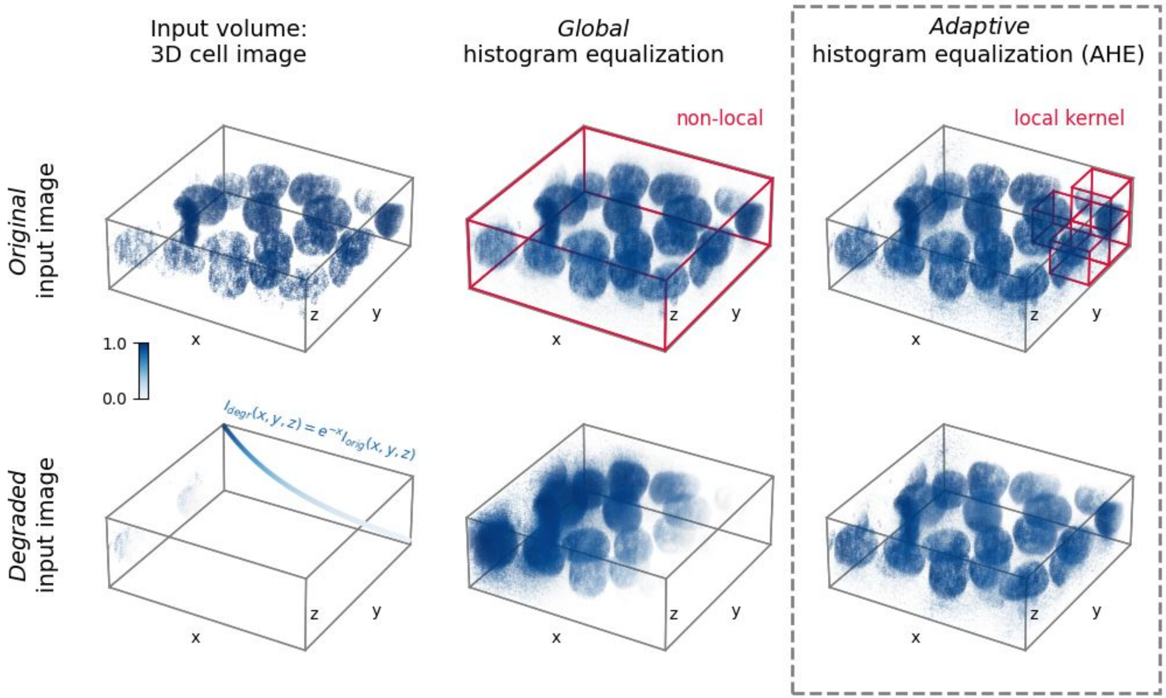
Global Histogram Equalization



Local (36x36) Histogram Equalization

Clip limit = 1% of kernel size

Contrast Limited Adaptive Histogram Equalization (CLAHE) - 3D Image



Reproduced from https://scikit-image.org/docs/stable/auto_examples/color_exposure/plot_adapt_hist_eq_3d.html#sphx-glr-auto-examples-color-exposure-plot-adapt-hist-eq-3d-py

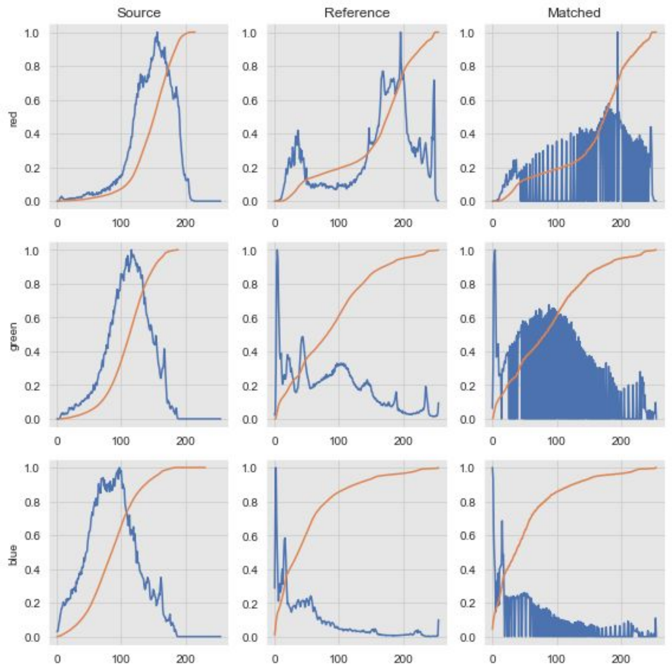
Histogram Matching (Specification)

Example

Reproduced from https://scikit-image.org/docs/stable/auto_examples/color_exposure/plot_histogram_matching.html?highlight=histogram%20equalization

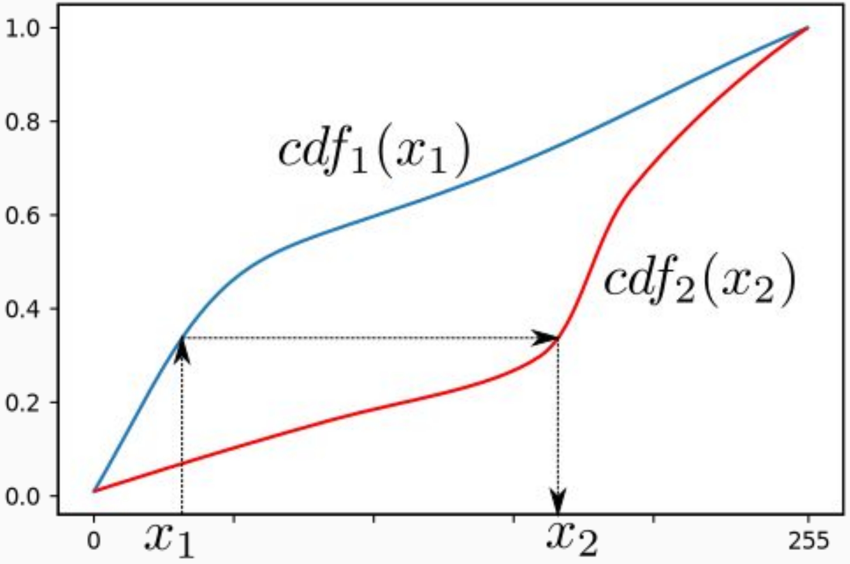
Histogram Matching (Specification)

Example



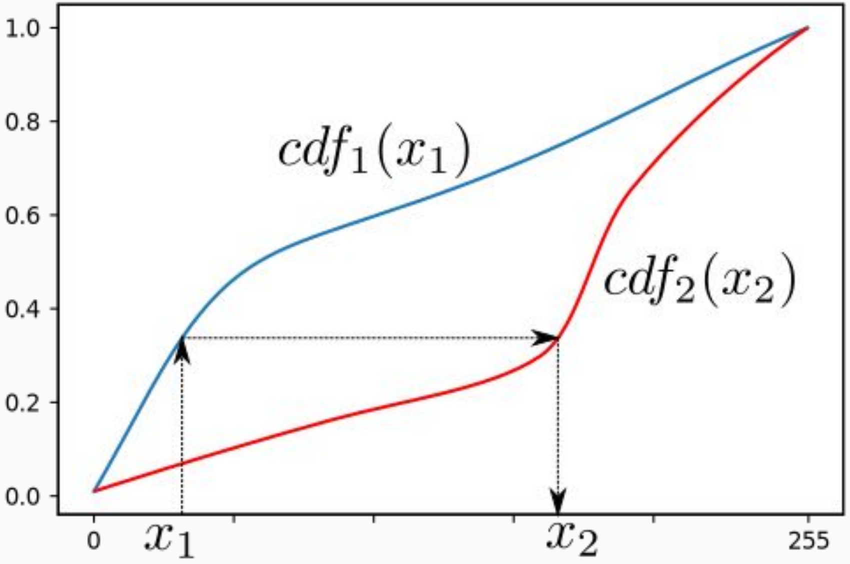
Reproduced from https://scikit-image.org/docs/stable/auto_examples/color_exposure/plot_histogram_matching.html?highlight=histogram%20equalization

Histogram Matching
How to



Histogram Matching

How to



$$x_2 = T(x_1) = cdf_2^{-1}(cdf_1(x_1))$$

Histogram Matching

Example

Thank You!