Synthesis techniques

i/p, o/p জানা থাকলে — তা হলে func থেকে ckt synthesis → using

1) Minterms → SOP

2) Maxterms → POS

3) K-Map

4) Multiplexers
   - → Simple / Noob way
     - → i/p হলো select pin
     - → o/p কে i/p of MUX করো।
   - → Smart way → take MSB of the i/p's as select pins and $\underbrace{\text{derive expressions of the select pi[ns]}}_{\text{MUX's i/p}}$
     - ↓ (use Shanon expansion theorem)
     - ↓ যা বেশিবার occur করে তাকে নিতে select pin(s).

5) Decoders
   - ↳ SOP
   - ↳ POS implement

6) use CPLD, FPGA etc boards.

7) use RAM, ROM chips.

8)

Boolean synthesis

Synthesis → finding the network that performs certain functional behaviour.

Minterm → complemented if 0 ; uncomplemented if 1

Maxterm → complemented if 1 ; uncomplemented if 0

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x_1	x_2	x_3	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

→ synthesis करना for the given i/ps and outputs
↓
design the network.

SOP → Sum of Products: → use Minterms

f = 1 के corresponding value इसको लेना।

$$f = f(x_1, x_2, x_3) = \bar{x}_1\bar{x}_2x_3 + \bar{x}_1x_2\bar{x}_3 + x_1\bar{x}_2\bar{x}_3 + x_1x_2x_3$$

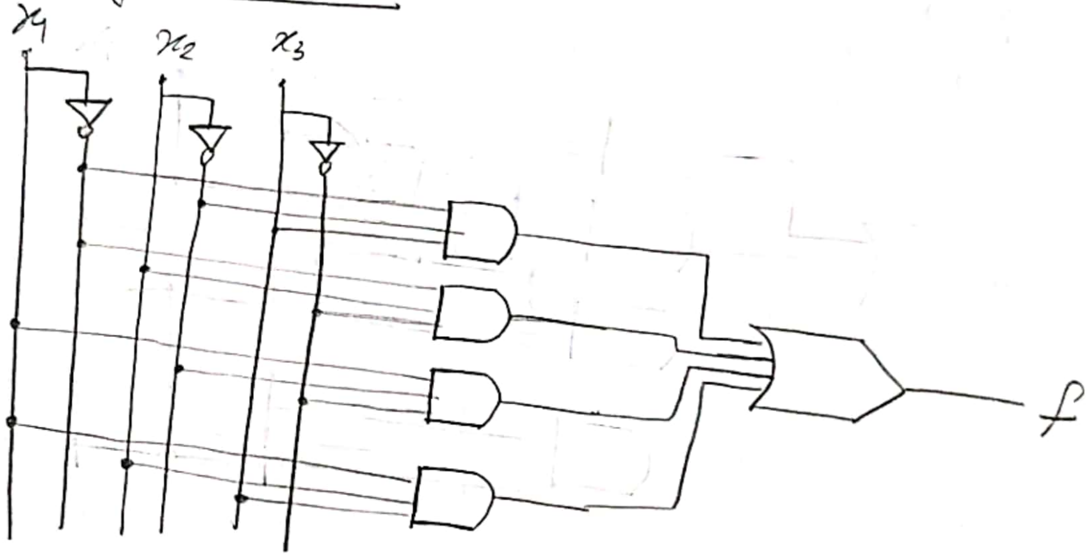
$$= m_1 + m_2 + m_4 + m_7$$

POS(Product of Sums): use Maxterms

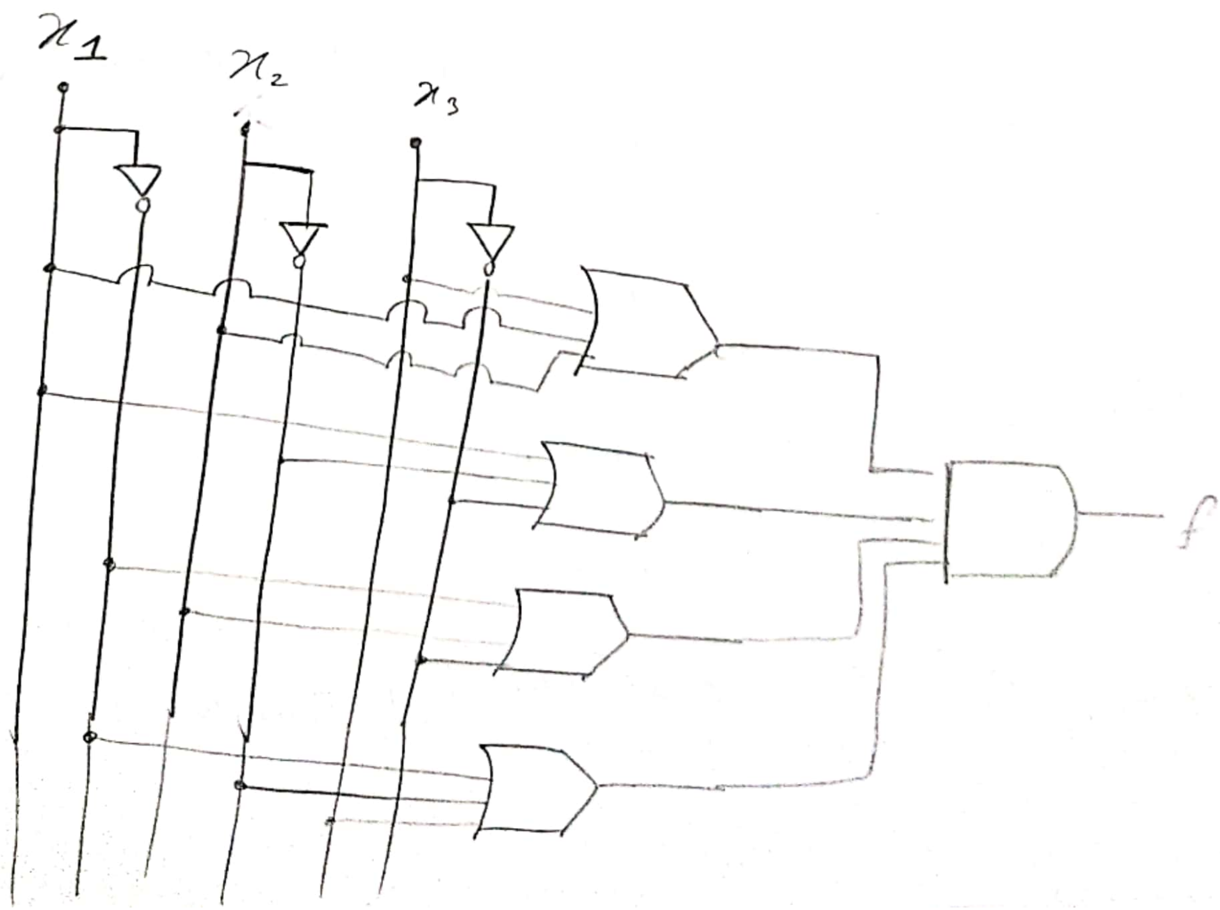
$f = M_0 \cdot M_3 \cdot M_5 \cdot M_6 \rightarrow$ f=0 এর respective terms নিব।

$= (x_1+x_2+x_3)\cdot(x_1+\overline{x_2}+\overline{x_3})\cdot(\overline{x_1}+x_2+\overline{x_3})\cdot(\overline{x_1}+\overline{x_2}+x_3)$

using SoP results :



using POS :



Logic Circuit Minimization

Karnaugh Map → to produce min. cost logic expression.

for →

x_1	x_2		f
0	0	m_0	1
0	1	m_1	1
1	0	m_2	0
1	1	m_3	1

$x_2 \backslash x_1$	0	1
0	m_0	m_2
1	m_1	m_3

for 3 variables →

$x_3 \backslash x_1x_2$	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5

for 4 variables :

$x_3x_4 \backslash x_1x_2$	00			
00				

---

for

x_1	x_2	f
0	0	0
0	1	0
1	0	1
1	1	1

K-map →

$x_1 \backslash x_2$	0	1
0	0	0
1	1	1

x_1 change হলে f changes.
x_2 change হলেও f = no change

SOP → $f = x_1\bar{x}_2 + x_1x_2 = x_1$

POS → $f = (x_1+x_2)\cdot(x_1+\bar{x}_2) = x_1$

f value x_1 dependent →

SOP → largest chain of 1 select
POS → " " " 0 "

$\left.\right\}$ 2^n where $n = 0, 1, 2, \ldots$

↑ element no. total in the chain or rectangle

x_1	x_2	f
0	0	1
0	1	1
1	0	0
1	1	1

$x_2 \backslash x_1$	0	1
0	1	0
1	1	1

→ x_2

↓ $\overline{x_1}$

$\therefore SOP = \overline{x_1} + x_2$

$2^1 = 2$ সংখ্যক elements in the chain

POS লিখতে চাইলে take chain of $0 \rightarrow (\overline{x_1} + x_2)$

for 3 variables:

→ Grey coding use

$x_3 \backslash x_1x_2$	00	01	11	10
0	1	1	1	1
1	0	0	1	1

→ $\overline{x_3}$

↓ $(x_1 + \overline{x_3})$ ↓ x_1

expressed in K-map

x_1	x_2	x_3	f
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

SOP → find chain of 1 (largest one) → the rectangle and chain also → $(x_1 + \overline{x_3}) = f$

diagonal chain not allowed.

POS → chain of 0 find → $f = (x_1 + \overline{x_3})$

useful info for verification

chain size $= 2^2 = 4$ for SOP and variable no. $= 3$টি

∴ per chain এ variable কমবে $= 4 - 3 = 1$টি var. $\leftarrow$ x_1 and $\bar{x}_3$

---

find K-map for $\Sigma m(1,4,5,6)$:

→ ওসব position এ 1 আছে ওদের ধরেছি।

$x_3 \backslash x_1x_2$	00	01	11	10
0	0 ($_0$)	0 ($_2$)	1 ($_6$)	1 ($_4$)
1	1 ($_1$)	0 ($_3$)	0 ($_7$)	1 ($_5$)

$(x_3 + x_1)$

$\rightarrow (\bar{x}_3 x_1)$

$\rightarrow x_1 \bar{x}_2$

$(\bar{x}_3 + \bar{x}_2)$

$\rightarrow x_3 \bar{x}_2$

as grey code = cyclic

এরা connected as 00 and 10 এ difference মাত্র 1টা in grey code hence এদেরও chain করা যাবে।

$\therefore$ SOP $\rightarrow f = x_1\bar{x}_3 + x_3\bar{x}_2$ → here, $x_1\bar{x}_2$ is redundant.

POS $\rightarrow f = (x_1 + x_3)\cdot(\bar{x}_3 + \bar{x}_2) = x_1\bar{x}_3 + x_1\bar{x}_2 + x_3\bar{x}_2$

---

for 4 variables → find POS and SOP from K-map

x_3x_4 \ x_1x_2	00	01	11	10
00	0	0	0	0
01	0	0	1	1
11	1	0	0	1
10	1	0	0	1

(x_1+x_3)

(x_3+x_4)

$x_1\bar{x}_3x_4$

invalid as 1, 0, 5, 8 ≠ 2^n, n=0,1,2 ..

$x_1\bar{x}_2x_4$

redundant term

$(\bar{x}_2+\bar{x}_3)$

→ rectangle chain exists due to cyclic grey code → $\bar{x}_2x_3$

$$SOP = (x_1\bar{x}_3x_4 + \bar{x}_2x_3 + \cancel{x_1\bar{x}_2x_4})$$

$$POS = (x_1+x_3)(x_3+x_4)(\bar{x}_2+\bar{x}_3)$$

$$=(x_1x_3 + x_1x_4 + x_3 + x_3x_4)(\bar{x}_2+\bar{x}_3)$$

$$= x_1\bar{x}_2x_3 + x_1\bar{x}_2x_4 + x_1\bar{x}_3x_4 + (\bar{x}_2x_3 + x_3\bar{x}_2x_4)$$

$$= \{\bar{x}_2x_3(\underbrace{1+x_1+x_4}_{1}) + x_1\bar{x}_2x_4 + x_1\bar{x}_3x_4\}$$

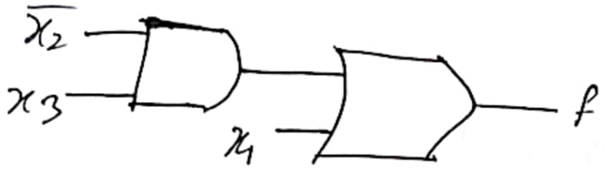
= SOP same

Cost হিসাব

cost = (# gates) + (# total inputs to all the gates)

Ex: $x_1 + \bar{x}_2 x_3$

for i/p variables $x_2, \bar{x}_2$ এর cost same, hence →



→ $\text{Cost} = \underset{\text{no. of gates}}{2} + (2+1+1) = 6$

Ex:

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00	0	0	0	0
01	0	0	1	1
11	1	1	1	1
10	1	1	1	1

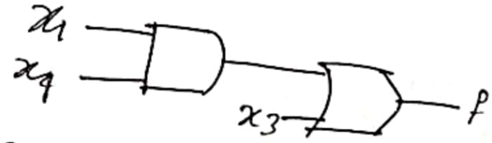
$(x_3 + x_4)$

$(x_1 + x_3)$

$x_1 x_4$

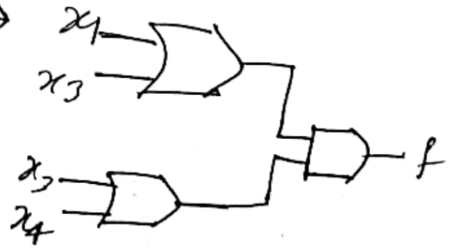
x_3

SOP → $f = x_1 x_4 + x_3$ →



Cost = $2 + (2+1+1) = 6$ হয়।

POS → $f = (x_1 + x_3)(x_3 + x_4)$ →



$x_1x_3 + x_1x_4 + x_3 + x_3x_4$

$= x_3(1 + x_1 + x_4) + x_1x_4$

= SOP হয়ে এর format

কিন্তু SOP format, not POS format

hence POS চাইলে $(x_1+x_3)(x_3+x_4)$ এই লিখে দিলে cost টা বেশি লাগবে।

Cost = $3 + 6 = 9$

3 টা gates এর

Ex: find cost in SOP and POS format and find the min. cost.

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00	1	1	1	0
01	1	1	1	0
11	0	0	1	1
10	0	0	1	1

$x_2\bar{x}_3$

$\bar{x}_1\bar{x}_3$

x_1x_3

x_1x_2

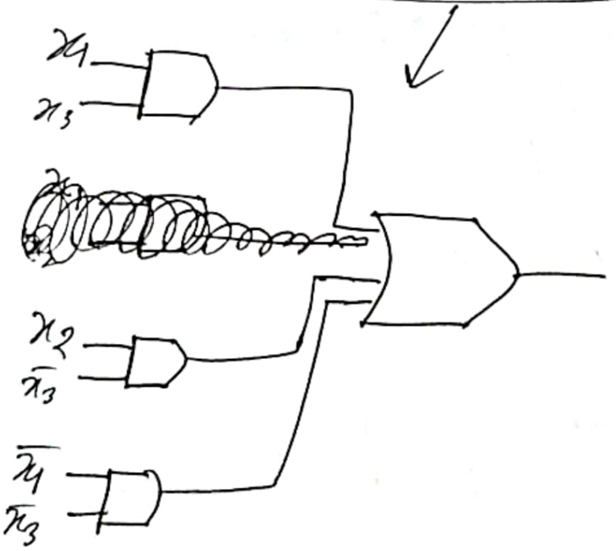
as 6টা 1 এর chain here and $6 \neq 2^n$ where, $n = 0, 1, 2, 3, \ldots$

$\therefore$ invalid chain এটা

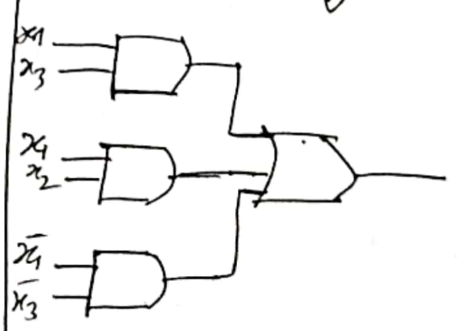
in SOP → $f = x_1x_3 + x_2\bar{x}_3 + \bar{x}_1\bar{x}_3$

highlighted $\begin{pmatrix}1\\1\end{pmatrix}$ কে include অন্য either chain or rect দিয়ে হলো

অথবা, $x_1x_3 + x_1x_2 + \bar{x}_1\bar{x}_3$



cost $= 4 + (9) = 13$



cost $= 4 + 9 = 13$ = same.

PTO.

the

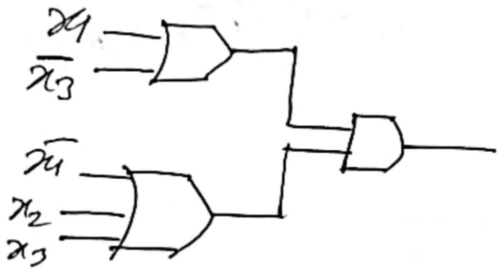
for POS format:

x_3x_4 \ x_1x_2	00	01	11	10
00	1	1	1	0
01	1	1	1	0
11	0	0	1	1
10	0	0	1	1

$\rightarrow (\bar{x}_1 + x_2 + x_3)$

$(x_1 + \bar{x}_3)$

$$f = (x_1 + \bar{x}_3)\cdot(\bar{x}_1 + x_2 + x_3)$$



$\rightarrow$ cost = 3 + 7 = 10 < SOP

Ex:

x_3x_4 \ x_1x_2	00	01	11	10
00	0	0	0	0
01	0	1	1	0
11	1	1	0	1
10	1	1	1	1

$\rightarrow x_2\bar{x}_3x_4$

$\bar{x}_1x_3$

$x_3\bar{x}_4$

$\bar{x}_2x_3$

SOP তে → $f = (\bar{x}_1 x_3 + x_3 \bar{x}_4 + \bar{x}_2 x_3 + x_2 \bar{x}_3 x_4)$

in POS:

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00	0	0	0	0
01	0	1	1	0
11	1	1	0	1
10	1	1	1	1

→ $(x_3 + x_4)$

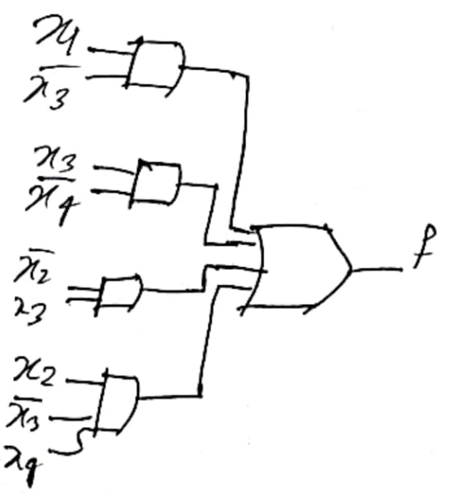
→ $(x_2 + x_3)$

→ এখন এটা → $(\bar{x}_1 + \bar{x}_2 + \bar{x}_3 + \bar{x}_4)$

Mr. নাম্বার = 4 − 0 = 4টা

$2^0 = 1$ element পাবো

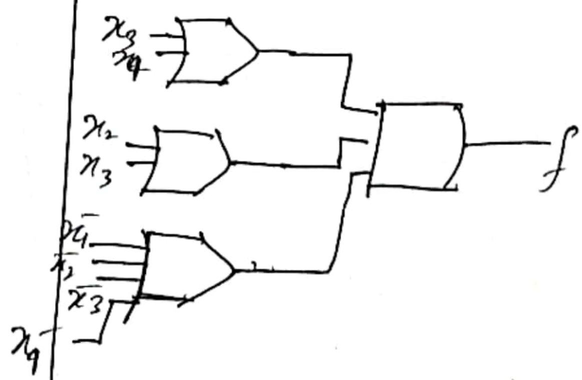
$$POS \rightarrow f = (x_3 + x_4)(x_2 + x_3) \cdot (\bar{x}_1 + \bar{x}_2 + \bar{x}_3 + \bar{x}_4)$$



Cost = 5 + 13 = (18)

for SOP

for POS:



Cost = 4 + 11 = 15

For 5 variables

$x_5 = 0$

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00				
01			1	1
11	1	1		
10	1	1		

$x_5 = 1$

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00				1
01			1	1
11	1	1		
10	1	1		

($x_1 \bar{x}_2 \bar{x}_3 x_5$)

4টার cube $\Rightarrow (x_1 \bar{x}_3 x_4)$

ঘর: মান = $\overset{16}{1}\overset{8}{0}\overset{4}{1}\overset{2}{0}\overset{1}{0}$ = 20

ঘর: মান $\Rightarrow \overset{16}{1}\overset{8}{1}\overset{4}{1}\overset{2}{0}\overset{1}{1}$ = 29

৮ এর cube এটি

$\downarrow$

$\bar{x}_1 x_3$

$\therefore$ SOP $\rightarrow f = (\bar{x}_1 x_3 + x_1 \bar{x}_3 x_4 + x_1 \bar{x}_2 \bar{x}_3 x_5)$

2টা block একটা আরেকটার উপর আছে in 3D.

Things to Note

1) K-map provides the minimized SOP or POS version but not the most minimized form.

2) If a bit changes, it doesn't matter. If it stays same, it matters.

3) include all 1's for SOP → use Minterm
   " " 0's for POS → use Maxterm

4) Select largest chain (lateral or vertical) or rectangle with size = 2^n where n = 0, 1, 2, 3, —— etc.

5) Chains may partially overlap

6) Grey coding is cyclic hence ends are connected.

K-Map - Part-2

Literal: each appearance of a variable either uncomplemented or complemented.

Ex: $x_1\bar{x}_2$ এর literal = 2

$x_1\bar{x}_3x_4$ " " = 3

$x_1\bar{x}_1x_2$ " " = নাই as result = 0 হয়ে গেছে।

Implicant: যতগুলো product term আছে যারা 1 দিতে পারে তারা সবাই একেকটি implicant.

Ex:

$x_3 \backslash x_1x_2$	00	01	11	10
0	1	1	0	0
1	1	1	1	0

∴ total = 11 টি implicant → see video. → not 12 ie নিচের 3টি 1 এর row বাদ as $3 \neq 2^n$; $n=0,1,2,\ldots$ etc.

Prime implicant: এমন implicants যাদেরকে আর বড় করা যাবে না।

$x_3 \backslash x_1x_2$	00	01	11	10
0	1	1	0	0
1	1	1	1	0

∴ total = 2টি

Ex:

Cover: যতগুলো implicants sum করে 1 বানানো যায় ওদের total যতগুলো minterm আছে function expression (SOP) এ।

x_3 \ x_1x_2	00	01	11	10
0	1	1	0	0
1	1	1	1	0

SOP form

$$f = \bar{x}_1\bar{x}_2 x_3 + \bar{x}_1\bar{x}_2 x_3 + x_1 \bar{x}_1 \bar{x}_3 + \bar{x}_1 x_2 x_3 + x_1 x_2 x_3$$

$$= \bar{x}_1\bar{x}_2 + \bar{x}_1 x_2 + x_2 x_3$$

$$= \bar{x}_1 + x_2 x_3 \longrightarrow \text{সবচেয়ে ছোট cover} = 2$$

সবচেয়ে বড় cover $= 5$

Cost

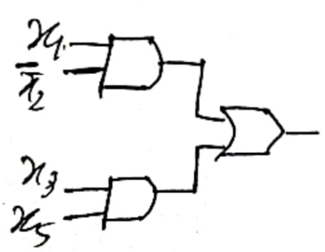
a) Cost = (#gates) + (# total i/p to all gates)

b) i/p side এ কোনো complement, uncomplemented same cost ধরে

$x_2, \bar{x}_2$ same ধরব, extra NOT বানানোর cost নাই

c) But, if inversion needed inside a ckt, not at i/p side then NOT gate and its i/p cost included.

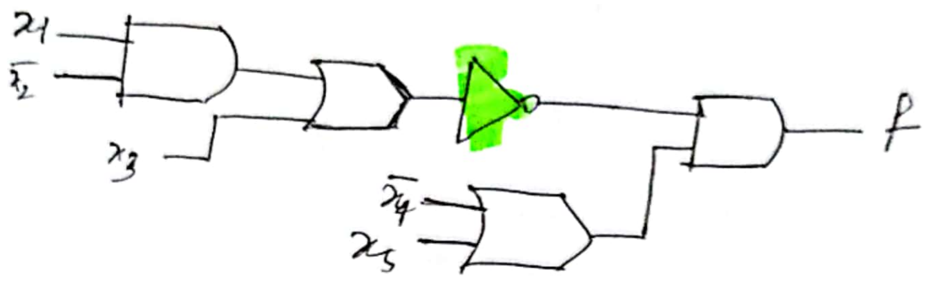
$$f = x_1\bar{x}_2 + x_3 x_5$$



Cost $= 3 + (6) = 9$

Ex:

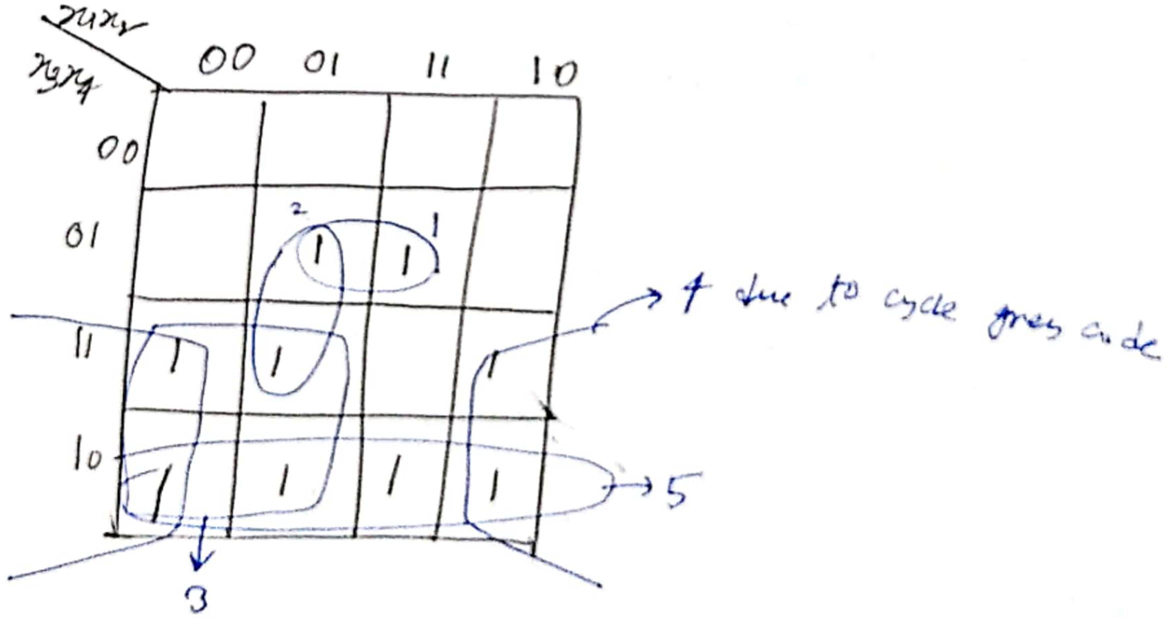
$$f = \overline{x_1\bar{x}_2 + x_3} \cdot (\bar{x}_4 + x_5)$$



Cost = #gates + #total i/p's to all gates
= 5 + {9} = 14

Essential Prime Implicant

কোনো prime implicant বাদ দিয়ে দিলে যদি ঐ implicant এর কোনো element এর আর কোনো implicant না হলে then that prime implicant is essential.



1 no. prime implicant বাদ দিলে left side 1 এর জন্য prime implicant = 2 no. exists but Right side 1 এর আর কেউ নাই hence 1 no. prime implicant is essential → $x_2\bar{x}_3x_4$

5 no also essential prime implicant as 5 no. বাদ দিলে বাম হতে ওই 1 এর আর কোনো implicant নাই। → $x_3\bar{x}_4$

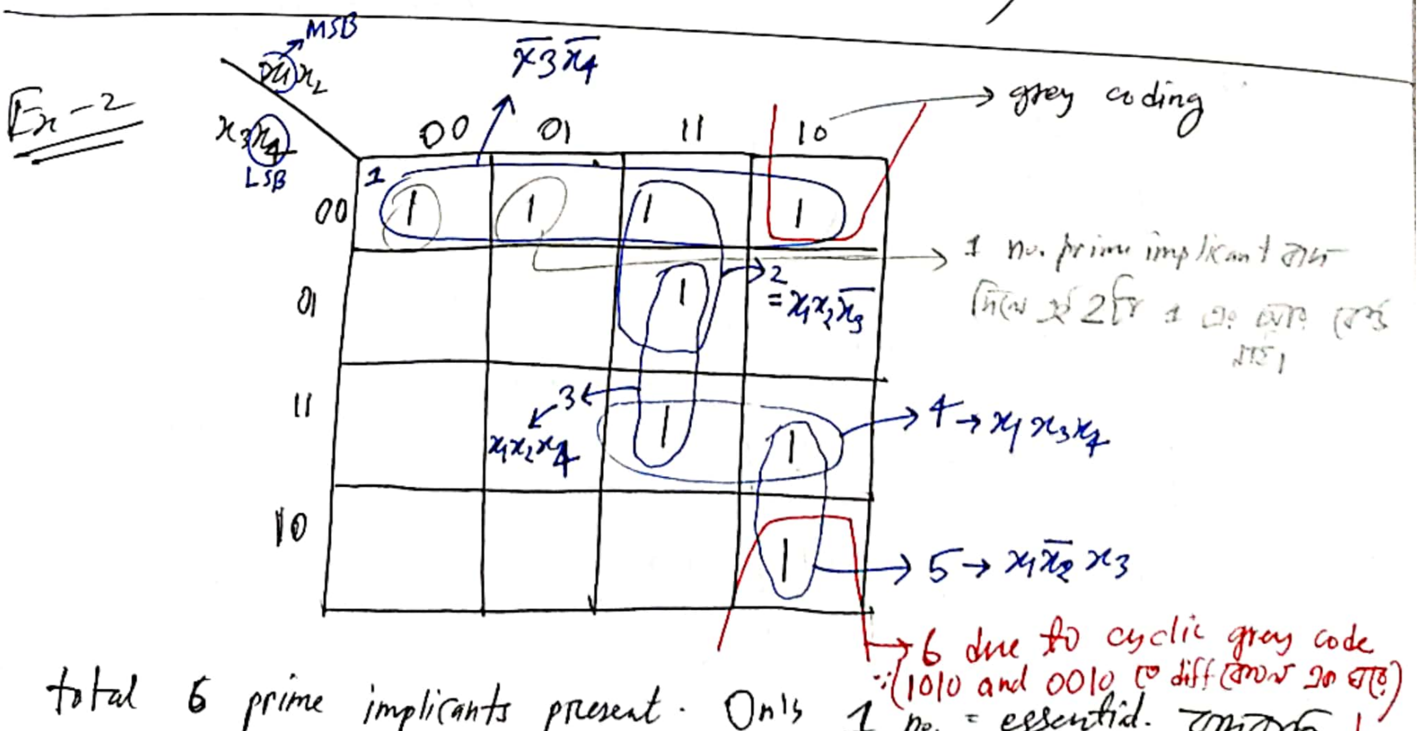
4 no. also essential as (3, 4) এ থাকা 1 এর আর কোনো prime implicant নেই অন্য গ্রহণ করায়। → $\bar{x}_2 x_3$

বাকি দুটো অর্থাৎ 2 no. and 3 no. prime implicants = not essential.

Non-essential এর ক্ষেত্রে take the largest prime implicant ie the 3 no. → $\bar{x}_1 x_3$ → to get minimized result.

$$\therefore f = SOP = (x_2\bar{x}_3x_4 + x_3\bar{x}_4 + \bar{x}_2x_3 + \bar{x}_1x_3)$$

Ex-2



total 6 prime implicants present. Only 1 no. = essential. কারণ অন্যান্যগুলোতেই 1 এর অন্য prime implicant এ স্থান পাওয়া যাচ্ছে।

কয়েকটা select করে করে দেখা যাবে:

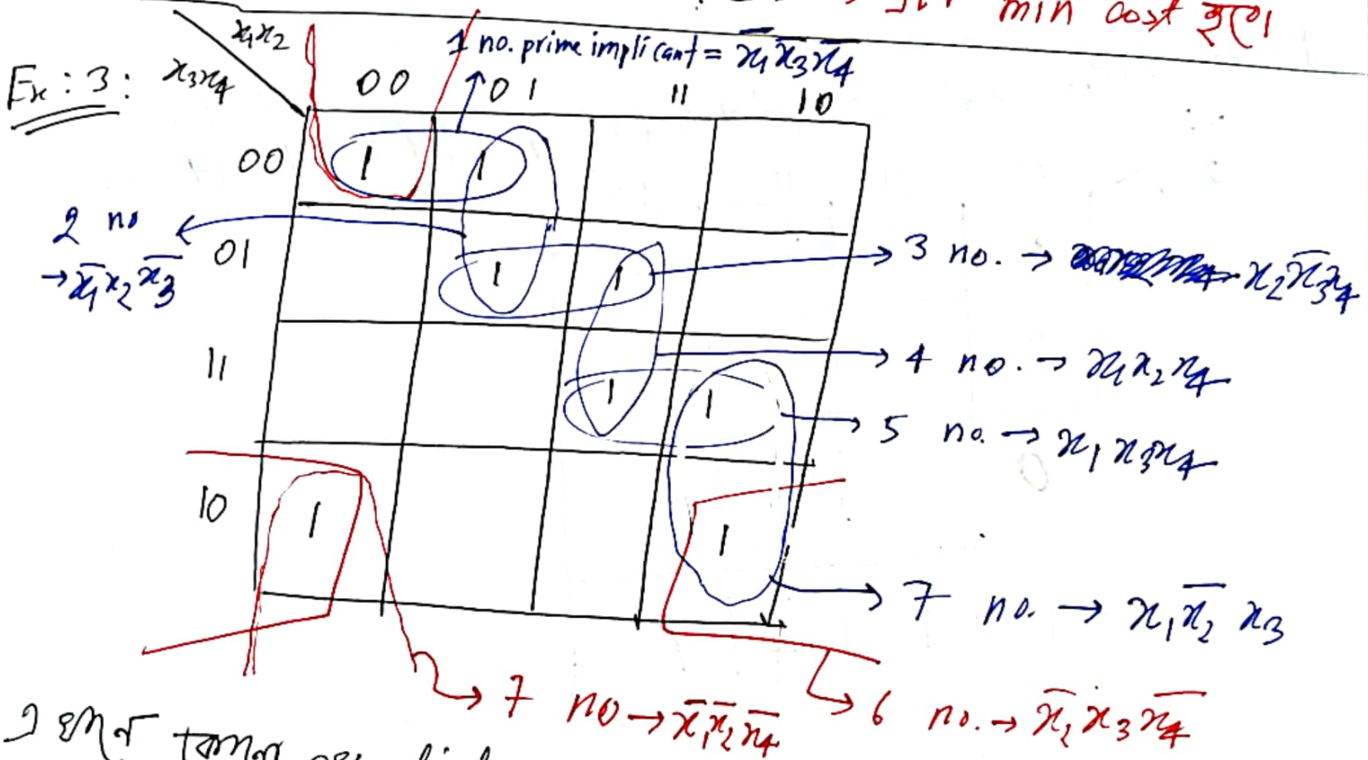
Way-1: $f = SOP = \text{essential} + 2 \text{ no.} + \cancel{3 \text{ no.}}$ দরকার নাই $+ 4 \text{ no.} + 5 \text{ no.}$

$$= \bar{x}_3\bar{x}_4 + x_1 x_2 \bar{x}_3 + x_1 x_3 x_4 + x_1 \bar{x}_2 x_3$$

Way 2: $f = 1 \text{ no.} + 3 \text{ no.} + 5 \text{ no.}$ নিলেই সব 1 cover হয়ে যাবে

$$= \bar{x}_3\bar{x}_4 + x_1 x_2 x_3 + x_1 \bar{x}_2 x_3$$ → এটা min cost হলো

Ex: 3:



এখানে কোনো essential prime implicant নাই কোনো। So, 1 কোনো একটা দিয়ে কাজ শুরু করি।

→ **Way-1:**

$$f = 1 \text{ no.} + 3 \text{ no.} + 5 \text{ no.} + 6 \text{ no.} = \bar{x}_1\bar{x}_3\bar{x}_4 + x_2\bar{x}_3 x_4 + x_1 x_3 x_4 + \bar{x}_2 x_3 \bar{x}_4$$

Way-2: $f = 7 \text{ no.} + 2 \text{ no.} + 4 \text{ no.} + 7 \text{ no}$

$$= \bar{x}_1\bar{x}_2\bar{x}_4 + \bar{x}_1 x_2 \bar{x}_3 + x_1 x_2 x_4 + x_1 \bar{x}_2 x_3$$

$\boxed{\text{Both same cost}}$

K-Map → Part 3

Incompletely specified function case এ K-map :

A function that has don't care condition(s) is incompletely specified func.

Ex: if two switch are such that both can't be closed simultaneously then only allowed i/p are 00, 01, 10 ; 11 i/p is invalid hence don't care state.

Ex: $f(x_1, \dots x_4) = \Sigma m(2,4,5,6,10) + D(12, 13, 14, 15)$ → মানে Don't care terms

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00	0	1	d=1	0
01	0	1	d=1	0
11	0	0	d	0
10	1	1	d=1	1

don't care term না থাকলে নিলাম → SOP → $f = \bar{x}_1 x_2 \bar{x}_3 + \bar{x}_1 x_3 \bar{x}_4 + \bar{x}_2 x_3 \bar{x}_4$

As don't care term আসতেছে না hence আমরা $d = 1$ হিসেবে নিতে পারি then →

$f = x_2 \bar{x}_3 + x_3 \bar{x}_4$

↳ বা $d=0$ - ও নিতে পারি ইচ্ছে

↓

much minimized form যে হবে পাইলাম।

Note that শুধু d এবং কে cycle নিতে হবে না। যতগুলো 1 cover হয় এমন ভাবে d=1 নিয়ে chain বানাবো।

POS form

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00	0	1	d	0
01	0	1	d	0
11	0	0	d	0
10	1	1	d	1

① → (x_2+x_3)

② → $(x_2+\bar{x}_3+\bar{x}_4)$

③ → $(x_1+\bar{x}_3+\bar{x}_4)$

don't care term হিসেবে না নিলে → $f = POS$

$$= (x_1+x_3)(x_1+\bar{x}_3+\bar{x}_4)(x_2+\bar{x}_3+\bar{x}_4)$$

---

if d are considered now :

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00	0	1	d	0
01	0	1	d	0
11	0	0	d=0 নিই	0
10	1	1	d	1

④ → (x_2+x_3)

⑤ → $(\bar{x}_3+\bar{x}_4)$

$f = POS = (x_2+x_3)\cdot(\bar{x}_3+\bar{x}_4)$ → more cost efficient

Multiple Output Ckt case

individual SOP

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00			1	1
01		1	1	1
11	1	1		
10	1	1		

(a) function f_1

$$f_1 = x_1\bar{x}_3 + \bar{x}_1 x_3 + x_2\bar{x}_3x_4$$

↓

cost = 14

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00			1	1
01			1	1
11	1	1	1	
10	1	1		

(b) function f_2

common parts

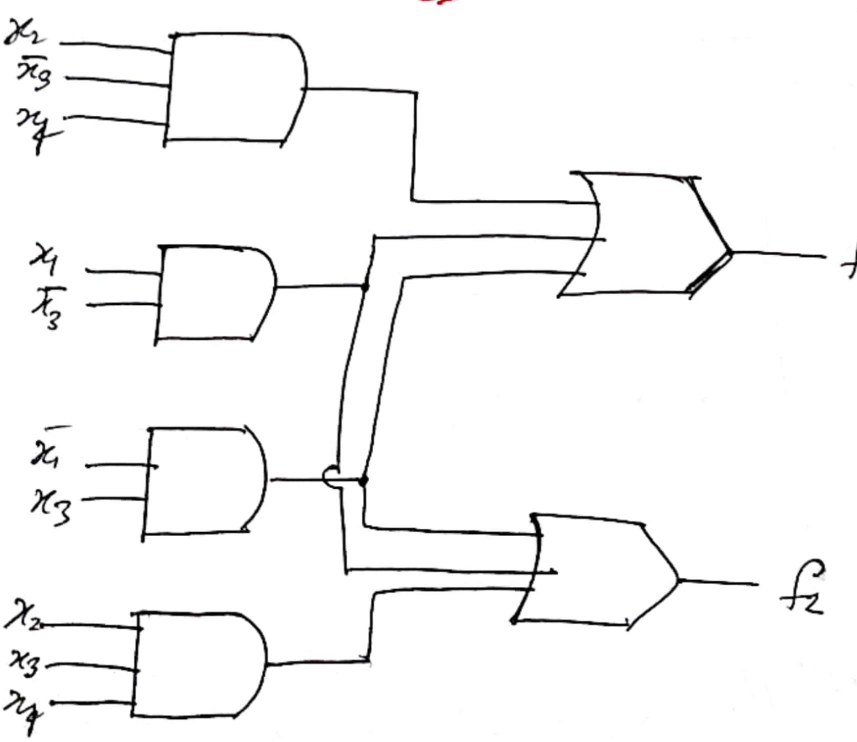
$$f_2 = \bar{x}_1x_3 + x_1\bar{x}_3 + x_2x_3x_4$$

↓

cost = 14

total cost = 14 + 14 = 28

Combined SOP



Cost = 6 + 16

= 22

Using POS

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00	0	0	1	1
01	0	1	1	1
11	1	1	0	0
10	1	1	0	0

$$f_1 = (x_1 + x_3 + x_4)\cdot(x_1 + x_2 + x_3)\cdot(\bar{x}_1 + \bar{x}_3)$$

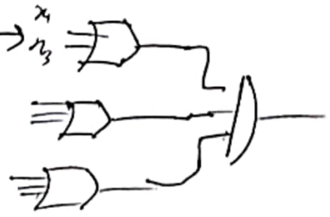
$$\text{Cost} = (3+3+2+3)$$
$$= 15$$

total = 30

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00	0	0	1	1
01	0	0	1	1
11	1	1	1	0
10	1	1	0	0

$$f_2 = (x_1 + x_3)(\bar{x}_1 + x_2 + \bar{x}_3)(\bar{x}_1 + \bar{x}_3 + x_4)$$

Cost →



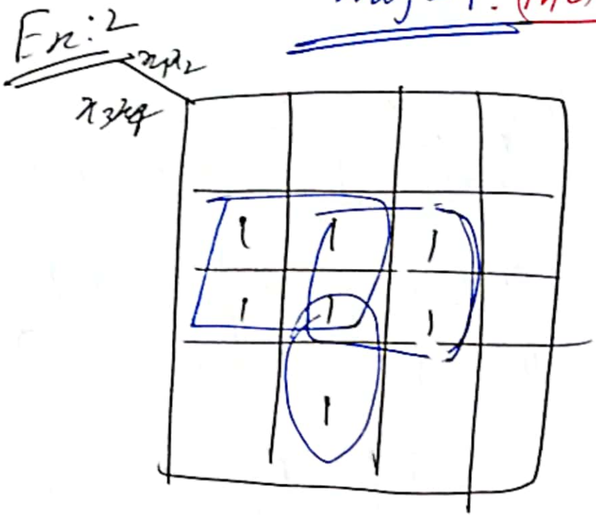
$= 15$

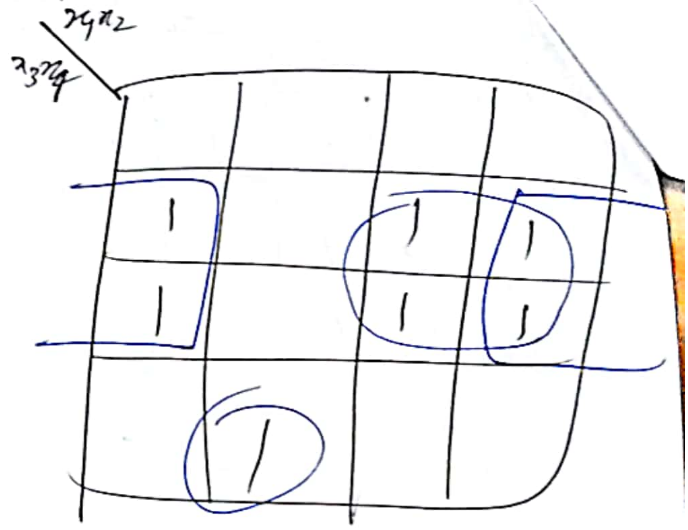
Combined করলে

No common parts. অনেক বড় হবে ckt. Cost > 30 হবে
যাতে চাইনা।

Ex: 2

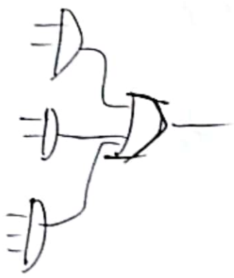
Way-1: (individual SOP)





$f_1 = \bar{x}_1 x_4 + x_2 x_4 + \bar{x}_1 x_2 x_3$

$f_2 = x_1 x_4 + \bar{x}_2 x_4 + \bar{x}_1 x_2 x_3 \bar{x}_4$

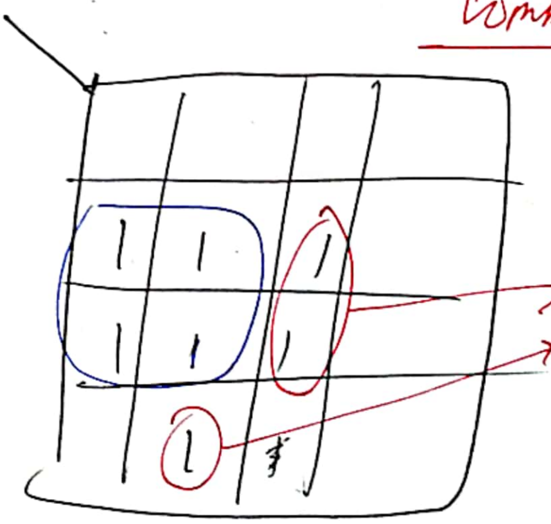


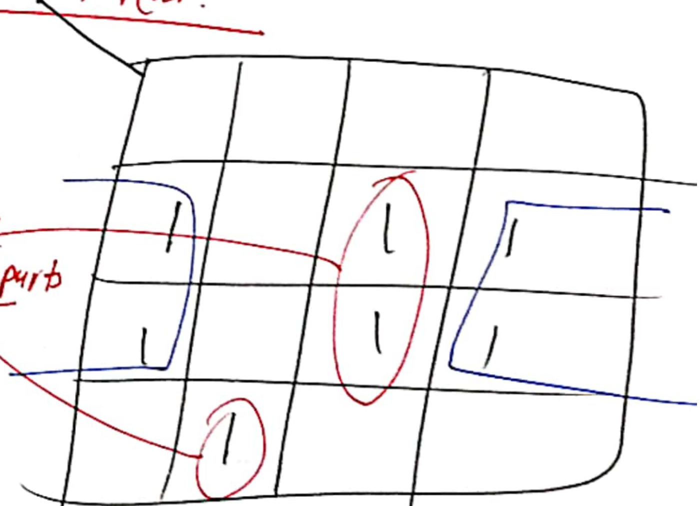
$\text{cost}_{f_1} = 4 + 10 = 14$

$\Rightarrow \text{cost} = 15$

total cost $= 14 + 15 = 29$

Combined SOP নিলে!

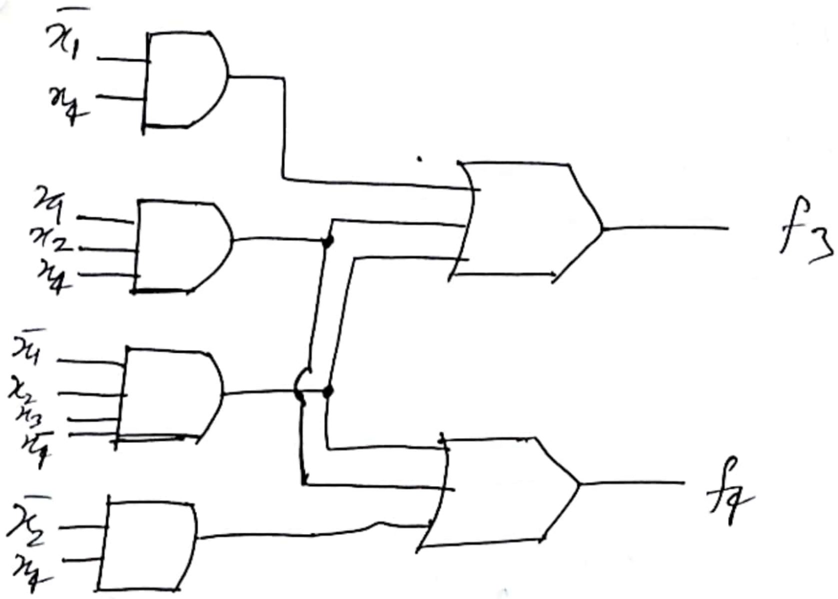




Common parts নিবে

$f_3 = \boxed{x_1 x_2 x_4 + \bar{x}_1 x_2 x_3 \bar{x}_4}$ $+ \bar{x}_1 x_4$

$f_4 = \boxed{x_1 x_2 x_4 + \bar{x}_1 x_2 x_3 \bar{x}_4}$ $+ \bar{x}_2 x_4$



Cost in combined = 6 + 17 = 23 → কম

POS নিলে :

x_3x_4 \ x_1x_2	00	01	11	10
00	0	0	0	0
01	1	1	1	0
11	1	1	1	0
10	0	1	0	0

$f_3 = (x_2 + x_4) . (x_3 + x_4) . (\bar{x}_1 + x_4) (\bar{x}_1 + x_2)$

x_3x_4 \ x_1x_2	00	01	11	10
00	0	0	0	0
01	1	0	1	1
11	1	0	1	1
10	0	1	0	0

$f_4 = (x_3 + x_4) (x_2 + x_4) (\bar{x}_1 + x_4) (\bar{x}_1 + x_2 + \bar{x}_4)$

Watch out for corner elements as well

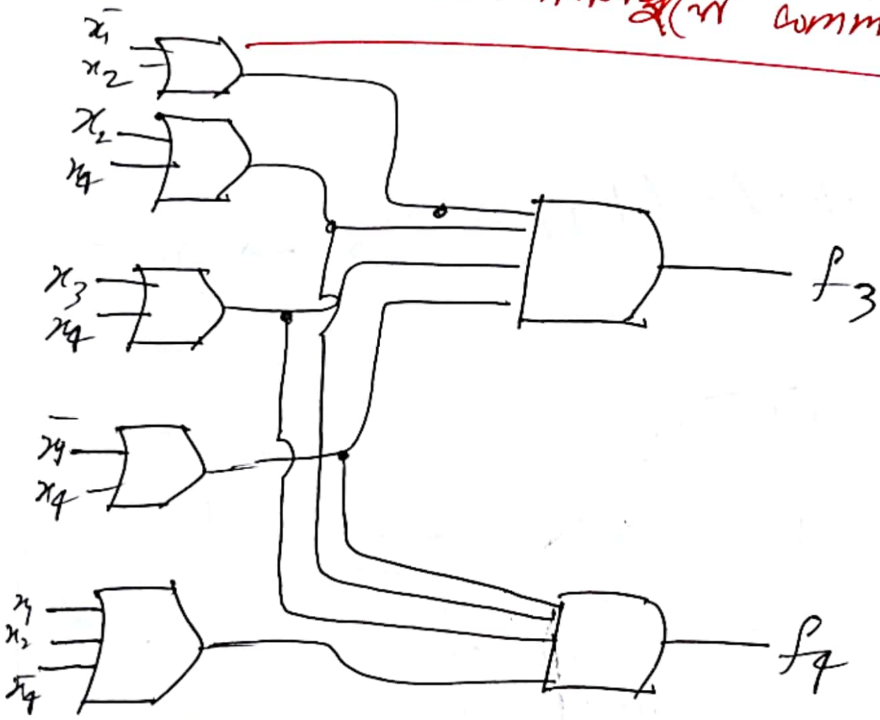
f_3 cost → $5+12 = 17$

f_4 " → $5+13 = 18$

total = 35

Combined POS

Common জিনিসগুলো common নিয়ে আসবো ⇒



Cost = $7 + 19 = 26$

শেষ