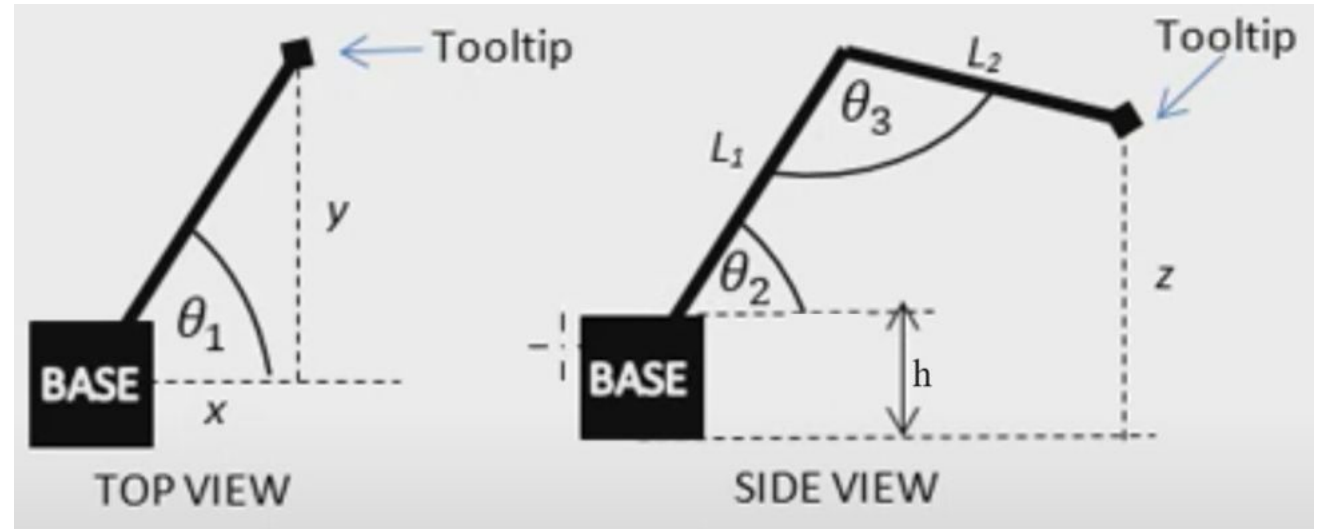
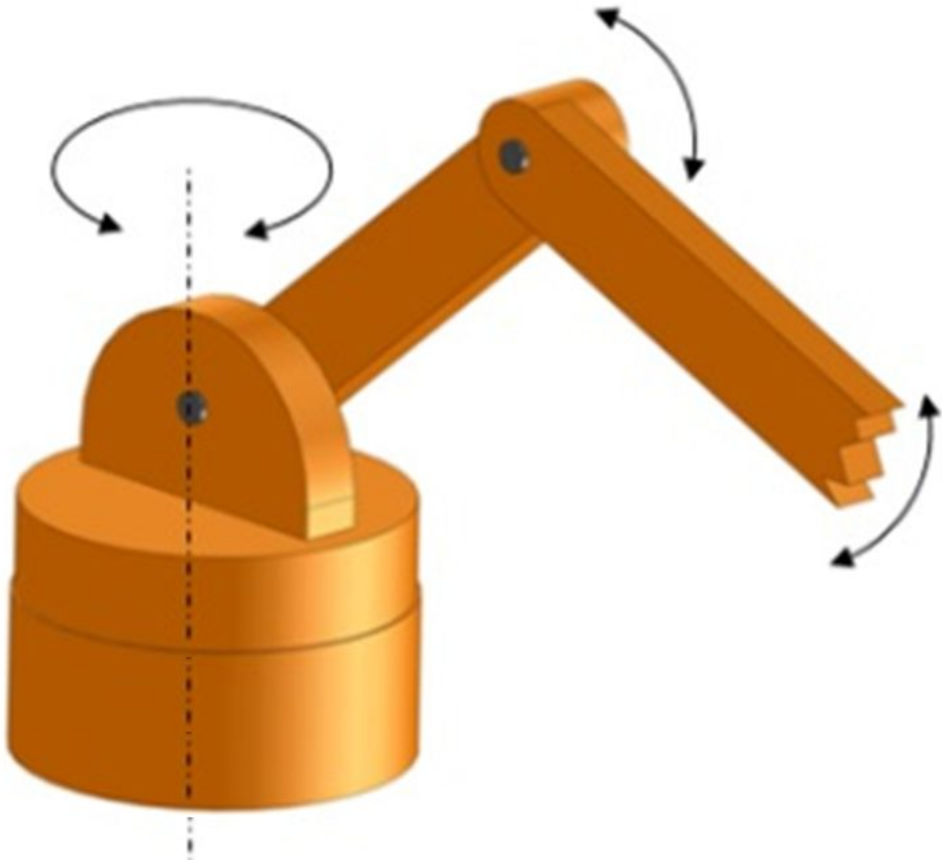


Kinematics

Md. Khalilur Rhaman

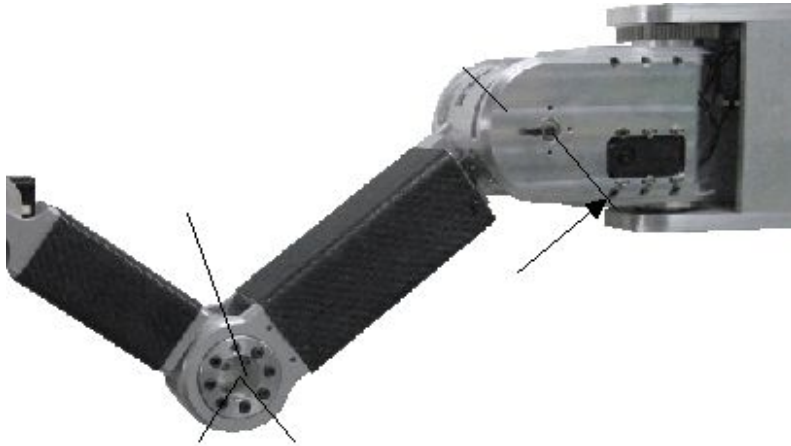
Forward Kinematics



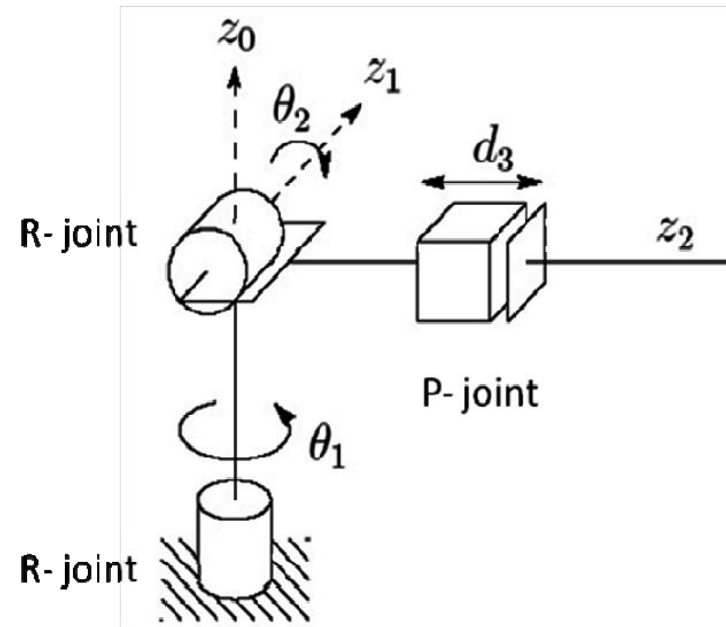
<https://www.youtube.com/watch?v=NRgNDIVtmz0>

Type of Joint

- Active Joint
- Passive Joint



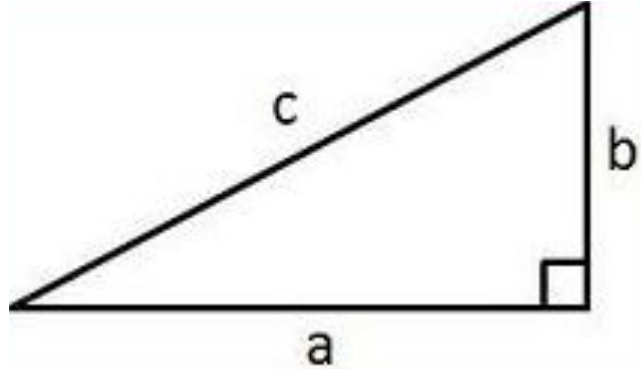
- Revolute
- Prismatic



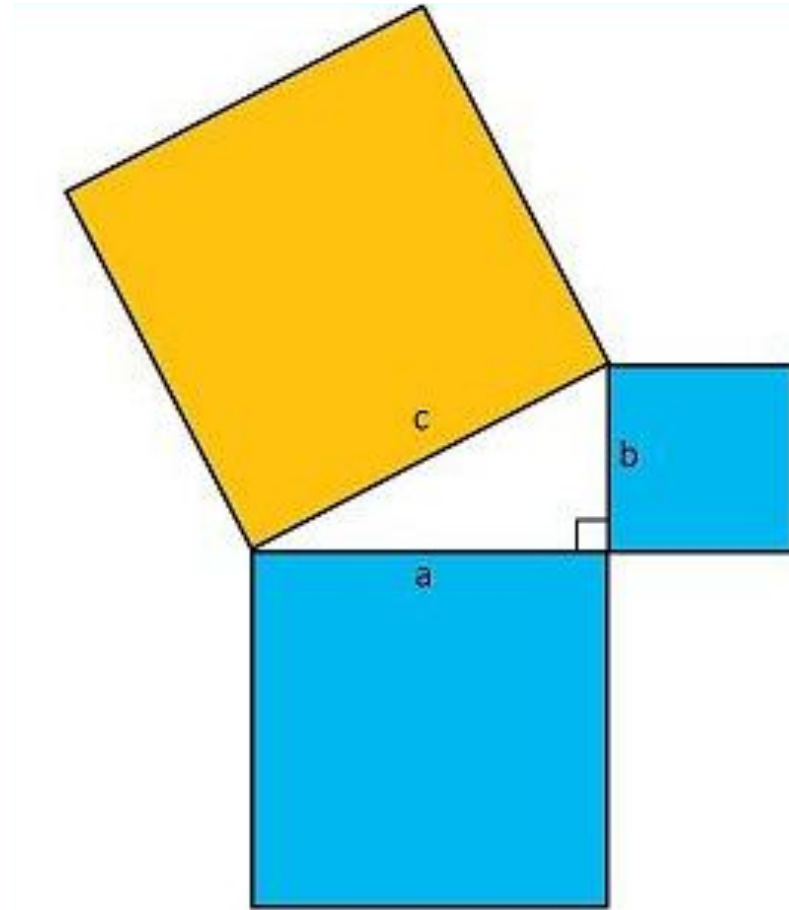
Forward kinematics using trigonometry

Pythagoras' Theorem for right triangle

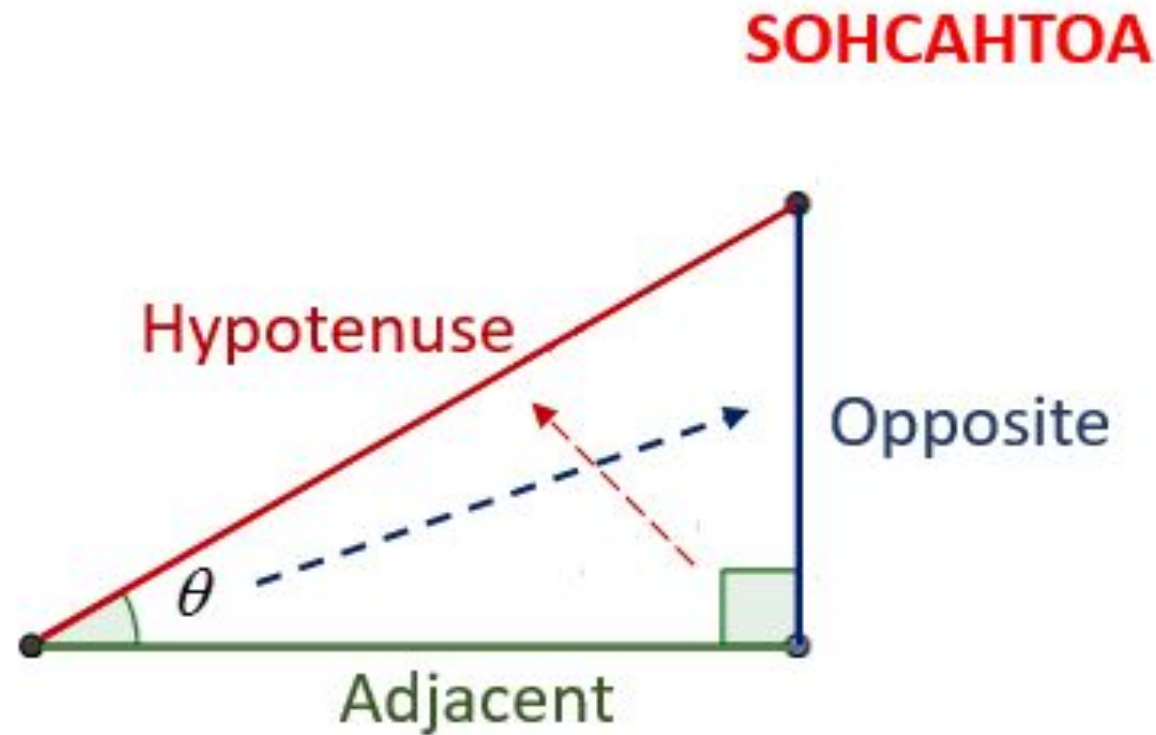
-



- $c^2 = a^2 + b^2$



Basic Trigonometric Functions



SOH $\sin \theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$

CAH $\cos \theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$

TOA $\tan \theta = \frac{\text{Opposite}}{\text{Adjacent}}$

The Sine and Cosine Rules

- *Sine Rule:*

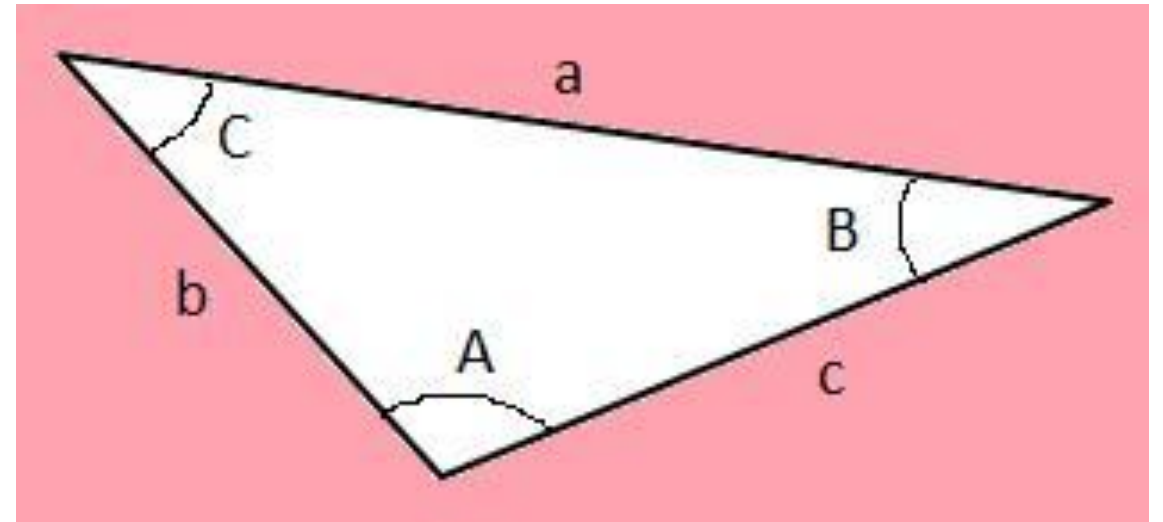
- $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

- *Cosine Rule:*

- $a^2 = b^2 + c^2 - 2bc \cos A$

- *or*

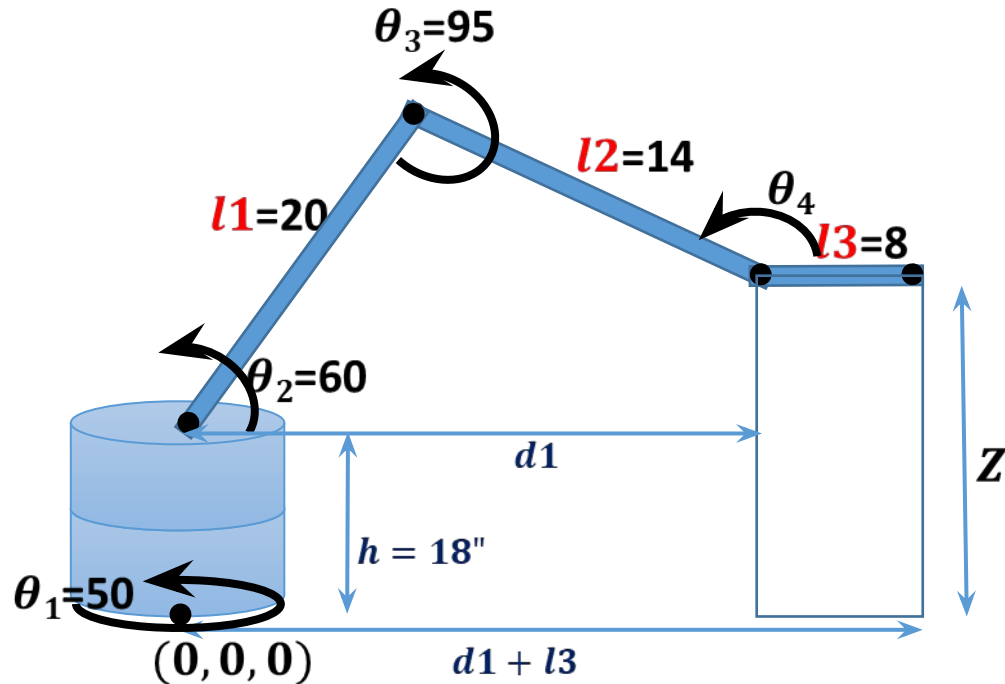
- $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$



Forward Kinematics

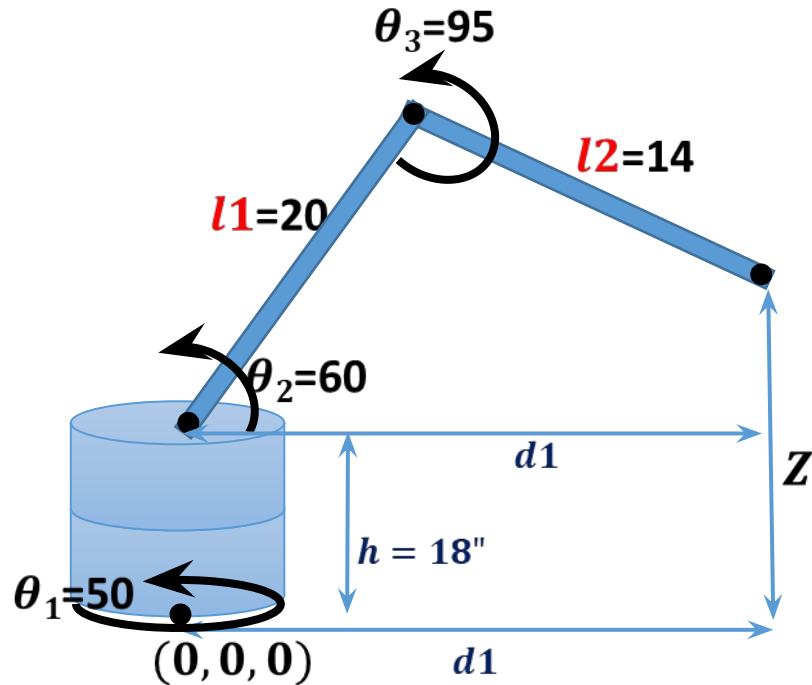
- Angles \rightarrow coordinate
- $(\theta_1, \theta_2, \theta_3) \rightarrow (X, Y, Z)$
- Where Height, length of arm-1 and Length of Arm-2 are fixed

4 DOF Arm Calculation



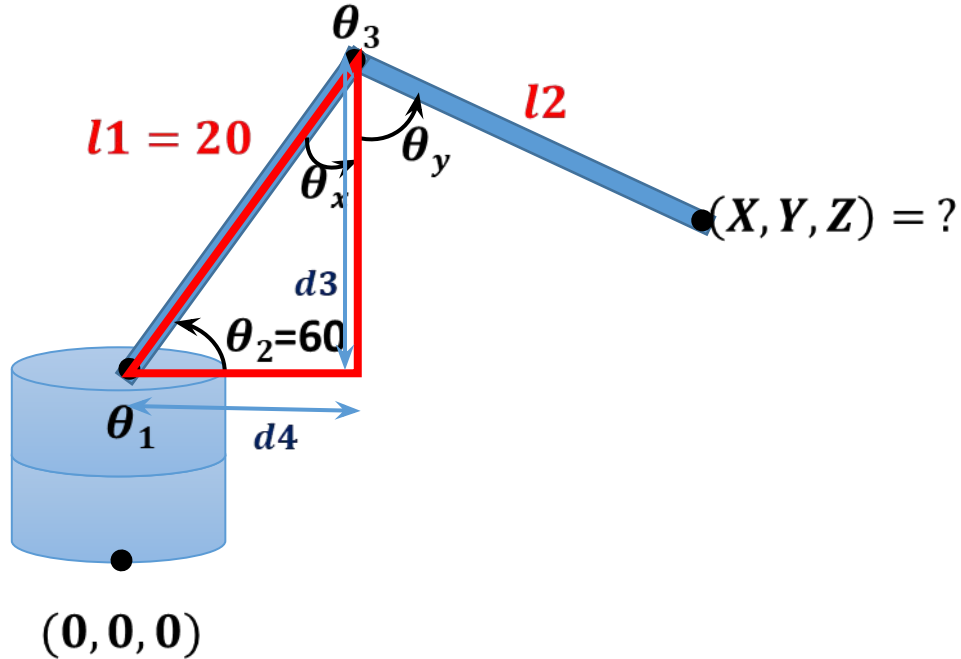
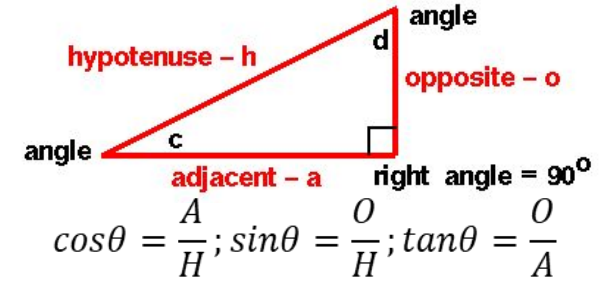
- l_1 = Length of first arm
- l_2 = Length of 2nd Arm
- l_3 = Length of 3rd Arm (end effector)
- h = height of base
- θ_1 = Angle of base rotation
- θ_2 = Angle of first arm from horizon
- θ_3 = Angle between 1st and 2nd arm
- θ_4 = Angle between 2nd arm and end effector

Example with a Value



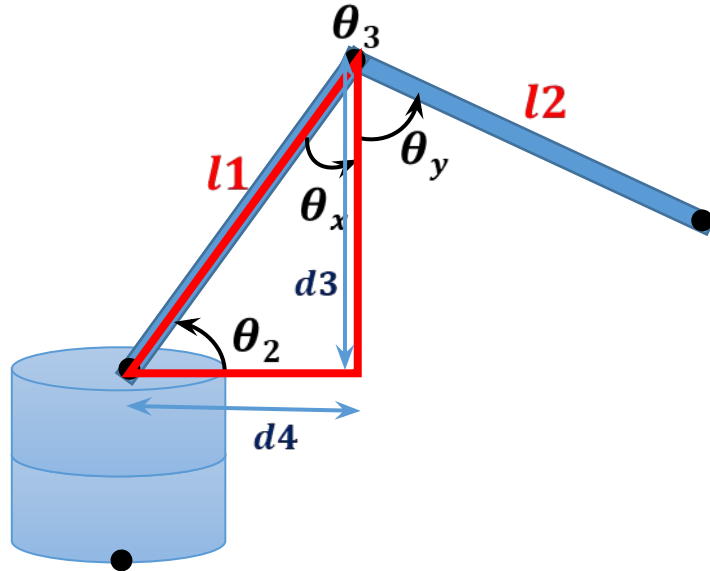
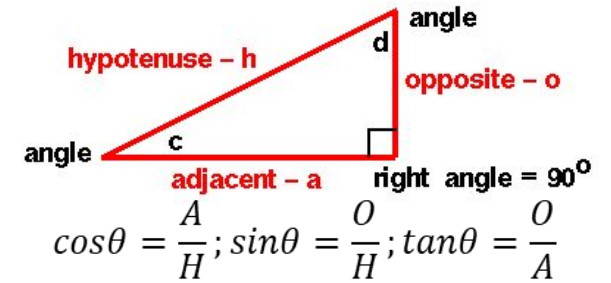
- $l_1 = 20''$
 - $l_2 = 14''$
 - $l_3 = 8''$
 - $h = 18''$
-
- $\theta_1 = 50$
 - $\theta_2 = 60$
 - $\theta_3 = 95$

Forward Kinematics Calculation



- If we draw a right angle triangle with first arm,
- θ_x is the angle with opposite – d_3
- θ_y is the angle with Adjacent – d_4
- So, $\theta_3 = \theta_x + \theta_y$
- Here l_1 is hypotenuse
- So, $\sin\theta = \frac{O}{H}; \sin\theta_2 = \frac{d_3}{l_1}$
- $d_3 = \sin\theta_2 * l_1$

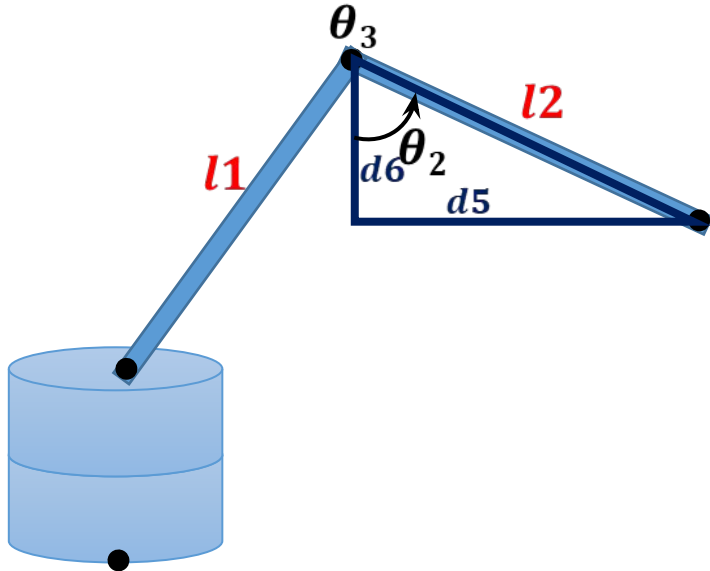
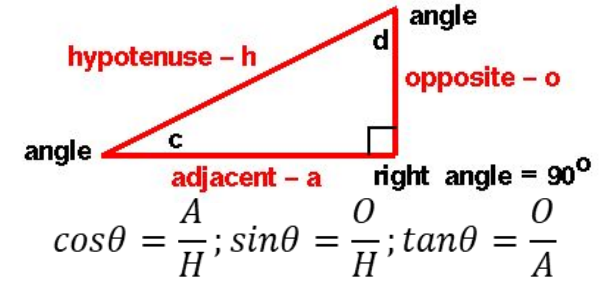
Example continuation...



- $\theta_x = 180 - 90 - \theta_2$
- $\theta_x = 180 - 90 - 60 = 30$
- $\theta_3 = \theta_x + \theta_y$
- $\theta_y = (\theta_3 - \theta_x)$
- $\theta_y = 95 - 30 = 65$
- $d3 = \sin\theta_2 * l1$
- $d3 = \sin 60 * l1$
- $d3 = 0.866 * 20 = 17.32$
- $d4 = \cos\theta_2 * l1$
- $d4 = \cos 60 * l1$
- $d4 = .5 * 20 = 10$

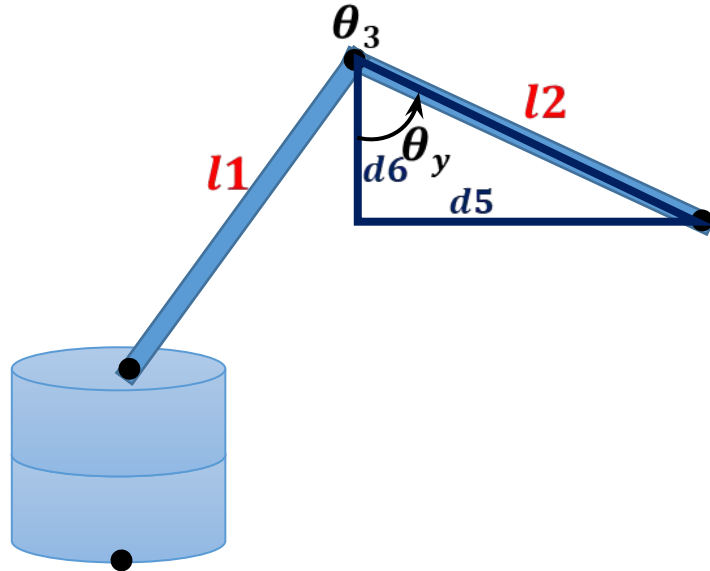
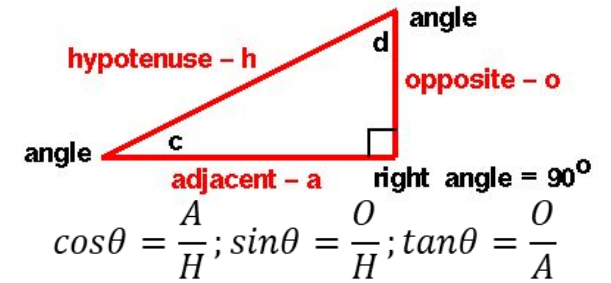
$l1 = 20''$
 $l2 = 14''$
 $l3 = 8''$
 $h = 18''$
 $\theta_1 = 50$
 $\theta_2 = 60$
 $\theta_3 = 95$

Forward Kinematics Calculation



- If we draw a right angle triangle with 2nd arm $l2$
- θ_y is the angle with opposite – $d6$
- $l2$ is hypotenuse
- So, $\cos\theta = \frac{A}{H}$; $\cos\theta_y = \frac{d6}{l2}$;
- $d6 = \cos\theta_y * l2$;
- $\sin\theta = \frac{O}{H}$; $\sin\theta_y = \frac{d5}{l2}$;
- $d5 = \sin\theta_y * l2$

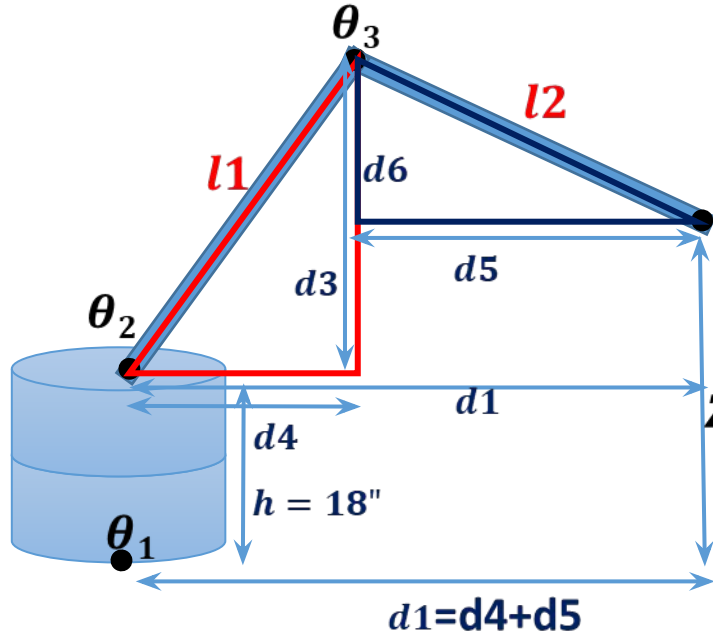
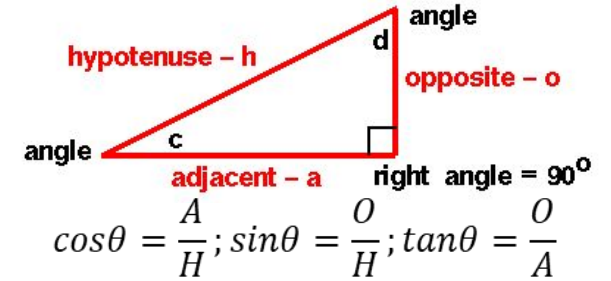
Example continuation...



- $d6 = \cos\theta_y * l2;$
- $d6 = \cos 65 * l2;$
- $d6 = 0.4226 * 14 = 5.91$
- $d5 = \sin\theta_y * l2$
- $d5 = \sin 65 * l2$
- $d5 = .9 * 14$
- $d5 = 12.69$

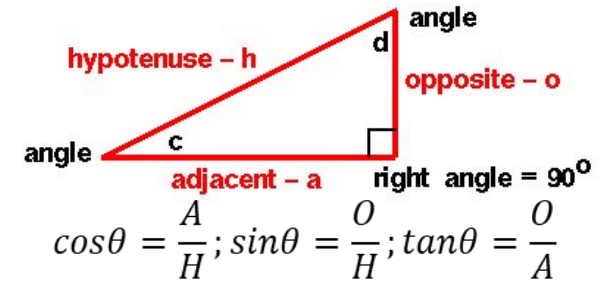
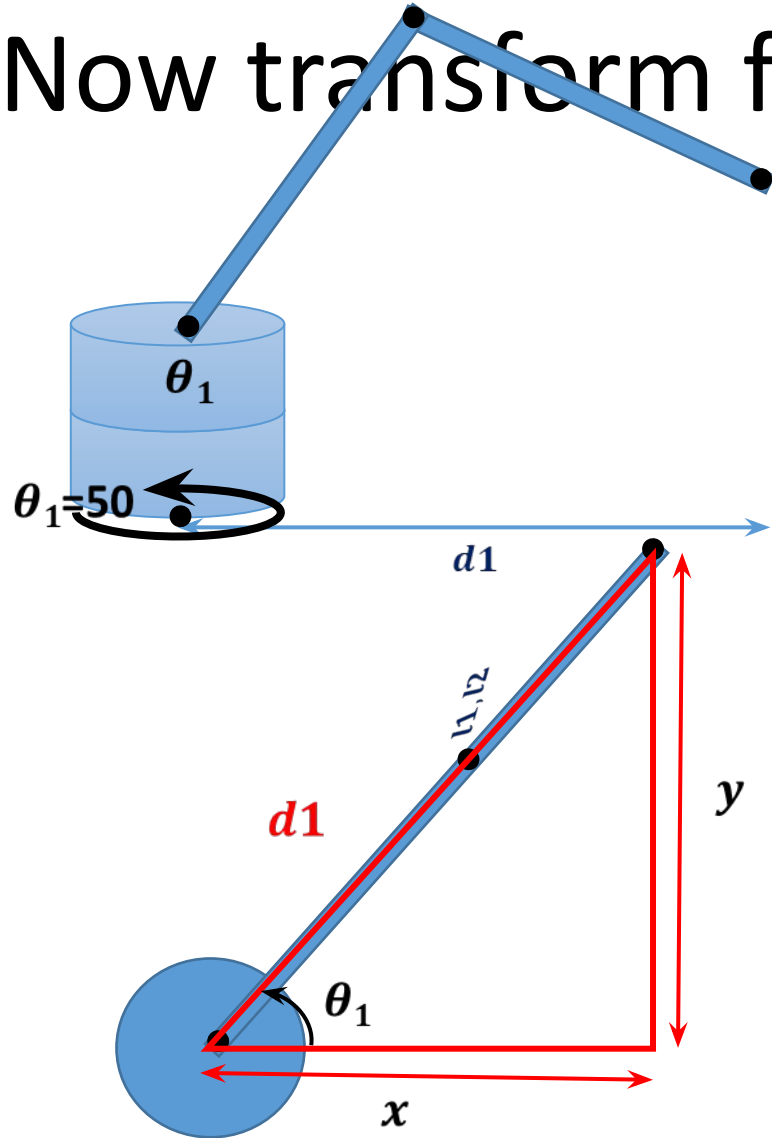
$l1 = 20''$
 $l2 = 14''$
 $l3 = 8''$
 $h = 18''$
 $\theta_1 = 50$
 $\theta_2 = 60$
 $\theta_3 = 95$
 $\theta_x = 30$
 $\theta_y = 65$
 $d3 = 17.32$
 $d4 = 10$

Forward Kinematics Calculation



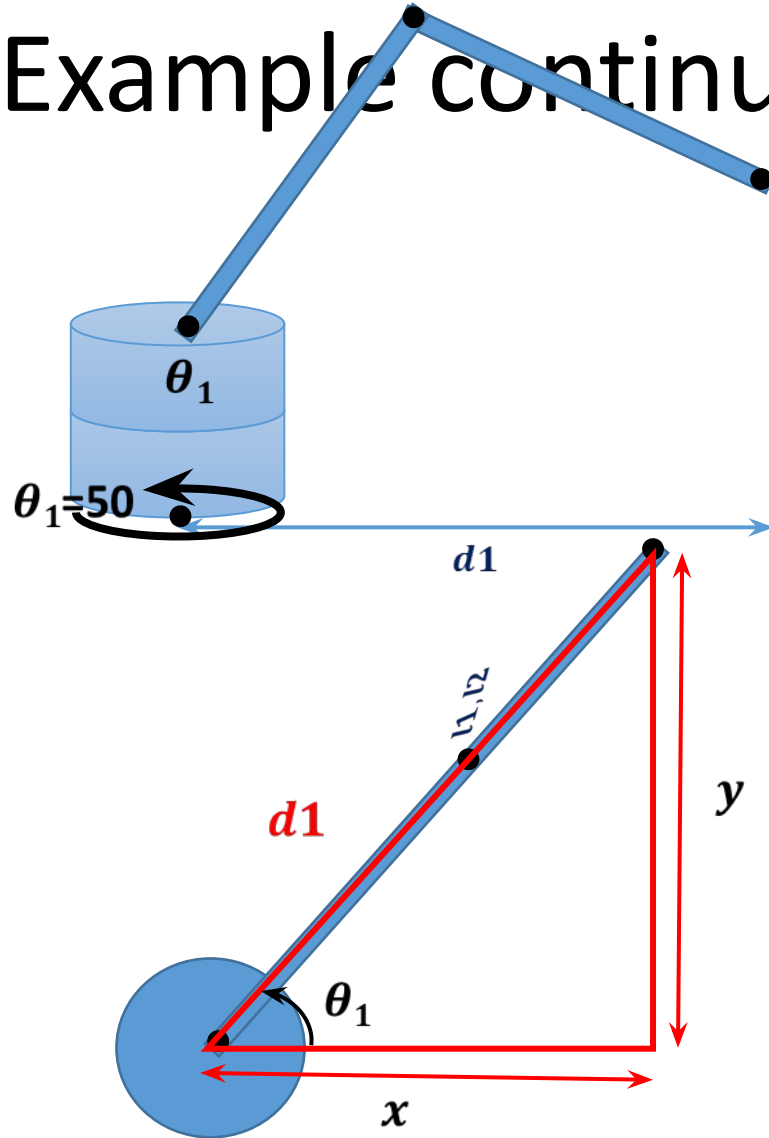
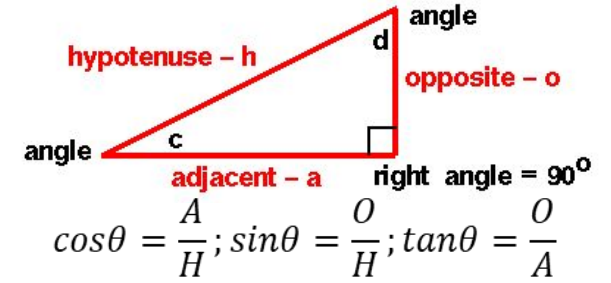
- $Z = h + (d_3 - d_6)$
- $d_1 = d_4 + d_5$
- So, d_1 is the current length of arm from base
- Z is the height of arm endpoint

Now transform from top view



- $\cos \theta_1 = \frac{x}{d_1}$;
- $x = \cos \theta_1 * d_1$
- $\sin \theta_1 = \frac{y}{d_1}$;
- $y = \sin \theta_1 * d_1$
- Position is: (x, y, z)

Example continuation...



- $Z = d2 + (d3 - d6)$
- $Z = 18 + 17.32 - 5.916 = 29.4$
- $d1 = d4 + d5 = 10 + 12.69 = 22.69$
- $x = \cos\theta_1 * d1(d1 + l3)$
- $x = \cos 50 * 22.69 = 14.58$
- $y = \sin 50 * 22.69 = 17.38$
- *Position is:*
- $(x, y, z) = (14.58, 17.38, 29.4)$

$l1 = 20''$
 $l2 = 14''$
 $l3 = 8''$
 $h = 18''$
 $\theta_1 = 50$
 $\theta_2 = 60$
 $\theta_3 = 95$
 $\theta_x = 30$
 $\theta_y = 65$
 $d3 = 17.32$
 $d4 = 10$
 $d6 = 5.91$
 $d5 = 12.69$

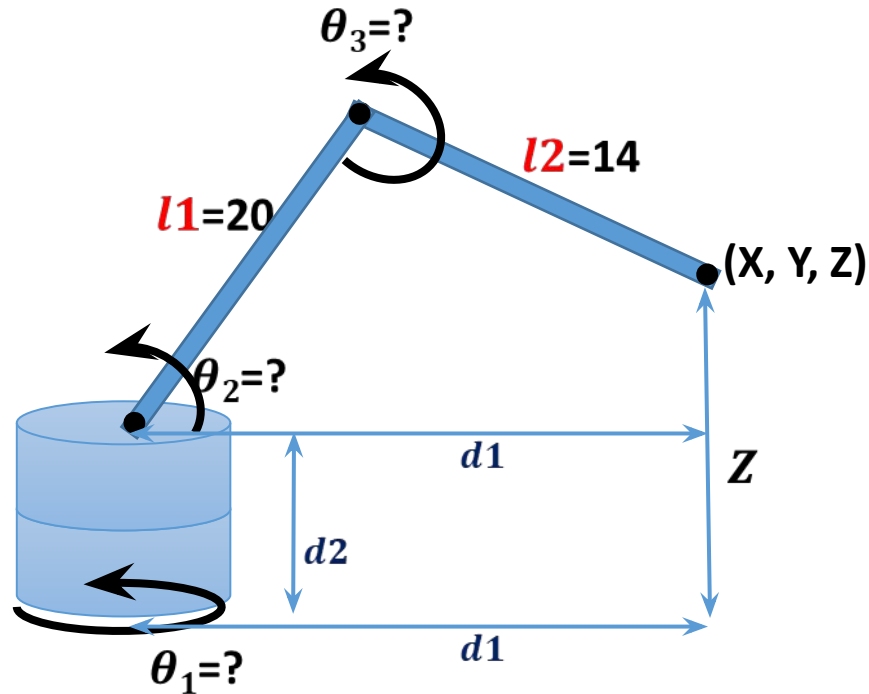
Inverse or Reverse kinematics using trigonometry

Coordinate \rightarrow Angle

$(X, Y, Z) \rightarrow (\theta_1, \theta_2, \theta_3)$

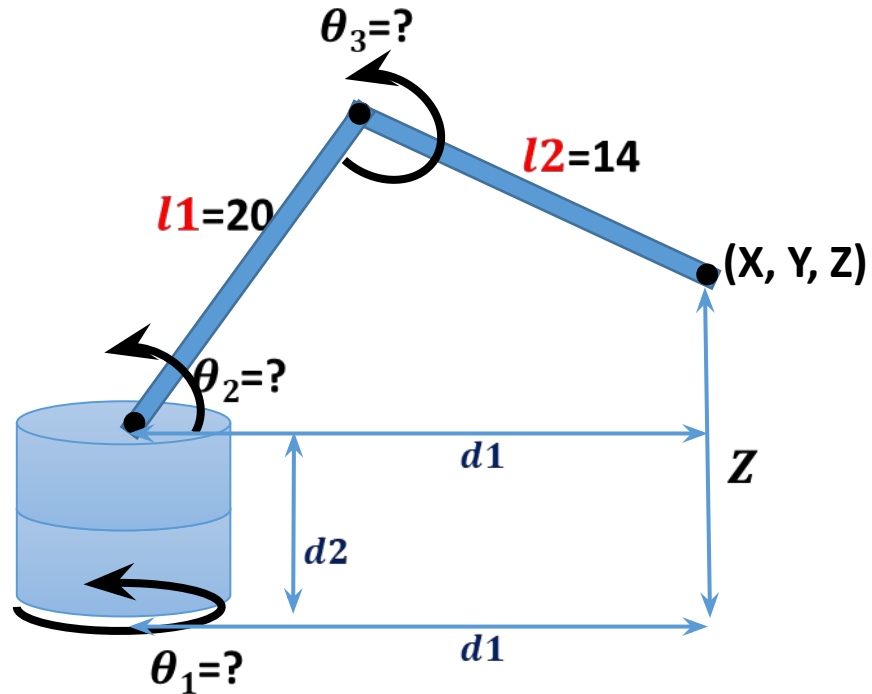
Where Height, length of arm-1 and Length of Arm-2 are fixed

Reverse Kinematics (coordinate to angle)



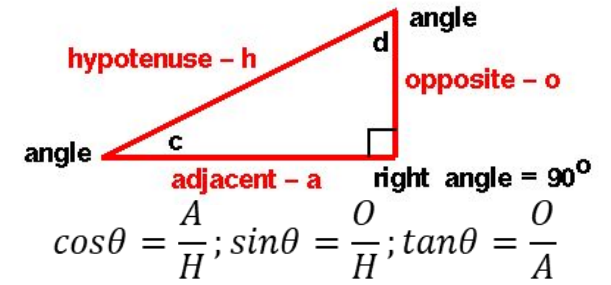
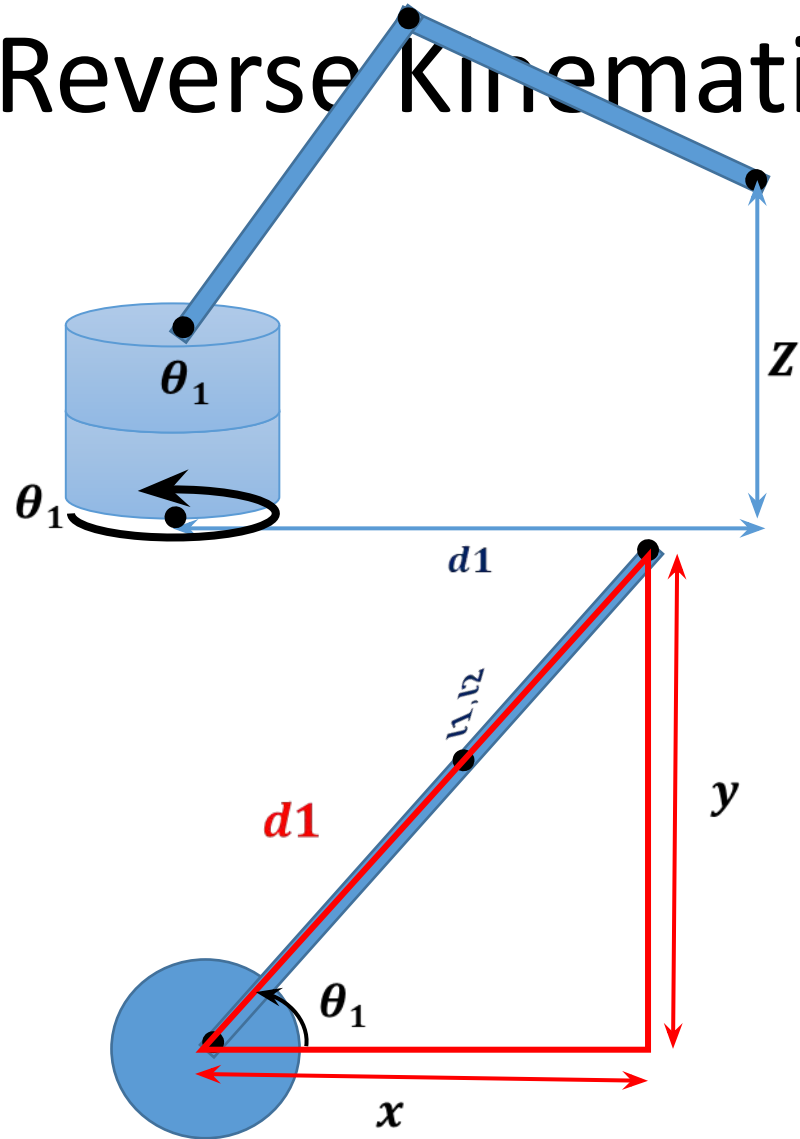
- x = Position on front
- y = Position on left or right
- z = Position on height
- l_1 = Length of first arm
- l_2 = Length of second arm
- l_3 = Length of third arm
- d_2 = Height of base from ground

Reverse Kinematics Example value



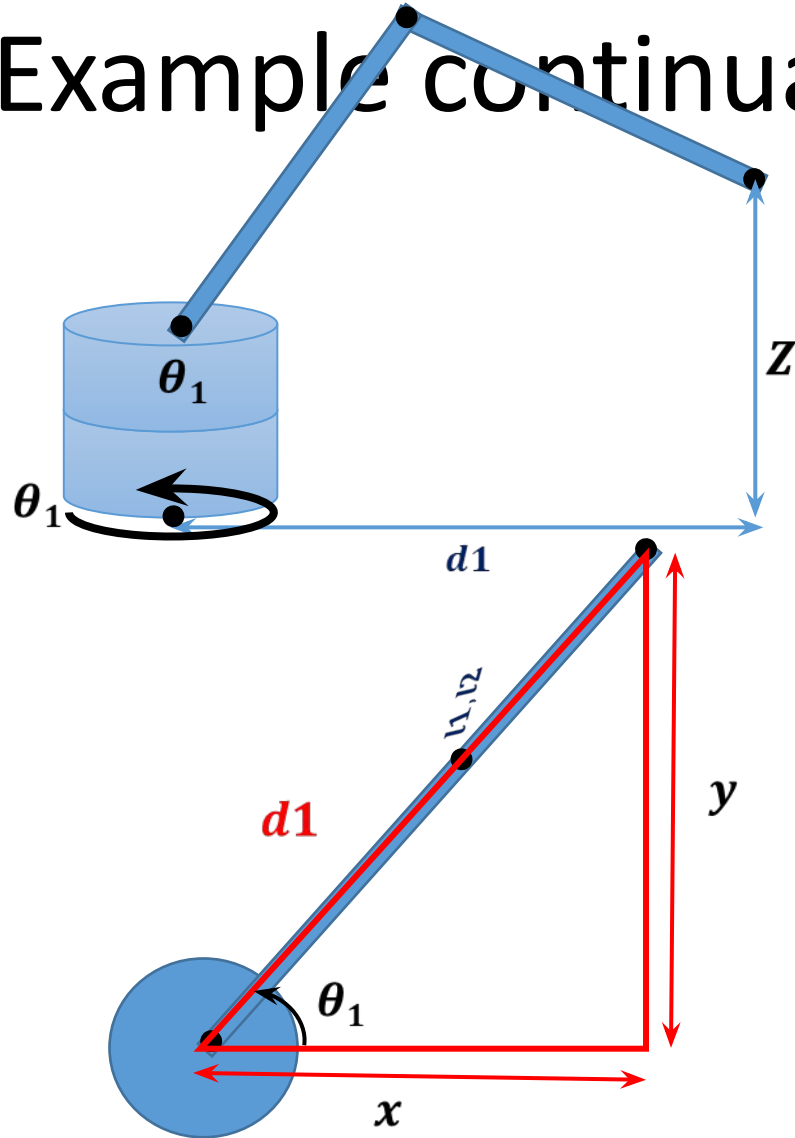
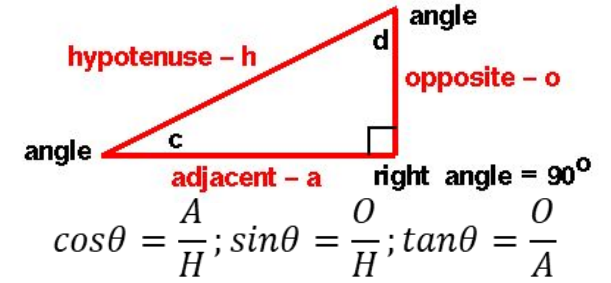
- $x = 14.58$
- $y = 17.38$
- $z = 29.4$
- $l_1 = 20$
- $l_2 = 14$
- $l_3 = 8$
- $d_2 = 18$

Reverse Kinematics



- $(d1 + l3)^2 = x^2 + y^2$
- $d1 = \sqrt{x^2 + y^2}$
- $\cos\theta_1 = \frac{x}{d1}$
- $\theta_1 = \cos^{-1}\left(\frac{x}{d1}\right)$

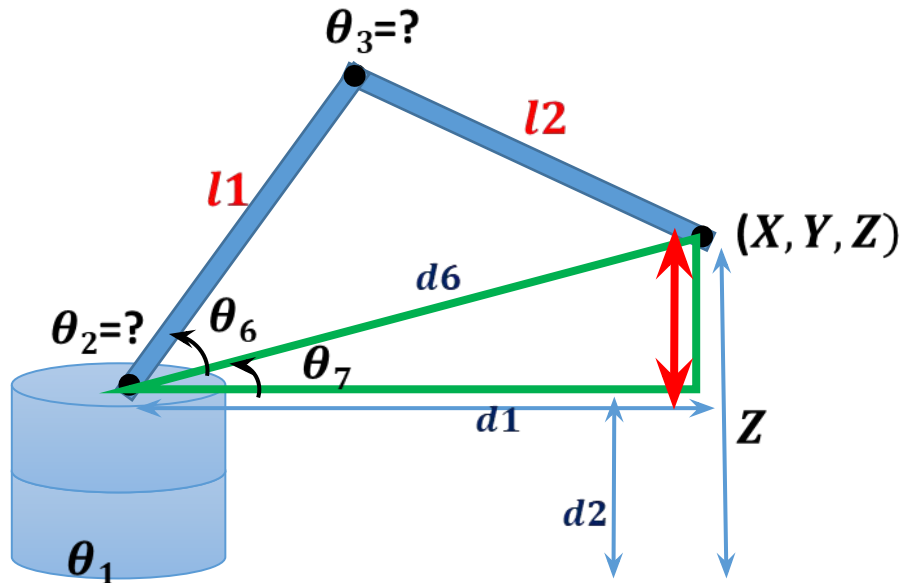
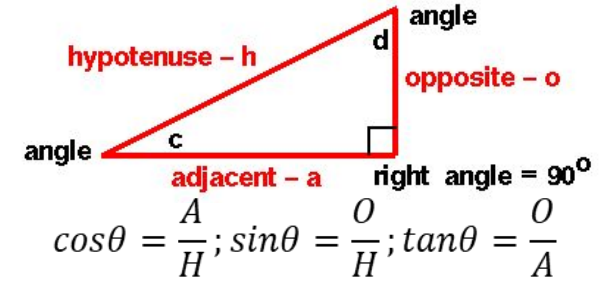
Example continuation...



- $d1 = \sqrt{x^2 + y^2}$
- $d1 = \sqrt{14.58^2 + 17.38^2}$
- $d1 = 22.69$
- $\theta_1 = \cos^{-1}\left(\frac{x}{d1}\right)$
- $\theta_1 = \cos^{-1}\left(\frac{14.58}{22.69}\right)$
- $\theta_1 = 50$

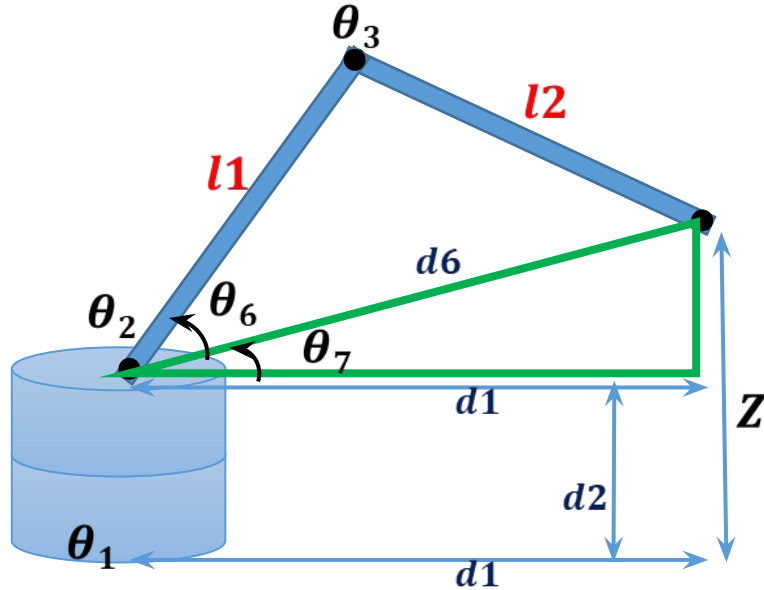
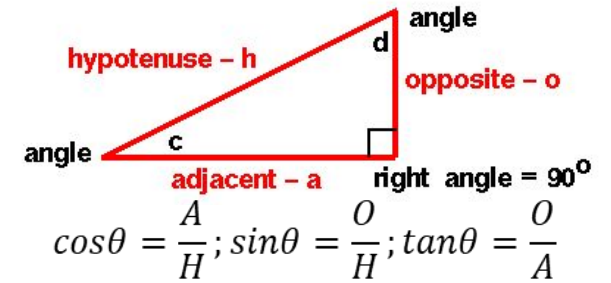
$x = 14.58$
 $y = 17.38$
 $z = 29.4$
 $l1 = 20$
 $l2 = 14$
 $l3 = 8$
 $d2 = 18$

Reverse Kinematics



- $d6^2 = d1^2 + (z - d2)^2$
- $d6 = \sqrt{d1^2 + (z - d2)^2}$
- $\cos\theta_7 = \frac{d1}{d6}$;
- $\theta_7 = \cos^{-1}\left(\frac{d1}{d6}\right)$

Example continuation...

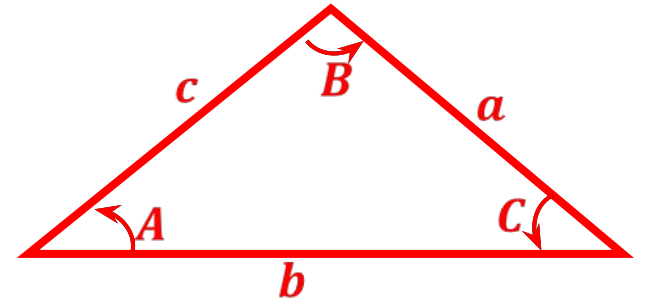
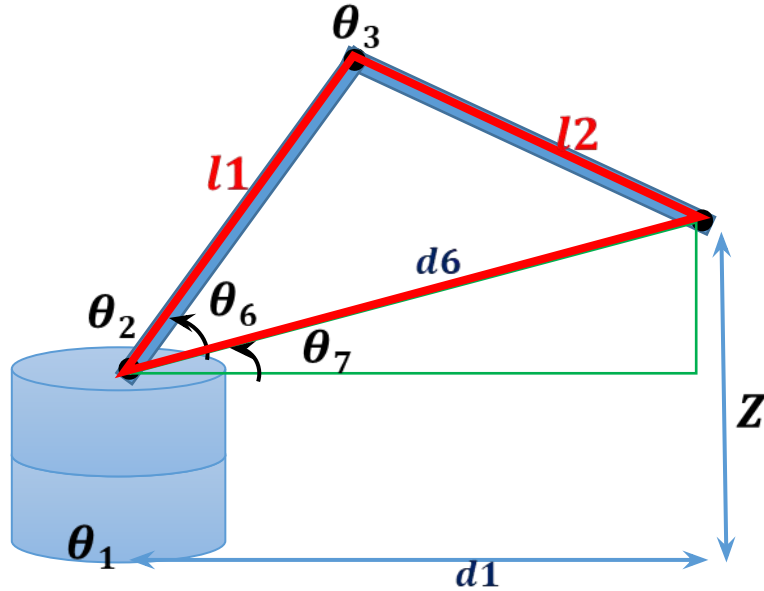


$$\begin{aligned}x &= 14.58 \\y &= 17.38 \\z &= 29.4 \\l_1 &= 20 \\l_2 &= 14 \\l_3 &= 8 \\d_2 &= 18 \\d_1 &= 22.69 \\\theta_1 &= 50 \\d_6 &= 25.4 \\\theta_7 &= 26.7\end{aligned}$$

- $d_6 = \sqrt{d_1^2 + (z - d_2)^2}$
- $d_6 = \sqrt{22.69^2 + (29.4 - 18)^2}$
- $d_6 = 25.4$
- $\theta_7 = \cos^{-1}\left(\frac{d_1}{d_6}\right)$
- $\theta_7 = \cos^{-1}\left(\frac{22.69}{25.4}\right)$
- $\theta_7 = 26.7$

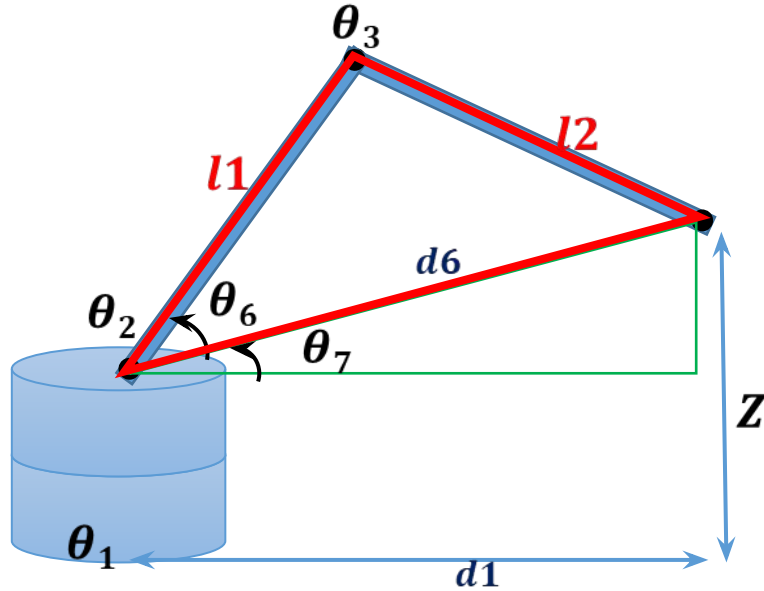
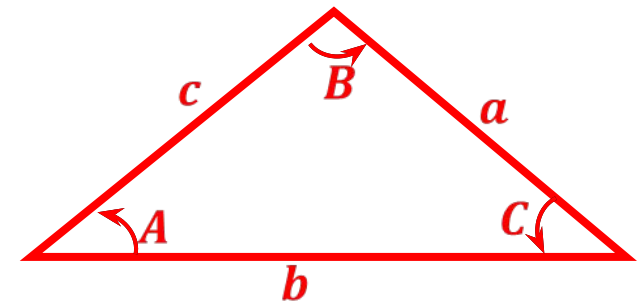
$$\begin{aligned}x &= 14.58 \\y &= 17.38 \\z &= 29.4 \\l_1 &= 20 \\l_2 &= 14 \\l_3 &= 8 \\d_2 &= 18 \\d_1 &= 22.69 \\\theta_1 &= 50\end{aligned}$$

Reverse Kinematics



- $b^2 = a^2 + c^2 - 2ac\cos B$
- $\cos \theta_3 = \left(\frac{l1^2 + l2^2 - d6^2}{2 * l1 * l2} \right);$
- $\theta_3 = \cos^{-1} \left(\frac{l1^2 + l2^2 - d6^2}{2 * l1 * l2} \right)$
- $\cos \theta_6 = \left(\frac{l1^2 + d6^2 - l2^2}{2 * l1 * d6} \right);$
- $\theta_6 = \cos^{-1} \left(\frac{l1^2 + d6^2 - l2^2}{2 * l1 * d6} \right)$
- $\theta_4 = \theta_2 + \theta_3$

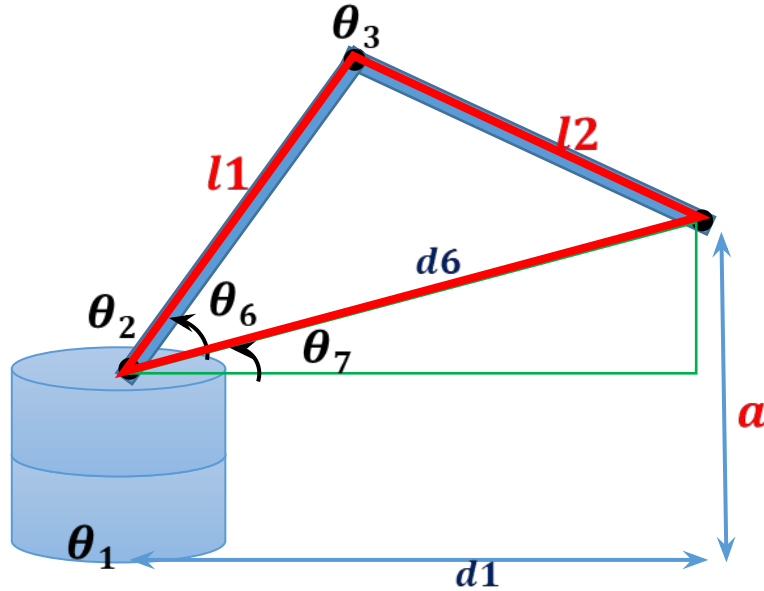
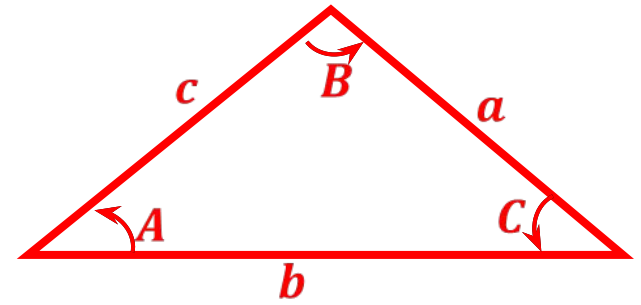
Example continuation...



- $\theta_3 = \cos^{-1} \left(\frac{l1^2 + l2^2 - d6^2}{2 * l1 * l2} \right)$
- $\theta_3 = \cos^{-1} \left(\frac{20^2 + 14^2 - 25.4^2}{2 * 20 * 14} \right)$
- $\theta_3 = 95$

$x = 14.58$
 $y = 17.38$
 $z = 29.4$
 $l1 = 20$
 $l2 = 14$
 $l3 = 8$
 $d2 = 18$
 $d1 = 22.69$
 $\theta_1 = 50$
 $d6 = 25.4$
 $\theta_7 = 26.7$

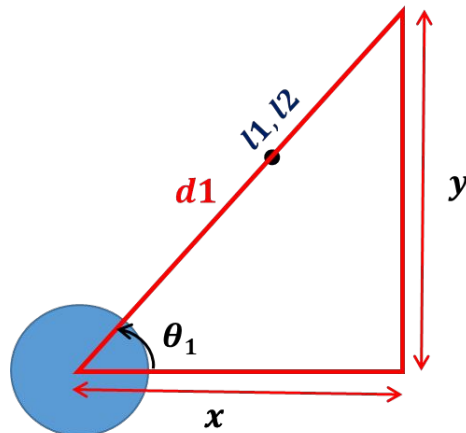
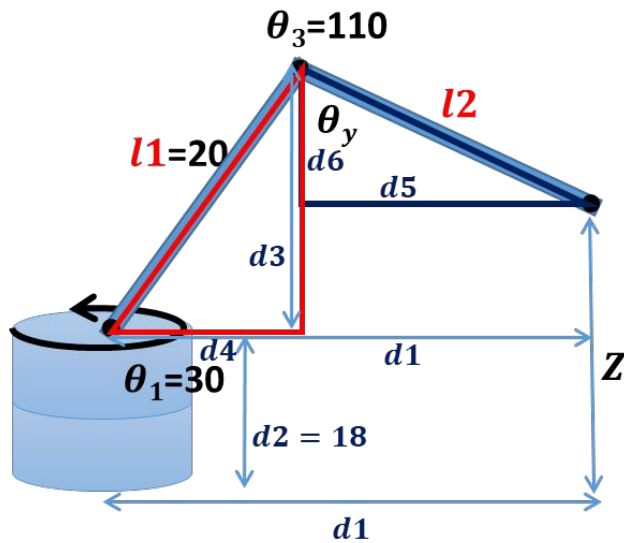
Example continuation...



- $\theta_6 = \cos^{-1} \left(\frac{l1^2 + d6^2 - l2^2}{2 * l1 * d6} \right)$
- $\theta_6 = \cos^{-1} \left(\frac{20^2 + 25.4^2 - 14^2}{2 * 20 * 25.4} \right)$
- $\theta_6 = 33.3$
- $\theta_2 = 33.3 + 26.7$
- $\theta_2 = 60$
- $(\theta_1, \theta_2, \theta_3, \theta_4)$
- $(50, 60, 95)$

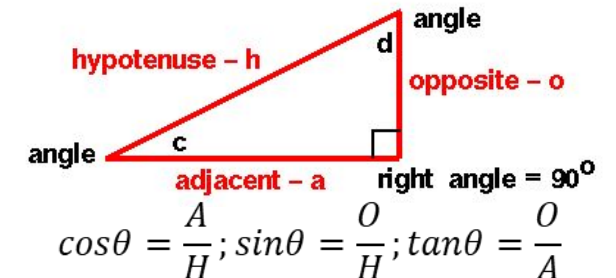
$x = 14.58$
 $y = 17.38$
 $z = 29.4$
 $l1 = 20$
 $l2 = 14$
 $l3 = 8$
 $d2 = 18$
 $d1 = 22.69$
 $\theta_1 = 50$
 $d6 = 25.4$
 $\theta_7 = 26.7$

Example-2



- $\cos \theta = a/h$
- $d4 = \cos 45^\circ * 20 = 14.14$
- $d3 = \sin 45^\circ * 20 = 14.14$
- $\theta_x = 180 - (90 + 45) = 45$
- $\theta_y = \theta_3 - \theta_x = 110 - 45 = 65$
- $d6 = \cos 65^\circ * 14 = 5.91$
- $d5 = \sin 65^\circ * 14 = 12.68$
- $d1 = d4 + d5 = 14.14 + 12.68 = 26.8$
- $Z = d2 + (d3 - d6) = 18 + (14.14 - 5.91) = 26.23$
- $X = \cos 30^\circ * d1 = \cos 30^\circ * 26.8 = 23.2$
- $Y = \sin 30^\circ * 26.8 = 13.41$
- $(X, Y, Z) = (23.2, 13.41, 26.23)$

- $l1 = 20''$
- $l2 = 14''$
- $l3 = 8''$
- $d2 = 18''$
- $\theta_1 = 30$
- $\theta_2 = 45$
- $\theta_3 = 110$



End of Forward and Inverse Kinematics using trigonometry

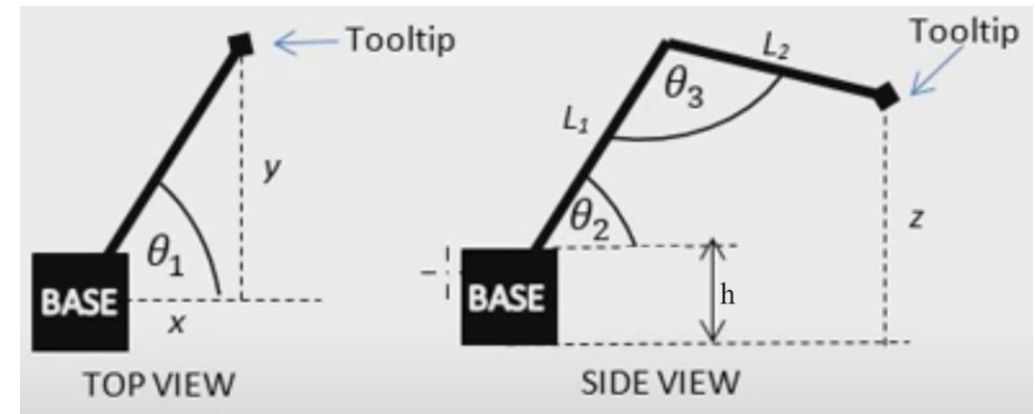
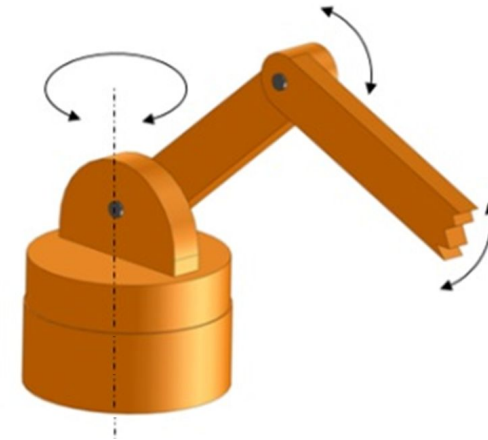
<https://www.youtube.com/watch?v=1-FJhmey7vk>

Forward kinematics using DH
parameter and Homogeneous
transformation matrix

Forward Kinematics

Forward kinematics refers to process of obtaining position and velocity of end effector, given the known joint angles and angular velocities.

For example, if shoulder and elbow joint angles are given for arm in sagittal plane, the goal is to find Cartesian coordinates of wrist/fist.



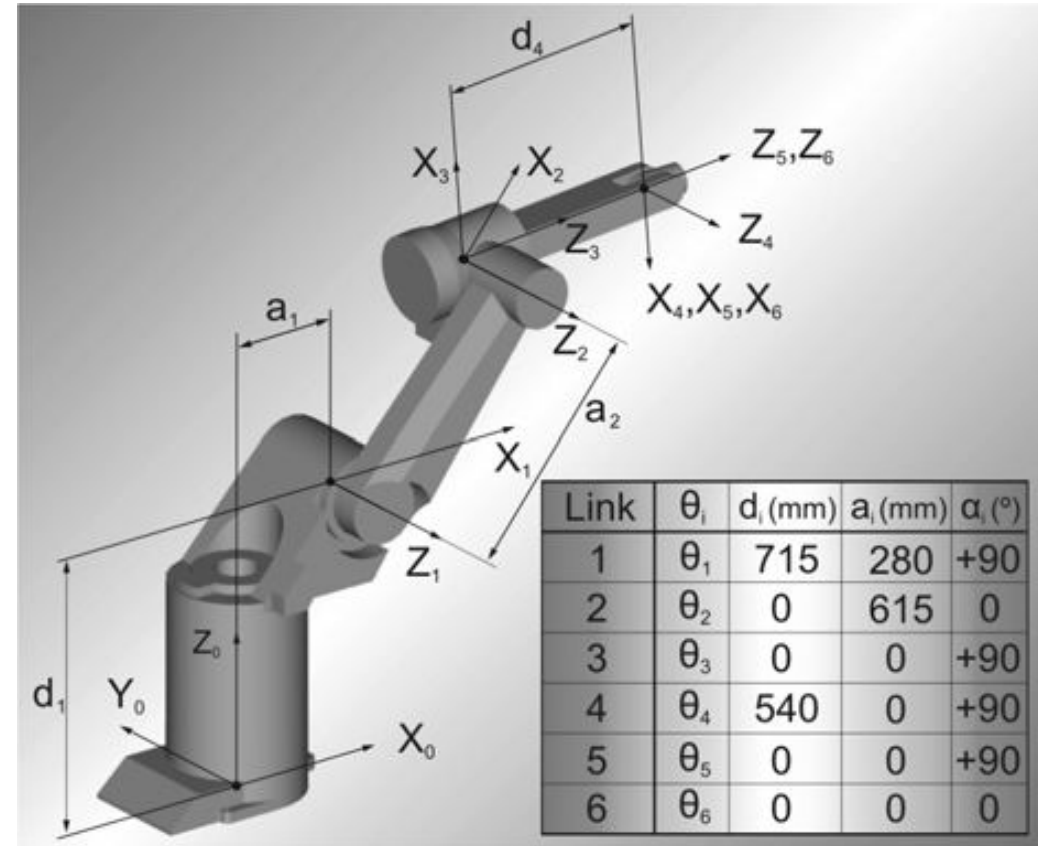
Forward Kinematics

- Angles \rightarrow coordinate
- $(\theta_1, \theta_2, \theta_3) \rightarrow (X, Y, Z)$
- Where Height, length of arm-1 and Length of Arm-2 are fixed

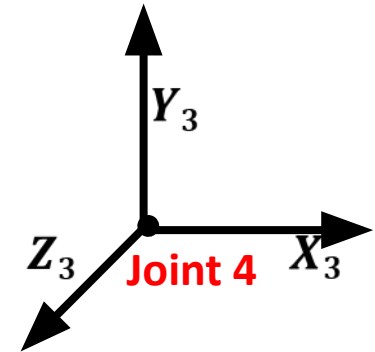
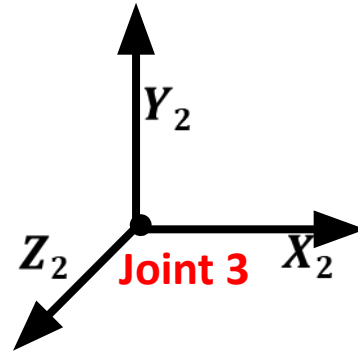
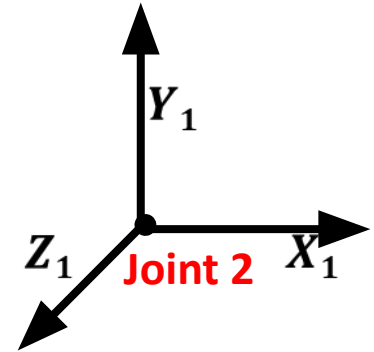
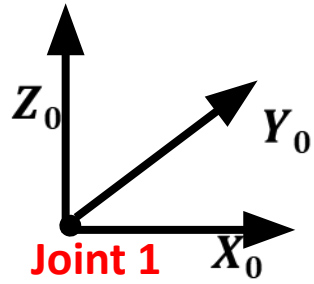
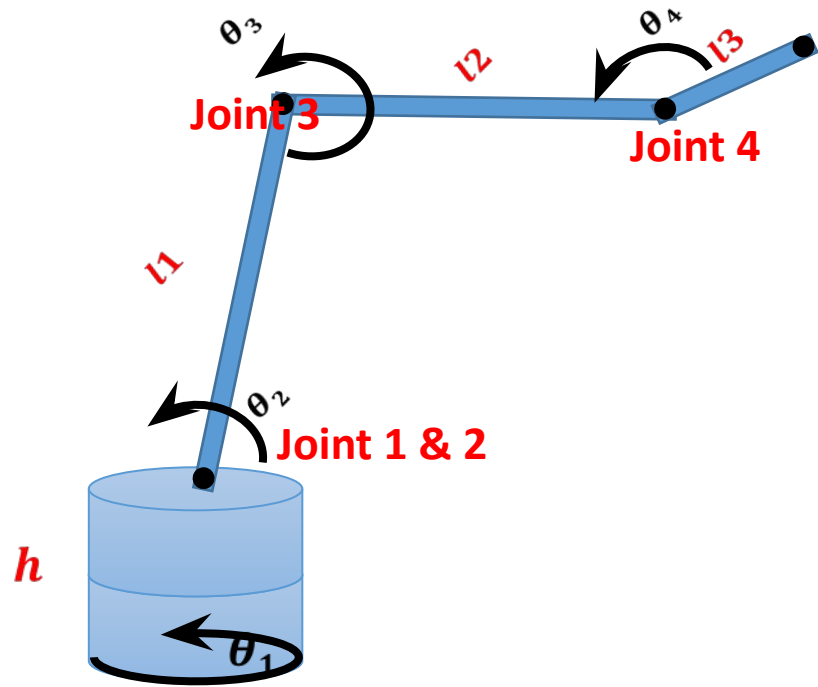
<https://www.youtube.com/watch?v=D3w3ZANOy3s>

DH (Denavit-Hartenberg) parameters

DH parameters are used in robot manipulator to describe the axis orientations and arm lengths. The 4 DH parameters are called theta, d, alpha and a. DOF stands for degrees of freedom.



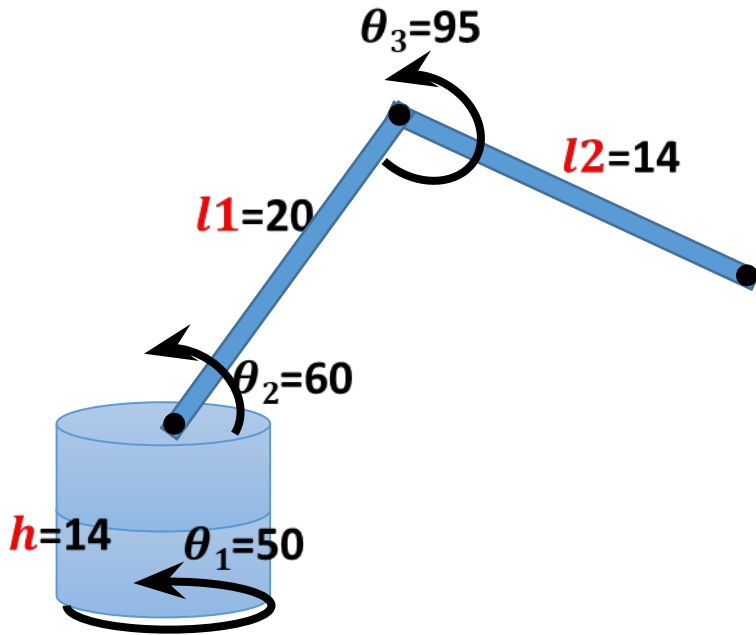
Example



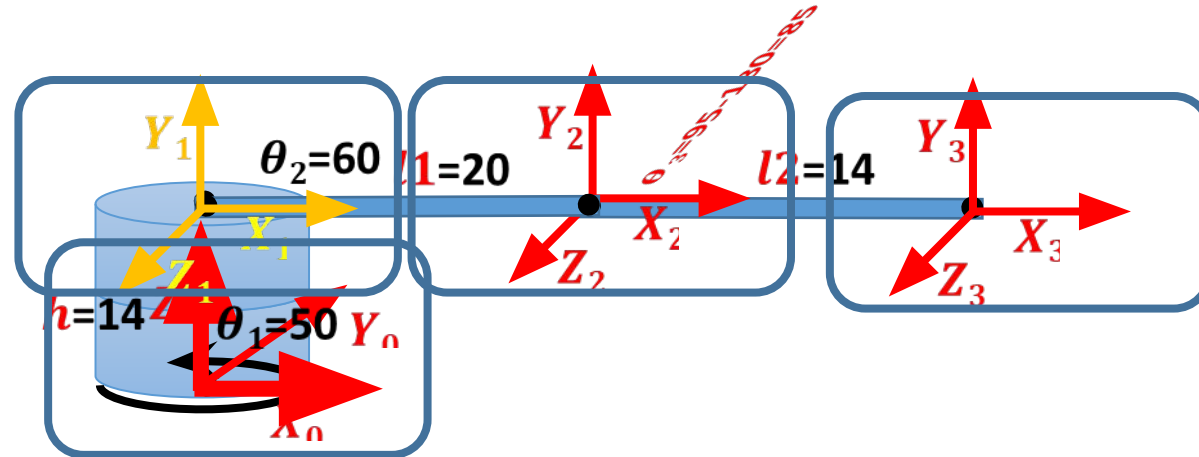
DH parameters calculation for two points

- θ_i : Joint angle
- α_i : Link Twist
- a_i : Link length
- d_i : Link offset
- DH convention link with the Homogeneous transformation T_i with Four Basic Transformation
- $T_i = \text{RotZ}\theta_i, \text{TransZ}d_i, \text{TransX}a_i, \text{RotX}\alpha_i$
- θ : Alignment of X_n with X_{n+1} with respect to rotating axis Z_n + Rotation of angle of θ_n
- d : Translation along Z axis
- a : Translation along X axis
- α : Alignment of Z_n with Z_{n-1} with respect to rotating axis X_n + Rotation of angle of θ_n
- Rotation of X for an angle of α_i

Transform arm to find DH parameters

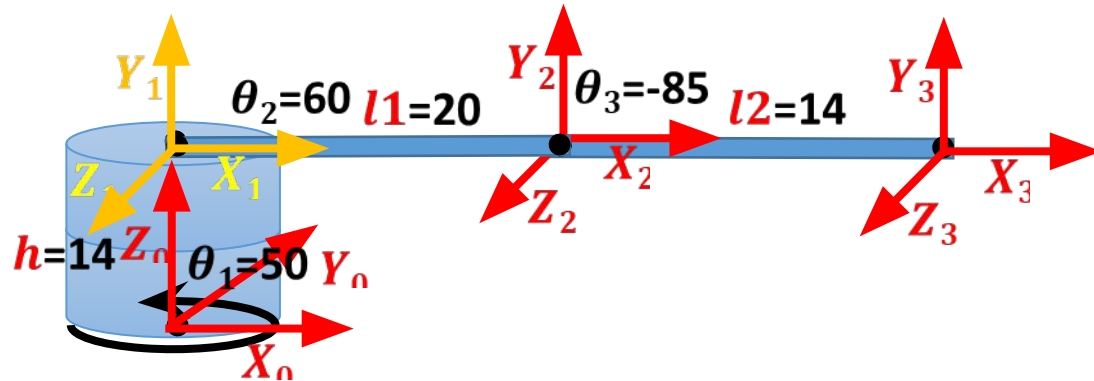


$$\begin{aligned} h &= 18'' \\ l1 &= 20'' \\ l2 &= 14'' \\ l3 &= 8'' \end{aligned} \quad \begin{aligned} \theta_1 &= 50 \\ \theta_2 &= 60 \\ \theta_3 &= 95 \end{aligned}$$



There are 4 frames in this arm

Example of DH parameters



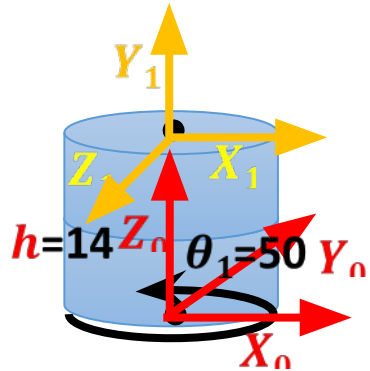
| Joint | | | | |
|-------|------|----|----|----|
| 1 | 0+50 | 90 | 0 | 18 |
| 2 | 0+60 | 0 | 20 | 0 |
| 3 | 0-85 | 0 | 14 | 0 |

Let us consider two frames: frame 0 and frame 1

- θ : defined as the rotation of X of second frame to align X of first frame around Z axis of first frame. It also includes the rotation of the joint.
- α : defined as the rotation of Z of first frame to align Z of second frame around X axis of second frame.
- In both cases anticlockwise rotation will indicate positive value and clockwise rotation will indicate negative value.
- a : is the distance between the center of first frame to second frame in the direction of X axis of second frame.
- d : is the distance between the center of first frame to second frame in the direction of Z axis of first frame.

DH parameters of first Joint

| Joint | | | | |
|-------|------|--|--|--|
| 1 | 0+50 | | | |
| 2 | | | | |
| 3 | | | | |



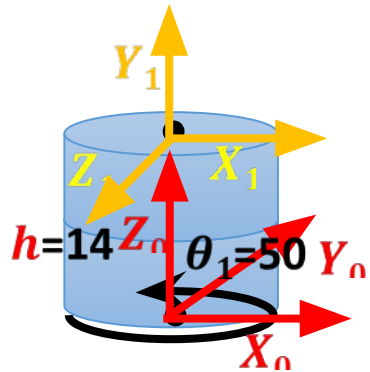
As we can see in the figure, the directions of X_0 and X_1 are same (on right direction) so we do not need to add any rotation around Z_0 . So, θ would be the value of only $\theta_1=50$.

Let us consider two frames: frame 0 and frame 1

- θ : defined as the rotation of X of second frame to align X of first frame around Z axis of first frame. It also includes the rotation of the joint.
- α : defined as the rotation of Z of first frame to align Z of second frame around X axis of second frame.
- In both cases anticlockwise rotation will indicate positive value and clockwise rotation will indicate negative value.
- a : is the distance between the center of first frame to second frame in the direction of X axis of second frame.
- d : is the distance between the center of first frame to second frame in the direction of Z axis of first frame.

DH parameters of first Joint

| Joint | | | | |
|-------|------|----|--|--|
| 1 | 0+50 | 90 | | |
| 2 | | | | |
| 3 | | | | |



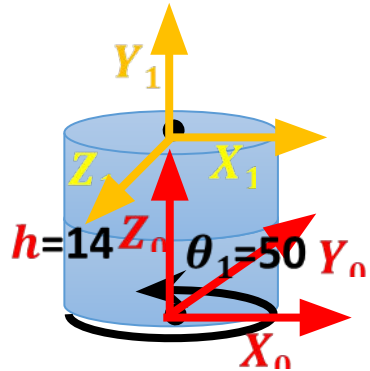
We have to rotate Z_0 anticlockwise 90 degree around X_1 to align with Z_1 . So the value of α will be 90.

Let us consider two frames: frame 0 and frame 1

- θ : defined as the rotation of X of second frame to align X of first frame around Z axis of first frame. It also includes the rotation of the joint.
- α : defined as the rotation of Z of first frame to align Z of second frame around X axis of second frame.
- In both cases anticlockwise rotation will indicate positive value and clockwise rotation will indicate negative value.
- a : is the distance between the center of first frame to second frame in the direction of X axis of second frame.
- d : is the distance between the center of first frame to second frame in the direction of Z axis of first frame.

DH parameters of first Joint

| Joint | | | | |
|-------|------|----|---|--|
| 1 | 0+50 | 90 | 0 | |
| 2 | | | | |
| 3 | | | | |



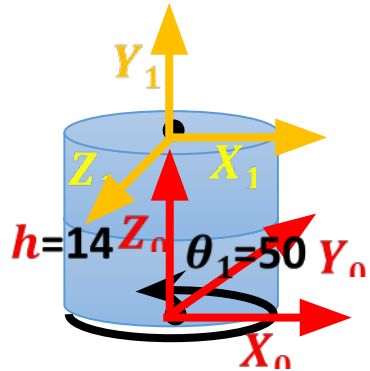
Center of first and second frame are in the same alignment in X_1 direction. So, the value of a will 0.

Let us consider two frames: frame 0 and frame 1

- θ : defined as the rotation of X of second frame to align X of first frame around Z axis of first frame. It also includes the rotation of the joint.
- α : defined as the rotation of Z of first frame to align Z of second frame around X axis of second frame.
- In both cases anticlockwise rotation will indicate positive value and clockwise rotation will indicate negative value.
- a : is the distance between the center of first frame to second frame in the direction of X axis of second frame.
- d : is the distance between the center of first frame to second frame in the direction of Z axis of first frame.

DH parameters of first Joint

| Joint | | | | |
|-------|------|----|---|----|
| 1 | 0+50 | 90 | 0 | 18 |
| 2 | | | | |
| 3 | | | | |



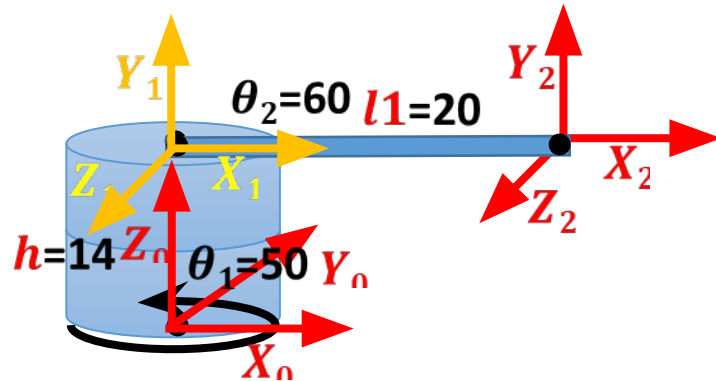
The distance between the center of first and second frame is 18 in Z_1 direction. So, the value of d will 18.

Let us consider two frames: frame 0 and frame 1

- θ : defined as the rotation of X of second frame to align X of first frame around Z axis of first frame. It also includes the rotation of the joint.
- α : defined as the rotation of Z of first frame to align Z of second frame around X axis of second frame.
- In both cases anticlockwise rotation will indicate positive value and clockwise rotation will indicate negative value.
- a : is the distance between the center of first frame to second frame in the direction of X axis of second frame.
- d : is the distance between the center of first frame to second frame in the direction of Z axis of first frame.

DH parameters of second Joint

| Joint | | | | |
|-------|------|----|---|----|
| 1 | 0+50 | 90 | 0 | 18 |
| 2 | 0+60 | | | |
| 3 | | | | |



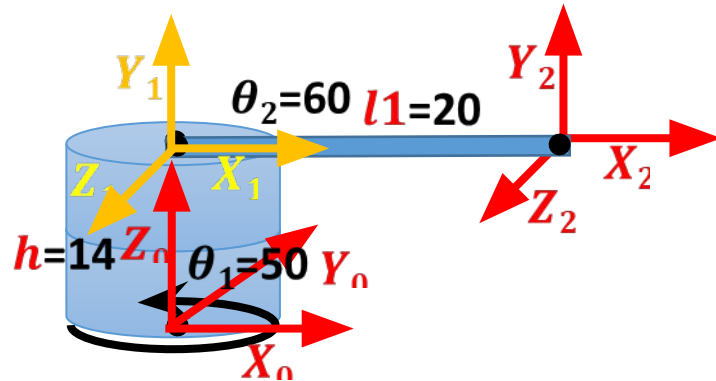
As we can see in the figure, the directions of X_1 and X_2 are same. So again we do not need to add any rotation around Z_1 . So, θ would be the value of only θ_2 that is 60.

Let us consider two frames: frame 0 and frame 1

- θ : defined as the rotation of X of second frame to align X of first frame around Z axis of first frame. It also includes the rotation of the joint.
- α : defined as the rotation of Z of first frame to align Z of second frame around X axis of second frame.
- In both cases anticlockwise rotation will indicate positive value and clockwise rotation will indicate negative value.
- a : is the distance between the center of first frame to second frame in the direction of X axis of second frame.
- d : is the distance between the center of first frame to second frame in the direction of Z axis of first frame.

DH parameters of second Joint

| Joint | | | | |
|-------|------|----|---|----|
| 1 | 0+50 | 90 | 0 | 18 |
| 2 | 0+60 | 0 | | |
| 3 | | | | |



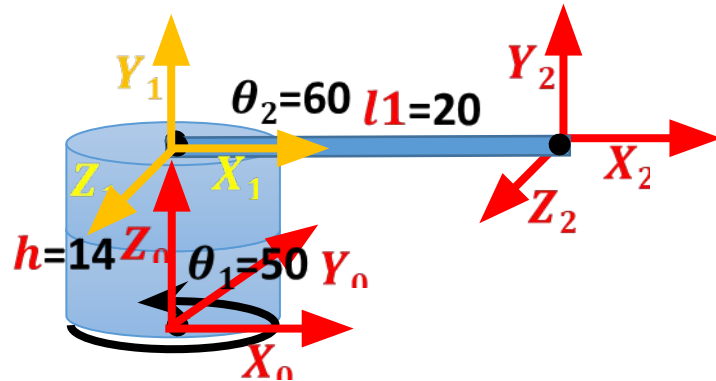
We do not need to rotate Z_1 around X_2 to align with Z_2 . So the value of α will be 0.

Let us consider two frames: frame 0 and frame 1

- θ : defined as the rotation of X of second frame to align X of first frame around Z axis of first frame. It also includes the rotation of the joint.
- α : defined as the rotation of Z of first frame to align Z of second frame around X axis of second frame.
- In both cases anticlockwise rotation will indicate positive value and clockwise rotation will indicate negative value.
- a : is the distance between the center of first frame to second frame in the direction of X axis of second frame.
- d : is the distance between the center of first frame to second frame in the direction of Z axis of first frame.

DH parameters of second Joint

| Joint | | | | |
|-------|------|----|----|----|
| 1 | 0+50 | 90 | 0 | 18 |
| 2 | 0+60 | 0 | 20 | |
| 3 | | | | |



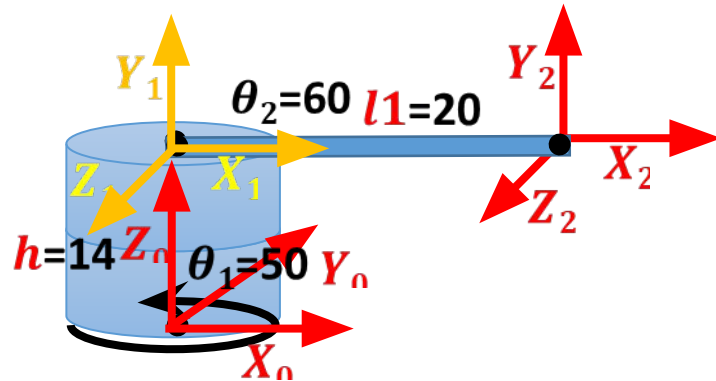
The distance between the center of first and second frame is 20 in X_2 direction. So, the value of a will 20.

Let us consider two frames: frame 0 and frame 1

- θ : defined as the rotation of X of second frame to align X of first frame around Z axis of first frame. It also includes the rotation of the joint.
- α : defined as the rotation of Z of first frame to align Z of second frame around X axis of second frame.
- In both cases anticlockwise rotation will indicate positive value and clockwise rotation will indicate negative value.
- a : is the distance between the center of first frame to second frame in the direction of X axis of second frame.
- d : is the distance between the center of first frame to second frame in the direction of Z axis of first frame.

DH parameters of second Joint

| Joint | | | | |
|-------|------|----|----|----|
| 1 | 0+50 | 90 | 0 | 18 |
| 2 | 0+60 | 0 | 20 | 0 |
| 3 | | | | |



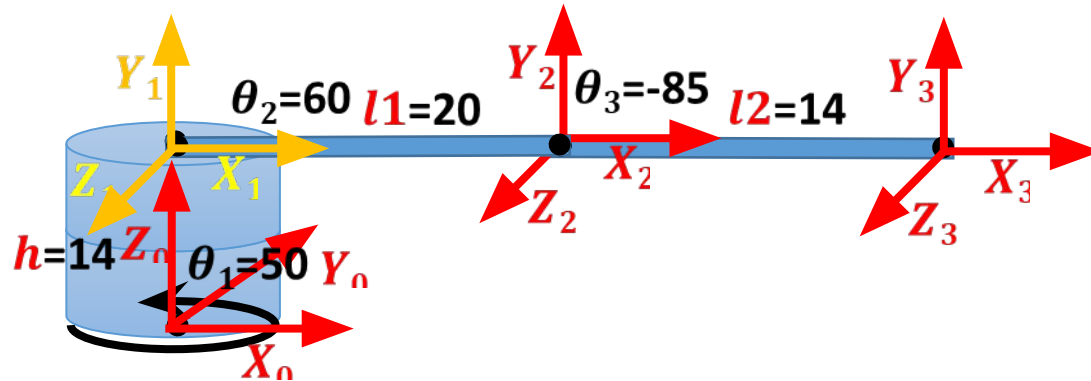
Center of first and second frame are in the same alignment in Z_2 direction. So, the value of d will 0.

Let us consider two frames: frame 0 and frame 1

- θ : defined as the rotation of X of second frame to align X of first frame around Z axis of first frame. It also includes the rotation of the joint.
- α : defined as the rotation of Z of first frame to align Z of second frame around X axis of second frame.
- In both cases anticlockwise rotation will indicate positive value and clockwise rotation will indicate negative value.
- a : is the distance between the center of first frame to second frame in the direction of X axis of second frame.
- d : is the distance between the center of first frame to second frame in the direction of Z axis of first frame.

DH parameters of third Joint

| Joint | | | | |
|-------|------|----|----|----|
| 1 | 0+50 | 90 | 0 | 18 |
| 2 | 0+60 | 0 | 20 | 0 |
| 3 | 0-85 | | | |



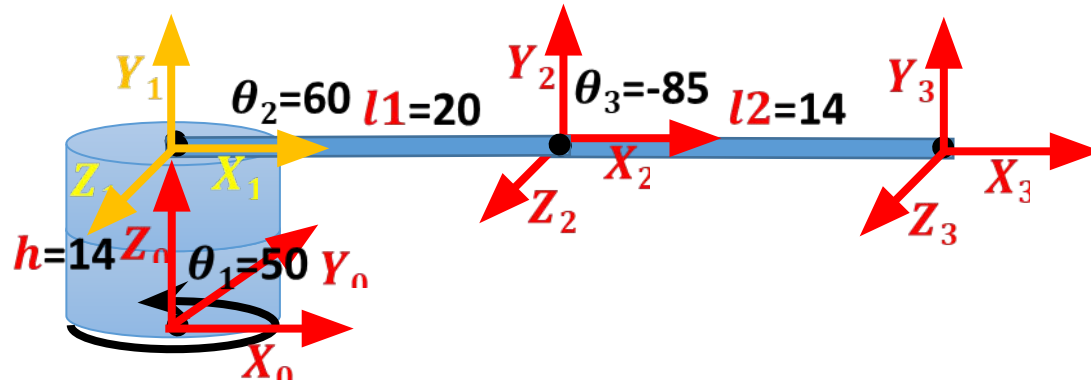
As we can see in the figure, the directions of X_2 and X_3 are same (on right direction) so we do not need to add any rotation around Z_2 . So, θ would be the value of θ_3 that is -85.

Let us consider two frames: frame 0 and frame 1

- θ : defined as the rotation of X of second frame to align X of first frame around Z axis of first frame. It also includes the rotation of the joint.
- α : defined as the rotation of Z of first frame to align Z of second frame around X axis of second frame.
- In both cases anticlockwise rotation will indicate positive value and clockwise rotation will indicate negative value.
- a : is the distance between the center of first frame to second frame in the direction of X axis of second frame.
- d : is the distance between the center of first frame to second frame in the direction of Z axis of first frame.

DH parameters of third Joint

| Joint | | | | |
|-------|------|----|----|----|
| 1 | 0+50 | 90 | 0 | 18 |
| 2 | 0+60 | 0 | 20 | 0 |
| 3 | 0-85 | 0 | 14 | 0 |



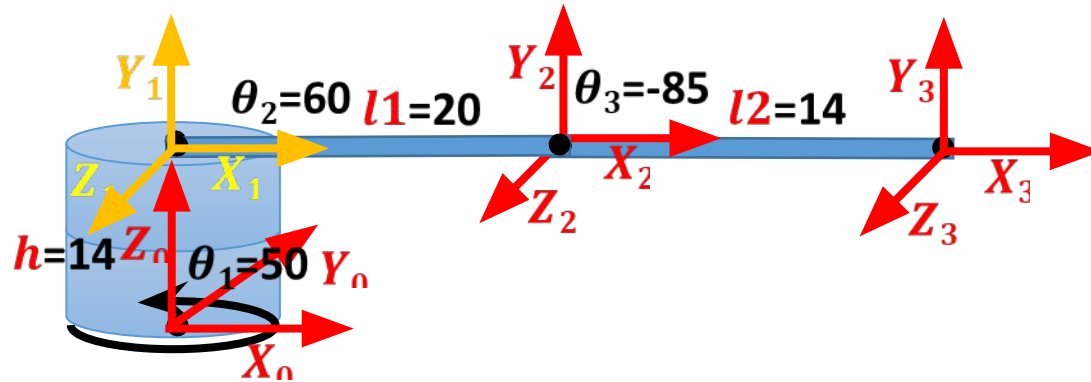
We do not need to rotate Z_2 around X_3 as it is already aligned with Z_3 . So the value of α will be 0.

Let us consider two frames: frame 0 and frame 1

- θ : defined as the rotation of X of second frame to align X of first frame around Z axis of first frame. It also includes the rotation of the joint.
- α : defined as the rotation of Z of first frame to align Z of second frame around X axis of second frame.
- In both cases anticlockwise rotation will indicate positive value and clockwise rotation will indicate negative value.
- a : is the distance between the center of first frame to second frame in the direction of X axis of second frame.
- d : is the distance between the center of first frame to second frame in the direction of Z axis of first frame.

DH parameters of third Joint

| Joint | | | | |
|-------|------|----|----|----|
| 1 | 0+50 | 90 | 0 | 18 |
| 2 | 0+60 | 0 | 20 | 0 |
| 3 | 0-85 | 0 | 14 | 0 |



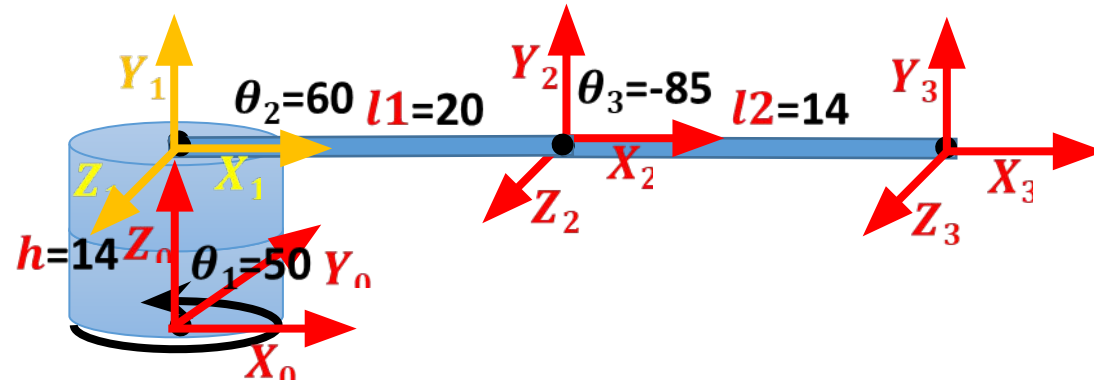
The distance between the center of third and fourth frame is 14 in X_3 direction. So, the value of a will be 14.

Let us consider two frames: frame 0 and frame 1

- θ : defined as the rotation of X of second frame to align X of first frame around Z axis of first frame. It also includes the rotation of the joint.
- α : defined as the rotation of Z of first frame to align Z of second frame around X axis of second frame.
- In both cases anticlockwise rotation will indicate positive value and clockwise rotation will indicate negative value.
- a : is the distance between the center of first frame to second frame in the direction of X axis of second frame.
- d : is the distance between the center of first frame to second frame in the direction of Z axis of first frame.

DH parameters of third Joint

| Joint | | | | |
|-------|------|----|----|----|
| 1 | 0+50 | 90 | 0 | 18 |
| 2 | 0+60 | 0 | 20 | 0 |
| 3 | 0-85 | 0 | 14 | 0 |

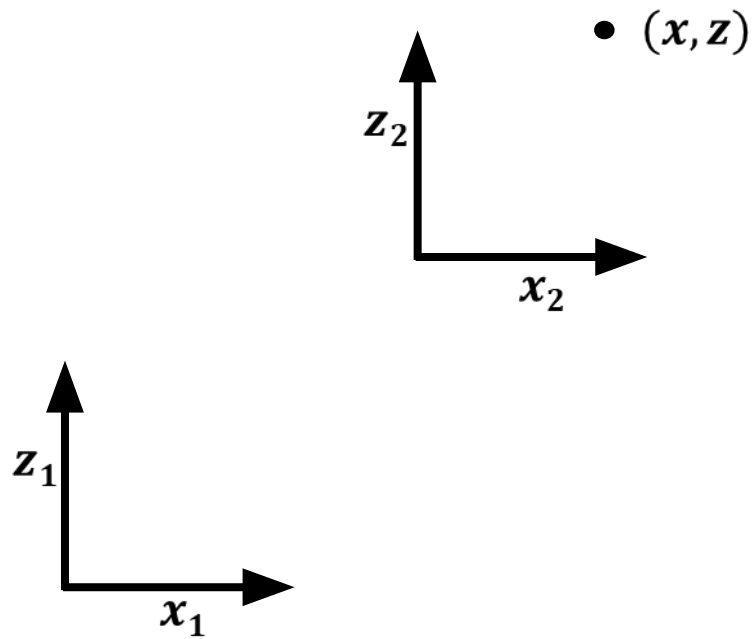


Center of first and second frame are in the same alignment in Z_3 direction. So, the value of d will 0.

Let us consider two frames: frame 0 and frame 1

- θ : defined as the rotation of X of second frame to align X of first frame around Z axis of first frame. It also includes the rotation of the joint.
- α : defined as the rotation of Z of first frame to align Z of second frame around X axis of second frame.
- In both cases anticlockwise rotation will indicate positive value and clockwise rotation will indicate negative value.
- a : is the distance between the center of first frame to second frame in the direction of X axis of second frame.
- d : is the distance between the center of first frame to second frame in the direction of Z axis of first frame.

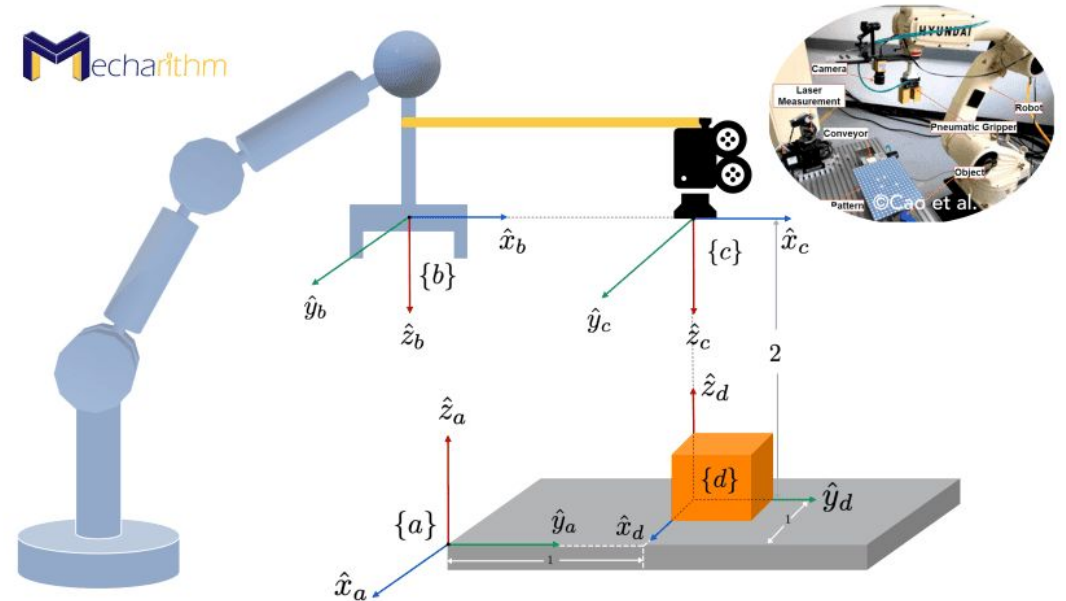
Homogeneous transformation



Homogeneous transformation matrix will transform the axis of point (x, z) from the (x_2, z_2) to (x_1, z_1)

Homogeneous transformation matrix

- The homogenous transformation matrix can act on a vector or a frame and displaces (rotating and translating) it. DH convention link with the Homogeneous transformation T_i is the combination of Four Basic Transformation
- $T_i = \text{RotZ}\theta_i, \text{TransZ}d_i, \text{TransX}a_i, \text{RotX}\alpha_i$



DH parameters to Homogeneous transformation

- $T_i = \text{RotZ}\theta_i, \text{TransZ}d_i, \text{TransX}a_i, \text{RotX}\alpha_i$
 - Rotation of angle of θ_i with respect to Z
 - Translation along Z with for value of d_i
 - Translation along X with for value of a_i
 - Rotation of X for an angle of α_i
- Where:
 - α_i : Z axis angle of two points (Link Twist)
 - a_i : Distance in Z axis considering the axis of Joint 4 (Link length)
 - d_i : Distance in Z axis considering the axis of Joint 3 (Link offset)
 - θ_i : X axis angle of two points (Joint angle)

$$\text{RotZ}\theta_i = \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & 0 \\ \sin\theta_i & \cos\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{TransX}a_i = \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{TransZ}d_i = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

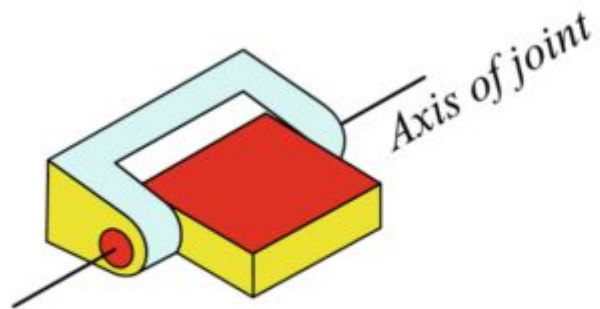
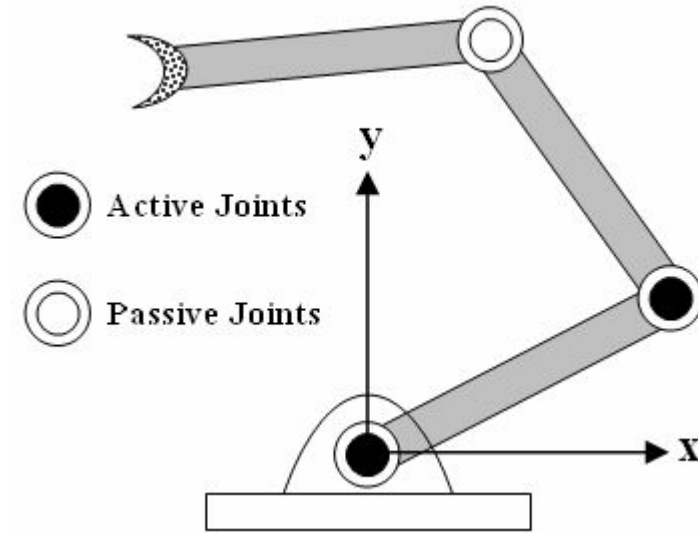
$$\text{RotX}\alpha_i = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha_i & -\sin\alpha_i & 0 \\ 0 & \sin\alpha_i & \cos\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_i = \text{RotZ}\theta_i, \text{TransZ}d_i, \text{TransX}a_i, \text{RotX}\alpha_i$$

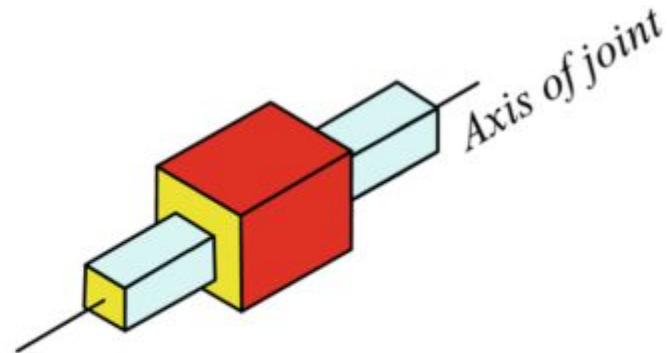
$$T_i = \begin{bmatrix} \cos\theta_i & -\cos\alpha_i\sin\theta_i & \sin\alpha_i\sin\theta_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\alpha_i\cos\theta_i & -\sin\alpha_i\cos\theta_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Type of joint

- Passive Joint (No movement)
- Active Joint
 - Revolute Joint (Rotation)
 - Prismatic Joint (Linear Shift)



Revolute joint



Prismatic joint

Homogeneous transformation matrixes for all joints

| Joint | | | | |
|-------|------|----|----|----|
| 1 | 0+50 | 90 | 0 | 18 |
| 2 | 0+60 | 0 | 20 | 0 |
| 3 | 0-85 | 0 | 14 | 0 |

$$T_i = \begin{bmatrix} \cos\theta_i & -\cos\alpha_i\sin\theta_i & \sin\alpha_i\sin\theta_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\alpha_i\cos\theta_i & -\sin\alpha_i\cos\theta_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

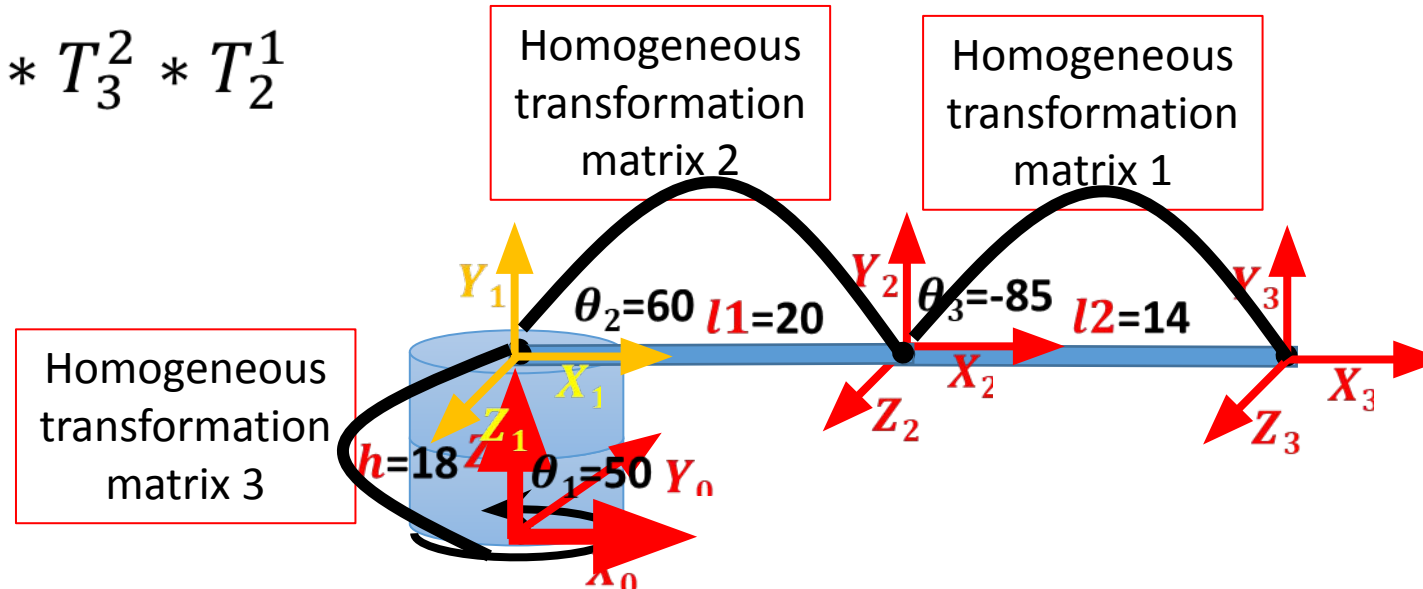
$$T_2^1 = \begin{bmatrix} \cos 50 & -\cos 90 \sin 50 & \sin 90 \sin 50 & 0 * \cos 50 \\ \sin 50 & \cos 90 \cos 50 & -\sin 90 \cos 50 & 0 * \sin 50 \\ 0 & \sin 90 & \cos 90 & 18 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.643 & 0 & 0.766 & 0 \\ 0.766 & 0 & -0.643 & 0 \\ 0 & 1 & 0 & 18 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^2 = \begin{bmatrix} \cos 60 & -\cos 0 \sin 60 & \sin 0 \sin 60 & 20 * \cos 60 \\ \sin 60 & \cos 0 \cos 60 & -\sin 0 \cos 60 & 20 * \sin 60 \\ 0 & \sin 0 & \cos 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.5 & -0.866 & 0 & 10 \\ 0.866 & 0.5 & 0 & 17.321 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_4^3 = \begin{bmatrix} \cos 95 & -\cos 0 \sin 95 & \sin 0 \sin 95 & 20 * \cos 95 \\ \sin 95 & \cos 0 \cos 95 & -\sin 0 \cos 95 & 20 * \sin 95 \\ 0 & \sin 0 & \cos 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.087 & 0.996 & 0 & 1.22 \\ -0.996 & 0.087 & 0 & -13.947 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Homogeneous transformation matrixes

- $T = T_4^3 * T_3^2 * T_2^1$



$$\begin{bmatrix} 0.087 & 0.996 & 0 & 1.22 \\ -0.996 & 0.087 & 0 & -13.947 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0.5 & -0.866 & 0 & 10 \\ 0.866 & 0.5 & 0 & 17.321 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0.643 & 0 & 0.766 & 0 \\ 0.766 & 0 & -0.643 & 0 \\ 0 & 1 & 0 & 18 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.583 & 0.272 & 0.766 & 14.584 \\ 0.694 & 0.324 & -0.643 & 17.38 \\ -0.423 & 0.906 & 0 & 29.404 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$(X, Y, Z) = (14.584, 17.38, 29.404)$$

$$\begin{bmatrix} 0.583 & 0.272 & 0.766 & 14.584 \\ 0.694 & 0.324 & -0.643 & 17.38 \\ -0.423 & 0.906 & 0 & 29.404 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$