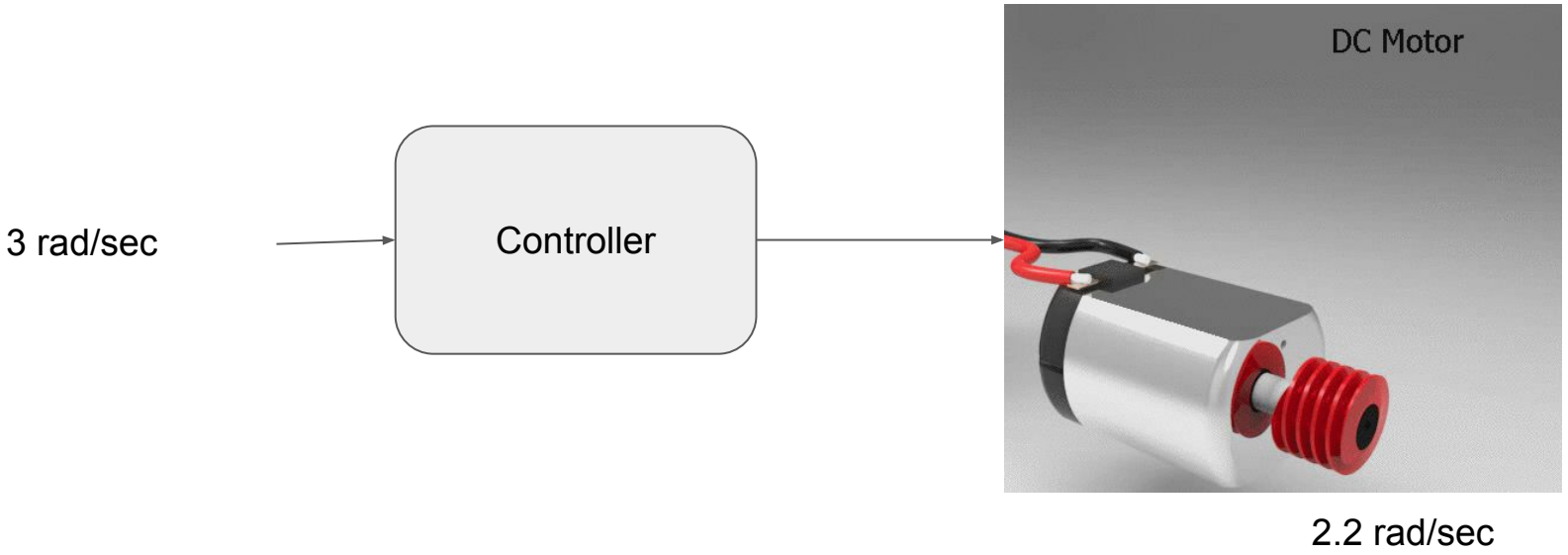
Introduction to Control System Theory

Control Theory

Control Theory

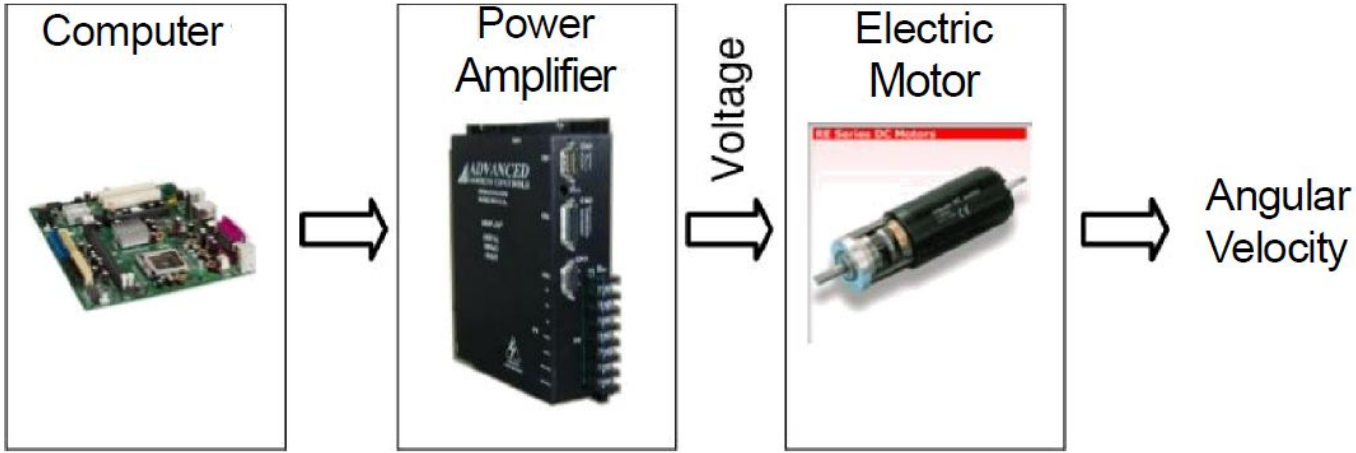
Definition: Control theory is a branch of engineering and mathematics that deals with the **behavior of dynamical systems**. It focuses on designing systems that can control the behavior of other systems to produce specific outputs.
Control theory develops **mathematical models** and **algorithms** to manipulate the behavior of systems, ensuring **they operate as desired.**

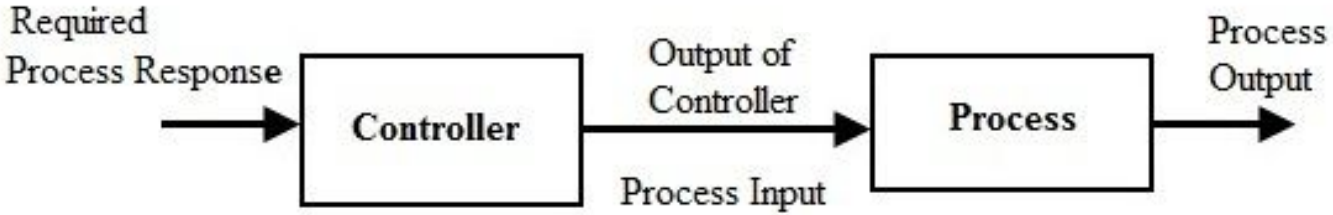
Control System

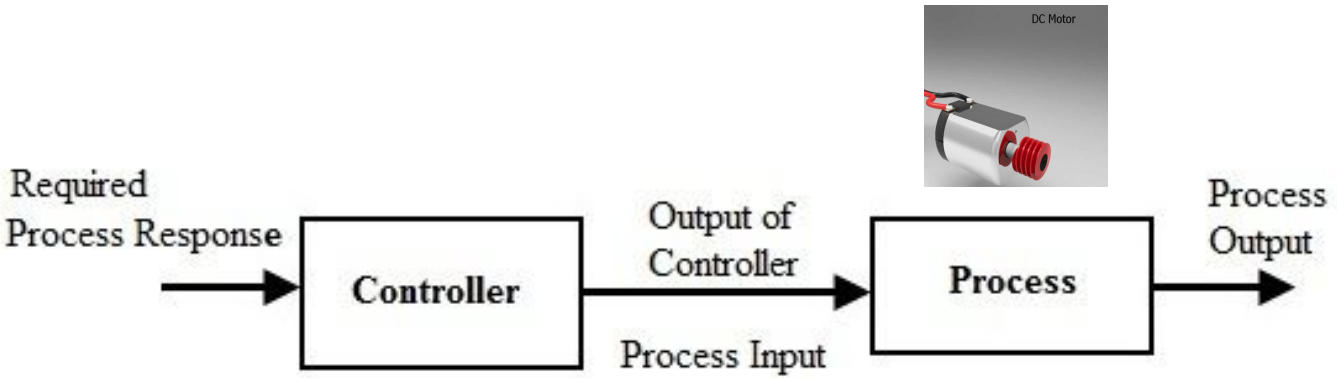


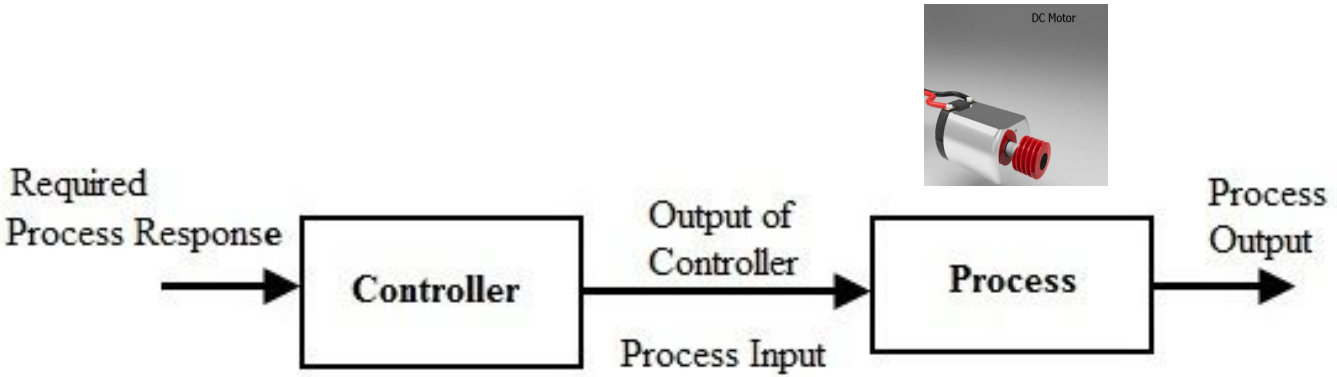
Open Loop

- We want to spin a motor at a given angular velocity. We can apply a fixed voltage to it, and never check to see if it is rotating properly.
- Called open loop.




Required
Process Response
Controller
Output of
Controller
Process Input
Process
Process
Output


DC Motor
Required
Process Response
Controller
Output of
Controller
Process Input
Process
Process
Output



3 rad/s

2.2 rad/s

Required
Process Response

Controller

Output of
Controller

Process Input

Process

Process
Output

Required
Velocity

3 rad/s

Output
Velocity

2.2 rad/s

Open Loop Control

Required
Process Response

Controller

Output of
Controller

Process Input

Process

Process
Output

Required
Velocity

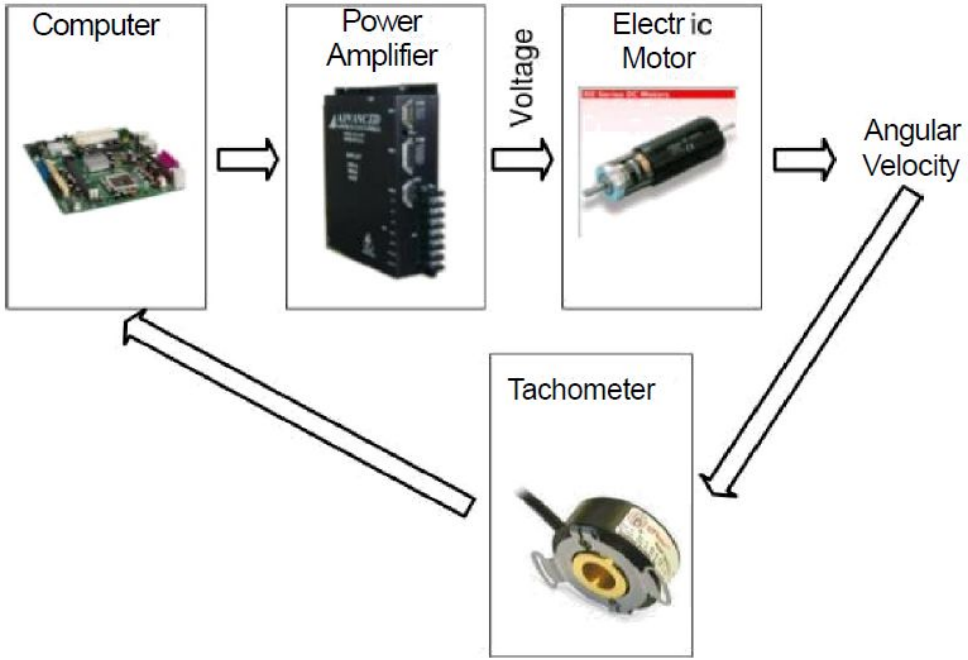
3 rad/s

Output
Velocity

2.2 rad/s

Closing the Loop

- Let's measure the actual angular velocities.
- Now we can compensate for changes in load by *feeding back* some information.



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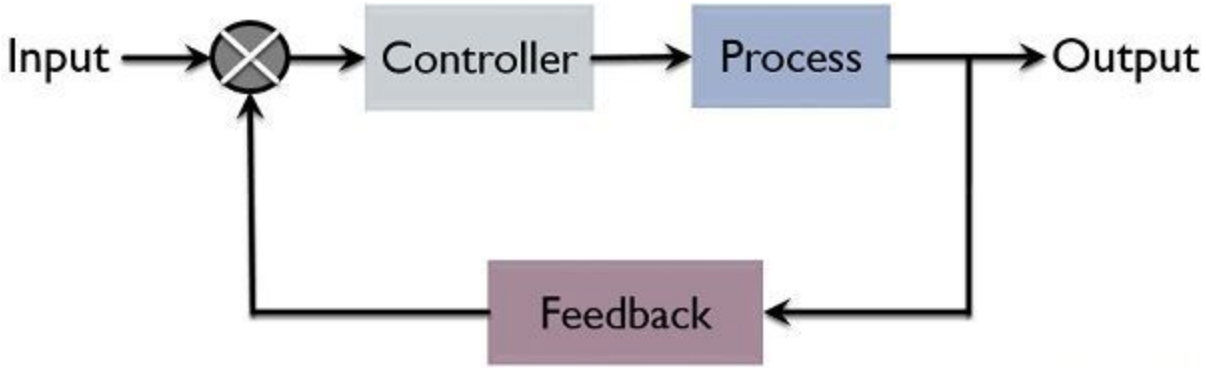
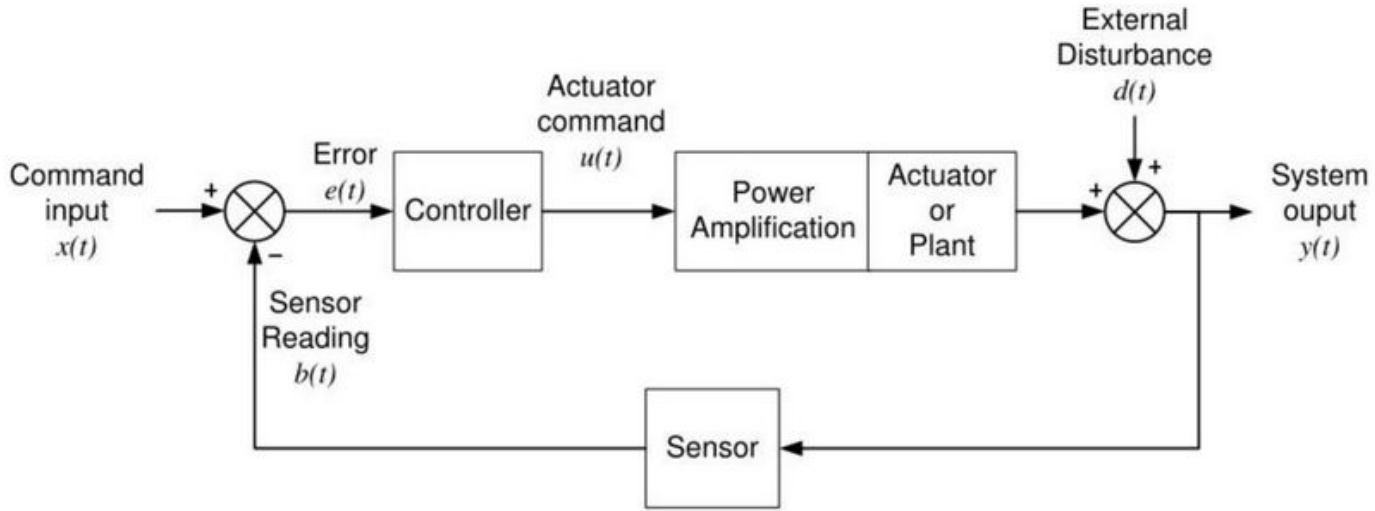
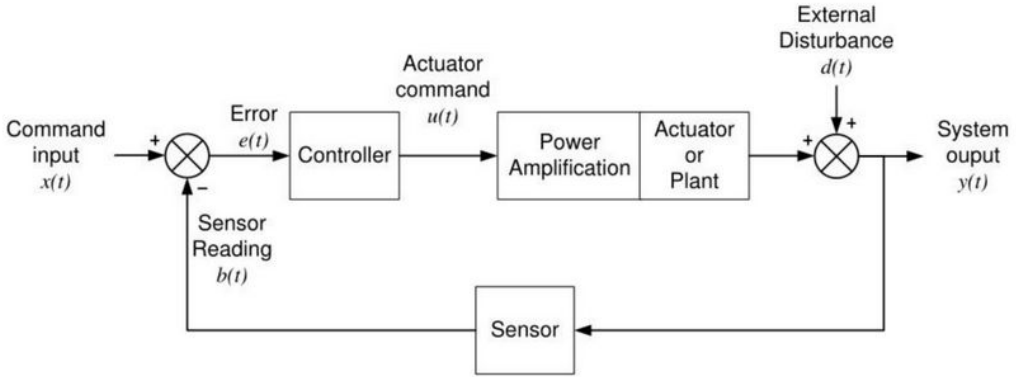
Closed Loop Control



Feedback Diagram





Characteristics of Feedback System

- Power amplification
- Actuator
- Feedback
  - measurement (sensor)
- Error signal
- Controller

Components

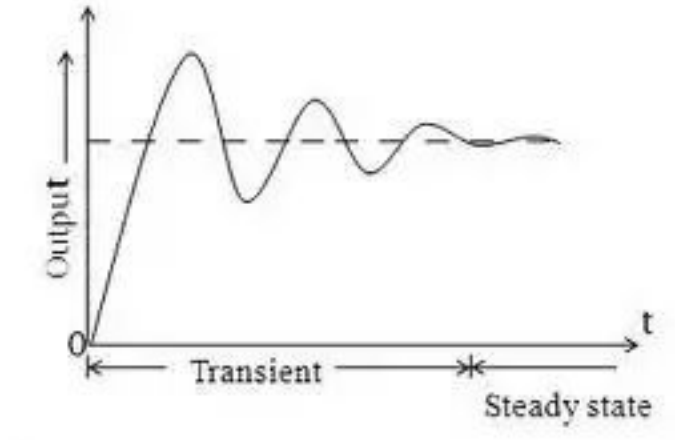
- **Input:** The signal or commands applied to the system.
- **Output**: The response of the system to the input signals.
- **Controller**: The component responsible for generating control signals based on the system's output and desired behavior.
- **Feedback**: Information about the system's output that is fed back to the controller to adjust the control signals.

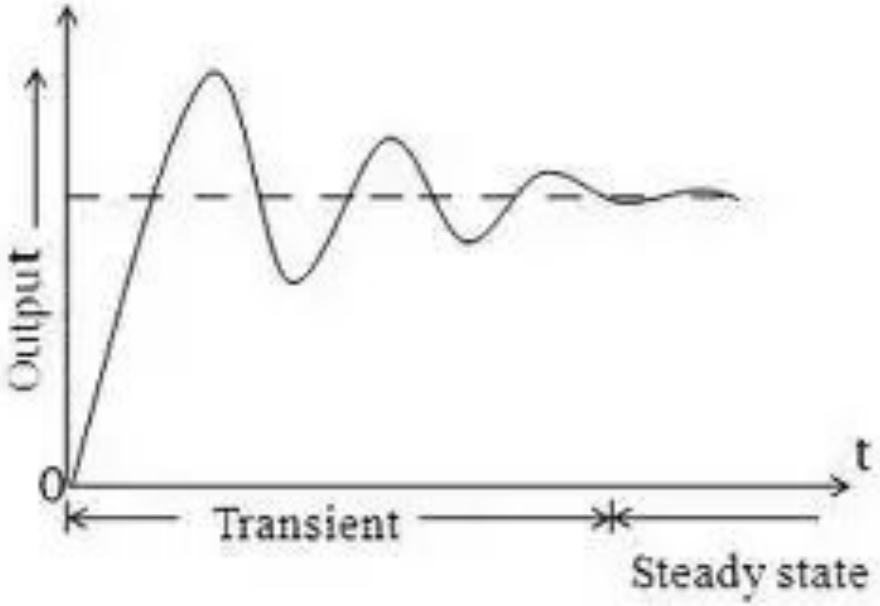
Theoretical Concepts

- **Feedback**: Feedback involves using information about the output of a system to adjust the control input, thereby regulating the system's behavior.
  Ex. Thermostat, measures the current room temperature(feedback), compares it to the desired temperature set by user, adjusts the heating or cooling.
- **Stability**: Stability ensures that the system's response remains bounded over time, even in the presence of disturbances or changes in operating conditions.
  Ex. balancing a pencil on the tip of a finger. Moving your finger slightly, the pencil wobbles but eventually return to its position. The pencil returns to equilibrium despite disturbances illustrates stability.

Theoretical Concepts

- **Transient Response:** refers to how a system behaves during the transition from one state to another after a change in inputs.
  Ex. Turning on a light switch, the time it takes for the light to reach its steady state and any oscillations or overshoot during this transition represent the transient response of the lighting system.
- **Steady-State Response:**describes the behavior of the system once it has settled into a stable operating condition.
  Ex. In an elevator system, when you press a floor button, the elevator initially accelerates, then moves at a constant speed, and finally decelerates as it reaches the desired floor.

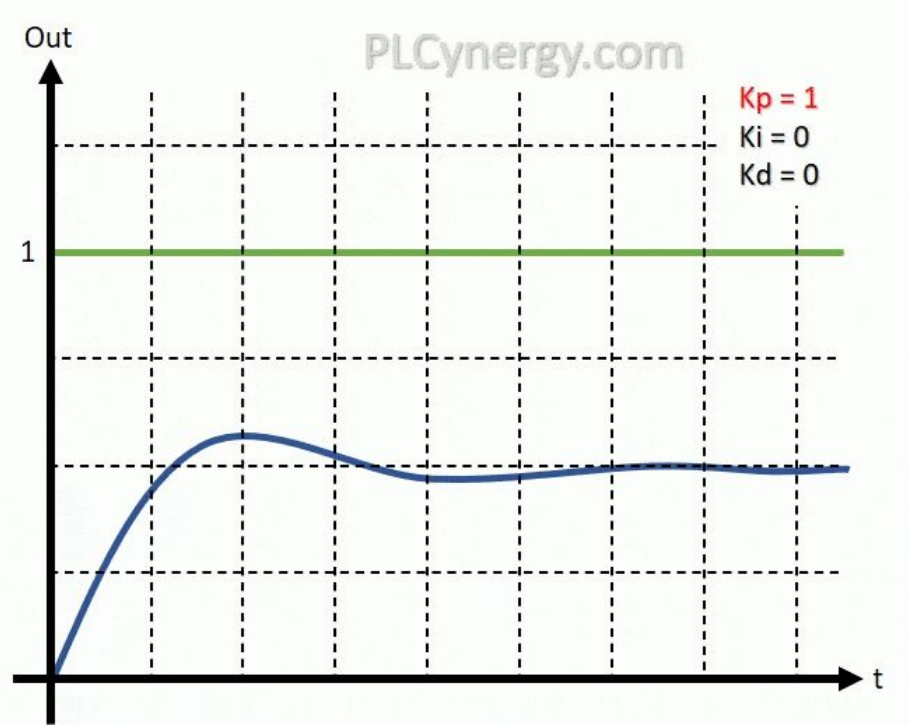




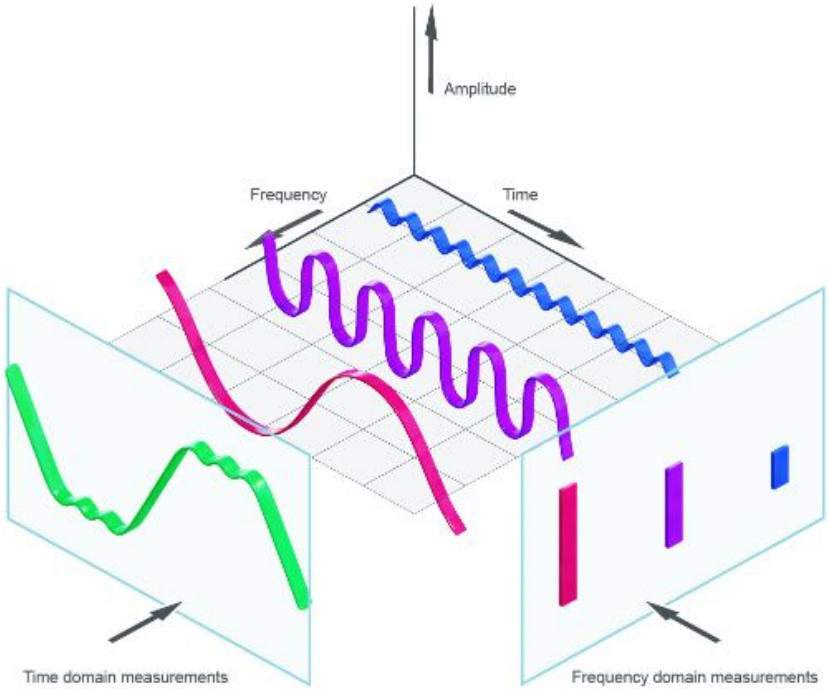
time domain

Effect of Controller Functions

- Proportional Action
  - Simplest Controller Function
- Integral Action
  - Eliminates steady-state error
  - Can cause oscillations
- Derivative Action ("rate control")
  - Effective in transient periods
  - Provides faster response (higher sensitivity)
  - Never used alone



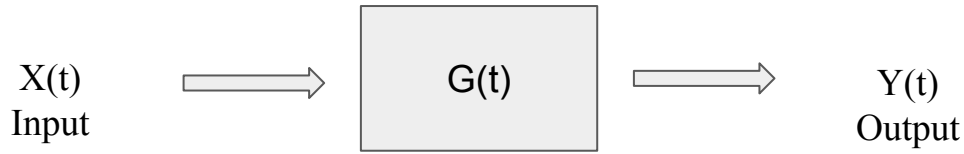
Time Domain to Frequency Domain



x(t)  x(s)

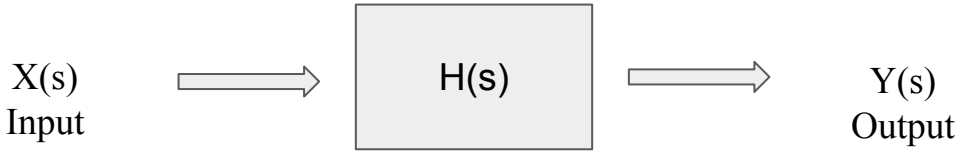
Laplace Transform

Block Diagram



$$Y(t) = X(t)*G(t)$$
$$Y(t)/X(t) = G(t)$$
$$\text{Gain} = Y(t)/X(t) = \text{Output/Input}$$

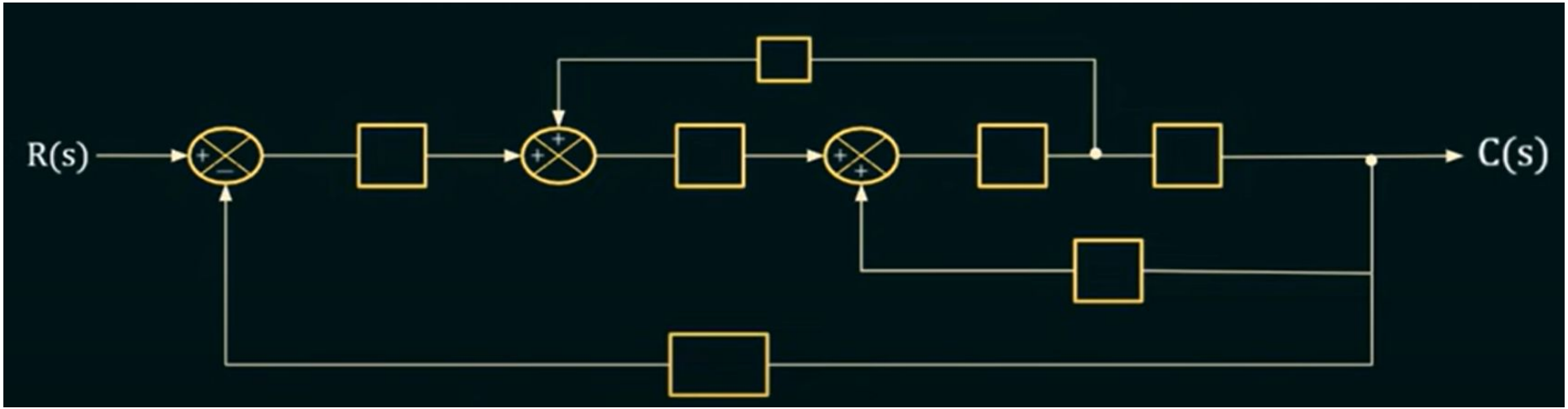
Transfer Function



- Definition: Transfer function is defined as the ratio of LT of output to the L.T of input. When all the initial condition assume to be zero.

$$H(s) = Y(s) / X(s)$$

- Relates the output of a linear system to its input.
- Describes how a linear system responds to an impulse, called impulse response


R(s)
C(s)

Next Class
Block Diagram Solving

Thank You