Introduction to Robotics
CSE 461

Class Topic: Introduction to Control System Theory (PID)

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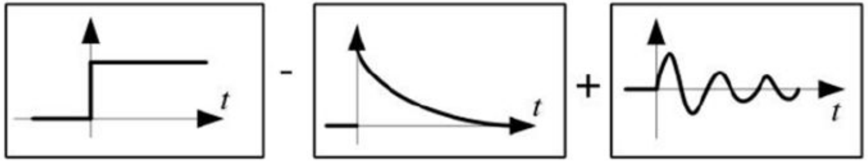
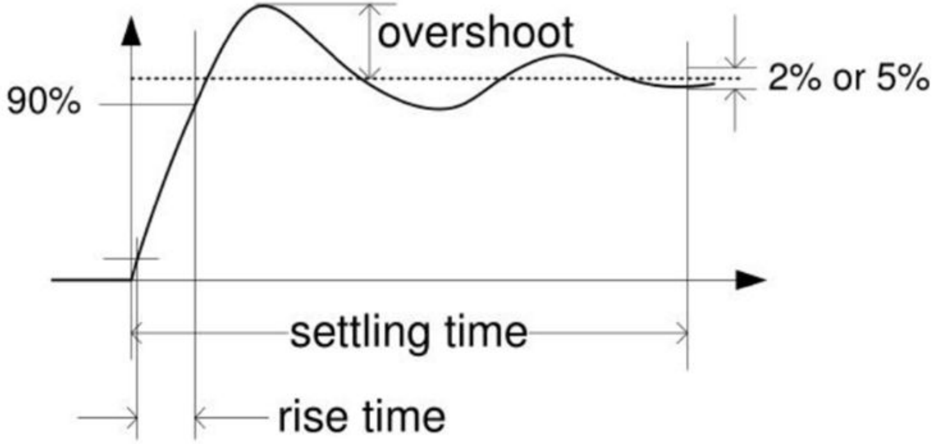
2nd order control system

- Typical response to step input is:





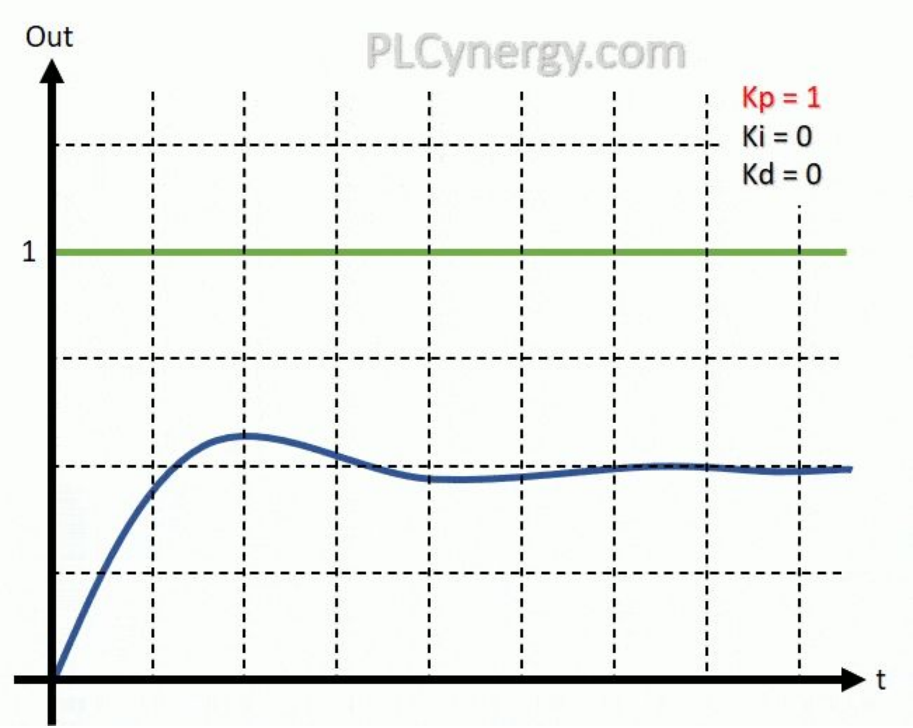
overshoot -- % of final value exceeded at first *oscillation*

rise time -- time to span from 10% to 90% of the final value

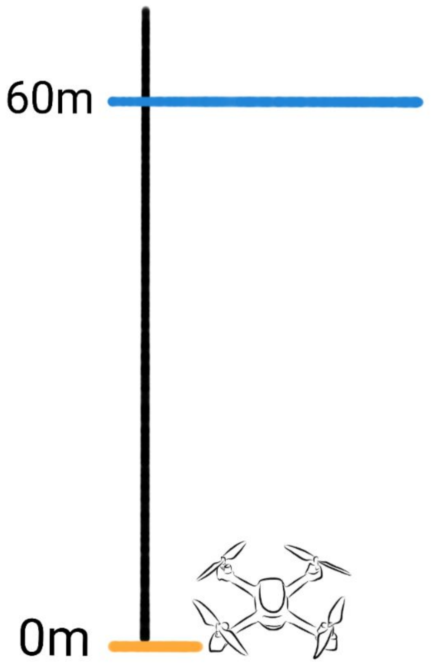
settling time -- time to reach within 2% or 5% of the final value

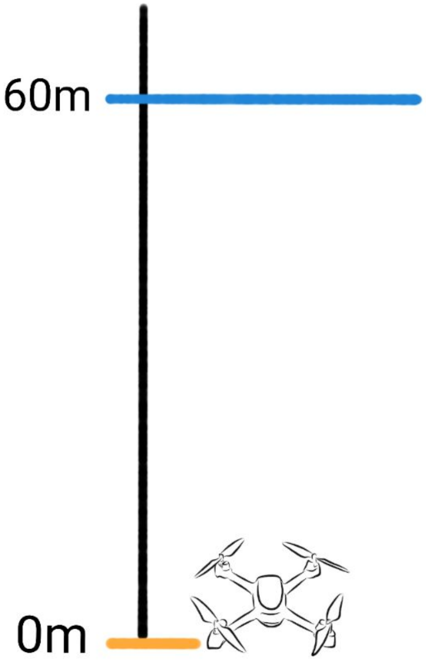
Effect of PID controller function

- Proportional Action
  - Simplest Controller Function
- Integral Action
  - Eliminates steady-state error
  - Can cause oscillations
- Derivative Action (“rate control”)
  - Effective in transient periods
  - Provides faster response (higher sensitivity)
  - Never used alone

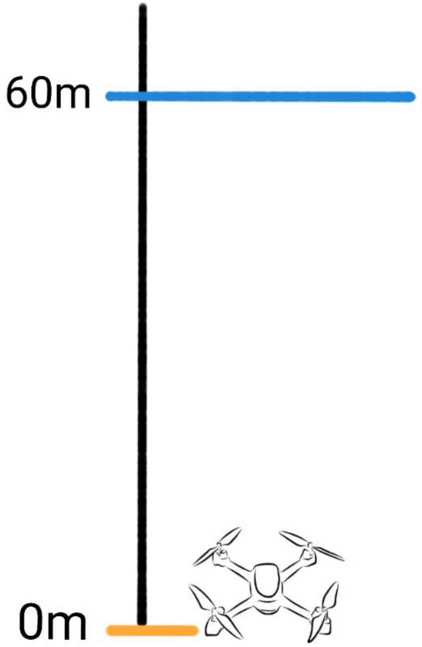


https://www.youtube.com/watch?v=wkfEZmsQqiA



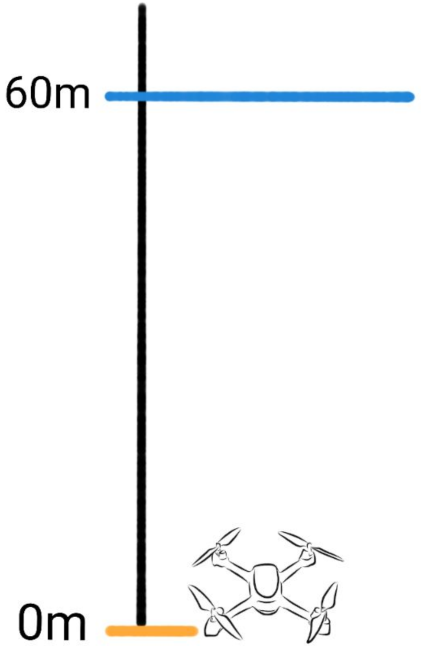


Desired Altitude = 60m
Actual Altitude = 0m



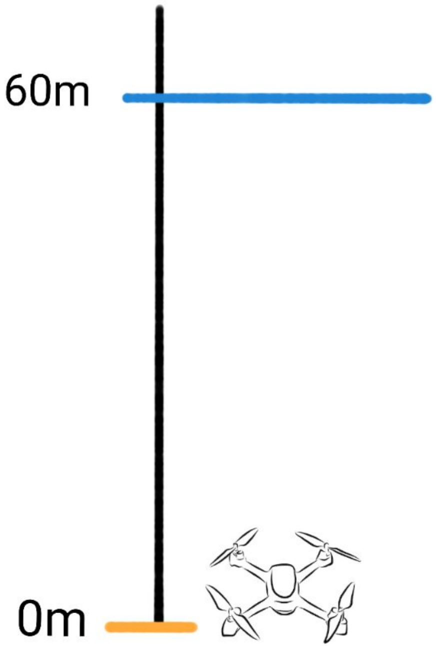
Desired Altitude = 60m
Actual Altitude = 0m

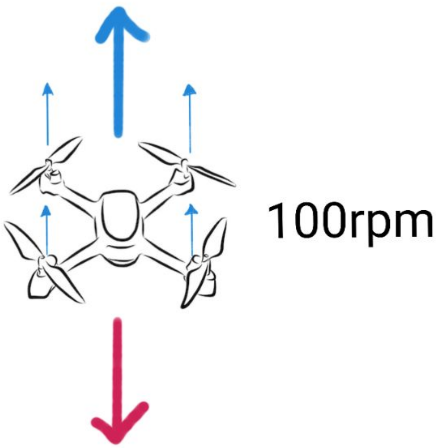
Desired Altitude - Actual Altitude
= 60m - 0m
= 60

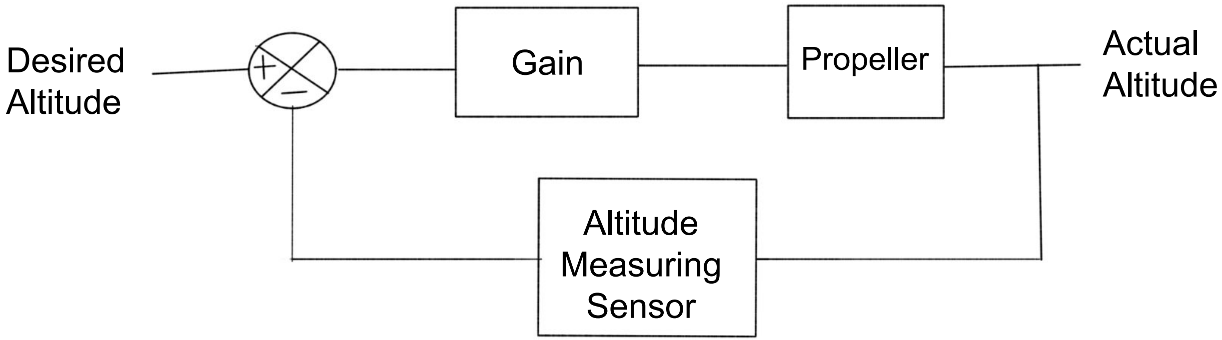


Desired Altitude = 60m
Actual Altitude = 0m

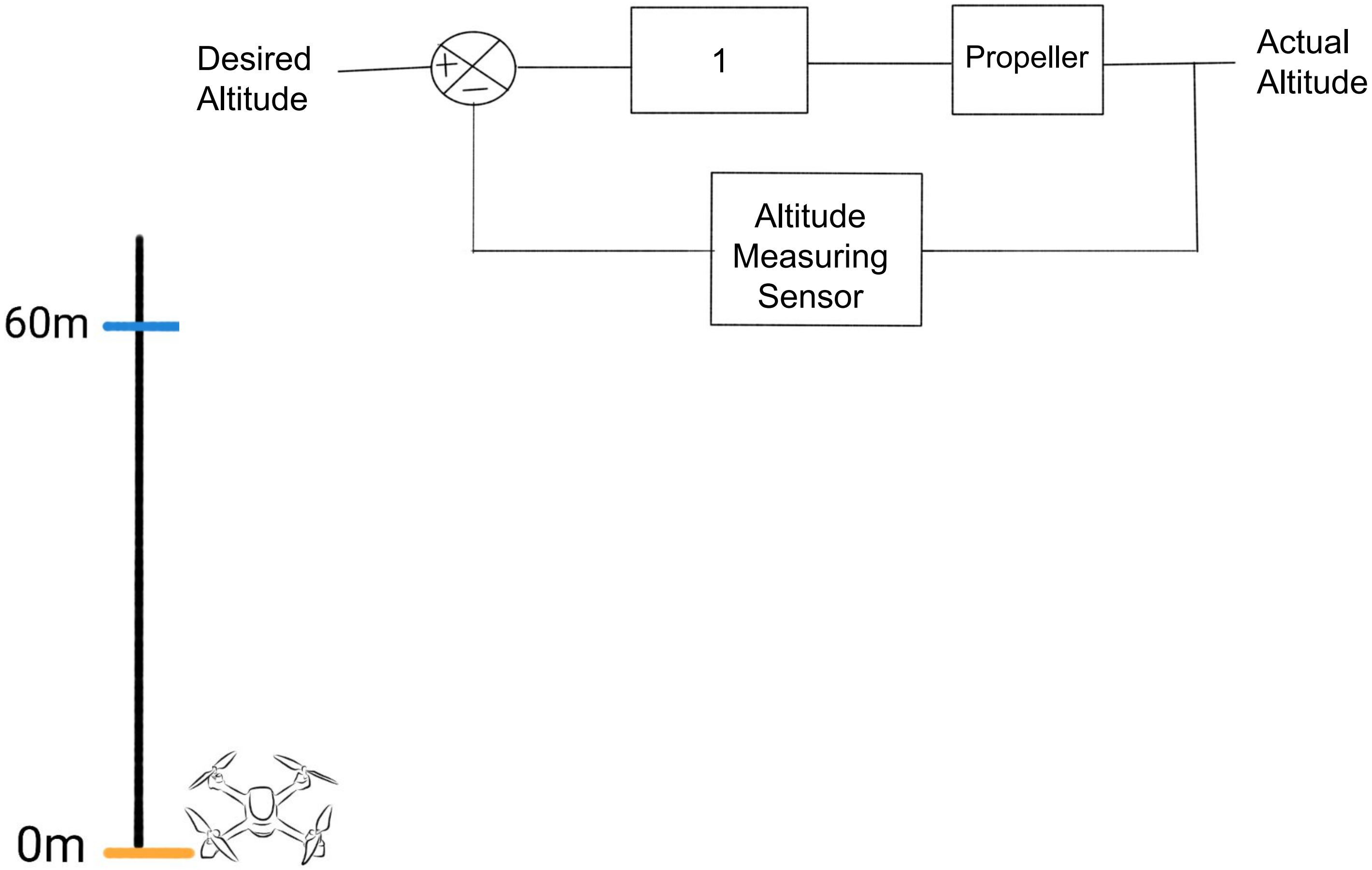
Desired Altitude - Actual Altitude
= 60m - 0m
= 60
= Error

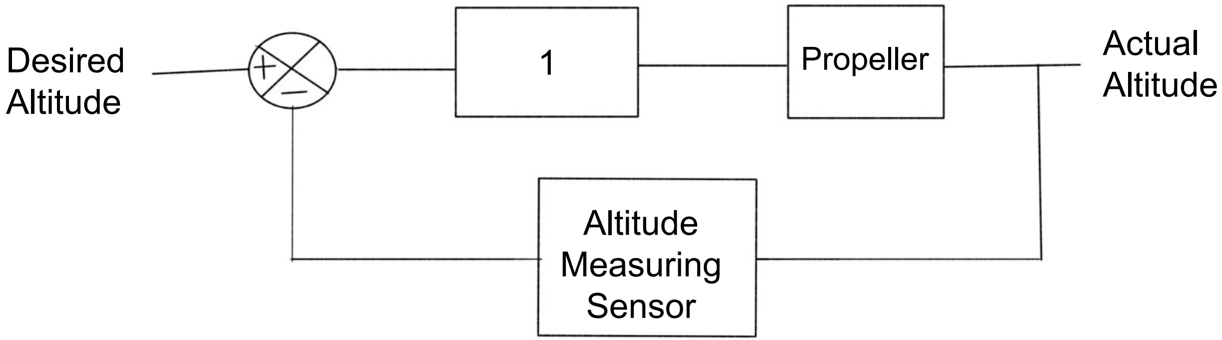

60m
0m

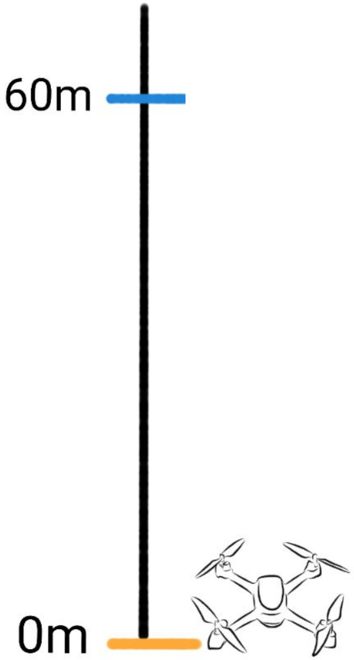

100rpm


Desired
Altitude
+
−
Gain
Propeller
Actual
Altitude
Altitude
Measuring
Sensor


60m
0m

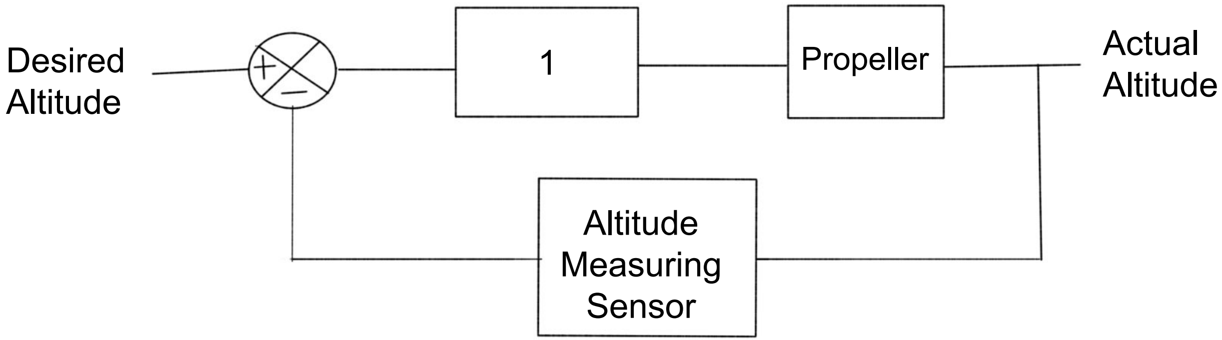

Desired
Altitude
1
Propeller
Actual
Altitude
Altitude
Measuring
Sensor
60m
0m

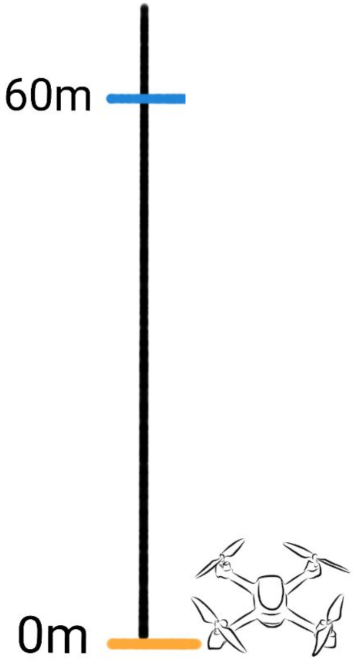




Desired Altitude - Actual Altitude
= 60m - 0m
= 60

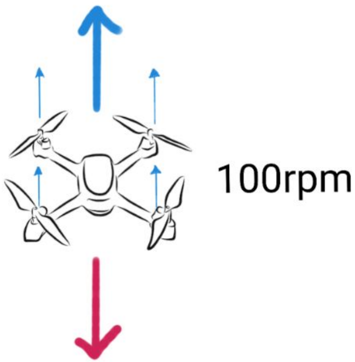
Propeller Speed = 60 * 1 = 60 rpm

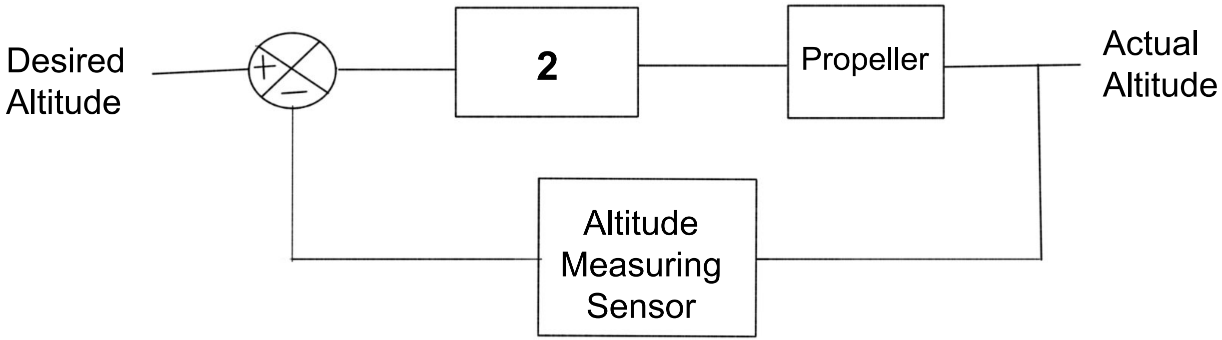


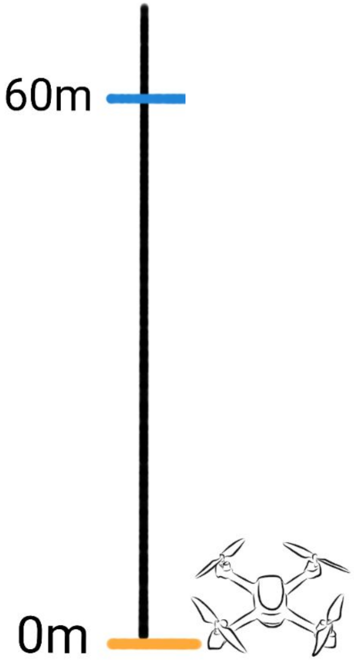


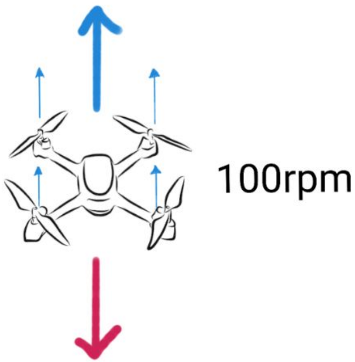
Desired Altitude - Actual Altitude
= 60m - 0m
= 60

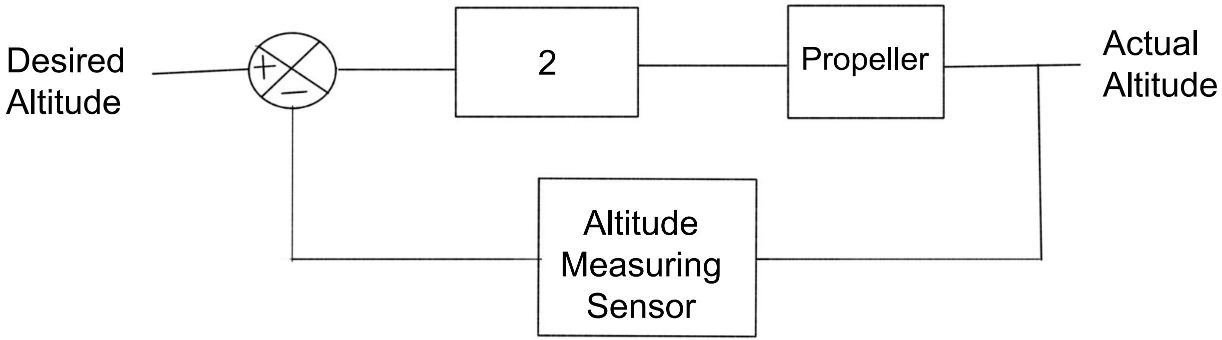
Propeller Speed = 60 * 1 = 60 rpm

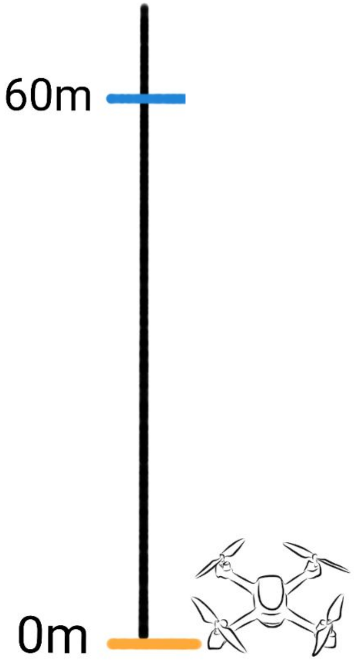



Desired
Altitude
2
Propeller
Actual
Altitude
Altitude
Measuring
Sensor


60m
0m

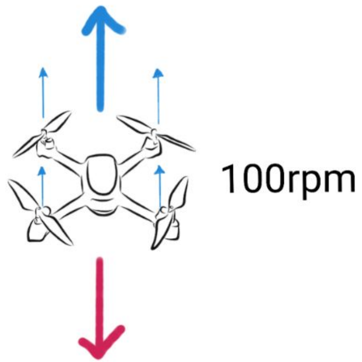

100rpm

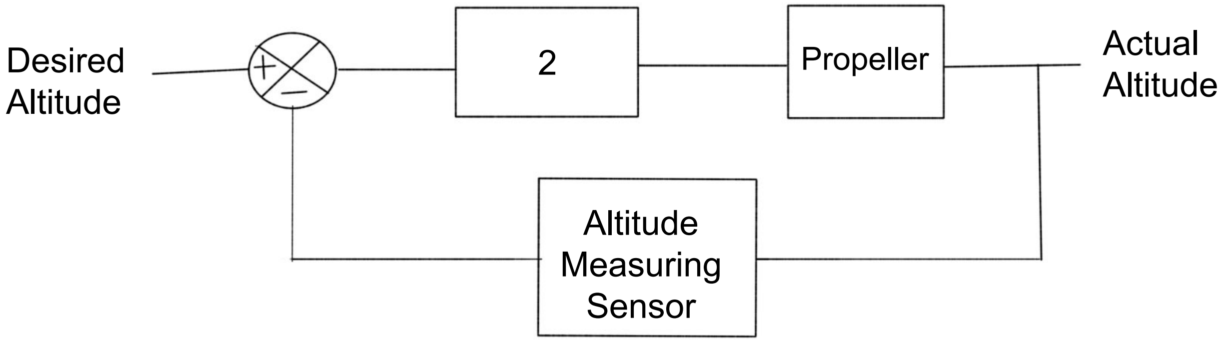


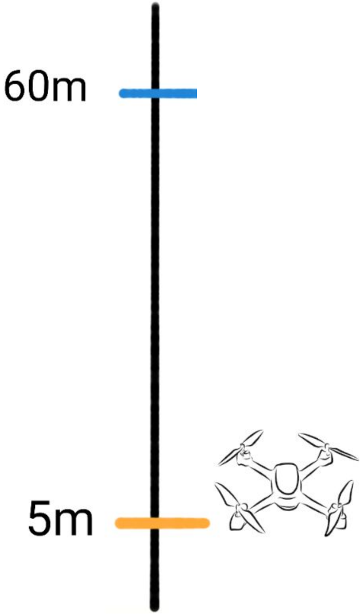


Desired Altitude - Actual Altitude
= 60m - 0m
= 60

Propeller Speed = 60 * 2 = 120 rpm

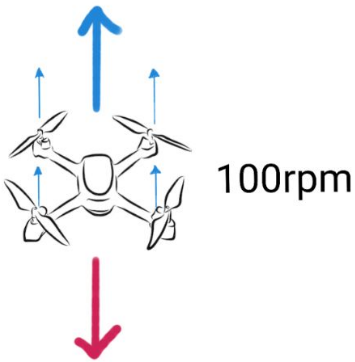


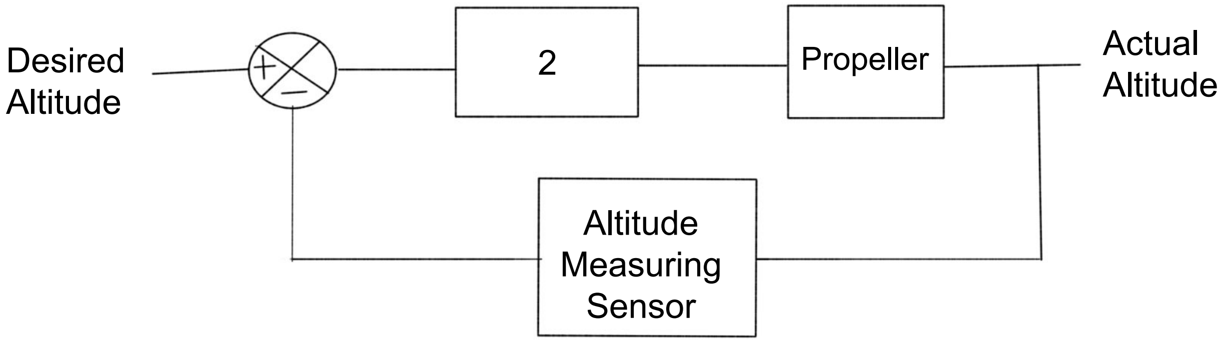


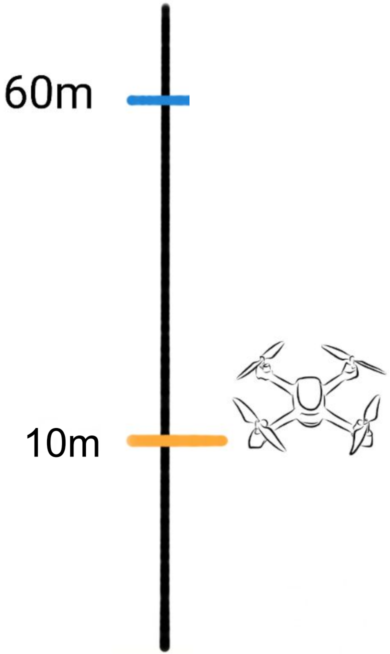


Desired Altitude - Actual Altitude
= 60m - 5m
= 55

Propeller Speed = 55 * 2 = 110 rpm

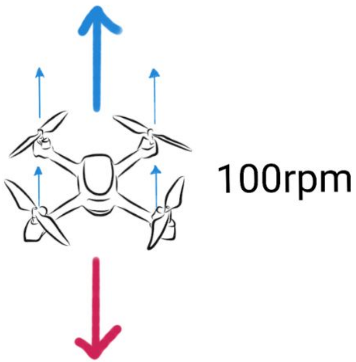


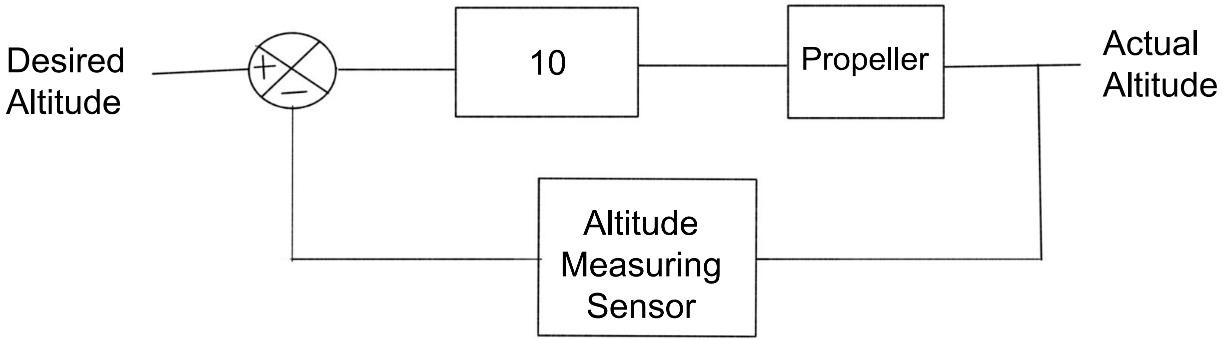


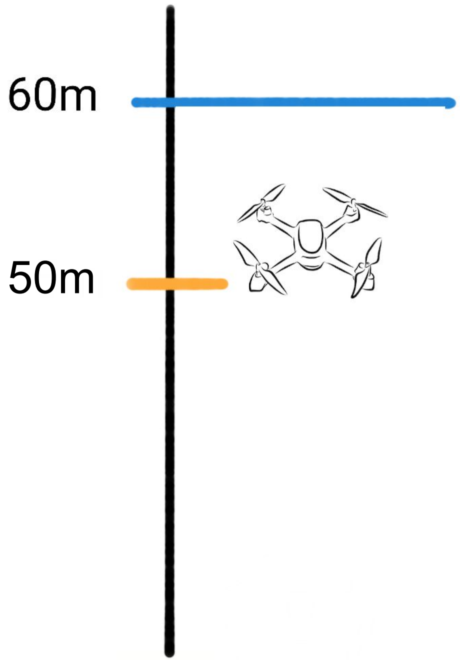


Desired Altitude - Actual Altitude
= 60m - 10m
= 50

Propeller Speed = 50 * 2 = 100 rpm

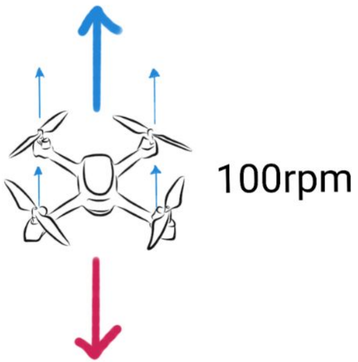


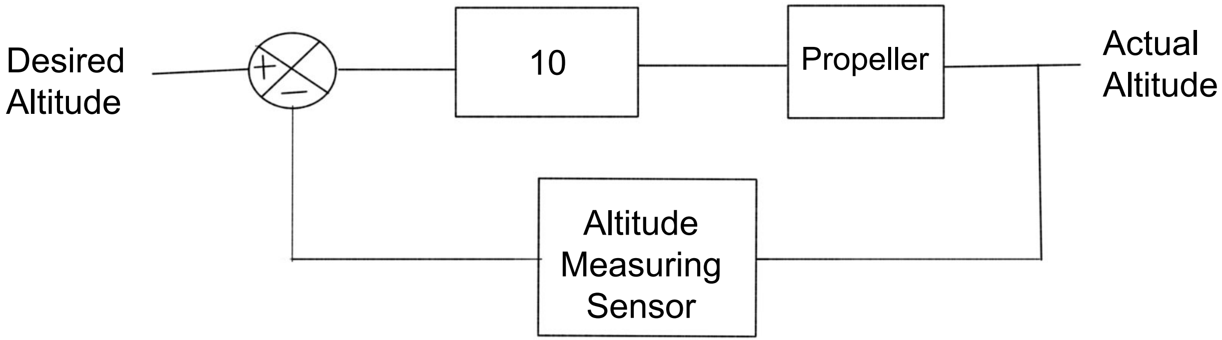


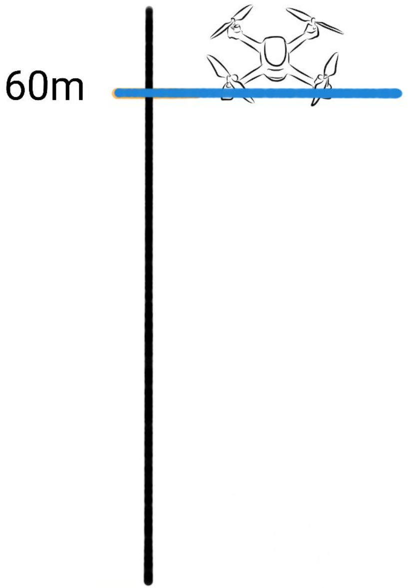


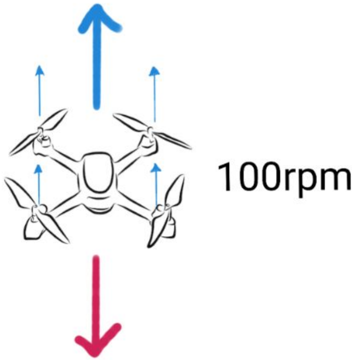
Propeller Speed = 10 * 10 = 100 rpm

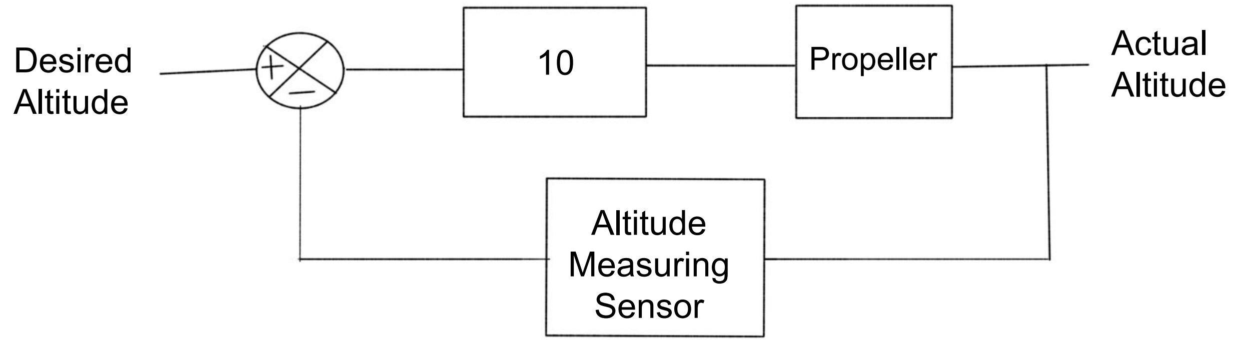
Error = 10

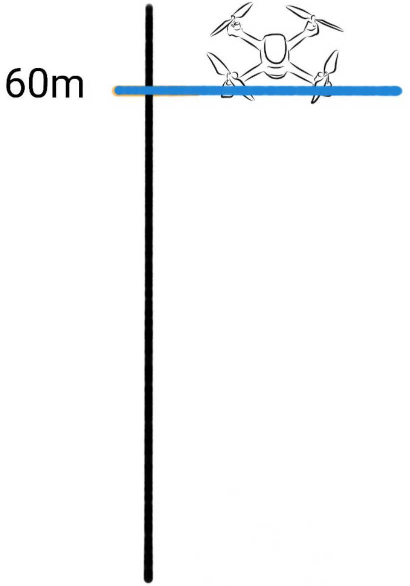



Desired Altitude
+
−
10
Propeller
Actual Altitude
Altitude Measuring Sensor


60m

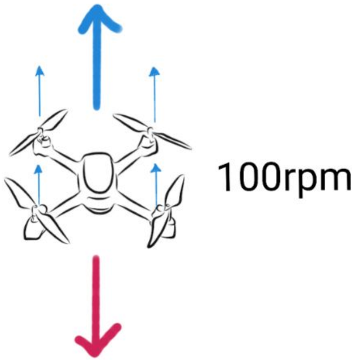

100rpm

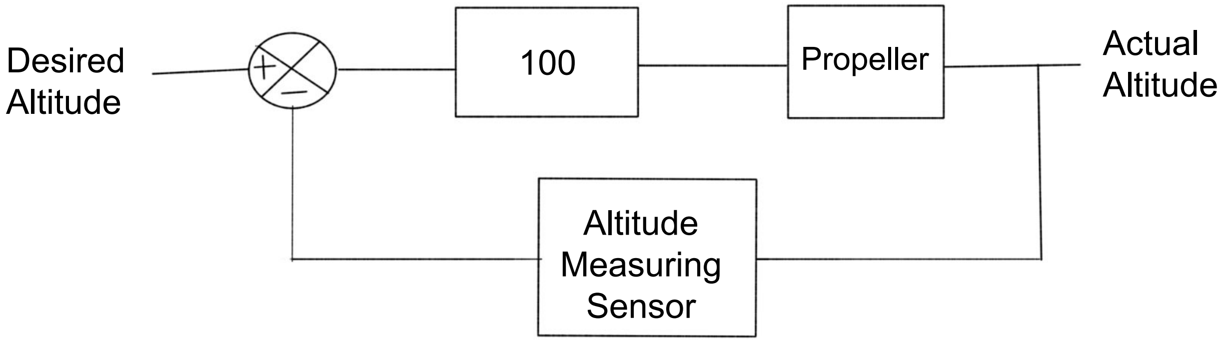


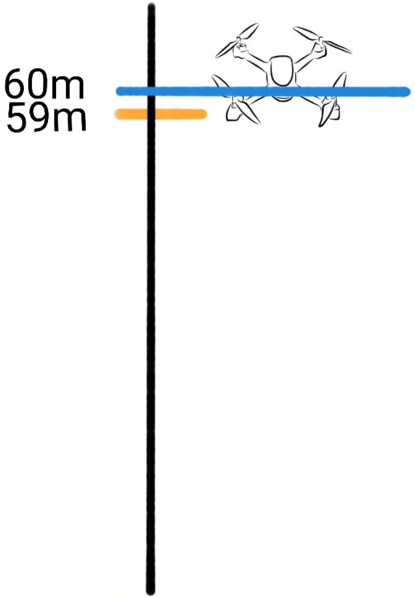


Desired Altitude - Actual Altitude
= 60m - 60m
= 0

Propeller Speed = 0 * 10 = 0 rpm

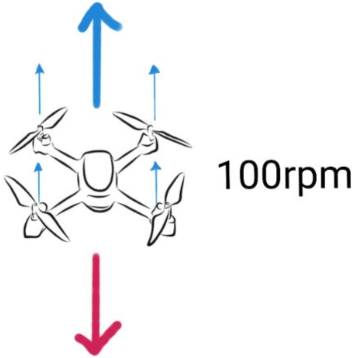


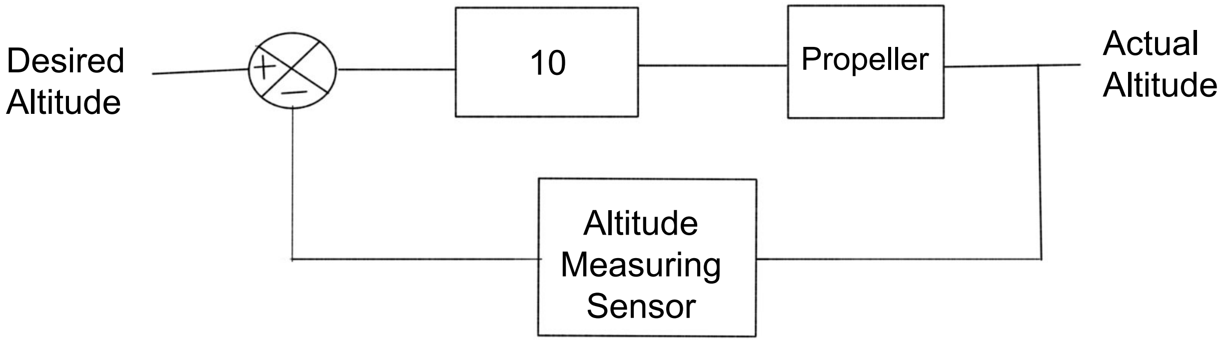


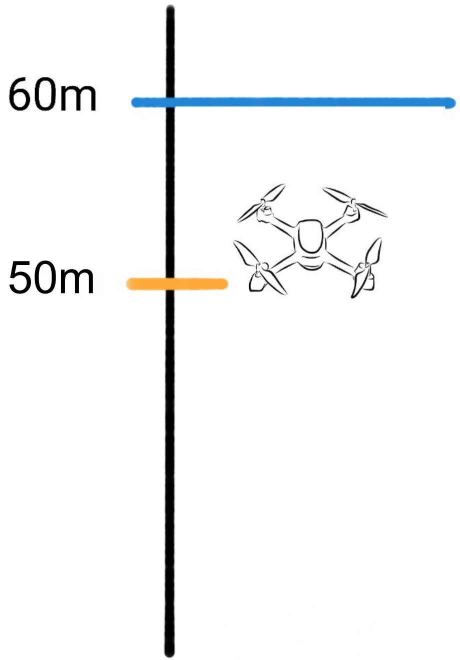


Desired Altitude - Actual Altitude
= 60m - 59m
= 1

Propeller Speed = 1 *100 = 100 rpm

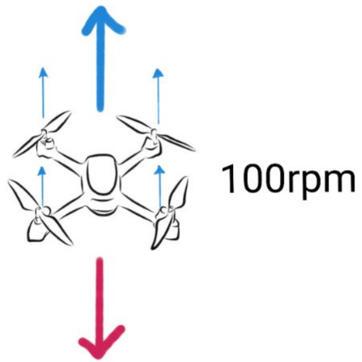




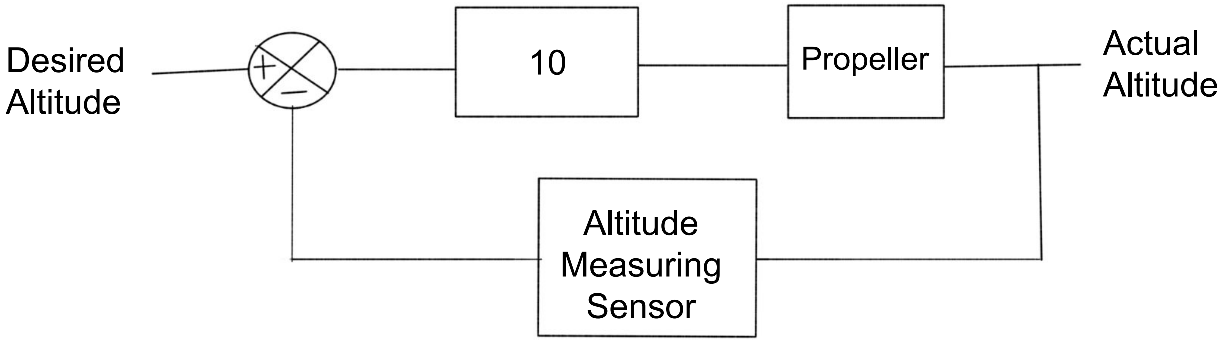


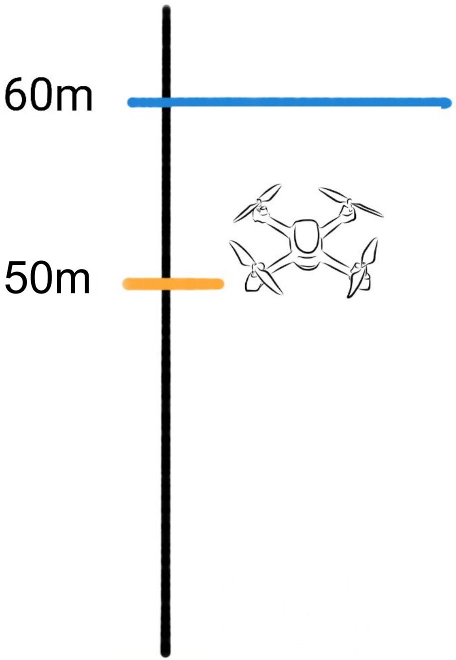
Propeller Speed = 10 * 10 = 100 rpm

Error = 10



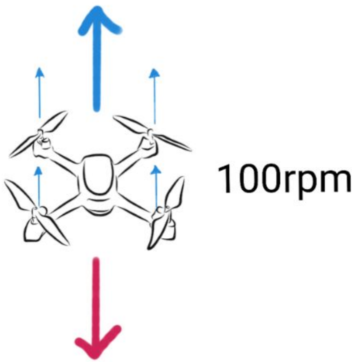
Output = Kp * error





Propeller Speed = 10 * 10 = 100 rpm

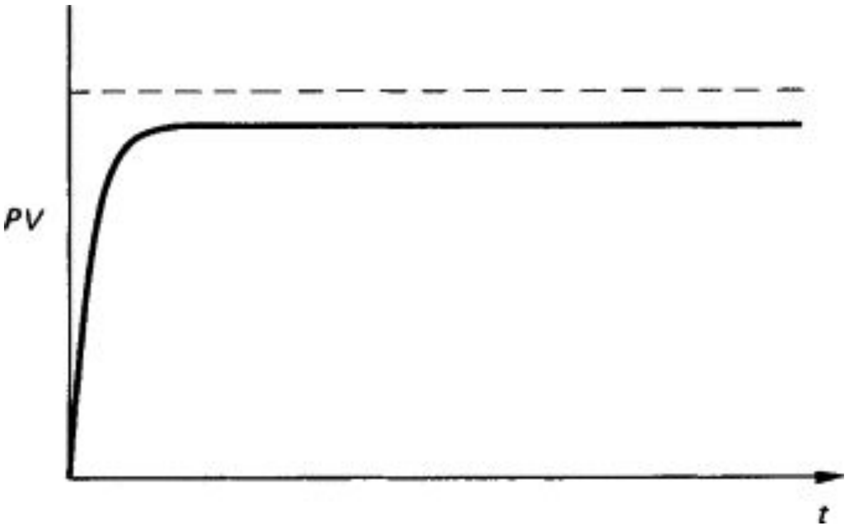
Error = 10

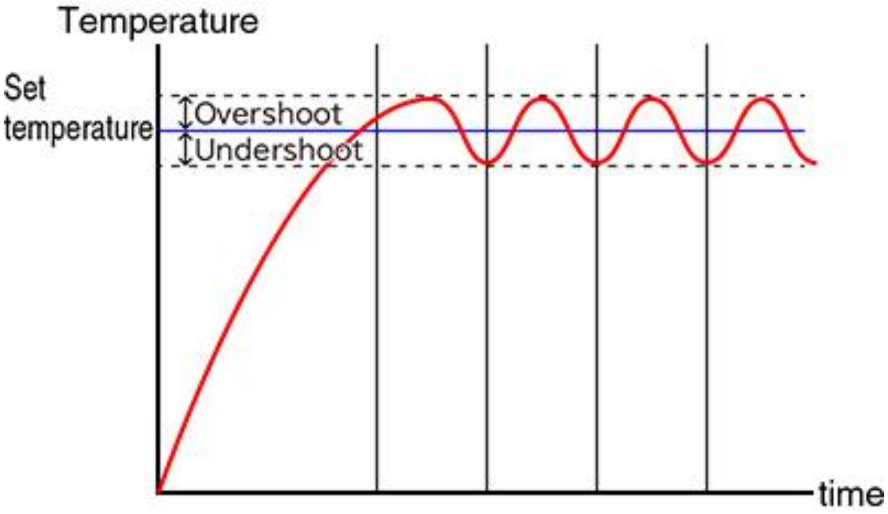


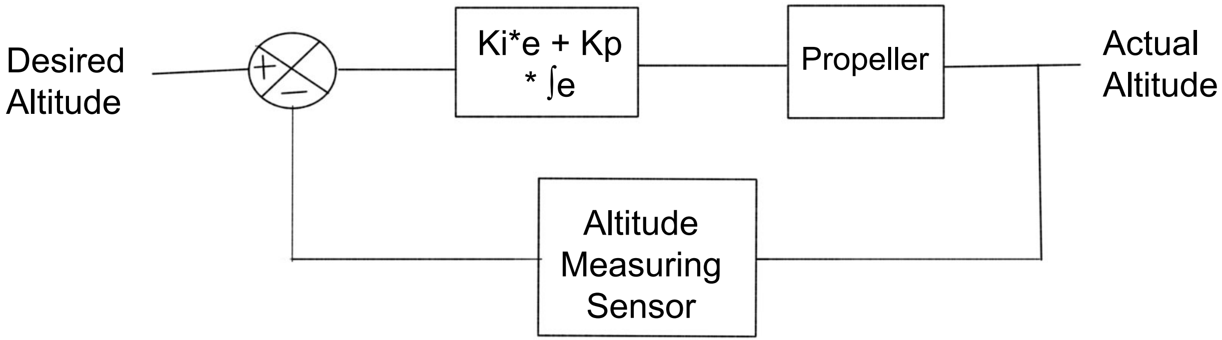
Output = Kp * error

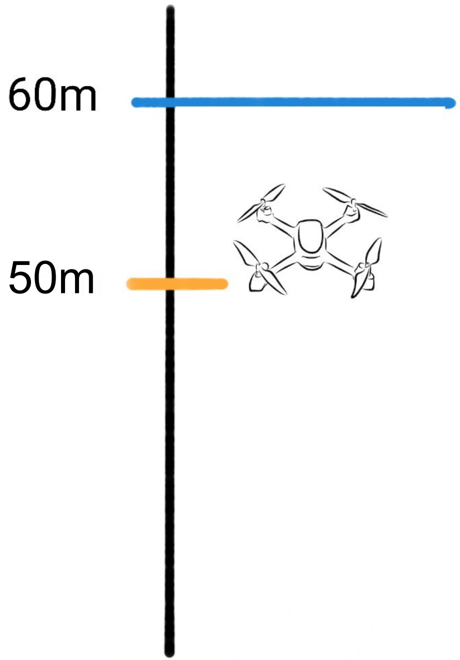
Proportional Control

Proportional Control



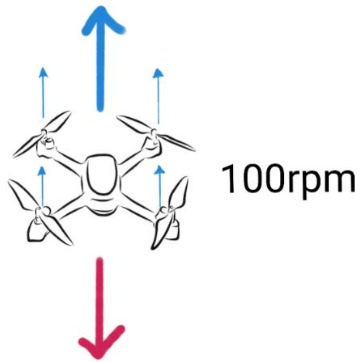


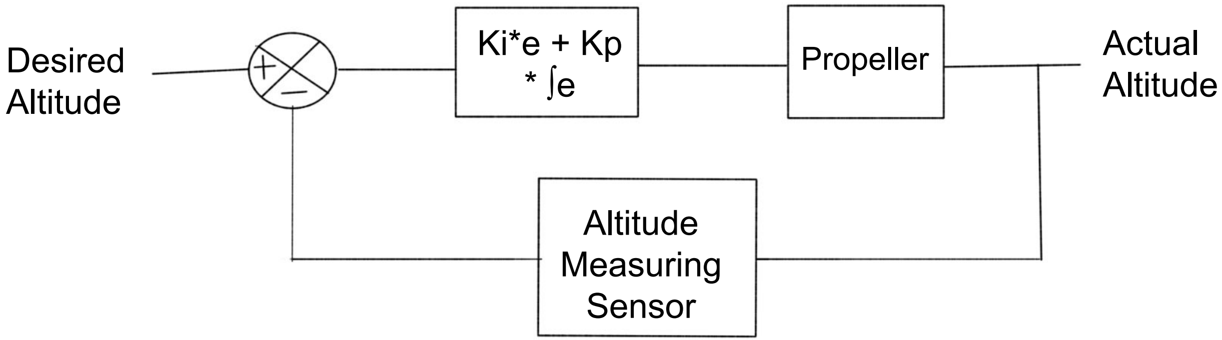


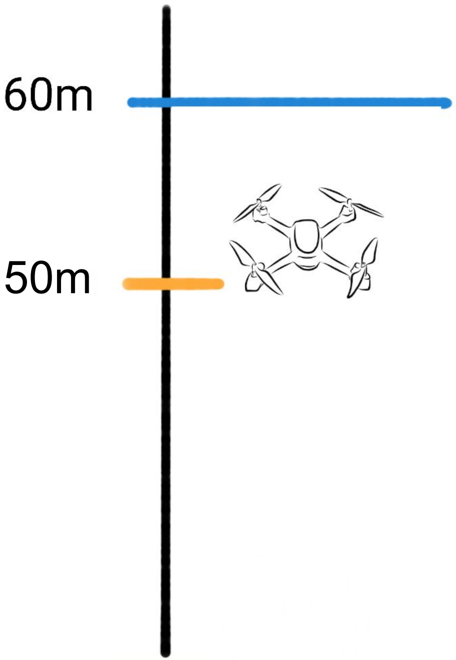


Propeller Speed = 10 * 10 = 100 rpm

Error = 10

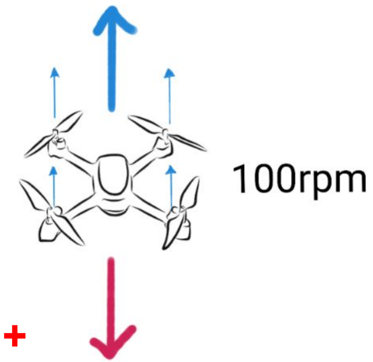




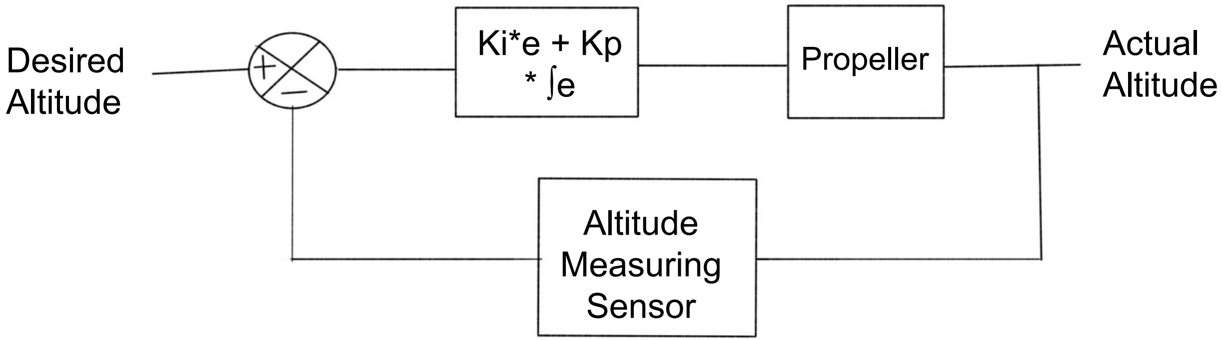


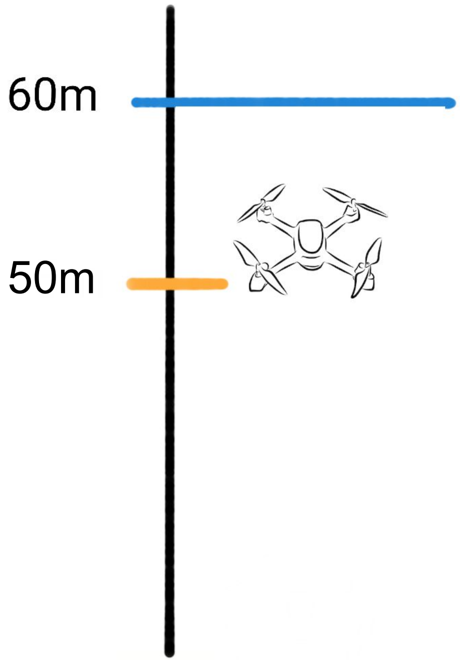
Propeller Speed = 10 * 10 = 100 rpm

Error = 10



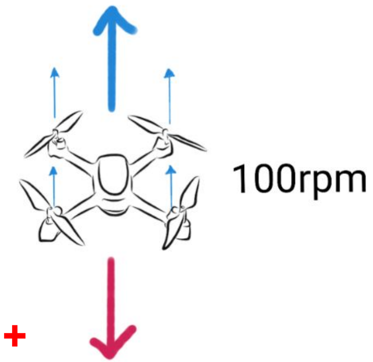
Output = Kp * error + Ki * (error1 + error2 + error3)





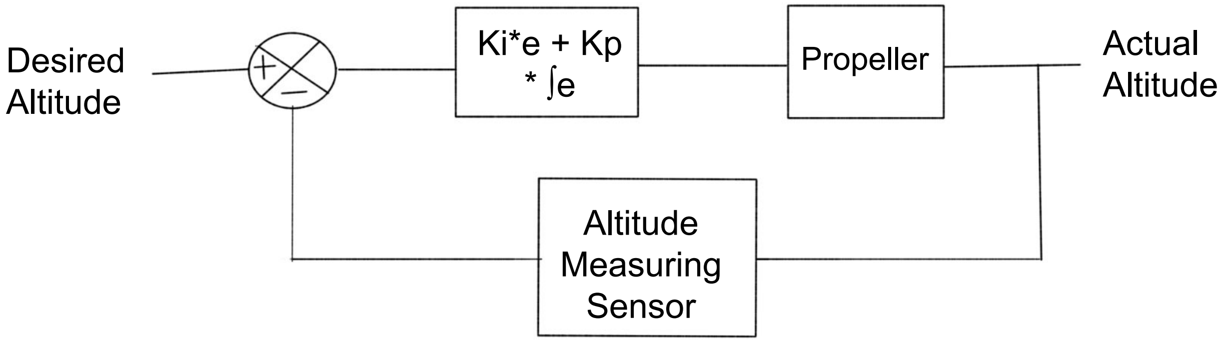
Propeller Speed = 10 * 10 = 100 rpm

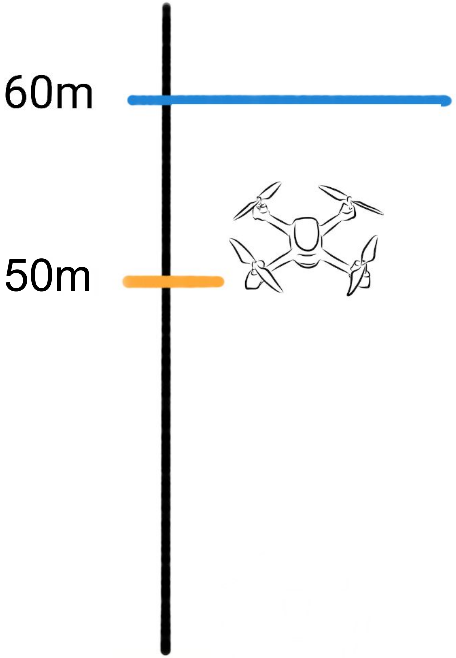
Error = 10

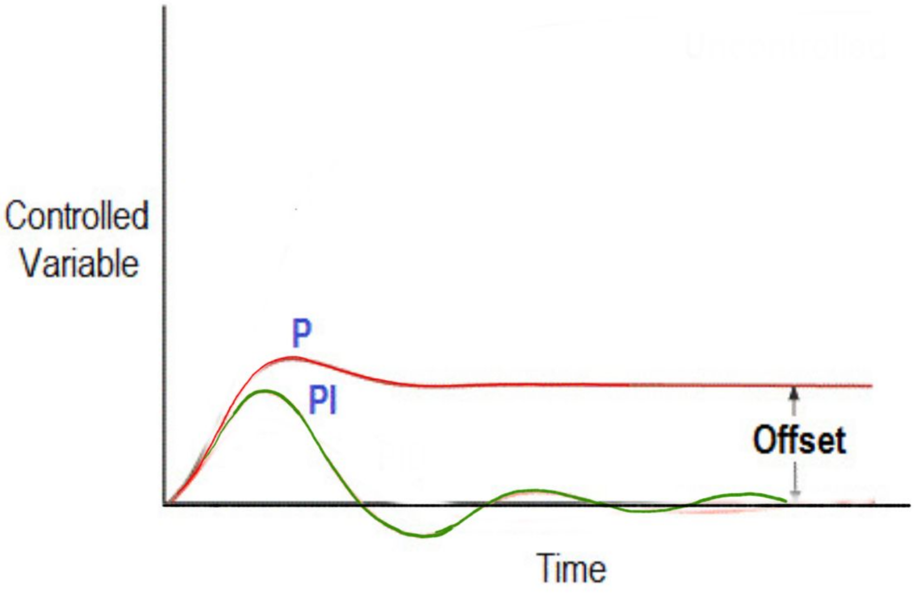


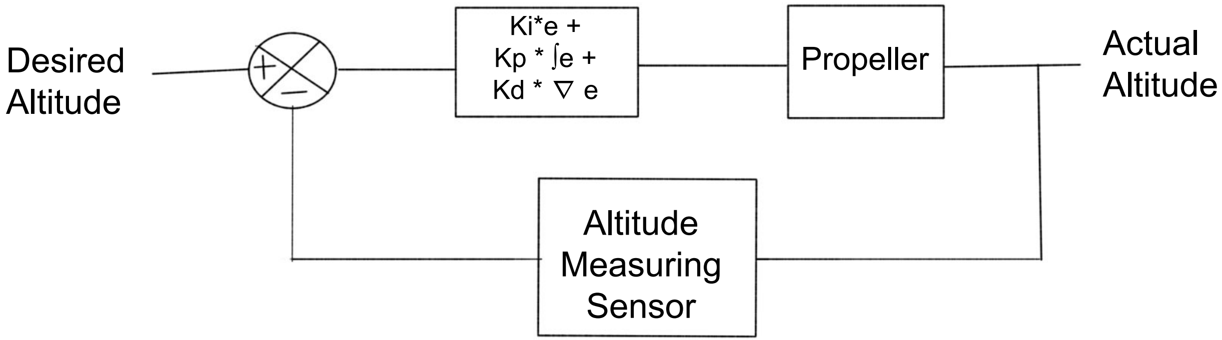
Output = Kp * error + Ki * (error1 + error2 + error3)

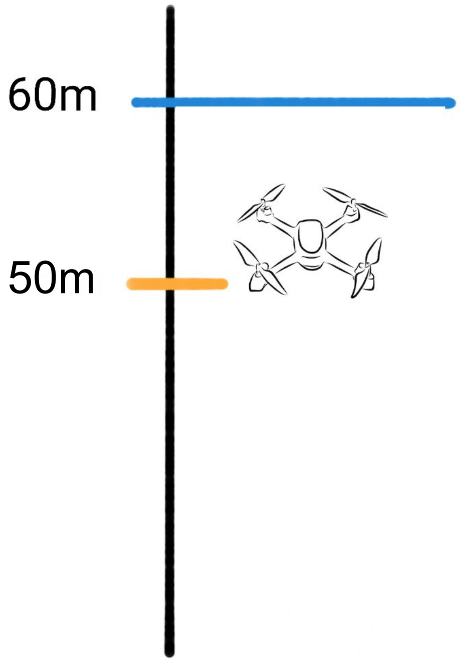
Proportional - Integral Control


Desired Altitude
+
−
Ki*e + Kp * ∫e
Propeller
Actual Altitude
Altitude Measuring Sensor


60m
50m

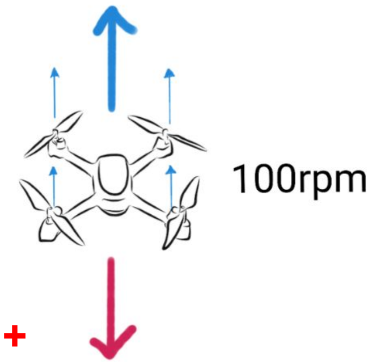

Controlled Variable
P
PI
Offset
Time



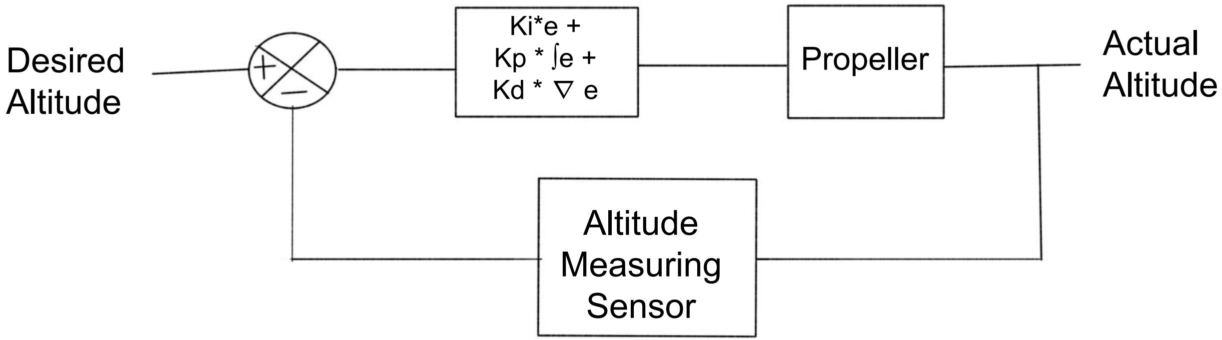


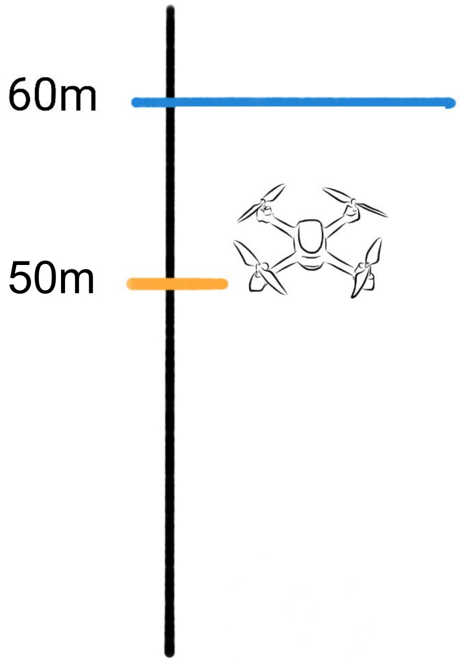
Propeller Speed = 10 * 10 = 100 rpm

Error = 10

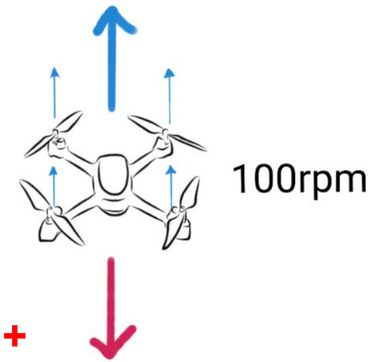


Output = Kp * error + Ki * (error1 + error2 + error3) + Kd * (error3- error 2)





Propeller Speed = 10 * 10 = 100 rpm

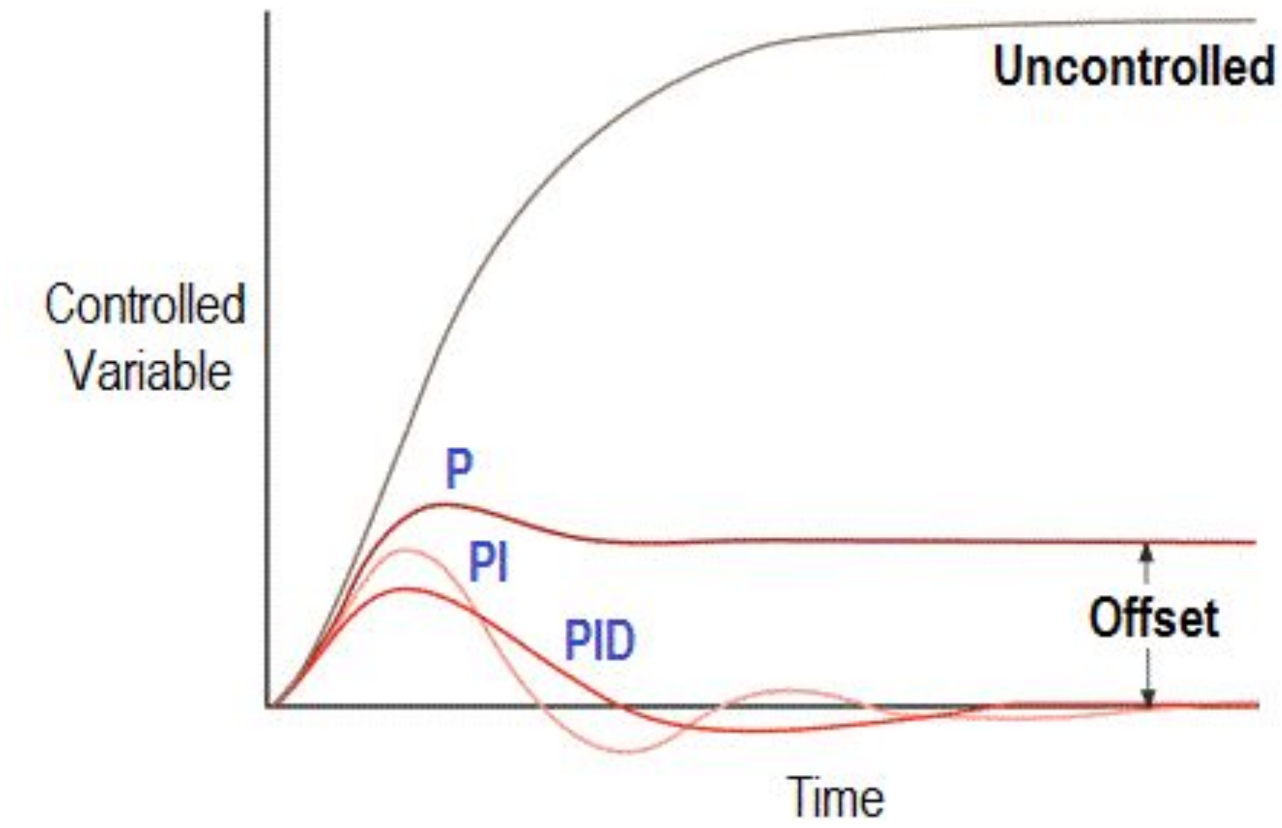


Error = 10

Output = Kp * error + Ki * (error1 + error2 + error3) + Kd * (error3- error 2)

Proportional - Derivative - Integral Control (PID) Control

Proportional - Derivative - Integral Control (PID) Control



PID Controller

$$\text{Proportion al control :} \quad u(t) = K_p e(t)$$

$$\text{Integral control :} \quad u(t) = K_i \int_0^t e(t)dt$$

$$\text{Differenti al control :} \quad u(t) = K_d \frac{d}{dt} e(t)$$

- It produces an output, which is the combination of the outputs of proportional , integral & derivative controllers

$$u(t) \propto e(\mathrm{t}) + \int e(t) + \frac{\mathrm{d}}{\mathrm{d}t} e(t)$$

$$\gg u(t) = K_P e(\mathrm{t}) + K_I \int e(t) + K_D \frac{\mathrm{d}}{\mathrm{d}t} e(t)$$

How to get the PID parameter values ?

- If we know the transfer function, analytical methods can be used (e.g., root-locus method) to meet the transient and steady-state specs.
- When the system dynamics are not precisely known, we must resort to experimental approaches.

Ziegler-Nichols Rules for Tuning PID Controller

Using only Proportional control, turn up the gain until the system oscillates without dying down, i.e., is marginally stable. Assume that K and P are the resulting gain and oscillation period, respectively.

Then Use

for P control	for PI control	for PID control
$K_p = 0.5\,K$	$K_p = 0.45\,K$	$K_p = 0.6\,K$
	$K_i = 1.2\,/\,P$	$K_i = 2.0\,/\,P$
		$K_d = P\,/\,8.0$

Ziegler-Nichols Tuning for second or higher order systems

Generating PID parameters

- **Auto-Tuning Algorithms**:
    - Relay-based Auto-Tuning
    - A relay is introduced in the control loop.
    - Observes the system's oscillatory response.
    - Calculates PID parameters based on oscillation characteristics.
- **Trial and Error:** Iteratively adjust parameters based on system response.

Advantages and Applications of PID control

Advantages:

- Simple: easy to understand.
- Effective: Accurate and stable, even in dynamic environments.
- Robust: Can be adopted to different robot systems

Application:

- Arm positioning
- Robot Navigation
- Speed Control
- Balance control

Conclusion

- PID control---most widely used control strategy today
- Over 90% of control loops employ PID control, often the derivative gain set to zero (PI control)
- The three terms are intuitive---a non -specialist can grasp the essentials of the PID controller’s action. It does not require the operator to be familiar with advanced math to use PID controllers
- Engineers prefer PID controls over untested solutions