



# Introduction to Robotics

## CSE 461

Chapter 2: Lecture 5 (Kinematics)

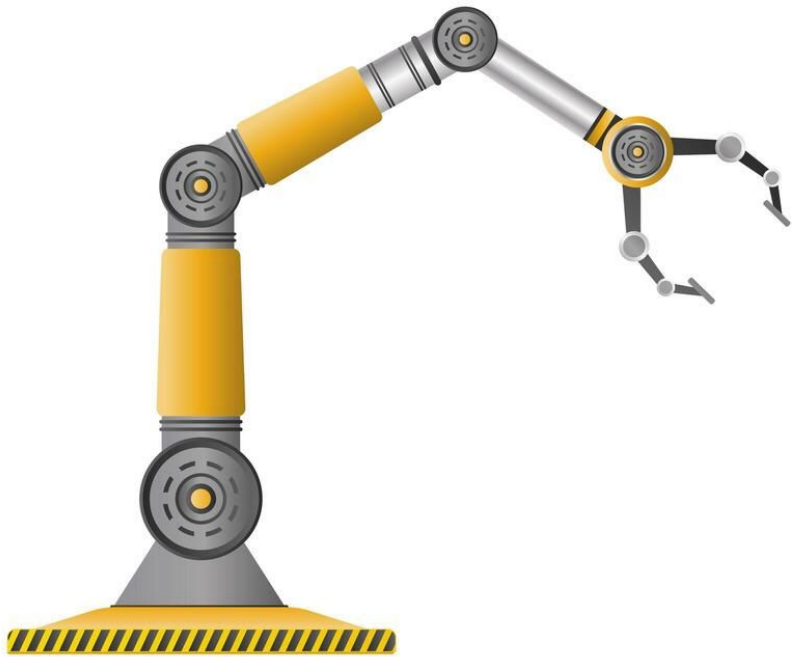
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# Today

Kinematics

Joints

D-H Parameters

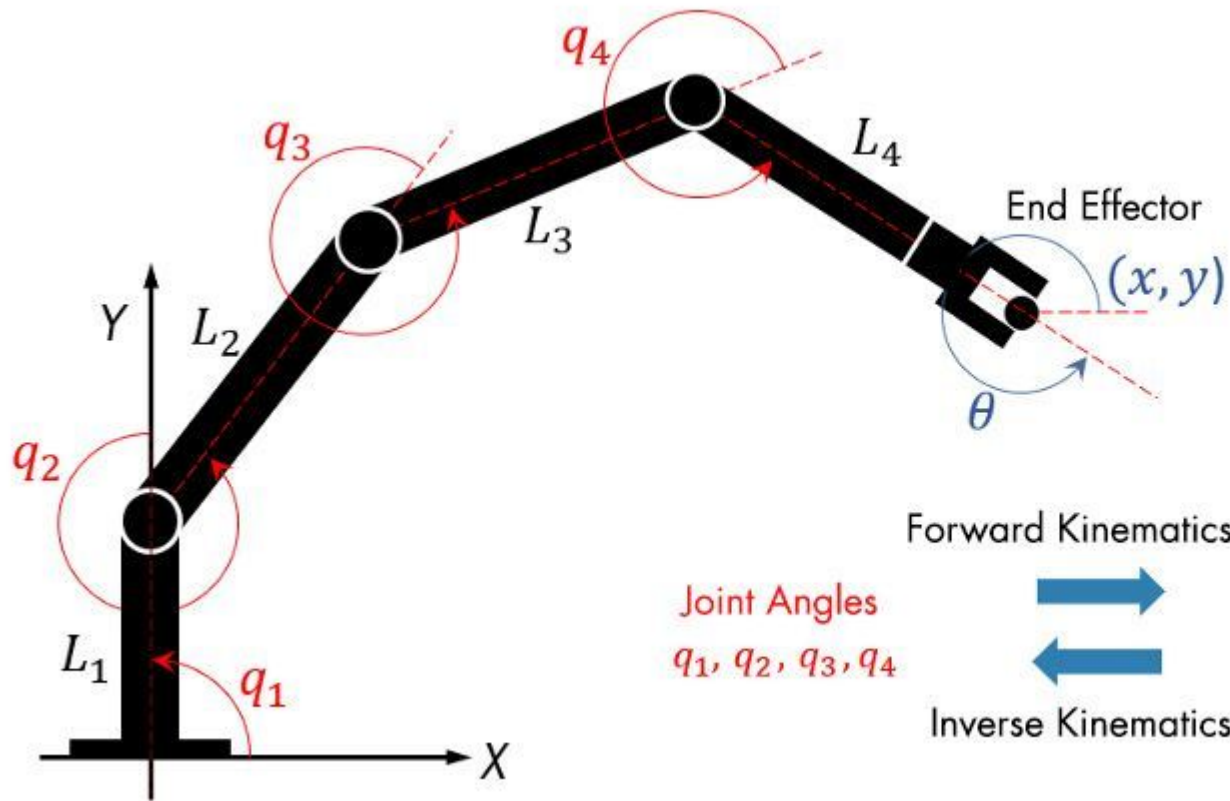


# Kinematics

Robot arm kinematics deals with the analytical study of the geometry of motion of a robot arm with respect to a fixed reference coordinate system as a function of time without regard to the force/moments that cause the motion.

There are two problems related with robot arm kinematics.

1. Direct Kinematics
2. Inverse Kinematics



Forward Kinematics (FK)



End Effector Pose

$x, y, \theta$

Joint Angles  
 $q_1, q_2, q_3, q_4$



Inverse Kinematics (IK)

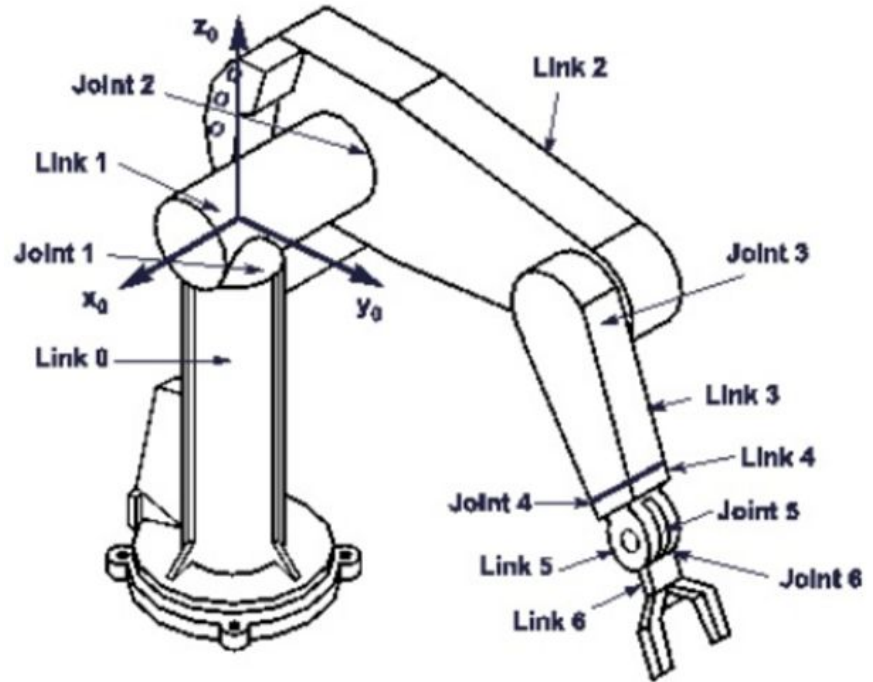
# Degrees of Freedom

Degrees of Freedom (DoF)

**General purpose robots:** Possesses 6 dof.

**Redundant robot:** Possesses more than 6 dof.

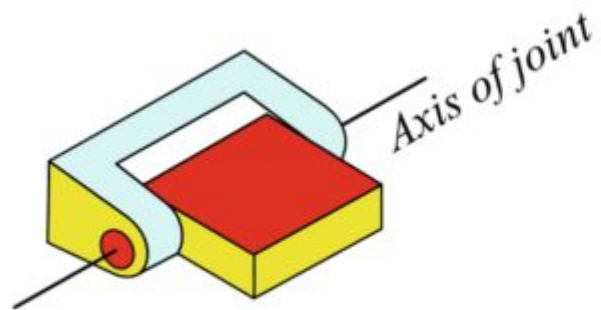
**Deficient robot :** Possesses less than 6 dof.



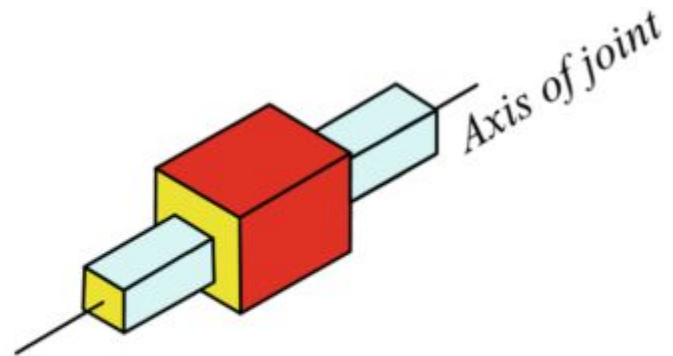
# Types of Joints

**Prismatic Joint, P:** Permits two paired elements to rotate with respect to each other about an axis that is defined by the geometry of the joint. It is sometimes called a sliding pair. It has 1-dof.

**Revolute Joint, R:** Permits two paired elements to rotate with respect to each other about an axis that is defined by the geometry of the joint. It is sometimes called a turning pair, a hinge or a pin point. It has 1-dof.



*Revolute joint*



*Prismatic joint*



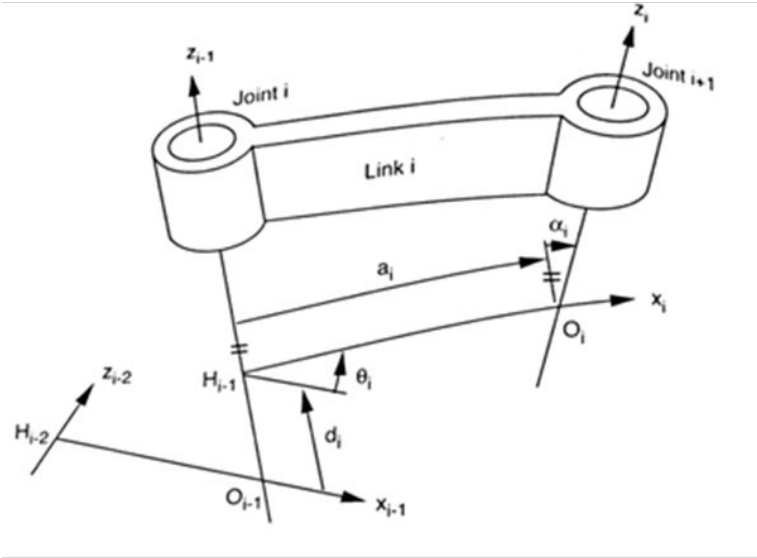
# Types of Joints

**Cylindrical Joint, C:** Permits rotation about, and independent translation along an axis, that is defined by the geometry of the joint. It has 2-dof.

**Helical Joint, H:** Allowed two paired elements to rotate about, and translation along, an axis defined by the geometry of the joint. However, the translation is related to the rotation by the pitch of a screw. It has 1-dof.

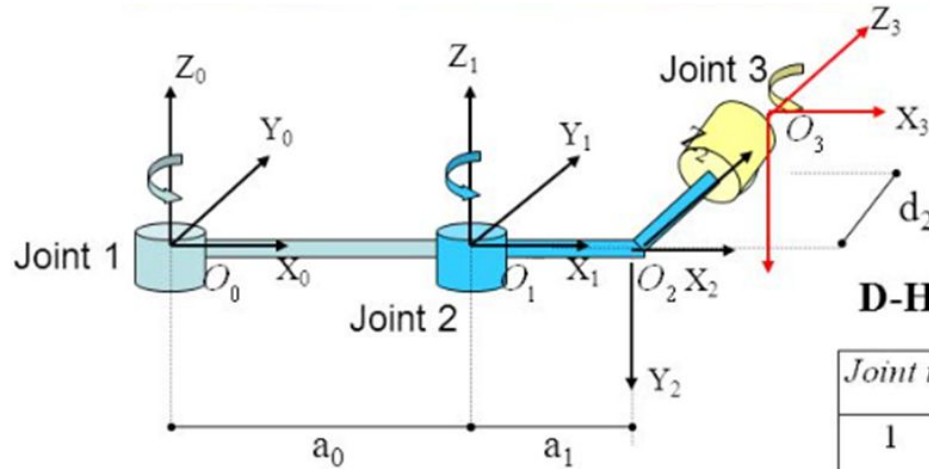
**Spherical Joint, S:** Allows one element to rotate freely with respect to the other about the center of the sphere in all possible orientations. It has 3-dof.

# Link parameters and joint variables



- **Joint angle**  $\theta_i$ : the angle of rotation from the  $X_{i-1}$  axis to the  $X_i$  axis about the  $Z_{i-1}$  axis. It is the joint variable if joint  $i$  is rotary.
- **Joint distance**  $d_i$ : the distance from the origin of the  $(i-1)$  coordinate system to the intersection of the  $Z_{i-1}$  axis and the  $X_i$  axis along the  $Z_{i-1}$  axis. It is the joint variable if joint  $i$  is prismatic.
- **Link length**  $a_i$ : the distance from the intersection of the  $Z_{i-1}$  axis and the  $X_i$  axis to the origin of the  $i$ th coordinate system along the  $X_i$  axis.
- **Link twist angle**  $\alpha_i$ : the angle of rotation from the  $Z_{i-1}$  axis to the  $Z_i$  axis about the  $X_i$  axis.

# Link Parameters and Joint Variables example



**D-H Link Parameter Table**

Joint $i$	$\alpha_i$	$a_i$	$d_i$	$\theta_i$
1	0	$a_0$	0	$\theta_1$
2	-90	$a_1$	0	$\theta_2$
3	0	0	$d_2$	$\theta_3$

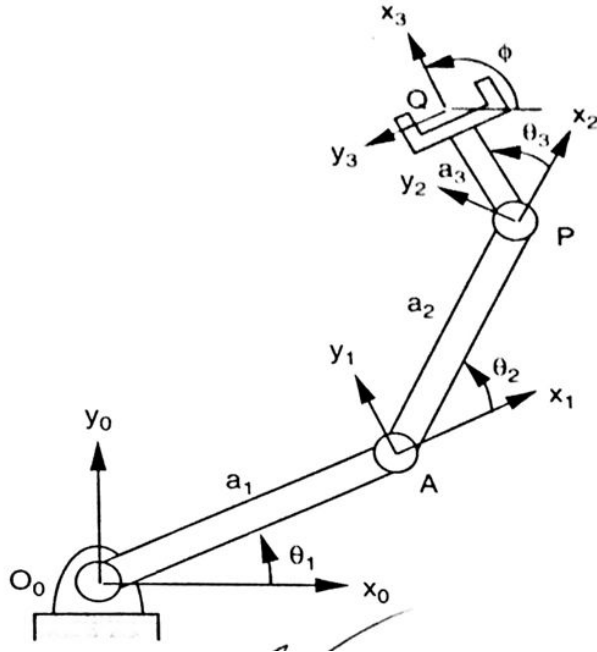
$\alpha_i$  : rotation angle from  $Z_{i-1}$  to  $Z_i$  about  $X_i$

$a_i$  : distance from intersection of  $Z_{i-1}$  &  $X_i$  to origin of  $i$  coordinate along  $X_i$

$d_i$  : distance from origin of  $(i-1)$  coordinate to intersection of  $Z_{i-1}$  &  $X_i$  along  $Z_{i-1}$

$\theta_i$  : rotation angle from  $X_{i-1}$  to  $X_i$  about  $Z_{i-1}$

# Parameters of a simply 3 DOF Manipulator



**TABLE 2.1. D-H Parameters of a 3-DOF Manipulator**

Joint $i$	$\alpha_i$	$a_i$	$d_i$	$\theta_i$
1	0	$a_1$	0	$\theta_1$
2	0	$a_2$	0	$\theta_2$
3	0	$a_3$	0	$\theta_3$

$\alpha_i$  : rotation angle from  $Z_{i-1}$  to  $Z_i$  about  $X_i$

$a_i$  : distance from intersection of  $Z_{i-1}$  &  $X_i$  to origin of  $i$  coordinate along  $X_i$

$d_i$  : distance from origin of  $(i-1)$  coordinate to intersection of  $Z_{i-1}$  &  $X_i$  along  $Z_{i-1}$

$\theta_i$  : rotation angle from  $X_{i-1}$  to  $X_i$  about  $Z_{i-1}$

2	-90	$a_1$	0	$\theta_2$
3	0	0	$d_2$	$\theta_3$

# Parameters of SCARA Arm

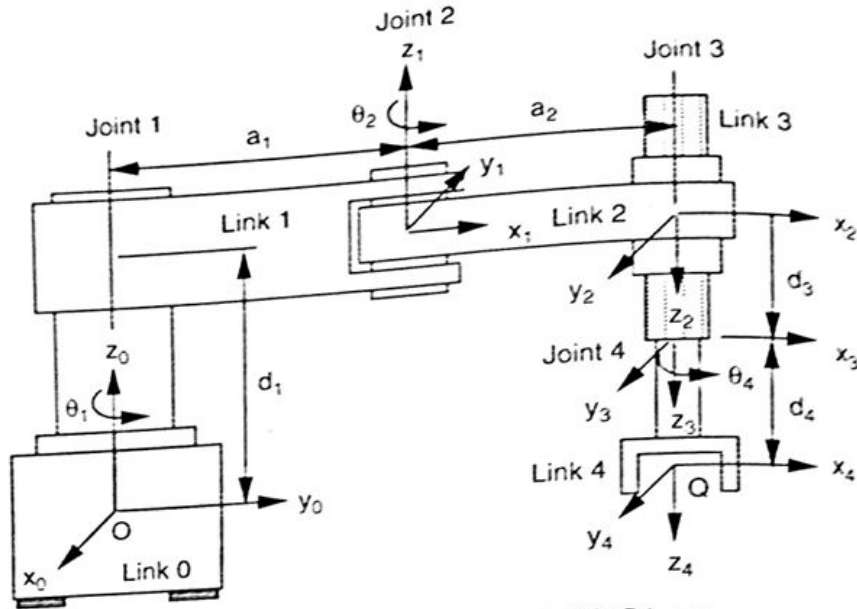


FIGURE 2.4. Schematic diagram of a SCARA arm.

TABLE 2.2. D-H Parameters of the SCARA Arm

Joint $i$	$\alpha_i$	$a_i$	$d_i$	$\theta_i$
1	0	$a_1$	$d_1$	$\theta_1$
2	$\pi$	$a_2$	0	$\theta_2$
3	0	0	$d_3$	0
4	0	0	$d_4$	$\theta_4$

$\alpha_i$  : rotation angle from  $Z_{i-1}$  to  $Z_i$  about  $X_i$

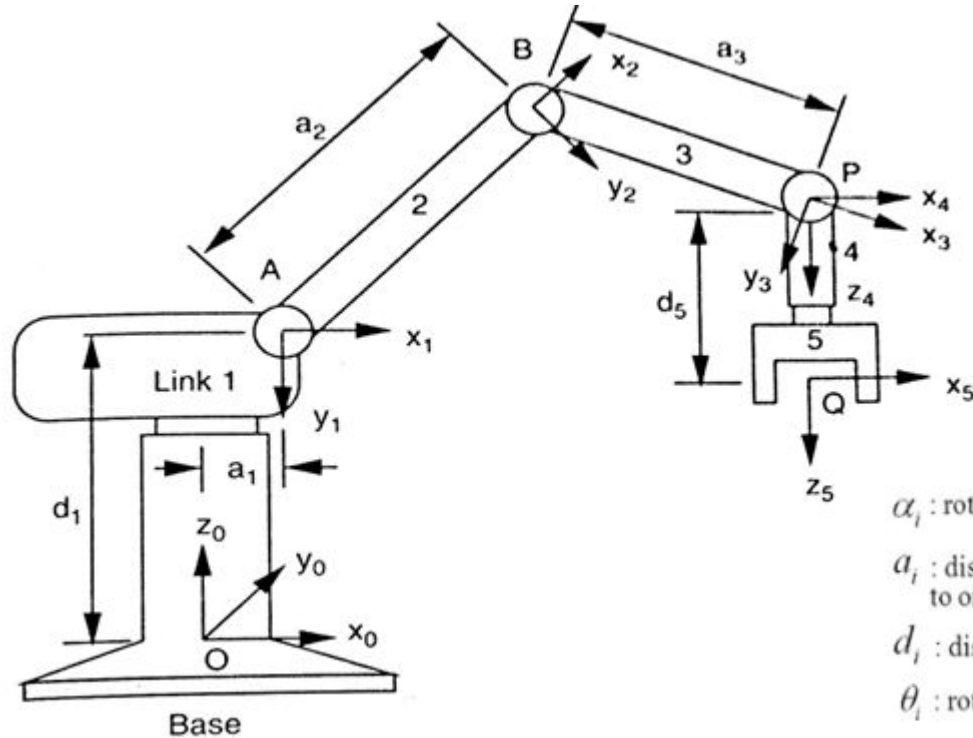
$a_i$  : distance from intersection of  $Z_{i-1}$  &  $X_i$  to origin of  $i$  coordinate along  $X_i$

$d_i$  : distance from origin of  $(i-1)$  coordinate to intersection of  $Z_{i-1}$  &  $X_i$  along  $Z_{i-1}$

$\theta_i$  : rotation angle from  $X_{i-1}$  to  $X_i$  about  $Z_{i-1}$

2	-90	$a_1$	0	$\theta_2$
3	0	0	$d_2$	$\theta_3$

# SCORBOT Robot (5 DOF Manipulator)



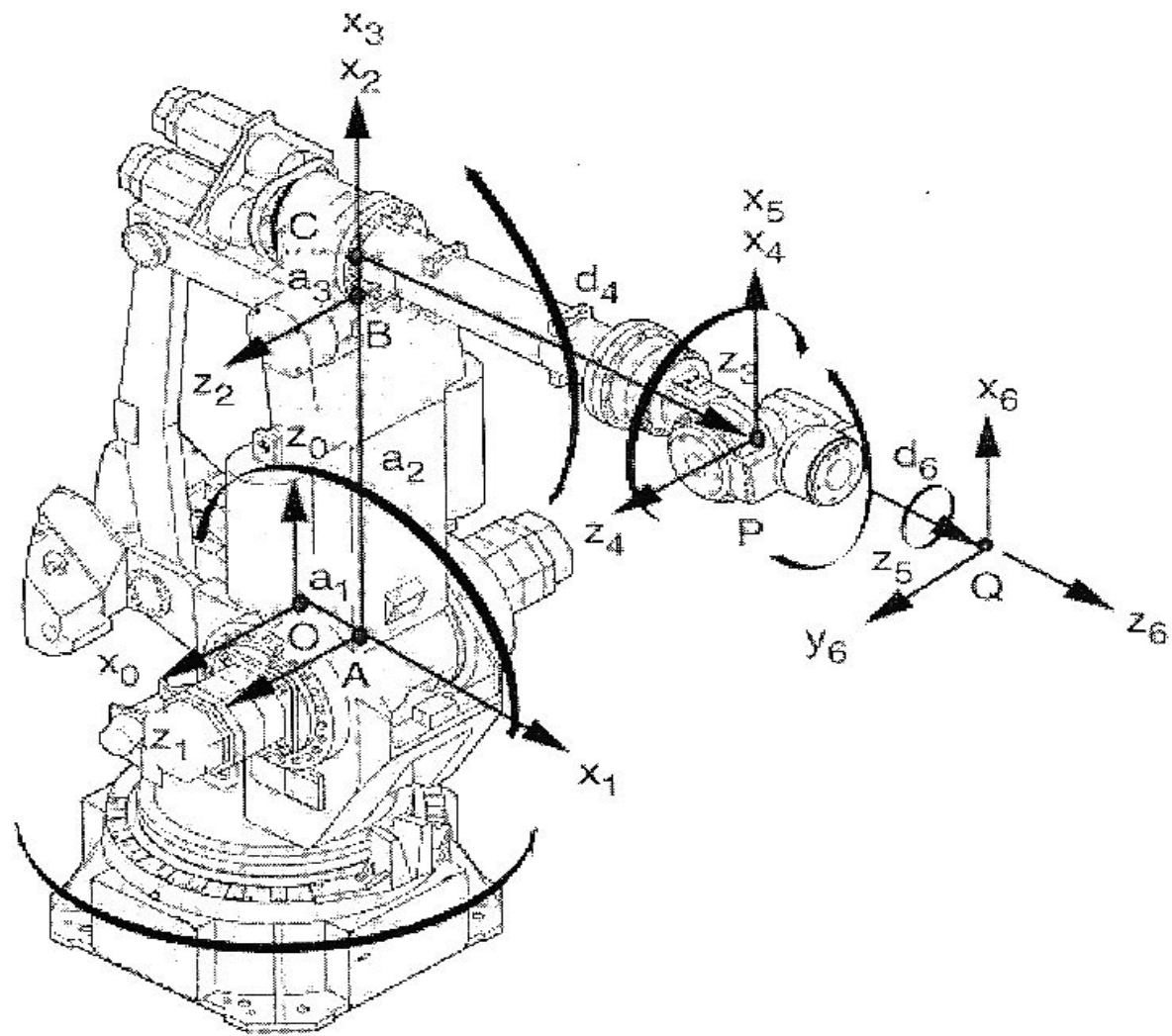
$\alpha_i$  : rotation angle from  $Z_{i-1}$  to  $Z_i$  about  $X_i$

$a_i$  : distance from intersection of  $Z_{i-1}$  &  $X_i$  to origin of  $i$  coordinate along  $X_i$

$d_i$  : distance from origin of  $(i-1)$  coordinate to intersection of  $Z_{i-1}$  &  $X_i$  along  $Z_{i-1}$

$\theta_i$  : rotation angle from  $X_{i-1}$  to  $X_i$  about  $Z_{i-1}$

2	-90	$a_1$	0	$\theta_2$
3	0	0	$d_2$	$\theta_3$



Thank You