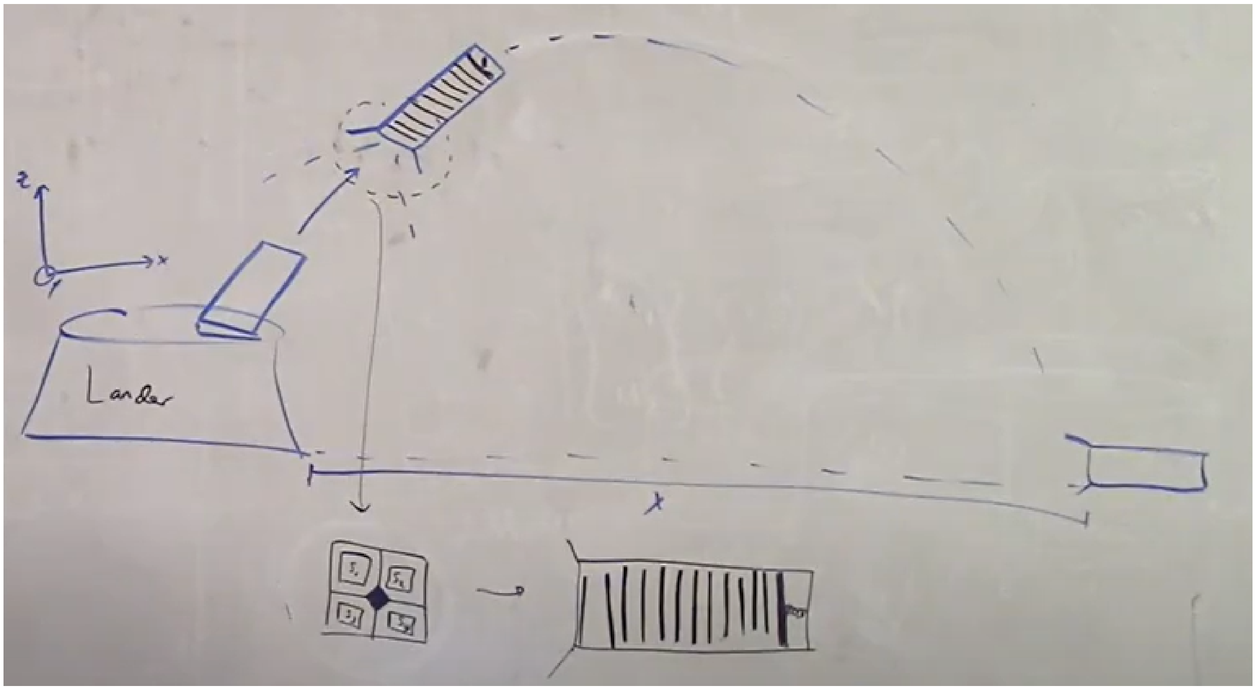
Colorado Space Grant Consortium - GLEE | Audrey Viland



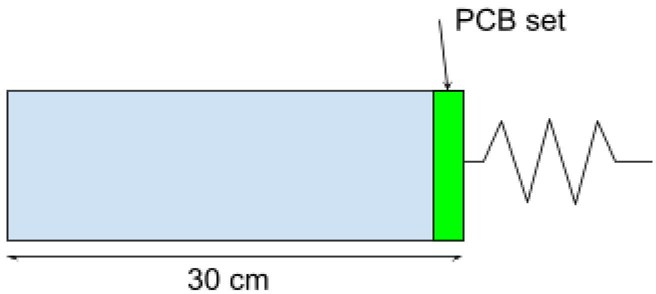

Spacecraft holding 500 satellites (PCBs) (in groups of 4 PCBs): 125 layers
Satellites collect information and communicate with each other via RF, solar panels to survive for 2 lunar weeks
Goal: deploy 500 satellites on the lunar surface within a specific dispersion parameter
3U Satellite: 30 cm long and 10 cm^2
While flying through the air: it's losing mass

Mass of container = 0.5 kg

Mass of all satellites = 2.5 kg

Model where the carrier will land and where the satellites will land
Dispersion parameter: 250 m^2
45 degree bumper launches small satellites left and right
Based on carrier's instantaneous orientation, where will all of the PCBs land?
Spring in carrier forces satellites out in opposite direction of motion

**Problem 1**: We want to test different spring constants in the carrier spring. So, we place the carrier horizontally on the ground where it is stationary. We load one satellite (set of PCBs) in the carrier and compress the spring fully to 30 cm. What is the spring constant if the relative satellite exit velocity from the carrier is 2 m/s and friction forces are negligible?




**Assume**

1. Carrier is stationary and horizontal
2. Friction is negligible

**Known**

| | |
|---|---|
| Initial velocity of PCB | $v_1 = 0\text{m/s}$ |
| Initial compression of spring | $s_1 = -0.30\text{m}$ |
| Final length of spring | $s_2 = 0\text{m}$ |
| Mass of one satellite (2.5 kg/500)*4 | $m = 0.02\text{kg}$ |
| Final velocity of satellite | $v_2 = 2\text{m/s}$ |

**Unknown**

| | |
|---|---|
| Spring constant | $k$ |

**Equations**

| | |
|---|---|
| Principle of Work and Energy | $T_1 + \sum U_{1-2} = T_2$ |
| Kinetic energy of a particle | $T = \frac{1}{2}mv^2$ |
| Spring work | $U_s = -\left(\frac{1}{2}ks_2^{\ 2} - \frac{1}{2}ks_1^{\ 2}\right)$ |

**Solution**

$$T_1 + \sum U_{1-2} = T_2$$

$$\frac{1}{2}m{v_1}^2 +- (\frac{1}{2}k{s_2}^2 - \frac{1}{2}k{s_1}^2) = \frac{1}{2}m{v_2}^2 \qquad v_1 = 0\text{m/s} ; s_2 = 0\text{m}$$

$$\frac{1}{2}k{s_1}^2 = \frac{1}{2}m{v_2}^2$$

$$k{s_1}^2 = m{v_2}^2$$

$$k = \frac{m{v_2}^2}{{s_1}^2} = \frac{(0.02kg)(2m/s)^2}{(-0.30m)^2} = 0.89\text{ N/m}$$

*EES Verify*

```
v_1 = 0 [m/s]
v_2 = 2 [m/s]
s_1 = -0.3 [m]
s_2 = 0 [m]
m = 0.02 [kg]

T_1 = (1/2)*m*v_1^2
T_2 = (1/2)*m*v_2^2
U_s = -((1/2)*k*s_2^2 - (1/2)*k*s_1^2)

T_1 + U_s = T_2
```

SOLUTION

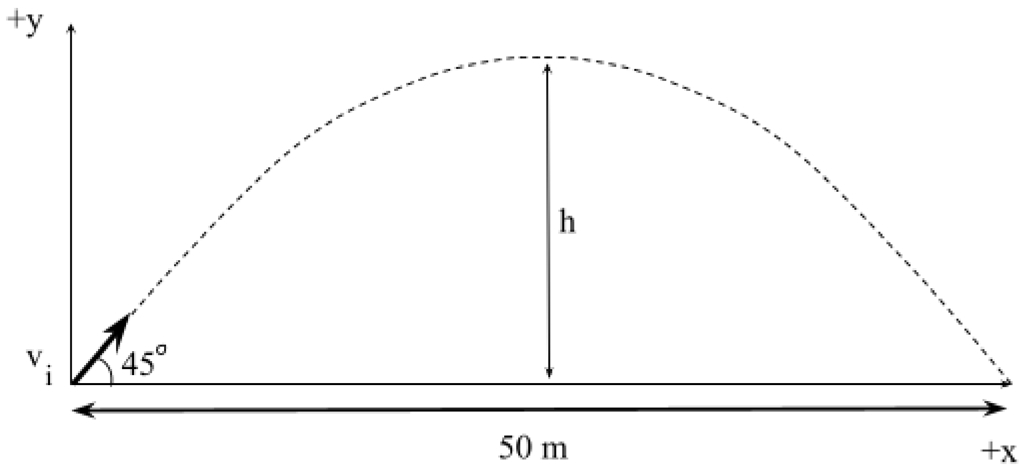
**Unit Settings: SI C kPa kJ mass deg**

| | | |
|---|---|---|
| k = 0.8889 [N/m] | m = 0.02 [kg] | $s_1$ = -0.3 [m] |
| $s_2$ = 0 [m] | $T_1$ = 0 [N*m] | $T_2$ = 0.04 [N*m] |
| $U_s$ = 0.04 [N*m] | $v_1$ = 0 [m/s] | $v_2$ = 2 [m/s] |

No unit problems were detected.

**Problem 2**

Now we want to start modeling to achieve the dispersion parameter of 250 $m^2$. In this early model, we assume that the mass of the container is constant throughout the duration of its flight. The lunar lander will launch the container at a 45° angle to the lunar surface.
What must be the initial velocity of the container as it leaves the craft in order to travel 50 m? What is the maximum height the container will reach?




**Assume**

1. Carrier acts as a point mass (no rotation during flight)
2. No air resistance (moon has no atmosphere)
3. Launch point starts and ends on the same horizontal plane (lunar surface) i.e. launch pad is not elevated or depressed relative to surroundings

**Known**

| | |
|---|---|
| Gravitational acceleration on the moon (vertical) | $a_y = -1.62\text{m/s}^2$ |
| Horizontal acceleration | $a_x = 0\text{m/s}^2$ |
| Horizontal distance traveled | $\Delta x = 50\text{m}$ |
| Launch angle | $\theta = 45°$ |
| y velocity at top of arc | $v_{y,h} = 0\text{m/s}$ |

**Unknown**

| | |
|---|---|
| Flight time | $t$ |
| Initial velocity | $v_0$ |
| Maximum height | $h$ |

**Equations**

| | |
|---|---|
| Constant acceleration kinematic equations | |
| Velocity, time, acceleration | $v = v_0 + a_c t$ |
| Position, time, velocity, acceleration | $s = s_0 + v_0 t + \frac{1}{2} a_c t^2$ |
| Velocity component in $x$ and $y$ | $v_x = v\cos\theta$<br>$v_y = v\sin\theta$ |
| Conservation of Energy Theorem | $T_1 + V_1 = T_2 + V_2$ |
| Kinetic energy of a particle | $T = \frac{1}{2} m v^2$ |
| Gravitational potential energy | $V_g = \pm Wy$ |

**Solution**

| | |
|---|---|
| Find total flight time based on vertical flight time | |
| $v_{0,y} = v_0 \sin\theta$ | |
| $v_y = v_{0,y} + a_y t = 0$ | Vertical velocity zero at top of arc<br>($t = \frac{1}{2} t_{total}$ since flight path is symmetrical) |
| $t = \frac{v_{0,y}}{-a_y} = \frac{1}{2} t_{total}$ | (1) |
| Distance traveled in horizontal direction based on horizontal velocity and flight time | |
| $s = s_0 + v_{0,x} t + \frac{1}{2} a_x t^2$ | $a_x = 0, s_0 = 0$ |
| $\Delta x = v_{0,x} t_{total}$ | (2) |
| $\Delta x = 2 v_{0,x} \frac{v_{0,y}}{-a_y}$ | Substitute (1) into (2) |
| $\Delta x = 2 v_0 \cos\theta \times v_0 \sin\theta / -a_y$ | Substitute x and y components of initial velocity |

| | |
|---|---|
| $\frac{\Delta x(-a_y)}{2\cos\theta\sin\theta} = v_0^{\ 2}$ | Solve for $v_0$ |
| $\sqrt{\frac{\Delta x(-a_y)}{2\cos\theta\sin\theta}} = v_0 = \sqrt{\frac{(50m)(1.62m/s^2)}{2\cos45\sin45}} =$9 m/s= $v_0$ | |
| Conservation of energy to find max height $h$ | |
| $T_1 + V_1 = T_2 + V_2$ | Using ground as reference point: the carrier starts off with all kinetic energy, then at maximum height all energy converted to potential |
| $T_{0,y} = V_h$ | Conservation of energy along $y$ axis |
| $\frac{1}{2}mv_{0,y}^{\ 2} = Wy =- ma_y h$ | |
| $\frac{1}{2}v_{0,y}^{\ 2} =- a_y h$ | |
| $h = \frac{v_{0,y}^{\ 2}}{-2a_y} = \frac{(v_0\sin\theta)^2}{-2a_y} = \frac{(9\ m/s\sin45)^2}{-2(-1.62m/s^2)} =$$12.5$m= $h$ | |

EES verify

```
a_y = -1.62 [m/s^2]
Delta_x = 50 [m]
theta = 45 [deg]

v_0_y = v_0 * sin(theta)
v_0_x = v_0 * cos(theta)

v_0_y + a_y*t = 0

Delta_x = 2*t*v_0_x

(1/2)*v_0_y^2 = -a_y*h
```

SOLUTION

**Unit Settings: SI C kPa kJ mass deg**

| | | |
|---|---|---|
| $a_y$ = -1.62 [m/s$^2$] | $\delta_x$ = 50 [m] | h = 12.5 [m] |
| t = 3.928 [s] | $\theta$ = 45 [deg] | $v_0$ = 9 [m/s] |
| $v_{0,x}$ = 6.364 [m/s] | $v_{0,y}$ = 6.364 [m/s] | |

No unit problems were detected.

**Problem 3**

Now that we have an estimate for our launch velocity (problem 2) and spring constant (problem 1), let's combine the two to determine how the carrier might behave while launching PCBs mid-flight. While these steps greatly simplify the problem, they help develop our understanding of the problem so that we can work up to more complex tools for modeling the intricacies of the craft's motion.

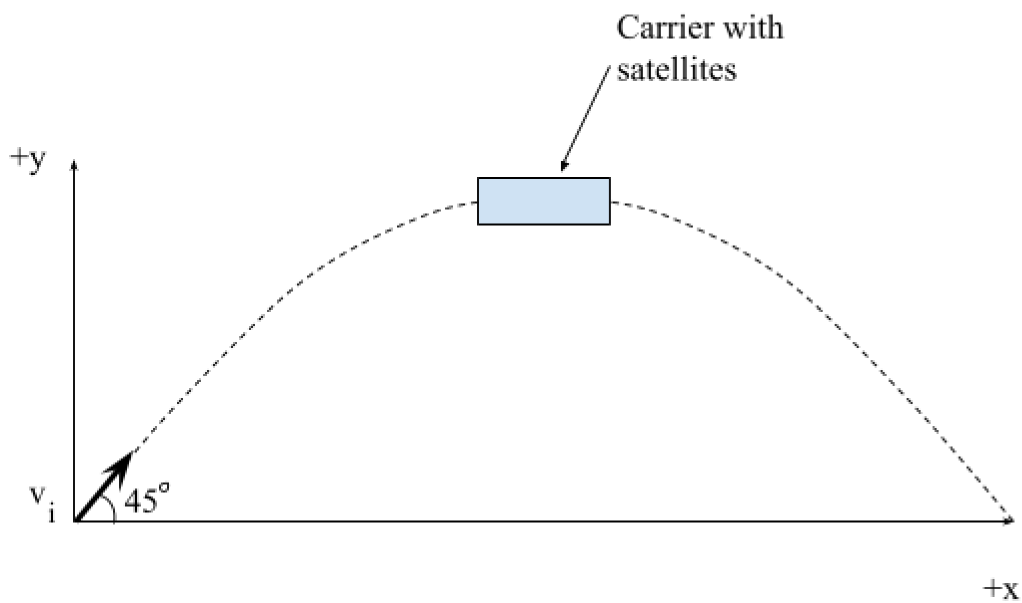
Let's say the carrier is at the top of its arc, mid-flight ($v_y = 0$). It is oriented parallel to the lunar surface. It was launched with the initial velocity and angle from Problem 2.

Like Problem 1, it is loaded with 1 satellite (set of 4 PCBs) having a mass of 0.02 kg. The spring is fully compressed (30 cm) and has the spring constant we found in problem 1. Once the carrier reaches the top of its arc, the spring is released, launching the PCBs in the opposite direction.

After deploying the PCBs:

What is the new velocity of the carrier, relative to the lunar surface?

What is the velocity of the PCB relative to the lunar surface?




**Assume**

1. Container is oriented horizontally (relative to the lunar surface) throughout the duration of deployment and vertical velocity is 0.

**Known**

| | |
|---|---|
| Initial velocity of carrier | $v_0 = 9$m/s |
| Velocity in y at top of arc | $v_y = 0$m/s |
| Using the spring constant in the carrier from earlier: will produce the same final velocity | $v_{B/A} =- 2$m/s |

| | |
|---|---|
| relative to the carrier. Include direction where → is the positive x direction. | |
| Launch angle | $\theta = 45°$ |
| Mass of PCB satellite | $m_{PCB} = 0.02\text{kg}$ |
| Mass of carrier | $m_{carrier} = 0.5\text{kg}$ |

**Equations**

| | |
|---|---|
| Conservation of Linear Momentum | $\sum(syst.\ mv)_1 = \sum(syst.\ mv)_2$ |
| Relative Motion | $v_B = v_A + v_{B/A}$ |
| Initial horizontal velocity of the carrier holding the PCB (no horizontal acceleration) | $v_x = v_0 \cos\theta$ |

**Solution**

*Note: all velocities are horizontal (no vertical component from assumption 1) unless otherwise noted

→ +x

← -x

| | |
|---|---|
| $(m_{carrier} + m_{PCB})v_x = m_{carrier}v_{carrier} + m_{PCB}v_{PCB}$ | Conservation of linear momentum |
| $v_{PCB} = v_{carrier} + v_{PCB/carrier}$ | Relative motion |
| where $v_{PCB/carrier} = -\ 2\text{m/s}$ | from Problem 1 |
| $v_x = v_0 \cos\theta$ (where $v_0$at 45°) | |
| $(m_{carrier} + m_{PCB})v_x = m_{carrier}v_{carrier} + m_{PCB}(v_{carrier} + v_{PCB/carrier})$ | Substitute into conservation of linear momentum |
| $(m_{carrier} + m_{PCB})v_x - m_{PCB}(v_{PCB/carrier}) = v_{carrier}(m_{carrier} + m_{PCB}$ | Solve for $v_{carrier}$ |
| $(v_x(m_{carrier} + m_{PCB}) - m_{PCB}(v_{PCB/carrier}))/(m_{carrier} + m_{PCB}) = v_{ca}$ | |

$$v_{carrier} = (v_0 \cos\theta(m_{carrier} + m_{PCB}) - m_{PCB}(v_{PCB/carrier}))/(m_{carrier} + m_{PCB})$$

$$v_{carrier} = ((9m/s)\cos 45(0.05kg + 0.02kg) - 0.02kg(-2m/s))/0.05kg + 0.02kg)$$
6.44 m/s

$$v_{PCB} = v_{carrier} + v_{PCB/carrier} = 6.44\,m/s + (-2\,m/s) =$$ 4.44 m/s

EES verify

```
v_0 = 9 [m/s]
v_B_A = -2 [m/s]
theta = 45 [deg]
m_B = 0.02 [kg]
m_A = 0.5 [kg]

(m_A+m_B)*v_x = m_A*v_A + m_B*v_B
v_x = v_0*cos(theta)
v_B = v_A+v_B_A
```

SOLUTION

**Unit Settings: SI C kPa kJ mass deg**

| | | |
|---|---|---|
| $m_A$ = 0.5 [kg] | $m_B$ = 0.02 [kg] | $\theta$ = 45 [deg] |
| $v_0$ = 9 [m/s] | $v_A$ = 6.441 [m/s] | $v_B$ = 4.441 [m/s] |
| $v_{B,A}$ = -2 [m/s] | $v_x$ = 6.364 [m/s] | |

No unit problems were detected.

**Problem 4**

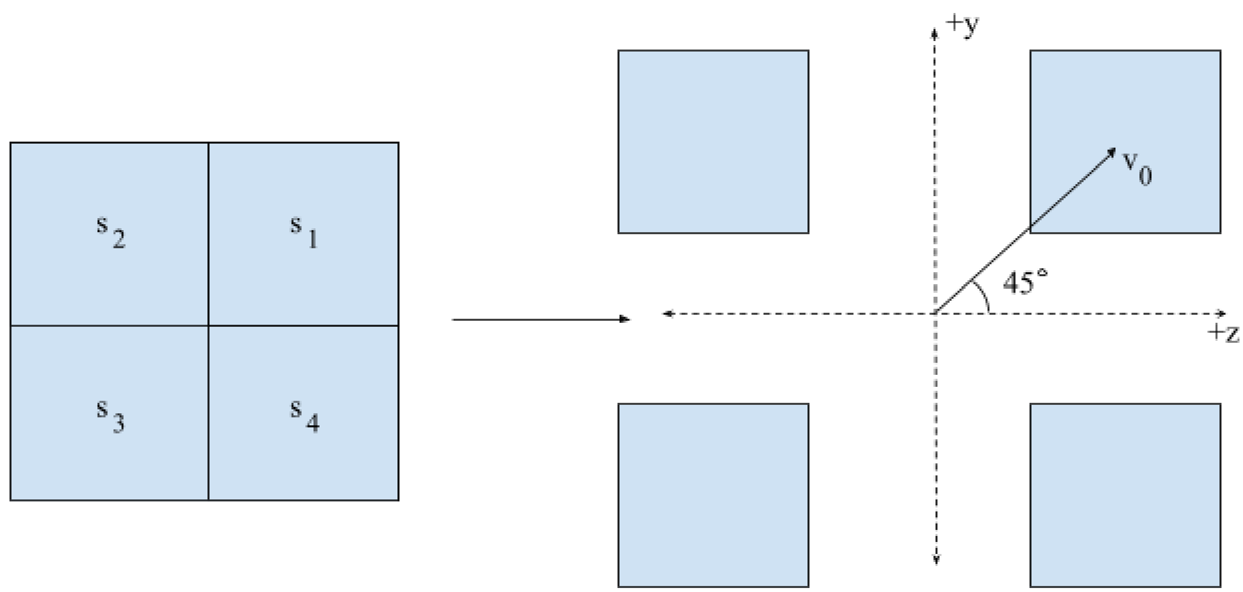
Now let’s look at the dispersion of a single PCB unit after being ejected from the carrier. Let’s use the same scenario as Problem 3 where the PCBs were ejected horizontally from the carrier. This means that they are oriented perpendicular to the lunar surface.

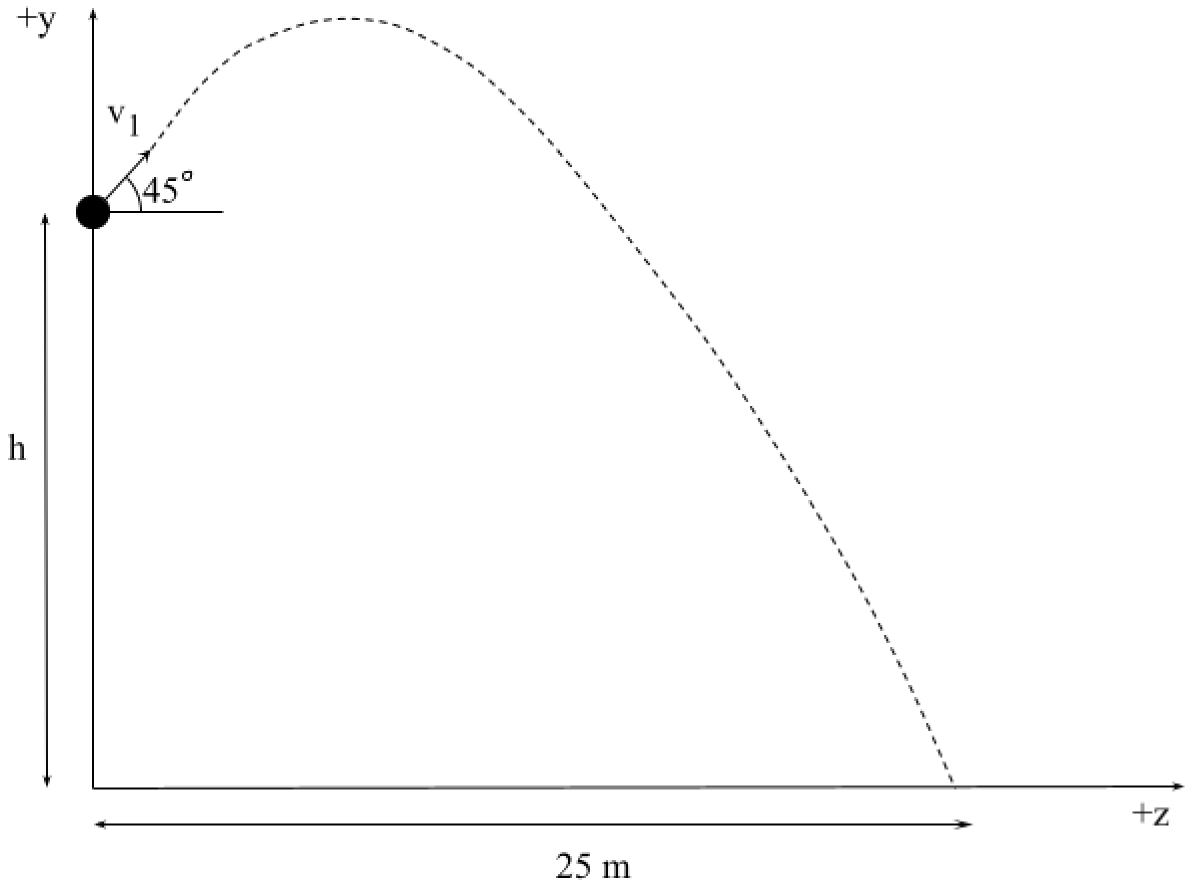
We also know from the problem setup that they will be launched at a 45 degree angle. Let’s assume that each PCB has the same mass so they will move symmetrically with the same initial velocity (conservation of linear momentum). Let’s also assume that none of the PCBs are rotating.

Finally, since we’re using the same scenario as Problem 3, the PCBs are launched at the top of the carrier’s arc. This means they start out with zero velocity in the y direction.

What must be the initial separation velocity of the PCBs in order to reach the 250 $m^2$ dispersion parameter? i.e. what does $v_0$ have to be in order for $s_1$ to travel 25 m in the +z direction before landing on the lunar surface?

*Note: for right now, we’re focusing on how to reach 25 m in +z. We aren’t concerned with the x component of the final landing position.






**Assume**

1. No air resistance (moon has no atmosphere)
2. Initial velocity in y-direction is 0 (after being ejected from carrier but before separation)
3. PCB is perpendicular to lunar surface
4. Treat each PCB as a point mass (translation only, no rotation)

**Known**

| | |
|---|---|
| Displacement in z | $\Delta z = 25\text{m}$ |
| Starting velocity direction (angle from horizontal) | $\theta = 45°$ |
| Initial velocity in y (after ejection, before PCB separation) | $v_{y,0} = 0$ |
| Initial velocity in z (after ejection, before PCB separation) | $v_{z,0} = 0$ |
| Ejection height (max height from problem 2) | $h = 12.5\text{m}$ |

| | |
|---|---|
| Gravitational acceleration on the moon (vertical) | $a_y = -1.62 \mathrm{m/s^2}$ |

**Unknown**

| | |
|---|---|
| Starting velocity magnitude | $v_1$ |
| In-air time after ejection | $t$ |

**Equations**

| | |
|---|---|
| y velocity component | $v_y = v \sin\theta$ |
| z velocity component | $v_z = v \cos\theta$ |
| Particle rectilinear motion, constant $a = a_c$ | $s = s_0 + v_0 t + \frac{1}{2} a_c t^2$ |

**Solution**

*Note: since $v_{y,0} = v_{z,0} = 0$, we do not have to worry about relative velocity in *y* or *z*

| | |
|---|---|
| $z - z_0 = v_z t + \frac{1}{2} a_z t^2 = v_1 \cos\theta t + \frac{1}{2} a_z t^2$<br>$\Delta z = v_1 t \cos\theta$ | Motion in *z*-direction |
| $t = \frac{\Delta z}{v_1 \cos\theta}$ | Solve for *t* |
| $y - y_0 = v_y t + \frac{1}{2} a_y t^2 = 0 - h$<br>$-h = v_y t + \frac{1}{2} a_y t^2$<br>$-h = v_1 t \sin\theta + \frac{1}{2} a_y t^2$ | Motion in *y*-direction |
| $-h = v_1 \frac{\Delta z}{v_1 \cos\theta} \sin\theta + \frac{1}{2} a_y (\frac{\Delta z}{v_1 \cos\theta})^2$ | Substitute $t$from motion in *z*-direction |
| $-h = \frac{\Delta z \sin\theta}{\cos\theta} + \frac{a_y \Delta z^2}{2 v_1^2 \cos^2\theta}$<br>$-h - \frac{\Delta z \sin\theta}{\cos\theta} = \frac{a_y \Delta z^2}{2 v_1^2 \cos^2\theta}$<br>$v_1^2(-h - \frac{\Delta z \sin\theta}{\cos\theta}) = \frac{a_y \Delta z^2}{2 \cos^2\theta}$ | Solve for $v_1$ |

$$v_1^{\,2} = \frac{a_y \Delta z^2}{2\cos^2\theta(-h - \frac{\Delta z \sin\theta}{\cos\theta})}$$

$$v_1^{\,2} = \frac{a_y \Delta z^2}{-2h\cos^2\theta - 2\Delta z\cos\theta\sin\theta}$$

$$v_1 = \sqrt{\frac{a_y \Delta z^2}{-2h\cos^2\theta - 2\Delta z\cos\theta\sin\theta}} = \sqrt{\frac{(-1.62 m/s^2)(25m)^2}{-2(12.5m)\cos^2 45 - 2(25m)\cos 45 \sin 45}} = 5.2 \text{ m/s} = v_1$$

EES Verify:

```
"Audrey Viland Lunar Lander P4"

Delta_z = 25 [m]
theta = 45 [deg]
h = 12.5 [m]
a_y = -1.62 [m/s^2]

v_z = v_1*cos(theta)
v_y = v_1*sin(theta)

{Motion in z-direction}
Delta_z = v_z*t

{Motion in y-direction}
-h=v_y*t+(1/2)*a_y*t^2
```

SOLUTION

**Unit Settings: SI C kPa kJ mass deg**

| | | |
|---|---|---|
| $a_y$ = -1.62 [m/s$^2$] | $\delta_z$ = 25 [m] | h = 12.5 [m] |
| t = 6.804 [s] | $\theta$ = 45 [deg] | $v_1$ = 5.196 [m/s] |
| $v_y$ = 3.674 [m/s] | $v_z$ = 3.674 [m/s] | |

No unit problems were detected.