Sam Nesmith Dynamics Problem

Speed governor

From video: Two masses on either side. Pulled together by a spring. As spins faster, masses want to move outward, creating frictional force to slow system

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Problem statement: Sam wants to create a speed governor for a set of automatic retracting blinds. His current design consists of a stationary housing with radius 2 cm. The springs inside have a spring constant of 250 N/m and a resting length of 1.5 cm. On the end of each spring is a 30 gram mass. The masses have a sliding coefficient of friction $\mu_k = 0.35$against the inside housing walls.

a. What is the minimum rotational velocity of the center axis before the masses contact the housing walls?
b. Assume the blind retraction mechanism causes a constant angular acceleration of 20 rad/s^2 at the center axis. What is the maximum angular velocity of the center axis?
c. Given the angular acceleration in b, plot the maximum angular velocity as a function of
    i. Spring constant
    ii. Mass
    iii. Initial length of spring

   with all other variables constant.

   Do each of these plots make sense? What do they tell us about the maximum angular velocity? How might Sam use this data to inform his design decisions?

Solution: part a

Assumptions

1. Approximate masses as point masses
2. Neglect air resistance
3. Neglect gravity

Known/given

Length of spring at rest	$l_0 = 1.5\text{cm}$
Radius of outside container	$r = 2\text{cm}$
Change in length of spring when mass contacts housing wall	$\Delta x = r - l_0 = 0.5\text{cm}$
Spring constant	$k = 250\text{N/m}$
Mass of one of the spinning masses	$m = 30\text{g}$
Sliding coefficient of friction between mass and container wall	$\mu_k = 0.35$

Unknown

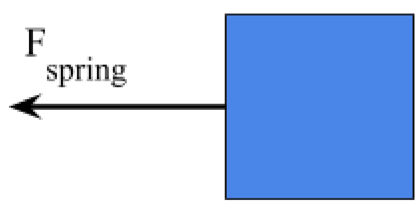
angular velocity of center axis	ω

Equations

Centripetal acceleration formula	$a_c = \frac{v^2}{r}$
Linear and rotational velocity relation	$v = \omega r$
Newton's second law	$F = ma$
Spring force equation	$F_s = kx$

Find minimum rotational velocity before masses contact housing wall

Free body diagram of mass before contacting housing wall



$$F_s = k\Delta x = ma_c$$

$$k\Delta x = mv^2/r$$

$$k\Delta x = m(\omega r)^2/r = m\omega^2 r$$

$$\omega^2 = \frac{k\Delta x}{mr}$$

$$\omega = \sqrt{\frac{k\Delta x}{mr}} = \sqrt{\frac{(250\ N/m)(0.005\ m)}{(0.03\ kg)(0.02\ m)}} = 45.6\ \text{rad/s}$$

EES verify:

```
l_0=1.5 [cm]*convert(cm,m)
r=2 [cm]*convert(cm,m)
delta_x=r-l_0
k=250 [N/m]
m=30 [g]*convert(g,kg)
mu_k=0.35

v=omega*r
a_c=v^2/r
F_s=k*delta_x
F_s=m*a_c
```

SOLUTION

Unit Settings: SI C kPa kJ mass deg

a_c = 41.67 [m/s^2]	δ_x = 0.005 [m]	F_s = 1.25 [N]
k = 250 [N/m]	l_0 = 0.015 [m]	m = 0.03 [kg]
μ_k = 0.35	ω = 45.64 [rad/s]	r = 0.02 [m]
v = 0.9129 [m/s]		

No unit problems were detected.

Solution: part b

Assumptions

4. Approximate masses as point masses
5. Neglect air resistance
6. Neglect gravity

Known/given

Angular acceleration of center axis	$\alpha = 20\text{rad/s}^2$
Length of spring at rest	$l_0 = 1.5\text{cm}$
Radius of outside container	$r = 2\text{cm}$
Change in length of spring when mass contacts housing wall	$\Delta x = r - l_0 = 0.5\text{cm}$
Spring constant	$k = 250\text{N/m}$
Mass of one of the spinning masses	$m = 30\text{g}$
Sliding coefficient of friction between mass and container wall	$\mu_k = 0.35$

Unknown

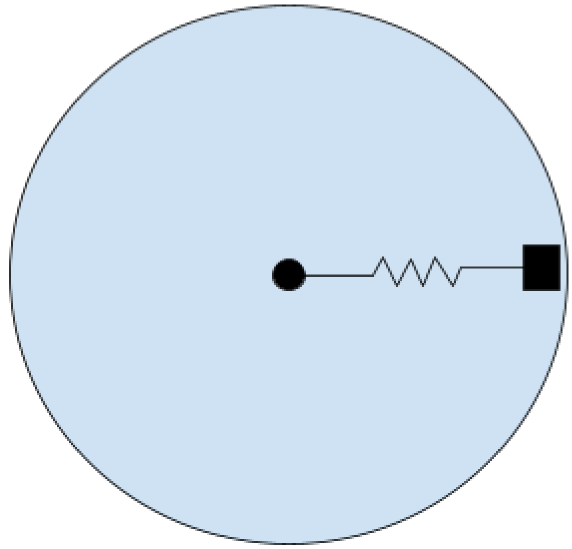
angular velocity of center axis	ω

Equations

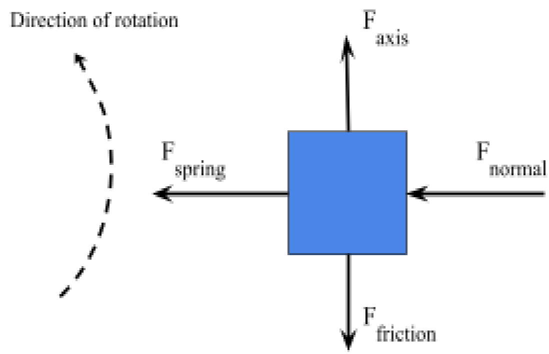
Centripetal acceleration formula	$a_c = \frac{v^2}{r}$
Linear and rotational velocity relation	$v = \omega r$
Newton’s second law	$F = ma$
Spring force equation	$F_s = kx$

Find maximum velocity

Simplified diagram of half the spring assembly and FBD of mass:



Free body diagram of mass in contact with housing wall



We can model the system using just one of the masses. Even though there will be another frictional force created by the second mass in contact with the housing, the F_{axis} will be the same on the other mass. Thus, the forces will cancel and only one needs to be modeled. If, say, we had to solve for the angular acceleration given an external force from the blinds, then we would need to model the force from both masses. However, since we are given the angular acceleration of the center axis, we only need to model one of the masses.

$F_{friction} = F_{axis}$	No longer accelerating in tangential direction: max velocity
$F_c = F_{spring} + F_{normal}$	Centripetal force (forces perpendicular to tangential velocity)
$F_{axis} = ma_t$	Tangential force caused by angular acceleration of center axis
$F_c = ma_c$	Centripetal force/acceleration
$a_c = \frac{v^2}{r}$	Centripetal acceleration
$F_{spring} = k\Delta x$	Spring force
$a_t = r\alpha$	Tangential acceleration caused by axial angular acceleration
$v = \omega r$	Tangential velocity to angular velocity

Algebra:

$$F_{axis} = F_{friction}$$

$$ma_t = \mu_k F_{normal}$$

$$F_c = F_{spring} + F_{normal}$$

$$mr\alpha = \mu_k (F_c - F_{spring})$$

$$F_{normal} = F_c - F_{spring}$$

$$mr\alpha = \mu_k (ma_c - k\Delta x)$$

$$mr\alpha = \mu_k (mv^2/r - k\Delta x)$$

$$v = \omega r$$

$$mr\alpha = \mu_k (m(\omega r)^2/r - k\Delta x) = mr\alpha = \mu_k (m\omega^2 r - k\Delta x)$$

$$mr\alpha/\mu_k + k\Delta x = m\omega^2 r$$

$$\omega^2 = \frac{\alpha}{\mu_k} + \frac{k\Delta x}{mr}$$

$$\omega = \sqrt{\frac{\alpha}{\mu_k} + \frac{k\Delta x}{mr}} = \sqrt{\frac{(20\,rad/s)}{0.35} + \frac{(250\,N/m)(0.005\,m)}{(0.03\,kg)(0.02\,m)}} = 46.27 \text{ rad/s}$$

EES Verify

```
l_0=1.5 [cm]*convert(cm,m)
r=2 [cm]*convert(cm,m)
delta_x=r-l_0
k=250 [N/m]
m=30 [g]*convert(g,kg)
mu_k=0.35
alpha=20 [rad/s^2]

F_c=F_spring+F_normal
F_spring=k*delta_x
F_c=m*a_c
a_c=v^2/r
a_t=r*alpha
F_axis=m*a_t
F_friction=mu_k*F_normal
F_friction=F_axis
v=omega*r
```

SOLUTION

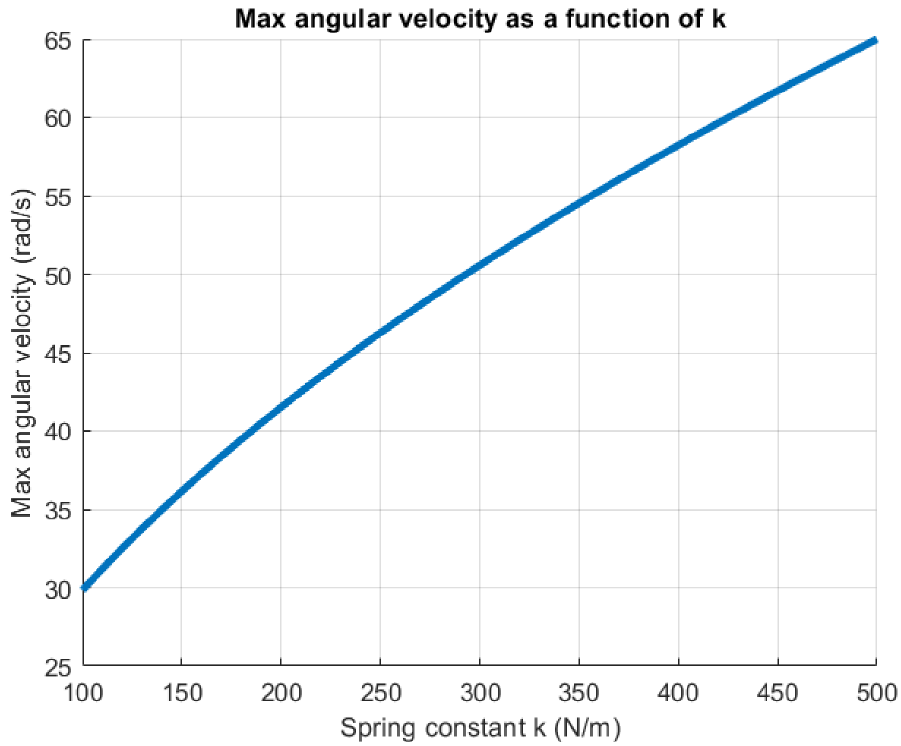
Unit Settings: SI C kPa kJ mass deg

α = 20 [rad/s^2]	a_c = 42.81 [m/s^2]	a_t = 0.4 [m/s^2]
δ_x = 0.005 [m]	F_{axis} = 0.012 [N]	F_c = 1.284 [N]
$F_{friction}$ = 0.012 [N]	F_{normal} = 0.03429 [N]	F_{spring} = 1.25 [N]
k = 250 [N/m]	l_0 = 0.015 [m]	m = 0.03 [kg]
μ_k = 0.35	ω = 46.27 [rad/s]	r = 0.02 [m]
v = 0.9253 [m/s]		

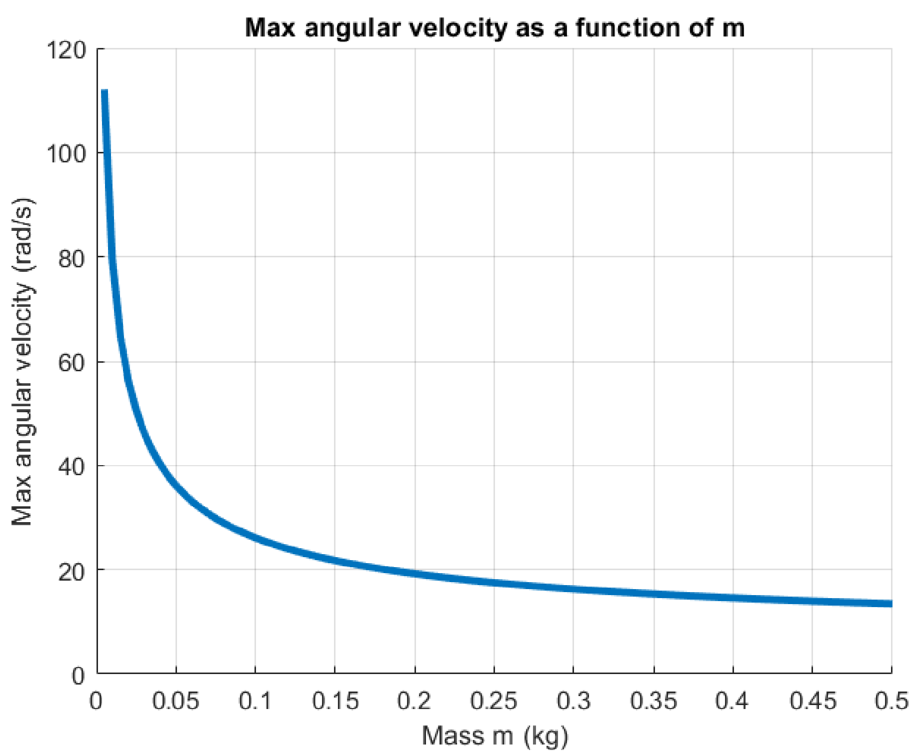
No unit problems were detected.

Solution: part c

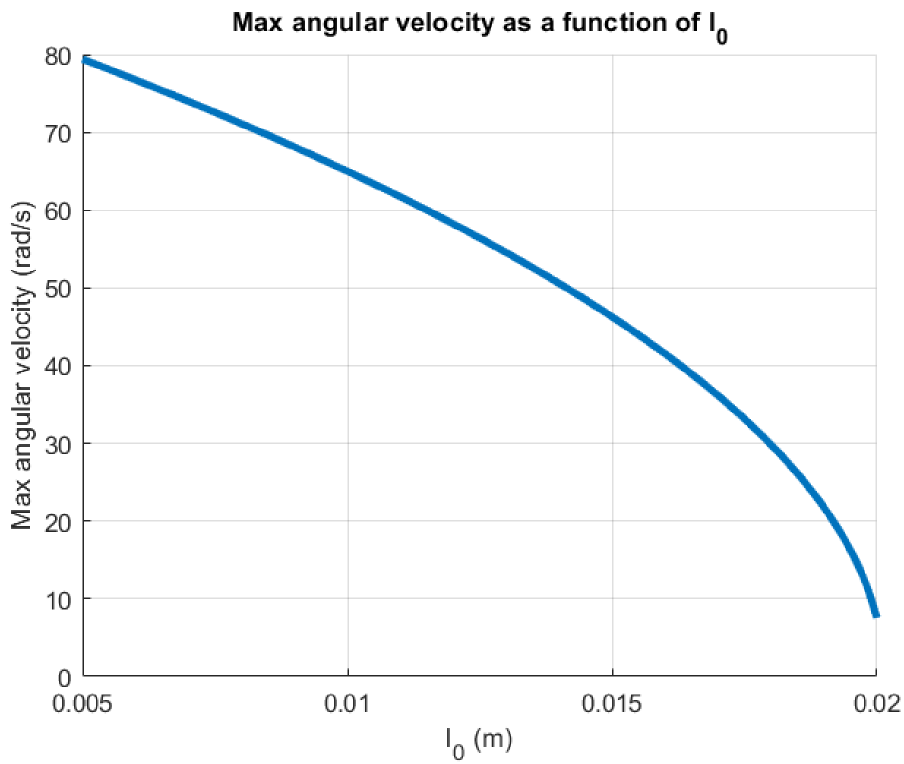
Do each of these plots make sense? What do they tell us about the maximum angular velocity? How might Sam use this data to inform his design decisions?



This plot shows that the maximum angular velocity increases as the spring constant increases. This makes sense because as the spring gets stiffer, the center axis will need to spin faster to force the masses outward and create friction to slow the angular velocity. So, if Sam wants the maximum retraction speed of the blinds to be faster, he can choose to implement a stiffer spring.



This plot reveals that, as the mass of the weights increases, the maximum angular velocity decreases. This makes sense because the centripetal force is related to the angular velocity, the friction force, and the mass. Once the masses reach the housing walls and the spring can no longer stretch, the spring force becomes constant. However, as the angular velocity increases, the centripetal force also increases ($F_c = ma_c = mv^2/r$). Since the inward spring force is constant, the only other way to increase the centripetal force is to increase the normal force on the mass from the housing walls. From the centripetal force equation, we can see that the centripetal force is directly proportional to the mass. So, with a larger mass, comes a larger centripetal force, and thus a larger normal force. Finally, as the normal force increases, so does the frictional force, thus preventing the center axis from reaching higher rotational velocity. Sam could take this information into his design consideration. If he wanted to reduce the mass and potentially save on cost, he must also take into account the increase in the maximum angular velocity of the center axis.



This graph shows that as the resting spring length increases, the maximum angular velocity decreases. This makes sense since the spring keeps the masses from contacting the housing walls before a certain angular velocity. So, as the spring gets longer (smaller Δx to reach the walls), it takes lower angular velocities before the masses reach the housing walls. So, the masses will contact the walls at lower angular velocities and start creating friction to slow the rotation of the center axis. Sam could design the speed governor with longer resting spring lengths if he wanted to lower the maximum angular velocity of the center axis.