AERO7970 - Trajectory Optimiztion

Project 01

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October 10, 2022

Abstract

This report details the implementation of a fuel-optimal close-proximity rendezvous maneuver. The solution was implemented in `Matlab` using both the built-in `solve` function and the `CVX` library. Included are performance comparisons and the resulting trajectories and control inputs.

1 Problem Modeling

We are given the following optimization problem:

$$\begin{aligned} \min_{X,U} \quad & \sum_{i=1}^{N-1} ||U_i||_{L1} \\ \text{s.t.} \quad & \dot{X}_t = AX_t + BU_t \\ & T_{min} \le U_i \le T_{max} \\ & X_0 = \begin{bmatrix} -1e3 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\ & X_N = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{aligned}$$

While the L1-norm is not strictly a linear performance criterion, we can introduce a set of slack variables $S \ge 0$ to bound the control input U and then minimize their sum as an LP problem. Additionally, we must convert the continuous-time dynamics constraint into a discretized form. We use a zero-order hold discretization to convert the dynamics:

$$\dot{X} = AX + BU \rightarrow X_{k+1} = expm(A * dt)X_k + B * dt * U_k$$

Restructuring the problem as such results in:

$$
\begin{aligned}
\min_{X,U,S} \quad & \sum_{i=1}^{N-1} S_i \\
\text{s.t.} \quad & X_{i+1} = expm(A*dt)*X_i + B*dt*U_i \\
& S_i \geq 0 \\
& -S_i \leq U_i \leq S_i \\
& T_{min} \leq U_i \leq T_{max} \\
& X_0 = \begin{bmatrix} -1e3 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
& X_N = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}
\end{aligned}
$$

which was then implemented in problem-based form in `Matlab` and `CVX`.

2 Results

After implementing the problem in `Matlab` and `CVX`, the formulation-to-solution code was timed and averaged over 10 runs. The average times are shown below in Table 1.

Solver	Average Time (s)
Linprog (Dual-Simplex)	0.41164
Linprog (Interior-Point)	0.22449
CVX	0.73741

Table 1: Average formulation-to-solution time for considered solvers

The resulting trajectories and control inputs are shown below in Figures 1 and 2.

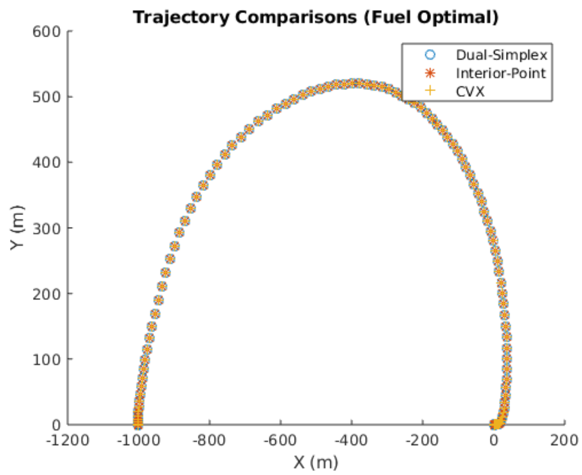

Figure 1: Generated fuel-optimal trajectories from each solver

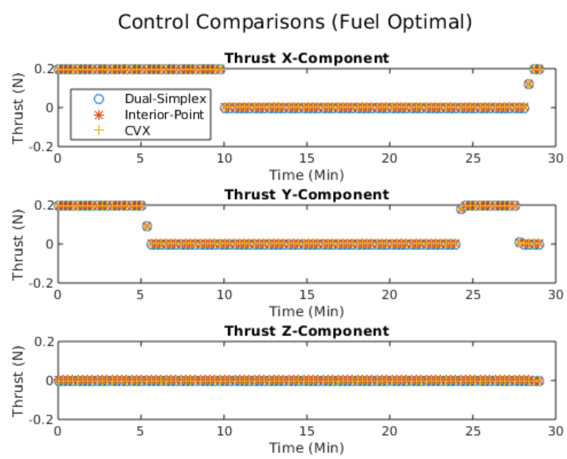

Figure 2: Generated fuel-optimal control inputs from each solver

Additionally, it was found that the minimum time needed to perform the maneuver was 21 minutes. The generated time-optimal trajectories and controls are shown below in Figures 3 and 4.

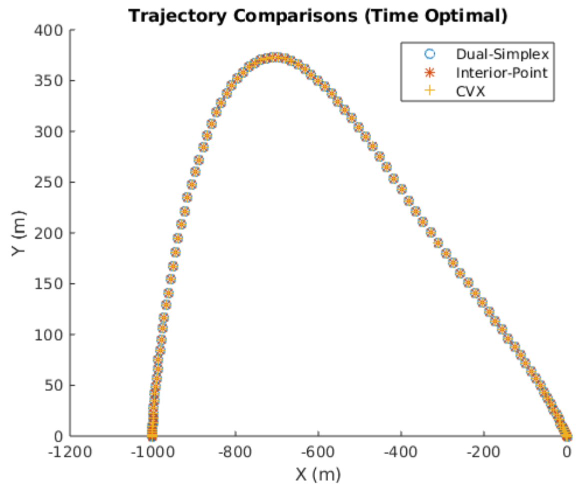

Figure 3: Generated time-optimal trajectories from each solver

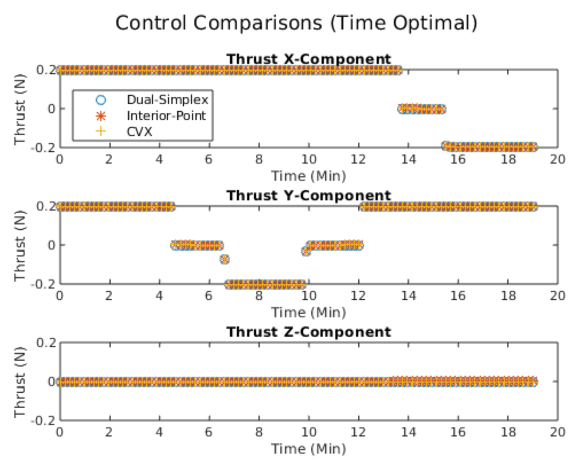

Figure 4: Generated time-optimal control inputs from each solver

Conclusion

The main challenges faced in this project were in formulating the optimization problem - it took several tries before `Matlab` recognized the L1 slack-variable formulation as an LP. Additionally, `CVX`'s documentation is sparse when it comes to LP problems and it took some time to get a feel for its error messages. What I learned from this project is both the value of being able to rearrange nonlinear problem components to fit into an LP problem and also that I *vastly* prefer problem-based modeling to calling solver functions like I have in the past. I spent approximately 8 hours on this project.