Close-Proximity Maneuver: Formation flying and close proximity maneuvers of satellites are identified as enabling technologies for many future NASA and U.S. Air Force missions. You are assigned with the task of formulating and solving a particular re-formulation of minimum-fuel, rendezvous-class maneuvers of spacecraft in close proximity using the techniques of *linear programming.*

Basic Formulation of the Linear Programming Problem: Let $\boldsymbol{x} = [x, y, z, v_x, v_y, v_z]^\top \in \mathbb{R}^6$ and $\boldsymbol{u} = [T_x, T_y, T_z]^\top \in \mathbb{R}^3$ denote the state and control vectors of a spacecraft. For the problem of interest in this project, a spacecraft (chaser) is assumed to be in close proximity to a main spacecraft (chief). The most common approximation for spacecraft dynamics are the linearized Hill's equations

$$\dot{\boldsymbol{x}} = \mathbb{A}\boldsymbol{x} + \mathbb{B}\boldsymbol{u}, \tag{1}$$

where $\mathbb{A}$ and $\mathbb{B}$ matrices are defined as

$$\mathbb{A} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 3\omega^2 & 0 & 0 & 0 & 2\omega & 0 \\ 0 & 0 & 0 & -2\omega & 0 & 0 \\ 0 & 0 & -\omega^2 & 0 & 0 & 0 \end{bmatrix}, \qquad \mathbb{B} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{1}{m} & 0 & 0 \\ 0 & \frac{1}{m} & 0 \\ 0 & 0 & \frac{1}{m} \end{bmatrix}. \tag{2}$$

Here, m denotes the spacecraft mass, which is assumed to be constant during the maneuver since the low-thrust propulsion system has a high specific impulse value. Note that the x coordinate is in the radial direction, the y coordinate is in the in-track direction, and the z component is in the out-of-plane direction (along the osculating angular momentum vector). The details of the derivation of the Hill's equations can be found, for instance, in Chapter 7 of the book by Howard Curtis – Orbital Mechanics for Engineering Students. Figure 1 depicts the scenario and the definition of the chaser and target spacecraft along with the co-moving LVLH frame. If the orbit of target vehicle is a circle, then the LVLH frame is called a Clohessy-Wiltshire (CW) frame. The relation for ω can be found in the same chapter. Under this assumption, the equations of motion of the chaser in the CW frame are given in Eq. (1). Figure 2 shows a simplified view of the CW frame.

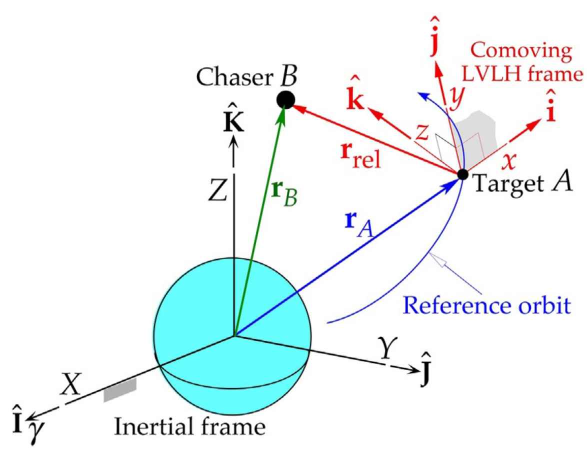

Figure 1: Position of chaser relative to the chief/target spacecraft and the definition of the Local-Vertical Local-Horizontal (LVLH) frame.

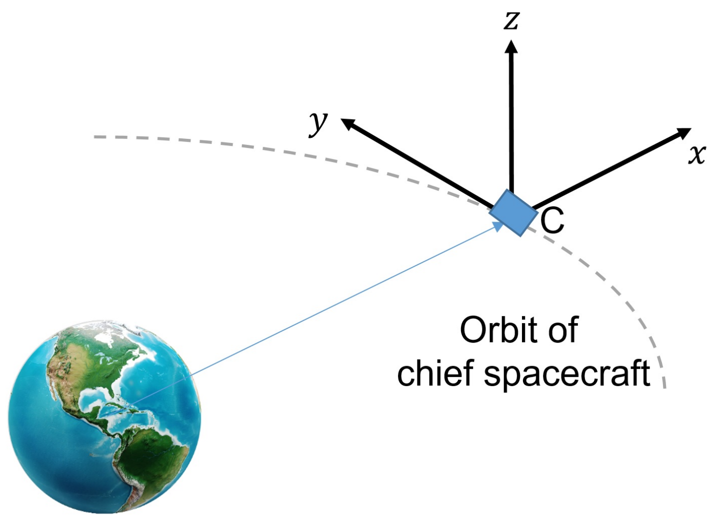

Figure 2: CW frame.

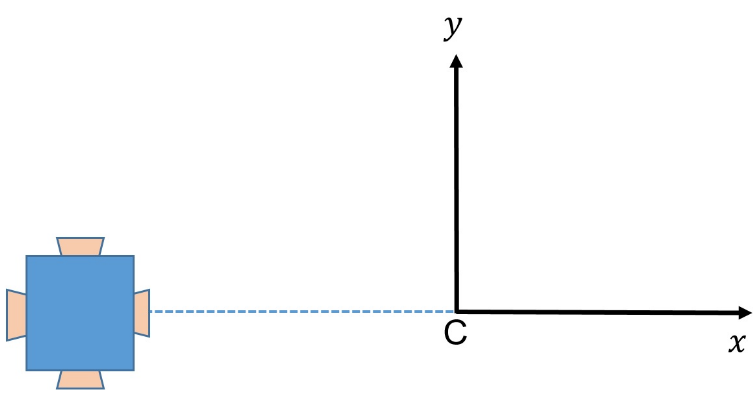

Figure 3: Initial location of the chaser.

The gravitational parameter of the Earth is $\mu = 398600$ km^3/s^2 and the radius of the Earth is assumed to be $Re = 6378$ km. The chief spacecraft is located at the origin of the CW frame, i.e., $\boldsymbol{x}_{\text{chief}} = [0, 0, 0, 0, 0, 0]^\top$ with its circular orbit at an altitude of 550 km. As is shown in Figure 3, the chaser with a mass of $m = 50$ kg is at a stationary position on the negative x axis of the CW frame with the given states, $\boldsymbol{x}_{\text{chaser}} = [-1 \text{ km}, 0, 0, 0, 0, 0]^\top$. The chaser has thrusters on all sides of its body that allows it to thrust along the x, y and z directions of the CW frame at a maximum value of $T_{\max} = 200$ miliNetwon. In other words, we are ignoring the rotational dynamics of the chaser, but allow independent thursting along x, y, and z coordinates of the CW frame as follows

$$
\begin{aligned}
-T_{\max} &\leq T_x \leq T_{\max}, \\
-T_{\max} &\leq T_y \leq T_{\max}, \\
-T_{\max} &\leq T_z \leq T_{\max}.
\end{aligned}
$$

The optimization problem that is considered for a minimum-fuel maneuver that chaser executes to rendezvous with the chief spacecraft is written as

$$
\begin{aligned}
J = \text{minimize} \sum_{i=1}^{N-1} \{|T_{x,i}| + |T_{y,i}| + |T_{z,i}|\}& \\
s.t.:& \\
Eq.\ (1),& \\
\boldsymbol{x}_{\text{chief}} = \boldsymbol{0},& \\
\boldsymbol{x}_{\text{chaser}} = [-1\ \text{km}, 0, 0, 0, 0, 0, 0]^\top.&
\end{aligned}
$$

where N is the number of points used to discretize the maneuver time, t_f. One interpretation of the considered cost is to minimize the sum of absolute thrust values in each direct at each instant of the discretized points. Note that the thrust value will be fixed until the next discrete point. This is referred to as a *zero-order hold* control. In this problem, the time of fight is fixed at $t_f = 30$ minutes and $N = 100$.

Tasks: Use the techniques of linear programming to convert the above minimum-fuel trajectory optimization problem into an LP problem and solve the resulting LP problem using the `problem-based` *problem-based* approach, i.e., using the *solve* command and the *CVX* solver.

What to Report: Your report should be typeset in Overleaf and your final report has to be in a pdf format. In addition, you should upload your code as a zip file with its main function named as 'MainRendezvousMILP.m'.

Item 1: Summarize the key steps in formulating the LP problem.

Item 2: Report the average time it takes for solving ten instances of the LP problem using both "dual-simple" and "interior-point" algorithms. In addition, formulate the same problem in MATLAB, but use CVX solver in MATLAB. CVX is a tool developed for solving convex optimization problems. You can download CVX solver from `http://cvxr.com/cvx/`.

Item 3: For the given parameters of the problem and spacecraft and values for N, what is the shortest (integer value) time of maneuver, $t_{\text{f,min}}$, below which it is not possible to find a feasible solution?

Item 4: Include a figure that overlays the $x - y$ view of the trajectories obtained using the "dual-simple" and "interior-point" algorithms. Use a different line style or color to distinguish the solution of the "dual-simple" and "interior-point" algorithms. Include a figure with 3 sub-plots that overlay the time histories of thrust components, i.e., T_x, T_y and T_z. Use a different line style or color to distinguish the solution of the "dual-simple" and "interior-point" algorithms.

Item 5: Conclusion/Summary: Summarize the main challenges you faced in this project and what you learned from this assignment. Report the total number of hours spent on this project.