AERO7970 - Trajectory Optimiztion

Project 02

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Abstract

This report presents a solution to a mixed-integer resource allocation problem. The solution was implemented in `Matlab` using the `intlinprog` solver.

1 Problem Modeling

We model the solution to the allocation problem as a 1000×12 binary matrix X, where each row is a 1-hot vector $[0, \ldots, 1, \ldots, 0]$ indicating which station the asteroid is allocated to. Letting A be the 1000×12 asteroid-station-mass matrix given in the problem, the asteroid mass at station i can be calculated as

$$M_i = A[:, i]^T \cdot X[:, i]$$

Additionally, maximizing the minimum station mass is not itself a linear objective, so instead we introduce a scalar S representing the lower bound of station masses and maximize it. This results in the optimization problem:

$$\begin{aligned} \max_{X,S} \quad & 10^{-10} * S \\ \text{s.t.} \quad & sum(X(i,:)) = 1 \\ & A[:, i]^T \cdot X[:, i] \geq S \end{aligned}$$

2 Results

After running the solver, we arrive at the station masses shown below in Figure 1.

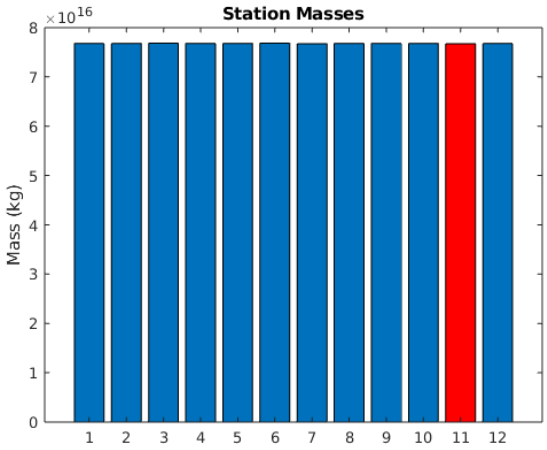

Figure 1: Allocated masses for each station

Station 11 was determined to have the minimum final mass of $7.6750055125e+16kg$, resulting in a final cost $J = 7675005.5125$. Figure 2 below shows the progress of the solver over time.

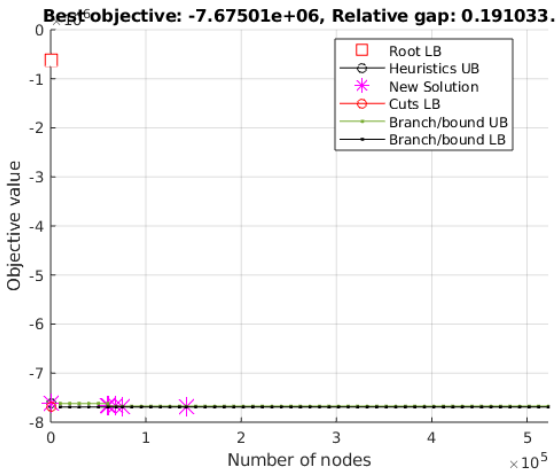

Figure 2: Solver progress

3 Conclusion

The main challenges of this project were in figuring out how to model the problem and how to get `Matlab` to actually solve it. I spent a lot of time trying to find an elegant way to have X be a vector and write the entire system as

an $AX = B$ form before deciding on the selection-matrix form. The fact that these optimization solvers allow fairly arbitrary variable declarations as long as they can be used linearly is a lifesaver. As for solving the problem, `intlinprog` is *extremely* slow to solve the problem so I took a lot of time tuning the solver options to try and speed it up.

I spent approximately 15 hours on this project.