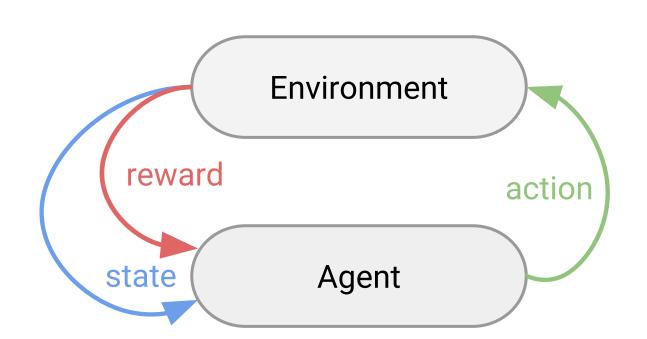
# Learning to Do

Supervised Learning: given data, predict labels

Unsupervised Learning: given data, learn about that data

Reinforcement Learning: given data, choose action to maximize expected long-term reward



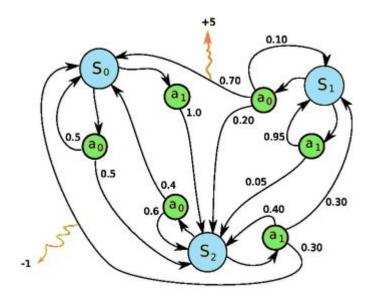


#### Episode: sequence of states and actions

$$s_0, a_0, r_0, s_1, a_1, r_1, \dots, s_{T-1}, a_{T-1}, r_{T-1}, s_T, r_T$$

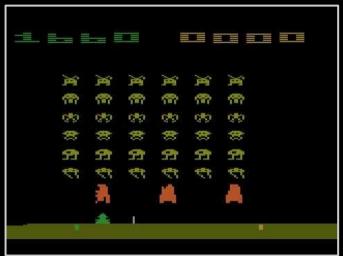
#### Transition function:

$$P(s_{t+1}, r_t \mid s_t, a_t)$$











We want to find a policy 
$$\pi(s)=p(a|s)$$
 hich maximizes 
$$\frac{r_0+r_1+r_2+\cdots+r_T}{r_0+\gamma r_1+\gamma^2 r_2+\cdots+\gamma^T r_T} \gamma OI$$

 $E \left| \mathbf{r}_0 + \gamma \mathbf{r}_1 + \gamma^2 \mathbf{r}_2 + \dots + \gamma^T \mathbf{r}_T \right|$ 

$$\max_{\pi} E \left[ \sum_{i=0}^{T} \gamma^{i} \mathbf{r}_{i} \right]$$

## Policy Learning

Value Learning

Find  $\pi(s)$ 

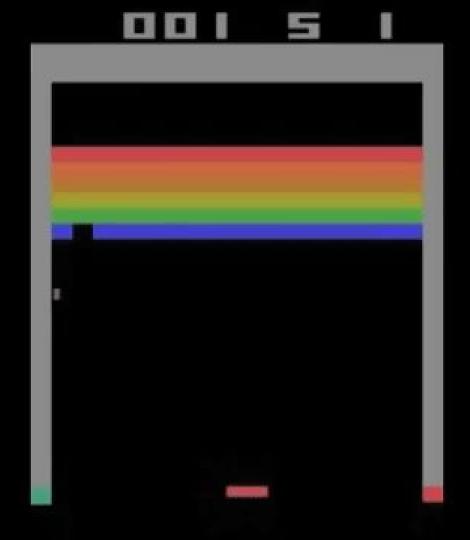
Find Q(s, a)

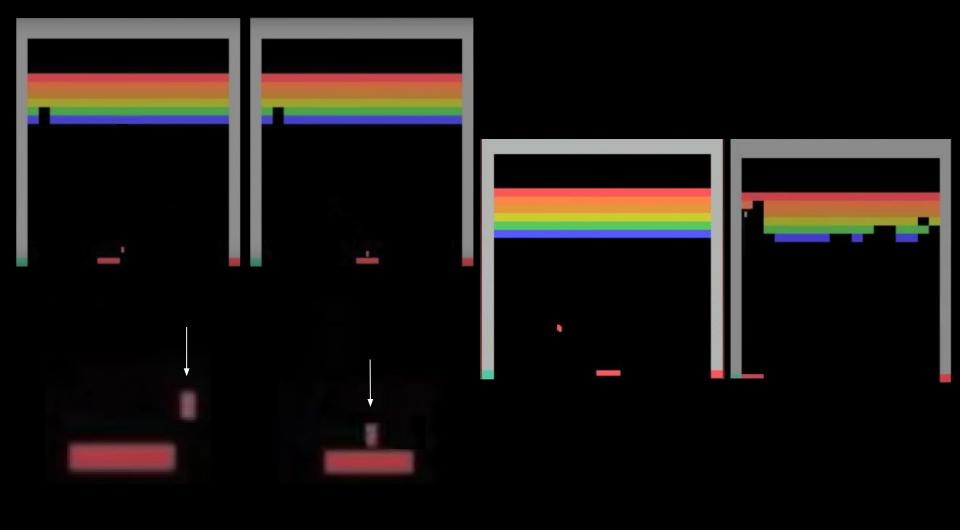
 $a \sim \pi(s)$ 

 $a = \arg\max_{a'} Q(s, a')$ 

$$Q^*(\mathbf{s_t}, a_t) = \max_{\pi} E \left[ \sum_{i=t}^{T} \gamma^i \mathbf{r_i} \right]$$

maximum expected future rewards starting at state  $s_i$ , choosing action  $a_i$ , and then following an optimal policy  $\pi^*$ 





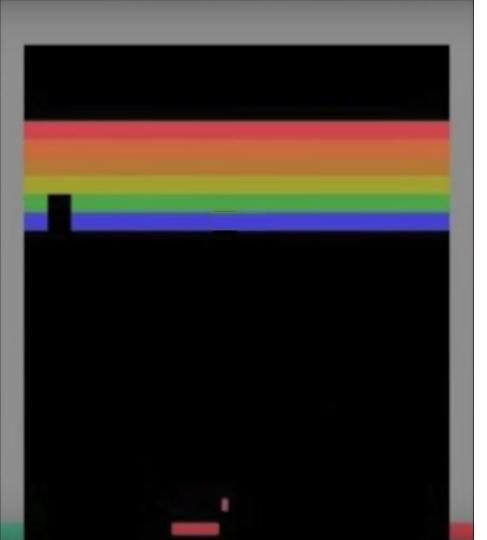
$$Q^*(\mathbf{s_t}, a_t) = E\left[\mathbf{r_t} + \gamma \max_{a'} Q^*(\mathbf{s_{t+1}}, a')\right]$$

The max future reward for taking action a<sub>t</sub> is the current reward plus the next step's max future reward from taking the best next action a'

$$\widehat{Q_{j+1}}(s_t, a_t) \leftarrow E\left[\frac{r_t}{r_t} + \gamma \max_{a'} \widehat{Q_j}(s_{t+1}, a')\right]$$

$$\widehat{Q_j} \to \widehat{Q_{j+1}} \to \widehat{Q_{j+2}} \to \cdots \to Q^*$$

But... how large is  $Q(\cdot)$  ?



# states: ~2<sup>96</sup>•60•60

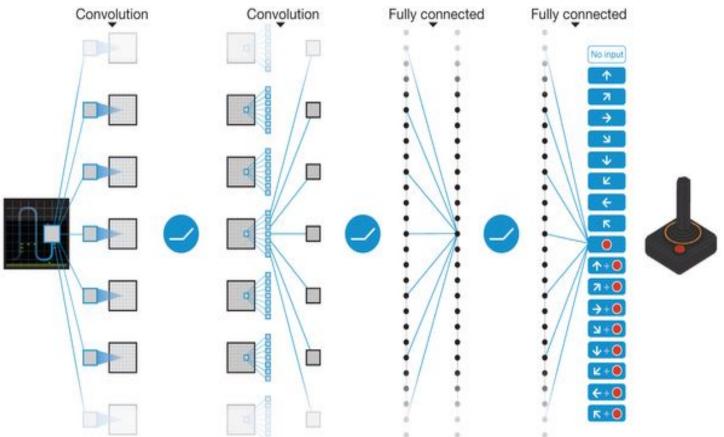
# actions: 3

# Q values: ~2<sup>111</sup>

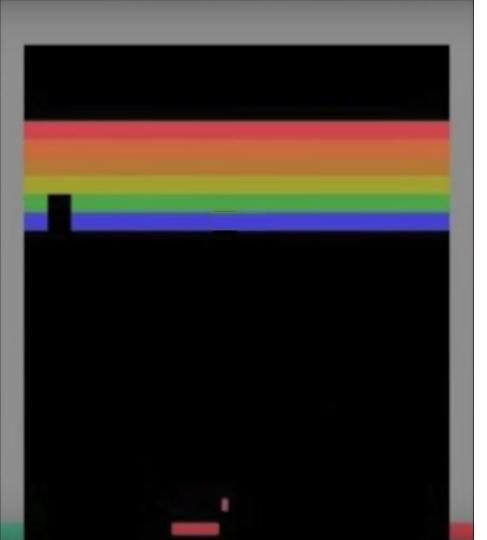
1957 - 2013

:(

#### **ENTER THE DEEP**



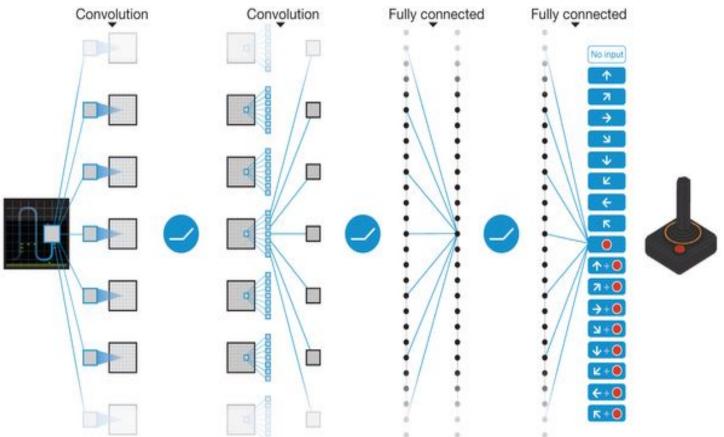
Mnih et al, 2013



# Features for Estimating Q

- Whether the paddle can reach the ball
- # remaining blocks
- How are the blocks spatially arranged?

#### **ENTER THE DEEP**



Mnih et al, 2013

## Define approximate Q\* function

$$\widehat{Q_{\theta}}(s, a|\theta) \sim Q^*(s, a)$$

## and choose $\theta$ to minimize

$$\min_{\theta} \sum_{e \in E} \sum_{t=0}^{T} \left\| \widehat{Q}(s_t, a_t | \theta) - \left( r_t + \gamma \max_{a'} \widehat{Q}(s_{t+1}, a' | \theta) \right) \right\|$$

#### 1: **function** Q-LEARNING

2: Initialize 
$$\theta$$
  
3:  $s = s_0$ 

6:

7:

5: Choose a from some policy 
$$\pi(s)$$
, and store results  $r, s_{new}$ 

Choose a from some Compute 
$$\nabla_a E_a = 1$$

ompute 
$$\nabla_{\theta} E_{O} = 1$$

Compute 
$$\nabla_{\theta} E_Q = \nabla_{\theta} \left\| \widehat{Q}(s, a|\theta) - \left( r + \gamma \max_{a'} \widehat{Q}(s, a'|\theta_{old}) \right) \right\|$$

$$heta = heta - \eta 
abla_{ heta} E_Q$$

$$s = s_{new}$$
 (or  $s_0$  if episode ended)

8: 
$$s = s_{new}$$
9:  $\theta_{old} = \theta$ 

$$= s_{new}$$

$$\nabla_{ heta} \| \widehat{Q}(s,$$

y 
$$\pi(s)$$
, and  $(s,a| heta)-\Big($ 

$$-\left(r\right)$$

ore results 
$$r$$



#### 1: **function** Q-LEARNING

2: Initialize 
$$\theta$$

5:

$$s = s_0$$

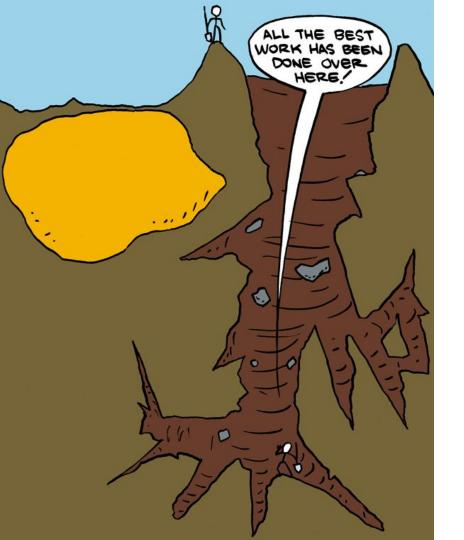
while not bored yet do

Choose a from some policy 
$$\pi(s)$$
, and store results  $r, s_{new}$ 

Compute 
$$\nabla_{\theta} E_Q = \nabla_{\theta} \left\| \widehat{Q}(s, a|\theta) - \left( r + \gamma \max_{a'} \widehat{Q}(s, a'|\theta_{old}) \right) \right\|$$
  
 $\theta = \theta - \eta \nabla_{\theta} E_Q$ 

$$s = s_{new}$$
 (or  $s_0$  if episode ended)

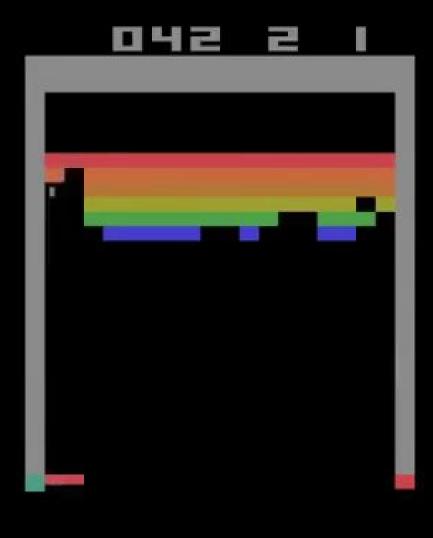
9: 
$$\theta_{old} = \theta$$

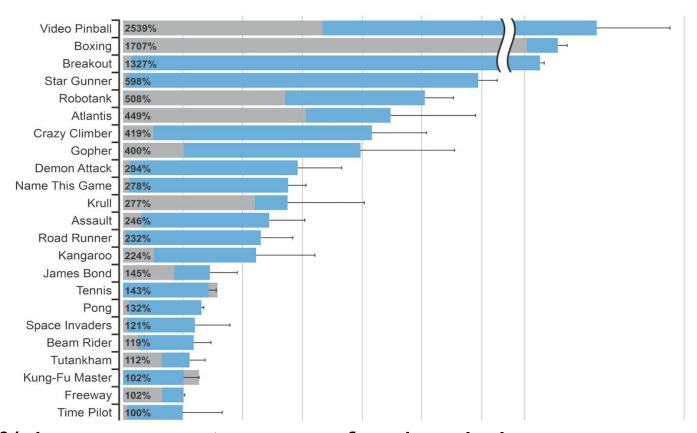


We need to balance exploration and exploitation

## $\epsilon$ -greedy exploration

With probability  $1 - \epsilon$ :
Pick  $a_{t+1} \sim \operatorname{soft} \max_{a'} \widehat{Q}(s_{t+1}, a')$ With probability  $\epsilon$ :
Pick  $a_{t+1}$  at random





% improvement over professional player Mnih et al, 2013



## Try it out!

http://selfdrivingcars.mit.edu/deeptraffic/

(Kudos to Lex Fridman & the 6.S094 team!)

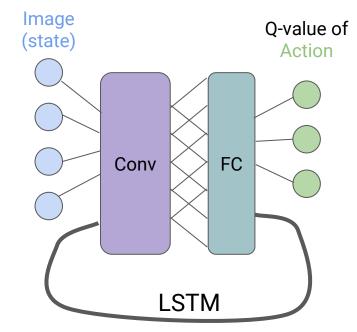
Road Overlay:

## The problems with Q-learning

- Restrictive Assumptions
- Handles long horizons poorly
- Requires a simulator



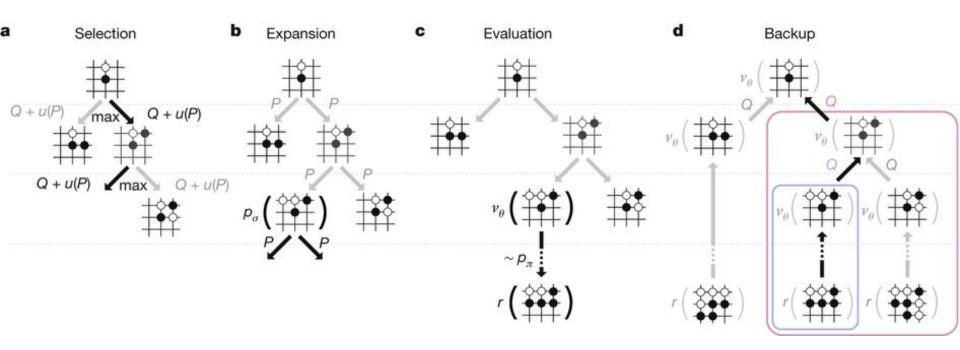
### LSTM RNNs!



## The problems with Q-learning

- Restrictive Assumptions
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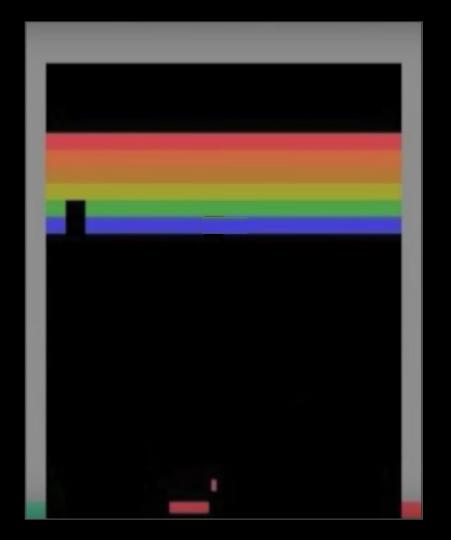
## The problems with Q-learning

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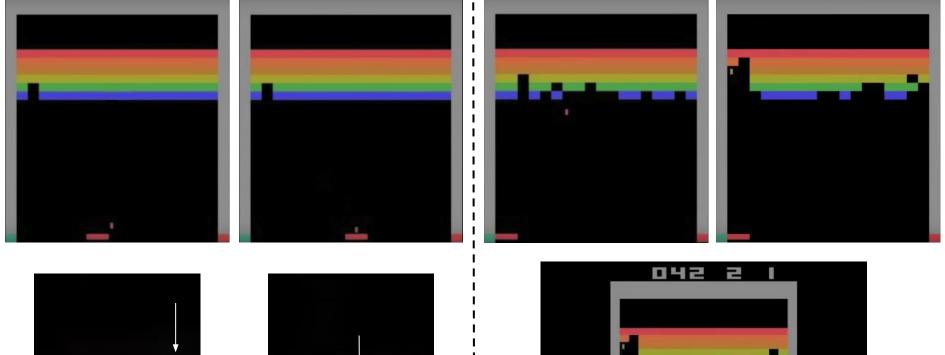


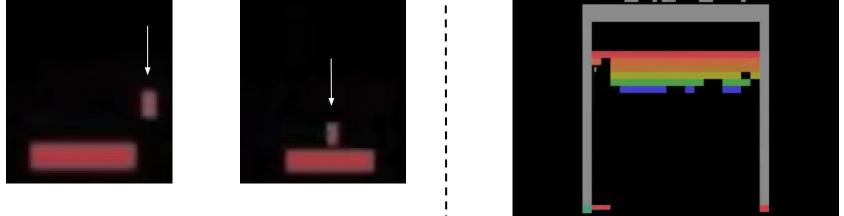


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## 1: **function** $\epsilon$ -GREEDY-Q-LEARNING

Initialize  $\theta$ 2:

 $\epsilon = \text{some tiny number}$ while not bored yet do

p = randf(0,1)5:

if  $p < \epsilon$  then

 $\theta = \theta - \eta \nabla_{\theta} E_{Q}$ 

 $\theta_{old} = \theta$ 

7: else 8:

9:

10:

11:

12:

13:

Choose random action a and store results  $r, s_{new}$ 

Choose  $a = \arg \max_{a'} Q(s, a')$ , and store results  $r, s_{new}$ 

Compute  $\nabla_{\theta} E_Q = \nabla_{\theta} \left\| \widehat{Q}(s, a|\theta) - \left( r + \gamma \max_{a'} \widehat{Q}(s, a'|\theta_{old}) \right) \right\|$ 

 $s = s_{new}$  (or  $s_0$  if episode ended)

