B+-TREE COSTS

Recall: Cost of Operations

- **Can we do better with indexes?**

- **B:** Number of data blocks
- **R:** Number of records per block
- **D:** Average time to read/write disk block

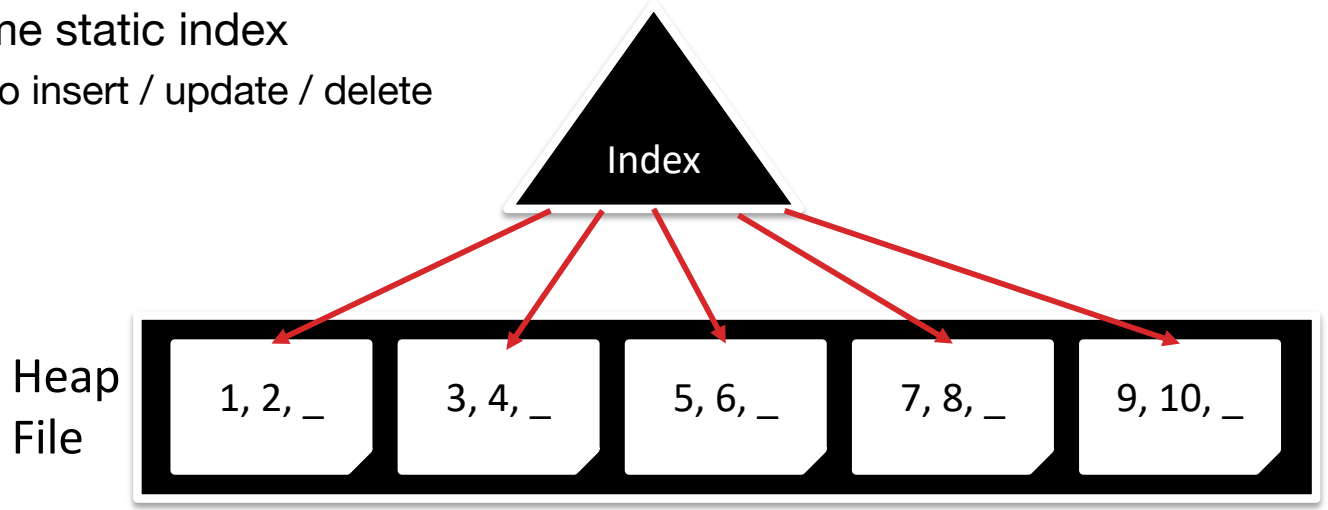
Recall we are interested in the average case

Both reading and writing to the disk cost I/Os!!

	Heap File	Sorted File
Scan all records	B*D	B*D
Equality Search	0.5*B*D	$(\log_2 B)*D$
Range Search	B*D	$((\log_2 B)+pages))*D$
Insert	2*D	$((\log_2 B) + B)*D$
Delete	(0.5*B+1)*D	$((\log_2 B) + B)*D$

Index Assumptions

- Store data by reference (Alternative 2)
- **Clustered** index with 2/3 full heap file pages
  - Clustered → Heap file is initially sorted
  - **Fan-out** (F) (i.e., branching factor) of tree internal node:
    - Page of <key, pointer> pairs ~ O(R) [R: Number of records per block]
    - in practice this is relatively large. Why?
  - Assume static index
    - No insert / update / delete



Cost of Operations

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	Heap File	Sorted File	Clustered Index
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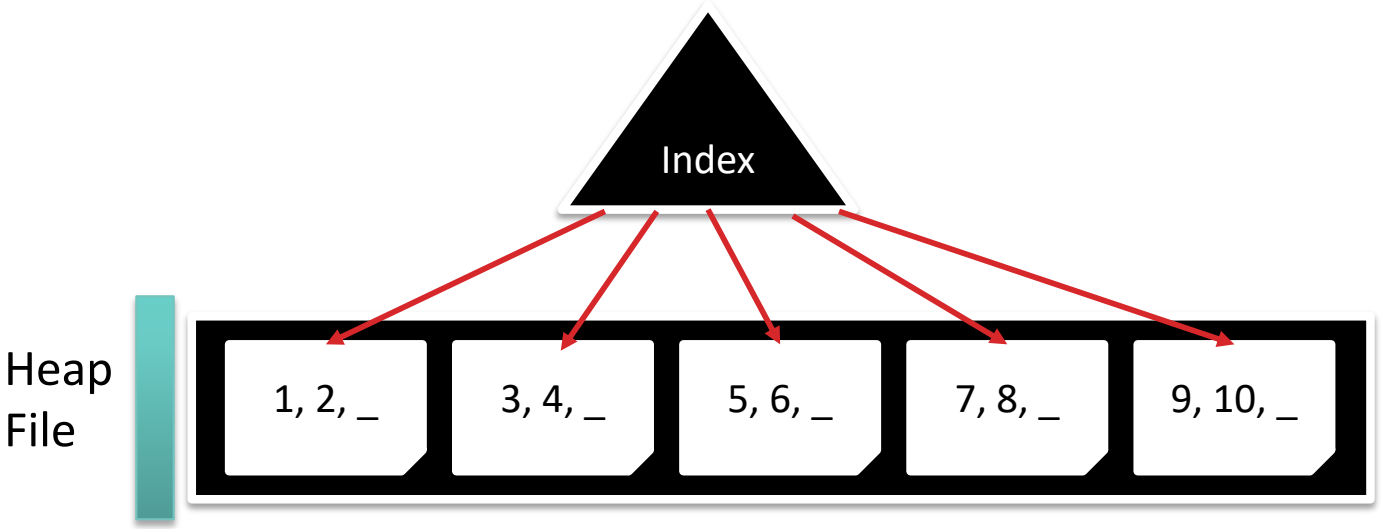
Range Search	B*D	$((\log_2 B)+pages))*D$	
Insert	2*D	$((\log_2 B) + B)*D$	
Delete	(0.5*B+1)*D	$((\log_2 B) + B)*D$	

Scan all the Records

- Do we need an Index?
  - No
- Cost? = 1.5 * B * D
  - Why?

B: Number of data blocks
D: Average time to read/write disk block

Recall assumption on clustered indexes: heap file pages are only **2/3** full.



Cost of Operations: Scan

	Heap File	Sorted File	Clustered Index
Scan all records	B*D	B*D	3/2 * B * D
Equality Search	0.5*B*D	$(\log_2 B)*D$	
Range Search	B*D	$((\log_2 B)+pages))*D$	
Insert	2*D	$((\log_2 B) + B)*D$	
Delete	(0.5*B+1)*D	$((\log_2 B) + B)*D$	

- **B:** Number of data blocks
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Cost of Operations: Equality Search?

	Heap File	Sorted File	Clustered Index
Scan all records	B*D	B*D	3/2 * B * D
Equality Search	0.5*B*D	$(\log_2 B)*D$	
Range Search	B*D	$((\log_2 B)+pages))*D$	
Insert	2*D	$((\log_2 B) + B)*D$	
Delete	(0.5*B+1)*D	$((\log_2 B) + B)*D$	

- **B:** Number of data blocks
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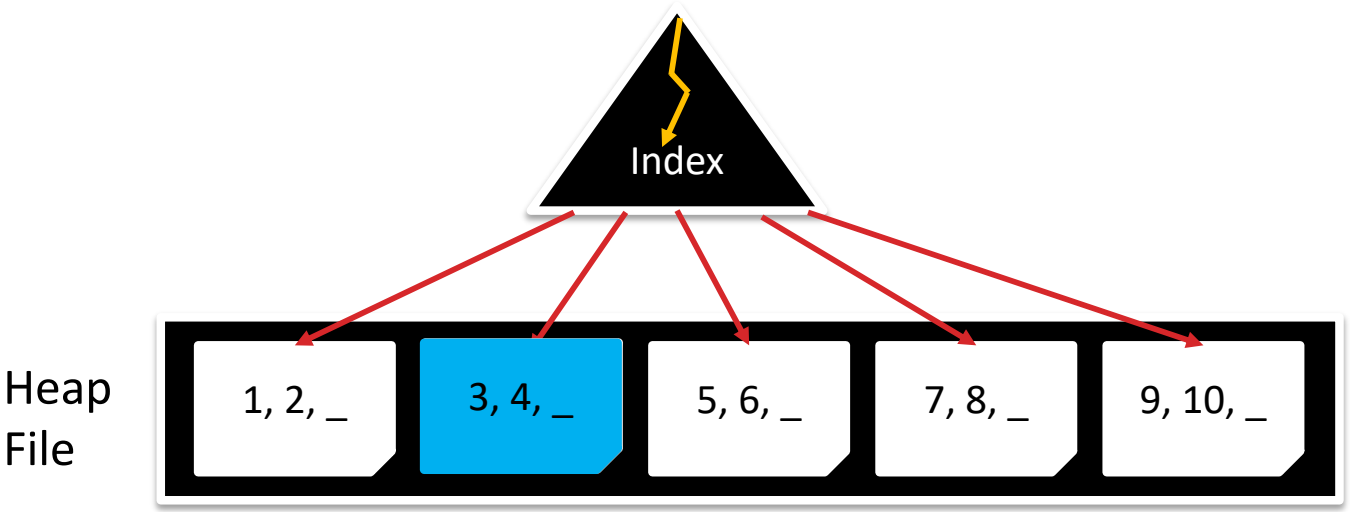
- **E:** Average # data entries per leaf

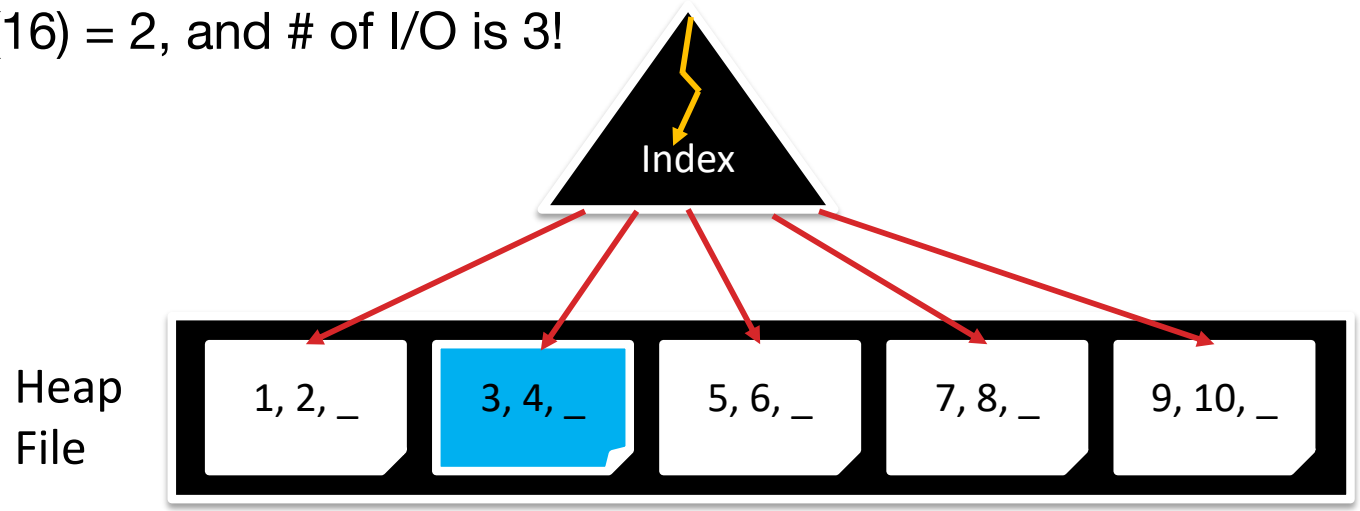
Find the record with key 3

- Two steps
  1. Search index to find the page and record-id
  2. Fetch record-id from heap file



Find the record with key 3

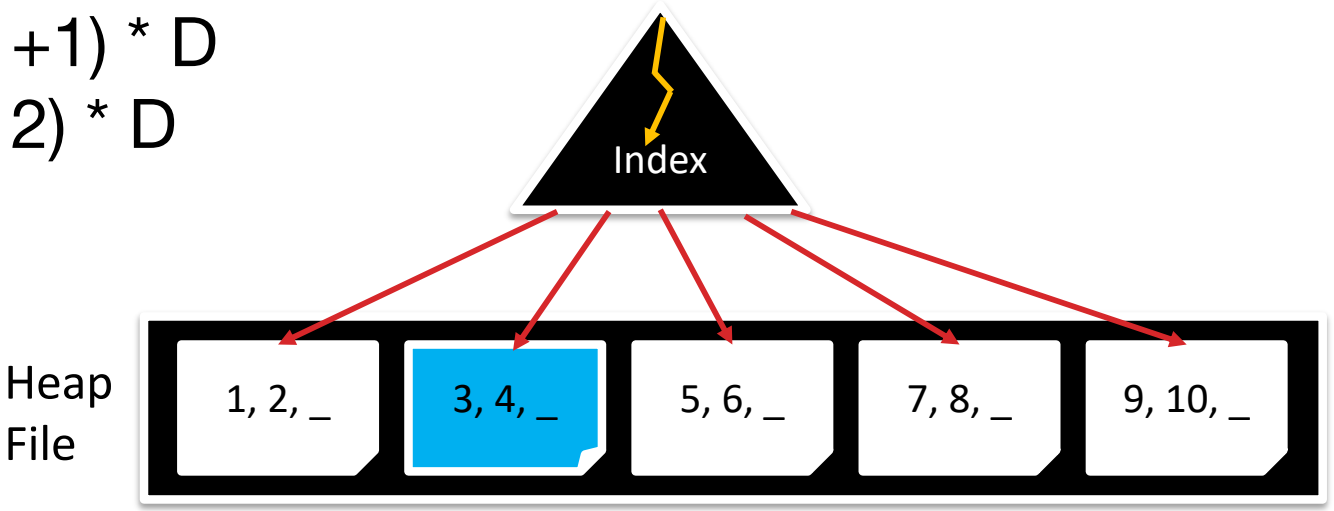
- I/Os for index search = height of index + 1
- = $\log_F$ (# of leaves) + 1 = $\log_F$ (B*R/E) + 1
  - B*R is the total number of records; E is the #records per leaf
  - Why +1? Catches the cost of fetching the leaf from index
    - E.g., F = 4, BR/E = 16: root → intermediate → leaf
    - But $\text{Log}_4(16) = 2$, and # of I/O is 3!



Find the record with key 3

- I/Os for lookup record in heap file by record-id: 1
  - Recall record-id = <page, slot #>

- Total cost: (# of I/Os) * D
  = (I/Os for index search + I/Os for heap file read) * D
  = ($\log_F$ (BR/E)+1+1) * D
  = ($\log_F$ (BR/E) + 2) * D



Cost of Operations: Equality Search

	Heap File	Sorted File	Clustered Index
Scan all records	B*D	B*D	3/2 * B * D
Equality Search	0.5*B*D	$(\log_2 B)*D$	$(\log_F(BR/E)+2)*D$
Range Search	B*D	$((\log_2 B)+pages))*D$	
Insert	2*D	$((\log_2 B) + B)*D$	
Delete	(0.5*B+1)*D	$((\log_2 B) + B)*D$	

- **B:** Number of data blocks
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Cost of Operations: Range Search?

	Heap File	Sorted File	Clustered Index
Scan all records	B*D	B*D	3/2 * B * D
Equality Search	0.5*B*D	$(\log_2 B)*D$	$(\log_F(BR/E)+2)*D$
Range Search	B*D	$((\log_2 B)+pages))*D$	
Insert	2*D	$((\log_2 B) + B)*D$	
Delete	(0.5*B+1)*D	$((\log_2 B) + B)*D$	

- **B:** Number of data blocks
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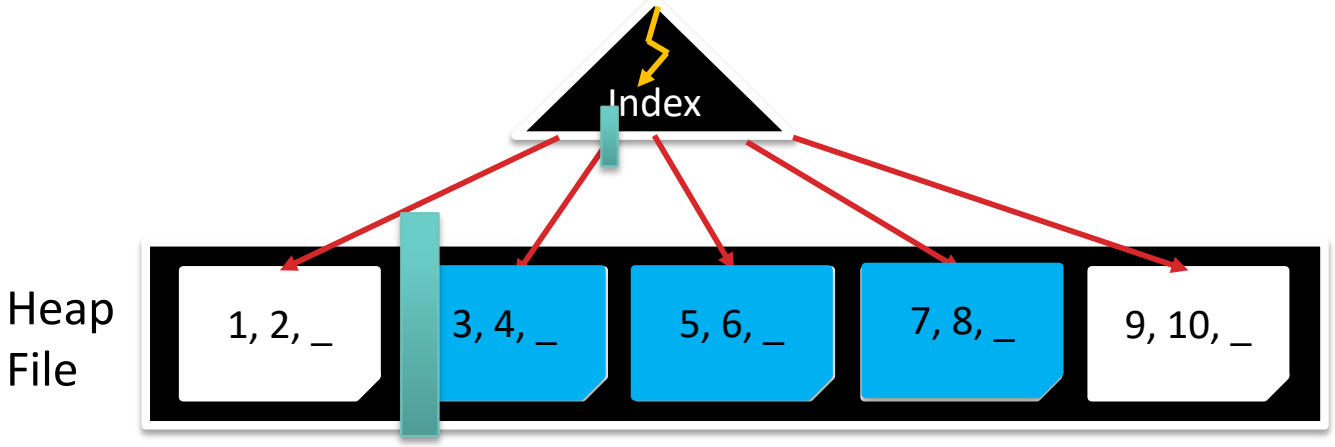
- **E:** Average # data entries per leaf

Find records with keys between 3 and 7

- Three steps:
  1. Search index to find first page to read
  2. Scan index leaf pages to find which heap file pages to read
  3. Read the corresponding records from heap file
- I/Os for 1: $\log_F (B*R/E) + 1$ [+1 for the index leaf page]
- I/Os for 3: (3/2 * #pages) [#pages: # of pages storing records between 3 and 7]
- I/Os for 2: (3/2 * #pages) [over-approximate and assume same as 2]
- Total cost : $((\log_F (B*R/E) + 1) + 2 * (3/2 * \#pages) - 1) * D = (\log_F (B*R/E) + 3 * \#pages) * D$
  - Why -1? We overcounted accessing the first leaf page in the index



Cost of Operations: Range Search

	Heap File	Sorted File	Clustered Index
Scan all records	B*D	B*D	3/2 * B * D
Equality Search	0.5*B*D	$(\log_2 B)*D$	$(\log_F(BR/E)+2)*D$
Range Search	B*D	$((\log_2 B)+pages))*D$	$(\log_F(BR/E)+3*pages)*D$
Insert	2*D	$((\log_2 B) + B)*D$	
Delete	(0.5*B+1)*D	$((\log_2 B) + B)*D$	

- **B:** Number of data blocks
- **R:** Number of records per block
- **D:** Average time to read/write disk block
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- **E:** Average # data entries per leaf

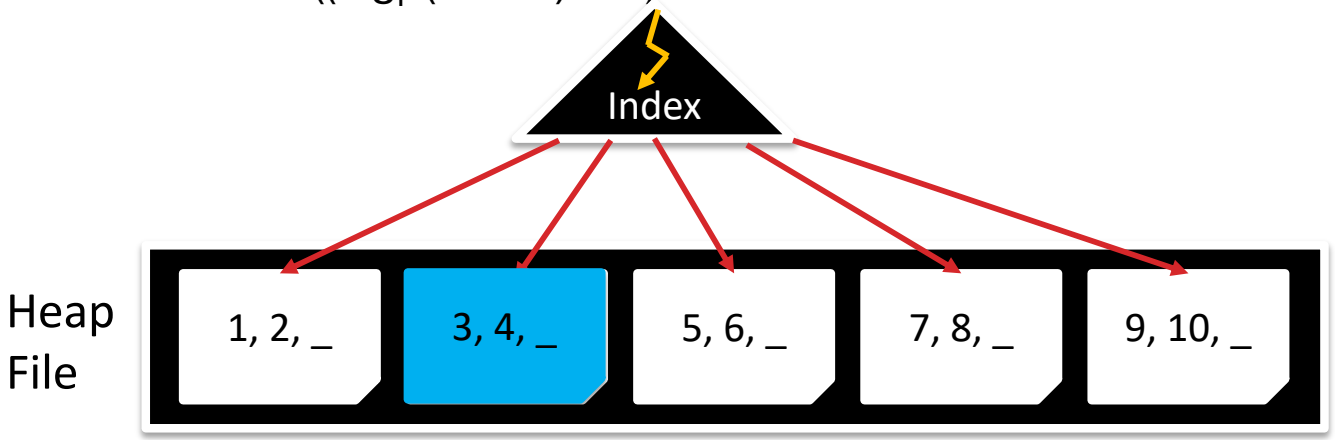
Cost of Operations: Insert?

	Heap File	Sorted File	Clustered Index
Scan all records	B*D	B*D	3/2 * B * D
Equality Search	0.5*B*D	$(\log_2 B)*D$	$(\log_F(BR/E)+2)*D$
Range Search	B*D	$((\log_2 B)+pages))*D$	$(\log_F(BR/E)+3*pages)*D$
Insert	2*D	$((\log_2 B) + B)*D$	
Delete	(0.5*B+1)*D	$((\log_2 B) + B)*D$	

- **B:** Number of data blocks
- **R:** Number of records per block
- **D:** Average time to read/write disk block
- **F:** Average internal node fanout
- **E:** Average # data entries per leaf

Insert record with key 4.5

- Three steps:
  1. Search index to find heap file page to modify
  2. Read corresponding page and modify
  3. Write back the modified index leaf and heap file pages
- I/Os for 1: $\log_F (B*R/E) + 1$
- I/Os for 2: 1
- I/Os for 3: 2 [1 for index leaf, 1 for heap file page]
- Total cost : $((\log_F (B*R/E) + 4) * D$



Cost of Operations: Insert

	Heap File	Sorted File	Clustered Index
Scan all records	B*D	B*D	3/2 * B * D
Equality Search	0.5*B*D	$(\log_2 B)*D$	$(\log_F(BR/E)+2)*D$
Range Search	B*D	$((\log_2 B)+pages))*D$	$(\log_F(BR/E)+3*pages)*D$
Insert	2*D	$((\log_2 B) + B)*D$	$(\log_F(BR/E)+4)*D$
Delete	(0.5*B+1)*D	$((\log_2 B) + B)*D$	

- **B:** Number of data blocks
- **R:** Number of records per block
- **D:** Average time to read/write disk block
- **F:** Average internal node fanout
- **E:** Average # data entries per leaf

Cost of Operations: Delete

	Heap File	Sorted File	Clustered Index
Scan all records	B*D	B*D	3/2 * B * D
Equality Search	0.5*B*D	$(\log_2 B)*D$	$(\log_F(BR/E)+2)*D$
Range Search	B*D	$((\log_2 B)+pages))*D$	$(\log_F(BR/E)+3*pages)*D$
Insert	2*D	$((\log_2 B) + B)*D$	$(\log_F(BR/E)+4)*D$
Delete	(0.5*B+1)*D	$((\log_2 B) + B)*D$	$(\log_F(BR/E)+4)*D$

- **B:** Number of data blocks
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- **F:** Average internal node fanout
- **E:** Average # data entries per leaf

Cost of Operations: Big O Notation

	Heap File	Sorted File	Clustered Index
Scan all records	O(B)	O(B)	O(B)
Equality Search	O(B)	$O(\log_2 B)$	$O(\log_F B)$
Range Search	O(B)	$O(\log_2 B)$	$O(\log_F B)$
Insert	O(1)	O(B)	$O(\log_F B)$
Delete	O(B)	O(B)	$O(\log_F B)$

- **B:** Number of data blocks
- **R:** Number of records per block
- **D:** Average time to read/write disk block
- **F:** Average internal node fanout
- **E:** Average # data entries per leaf

Constant factors

- Assume you can do 100 sequential I/Os in the time of 1 random I/O

- For a particular lookup, is a B+-tree better than a full-table scan?
  - Better be very “selective”!
    - Visit < ~1% of pages!
  - Two ways to make that happen:
    - Use a clustered index so that most reads are sequential
    - Use SSD so that random and sequential reads have the same cost

Summary

- Query Structure
  - Understand composite search keys
  - Lexicographic order and search key prefixes
- Data Storage
  - Data Entries: Alt 1 (tuples), Alt 2 (recordIds), Alt 3 (lists of recordIds)
  - Clustered vs. Unclustered
    - Only Alt 2 & 3!

Summary

- Variable length key refinements
  - Fill factors for variable-length keys
  - Prefix and suffix key compression
- B+-tree Cost Model
  - Attractive big-O
  - But don’t forget constant factors of random I/O
    - Hard to beat sequential I/O of scans unless very selective
  - Indexes beyond B+-trees for more complex searches