

CS 188: Artificial Intelligence

HMMs, Particle Filters, and Applications



[These slides were created by Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley. All CS188 materials are available at <http://ai.berkeley.edu>.]

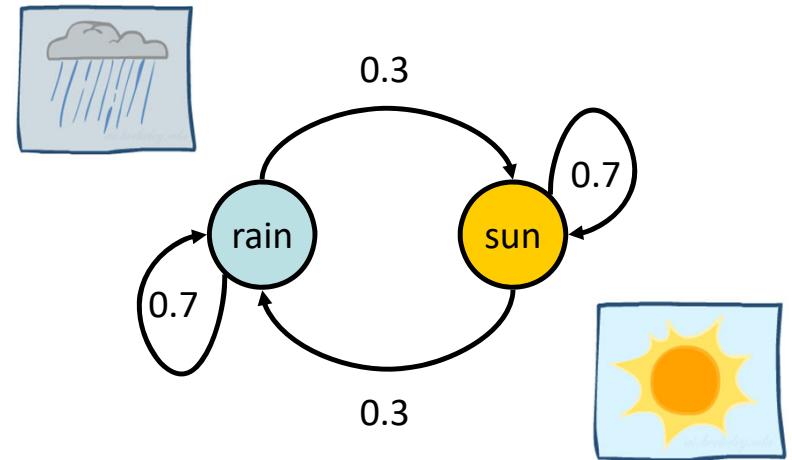
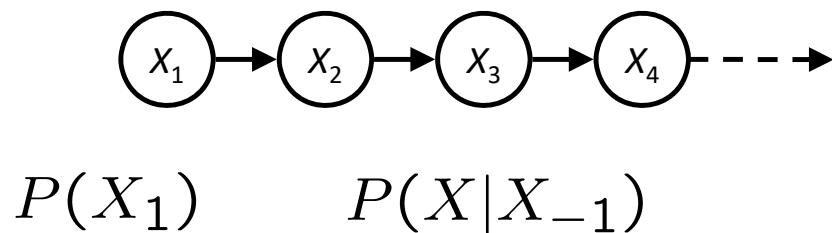
Today

- HMMs
 - Particle filters
 - Demos!
 - Most-likely-explanation queries
- Applications:
 - Robot localization / mapping
 - Speech recognition (later)

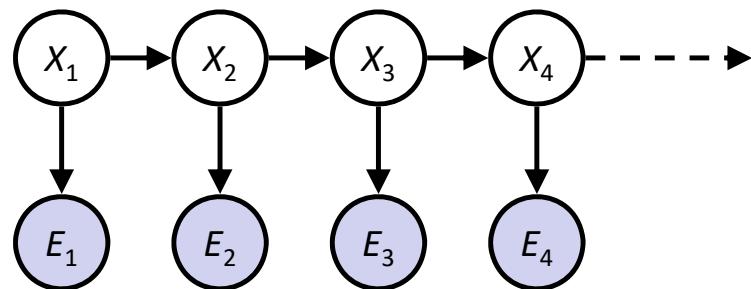
[Demo: Ghostbusters Markov Model (L15D1)]

Recap: Reasoning Over Time

- Markov models



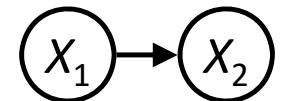
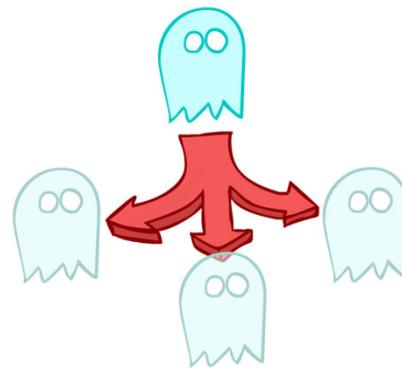
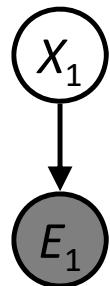
- Hidden Markov models



$$P(E|X)$$

X	E	P
rain	umbrella	0.9
rain	no umbrella	0.1
sun	umbrella	0.2
sun	no umbrella	0.8

Inference: Base Cases



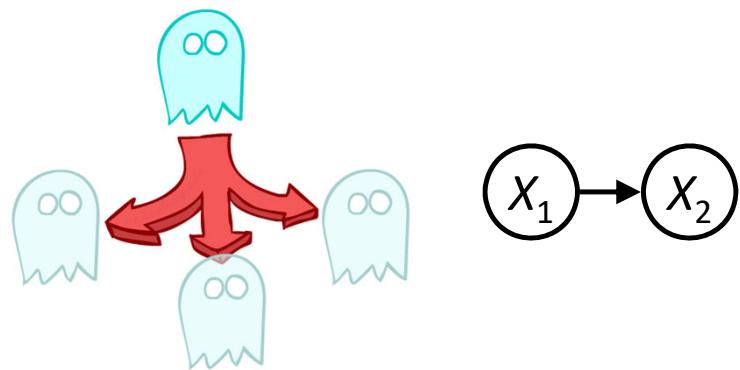
$$P(X_1|e_1)$$

$$\begin{aligned} P(x_1|e_1) &= P(x_1, e_1)/P(e_1) \\ &\propto_{X_1} P(x_1, e_1) \\ &= P(x_1)P(e_1|x_1) \end{aligned}$$

$$P(X_2)$$

$$\begin{aligned} P(x_2) &= \sum_{x_1} P(x_1, x_2) \\ &= \sum_{x_1} P(x_1)P(x_2|x_1) \end{aligned}$$

Inference: Base Cases



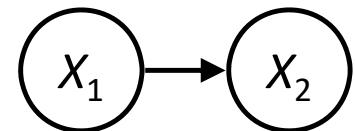
$$P(X_2)$$

$$\begin{aligned} P(x_2) &= \sum_{x_1} P(x_1, x_2) \\ &= \sum_{x_1} P(x_1)P(x_2|x_1) \end{aligned}$$

Passage of Time

- Assume we have current belief $P(X | \text{evidence to date})$

$$B(X_t) = P(X_t | e_{1:t})$$



- Then, after one time step passes:

$$\begin{aligned} P(X_{t+1} | e_{1:t}) &= \sum_{x_t} P(X_{t+1}, x_t | e_{1:t}) \\ &= \sum_{x_t} P(X_{t+1} | x_t, e_{1:t}) \color{red}{P(x_t | e_{1:t})} \\ &= \sum_{x_t} P(X_{t+1} | x_t) \color{red}{P(x_t | e_{1:t})} \end{aligned}$$

- Basic idea: beliefs get “pushed” through the transitions

- With the “B” notation, we have to be careful about what time step t the belief is about, and what evidence it includes

- Or compactly:

$$B'(X_{t+1}) = \sum_{x_t} P(X' | x_t) \color{red}{B(x_t)}$$

Example: Passage of Time

- As time passes, uncertainty “accumulates”

<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
<0.01	<0.01	1.00	<0.01	<0.01	<0.01
<0.01	<0.01	<0.01	<0.01	<0.01	<0.01

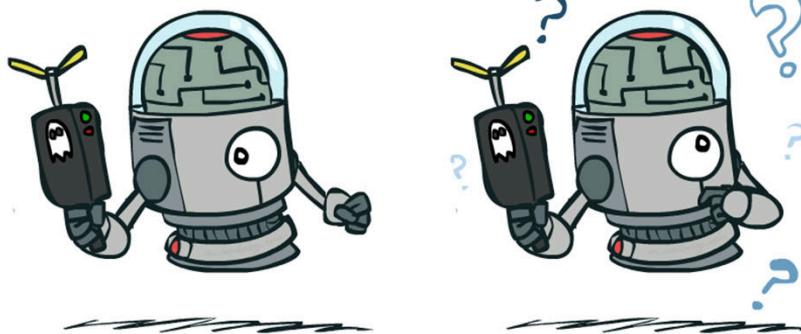
$T = 1$

<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
<0.01	<0.01	0.06	<0.01	<0.01	<0.01
<0.01	0.76	0.06	0.06	<0.01	<0.01
<0.01	<0.01	0.06	<0.01	<0.01	<0.01

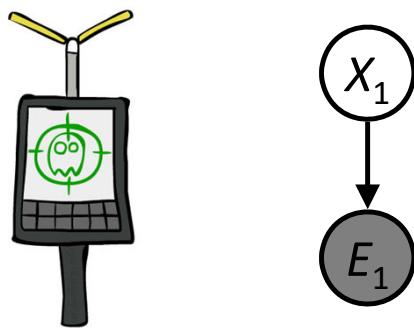
$T = 2$

0.05	0.01	0.05	<0.01	<0.01	<0.01
0.02	0.14	0.11	0.35	<0.01	<0.01
0.07	0.03	0.05	<0.01	0.03	<0.01
0.03	0.03	<0.01	<0.01	<0.01	<0.01

$T = 5$



Inference: Base Cases



$$P(X_1|e_1)$$

$$\begin{aligned} P(x_1|e_1) &= P(x_1, e_1)/P(e_1) \\ &\propto_{X_1} P(x_1, e_1) \\ &= P(x_1)P(e_1|x_1) \end{aligned}$$

Observation

- Assume we have current belief $P(X \mid \text{previous evidence})$:

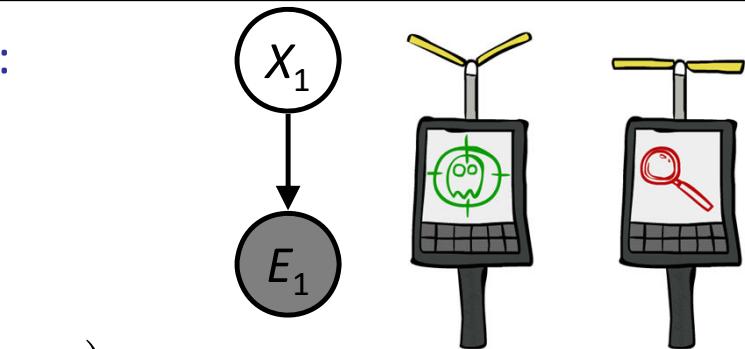
$$B'(X_{t+1}) = P(X_{t+1} | e_{1:t})$$

- Then, after evidence comes in:

$$\begin{aligned} P(X_{t+1} | e_{1:t+1}) &= P(X_{t+1}, e_{t+1} | e_{1:t}) / P(e_{t+1} | e_{1:t}) \\ &\propto_{X_{t+1}} P(X_{t+1}, e_{t+1} | e_{1:t}) \\ &= P(e_{t+1} | e_{1:t}, X_{t+1}) B'(X_{t+1}) \\ &= P(e_{t+1} | X_{t+1}) B'(X_{t+1}) \end{aligned}$$

- Or, compactly:

$$B(X_{t+1}) \propto_{X_{t+1}} P(e_{t+1} | X_{t+1}) B'(X_{t+1})$$



- Basic idea: beliefs “reweighted” by likelihood of evidence
- Unlike passage of time, we have to renormalize

Example: Observation

- As we get observations, beliefs get reweighted, uncertainty “decreases”

0.05	0.01	0.05	<0.01	<0.01	<0.01
0.02	0.14	0.11	0.35	<0.01	<0.01
0.07	0.03	0.05	<0.01	0.03	<0.01
0.03	0.03	<0.01	<0.01	<0.01	<0.01

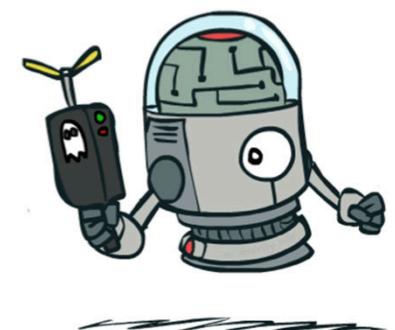
Before observation

<0.01	<0.01	<0.01	<0.01	0.02	<0.01
<0.01	<0.01	<0.01	0.83	0.02	<0.01
<0.01	<0.01	0.11	<0.01	<0.01	<0.01
<0.01	<0.01	<0.01	<0.01	<0.01	<0.01

After observation



$$B(X) \propto P(e|X)B'(X)$$



Filtering

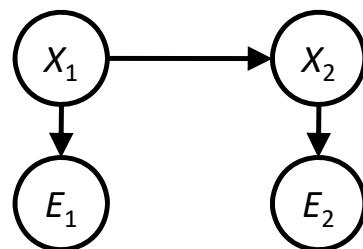
Elapse time: compute $P(X_t | e_{1:t-1})$

$$P(x_t | e_{1:t-1}) = \sum_{x_{t-1}} P(x_{t-1} | e_{1:t-1}) \cdot P(x_t | x_{t-1})$$

Observe: compute $P(X_t | e_{1:t})$

$$P(x_t | e_{1:t}) \propto P(x_t | e_{1:t-1}) \cdot P(e_t | x_t)$$

<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
<0.01	<0.01	0.06	<0.01	<0.01	<0.01
<0.01	0.76	0.06	0.06	<0.01	<0.01
<0.01	<0.01	0.06	<0.01	<0.01	<0.01



Belief: $\langle P(\text{rain}), P(\text{sun}) \rangle$

$P(X_1)$ <0.5, 0.5> *Prior on X_1*

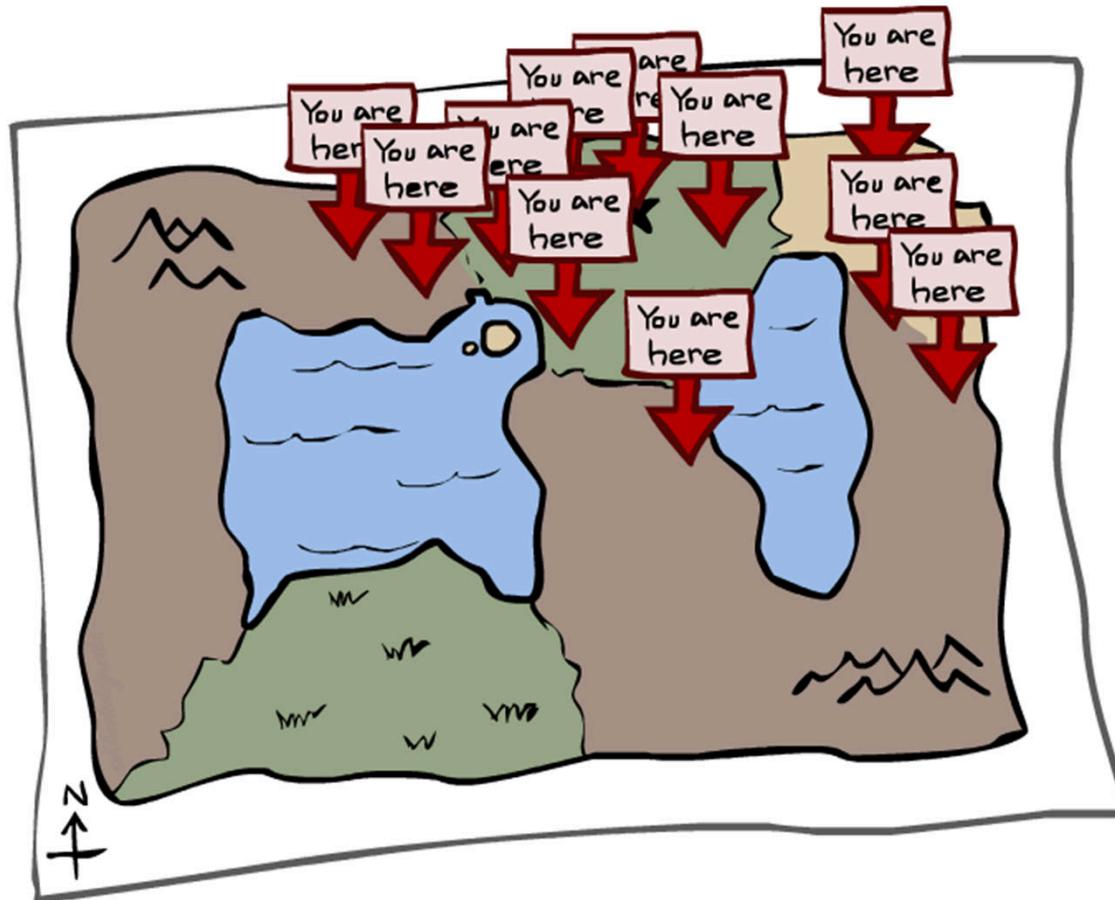
$P(X_1 | E_1 = \text{umbrella})$ <0.82, 0.18> *Observe*

$P(X_2 | E_1 = \text{umbrella})$ <0.63, 0.37> *Elapse time*

$P(X_2 | E_1 = \text{umb}, E_2 = \text{umb})$ <0.88, 0.12> *Observe*

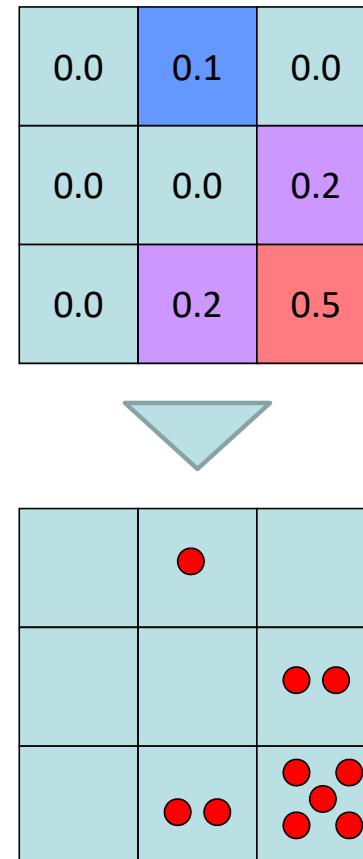
[Demo: Ghostbusters Exact Filtering (L15D2)]

Particle Filtering



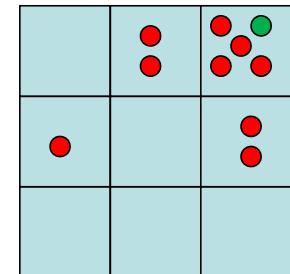
Particle Filtering

- Filtering: approximate solution
- Sometimes $|X|$ is too big to use exact inference
 - $|X|$ may be too big to even store $B(X)$
 - E.g. X is continuous
- Solution: approximate inference
 - Track samples of X , not all values
 - Samples are called particles
 - Time per step is linear in the number of samples
 - But: number needed may be large
 - In memory: list of particles, not states
- This is how robot localization works in practice
- Particle is just new name for sample



Representation: Particles

- Our representation of $P(X)$ is now a list of N particles (samples)
 - Generally, $N \ll |X|$
 - Storing map from X to counts would defeat the point
- $P(x)$ approximated by number of particles with value x
 - So, many x may have $P(x) = 0$!
 - More particles, more accuracy
- For now, all particles have a weight of 1



Particles:
(3,3)
(2,3)
(3,3)
(3,2)
(3,3)
(3,2)
(1,2)
(3,3)
(3,3)
(2,3)

Particle Filtering: Elapse Time

- Each particle is moved by sampling its next position from the transition model

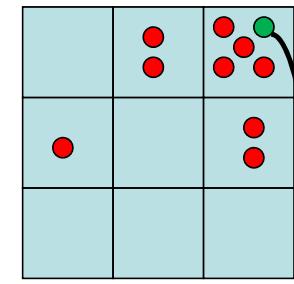
$$x' = \text{sample}(P(X'|x))$$

- This is like prior sampling – samples' frequencies reflect the transition probabilities
- Here, most samples move clockwise, but some move in another direction or stay in place

- This captures the passage of time
 - If enough samples, close to exact values before and after (consistent)

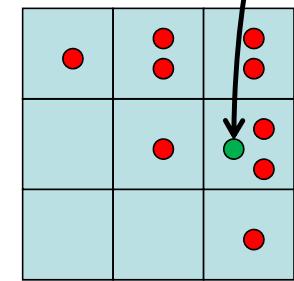
Particles:

- (3,3)
- (2,3)
- (3,3)
- (3,2)
- (3,3)
- (3,2)
- (1,2)
- (3,3)
- (3,3)
- (2,3)



Particles:

- (3,2)
- (2,3)
- (3,2)
- (3,1)
- (3,3)
- (3,2)
- (1,3)
- (2,3)
- (3,2)
- (2,2)



Particle Filtering: Observe

- Slightly trickier:

- Don't sample observation, fix it
- Similar to likelihood weighting, downweight samples based on the evidence

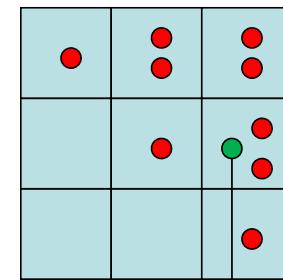
$$w(x) = P(e|x)$$

$$B(X) \propto P(e|X)B'(X)$$

- As before, the probabilities don't sum to one, since all have been downweighted (in fact they now sum to (N times) an approximation of $P(e)$)

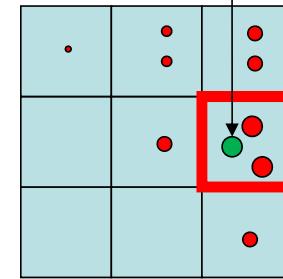
Particles:

(3,2)
(2,3)
(3,2)
(3,1)
(3,3)
(3,2)
(1,3)
(2,3)
(3,2)
(2,2)



Particles:

(3,2) w=.9
(2,3) w=.2
(3,2) w=.9
(3,1) w=.4
(3,3) w=.4
(3,2) w=.9
(1,3) w=.1
(2,3) w=.2
(3,2) w=.9
(2,2) w=.4

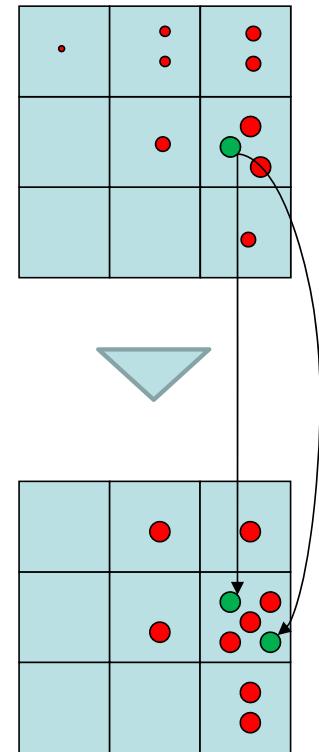


Particle Filtering: Resample

- Rather than tracking weighted samples, we resample
- N times, we choose from our weighted sample distribution (i.e. draw with replacement)
- This is equivalent to renormalizing the distribution
- Now the update is complete for this time step, continue with the next one

Particles:

(3,2) w=.9
(2,3) w=.2
(3,2) w=.9
(3,1) w=.4
(3,3) w=.4
(3,2) w=.9
(1,3) w=.1
(2,3) w=.2
(3,2) w=.9
(2,2) w=.4

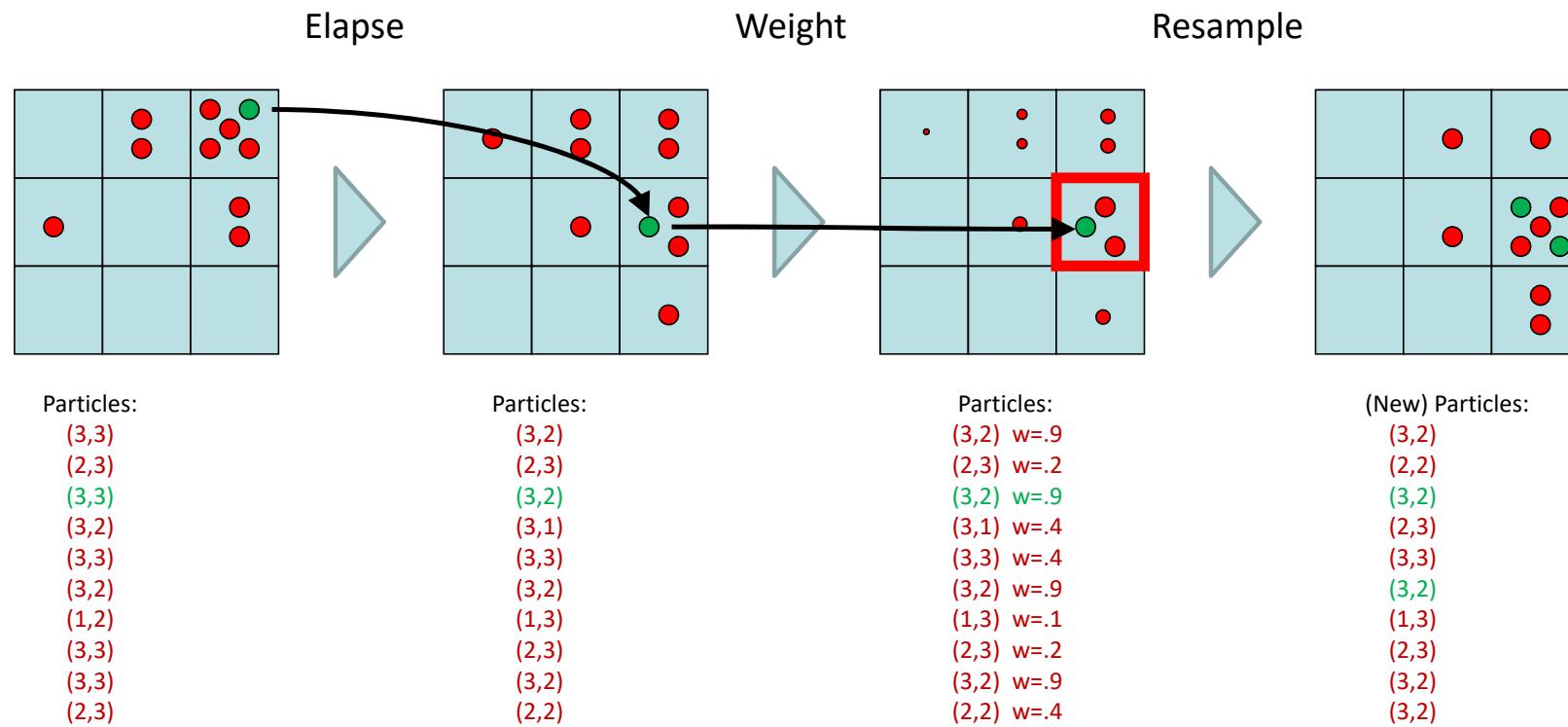


(New) Particles:

(3,2)
(2,2)
(3,2)
(2,3)
(3,3)
(3,2)
(1,3)
(2,3)
(3,2)
(3,2)

Recap: Particle Filtering

- Particles: track samples of states rather than an explicit distribution

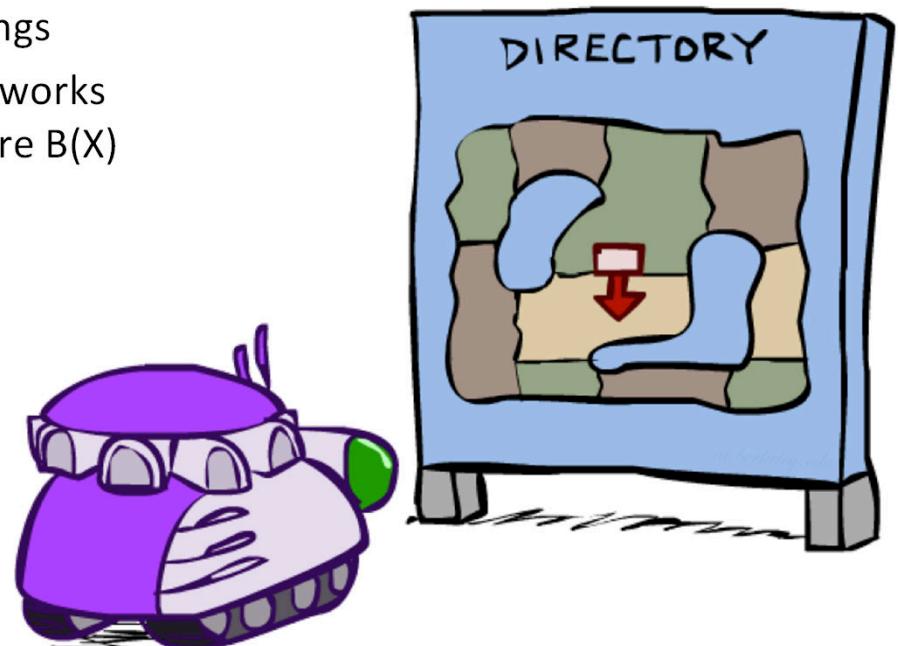
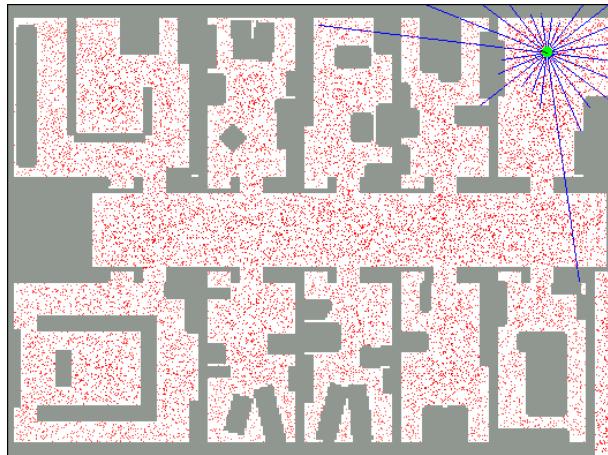


[Demos: ghostbusters particle filtering (L15D3,4,5)]

Robot Localization

- In robot localization:

- We know the map, but not the robot's position
- Observations may be vectors of range finder readings
- State space and readings are typically continuous (works basically like a very fine grid) and so we cannot store $B(X)$
- Particle filtering is a main technique

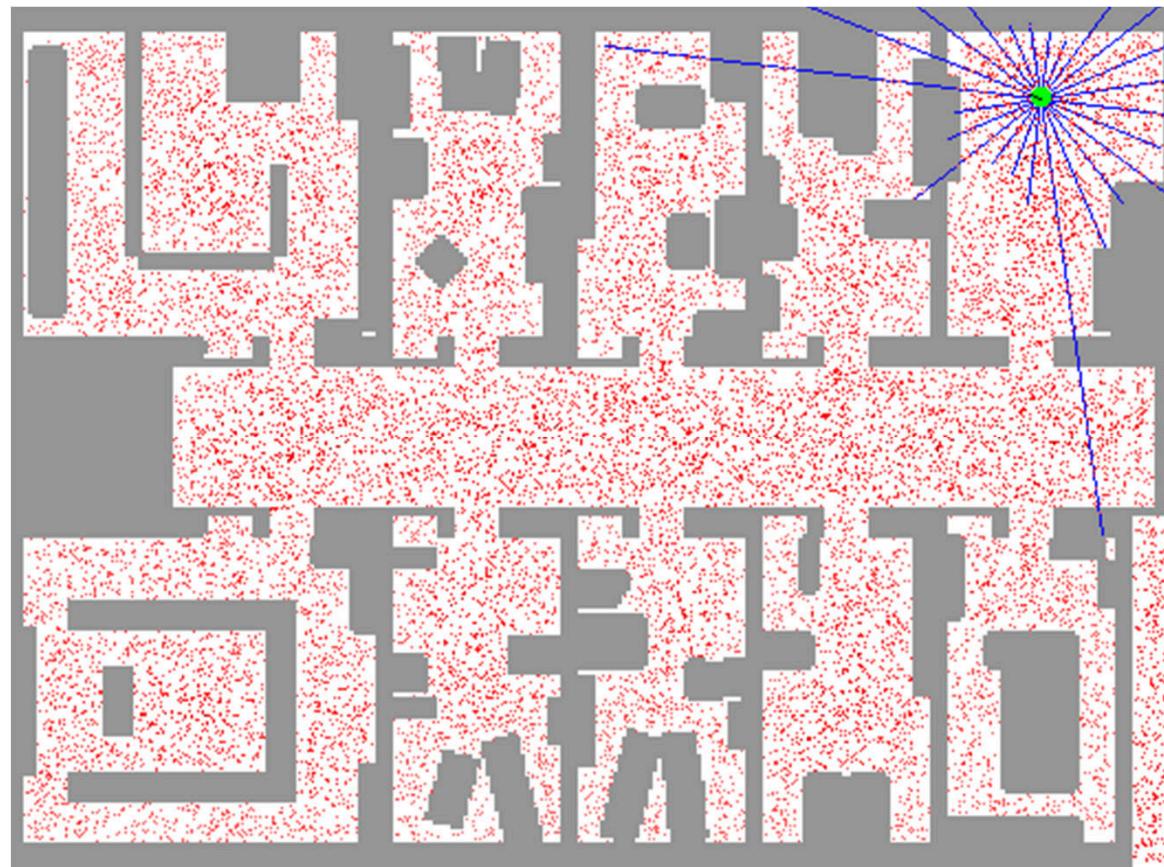


Particle Filter Localization (Sonar)



[Video: global-sonar-uw-annotated.avi]

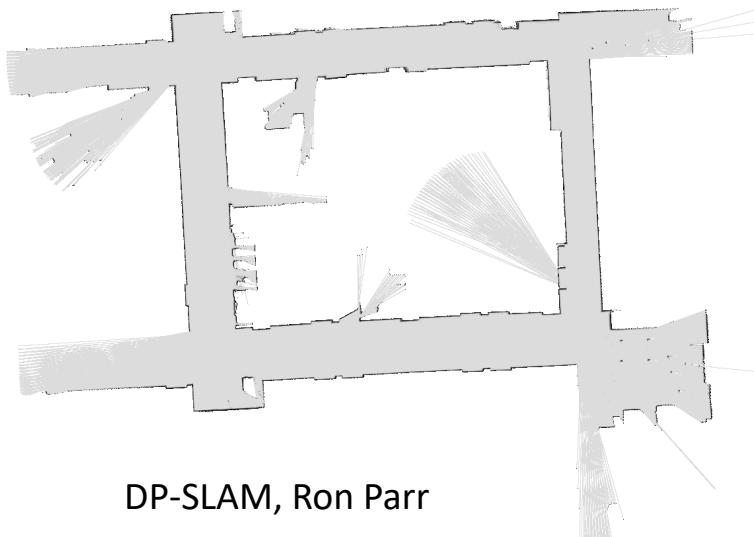
Particle Filter Localization (Laser)



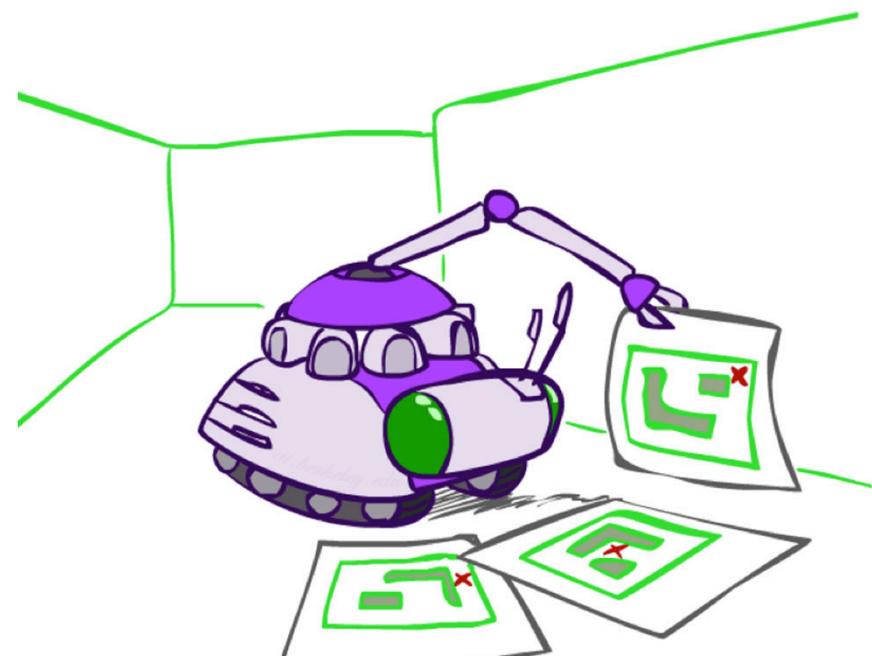
[Video: global-floor.gif]

Robot Mapping

- SLAM: Simultaneous Localization And Mapping
 - We do not know the map or our location
 - State consists of position AND map!
 - Main techniques: Kalman filtering (Gaussian HMMs) and particle methods

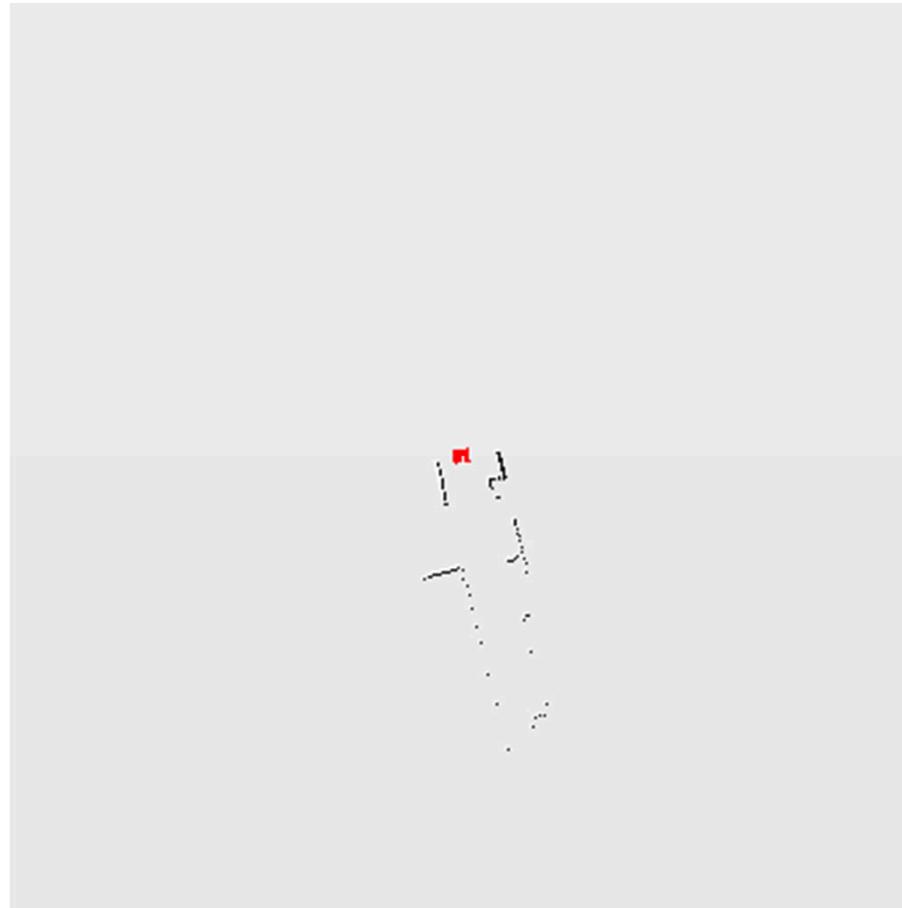


DP-SLAM, Ron Parr



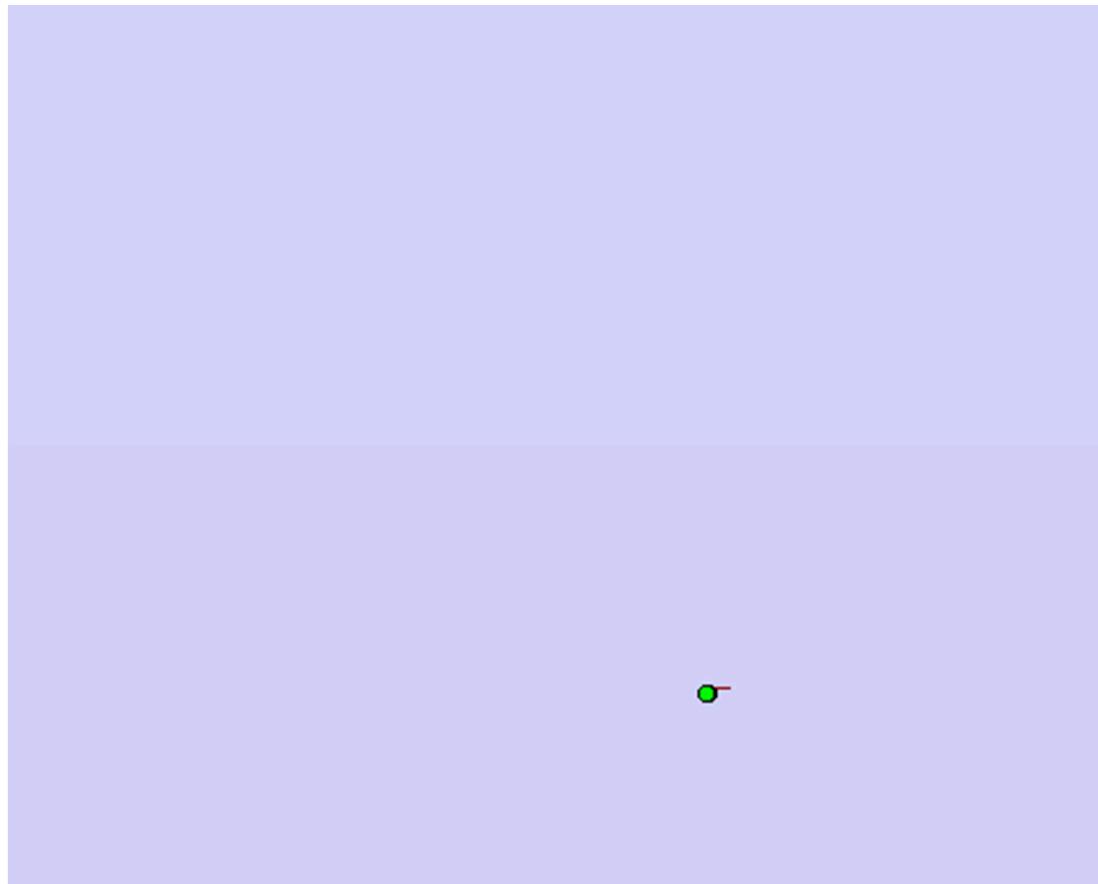
[Demo: PARTICLES-SLAM-mapping1-new.avi]

Particle Filter SLAM – Video 1



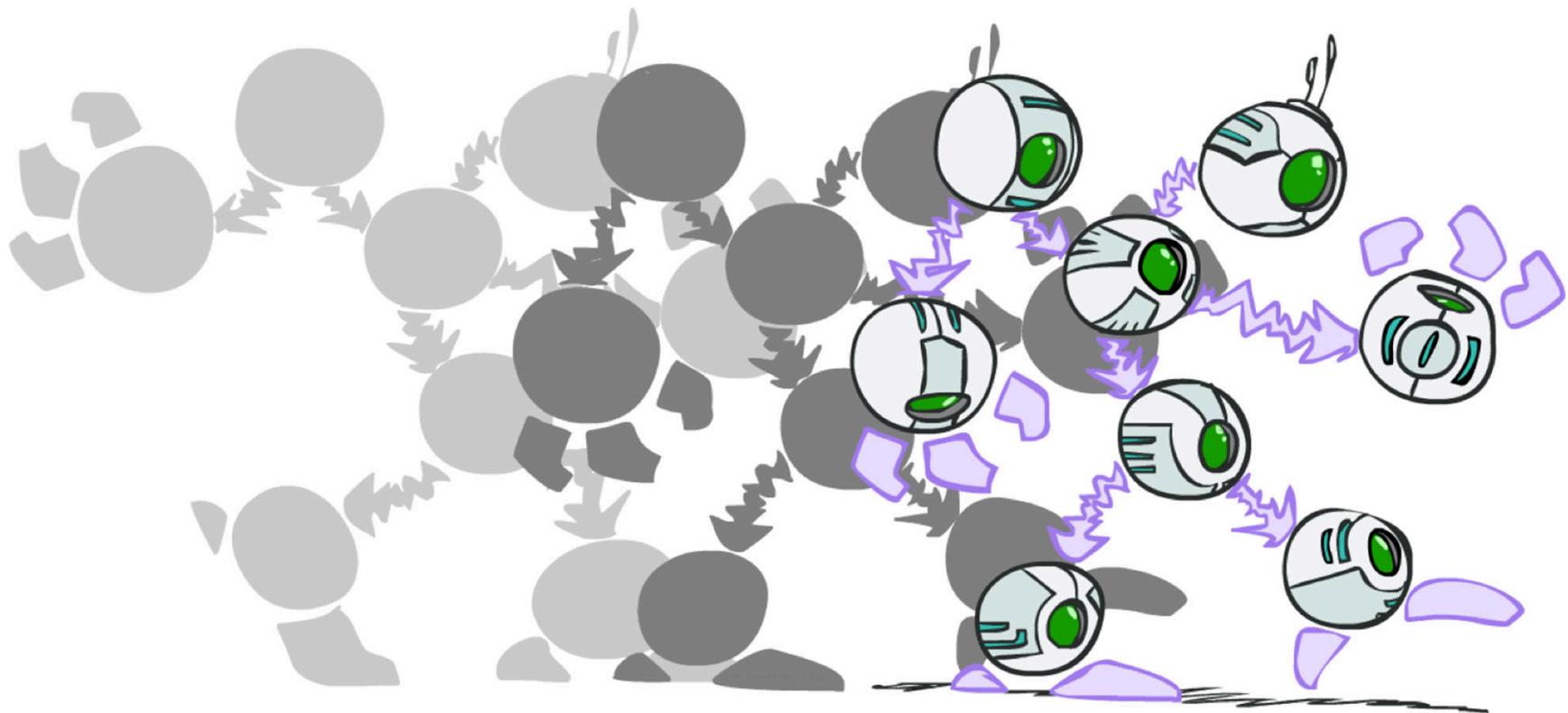
[Demo: PARTICLES-SLAM-mapping1-new.avi]

Particle Filter SLAM – Video 2



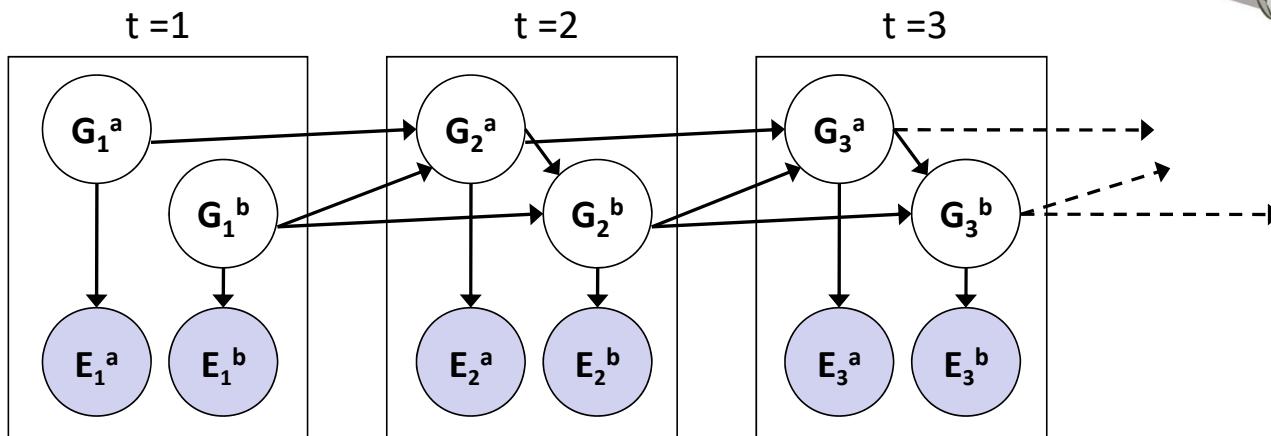
[Demo: PARTICLES-SLAM-fastslam.avi]

Dynamic Bayes Nets



Dynamic Bayes Nets (DBNs)

- We want to track multiple variables over time, using multiple sources of evidence
- Idea: Repeat a fixed Bayes net structure at each time
- Variables from time t can condition on those from $t-1$



- Dynamic Bayes nets are a generalization of HMMs

[Demo: pacman sonar ghost DBN model (L15D6)]

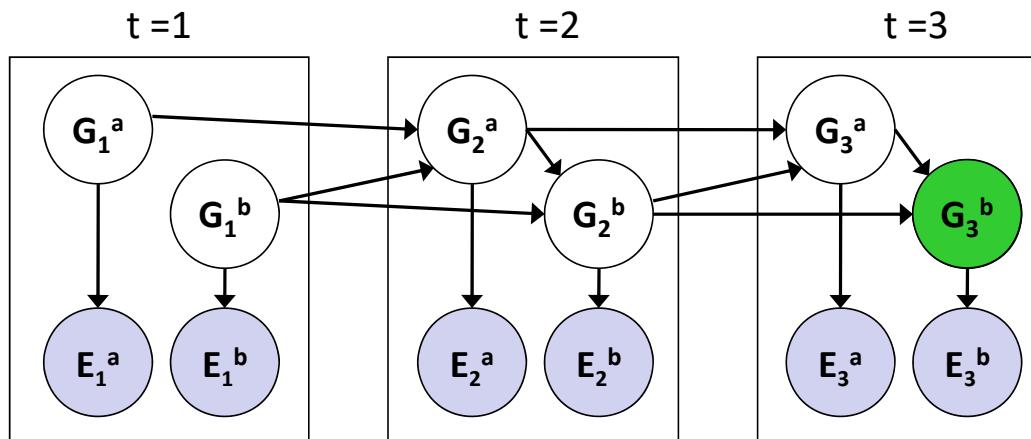
Pacman – Sonar (P4)



[Demo: Pacman – Sonar – No Beliefs(L14D1)]

Exact Inference in DBNs

- Variable elimination applies to dynamic Bayes nets
- Procedure: “unroll” the network for T time steps, then eliminate variables until $P(X_T | e_{1:T})$ is computed



- Online belief updates: Eliminate all variables from the previous time step; store factors for current time only

DBN Particle Filters

- A particle is a complete sample for a time step
- **Initialize:** Generate prior samples for the t=1 Bayes net
 - Example particle: $\mathbf{G}_1^a = (3,3)$ $\mathbf{G}_1^b = (5,3)$
- **Elapse time:** Sample a successor for each particle
 - Example successor: $\mathbf{G}_2^a = (2,3)$ $\mathbf{G}_2^b = (6,3)$
- **Observe:** Weight each entire sample by the likelihood of the evidence conditioned on the sample
 - Likelihood: $P(E_1^a | \mathbf{G}_1^a) * P(E_1^b | \mathbf{G}_1^b)$
- **Resample:** Select prior samples (tuples of values) in proportion to their likelihood

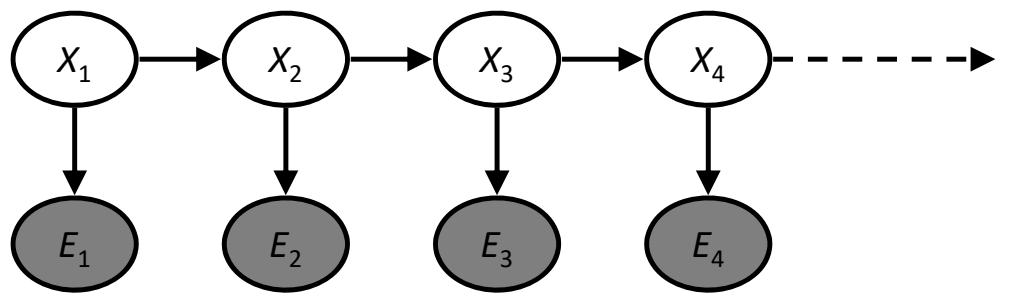
Most Likely Explanation



HMMs: MLE Queries

- HMMs defined by

- States X
- Observations E
- Initial distribution: $P(X_1)$
- Transitions: $P(X|X_{-1})$
- Emissions: $P(E|X)$



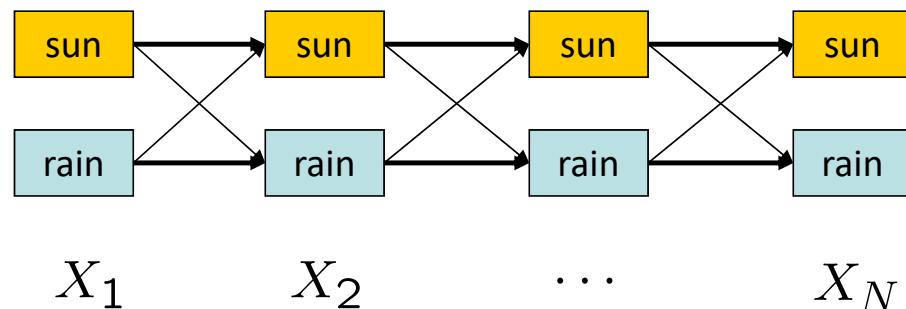
- New query: most likely explanation:

$$\arg \max_{x_{1:t}} P(x_{1:t}|e_{1:t})$$

- New method: the Viterbi algorithm

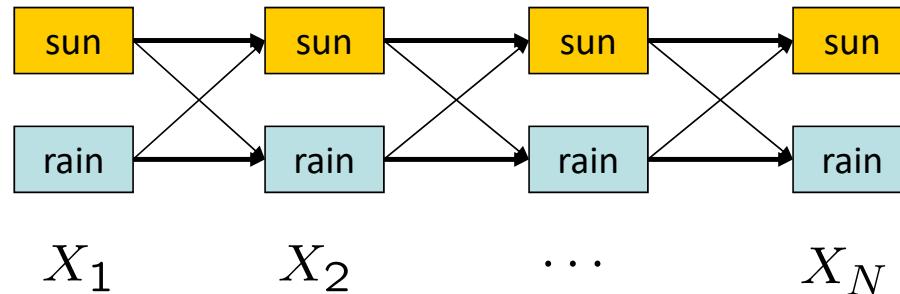
State Trellis

- State trellis: graph of states and transitions over time



- Each arc represents some transition $x_{t-1} \rightarrow x_t$
- Each arc has weight $P(x_t|x_{t-1})P(e_t|x_t)$
- Each path is a sequence of states
- The product of weights on a path is that sequence's probability along with the evidence
- Forward algorithm computes sums of paths, Viterbi computes best paths

Forward / Viterbi Algorithms



Forward Algorithm (Sum)

$$f_t[x_t] = P(x_t, e_{1:t})$$

$$= P(e_t|x_t) \sum_{x_{t-1}} P(x_t|x_{t-1}) f_{t-1}[x_{t-1}]$$

Viterbi Algorithm (Max)

$$m_t[x_t] = \max_{x_{1:t-1}} P(x_{1:t-1}, x_t, e_{1:t})$$

$$= P(e_t|x_t) \max_{x_{t-1}} P(x_t|x_{t-1}) m_{t-1}[x_{t-1}]$$