Lecture 5: Value Function Approximation

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CS234 Reinforcement Learning.

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The value function approximation structure for today closely follows much of David Silver's Lecture 6.

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Class Structure

- Last time: Control (making decisions) without a model of how the world works
- **This time: Value function approximation**
- Next time: Deep reinforcement learning

Last time: Model-Free Control

- Last time: how to learn a good policy from experience
- So far, have been assuming we can represent the value function or state-action value function as a vector/ matrix
  - Tabular representation
- Many real world problems have enormous state and/or action spaces
- Tabular representation is insufficient

Recall: Reinforcement Learning Involves

- Optimization
- Delayed consequences
- Exploration
- Generalization

Today: Focus on Generalization

- Optimization
- Delayed consequences
- Exploration
- **Generalization**

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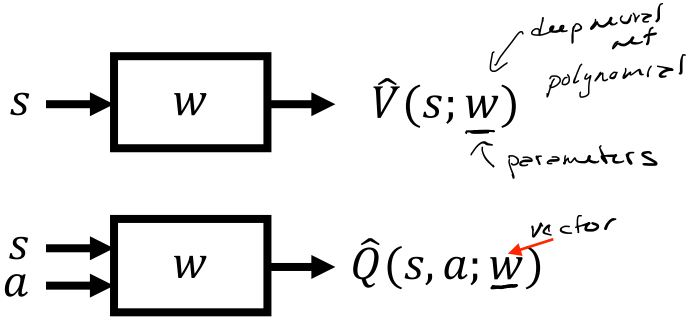
Value Function Approximation (VFA)

- Represent a (state-action/state) value function with a parameterized function instead of a table



Motivation for VFA

- Don't want to have to explicitly store or learn for every single state a
  - Dynamics or reward model
  - Value
  - State-action value
  - Policy
- Want more compact representation that generalizes across state or states and actions

Benefits of Generalization

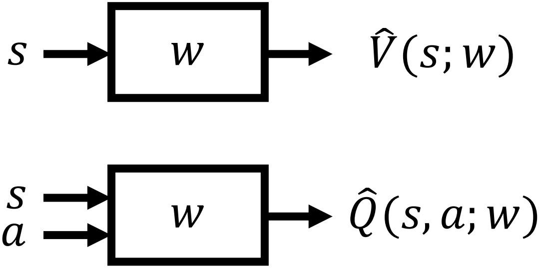
representational capacity vs memory computation data

- Reduce memory needed to store $(P, R)/V/Q/\pi$
- Reduce computation needed to compute $(P, R)/V/Q/\pi$
- Reduce experience needed to find a good $P, R/V/Q/\pi$

how much data is needed

Value Function Approximation (VFA)

- Represent a (state-action/state) value function with a parameterized function instead of a table

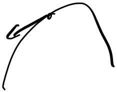


- **Which function approximator?**

Function Approximators

- Many possible function approximators including
  - Linear combinations of features
  - Neural networks
  - Decision trees   highly interpretable
  - Nearest neighbors
  - Fourier/ wavelet bases
- In this class we will focus on function approximators that are differentiable (Why?)   easier to optimize for
- Two very popular classes of differentiable function approximators
  - Linear feature representations (Today)
  - Neural networks (Next lecture)

Review: Gradient Descent



- Consider a function $J(\boldsymbol{w})$ that is a differentiable function of a parameter vector $\boldsymbol{w}$
- Goal is to find parameter $\boldsymbol{w}$ that minimizes J
- The gradient of $J(\boldsymbol{w})$ is

$$\nabla_w J(w) = \left[\frac{\partial J(w)}{\partial w_1} \quad \frac{\partial J(w)}{\partial w_2} \quad \cdots \quad \frac{\partial J(w)}{\partial w_N} \right]$$

$$\vec{w} \leftarrow \vec{w} - \underbrace{\alpha}_{\text{learning rate}} \nabla_w J(w)$$

local optima

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Value Function Approximation for Policy Evaluation with an Oracle

$\pi(s) \to a$
could be stochastic
$s \to p(a)$

- First assume we could query any state s and an oracle would return the true value for $V^\pi(s)$ $(s, V^\pi(s))$
- The objective was to find the best approximate representation of V^π given a particular parameterized function

Stochastic Gradient Descent

- Goal: Find the parameter vector $\boldsymbol{w}$ that minimizes the loss between a true value function $V^\pi(s)$ and its approximation $\hat{V}(s;\boldsymbol{w})$ as represented with a particular function class parameterized by $\boldsymbol{w}$.
- Generally use mean squared error and define the loss as

parameterized

$$J(\boldsymbol{w}) = \mathbb{E}_\pi[(\underline{V^\pi(s)} - \underline{\hat{V}(s;\boldsymbol{w})})^2]$$

- Can use gradient descent to find a local minimum

$$\Delta \boldsymbol{w} = -\frac{1}{2}\alpha \nabla_{\boldsymbol{w}} J(\boldsymbol{w})$$

- Stochastic gradient descent (SGD) samples the gradient:

$$\nabla_w J(w) = E_\pi[2(V^\pi(s) - \hat{V}(s,w))] \nabla_w V$$

$$\underbrace{\Delta w}_{\text{updating } w} = \alpha (V^\pi(s) - \hat{V}(s,w)) \nabla_w V(s)$$

single point

- Expected SGD is the same as the full gradient update

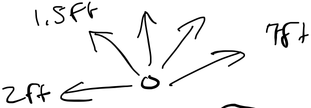
Model Free VFA Policy Evaluation

- Don't actually have access to an oracle to tell true $V^\pi(s)$ for any state s
- Now consider how to do model-free value function approximation for prediction / evaluation / policy evaluation without a model

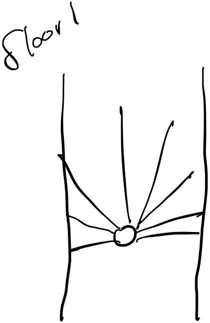
Model Free VFA Prediction / Policy Evaluation

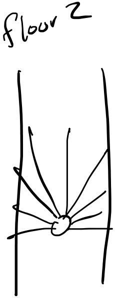
- Recall model-free policy evaluation (Lecture 3)
  - Following a fixed policy π (or had access to prior data)
  - Goal is to estimate V^π and/or Q^π
- Maintained a look up table to store estimates V^π and/or Q^π
- Updated these estimates after each episode (Monte Carlo methods) or after each step (TD methods)
- **Now: in value function approximation, change the estimate update step to include fitting the function approximator**

Feature Vectors



- Use a feature vector to represent a state s





$$\mathbf{x}(s) = \begin{pmatrix} x_1(s) \\ x_2(s) \\ \dots \\ x_n(s) \end{pmatrix}$$

Linear Value Function Approximation for Prediction With An Oracle

- Represent a value function (or state-action value function) for a particular policy with a weighted linear combination of features

$$\hat{V}(s;\mathbf{w}) = \sum_{j=1}^{n} x_j(s) w_j = \underbrace{\mathbf{x}(s)^T \overset{\text{vector}}{\mathbf{w}}}$$

Q

- Objective function is

$$J(\mathbf{w}) = \mathbb{E}_{\pi}[(V^{\pi}(s) - \hat{V}(s;\mathbf{w}))^2]$$

$s \sim \pi$

$\underbrace{x^T w}$

无论是 V-xw, 还是xw-V,
最后的偏导数都应该是
2(xw-V)x, 这老师说的有
问题

- Recall weight update is

$$\Delta \mathbf{w} = -\frac{1}{2}\alpha \nabla_{\mathbf{w}} J(\mathbf{w})$$

true

感觉不对啊，不应该是 -x(s) ?

- Update is:

$$\Delta w = \underbrace{-1/2\,\alpha}\,(2\,(\underbrace{V^{\pi}(s) - \hat{V}(s;w)})\,\underbrace{x(s)})$$

- Update = step-size × prediction error × feature value

Monte Carlo Value Function Approximation

- Return G_t is an unbiased but noisy sample of the true expected return $V^\pi(s_t)$

this is how the Pong tutorial doing.

- $\boxed{\text{Therefore can reduce MC VFA to doing supervised learning}}$ on a set of (state,return) pairs: $\langle s_1, G_1\rangle, \langle s_2, G_2\rangle, \ldots, \langle s_T, G_T\rangle$ ← estimate of $V^\pi(s_T)$
  - Substitute G_t for the true $V^\pi(s_t)$ when fit function approximator
- Concretely when using linear VFA for policy evaluation

episodic

$$
\begin{aligned}
\Delta \boldsymbol{w} &= \alpha(G_t - \hat{V}(s_t; \boldsymbol{w}))\nabla_{\boldsymbol{w}} \hat{V}(s_t; \boldsymbol{w}) \\
&= \alpha(G_t - \hat{V}(s_t; \boldsymbol{w}))\boldsymbol{x}(s_t) \\
&= \alpha(G_t - \underbrace{\boldsymbol{x}(s_t)^T \boldsymbol{w}})\boldsymbol{x}(s_t)
\end{aligned}
$$

- Note: G_t may be a very noisy estimate of true return

MC Linear Value Function Approximation for Policy Evaluation

$\gamma = 1$

```
Initialize w = 0, k = 1
loop
    Sample k-th episode (s_{k,1}, a_{k,1}, r_{k,1}, s_{k,2}, ..., s_{k,L_k}) given π
    for t = 1, ..., L_k do
        if First visit to (s) in episode k then
            G_t(s) = Σ_{j=t}^{L_k} r_{k,j} γ^{j-t}
            Update weights:
        end if
    end for
    k = k + 1
end loop
```

Initialize $\mathbf{w} = \mathbf{0}$, $k = 1$

loop

Sample k-th episode $(s_{k,1}, a_{k,1}, r_{k,1}, s_{k,2}, \ldots, s_{k,L_k})$ given π

for $t = 1, \ldots, L_k$ **do**

if First visit to (s) in episode k **then** could also do every visit

$G_t(s) = \sum_{j=t}^{L_k} r_{k,j} \boxed{\gamma^{j-t}}$ $X^{(s)}_{\omega}$

Update weights:

$\omega = \omega - \underbrace{\alpha}\left(G_t(s) - \hat{V}(s, \omega)\right) X(s)$

Generally in episodic cases, you will set γ=1 because in episodic cases things are always guaranteed to be bounded in terms of their value function.

end if

end for

$k = k + 1$

老师在这里还提到，某个状态visit的先后，并不会影响算法的执行。

end loop

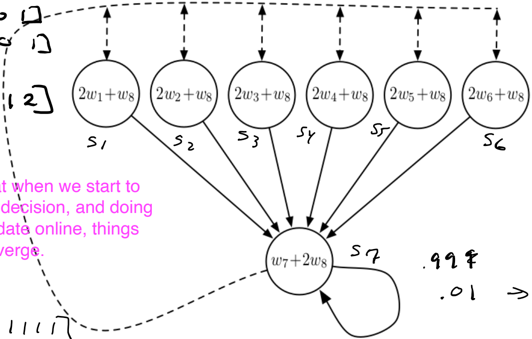
Baird (1995)-Like Example with MC Policy Evaluation[1]

s_1 [2 0 0 0 0 0 0 1]
s_2 [0 2 0 0 0 0 0 1]
s_3 [0 0 2 0 0 0 0 1]
⋮
s_7 [0 0 0 0 0 0 1 2]

rewards 0

has 2 actions
take a1 goto s7
take a2 goto s1~6.

a_1 —
a_2 - - -
w/prob 1/6 s_i
$i = 1:6$



This example turns out that when we start to combine VFA with making decision, and doing this sort of incremental update online, things can start to go bad, say diverge.

.99
.01 → terminate

init w: $w_0 = [1\ 1\ 1\ 1\ 1\ 1\ 1\ 1]$

$\alpha = .5$

- MC update: $\Delta \mathbf{w} = \alpha(G_t - \mathbf{x}(s_t)^T \mathbf{w})\mathbf{x}(s_t)$
- Small prob s_7 goes to terminal state, $\mathbf{x}(s_7)^T = [0\ 0\ 0\ 0\ 0\ 0\ 1\ 2]$

ex: s_1 a_1 0 s_7 a_1 0 s_7 a_1 0 terminate

s_1 $G = 0$ $V(s_1) = 3 = x^T w$

$\Delta w = 0.5\ (0 - 3)\ x\ = -1.5\ [2 \ldots 1] = [-3\ 0\ 0\ 0\ 0\ 0\ 0\ -1.5]$ = - 0.5·(3-0)x

$w = w + \Delta w = [-2\ 1\ 1\ 1\ 1\ 1\ 1\ -.5]$

Convergence Guarantees for Linear Value Function Approximation for Policy Evaluation: Preliminaries

- The Markov Chain defined by a MDP with a particular policy will eventually converge to a probability distribution over states $d(s)$
- $d(s)$ is called the stationary distribution over states of π
- $\sum_s d(s) = 1$
- $d(s)$ satisfies the following balance equation:

$$\cancel{d(s) = \sum_{s'} \sum_{a} \pi(s'|a) p(s'|s, a) d(s')}$$

$$d(s') = \sum_s \sum_a \pi(a|s)\, p(s'|s, a)\, d(s)$$

Convergence Guarantees for Linear Value Function Approximation for Policy Evaluation[2]

- Define the mean squared error of a linear value function approximation for a particular policy π relative to the true value as

$$MSVE(\boldsymbol{w}) = \sum_{s \in S} d(s)(V^\pi(s) - \hat{V}^\pi(s; \boldsymbol{w}))^2$$

- where
  - $d(s)$: stationary distribution of π in the true decision process
  - $\hat{V}^\pi(s; \boldsymbol{w}) = \boldsymbol{x}(s)^T \boldsymbol{w}$, a linear value function approximation

[2] Tsitsiklis and Van Roy. An Analysis of Temporal-Difference Learning with Function Approximation. 1997.https://web.stanford.edu/ bvr/pubs/td.pdf

Convergence Guarantees for Linear Value Function Approximation for Policy Evaluation[1]

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  - $d(s)$: stationary distribution of π in the true decision process
  - $\hat{V}^\pi(s; \boldsymbol{w}) = \boldsymbol{x}(s)^T \boldsymbol{w}$, a linear value function approximation
- Monte Carlo policy evaluation with VFA converges to the weights $\boldsymbol{w}_{MC}$ which has the minimum mean squared error possible:

$$MSVE(\boldsymbol{w}_{MC}) = \min_{\boldsymbol{w}} \sum_{s \in S} d(s)(V^\pi(s) - \hat{V}^\pi(s; \boldsymbol{w}))^2$$

[1] Tsitsiklis and Van Roy. An Analysis of Temporal-Difference Learning with Function Approximation. 1997.https://web.stanford.edu/ bvr/pubs/td.pdf

Batch Monte Carlo Value Function Approximation

- May have a set of episodes from a policy π
- Can analytically solve for the best linear approximation that minimizes mean squared error on this data set
- Let $G(s_i)$ be an unbiased sample of the true expected return $V^\pi(s_i)$

set of data

$$\arg\min_{\boldsymbol{w}} \sum_{i=1}^{N} (G(s_i) - \boldsymbol{x}(s_i)^T \boldsymbol{w})^2$$

- Take the derivative and set to 0

$$\boldsymbol{w} = (X^T X)^{-1} X^T \boldsymbol{G}$$

- where $\boldsymbol{G}$ is a vector of all N returns, and X is a matrix of the features of each of the N states $\boldsymbol{x}(s_i)$
- Note: not making any Markov assumptions

- Uses bootstrapping and sampling to approximate V^π
- Updates $V^\pi(s)$ after each transition (s, a, r, s'):

$$V^\pi(s) \underset{\text{sampling}}{=} V^\pi(s) + \alpha(\underbrace{r + \gamma V^\pi(s')}_{} - V^\pi(s))$$

bootstrap

- Target is $r + \gamma V^\pi(s')$, a biased estimate of the true value $V^\pi(s)$
- Represent value for each state with a separate table entry

Temporal Difference (TD(0)) Learning with Value Function Approximation

- Uses bootstrapping and sampling to approximate true V^π
- Updates estimate $V^\pi(s)$ after each transition (s, a, r, s'):

$$V^\pi(s) \quad = \quad V^\pi(s) + \alpha(r + \gamma V^\pi(s') - V^\pi(s))$$

- Target is $r + \gamma V^\pi(s')$, a biased estimate of of the true value $V^\pi(s)$
- In value function approximation, target is $r + \gamma \hat{V}^\pi(s'; \boldsymbol{w})$, a biased and approximated estimate of of the true value $V^\pi(s)$
- 3 forms of approximation:

  - function approx
  - bootstrapping
  - sampling

  but still on policy

Temporal Difference (TD(0)) Learning with Value Function Approximation

- In value function approximation, target is $r + \gamma \hat{V}^{\pi}(s'; \boldsymbol{w})$, a biased and approximated estimate of the true value $V^{\pi}(s)$
- Can reduce doing TD(0) learning with value function approximation to supervised learning on a set of data pairs:
  - $\langle s_1, r_1 + \gamma \hat{V}^{\pi}(s_2; \boldsymbol{w}) \rangle, \langle s_2, r_2 + \gamma \hat{V}(s_3; \boldsymbol{w}) \rangle, \ldots$
- Find weights to minimize mean squared error

$$J(\boldsymbol{w}) = \mathbb{E}_{\pi}[(r_j + \gamma \hat{V}^{\pi}(s_{j+1}, \boldsymbol{w}) - \hat{V}(s_j; \boldsymbol{w}))^2]$$

Temporal Difference (TD(0)) Learning with Value Function Approximation

- In value function approximation, target is $r + \gamma \hat{V}^{\pi}(s'; \boldsymbol{w})$, a biased and approximated estimate of the true value $V^{\pi}(s)$
- Supervised learning on a different set of data pairs: $\langle s_1, r_1 + \gamma \hat{V}^{\pi}(s_2; \boldsymbol{w}) \rangle, \langle s_2, r_2 + \gamma \hat{V}(s_3; \boldsymbol{w}) \rangle, \ldots$
- In linear TD(0)

$$
\begin{aligned}
\Delta \boldsymbol{w} &= \alpha(\overbrace{r + \gamma \hat{V}^{\pi}(s'; \boldsymbol{w})}^{\text{TD target}} - \overbrace{\hat{V}^{\pi}(s; \boldsymbol{w})})\nabla_{\boldsymbol{w}} \hat{V}^{\pi}(s; \boldsymbol{w}) \\
&= \alpha(r + \gamma \hat{V}^{\pi}(s'; \boldsymbol{w}) - \hat{V}^{\pi}(s; \boldsymbol{w}))\boldsymbol{x}(s) \\
&= \alpha(\underbrace{r + \gamma \underbrace{\boldsymbol{x}(s')^T \boldsymbol{w}}}_{\text{vs } G(s)} - \underbrace{\boldsymbol{x}(s)^T \boldsymbol{w}})\underbrace{\boldsymbol{x}(s)}
\end{aligned}
$$

$- (x^T w - G)*(-x) = (x^T w - G)\, x$

TD(0) Linear Value Function Approximation for Policy Evaluation

Initialize $\mathbf{w} = \mathbf{0}$, $k = 1$

loop

  Sample tuple (s_k, a_k, r_k, s_{k+1}) given π

  Update weights:

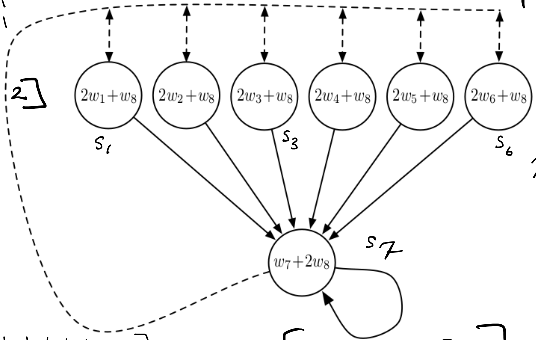
$$\boldsymbol{w} = \boldsymbol{w} + \alpha(r + \gamma \underbrace{\boldsymbol{x}(s')^T \boldsymbol{w}} - \underbrace{\boldsymbol{x}(s)^T \boldsymbol{w}})\boldsymbol{x}(s)$$

  $k = k + 1$

end loop

Baird Example with TD(0) On Policy Evaluation [1]

s_1 2 0 0 0 0 0 0 1
0 2 0 0 0 0 0 1
s_3 0 0 2 0 0 0 0 1
⋮
s_7 [0 0 0 0 0 0 1 2]



s_1 s_3 s_6 s_7

rewards all 0

$\alpha = .5$

$\gamma = .9$

a_1 —

a_2 - - -

$w_0 = [1\ 1\ 1\ 1\ 1\ 1\ 1\ 1]$

$[s_1, a_1, 0, s_7]$

- TD update: $\Delta \mathbf{w} = \alpha(r + \gamma \underbrace{\mathbf{x}(s')}_{s_7}{}^T \mathbf{w} - \underbrace{\mathbf{x}(s)^T \mathbf{w}}_{x(s_1)w = 3})\mathbf{x}(s)$

new w? vs MC?

$\Delta w = \alpha(0 + .9 \cdot 3 - 3) = \alpha \cdot -.3$

[1] Figure from Sutton and Barto 2018

Convergence Guarantees for Linear Value Function Approximation for Policy Evaluation

- Define the mean squared error of a linear value function approximation for a particular policy π relative to the true value as

$$MSVE(\boldsymbol{w}) = \sum_{s \in S} d(s)(V^{\pi}(s) - \hat{V}^{\pi}(s; \boldsymbol{w}))^2$$

- where
  - $d(s)$: stationary distribution of π in the true decision process
  - $\hat{V}^{\pi}(s; \boldsymbol{w}) = \boldsymbol{x}(s)^T \boldsymbol{w}$, a linear value function approximation
- TD(0) policy evaluation with VFA converges to weights $\boldsymbol{w}_{TD}$ which is within a constant factor of the minimum mean squared error possible:

$$MSVE(\boldsymbol{w}_{TD}) \leq \frac{1}{1-\gamma} \min_{\boldsymbol{w}} \sum_{s \in S} d(s)(V^{\pi}(s) - \hat{V}^{\pi}(s; \boldsymbol{w}))^2$$

Check Your Understanding

- Monte Carlo policy evaluation with VFA converges to the weights $\mathbf{w}_{MC}$ which has the minimum mean squared error possible:

$$MSVE(\mathbf{w}_{MC}) = \min_{\mathbf{w}} \sum_{s \in S} d(s) \underbrace{(V^\pi(s) - \hat{V}^\pi(s; \mathbf{w}))^2}_{=0}$$

- TD(0) policy evaluation with VFA converges to weights $\mathbf{w}_{TD}$ which is within a constant factor of the minimum mean squared error possible:

$$MSVE(\mathbf{w}_{TD}) \leq \frac{1}{1-\gamma} \overbrace{\min_{\mathbf{w}} \sum_{s \in S} d(s)(V^\pi(s) - \hat{V}^\pi(s; \mathbf{w}))^2}^{= MSVE(w_{MC})}$$

- If the VFA is a tabular representation (one feature for each state), what is the MSVE for MC and TD? 0

Convergence Rates for Linear Value Function Approximation for Policy Evaluation

- Does TD or MC converge faster to a fixed point?
- Not (to my knowledge) definitively understood
- Practically TD learning often converges faster to its fixed value function approximation point

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Control using Value Function Approximation

- Use value function approximation to represent state-action values $\hat{Q}^{\pi}(s, a; \boldsymbol{w}) \approx Q^{\pi}$
- Interleave
  - Approximate policy evaluation using value function approximation
  - Perform ϵ-greedy policy improvement
- Can be unstable. Generally involves intersection of the following:
  - Function approximation
  - Bootstrapping
  - **Off-policy learning**

sampling

Action-Value Function Approximation with an Oracle

- $\hat{Q}^\pi(s, a; \boldsymbol{w}) \approx Q^\pi$
- Minimize the mean-squared error between the true action-value function $Q^\pi(s, a)$ and the approximate action-value function:

$$J(\boldsymbol{w}) = \mathbb{E}_\pi[(Q^\pi(s, a) - \hat{Q}^\pi(s, a; \boldsymbol{w}))^2]$$

- Use stochastic gradient descent to find a local minimum

$$\begin{aligned} -\frac{1}{2}\nabla_{\boldsymbol{W}} J(\boldsymbol{w}) &= \mathbb{E}\left[(Q^\pi(s, a) - \hat{Q}^\pi(s, a; \boldsymbol{w}))\nabla_{\boldsymbol{w}}\hat{Q}^\pi(s, a; \boldsymbol{w})\right] \\ \Delta(\boldsymbol{w}) &= -\frac{1}{2}\alpha\nabla_{\boldsymbol{w}} J(\boldsymbol{w}) \end{aligned}$$

- Stochastic gradient descent (SGD) samples the gradient

Linear State Action Value Function Approximation with an Oracle

- Use features to represent both the state and action

VFA feature: the distance between pacman and dot

QFA feature: the distance between pacman and dot AFTER taking action a

$$\mathbf{x}(s,a) = \begin{pmatrix} x_1(s,a) \\ x_2(s,a) \\ \dots \\ x_n(s,a) \end{pmatrix}$$

- Represent state-action value function with a weighted linear combination of features

$$\hat{Q}(s,a;\mathbf{w}) = \mathbf{x}(s,a)^T\mathbf{w} = \sum_{j=1}^{n} x_j(s,a)w_j$$

- Stochastic gradient descent update:

$$\nabla_{\mathbf{w}} J(\mathbf{w}) = \nabla_{\mathbf{w}} \mathbb{E}_\pi[(Q^\pi(s,a) - \hat{Q}^\pi(s,a;\mathbf{w}))^2]$$

Incremental Model-Free Control Approaches

- Similar to policy evaluation, true state-action value function for a state is unknown and so substitute a target value
- In Monte Carlo methods, use a return $\underset{(s,a)}{G_t}$ as a substitute target

$$\Delta \boldsymbol{w} = \alpha(G_t - \hat{Q}(s_t, a_t; \boldsymbol{w}))\nabla_{\boldsymbol{w}} \hat{Q}(s_t, a_t; \boldsymbol{w})$$

- For SARSA instead use a TD target $r + \gamma \underbrace{\hat{Q}(s', a'; \boldsymbol{w})}$ which leverages the current function approximation value

$$\Delta \boldsymbol{w} = \alpha(r + \gamma \hat{Q}(s', a'; \boldsymbol{w}) - \hat{Q}(s, a; \boldsymbol{w}))\nabla_{\boldsymbol{w}} \hat{Q}(s, a; \boldsymbol{w})$$

Incremental Model-Free Control Approaches

- Similar to policy evaluation, true state-action value function for a state is unknown and so substitute a target value
- In Monte Carlo methods, use a return G_t as a substitute target

$$\Delta \boldsymbol{w} = \alpha(G_t - \hat{Q}(s_t, a_t; \boldsymbol{w}))\nabla_{\boldsymbol{w}}\hat{Q}(s_t, a_t; \boldsymbol{w})$$

- For SARSA instead use a TD target $r + \gamma \hat{Q}(s', a'; \boldsymbol{w})$ which leverages the current function approximation value

$$\Delta \boldsymbol{w} = \alpha(r + \gamma \hat{Q}(s', a'; \boldsymbol{w}) - \hat{Q}(s, a; \boldsymbol{w}))\nabla_{\boldsymbol{w}}\hat{Q}(s, a; \boldsymbol{w})$$

- For Q-learning instead use a TD target $r + \gamma \max_{a'} \hat{Q}(s', a'; \boldsymbol{w})$ which leverages the max of the current function approximation value

$$\Delta \boldsymbol{w} = \alpha(r + \gamma \max_{a'} \underbrace{\hat{Q}(s', a'; \boldsymbol{w})}_{x(s',a')w} - \underset{x(s,a)w}{\hat{Q}(s, a; \boldsymbol{w})})\nabla_{\boldsymbol{w}}\hat{Q}(s, a; \boldsymbol{w})$$

Convergence of TD Methods with VFA

Sutton & Barto chp 11

- TD with value function approximation is not following the gradient of an objective function
- Informally, updates involve doing an (approximate) Bellman backup followed by best trying to fit underlying value function to a particular feature representation
- Bellman operators are contractions, but value function approximation fitting can be an expansion

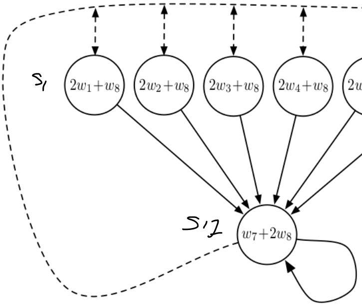
Geoff Gordon 1995

$$|OV - OV'|_\infty \leq |V - V'|$$

operator

$$\cancel{\$}\ P\,OV$$

Challenges of Off Policy Control: Baird Example [1]



a_1

$\pi(\text{solid}|\cdot) = 1$ evaluate

a_2

$\mu(\text{dashed}|\cdot) = 6/7$

$\mu(\text{solid}|\cdot) = 1/7$ behavior

a_1

exploration

$\gamma = 0.99$

generate data under behavior policy μ

- Behavior policy and target policy are not identical
- Value can diverge (s, a, r, s')

throw away data if $a \neq \pi(s)$

because if you do that (throw away data), your distribution data is not same as the data you'd get under your desired target policy.

Convergence of Control Methods with VFA

you basically converge
but you might be some
oscillation

Algorithm	Tabular	Linear VFA	Nonlinear VFA
Monte-Carlo Control	✓	(✓)	✗
Sarsa	✓	(✓)	✗
Q-learning	✓	✗	✗

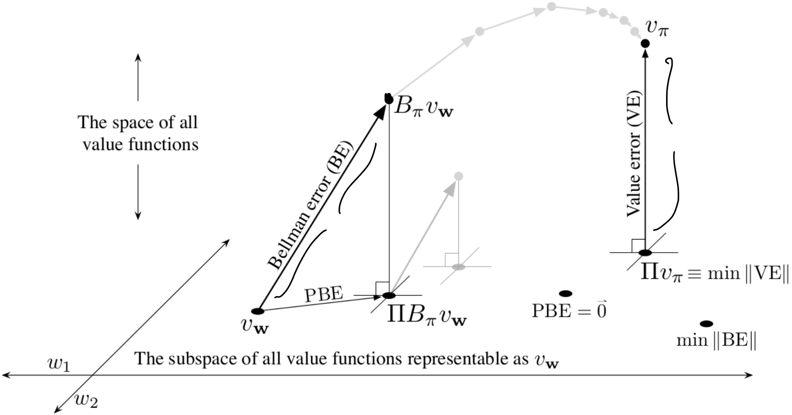
off policy

can diverge

Hot Topic: Off Policy Function Approximation Convergence

- Extensive work in better TD-style algorithms with value function approximation, some with convergence guarantees: see Chp 11 S& B
- Exciting recent work on batch RL that can converge with nonlinear VFA (Dai et al. ICML 2018): uses primal dual optimization
- An important issue is not just whether the algorithm converges, but **what** solution it converges too
- Critical choices: **objective function and feature representation**

Linear Value Function Approximation[3]



[3] Figure from Sutton and Barto 2018

What You Should Understand

- Be able to implement TD(0) and MC on policy evaluation with linear value function approximation
- Be able to define what TD(0) and MC on policy evaluation with linear VFA are converging to and when this solution has 0 error and non-zero error.
- Be able to implement Q-learning and SARSA and MC control algorithms
- List the 3 issues that can cause instability and describe the problems qualitiatively: function approximation, bootstrapping and off policy learning

Class Structure

- Last time: Control (making decisions) without a model of how the world works
- This time: Value function approximation
- **Next time**: Deep reinforcement learning