MAP562 Optimal design of structures

by Samuel Amstutz, Beniamin Bogosel

École Polytechnique

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Instructions Upload your solutions as `FreeFem++` files to the course Moodle by February 25th.

Exercise 4 Optimization of an inclusion

This exercise is related to the exercise 4 in class. Question 1 is particularly useful, as well as the `FreeFem++` example shown on Moodle.

1. **(Homework)** Consider the following types of parametric curves:

   1. **Stadiums:** rectangles of dimensions $a \times b$ with a half disk of diameter b glued to the two opposite sides of length b. Optimization variables: a, b and the center.
   2. **Superellipse:** parametric curve defined by $\frac{|x|^p}{a^p} + \frac{|y|^p}{b^p} = 1$ where $p = 4$. Optimization variables: a, b and the center.
   3. **Generic parametric curve:** curve with given radial parametrization $\rho(\beta) = 1 + a\cos(2\beta) + b\cos(3\beta)$. Recall that given a radial function $[0, 2\pi] \ni \beta \mapsto \rho(\beta) \in (0, \infty)$ the corresponding parametric curve is defined by $(x(\beta), y(\beta)) = (\rho(\beta)\cos\beta, \rho(\beta)\sin\beta)$. (notice that this is one of the standard ways to define a curve in `FreeFem++`) You may suppose that $|a| \leq 0.5, |b| \leq 0.5$ and the bounding box is the square $(-2.5, 2.5)^2$. For this type of curve you may assume that the center for the radial parametrization is fixed. The optimization variables are: a, b. Note that in the computation of the derivative you may need the value of the angle β on the boundary of the curve $\partial\omega$. You may use the `FreeFem++` function `atan2` to do this (an example will be posted on Moodle).

   Choose one of the categories of curves defined above and compute the partial derivatives of $J(\omega)$ with respect to the relevant parameters. As above, it is useful to find what is the corresponding vector field θ when varying the desired parameter and then use the general shape derivative formula found at Question 1.