# MAP562 Optimal design of structures

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## Homework Sheet 3, Jan 22nd, 2020

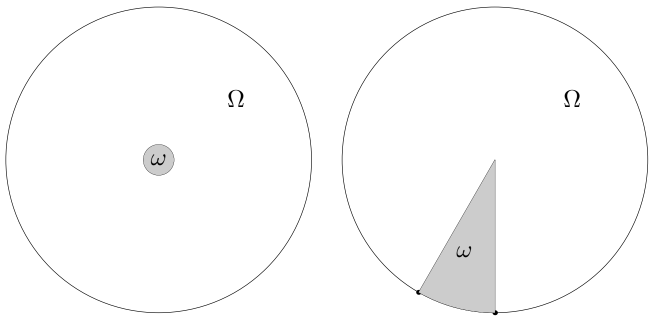
**Instructions** The exercise is due on January 28th, 2020. Upload your solutions as separate `FreeFEM` files, including detailed comments, to the course Moodle.

**Important.** Comments regarding your numerical experiments should be present either in a separate pdf file either in the `FreeFEM` codes. Students submitting codes without pertinent comments can only get grades lower than B.

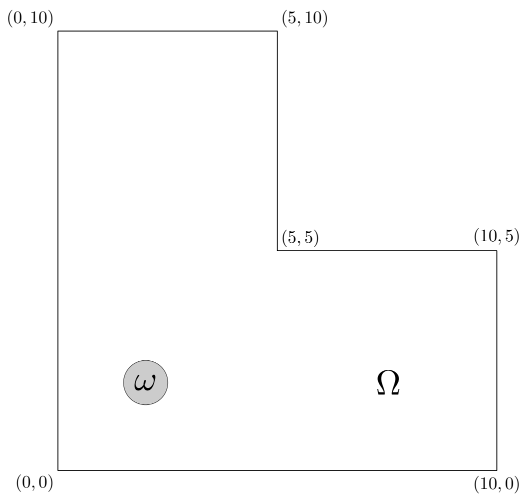
## Exercise 1

We denote by $(x_1, x_2)$ the coordinates of a point $x \in \mathbb{R}^2$ and by $B(c, r)$ the ball centered at $c \in \mathbb{R}^2$ of radius $r \in \mathbb{R}$.

1. For a **constant heat source** $v \in \mathbb{R}$, using a gradient method, implement in `FreeFem++` the minimization of the functional:
$$J(v) = \int_\Omega |T - T_0|^2 \, dx$$
where
$$\begin{cases} -\Delta T + u \cdot \nabla T & = & 1_\omega v & \text{in } \Omega \\ T & = & 0 & \text{on } \partial\Omega. \end{cases}$$
in which the desired temperature is the constant function $T_0 = 10$.
The following geometries and velocity fields will be considered:
   - $\Omega = B((0,0),1)$, $u(x) = (-x_2, x_1)^\top$
     - $\omega = B((0,0), 0.1)$,
     - $\omega$ is a *slice* of $\Omega$ with angle $\pi/6$ (see the drawing below).




   - $\Omega$ is the L shaped domain (see the drawing below), $u = \nabla\Phi$ where $\Phi$ is a solution of
$$\Delta\Phi = 0 \text{ in } \Omega, \text{ and } \frac{\partial\Phi}{\partial n} = \begin{cases} -1 & \text{if } x_1 = 0 \text{ and } 0.5 < x_2 < 1; \\ 1 & \text{if } x_1 = 10; \\ 0 & \text{otherwise.} \end{cases} \quad (1)$$
     - $\omega = B((2,2),1)$
     - vary the position of $\omega$ and observe the behavior of the solution.



2. Compare your result to the case without convection ($u = [0, 0]$): run the simulation and write a short comment on the influence of the velocity field.