MAP562 Optimal design of structures

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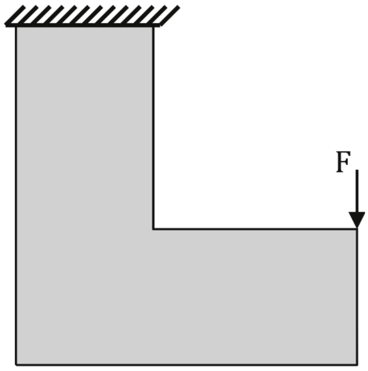
Instructions Upload your solutions as separate files to the course Moodle. Do not forget to remove old comments from the `FreeFem++` scripts and to add your own in order to explain what you are doing. The numbering of the exercises below corresponds to the numbers in the Exercise Sheet 1.

- Exercises 2 and 3 are due on January 14th, 2020.
- Exercise 4 is due on January 21st, 2020.

Exercise 2

We take the elasticity program `elasticity.edp` and investigate further properties. [**Hint:** Review first for yourself the variational formulation of elasticity from the lecture notes.]

1. Study the case of the L-beam for different mesh sizes with boundary conditions and loadings illustrated in the figure below. Trace the displacements and the norm of the constraints tensor. You should observe that the displacements converge, but the norm of the tensor of constraints diverges. Use the command `adaptmesh` in order to improve the results.



2. Change the Lamé coefficients and investigate in particular the limit case $\nu \to 0.5$ (where Poisson's ratio is defined by $\nu := \frac{\lambda}{2(\lambda+\mu)}$) . What is the physical interpretation of this case? Why does it pose numerical problems?

Exercise 3

1. In the numerical implementation of the problem stated in Exercise 4 from Session 1 try different values of the constant k_1. What happens for $k_1 \gg 1$ and $0 < k_1 \ll 1$?
2. Solve the problem by constructing the matrix and right hand side explicitly in `FreeFem++` instead of using the keyword `problem`. (consult the file `FreeFem_matrices.edp` for an example)

Exercise 4

1. Implement in `FreeFem++` the problem with nonlocal boundary conditions (related to Exercise 4 from Session 1).
   Hint: To implement the multiplication of two boundary integrals, you can evaluate the first one and create the corresponding vector, let us say b. Then you perform $b * b^\top$. This is a matrix that you can add to the other parts required for the total stiffness matrix M. Also you create explicitely the right hand side vector L. Finally you solve with

   ```
   set(M,solver=sparsesolver);
   u[]=M^-1*L;
   ```

   in order to obtain the discrete solution u.