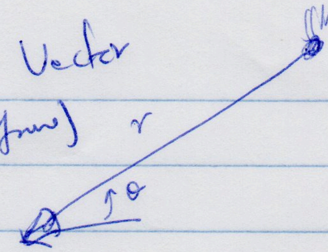
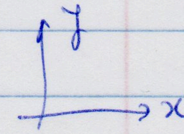
①

2.127, (velocity vector wrt. x,y frame)





Given

$r = 21000$ ft

$\dot{r} = -22{,}440$ ft/s

$\theta = 40° = \frac{40 \times \pi}{180} = 0.6981$ rad

$\dot{\theta} = -2.935$ rad/s

what is $\dot{V}$ in x, y, frame??

1) use $\vec{V} = \dot{r}\,\hat{u}_r + r\dot{\hat{u}}_r$

$\vec{V} = \dot{r}\,\hat{u}_r + \vec{\omega} \times \vec{r}$

2) $\hat{u}_r = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$

$\vec{\omega} = \omega\,\hat{k} = \dot{\theta}\,\hat{k}$

$\vec{V} = \dot{r}\,\hat{u}_r + \dot{\theta}\,\hat{k} \times \vec{r}$

$= \dot{r}(\cos\theta\,\hat{i} + \sin\theta\,\hat{j}) + \dot{\theta}\,\hat{k} \times (r\cos\theta\,\hat{i} + r\sin\theta\,\hat{j})$

$= \dot{r}(\cos\theta\,\hat{i} + \sin\theta\,\hat{j}) + r\dot{\theta}(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j})$

As $\hat{k} \times \hat{j} = -\hat{i}$

& $\hat{k} \times \hat{i} = \hat{j}$

$= (\dot{r}\cos\theta - r\dot{\theta}\sin\theta)\,\hat{i} + (\dot{r}\sin\theta + r\dot{\theta}\cos\theta)\,\hat{j}$

$= \left[-22{,}440\cos(0.6981) - 21000 \times -2.935 \times \sin(0.6981)\right]\hat{i}$

$+ \left[-22{,}440\sin(0.6981) + 21000 \times -2.935 \times \cos(0.6981)\right]\hat{j}$

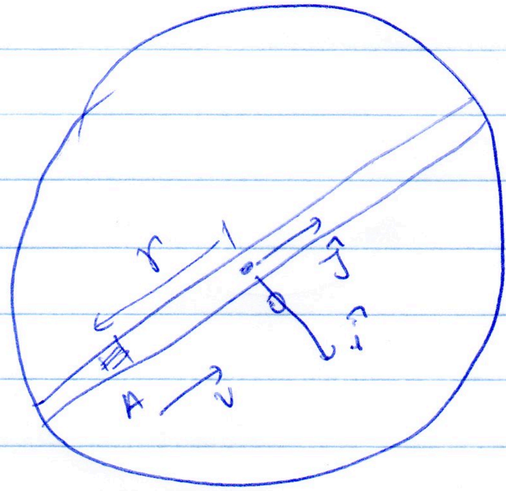
$\Rightarrow \vec{V} = (22{,}430\,\hat{i} - 61{,}640\,\hat{j})$ ft/s

2.135

2

find $\vec{a}$ when,

given
$V = 14\ m/s$
$r = 0$
$\omega = 4\ rad/s$
$a_r = 5\ m/s^2$



In general.

1 $$\ddot{\vec{r}} = \ddot{r}\,\hat{u}_r + 2\dot{r}\,(\vec{\omega}\times\hat{u}_r) + \dot{\vec{\omega}}\times\vec{r} + \vec{\omega}\times(\vec{\omega}\times\vec{r})$$

General form

at O, $\vec{r} = 0$, also as ω is constant, $\dot{\vec{\omega}} = 0$

also in given system,
$$\hat{u}_r = -\hat{j}$$

$$\Rightarrow \ddot{\vec{r}} = -\ddot{r}\,\hat{j} - 2\dot{r}\,(\vec{\omega}\times\hat{j})$$

2 $$\vec{\omega} = \omega\,\hat{k} = \dot{\theta}\,\hat{k}$$

$$\Rightarrow \ddot{\vec{r}} = -\ddot{r}\,\hat{j} - 2\dot{r}\,(\dot{\theta}\,\hat{k}\times\hat{j})$$

Specific eqn

$$= -\ddot{r}\,\hat{j} + 2\dot{r}\,\dot{\theta}\,\hat{i}$$

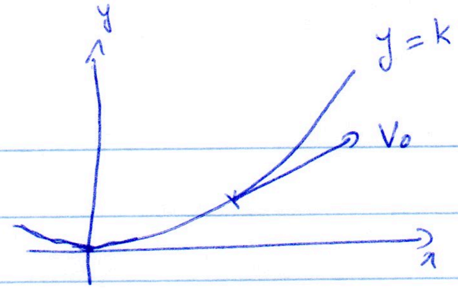
3 $\dot{r} = -V$ & $\ddot{r} = -a_r$ (Note, r & $\hat{j}$ are in opposite directions)

Substituting

$$\Rightarrow \vec{a}_A\Big|_{r=0} = \ddot{\vec{r}} = (2\times(-14)\times 4)\,\hat{i} \underbrace{-(5)}_{-(-5)}\hat{j} = \underline{(-112\,\hat{i} + 5\,\hat{j})\ m/s^2}$$

2.153

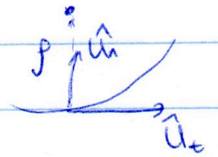
3



~~$V_0 = 180$ mph~~

get $\vec{a}$

Direct $\vec{a}$ calculation



$$\vec{V} = V\hat{u}_t$$

$$\dot{\vec{V}} = \dot{V}\hat{u}_t + V\dot{\hat{u}}_t$$

$$= \dot{V}\hat{u}_t + V\frac{d\hat{u}_t}{ds}\frac{ds}{dt}$$

$$= \dot{V}\hat{u}_t + V^2\underbrace{\frac{d\hat{u}_t}{ds}}$$

curvature → measure of "bendiness" of path.

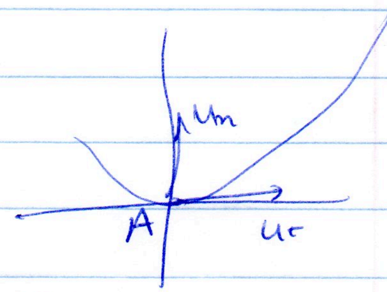
$$\hookrightarrow k\hat{u}_n = \frac{1}{\rho}\hat{u}_n$$

$$\boxed{\dot{\vec{V}} = \vec{a} = \dot{V}\hat{u}_t + \frac{V^2}{\rho}\hat{u}_n}$$

$$\boxed{K = \frac{1}{\rho} = \frac{|d^2y/dx^2|}{\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{3/2}}}$$

(4)

1) $\vec{a}_A = \dot{V}\hat{u}_t + \frac{V^2}{\rho}\hat{u}_n$



given $\dot{V} = 0$

$y = Kx^2$

$V = 180$ mph

find K so that $|\vec{a}_A| = 1.5g$

1) calculate ρ

$$\rho = \frac{\left(1+\left(\frac{dy}{dx}\right)^2\right)^{3/2}}{|d^2y/dx^2|} \Rightarrow$$

$y = kx^2$

$\frac{dy}{dx} = 2kx$, $\frac{d^2y}{dx^2} = 2k$.

$$\Rightarrow \rho = \frac{(1 + \cancel{4}k^2x^2)^{3/2}}{2k}$$

$$\rho_A = \rho(x=0) = \frac{1}{2k}$$

2) $\vec{a}_A = 0\hat{u}_t + \frac{V^2}{(1/2k)}\hat{u}_n$

$\vec{a}_A = 2kV^2\hat{u}_n$

$|\vec{a}_A| = 1.5g \Rightarrow 2kV^2 = 1.5g$

3) $V = 180$ mph $= 180 \times \frac{5280}{3600}$ ft/s

5

$$\Rightarrow \quad 2 \times K\left(\frac{180 \times 5280}{3600}\right)^2 = 1.5 \times 32.2$$

$$\Rightarrow \quad \boxed{K = 0.3465\ ft^{-1}}$$