Chapter 2: Particle Kinematics

MEC262: Engineering Dynamics

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Sections (2.1 to 2.8)

Position, Velocity and acceleration representations in various coordinate frames

2.1 Kinematics

- Kinematics describe the motion of a particle or a body without accounting for the forces that may cause the motion.
- Kinematics: Position, velocity and acceleration.

Relation between position, velocity and derivative

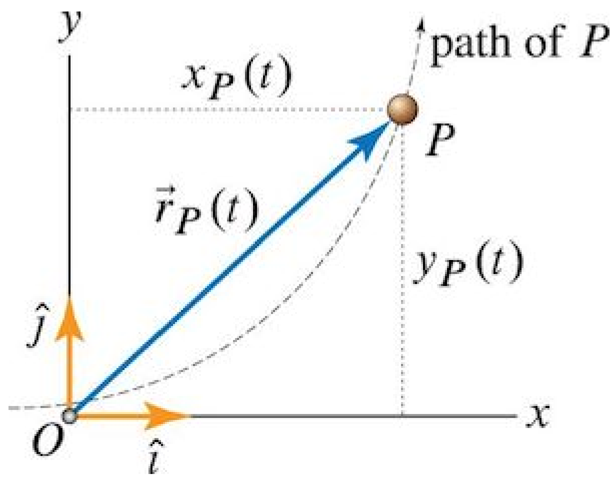
- Velocity is derivative of position, and acceleration is derivative of velocity.
- Given position vector $\vec{R}$,

$$Velocity: \ \vec{V} = \frac{d\vec{R}}{dt}$$
$$Acceleration: \ \vec{A} = \frac{d\vec{V}}{dt} = \frac{d^2\vec{R}}{dt^2}$$

Position vector in cartesian coordinates (x-y).

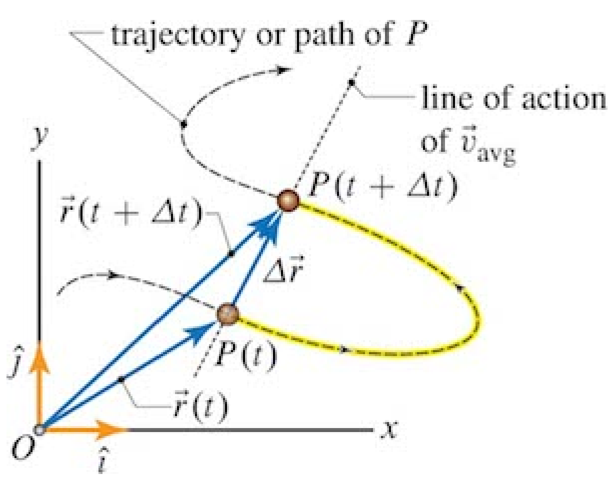
Given a point (x_P, y_P) the position vector is given by,

$$\vec{R}_P = x_P\hat{j} + y_P\hat{j}$$



Trajectory and displacement

- Trajectory is time-history (position vs time) or $(\vec{r}(t))$
- Displacement is difference in position vector between two given time instants. In the figure below, $\Delta\vec{r}$ is the displacement between times t and $t + \Delta t$.
$$\Delta\vec{r} = \vec{r}(t + \Delta t) - \vec{r}(t)$$
- Note, displacement of a particle can be zero even if the net distance traveled by the particle is non-zero. For example, a full rotation about a circle brings the particle back to the same position.
- Trajectory plotted in the cartesian space is called path (x vs y).



Velocity and speed

- Velocity is the rate of change of position.
- Speed is the magnitude of velocity.
- Average velocity is displacement over time. Note, average velocity can be zero, even if the velocity at the intermediate points is not zero.

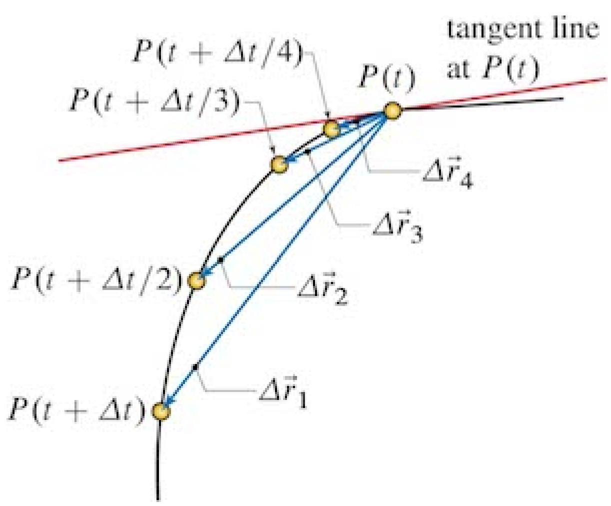
Average velocity

$$Average\ Velocity = \frac{Displacement}{time}$$
$$Displacement = \Delta\vec{r} = \vec{r}(t+\Delta t) - \vec{r}(t)$$
$$Average\ Velocity = \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}(t+\Delta t) - \vec{r}(t)}{\Delta t}$$

Velocity vector

As $\Delta t \rightarrow 0$, the average velocity approaches the true velocity of the particle at a given instant. Therefore, velocity can be computed as,

$$\vec{v}(t) = \underbrace{Lim}_{\Delta t \rightarrow 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} = \dot{\vec{r}}$$



- Velocity vector is always tangent to the path.

Speed along a path

Given s path length along a path, the speed along the path is given by,

$$v = \frac{ds}{dt}$$

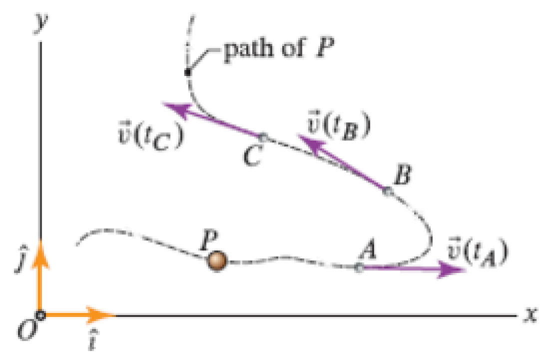
Average speed along the track,

$$v_{avg} = \frac{s_2 - s_1}{t_2 - t_1}$$

Acceleration

Acceleration is the rate of change of velocity.

$$\vec{a}(t) = \dot{\vec{v}} = \underbrace{Lim}_{\Delta t \to 0} \frac{\vec{v}(t + \Delta t) - \vec{v}(t)}{\Delta t}$$



Average and instantaneous acceleration

Given the velocity vector $\vec{v}$, the instantaneous acceleration is given by,

$$\vec{a} = \frac{d\vec{v}}{dt}$$

Average acceleration along the track,

$$\vec{a}_{avg} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$$

Elementary motions

1. Linear or rectilinear motion, motion along a straight line
2. Rotational motion, rotation of a point around a circle.

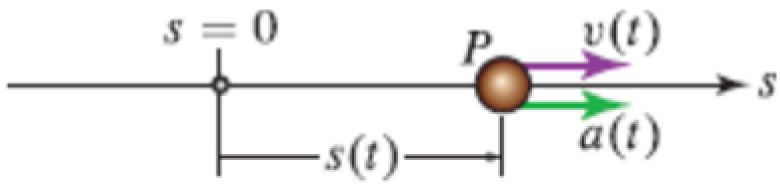
Linear or Rectilinear motion

For motion in a straight line, we can drop the direction vector and replace it with distance s from start.

$$v = \frac{ds}{dt}$$

$$a = \frac{dv}{dt} = \frac{d^2 s}{dt^2} = v\frac{dv}{ds}$$

The form of equation to choose depends on the quantify we are interested in and the given expression.



For constant acceleration,

$$v(t) = v(t_0) + a_c(t - t_0)$$

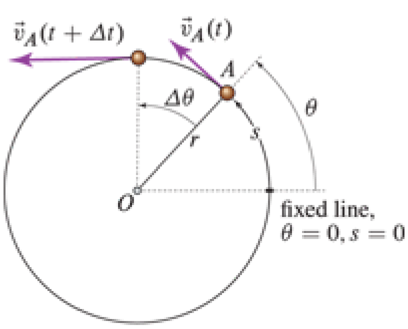
$$s(t) = s(t_0) + v(t_0)(t - t_0) + \frac{1}{2}a_c(t - t_0)^2$$

$$v(t)^2 = v(t_0)^2 + 2a_c(s(t) - s(t_0))$$

Circular motion.

All relations for rectilinear motion apply directly to the ANGULAR QUANTITIES (angle, angular velocity and angular acceleration).

NOTE: THE RECTILINEAR EQUATIONS DO NOT APPLY TO THE LINEAR KINEMATICS OF A PARTICLE MOVING IN A CIRCLE.



Given angle Θ, angular velocity is given by,

$$\omega = \frac{d\Theta}{dt} = \dot{\Theta}$$

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\Theta}{dt^2} = \omega\frac{\omega}{d\Theta} = \ddot{\Theta}$$

Projectile motion

Projectile motion is motion where a particle moves solely under the action of gravity (and sometimes drag). The accelerations, assuming no drag are given by

$$a_x = 0,\ a_y = -g = -9.8\ m/s^2$$

Given a projectile thrown with a velocity $v_x(0)$ and $v_y(0)$, the velocity is given by,

$$v_x(t) = v_x(0)$$
$$v_y(t) = v_y(0) - gt$$

Given a projectile thrown with a velocity $v_x(0)$ and $v_y(0)$ from positon $s_x(0)$ and $s_y(0)$, distance traveled by the projectile is given by,

$$s_x(t) = s_x(0) + v_x(0)t$$
$$s_y(t) = s_y(0) + v_y(0)t - \frac{1}{2}gt^2$$

EXAMPLES