

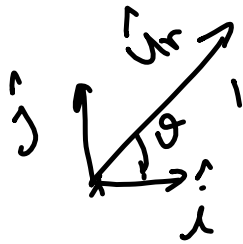
$$\vec{r} = r \hat{u}_r$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d(r\hat{u}_r)}{dt}$$



$$= \frac{dr}{dt} \hat{u}_r + r \frac{d\hat{u}_r}{dt}$$

$\underbrace{\frac{dr}{dt}}_{\text{radial velocity}}$        $\frac{d\hat{u}_r}{dt}$   $\swarrow$  change in direction

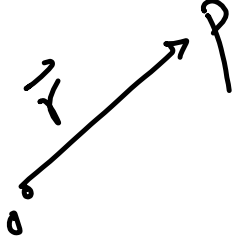


$$\begin{aligned}\hat{i} \times \hat{j} &= \hat{k} \\ \hat{j} \times \hat{k} &= \hat{i} \\ \hat{k} \times \hat{i} &= \hat{j}\end{aligned}$$

$$\begin{aligned}\hat{u}_r &= \cos \theta \hat{i} + \sin \theta \hat{j} \\ \frac{d\hat{u}_r}{dt} &= -\sin \theta \dot{\theta} \hat{i} + \cos \theta \dot{\theta} \hat{j} \\ &= \dot{\theta} (\cos \theta \hat{j} - \sin \theta \hat{i}) \\ &= \dot{\theta} (\cos \theta \hat{k} \times \hat{i} + \sin \theta \hat{k} \times \hat{j}) \\ &= (\dot{\theta} \hat{k}) \times (\cos \theta \hat{i} + \sin \theta \hat{j})\end{aligned}$$

$$\dot{\hat{u}}_r = \frac{d\hat{u}_r}{dt} = \vec{\omega} \times \hat{u}_r$$

$$\begin{aligned}\vec{v} &= \frac{dr}{dt} \hat{u}_r + r \frac{d\hat{u}_r}{dt} \\ &= \frac{dr}{dt} \hat{u}_r + r(\vec{\omega} \times \hat{u}_r)\end{aligned}$$



A diagram showing a vector  $\vec{r}$  originating from a point  $O$  and pointing to a point  $P$ . The vector is labeled  $\vec{r}$  above it.

$$\vec{V} = \underbrace{\frac{dr}{dt} \hat{u}_r}_{\text{Change in length}} + \underbrace{\vec{\omega} \times \vec{r}}_{\substack{\text{Change in} \\ d\tau}}$$