Learning The-Self: Leveraging Proprioception to Guide the Autonomous Discovery of the Robotic Body Schema

Fernando Díaz Ledezma, M.Sc.

Ph.D. Thesis

TECHNISCHE UNIVERSITÄT MÜNCHEN
Munich Institute of Robotics and Machine Intelligence

Learning The-Self: Leveraging Proprioception to Guide the Autonomous Discovery of the Robotic Body Schema

Fernando Díaz Ledezma, M.Sc.

Vollständiger Abdruck der von der TUM School of Computation, Information and Technology der Technischen Universität München zur Erlangung des akademischen

Grades eines

Doktor-Ingenieurs (Dr.-Ing.)

genehmigten Dissertation.

Vorsitzender:		Prof. Dr. XXX
Prüfer der Dissertation:	1.	Prof. Dr.-Ing. Sami Haddadin
	2.	Prof. XXX
	3.	Prof. XXX

Die Dissertation wurde am **XX.XX.2024** bei der Technischen Universität München eingereicht und durch die TUM School of Computation, Information and Technology am **XX.XX.2024** angenommen.

I am not Ultron, I am not Jarvis, I am… I am.

Vision, Avengers: Age of Ultron

Abstract

As robots become increasingly integral to human life, the imperative emerges for them to autonomously explore and construct models of their bodies. Robots should take cues from human capabilities, aspiring to build and utilize their body schema for advanced locomotion, finer manipulation, and adaptive interactions. Thus, a crucial foundation lies in seamlessly integrating the body schema to elevate learning, motor control, coordination, and spatial awareness. Furthermore, future robots should become self-sufficient entities that conduct monitoring, calibration, and adaptation exclusively through onboard sensing modalities. Standardizing constant self-monitoring nurtures spatial awareness and facilitates rapid error detection and correction. A profound understanding of their body structure will undoubtedly lead to enhanced, safe, and energy-aware interactions. However, current robot learning approaches encounter limitations, such as suboptimal generalization and sample efficiency, exhibiting a need for more structural knowledge. Versatile methods, like neural networks, confront challenges related to data and topology, confining learning to specific regions. On the other hand, learning robot physical attributes still rely on a presumed knowledge of the mechanical topology, often involving calibration and offline identification in controlled environments with a persistent reliance on external measurements, such as vision and motion-capturing systems. The research landscape generally reveals the lack of a unified framework that enables robots to build representations of their body schema to achieve improved body awareness and interaction capabilities. This study addresses these challenges by consolidating necessary and sufficient proprioceptive signal quantities, enabling robots to autonomously acquire knowledge about their body structure without relying on exteroceptive off-body sensors. It introduces an approach that reformulates robot kinematic calibration and system identification as a modular computational graph amenable to machine learning. This abstracted architecture, applied in online learning phases, seamlessly merges proprioceptive signals with first-order principles, extracting fundamental features of the robot body schema. Characterizing morphological properties of tree-like structures, the study infers mechanical topology through information-theoretic measures, validating and applying it independently of off-robot calibration. The research extends its scope

by complementing the robot body schema by instantiating inertial properties, ensuring online learning and physical feasibility. Ultimately, this work challenges the uncritical application of end-to-end learning in physical systems, urging a reevaluation of its limitations when excluding principled knowledge. It underscores opportunities for machine learning frameworks in embodied systems, emphasizing the untapped potential of synergizing structural knowledge with data-driven methods. This study catalyzes future research in an incipient field that underscores building and maintaining a body schema by demonstrating that fundamental properties of a robot's morphology can be deduced from proprioceptive signals. Its implications are far-reaching, addressing the needs of conventional and dynamic robotic structures with diverse sensory modalities that require a more profound sense of self.

Contents

Abbreviations and Symbols

In this thesis, scalar quantities are written as plain letters, e.g., λ, c, K. Vectors and matrices are represented by bold letters, whereas vectors have small letters, e.g., $\boldsymbol{q}$, $\boldsymbol{\theta}$ and matrices capital letters $\boldsymbol{K}$, $\boldsymbol{M}$. The total derivative w.r.t. time is indicated by a dot above the symbol, i.e., $\dot{\boldsymbol{x}} = \frac{d}{dt}\boldsymbol{x}, \ddot{\boldsymbol{x}} = \frac{d^2}{dt^2}\boldsymbol{x}$. The Euclidian norm of a vector q is denoted by $|q|$, the dot product of two vectors a and b by (a, b) and their cross product by $a \times b$.

All symbols are introduced in the text before they are used. In some sections, the argument is omitted for brevity. Several variables appear with different subscripts, superscripts, additional symbols and dimensions. In the following list, the quantities are generally described without being further specified. The specific meaning becomes apparent when the respective variable is introduced in the text. Please note that the list of symbols is not complete, but it contains symbols that appear frequently or are of major importance in this thesis.

List of Symbols

General Symbols

$\boldsymbol{q}$	Joint positions
$\dot{\boldsymbol{q}}$	Joint velocities
$\ddot{\boldsymbol{q}}$	Joint acceleration
$\boldsymbol{T}$	Pose in matrix form
$\boldsymbol{T}_d$	Desired pose in matrix form
$\boldsymbol{x}$	Pose in vector form
$\boldsymbol{x}_d$	Desired pose in vector form
$\dot{\boldsymbol{x}}$	Twist

$\dot{\boldsymbol{x}}_d$	Desired twist
$\ddot{\boldsymbol{x}}$	Cartesian acceleration
$\ddot{\boldsymbol{x}}_d$	Desired Cartesian acceleration
$\boldsymbol{f}$	Wrench
$\boldsymbol{f}_d$	Desired wrench
$\boldsymbol{f}_{ff}$	Feed-forward wrench
$\boldsymbol{f}_{\text{ext}}$	External wrench
$\boldsymbol{\tau}$	Joint motor torques
$\boldsymbol{\tau}_d$	Desired joint motor torques
$\boldsymbol{\tau}_{\text{ext}}$	External joint torque
$\boldsymbol{K}_x$	Cartesian stiffness
$\boldsymbol{K}$	Joint stiffness
$\boldsymbol{D}_x$	Cartesian damping
$\boldsymbol{D}$	Joint damping
$\boldsymbol{J}_x$	Jacobian
$\boldsymbol{M}$	Mass matrix
$\boldsymbol{C}$	Coriolis vector
$\boldsymbol{g}$	Gravity vector
t	Time
$\mathcal{U}(().)$	Environment around a pose

Process, Tactile Skill, and Taxonomy

ς	Skill
ι	Task
π_d	Tactile policy
Π	Set of tactile policies
p	Process
$\mathcal{P}$	Set of processes

$\mathcal{T}$	Taxonomic synthesis algorithm
o	Object
$\mathcal{O}$	Set of objects
s	Process state
s_0	Initial process state
s_1	Final process state
s_e	Error process state
s_π	Policy process state
$\mathcal{C}_{\text{pre}}$	Process pre condition
$\mathcal{C}_{\text{err}}$	Process error condition
$\mathcal{C}_{\text{suc}}$	Process success condition
δ	Process step transition
Δ	Set of process step transition
$\boldsymbol{\theta}$	Parameters
$\boldsymbol{\theta}_\pi$	Skill parameters
$\boldsymbol{\theta}_c$	Controller parameters

GGTWreP framework

w	World state
$\mathcal{W}$	World state space
E	Effects of skill execution on environment
$\mathcal{R}$	Requirements for skill execution
$\mathbb{D}$	Parameter domain
$\mathbb{C}$	Skill constraints
$\mathfrak{D}$	Dataset from a learning experiment
$\mathcal{Q}$	Quality metric
$\mathcal{C}$	Tactile policy commands

Planning

$\mathcal{A}$	Assembly
ρ	Assembly part
Γ	Set of assembly parts
a	Assembly action
α	Sequence of assembly actions
w	Agent
W	Set of agents
ζ	Assembly state
$\mathcal{Z}$	Set of assembly states

Abbreviations

ADAM	Adaptive Momentum
AI	Artificial intelligence
CGL	Combinatorial graph Laplacian
DL	Deep learning
DOF	Degrees of freedom
EE	End effector
e.g.	*Exemplis grata* (for example)
ESN	Echo state network
etc.	*Et cetera* (and so forth)
F/T sensor	Force/torque sensor
GGTWreP	Graph-Guided Twist-Wrench Policy Approximation
GP	Gaussian process
ICRA	International Conference on Robotics and Automation
i.e.	*Id est* (that is)
IEEE	Institute of Electrical and Electronics Engineers

LbD	Learning by demonstration
LWR	Lightweight Robot
MI	Mutual information
MIRMI	Munich Institute of Robotics and Machine Intelligence
ML	Machine learning
MST	Maximum spanning tree
NTI	Network topology inference
ODE	Ordinary differential equation
PC	Personal Computer
RL	Reinforcement Learning
ROI	Region of interest
ROS	Robot Operating System
SPD	Symmetric Positive Definite
SVM	Support vector machine
TMI	Total mutual information
TUM	Technical University Munich
UAV	Unmanned aerial vehicle
w.r.t.	With respect to

1

Introduction

Imagine your body; do not look at it. Close your eyes and tell me what you see. What is the pose of your body right now? Are you standing, seated, lying? Where are your arms and hands? Do not open your eyes just yet. Now, touch your left knee with your right hand. Did you struggle to find your knee? Chances are you did not. How can we manage to know where our body parts are in space at all times without even looking at them? It is safe to assume that we humans have what is called an internal model of our physical self. That is, we have a representation of our body. Such a representation allows us to accomplish remarkable feats. Have you ever caught an object in mid-air? Certainly, succeeding in that involved not only estimating the trajectory of the object but also the motion that your arm and hand needed to accomplish to reach the catching point. Had your arm dimensions been different from the ones you *have in mind*, or had the accuracy of your knowledge about your arm's pose been insufficient, you would not have caught the object. This internal model of yourself is called the body schema in psychology and neuroscience. It plays a definite role in knowing about the state of our body in space and also in calculating how we move. Consider a different scenario. You are in a pool and have been floating around for quite some time now. By the time you leave the pool, you feel heavier, like you need to put more effort into moving your body. A similar example happens during a workout at the gym. You lift a heavy dumbbell for a number of repetitions, and then, when you are done, it feels like bending your arm takes no effort at all. Your body schema adapted its representation to the situation. The accuracy of this internal model and, certainly, its plasticity play an important role in the kinematic and dynamic control of our bodies. Not only that, but the schema is very much related to our ability to use tools, say a pencil, a hammer, or a golf club. Our body schema is plastic and can temporarily integrate external objects, giving us the capability to manipulate those tools with such dexterity as if they were parts of our own bodies.

Robots have been driving mass production in industries for a good number of decades. They are an essential part of assembly lines in factories. Teams of them take care of tasks demanding high precision. Yet, even as they have been undoubtedly the workforce of automation, they are not as autonomous as we would like them to be. They are designed, constructed, and programmed to be accurate. To really ensure accuracy in their tasks, it is not only the robots that need to move with high precision, but also their environments need to be controlled. Every possibility of interaction needs to be predefined, and disturbances are to be minimized if not completely eliminated. This is precisely the reason why robots in industries normally execute their tasks in confined spaces, having little or no interaction at all with the external environments, let alone with humans. However, in recent decades, there has been a shift. Technology has evolved enough so that robots are now leaving these enclosed and protected environments and are aiming at increased interaction with their surroundings, other robots, and people. This advanced interaction comes at a price: uncertainty. The world cannot be modeled; every single potential interaction scenario cannot be anticipated and controlled. Robots, if they are to succeed in the human world, need to improvise strategies, adapt to the situation, and overcome constant, ever-changing challenges. But robots are not (at least until now) adaptable; the ways in which they are modeled are fixed and rarely subject to change. Their control paradigms also do not account for changes. Naturally, their very own bodies are not supposed to change. But change is there, a constant factor. Wear and tear is a typical phenomenon. A localized malfunction is an event that, although undesired, could occur, and a robot should be able to handle it. Perhaps the task that it needs to execute has parameters slightly different from what was anticipated; tools could be different, and the environment could be different. To put it bluntly, modern robots need to have plasticity in their models, just as humans do.

Let us go back to our imagination. Say you are left-handed. Unfortunately, you had a light accident and broke your left arm. Nothing serious, but you will not be using that arm for a couple of weeks. You can adapt your internal model to account for this situation and will probably become dexterous with that good right arm. If a humanoid robot is programmed to accomplish tasks with both arms and one ceases to operate for some reason, it is unlikely that I will carry on with the task. At this point, the engineers responsible for this robot may determine that the robot can be repurposed while the arm is repaired. They adapt the model of the robot, tune some parameters, and off the robot goes to work on its new tasks. But this is not the level of autonomy we imagine when we see a future in which robots coexist with us and assist us in a myriad of things. Once again, robots need to adapt. And the very first aspect that needs to be adapted is their own body. For that, robots need to have the capability to understand and construct a model of their bodies. This brings back the discussion to the idea that a robot can certainly benefit from the understanding of its physical self, and for that, it would need to leverage information coming from its on-board sensors and fuse them with ground truths to make sense of its body and the world around it. But how?

1.1 Motivation and vision

Empowering Robots to Learn and Control their Bodies As the integration of robots into various facets of human life becomes more pronounced, there arises a crucial imperative for these machines to proactively participate in the exploration and development of models for their own bodies. At the core of this autonomous self-discovery is the development of an awareness of the physical self by developing and maintaining a body schema. This self-awareness represents a pivotal step towards endowing robots with a profound understanding of their own embodiment and becomes the foundation for the integration of sensory information and motor control. The robot body schema, akin to the internal representation of the human body in the brain, becomes a dynamic and evolving map that serves as a reference point for the robot's interactions with its surroundings. The establishment of a body schema contributes significantly to the enhancement of robot motor control. Robots, equipped with a coherent internal model of their bodies, can execute movements with a heightened level of precision and coordination. The fusion of sensory information and motor control within the body schema lays the scaffolding for efficient learning. Additionally, autonomous self-discovery is not a static process. It involves ongoing refinement and adaptation. As robots encounter diverse environments and engage in various tasks, their body schema adapts and expands, allowing for an extended understanding of their physical capabilities. This adaptability is crucial for robots to navigate the intricacies of real-world scenarios, adjusting their responses based on the context in which they operate. The significance of this autonomous self-discovery transcends the technical aspects of robotics; it resonates with the broader narrative of robots becoming integrated and adaptive participants in human-centric environments. By actively engaging in the exploration of their own bodies, robots pave the way for a future where they seamlessly navigate, interact, and adapt in tandem with human activities. This not only enhances their functional capabilities but also fosters a harmonious integration of robots into diverse and dynamic human-centric spaces.

Learning and the body schema The seamless integration of the body schema is indispensable for the evolution of robots, forming the foundational underpinning upon which multifaceted capabilities are constructed. This integration spans across various domains, encompassing learning for physical awareness, motor control, coordination, and interaction. Learning, a hallmark of intelligent systems, coexists in a dual relationship with the body schema: learning not only contributes to the development of the body schema but, reciprocally, the body schema enhances learning. Primarily, it plays a pivotal role in detecting the structure within the myriad afferent and efferent signals of the robot's sensorimotor system, facilitating the construction of the body schema. Simultaneously, the incorporation of the body schema into learning frameworks allows robots to explore their sensorimotor maps and develop models of their morphology. This understanding becomes a scaffold for acquiring new skills, refining existing ones, and assimilating knowledge gained from interactions with the environment. The integration of the body schema thus catalyzes a learning process that is not only adaptive but also inherently tied to the robot's physical embodiment. Moving to motor control, another pivotal aspect of intelligent systems, there exists an intricate link to learning the body schema. Internal body representations that can adapt through learning mechanisms contribute to the development of superior forward and inverse models, ultimately refining control precision. Lastly, a seamlessly integrated body schema empowers robots to learn diverse tasks, surpassing the limitations of rigid, pre-programmed functionalities. Ultimately, a plastic body representation provides versatility crucial in dynamic and unpredictable settings, where the cohesive ability to learn, control movements, coordinate actions, and perceive space allows robots to thrive in a myriad of scenarios.

Enhancing locomotion, manipulation, and adaptability. Taking inspiration from the nuanced abilities of humans, the vision for future robots is set to take advantage of the enhanced bodily awareness that the integration of a body schema will bring about, thereby improving adaptability and interaction in diverse situations. The development and maintenance of the robot body schema will unlock a spectrum of capabilities that enables precise and coordinated movements, fostering in particular advanced locomotion, motion planning, and intricate manipulation. Bodily awareness is expected to revolutionize the locomotive prowess of robots. Future machines, informed by this internal map of their physical structure, will navigate environments with an unprecedented level of sophistication. This extends beyond basic movement, enabling robots to traverse complex terrains, negotiate obstacles, and adapt seamlessly to changes in their surroundings. Motion planning, a core element of robotics, will harness the robot body schema as a dynamic blueprint, supporting not only precise and efficient movements but also the ability to determine near-optimal trajectories in real-time. The result is a more adaptive and resourceful approach to navigating intricate spaces and executing tasks with heightened precision. The mastery of a body schema extends to manipulation with profound implications. Future robots, leveraging this internal map, will exhibit a level of dexterity and precision in manipulating objects that mirrors the intricacies of human hand-eye coordination. This enhanced capability will represent a breakthrough in applications requiring delicate and precise interactions, from handling diverse items to executing complex manufacturing tasks. Moreover, coordination and collaboration with other robots or humans becomes more refined through a well-integrated body schema, as it allows robots to anticipate and adapt their interactions with other agents. This anticipatory and adaptive capability is fundamental for fostering safe, effective, and harmonious interactions.

Constant self-monitoring for autonomy Continuous self-monitoring signifies a fundamental imperative for future robotic systems. It is achieved through the amalgamation of internal models and the uninterrupted stream of sensorimotor signals. This seamless fusion enables a dynamic and real-time understanding of the robot's own state, creating a powerful feedback loop that is integral to the system's autonomy. Perpetual monitoring sets in motion successive phases of error detection and correction. This iterative process ensures that the robot is not only aware of its physical state but also capable of recognizing and rectifying discrepancies between intended actions and actual outcomes. The ability to identify errors in real-time positions future robots on a trajectory towards enhanced reliability and precision in its interactions. The profound impact of this ongoing adaptation is most evident in the rapid formulation and execution of contingency motion strategies. Armed with an enriched spatial awareness and a continuously evolving set of internal models, the robot becomes adept at anticipating and responding to unforeseen challenges. In dynamic and unpredictable environments, the ability to swiftly devise and implement contingency plans allows the robot to navigate complex scenarios with agility and efficiency. This advanced interaction capability with the environment is a hallmark of the paradigm of continuous self-monitoring. The robot not only perceives its surroundings in real-time but also possesses the foresight to proactively engage with its environment. This goes beyond mere reactionary responses; it encapsulates a proactive and intelligent engagement that significantly elevates the robot's efficacy in accomplishing tasks and navigating diverse scenarios.

Onboard sensing for self-sufficiency Achieving true autonomy requires robots to evolve into self-sufficient entities capable of independent learning, calibration, monitoring, and adaptation of their body representation. This transformation is predicated on the exclusive reliance on onboard sensing modalities, a fundamental transition that empowers robots with heightened versatility and adaptability. At the core of this self-sufficiency lie two fundamental sensory modalities: somatosensation, encompassing proprioception and touch, and vision. These modalities collectively provide the robot understanding of its own body and the surrounding environment. Proprioception provides the robot awareness of its own body in space. The sense of touch complements the understanding of the body and allows the robot to distinguish itself from its immediate environment. Vision, another cornerstone modality, extends the robot's perception beyond immediate physical contact. By abstaining from off-board sensing devices, robots liberate themselves from external dependencies and enhance their self-sufficiency. Leveraging onboard sensing modalities empowers robots to dynamically respond to changes in their surroundings in real-time. Whether it's navigating through a cluttered environment, adjusting movements for safety, or adapting to unforeseen obstacles, the reliance on somatosensation and vision will enable future robot to operate with a level of autonomy and adaptability previously unseen.

Safety- and energy-awareness Comprehending their own body structure empowers robots to engage in more effective and nuanced interactions with both their robotic and human counterparts. The body schema serves as a predictive tool, endowing robots with the foresight to anticipate potential challenges. This heightened understanding facilitates dynamic adjustments in their movements, prioritizing safety and fostering efficiency in diverse collaborative scenarios. When interacting with other robots, this capability enables seamless coordination, averting collisions or disruptions during joint tasks. Likewise, in human-robot interactions, the ability to adapt movements ensures a safer environment, mitigating the risk of accidental impacts or collisions. Moreover, the comprehension of their own body structure provides robots with a unique advantage in optimizing energy consumption. By adapting their motions based on the inherent physical properties of their structures, robots can execute tasks with greater efficiency. This optimization is particularly crucial in mobile robotics, where energy conservation is paramount given the limited resources. The dual capability of enhancing safety in interactions and contributing to energy-aware robotics underscores the significance of robots understanding their own body structure in fostering a more efficient, collaborative, and environmentally conscious robotic landscape.

1.2 Problem Statement

Navigating the complexities of learning a robot's physical attributes in the field of robotics reveals intricate challenges that demand a closer look. Traditional calibration routines, tailored for known kinematic structures and offline system identification methods, face limitations concerning generalization capabilities, sample efficiency, and the reliance on a predetermined mechanical topology. This is particularly evident in the context of global and local machine learning frameworks for physical systems, where the exclusion of structural knowledge often results in operational inefficiencies.

Compounding these challenges is the substantial reliance on off-robot measurement devices, such as vision and motion-capturing systems, during calibration and identification processes. Despite the wealth of sensor signals from modern robots, determining the minimum set required for constructing a body model based solely on robot sensing remains a persistent and unresolved challenge. This dependence on external measurements not only introduces limitations but also raises questions about the practicality and applicability of learned models in real-world scenarios.

Venturing into alternative learning methods, such as neural networks, introduces a unique set of challenges. These approaches frequently lack crucial information about the robot's body structure, necessitating vast amounts of data for effective learning. The design of neural networks further compounds the issue, as it becomes an expert-driven task requiring meticulous determination of topology. Often, these approaches suffer from generalization limitations, confining their learning capabilities to specific input-output regions

Examining the research landscape reveals substantial gaps, including an unclear understanding of how object handling extends the robotic body schema. Furthermore, limited exploration into the mechanical arrangement of joints and links, known as the mechanical topology, underscores untapped potential in understanding the intricate details of robotic physical structure. These research gaps collectively contribute to the overarching challenge of lacking a unifying scheme that seamlessly integrates all learning stages, hindering the development of a fully characterized robotic body schema based solely on knowledge about sensorimotor signals.

To address these multifaceted challenges effectively, there is a compelling need to bridge the existing gaps and explore innovative learning approaches. This includes the development of methods capable of incorporating structural knowledge, reducing reliance on external measurements, and enhancing the generalization capabilities of machine learning frameworks in the realm of robotics. Moreover, the establishment of a comprehensive unifying scheme stands as a pivotal step toward advancing the field and achieving a fully characterized robotic body schema.

1.3 Research Questions and Contribution

1.4 State of the Art

1.5 Thesis Structure

1.6 Curriculum Vitae

2

Related Work

One fundamental characteristic of humans is the awareness of the body, which allows its control with dexterity and plasticity. We know where our body is in space, we know much of that space our body takes, and we know how to take it from its present situation at time t, its current *state* $\boldsymbol{s}_t$, to a predicted or expected future state $\boldsymbol{s}_{t+1}$ at time $t+1$ given a carefully chosen *action* $\boldsymbol{a}_t$. The ability to anticipate the future and to determine the actions that can take us there is connected to the notions of forward and inverse models in cognitive science and robotics [**?**, **?**, **?**]. An alternative and complementary interpretation is that of sensorimotor contingencies [**?**, **?**, **?**], which denote the structured relations between the actions of an agent and the ensuing sensory inputs resulting from interaction. The potential sensorimotor interactions are connected to the concept of *embodiment*, which, as expressed by Pfeifer et al. [**?**], deals with "how the body shapes the way we think," putting a premium on the morphology and capacities of an agent. Last, a few strongly related concepts are pertinent to the discussion: the *ecological self* REFERENCE, the *physical self* REFERENCE, and the *body schema* REFERENCE. They are different interpretations of *body models* that describe its physical structure and capabilities, and integrate sensory information for prediction and action. Research in cognitive science tells us that we construct and maintain representations involving the previous concepts from the moment that we are in womb and during all our conscious existence. Maintaining those models is not only a matter of refining and tuning them but also may involve adapting them to accommodate unexpected drastic changes. The level of bodily awareness encoded in those representations allows to achieve remarkable feats, like reach for things without even looking, catching an object in midair, or contorting our bodies to maneuver in tight spaces or negotiate obstacles.

As discussed in the Introduction the vision for future robots involves giving them bodily awareness capabilities inspired in humans. To contextualize what is the state of the art towards realizing that vision, this chapter provides a review and discussion of relevant works that address the field of modeling in robotics; in particular, model learning. The discussion will include a brief review of the fundamental methods to model robots and will gradually move to the paradigm of learning local and global models with and without prior knowledge. The discussion will connect with the above mentioned concepts pertaining body models and present pertinent works that have presented methods to learn the body schema and sensorimotor representations in robotics

2.1 Model learning in robotics

Existing learning frameworks often focus on developing forward and inverse models using either a global or local approach to capture input-output relationships [**?**].

Global methods risk overfitting and computational overload, while local methods suffer from limited generalization and hyperparameter sensitivity [**?**, **?**]. Despite advancements in computational power and data availability, deep learning faces challenges due to the neglect of prior principled knowledge, making it difficult to determine dedicated neural network architectures [**?**, **?**].

2.1.1 End-to-end learning

2.1.1.1 Classical neural networks model learning

Model-learning problems using neural networks (NN) mainly involves:

1. Input/output data collection: assumed to be available
2. Architecture design: usually found by trial and error
3. Parameter optimization/learning: via well understood schemes, e.g., backpropagation and variants

Designing NN for a particular problem requires experts to determine the best topology, i.e. the number of nodes and layers, connectivity, and activation functions [**?**]. Furthermore, generalization is difficult as the architecture needs to balance achieving accuracy while avoiding overfitting [?, ?, ?, ?, ?, ?]. Therefore, if a NN is used without any model information, large amounts of training data are required to generalize to unknown data [?].

2.1.1.2 Topology learning related works

Finding NN topologies is an important and challenging step [?, ?, ?]. Normally, function approximation via NN uses empirical topologies that rely on numerous parameters and do not lend themselves to interpretation. Such models provide no insight into the actual relation between the system variables. Recent works have aimed to find optimal topologies automatically. For example, evolutionary methods have been utilized to optimize the topology of Feed Forward NN (FFNN) [?, ?] as well as deep NN [?], by adding/deleting connections and weights. Constructive methods [?] and pruning methods [?] have also been applied to FFNN. Another method used for FFNN represents the network as a graph and reduces its degrees-of-freedom (DoF) [?]. Furthermore, reinforcement learning (through Q-learning) and topology learning (using variance analysis) have also been implemented to generate architectures [?, ?]. Noticeably, for learning complex dynamical systems, such as articulated robot structures, results have been limited in accuracy and generalization capabilities [?,?,?].

2.1.1.3 Robot inverse dynamics estimation via classical NN

NN have been applied in numerous variants to model robot inverse dynamics. In [?], Hopfield NN were applied to identify the inertial parameters. Likewise, in [?] a FFNN that used the regressor matrix as training samples was applied. Extreme Learning Machines were utilized in [?] with the same purpose. More recently a two-hidden-layers network with rectified linear activation units (ReLU) was used in [?]. Similarly, recurrent NN have been used to account for the sequential nature of the data. In [?], a recurrent NN in the hidden layer of an otherwise conventional three-layer FFNN was proposed. Additionally, self-organizing-networks, in conjunction with echo state networks, were used in [?] via a real-time deep learning algorithm.

2.2 Data-driven learning with structure information

In lieu of the challenges faced by deep learning to capture the intricacies of complex systems from scratch, Issues such as low sample efficiency, extended training times, and limited generalization highlight the necessity of balancing data-driven and principle-driven approaches [?, ?]. Recently, there has been a growing acknowledgment of the importance of integrating structure into the learning of physical systems [?, ?].

2.3 Model learning and the body schema

Building on the significance of structure in model learning for robotics, this dissertation addresses the inference of essential morphological properties in tree-like floating base structures, mimicking the development of a body schema. Efforts in cognitive robotics stress the pivotal role of internal body models in enhancing spatial awareness, motor control, and adaptability [?, ?]. However, consensus is lacking on what constitutes a robot's body schema. Some approaches focus solely on learning the kinematic structure, relying predominantly on off-body vision [?, ?, ?, ?, ?, ?]. Others explore sensorimotor associations between proprioceptive, tactile, and visual modalities [?, ?, ?, ?, ?], but they provide limited insights into the robot's physical structure.

Model-based robotics offers reliable methods for identifying physical attributes of robots based on known mechanical topologies. Conventional calibration routines [?] and offline system identification methods [?, ?] are effective for known kinematic structures in controlled environments. However, these methods face challenges when applied to floating base robots without standardized identification procedures [?, ?]. Importantly, these conventional methods were not initially designed for integration into online learning frameworks. While model-based robotics addresses kinematic calibration and forward/inverse kinematics, it provides limited insights into the comprehensive understanding of joint and link arrangement, known as mechanical topology. In cognitive robotics, only a few studies have approached this problem for self-modeling and monitoring, relying on exteroceptive vision [?, ?]. Regardless of the approach taken—black-box machine learning, cognitive methods, or model-based robotics—reliance on external measurement devices persists, overlooking embodied sensing modalities.

In summary, current robotics research reveals gaps in understanding and methods for refining body models. A comprehensive interpretation of the robot body schema and the determination of essential features are crucial. Identifying the fundamental set of necessary signals, both proprioceptive and embodied exteroceptive, is paramount. Integrating advanced machine learning with prior information and first-order principles shows promise for enhanced body models, addressing data requirements and generalization issues. However, the lack of synergy between modeling and learning approaches, along with the absence of a unified scheme for relevant learning stages, represents notable gaps requiring attention to advance robotics into more sophisticated and adaptable embodied systems.

2.4 Model learning in robotics

2.4.1 Classical and recent works in system identification

2.4.2 Local and global models linear models

2.4.3 End-to-end learning (black box models)

2.5 Data-driven learning with structure information

2.6 Model learning and the body schema

2.6.1 Internal representations

2.6.2 Sensorimotor maps

3

Theoretical Foundations

3.1 Robot equations of motion

3.1.1 Forward and inverse dynamics

3.1.2 The Newton-Euler formulation of the inverse dynamics

Extension for floating base robots with flexible joints and links have been presented elsewhere [**?**]

3.1.3 Composition of the kinematics and dynamics

3.2 Robot proprioception

3.2.1 Definition

3.2.2 Human and robot proprioception

3.2.3 A sensor suite for robot proprioception

3.3 State-of-the-art Gradient Descent

3.3.1 Fundamentals

3.3.2 Momentum gradient descent

3.3.3 ADAM gradient descent

3.3.4 AMS gradient descent

3.4 Fundamentals of graph theory

3.4.1 Definition of a graph

A graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is a mathematical construct the represents the interconnections between the elements of a system. These elements are represented as a set $\mathcal{V} = \{v_i\}_{i=1}^{n}$ of *n nodes* (also called *vertices*) and their connections are depicted as a set of *edges* $\mathcal{E} = \{e_i\}_{i=1}^{m} \subseteq \mathcal{V} \times \mathcal{V}$. Note that it is possible that an edge connects a node to itself, in which case the edge defines a self loop.

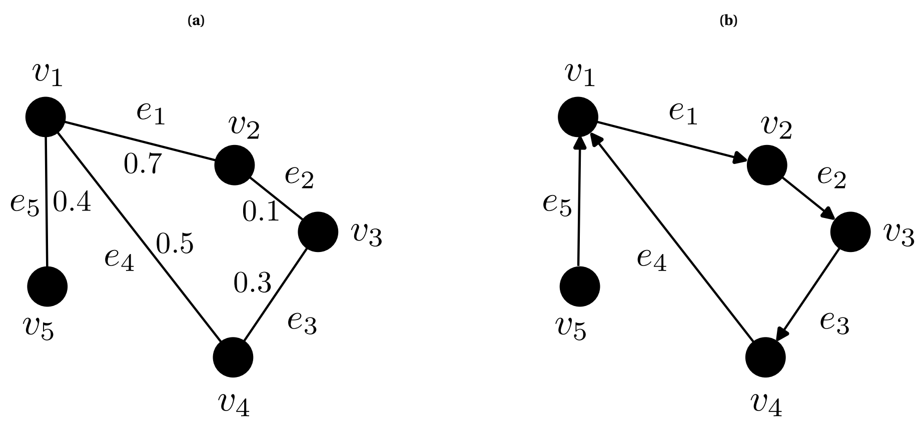

Figure 3.1: Two basic types of graphs. (a) An undirected and (b) a directed graph.

Two nodes i and j are said to be *adjacent* if there is an edge e connecting them. Similarly, the edge e connecting the nodes is called *incident* to i and j. For example, the nodes v_1 and v_4 in Fig. 3.1a are adjacent to each other and the edge e_4 is incident to both nodes. The number of edges that are incident to a node is the *degree* of the vertex.

A *path* represents a potential sequence of edges connecting any two given nodes. The number of edges determines the *length* of the path. A graph is said to be *connected* if there is at least one path between every pair of nodes; otherwise, it is labeled *disconnected.* A *cycle* constitutes a path comprising a minimum of three edges, where the initial and final vertices are identical, and no vertices are repeated in between. Lastly, a graph is *complete* if there exists a path connecting every pair of nodes. Notice that the maximum number of edges in an undirected graph with n nodes is $n(n-1)/2$

An *undirected* graph lacks any specified direction assigned to its edges, meaning the connections between nodes are bidirectional. On the other hand, a *directed* graph, commonly referred to as a *digraph*, introduces directionality to its edges. In a directed graph, each edge has an explicit direction, indicating a one-way flow from one node to another. This directional information adds an extra layer of complexity to the relationships within the graph, as opposed to the bidirectional nature of edges in an undirected graph.

A *weighted* graph $\mathcal{G}\big(\mathcal{V},\mathcal{E},\boldsymbol{W}\big)$ associates numerical value, a weight, to each edge. The weights with weights $\boldsymbol{W} : \mathcal{V}\times\mathcal{V} \to \mathbb{R}_+$ express some quantitative measure such as distance, cost, time, or any other relevant metric depending on the context of the graph. Unlike an unweighted graph, where edges simply represent connections between nodes, a weighted graph provides additional information about the relationships between nodes. For a weighted graph, the *strength* of a vertex corresponds to the sum of the edge weights associated with it.

3.4.2 Algebraic representation of a graph

A graph with n vertices can be represented as a square $n \times n$ matrix $\boldsymbol{A}$. This algebraic representation is called the *adjacency matrix* and depicts the graph's connectivity pattern; that is, the elements of the matrix indicate whether pairs of vertices $\big(i, j\big)$ are adjacent or not in the graph. Formally, the elements of a *binary* adjacency matrix $\boldsymbol{A}$ are determined as follows

$$(\boldsymbol{A})_{i,j} = \begin{cases} 1 & \text{if and edge connects nodes } i \text{ and } j \\ 0 & \text{otherwise.} \end{cases} \tag{3.1}$$

By extension, a *weighted* adjacency matrix $\boldsymbol{W}$ denotes a weighted connection for some of the $\big(i, j\big)$ entries, i.e $(\boldsymbol{W})_{i,j} \in \mathbb{R}_+$. If the graph is undirected, the adjacency matrix is symmetric, which implies that $\boldsymbol{A} = \boldsymbol{A}^\intercal$ and $\boldsymbol{W} = \boldsymbol{W}^\intercal$respectively. Note, however, that for the case of digraphs, the adjacency matrix is not necessarily symmetric. To illustrate this, refer to the graph in Fig. 3.1a, the corresponding binary and weighted adjacency matrices are

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

and

$$\boldsymbol{W} = \begin{bmatrix} 0 & 0.7 & 0 & 0.5 & 0.4 \\ 0.7 & 0 & 0.1 & 0 & 0 \\ 0 & 0.1 & 0 & 0.3 & 0 \\ 0.5 & 0 & 0.3 & 0 & 0 \\ 0.4 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Similarly, for the digraph in Fig. 3.1b, the adjacency matrix is

$$\boldsymbol{A}_D = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Notice that unless there are self loops in the graph, the main diagonal of an adjacency matrix contains only zero entries. Two other important matrices related to a graph $\mathcal{G}$ are the degree matrix $\boldsymbol{D} : (\boldsymbol{D}_{ii}) = \sum_{j=1}^{m} w_{ij}$, a diagonal matrix whose entries are the sum of the rows of $\boldsymbol{W}$, and the combinatorial graph Laplacian (CGL), defined as $\boldsymbol{L}_{\text{CGL}} = \boldsymbol{D} - \boldsymbol{W}$ [**?**].

3.4.2.1 The normalized adjacency matrix

The normalized adjacency matrix, defined as

$$\mathcal{W} = \boldsymbol{D}^{-\frac{1}{2}} \boldsymbol{W} \boldsymbol{D}^{-\frac{1}{2}}, \tag{3.2}$$

is particularly employed in spectral graph theory [**?**] and related analyses. Normalization helps account for variations in node degrees and provides a more balanced representation of the graph structure. One common application is in spectral clustering algorithms [**?**], where the normalized adjacency matrix is used to compute eigenvectors and eigenvalues, aiding in the identification of clusters or communities within a graph.

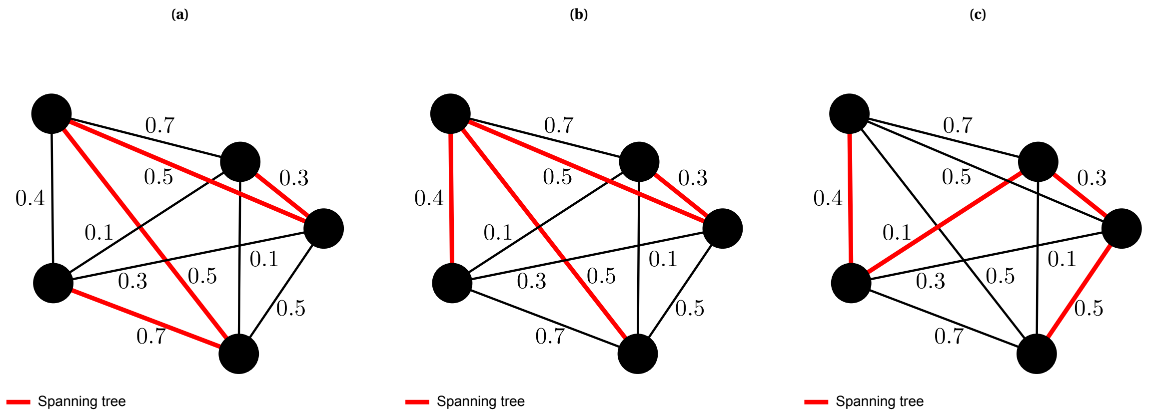

Figure 3.2: Different spanning trees for the same graph.

3.4.3 Subgraphs and spanning trees

A *subgraph* $\mathcal{G}_1$ is obtained by selecting a subset of the nodes of another graph $\mathcal{G}$ and a corresponding subset of the edges connecting those nodes. Formally, a graph $\mathcal{G}_1(\mathcal{V}_1, \mathcal{E}_1)$ is a subgraph of $\mathcal{G}$ if and only if $\mathcal{V}_1 \subseteq \mathcal{V}$ and $\mathcal{E}_1 \subseteq \mathcal{E}$. This is written $\mathcal{G}_1 \subseteq \mathcal{G}$. One particular subgraph of interest in this work are spanning trees. First, a *tree graph* is a graph that does not contain cycles. Such structure usually depicts a hierarchical arrangement. Consequently, a *spanning tree* $\mathcal{G}_1$ of a graph $\mathcal{G}$ is a subgraph containing all the nodes in $\mathcal{G}$ and whose edges form a tree that defines paths that ensure that all the nodes remain connected. The edges of $\mathcal{G}_1$ are denoted as the branches of the tree. In general, a graph $\mathcal{G}_1$ is said to be a spanning subgraph of $\mathcal{G}$ if and only if $\mathcal{V}_1 = \mathcal{V}$ and $\mathcal{E}_1 \subseteq \mathcal{E}$. It is worth noting that every connected graph has a spanning tree and that there are n^{n-2} distinct spanning trees with $n-1$ edges on a connected graph with n vertices [**?**]. Examples of spanning trees are shown in Fig. 3.2.

3.4.3.1 Minimum spanning tree and Kruskal's algorithm

The minimum spanning tree of an undirected, connected, and weighted graph is the spanning tree whose edge weight sum is less than or equal to that of all other spanning trees [**?**]. Kruskal's algorithm [**?**] is the standard method to find th minimum spanning tree of a graph. In this work we use an equivalent concept, the *maximum spanning tree* (MST), which corresponds to the spanning tree with maximum edge weight sum.

3.4.4 Metrics for graph comparison

TODO

3.5 Network topology inference

The process of identifying and visually representing relationships among various elements within a system, based on the available measurements, is termed *network topology inference* (NTI) [?]. Unveiling the structure of a network or graph through topology learning facilitates the analysis of interactions among entities.

3.5.1 Types connectivity

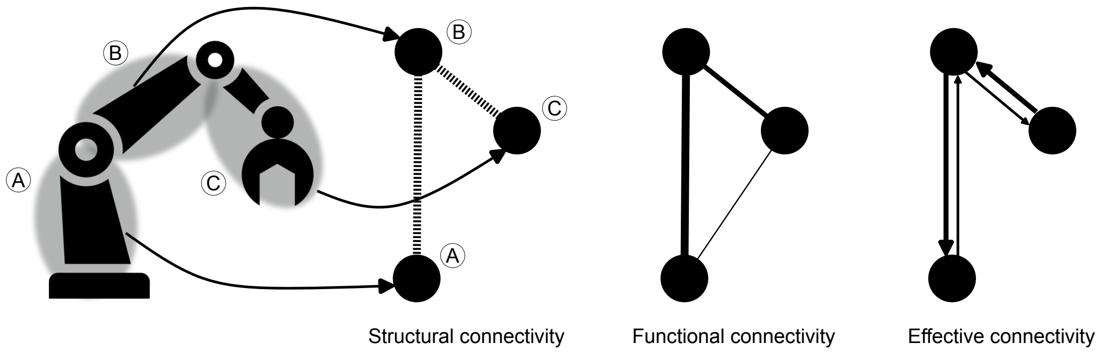

Figure 3.3: Types of connectivity. Every rigid body in the robot represents a node in the graphs.

Often used in the field of neuroscience [?], three different types of connectivity are distinguished [?, ?]:

Structural Connectivity (SC). Refers to the physical connections between different elements in a network. In the context of neuroscience, it typically involves the anatomical connections between brain regions. It is measured with techniques such as diffusion-weighted imaging (DWI) or diffusion tensor imaging (DTI) are commonly used to infer structural connectivity in the brain.

Functional Connectivity (FC). Refers to the statistical dependencies or temporal correlations between the activity of different elements in a network. As explained in [?], FC is an information-theoretic metric that characterizes dependencies using probability distributions of observed signals. It can be categorized into non-directed and directed forms, with the latter being associated with statistical causation derived from the data [?].

Effective Connectivity (EC). Specifically addresses the dynamic, state-dependent influence that one network element has on another within a particular causal dynamics model. Essentially, effective connectivity denotes coupling or directed causal influence [?]. Exemplary applications of these concepts can be observed in the fields of biology [?] and neuroscience [?, ?].

To understand the intricate relationships among the three types of connectivity refer to Fig. 3.3. The figure depicts a simple robot arm as an example. The nodes represent the three links that compose the robot. Regarding SC and FC, a nuanced connection exists, albeit not strictly one-to-one. For example, the SC reflect the physical composition of the body, three bodies connected by two joints (the edges in the SC graph). Sensory signals coming from these bodies might lead to potential functional interactions of varied degree (the edges in the FC graph) that have a foundation on the bodily structure of the robot; yet, it does not ensure their occurrence. Actually, functional relationships can appear even in the absence of direct structural connections, not the light edge between nodes A and C in the FC graph. Moving to EC and FC, the former extends the latter by seeking to model the direction and strength of influence between different elements. While FC identifies statistical associations, EC strives to unveil the underlying causal relationships, the directed edges in the EC graph. In summary, these three connectivity types are interrelated components in the complex landscape of NTI. Structural connections provide the anatomical substrate, functional connections depict statistical dependencies, and effective connections aspire to model causal relationships within the network.

3.5.2 Inferring the connectivity

In NTI the elements $\mathcal{V}$ of a graph $\mathcal{G}$ are known but its connectivity (i.e. how these elements relate to each other) is unknown. Then, the graph topology inference problem consists in finding the edges $\mathcal{E}$ that best explain the relationships among the nodes $\mathcal{V}$ given some prior knowledge, such as data distribution, the location of the sensors, the physical relationships between the signals, or the data similarity [**?**, **?**].

With **mutual information** [**?**]

With **GSP** [**?**]. We consider a method [**?**] which learns a relational matrix $\boldsymbol{W}_{GSP}$ without prior structural information considering the signals in $\boldsymbol{x}(t)$ as graph signals [**?**]. $\boldsymbol{W}_{GSP}$ is derived under the assumption that the signals on the graph change smoothly between connected nodes. Likewise, the Graph Signal Processing Toolbox (GSPBOX) from [**?**] was used to compute $\boldsymbol{W}_{GSP}$ using $\alpha = 0.6$, $\beta = 1$ (parameters that control the edge weight magnitude and the sparsity of $\boldsymbol{W}_{GSP}$, respectively) and normalizing the required pairwise distance matrix $\boldsymbol{Z}$ between $[0, 1]$.

With **correlation**. The first technique [**?**] upgrades standard correlation-based NTI by searching for an inverse covariance matrix with Laplacian (to find valid adjacency matrices $\boldsymbol{W}_{cor}$) and structural constraints (requiring a sparse matrix to reduce the graph edge density). We used the Graph Laplacian Learning (GLL) package [**?**] to calculate $\boldsymbol{W}_{cor}$, with regularization parameter $\gamma = 0.07$ and using a matrix $\boldsymbol{W}_0$ as connectivity prior with zero diagonal elements and ones elsewhere (denoting lack of structural knowledge).

3.5.2.1 Challenges in NTI

NTI brings with it challenges such as noisy measurements, lack of ground truth, large parameter spaces, and varying model complexity [?]. Moreover, inferring graph topology only from data is an ill-posed problem, having statistical models (e.g. correlation, entropy, mutual information) and physically motivated models (e.g. network diffusion) as general approaches [?]. Finally, the recently introduced paradigm of Graph Signal Processing (GSP) [?] considers samples from the signals at a given time as *graph signals*, whose properties are a consequence of the underlying graph.

3.5.3 Detecting linear dependencies with covariance

3.5.4 Graph signal processing

The fairly recent field of Graph Signal Processing (GSP) [?]

Under the assumption that the signals are related to the topology of the graph where they are supported, the goal of GSP is to develop algorithms that fruitfully leverage this relational structure and can make inferences about these relationships even when they are only partially observed.

A network may represent a conceptual model of pairwise relationships

A fundamental question in GSP is how to use the graph signals to infer the underlying structure of the network.

3.5.5 Based on statistic measures

3.6 Information theory

As discussed in [?] information is... Â¸

3.6.1 The meaning information

3.6.2 Entropy of random variables

3.6.3 Mutual Information: The correlation of the 21st century

Mutual information

The mutual information between two random variables is a symmetric measure of information computed as:

$$I(X;Y) = I(Y;X) = H(X) + H(Y) - H(X,Y) \tag{3.3}$$

with the Shannon's entropy of a variable X defined by

$$H(X) = -\sum_{i=1}^{n} p(x_i)\log_2\left(p(x_i)\right) \tag{3.4}$$

and the joint entropy between X and Y expressed as

$$H(X,Y) = -\sum_{i=1}^{n}\sum_{j=1}^{n} p(x_i, y_j)\log_2\left(p\left(x_i, y_j\right)\right). \tag{3.5}$$

3.6.3.1 Unexplored alternatives

The transfer entropy [**?**] is

3.7 Differential geometry

3.7.1 Fundamentals of differential geometry

3.7.2 Manifolds and the tangent space

3.7.3 Riemannian geometry and the metric

3.7.4 Applications in robotics

"The Symmetric Positive Definite (SPD) manifold is one specific type of Riemannian manifold. It is a smooth manifold where the tangent space is endowed with a Riemannian metric [2]. The Riemannian metric allows us to define various geometric notions such as the geodesic distance."