Learning The-Self: Leveraging Proprioception to Guide the Autonomous Discovery of the Robotic Body Schema

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Ph.D. Thesis

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Munich Institute of Robotics and Machine Intelligence

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Grades eines

Doktor-Ingenieurs (Dr.-Ing.)

genehmigten Dissertation.

Vorsitzender: Prof. Dr. XXX

Prüfer der Dissertation: 1. Prof. Dr.-Ing. Sami Haddadin

                             2. Prof. XXX

                             3. Prof. XXX

Die Dissertation wurde am **XX.XX.2024** bei der Technischen Universität München eingereicht und durch die TUM School of Computation, Information and Technology am **XX.XX.2024** angenommen.

I am not Ultron, I am not Jarvis, I am… I am.

Vision, Avengers: Age of Ultron

Abstract

As robots become increasingly integral to human life, the imperative emerges for them to autonomously explore and construct models of their bodies. Robots should take cues from human capabilities, aspiring to build and utilize their body schema for advanced locomotion, finer manipulation, and adaptive interactions. Thus, a crucial foundation lies in seamlessly integrating the body schema to elevate learning, motor control, coordination, and spatial awareness. Furthermore, future robots should become self-sufficient entities that conduct monitoring, calibration, and adaptation exclusively through onboard sensing modalities. Standardizing constant self-monitoring nurtures spatial awareness and facilitates rapid error detection and correction. A profound understanding of their body structure will undoubtedly lead to enhanced, safe, and energy-aware interactions. However, current robot learning approaches encounter limitations, such as suboptimal generalization and sample efficiency, exhibiting a need for more structural knowledge. Versatile methods, like neural networks, confront challenges related to data and topology, confining learning to specific regions. On the other hand, learning robot physical attributes still rely on a presumed knowledge of the mechanical topology, often involving calibration and offline identification in controlled environments with a persistent reliance on external measurements, such as vision and motion-capturing systems. The research landscape generally reveals the lack of a unified framework that enables robots to build representations of their body schema to achieve improved body awareness and interaction capabilities. This study addresses these challenges by consolidating necessary and sufficient proprioceptive signal quantities, enabling robots to autonomously acquire knowledge about their body structure without relying on exteroceptive off-body sensors. It introduces an approach that reformulates robot kinematic calibration and system identification as a modular computational graph amenable to machine learning. This abstracted architecture, applied in online learning phases, seamlessly merges proprioceptive signals with first-order principles, extracting fundamental features of the robot body schema. Characterizing morphological properties of tree-like structures, the study infers mechanical topology through information-theoretic measures, validating and applying it independently of off-robot calibration. The research extends its scope by complementing the robot body schema by instantiating inertial properties, ensuring on-

line learning and physical feasibility. Ultimately, this work challenges the uncritical application of end-to-end learning in physical systems, urging a reevaluation of its limitations when excluding principled knowledge. It underscores opportunities for machine learning frameworks in embodied systems, emphasizing the untapped potential of synergizing structural knowledge with data-driven methods. This study catalyzes future research in an incipient field that underscores building and maintaining a body schema by demonstrating that fundamental properties of a robot's morphology can be deduced from proprioceptive signals. Its implications are far-reaching, addressing the needs of conventional and dynamic robotic structures with diverse sensory modalities that require a more profound sense of self.

Contents

Abbreviations and Symbols

In this thesis, scalar quantities are written as plain letters, e.g., λ, c, K. Vectors and matrices are represented by bold letters, whereas vectors have small letters, e.g., $\boldsymbol{q}$, $\boldsymbol{\theta}$ and matrices capital letters $\boldsymbol{K}$, $\boldsymbol{M}$. The total derivative w.r.t. time is indicated by a dot above the symbol, i.e., $\dot{\boldsymbol{x}} = \frac{d}{dt}\boldsymbol{x}, \ddot{\boldsymbol{x}} = \frac{d^2}{dt^2}\boldsymbol{x}$. The Euclidian norm of a vector q is denoted by $|q|$, the dot product of two vectors a and b by (a, b) and their cross product by $a \times b$.

All symbols are introduced in the text before they are used. In some sections, the argument is omitted for brevity. Several variables appear with different subscripts, superscripts, additional symbols and dimensions. In the following list, the quantities are generally described without being further specified. The specific meaning becomes apparent when the respective variable is introduced in the text. Please note that the list of symbols is not complete, but it contains symbols that appear frequently or are of major importance in this thesis.

List of Symbols

General Symbols

$\boldsymbol{q}$	Joint positions
$\dot{\boldsymbol{q}}$	Joint velocities
$\ddot{\boldsymbol{q}}$	Joint acceleration
$\boldsymbol{T}$	Pose in matrix form
$\boldsymbol{T}_d$	Desired pose in matrix form
$\boldsymbol{x}$	Pose in vector form
$\boldsymbol{x}_d$	Desired pose in vector form
$\dot{\boldsymbol{x}}$	Twist
$\dot{\boldsymbol{x}}_d$	Desired twist

$\ddot{\boldsymbol{x}}$	Cartesian acceleration
$\ddot{\boldsymbol{x}}_d$	Desired Cartesian acceleration
$\boldsymbol{f}$	Wrench
$\boldsymbol{f}_d$	Desired wrench
$\boldsymbol{f}_{ff}$	Feed-forward wrench
$\boldsymbol{f}_{\text{ext}}$	External wrench
$\boldsymbol{\tau}$	Joint motor torques
$\boldsymbol{\tau}_d$	Desired joint motor torques
$\boldsymbol{\tau}_{\text{ext}}$	External joint torque
$\boldsymbol{K}_x$	Cartesian stiffness
$\boldsymbol{K}$	Joint stiffness
$\boldsymbol{D}_x$	Cartesian damping
$\boldsymbol{D}$	Joint damping
$\boldsymbol{J}_x$	Jacobian
$\boldsymbol{M}$	Mass matrix
$\boldsymbol{C}$	Coriolis vector
$\boldsymbol{g}$	Gravity vector
t	Time
$\mathcal{U}(().)$	Environment around a pose

Process, Tactile Skill, and Taxonomy

ς	Skill
ι	Task
$\boldsymbol{\pi}_d$	Tactile policy
Π	Set of tactile policies
p	Process
$\mathcal{P}$	Set of processes
$\mathcal{T}$	Taxonomic synthesis algorithm

o	Object
$\mathcal{O}$	Set of objects
s	Process state
s_0	Initial process state
s_1	Final process state
s_e	Error process state
s_π	Policy process state
$\mathcal{C}_{\mathrm{pre}}$	Process pre condition
$\mathcal{C}_{\mathrm{err}}$	Process error condition
$\mathcal{C}_{\mathrm{suc}}$	Process success condition
δ	Process step transition
Δ	Set of process step transition
$\boldsymbol{\theta}$	Parameters
$\boldsymbol{\theta}_\pi$	Skill parameters
$\boldsymbol{\theta}_c$	Controller parameters

GGTWreP framework

w	World state
$\mathcal{W}$	World state space
E	Effects of skill execution on environment
$\mathcal{R}$	Requirements for skill execution
$\mathbb{D}$	Parameter domain
$\mathbb{C}$	Skill constraints
$\mathfrak{D}$	Dataset from a learning experiment
$\mathcal{Q}$	Quality metric
$\mathcal{C}$	Tactile policy commands

Planning

$\mathcal{A}$	Assembly
ρ	Assembly part
Γ	Set of assembly parts
a	Assembly action
α	Sequence of assembly actions
w	Agent
W	Set of agents
ζ	Assembly state
$\mathcal{Z}$	Set of assembly states

Abbreviations

ADAM	Adaptive Momentum
AI	Artificial intelligence
CGL	Combinatorial graph Laplacian
DL	Deep learning
DOF	Degrees of freedom
EE	End effector
e.g.	*Exemplis grata* (for example)
ESN	Echo state network
etc.	*Et cetera* (and so forth)
F/T sensor	Force/torque sensor
GD	Gradient descent
GP	Gaussian process
ICRA	International Conference on Robotics and Automation
i.e.	*Id est* (that is)
IEEE	Institute of Electrical and Electronics Engineers

LbD	Learning by demonstration
LWR	Lightweight Robot
MI	Mutual information
MIRMI	Munich Institute of Robotics and Machine Intelligence
ML	Machine learning
MST	Maximum spanning tree
NTI	Network topology inference
ODE	Ordinary differential equation
PC	Personal Computer
RL	Reinforcement Learning
ROI	Region of interest
ROS	Robot Operating System
SPD	Symmetric Positive Definite
SVM	Support vector machine
TMI	Total mutual information
TUM	Technical University Munich
UAV	Unmanned aerial vehicle
w.r.t.	With respect to

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Introduction

Imagine your body; do not look at it. Close your eyes and tell me what you see. What is the pose of your body right now? Are you standing, seated, lying? Where are your arms and hands? Do not open your eyes just yet. Now, touch your left knee with your right hand. Did you struggle to find your knee? Chances are you did not. How can we manage to know where our body parts are in space at all times without even looking at them? It is safe to assume that we humans have what is called an internal model of our physical self. That is, we have a representation of our body. Such a representation allows us to accomplish remarkable feats. Have you ever caught an object in mid-air? Certainly, succeeding in that involved not only estimating the trajectory of the object but also the motion that your arm and hand needed to accomplish to reach the catching point. Had your arm dimensions been different from the ones you *have in mind*, or had the accuracy of your knowledge about your arm's pose been insufficient, you would not have caught the object. This internal model of yourself is called the body schema in psychology and neuroscience. It plays a definite role in knowing about the state of our body in space and also in calculating how we move. Consider a different scenario. You are in a pool and have been floating around for quite some time now. By the time you leave the pool, you feel heavier, like you need to put more effort into moving your body. A similar example happens during a workout at the gym. You lift a heavy dumbbell for a number of repetitions, and then, when you are done, it feels like bending your arm takes no effort at all. Your body schema adapted its representation to the situation. The accuracy of this internal model and, certainly, its plasticity play an important role in the kinematic and

dynamic control of our bodies. Not only that, but the schema is very much related to our ability to use tools, say a pencil, a hammer, or a golf club. Our body schema is plastic and can temporarily integrate external objects, giving us the capability to manipulate those tools with such dexterity as if they were parts of our own bodies.

Robots have been driving mass production in industries for a good number of decades. They are an essential part of assembly lines in factories. Teams of them take care of tasks demanding high precision. Yet, even as they have been undoubtedly the workforce of automation, they are not as autonomous as we would like them to be. They are designed, constructed, and programmed to be accurate. To really ensure accuracy in their tasks, it is not only the robots that need to move with high precision, but also their environments need to be controlled. Every possibility of interaction needs to be predefined, and disturbances are to be minimized if not completely eliminated. This is precisely the reason why robots in industries normally execute their tasks in confined spaces, having little or no interaction at all with the external environments, let alone with humans. However, in recent decades, there has been a shift. Technology has evolved enough so that robots are now leaving these enclosed and protected environments and are aiming at increased interaction with their surroundings, other robots, and people. This advanced interaction comes at a price: uncertainty. The world cannot be modeled; every single potential interaction scenario cannot be anticipated and controlled. Robots, if they are to succeed in the human world, need to improvise strategies, adapt to the situation, and overcome constant, ever-changing challenges. But robots are not (at least until now) adaptable; the ways in which they are modeled are fixed and rarely subject to change. Their control paradigms also do not account for changes. Naturally, their very own bodies are not supposed to change. But change is there, a constant factor. Wear and tear is a typical phenomenon. A localized malfunction is an event that, although undesired, could occur, and a robot should be able to handle it. Perhaps the task that it needs to execute has parameters slightly different from what was anticipated; tools could be different, and the environment could be different. To put it bluntly, modern robots need to have plasticity in their models, just as humans do.

Let us go back to our imagination. Say you are left-handed. Unfortunately, you had a light accident and broke your left arm. Nothing serious, but you will not be using that arm for a couple of weeks. You can adapt your internal model to account for this situation and will probably become dexterous with that good right arm. If a humanoid robot is programmed to accomplish tasks with both arms and one ceases to operate for some reason, it is unlikely that I will carry on with the task. At this point, the engineers responsible for this robot may determine that the robot can be repurposed while the arm is repaired. They adapt the model of the robot, tune some parameters, and off the robot goes to work on its new tasks. But this is not the level of autonomy we imagine when we see a future in which robots coexist with us and assist us in a myriad of things. Once again, robots need to adapt. And the very first

aspect that needs to be adapted is their own body. For that, robots need to have the capability to understand and construct a model of their bodies. This brings back the discussion to the idea that a robot can certainly benefit from the understanding of its physical self, and for that, it would need to leverage information coming from its on-board sensors and fuse them with ground truths to make sense of its body and the world around it. But how?

1.1 Motivation and vision

Empowering Robots to Learn and Control their Bodies As the integration of robots into various facets of human life becomes more pronounced, there arises a crucial imperative for these machines to proactively participate in the exploration and development of models for their own bodies. At the core of this autonomous self-discovery is the development of an awareness of the physical self by developing and maintaining a body schema. This self-awareness represents a pivotal step towards endowing robots with a profound understanding of their own embodiment and becomes the foundation for the integration of sensory information and motor control. The robot body schema, akin to the internal representation of the human body in the brain, becomes a dynamic and evolving map that serves as a reference point for the robot's interactions with its surroundings. The establishment of a body schema contributes significantly to the enhancement of robot motor control. Robots, equipped with a coherent internal model of their bodies, can execute movements with a heightened level of precision and coordination. The fusion of sensory information and motor control within the body schema lays the scaffolding for efficient learning. Additionally, autonomous self-discovery is not a static process. It involves ongoing refinement and adaptation. As robots encounter diverse environments and engage in various tasks, their body schema adapts and expands, allowing for an extended understanding of their physical capabilities. This adaptability is crucial for robots to navigate the intricacies of real-world scenarios, adjusting their responses based on the context in which they operate. The significance of this autonomous self-discovery transcends the technical aspects of robotics; it resonates with the broader narrative of robots becoming integrated and adaptive participants in human-centric environments. By actively engaging in the exploration of their own bodies, robots pave the way for a future where they seamlessly navigate, interact, and adapt in tandem with human activities. This not only enhances their functional capabilities but also fosters a harmonious integration of robots into diverse and dynamic human-centric spaces.

Learning and the body schema The seamless integration of the body schema is indispensable for the evolution of robots, forming the foundational underpinning upon which multifaceted capabilities are constructed. This integration spans across various domains, encompassing learning for physical awareness, motor control, coordination, and interaction.

Learning, a hallmark of intelligent systems, coexists in a dual relationship with the body schema: learning not only contributes to the development of the body schema but, reciprocally, the body schema enhances learning. Primarily, it plays a pivotal role in detecting the structure within the myriad afferent and efferent signals of the robot's sensorimotor system, facilitating the construction of the body schema. Simultaneously, the incorporation of the body schema into learning frameworks allows robots to explore their sensorimotor maps and develop models of their morphology. This understanding becomes a scaffold for acquiring new skills, refining existing ones, and assimilating knowledge gained from interactions with the environment. The integration of the body schema thus catalyzes a learning process that is not only adaptive but also inherently tied to the robot's physical embodiment. Moving to motor control, another pivotal aspect of intelligent systems, there exists an intricate link to learning the body schema. Internal body representations that can adapt through learning mechanisms contribute to the development of superior forward and inverse models, ultimately refining control precision. Lastly, a seamlessly integrated body schema empowers robots to learn diverse tasks, surpassing the limitations of rigid, pre-programmed functionalities. Ultimately, a plastic body representation provides versatility crucial in dynamic and unpredictable settings, where the cohesive ability to learn, control movements, coordinate actions, and perceive space allows robots to thrive in a myriad of scenarios.

Enhancing locomotion, manipulation, and adaptability. Taking inspiration from the nuanced abilities of humans, the vision for future robots is set to take advantage of the enhanced bodily awareness that the integration of a body schema will bring about, thereby improving adaptability and interaction in diverse situations. The development and maintenance of the robot body schema will unlock a spectrum of capabilities that enables precise and coordinated movements, fostering in particular advanced locomotion, motion planning, and intricate manipulation. Bodily awareness is expected to revolutionize the locomotive prowess of robots. Future machines, informed by this internal map of their physical structure, will navigate environments with an unprecedented level of sophistication. This extends beyond basic movement, enabling robots to traverse complex terrains, negotiate obstacles, and adapt seamlessly to changes in their surroundings. Motion planning, a core element of robotics, will harness the robot body schema as a dynamic blueprint, supporting not only precise and efficient movements but also the ability to determine near-optimal trajectories in real-time. The result is a more adaptive and resourceful approach to navigating intricate spaces and executing tasks with heightened precision. The mastery of a body schema extends to manipulation with profound implications. Future robots, leveraging this internal map, will exhibit a level of dexterity and precision in manipulating objects that mirrors the intricacies of human hand-eye coordination. This enhanced capability will represent a breakthrough in applications requiring delicate and precise interactions, from han-

dling diverse items to executing complex manufacturing tasks. Moreover, coordination and collaboration with other robots or humans becomes more refined through a well-integrated body schema, as it allows robots to anticipate and adapt their interactions with other agents. This anticipatory and adaptive capability is fundamental for fostering safe, effective, and harmonious interactions.

Constant self-monitoring for autonomy Continuous self-monitoring signifies a fundamental imperative for future robotic systems. It is achieved through the amalgamation of internal models and the uninterrupted stream of sensorimotor signals. This seamless fusion enables a dynamic and real-time understanding of the robot's own state, creating a powerful feedback loop that is integral to the system's autonomy. Perpetual monitoring sets in motion successive phases of error detection and correction. This iterative process ensures that the robot is not only aware of its physical state but also capable of recognizing and rectifying discrepancies between intended actions and actual outcomes. The ability to identify errors in real-time positions future robots on a trajectory towards enhanced reliability and precision in its interactions. The profound impact of this ongoing adaptation is most evident in the rapid formulation and execution of contingency motion strategies. Armed with an enriched spatial awareness and a continuously evolving set of internal models, the robot becomes adept at anticipating and responding to unforeseen challenges. In dynamic and unpredictable environments, the ability to swiftly devise and implement contingency plans allows the robot to navigate complex scenarios with agility and efficiency. This advanced interaction capability with the environment is a hallmark of the paradigm of continuous self-monitoring. The robot not only perceives its surroundings in real-time but also possesses the foresight to proactively engage with its environment. This goes beyond mere reactionary responses; it encapsulates a proactive and intelligent engagement that significantly elevates the robot's efficacy in accomplishing tasks and navigating diverse scenarios.

Onboard sensing for self-sufficiency Achieving true autonomy requires robots to evolve into self-sufficient entities capable of independent learning, calibration, monitoring, and adaptation of their body representation. This transformation is predicated on the exclusive reliance on onboard sensing modalities, a fundamental transition that empowers robots with heightened versatility and adaptability. At the core of this self-sufficiency lie two fundamental sensory modalities: somatosensation, encompassing proprioception and touch, and vision. These modalities collectively provide the robot understanding of its own body and the surrounding environment. Proprioception provides the robot awareness of its own body in space. The sense of touch complements the understanding of the body and allows the robot to distinguish itself from its immediate environment. Vision, another cornerstone modality, extends the robot's perception beyond immediate physical contact. By abstaining

from off-board sensing devices, robots liberate themselves from external dependencies and enhance their self-sufficiency. Leveraging onboard sensing modalities empowers robots to dynamically respond to changes in their surroundings in real-time. Whether it's navigating through a cluttered environment, adjusting movements for safety, or adapting to unforeseen obstacles, the reliance on somatosensation and vision will enable future robot to operate with a level of autonomy and adaptability previously unseen.

Safety- and energy-awareness Comprehending their own body structure empowers robots to engage in more effective and nuanced interactions with both their robotic and human counterparts. The body schema serves as a predictive tool, endowing robots with the foresight to anticipate potential challenges. This heightened understanding facilitates dynamic adjustments in their movements, prioritizing safety and fostering efficiency in diverse collaborative scenarios. When interacting with other robots, this capability enables seamless coordination, averting collisions or disruptions during joint tasks. Likewise, in human-robot interactions, the ability to adapt movements ensures a safer environment, mitigating the risk of accidental impacts or collisions. Moreover, the comprehension of their own body structure provides robots with a unique advantage in optimizing energy consumption. By adapting their motions based on the inherent physical properties of their structures, robots can execute tasks with greater efficiency. This optimization is particularly crucial in mobile robotics, where energy conservation is paramount given the limited resources. The dual capability of enhancing safety in interactions and contributing to energy-aware robotics underscores the significance of robots understanding their own body structure in fostering a more efficient, collaborative, and environmentally conscious robotic landscape.

1.2   Problem Statement

Navigating the complexities of learning a robot's physical attributes in the field of robotics reveals intricate challenges that demand a closer look. Traditional calibration routines, tailored for known kinematic structures and offline system identification methods, face limitations concerning generalization capabilities, sample efficiency, and the reliance on a predetermined mechanical topology. This is particularly evident in the context of global and local machine learning frameworks for physical systems, where the exclusion of structural knowledge often results in operational inefficiencies.

Compounding these challenges is the substantial reliance on off-robot measurement devices, such as vision and motion-capturing systems, during calibration and identification processes. Despite the wealth of sensor signals from modern robots, determining the minimum set required for constructing a body model based solely on robot sensing remains a persistent and unresolved challenge. This dependence on external measurements not only

introduces limitations but also raises questions about the practicality and applicability of learned models in real-world scenarios.

Venturing into alternative learning methods, such as neural networks, introduces a unique set of challenges. These approaches frequently lack crucial information about the robot's body structure, necessitating vast amounts of data for effective learning. The design of neural networks further compounds the issue, as it becomes an expert-driven task requiring meticulous determination of topology. Often, these approaches suffer from generalization limitations, confining their learning capabilities to specific input-output regions

Examining the research landscape reveals substantial gaps, including an unclear understanding of how object handling extends the robotic body schema. Furthermore, limited exploration into the mechanical arrangement of joints and links, known as the mechanical topology, underscores untapped potential in understanding the intricate details of robotic physical structure. These research gaps collectively contribute to the overarching challenge of lacking a unifying scheme that seamlessly integrates all learning stages, hindering the development of a fully characterized robotic body schema based solely on knowledge about sensorimotor signals.

To address these multifaceted challenges effectively, there is a compelling need to bridge the existing gaps and explore innovative learning approaches. This includes the development of methods capable of incorporating structural knowledge, reducing reliance on external measurements, and enhancing the generalization capabilities of machine learning frameworks in the realm of robotics. Moreover, the establishment of a comprehensive unifying scheme stands as a pivotal step toward advancing the field and achieving a fully characterized robotic body schema.

1.3 Research Questions and Contribution

1.4 State of the Art

1.5 Thesis Structure

1.6 Curriculum Vitae

2

Related Work

One fundamental characteristic of humans is the awareness of the body, which allows its control with dexterity and plasticity. We know where our body is in space, we know much of that space our body takes, and we know how to take it from its present situation at time t, its current *state s_t*, to a predicted or expected future state s_{t+1} at time $t+1$ given a carefully chosen *action a_t*. The ability to anticipate the future and to determine the actions that can take us there is connected to the notions of forward and inverse models in cognitive science and robotics [1–3]. An alternative and complementary interpretation is that of sensorimotor contingencies [4–6], which denote the structured relations between the actions of an agent and the ensuing sensory inputs resulting from interaction. The potential sensorimotor interactions are connected to the concept of *embodiment*, which, as expressed by Pfeifer et al. [7], deals with "how the body shapes the way we think," putting a premium on the morphology and capacities of an agent. Last, a few strongly related concepts are pertinent to the discussion: the *ecological self* REFERENCE, the *physical self* REFERENCE, and the *body schema* REFERENCE. They are different interpretations of *body models* that describe its physical structure and capabilities, and integrate sensory information for prediction and action. Research in cognitive science tells us that we construct and maintain representations involving the previous concepts from the moment that we are in womb and during all our conscious existence. Maintaining those models is not only a matter of refining and tuning them but also may involve adapting them to accommodate unexpected drastic changes. The level of bodily awareness encoded in those representations allows to achieve remarkable

feats, like reach for things without even looking, catching an object in midair, or contorting our bodies to maneuver in tight spaces or negotiate obstacles.

As discussed in the Introduction the vision for future robots involves giving them bodily awareness capabilities inspired in humans. To contextualize what is the state of the art towards realizing that vision, this chapter provides a review and discussion of relevant works that address the field of modeling in robotics; in particular, model learning. The discussion will include a brief review of the fundamental methods to model robots and will gradually move to the paradigm of learning local and global models with and without prior knowledge. The discussion will connect with the above mentioned concepts pertaining body models and present pertinent works that have presented methods to learn the body schema and sensorimotor representations in robotics

It is also important to emphasize internal models for awareness [8]

2.1   Model learning in robotics

Existing learning frameworks often focus on developing forward and inverse models using either a global or local approach to capture input-output relationships [3].

Global methods risk overfitting and computational overload, while local methods suffer from limited generalization and hyperparameter sensitivity [9,10]. Despite advancements in computational power and data availability, deep learning faces challenges due to the neglect of prior principled knowledge, making it difficult to determine dedicated neural network architectures [11,12].

2.1.1   End-to-end learning

2.1.1.1   Classical neural networks model learning

Model-learning problems using neural networks (NN) mainly involves:

1. Input/output data collection: assumed to be available

2. Architecture design: usually found by trial and error

3. Parameter optimization/learning: via well understood schemes, e.g., backpropagation and variants

Designing NN for a particular problem requires experts to determine the best topology, i.e. the number of nodes and layers, connectivity, and activation functions [13]. Furthermore, generalization is difficult as the architecture needs to balance achieving accuracy while avoiding overfitting [13–18]. Therefore, if a NN is used without any model information, large amounts of training data are required to generalize to unknown data [19].

2.1.1.2 Topology learning related works

Finding NN topologies is an important and challenging step [11, 14, 20]. Normally, function approximation via NN uses empirical topologies that rely on numerous parameters and do not lend themselves to interpretation. Such models provide no insight into the actual relation between the system variables. Recent works have aimed to find optimal topologies automatically. For example, evolutionary methods have been utilized to optimize the topology of Feed Forward NN (FFNN) [13, 14] as well as deep NN [20], by adding/deleting connections and weights. Constructive methods [16] and pruning methods [21] have also been applied to FFNN. Another method used for FFNN represents the network as a graph and reduces its degrees-of-freedom (DoF) [15]. Furthermore, reinforcement learning (through Q-learning) and topology learning (using variance analysis) have also been implemented to generate architectures [11, 22]. Noticeably, for learning complex dynamical systems, such as articulated robot structures, results have been limited in accuracy and generalization capabilities [3, 23, 24].

2.1.1.3 Robot inverse dynamics estimation via classical NN

NN have been applied in numerous variants to model robot inverse dynamics. In [25], Hopfield NN were applied to identify the inertial parameters. Likewise, in [26] a FFNN that used the regressor matrix as training samples was applied. Extreme Learning Machines were utilized in [27] with the same purpose. More recently a two-hidden-layers network with rectified linear activation units (ReLU) was used in [28]. Similarly, recurrent NN have been used to account for the sequential nature of the data. In [29], a recurrent NN in the hidden layer of an otherwise conventional three-layer FFNN was proposed. Additionally, self-organizing-networks, in conjunction with echo state networks, were used in [30] via a real-time deep learning algorithm.

2.2 Data-driven learning with structure information

In lieu of the challenges faced by deep learning to capture the intricacies of complex systems from scratch, Issues such as low sample efficiency, extended training times, and limited generalization highlight the necessity of balancing data-driven and principle-driven approaches [31, 32]. Recently, there has been a growing acknowledgment of the importance of integrating structure into the learning of physical systems [33, 34].

2.3   Model learning and the body schema

Building on the significance of structure in model learning for robotics, this dissertation addresses the inference of essential morphological properties in tree-like floating base structures, mimicking the development of a body schema. Efforts in cognitive robotics stress the pivotal role of internal body models in enhancing spatial awareness, motor control, and adaptability [35,36]. However, consensus is lacking on what constitutes a robot's body schema. Some approaches focus solely on learning the kinematic structure, relying predominantly on off-body vision [37–42]. Others explore sensorimotor associations between proprioceptive, tactile, and visual modalities [43–47], but they provide limited insights into the robot's physical structure.

Model-based robotics offers reliable methods for identifying physical attributes of robots based on known mechanical topologies. Conventional calibration routines [48] and offline system identification methods [49, 50] are effective for known kinematic structures in controlled environments. However, these methods face challenges when applied to floating base robots without standardized identification procedures [51, 52]. Importantly, these conventional methods were not initially designed for integration into online learning frameworks. While model-based robotics addresses kinematic calibration and forward/inverse kinematics, it provides limited insights into the comprehensive understanding of joint and link arrangement, known as mechanical topology. In cognitive robotics, only a few studies have approached this problem for self-modeling and monitoring, relying on exteroceptive vision [53, 54]. Regardless of the approach taken—black-box machine learning, cognitive methods, or model-based robotics—reliance on external measurement devices persists, overlooking embodied sensing modalities.

In summary, current robotics research reveals gaps in understanding and methods for refining body models. A comprehensive interpretation of the robot body schema and the determination of essential features are crucial. Identifying the fundamental set of necessary signals, both proprioceptive and embodied exteroceptive, is paramount. Integrating advanced machine learning with prior information and first-order principles shows promise for enhanced body models, addressing data requirements and generalization issues. However, the lack of synergy between modeling and learning approaches, along with the absence of a unified scheme for relevant learning stages, represents notable gaps requiring attention to advance robotics into more sophisticated and adaptable embodied systems.

2.4 Model learning in robotics

2.4.1 Classical and recent works in system identification

2.4.2 Local and global models linear models

2.4.3 End-to-end learning (black box models)

2.5 Data-driven learning with structure information

2.6 Model learning and the body schema

2.6.1 Internal representations

2.6.2 Sensorimotor maps

3

Theoretical Foundations

As mentioned already, the ability of robots to acquire, refine, and adapt body models of themselves is paramount for enhancing their locomotion, manipulation, and interaction capabilities. Despite this importance, existing learning frameworks in robotics often overlook the robot physical structure. Cognitive robotics emphasizes the significance of the body schema. Yet, approaches interpret a robot body schema as learning kinematic structure properties, while others focus on sensorimotor associations between proprioceptive, tactile, and visual modalities. The process of building a robot body schema is inherently tied to model-based robotics, providing methods for identifying physical attributes grounded in known kinematic structures. However, challenges persist, especially for floating base robots, and existing methods lack integration into incremental learning frameworks. This chapter discusses concepts and methods from different disciplines that are relevant for the definition of what is considered in this work as the robot body schema. Furthermore, the connections between the different methods will be outlined establishing their importance as a fundamental tool for building the robotic body schema.

As an anchoring point the concept of physical self will be briefly introduced and the relevant features that are pertinent for the discussion will be highlighted.

First of all this chapter will offer notions about the different interpretations of body knowledge. These encompass the definitions of the physical self and related concepts like emboidment, body morpholgy and the body schema. With the concepts extablished, a RECOPILACION of the important or fundamental features that are depicted by these definitions will

be offered. These features will be contrasted against what is conventionally performed in model-based robotics to asses if methods exist to learn these features. To complete the picture, supporting methods to complement inconplete existing methodologies or to find aspecs of the body structure that have not been fully addressed are provided.

3.1   Body concepts

3.1.1   The physical self

The concept of the physical self is rooted in human psychology and consciousness [55, 56]. It typically refers to an individual's perception and evaluation of their own physical body, encompassing appearance, abilities, and overall physicality. It involves the subjective experience of one's body and its relation to the environment, supported by sensory modalities like proprioception, touch, and interoception (sense of internal bodily sensations). This concept is explored in psychology, cognitive science, and related fields to understand how individuals perceive and interact with their bodies[1].

In the context of robotics, although robots lack subjective experiences, self-awareness, and consciousness, the physical self remains a critical concept. Robots must perceive their bodies and positions in space to operate effectively, particularly in unstructured or dynamic environments. Aspects related to the physical self, such as body morphology, embodiment, and the body schema, are crucial considerations in robotics, see Fig. 3.1. Body morphology, the physical structure of a robot, significantly influences its design based on intended tasks. For example, a robot designed for exploration might feature legs or wheels, while a robot for assembly line tasks may have a robotic arm [57]. Embodiment in robotics emphasizes how a robot's physical form and sensory-motor capabilities influence its interaction with the environment, shaping its perception and actions within its surroundings [58]. The concept of the physical self in robotics aligns closely with the body schema, representing an awareness of the robot's body, including the location, orientation, and relative motion of its parts.

In robotics, achieving a sense of the physical self is vital, especially for tasks in dynamic environments. Sensors provide information about the robot's orientation, speed, and acceleration, while actuators enable the robot's body to move, allowing it to learn about its physical capabilities through observed effects on the environment. This information contributes to the creation of a model of the robot's body within its internal representation of the world.
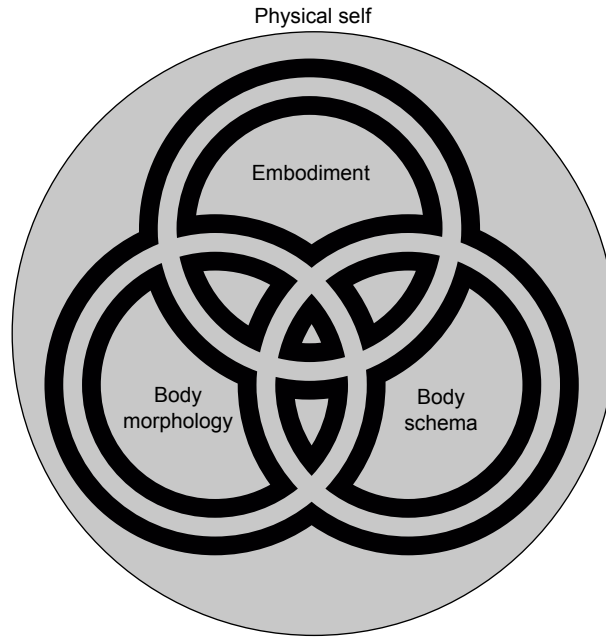
Figure 3.1: Overlapping concepts. Embodiment, body morphology, and body schema are three intersection concepts all involved in the definition of the physical self.

3.1.2 The body morphology

The robot body morphology encompasses its physical structure, including the configuration of joints, limbs, sensors, and actuators [59], playing a pivotal role in determining its capabilities and performance. The joints, serving as points of articulation, allow varied motion, with types like revolute, prismatic, and universal joints offering different degrees of flexibility. Limbs, the appendages extending from the body, consist of links and joints, defining the robot's range of motion and dexterity. Sensors provide information about the environment and internal state, influencing perception and decision-making. Actuators converting energy into mechanical motion and drive joint and limb movement, determining strength and endurance.

Design considerations for robot morphology involve stability, ensuring the robot withstands external forces and maintains balance. Dexterity is crucial for precise movements, achieved through joint, limb, and gripper design. Efficiency minimizes energy consumption for optimal performance, which is vital for autonomous or remote operation. Adaptability allows the robot to meet diverse task requirements or environmental conditions through modular designs or reconfigurable structures. Safety considerations prioritize minimizing risks to the robot and humans around it, incorporating protective tactile elements and elastic joints. Overall, a well-designed robot morphology integrates these key aspects and considerations to enhance its functionality and applicability.

The body morphology of a robot significantly influences its functionality and adaptability. Research in the field of robotics has emphasized the importance of the robot's body in

achieving stable locomotion, efficient control, and adaptation to environmental challenges. For instance, the design and control of soft robots have focused on exploiting dynamics and morphology, such as the shape and length of the body and feet, to achieve stable walking [59]. Additionally, studies have shown that morphological variations can be a powerful technique for achieving resilient robots [60]. Furthermore, it has been hypothesized that the morphological complexity of a robot should scale in relation to the complexity of its task environment [61].

In summary, the body morphology of a robot plays a crucial role in its locomotion, adaptability, and task-specific functionality. The research in this field underscores the importance of considering the dynamic interaction between a robot's body and its environment and the potential for co-optimizing morphology and control to enhance the robot's overall performance.

3.1.3   The body schema

3.1.3.1   General intuition

The human body relies on a sophisticated neural representation known as the *body schema* to navigate and interact with the surrounding environment, playing a foundational role in shaping the human experience of the physical self and the external world. This dynamic internal map within the brain encodes a multitude of attributes, ensuring the ability to perceive, move, and coordinate the body effectively.

The concept of the body schema has been defined in various ways in the scientific literature. One common definition is that it consists of sensorimotor representations guiding actions [62]. Another widely accepted definition emphasizes the requirement for internal representations of the body's current posture and spatial extension [63]. Fundamental to the body schema is the representation of the size and shape of different body parts, allowing for precise and coordinated movements. The brain's ability to accurately map the dimensions of the body contributes to the fluidity of human motion. Additionally, the body schema has been described as a means of obtaining information regarding body position in space from tactile, proprioceptive, visual, and other sensory modalities [64]. These definitions illustrate the multifaceted nature of the body schema, encompassing sensorimotor guidance, spatial representation, and the integration of multisensory information.

The body schema, a complex and dynamic representation, plays a fundamental role in facilitating our effective interaction with the world. This intricate construct incorporates various properties crucial for our sensory and motor experiences. Spatial properties, encompassing the position, posture, overall shape, and size of individual body parts, as well as the body's relationship to gravity and the environment, form the foundation of the body schema. Kinesthetic properties involve the sensations of movement, touch, generated mus-

cle force, and the resistance encountered during interactions. Vestibular properties contribute to our sense of balance, orientation in space, and awareness of the body's movement through space. Somatosensory properties, including the perception of temperature, pain, and pressure, as well as awareness of the body's surface and internal organs, further enrich the body schema. Additionally, the representation of mass, inertia, muscle forces, object properties, environmental properties, body ownership, and body boundaries collectively shape this intricate representation. Adaptability and plasticity enable adjustments through experience and learning. By integrating these diverse elements, the body schema enables humans to navigate their surroundings, interact with objects, and maintain a coherent sense of self in dynamic environments [65–67].

3.1.3.2   The robot body schema

Generally, the notion of body schema comprises sensorimotor representations of the body used to guide movement and action. For robotics this is, however, a far too general definition and incites the the question: *what concretely is the body schema in robotics?*. From the various takes on this topic, efforts to consolidate an answer to this baffling question were provided in the comprehensive work by Hoffmann et al. [36]. Relevant to this dissertation is the discussion that the authors provide on explicit, articulated, and implicit models of the body schema. The first two categories pertain models where there is a direct correspondence between the body parts of the robot and their representation in the sensors use to instantiate them measure variables associated to properties of the physical body. The main difference being that articulated models [68] emphasize that the signals ought to come from embodied sensors. This latter property is referred as subjective or situated body schema by [37]. The third category, implicit body schema, is more inline with traditional model learning methods in robotics and is aimed at constructing sensorimotor mappings.

As accentuated by the rich literature in model-based robotics, models of robots are required to achieve fine control and perform complex tasks. These models should include as much information as possible, explicit or implicit, of their body morphology (its embodiment). Most of the efforts associated with learning the body schema agree on the argument that an embodied robotic agent must autonomously learn and refine its body schema mainly proprioceptive information [36, 66]. Most the focus on calibration of a known kinematic structure REF with only few addressing the discovery of the body topology REF. It is worth mentioning that there is no solid consensus in the literature about whether the body schema includes the inertial description of the parts composing the body. Yet, given that the body schema is used for planning and executing movements and that the sensation of effort (force and torque) is part of proprioception, it seems reasonable to assume the body schema must include such a representation. Therefore, the question remains of what constitutes a

body schema in robotics.

To continue the discussion in this work the following conceptual definition of the robot body schema will be used:

Definition 3.1. The robot body schema is a representation of its body morphology primarily constructed from proprioceptive information. This representation encompasses the arrangement, geometry, and inertial properties of its parts, providing the capacity to perceive the relative positions of limbs and to plan and execute movements without relying on visual cues. Adaptive and self-acquired, the robot body schema plays a fundamental role in forward and inverse models, aiding in the planning and prediction of sensorimotor interactions beyond proprioception.

3.1.4 Embodiment

Embodiment, rooted in the notion that intelligence necessitates a body [7], is a key concept in robotics. In this context, it refers to how a robot's physical form and sensory-motor capabilities shape its interactions within its environment, influencing perception and actions [70]. This involves considerations of the robot's physical structure, situatedness, and morphology, extending beyond functionality to impact interactions with the external world [57].

At the core of embodiment is the physical form and morphology of a robot, including joints, limbs, sensors, and actuators. Beyond aesthetics, the design of a robot's physical form significantly influences movement, interaction capabilities, and overall functionality [73]. Engineers delve into nuances of body morphology to tailor robots for specific tasks, whether exploration or assembly line operations.

Embodying robots goes beyond external structure, emphasizing that artificial agents have physical bodies interacting with the environment through seamless sensorimotor integration [74]. Sensors act as receptors conveying information, and actuators enable the robot to act upon sensory input, resulting in adaptive behavior [59,75]. This sensorimotor integration distinguishes embodied robotics from abstract computational systems.

Embodiment also involves situatedness, where robots directly interact with their environment, enhancing autonomy and adaptability [71]. Incorporating action and sensory-motor experiences as sources of statistical regularities is crucial for cognitive robots [72].

The concept of body schema, closely linked to embodiment, represents an internal model encompassing a robot's body arrangement, geometry, and movements. It serves as a dynamic tool for spatial awareness. Adaptability is a hallmark of embodied robots, stemming from the close coupling of their physical bodies and control systems, enabling dynamic responses to environmental changes.

In essence, embodiment in robotics transcends traditional boundaries, transforming robots into physically present, interactive agents. The intricate dance between physical form, sen-
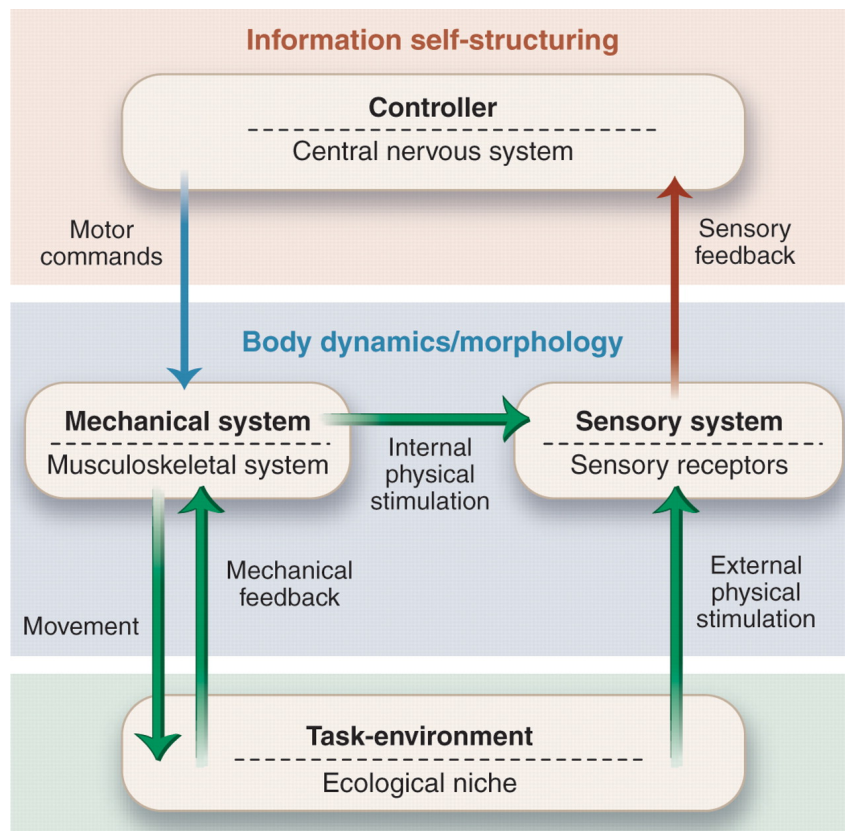
Figure 3.2: Implications of embodiment. From Pfeifer et al. [59]. Reprinted with permission from AAAS.

sorimotor integration, body schema, and adaptability enriches robotics, fostering the development of sophisticated, versatile, and context-aware systems.

3.1.4.1 Embodiment and information structure

TODO According to [69]

Concept	Definition	Relationship
Physical self	Our awareness of our own body and its interactions with the environment	Foundation of embodiment
Embodiment	The idea that our thoughts, feelings, and experiences are shaped by our physical bodies	Dynamic representation of body's physical structure and position in space
Body schema	Our internal model of our body's physical structure and its position in space	Manifestation of embodiment
Body morphology	The physical structure of our body, including its size, shape, and proportions	Influences embodiment and the physical self

3.2 Robot equations of motion

3.2.1 Forward and inverse dynamics

3.2.2 The Newton-Euler formulation of the inverse dynamics

Extension for floating base robots with flexible joints and links have been presented elsewhere [76]

3.2.3 Composition of the kinematics and dynamics

3.3 Robot proprioception

TODO Paraphrase and reference this section. Use this [67]

Proprioception is required to ensure the plasticity of the body schema [77]

3.3.1 The role of proprioception

Much of this knowledge about position and movement of the limbs and trunk is provided by sensations arising in propriocep- tors. The information they provide allows us to maneuver our way around obstacles in the dark and be able to manip- ulate objects out of view.

Traditionally, however, the term propri- oceptor has been restricted to receptors concerned with conscious sensations, and these include the senses of limb position and movement, the sense of tension or force, the sense of effort, and the sense of balance.

To motivate the importance of robot proprioception, this section first discusses the role it plays in living systems. This section relies on the material presented in As proprioceptive sensations, the senses of effort, force, and heaviness are distinct in that they are always associated with motor commands, while kinesthetic sensations can arise in a passive limb. It implies that for the senses of force and heavi- ness the peripheral input is always reafferent in origin. An- other issue to consider is the meanings of the terms. What do we mean by a sense of effort as distinct from a sense of tension?

It is concluded that a sense of effort, arising centrally in associ- ation with motor commands, is able to contribute to a num- ber of proprioceptive sensations including the sense of limb position and the sense of limb movement.

3.3.2   Definition

3.3.3   Human and robot proprioception

3.3.4   A sensor suite for robot proprioception

3.4   The body schema in robotics

3.4.1   Biological body schema

The biological body schema refers to the internal representation that the brain maintains of the body's spatial configuration and the relationship between its parts. It is a dynamic and flexible neural map that integrates sensory information, motor commands, and past experiences to create a coherent and updated perception of one's own body. The body schema plays a crucial role in motor control, spatial awareness, and the ability to interact effectively with the environment.

Key elements of the biological body schema include proprioception (awareness of the body's position in space), tactile sensations, and motor control. Proprioceptors, which are sensory receptors in muscles, tendons, and joints, provide information about the body's position and movement. This information is continuously processed by the brain to maintain an accurate and up-to-date representation of the body.

The biological body schema is not static; it can be modified through experiences, learning, and adaptation. It is essential for various cognitive functions, including spatial orientation, hand-eye coordination, and the execution of complex motor tasks. Disruptions to the body schema, such as neurological disorders or injuries, can lead to difficulties in motor control and spatial perception. Understanding the biological body schema is crucial in fields like neuroscience, psychology, and rehabilitation, as it provides insights into how the brain perceives and interacts with the body and the external environment.

3.4.2   Body features encoded in the body schema

The body schema representation in the brain encodes various attributes of the body to facilitate perception and interaction with the environment. These attributes include:

Body Size and Shape   The brain has a representation of the size and shape of different body parts. This allows for accurate and coordinated movements and interactions.

2. **Body Position:** The body schema includes information about the spatial orientation and position of body parts in relation to each other. This is essential for coordinating movements and maintaining balance.

3. **Proprioception:** This is the sense of the relative position of one's own body parts and the effort being employed in movement. Proprioceptive information helps in knowing the positions of limbs without having to visually observe them.

4. **Joint Angles:** The body schema encodes information about the angles and movements of joints, allowing for smooth and coordinated motor control.

5. **Touch and Tactile Sensations:** Sensations related to touch, pressure, temperature, and pain are integrated into the body schema. This information helps in perceiving and responding to stimuli from the external environment.

6. **Spatial Awareness:** The body schema contributes to spatial awareness, allowing individuals to navigate and interact with the surrounding space effectively.

7. **Body Image:** While related, body image is a more complex concept that involves cognitive and emotional aspects of how an individual perceives their own body. The body schema contributes to this perception but is not the sole factor.

8. **Motor Programs:** The body schema is closely tied to motor control and the execution of movement. It helps in generating and executing motor programs for voluntary actions.

These attributes collectively contribute to a coherent and dynamic representation of the body, enabling individuals to interact with their environment in a purposeful and adaptive manner. The body schema is not fixed but can be modified based on experiences, learning, and changes in the body itself.

3.4.3   Some context

"The body schema is viewed as a set of fronto-parietal networks that integrate information originating from regions of the body and external space in a way that is functionally relevant to specific actions performed by different body parts... It represents the body's spatial properties, including the length of limbs and limb segments, their arrangement, the configuration of the segments in space, and the shape of the body surface... The body schema is

characterized by short-term plasticity and reorganization, as shown by the quick integration of tools into the body schema" [66]

"The concept of body schema is related to model learning in robotics as it provides a framework for robots to understand and represent their own bodies and the surrounding environment. Research in this area aims to enable robots to improve their capabilities by automatically synthesizing, extending, or adapting a model of their body, similar to how the human body schema allows for adaptive and coordinated movement. This involves integrating information from multiple sensory modalities and controlling complex bodies, which are desirable capabilities for robots. By studying the body representations in biology from a functional or computational perspective, researchers seek to develop robotic systems that can effectively model their own bodies and interact with the environment in a more human-like manner. Therefore, the concept of body schema plays a crucial role in the development of robotic systems that can learn and adapt to their own physical characteristics and the surrounding world."

The concept of body schema and its relevance to model learning in robotics is well-supported in the scientific literature. While there are no specific sources directly linking the concept of body schema to model learning in robotics, the following sources provide relevant information about the body schema and its implications:

(1) The article "Influence of the Body Schema on Multisensory Integration: Evidence from the Mirror Box Illusion" in Scientific Reports - Nature discusses the influence of the body schema on multisensory integration, which is a fundamental aspect of robotic model learning.

(2) The paper "Neuroplasticity in the blind and sensory substitution for vision" on ResearchGate explores neuroplasticity in the blind, which is relevant to understanding how the body schema is developed and adapted in the absence of vision, a concept that can be applied to robotics.

(3) The article "Does visual experience influence arm proprioception and its lateralization? Evidence from passive matching performance in congenitally-blind and sighted adults" in ScienceDirect provides insights into the relationship between visual experience and proprioception, which is foundational to the development of the body schema and has implications for robotics.

(4) The study "Degraded vision affects mental representations of the body" published on Taylor & Francis Online discusses the impact of degraded vision on mental representations of the body, including the shift from a pictorial representation of the body to a somatosensory one, which is relevant to understanding the body schema in the context of robotic model learning.

While these sources do not directly address the application of body schema to model learning in robotics, they provide valuable insights into the development and influence of

the body schema, which can be extrapolated to understand its potential relevance to model learning in robotics.
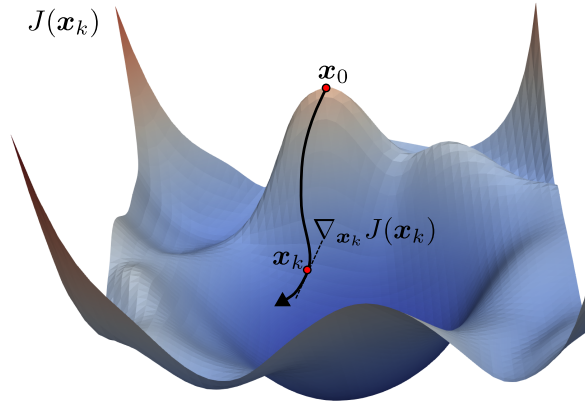
3.5   Driving online learning: Gradient descent

Figure 3.3: Concept art gradient descent. Image credits: `https://www.cs.umd.edu/~tomg/projects/landscapes/`

3.5.1   Offline and online learning

In the realm of machine learning, online learning and offline learning represent distinct approaches to model training. Online learning, or incremental learning, involves learning the model as new data becomes available, typically processing one observation at a time. In this scenario, the learning algorithm's parameters are updated after each individual training instance, making it particularly advantageous for systems dealing with a continuous flow of data that must rapidly adapt to changing conditions. On the other hand, offline learning, also known as batch learning, entails training the model over all observations in a dataset $\mathcal{D}$ simultaneously. The model is trained with a static dataset, and the parameters are updated once the learning process is complete for the entire dataset.

3.5.2   Fundamentals of gradient descent

Gradient descent (GD) is in essence an optimization algorithm to sequentially minimize a *cost function* $J(\boldsymbol{x})$ with respect to a given set of parameters $\boldsymbol{x} \in \mathbb{R}^d$ moving in the direction opposite to its gradient $\nabla_{\boldsymbol{x}} J(\boldsymbol{x})$ at the current point $\boldsymbol{x}_k$. GD is considered a local optimizer as it primarily finds a local minimum of J. The algorithm can be summarized in a iterative process expressed in the simple equation

$$\boldsymbol{x}_{k+1} = \boldsymbol{x}_k - \eta \nabla_{\boldsymbol{x}_k} J(\boldsymbol{x}_k; \mathcal{D}), \tag{3.1}$$

which gives the update x_{k+1} to the optimization variables using a *learning rate η* that controls the size of the step given in the negative direction of the gradient. If this hyperparameter is too small, the updates x_{k+1} will differ slightly from x_k and convergence will take a long time. On the contrary, if η is too large, the parameter updates may oscillate around a local optima and even diverge.

From the number of data points from an already-collected dataset $\mathcal{D}$ that are used to represent the optimization cost function J, three variants of GD are typical: batch, stochastic, and mini-batch GD. If all data in $\mathcal{D}$ is used to compute the gradient, it is called batch GD. It relies on a static dataset and substantial computational resources (CPU, memory, and storage) for training and thus lacks the flexibility to adapt incrementally to new data. This inability to incorporate new observations in real-time hinders its applicability for online learning in dynamic environments with constantly evolving data streams.

Stochastic Gradient Descent (SGD) [78] serves as a simplification of batch GD and is widely employed machine in machine learning for online learning due to its efficiency with large-scale datasets and its ability to adapt to new data one observation at a time. Its advantages lie in its speed, processing quicker parameter updates, and memory efficiency. A recent review [79] discusses the the properties of SGD as a standard optimization algorithm in deep learning. Despite its ability to escape local minima, the noisy updates of SGD and bouncing behavior near the minimum can yield good but non-optimal solutions REF. When the gradient of the cost function is computed using a small subset or mini-batch of $\mathcal{D}$, rather than a single example or the entire dataset, SGD becomes mini-batch GD. This approach reduces the noise in the parameter updates compared to SGD, leading to more stable convergence. It allows for computational efficiency as the mini-batch can be processed in parallel, and it is typically faster than batch GD for convergence.

3.5.3 Improved gradient descent

The challenges associated with standard (commonly named vanilla) GD encompass critical issues. First, there is the already mentioned sensitivity to the selected learning rate η. Noisy gradients are a particular problem of SGD and mini-batch GD. These may lead to slower and less stable training compared to batch GD. The algorithms may also encounter obstacles like local minima and saddle points, particularly problematic in non-convex optimization problems, hindering the discovery of the global minimum.

Various improvements have been proposed to improve vanilla GD in recent years [79, 80]. Noteworthy among these enhancements are the incorporation of a momentum term, adaptive learning rate mechanisms, and measures to handle very small step sizes. The momentum term, functioning as a moving average over past gradients, serves to smooth out steps in gradient descent, thereby mitigating oscillations and expediting convergence. Its purpose is

to combat issues such as bouncing around the search space and getting stuck in flat regions, ultimately elevating the optimization algorithm's efficacy and yielding superior final results.

The adaptive learning rate, tailored individually for each parameter based on the square root of the sum of the squares of historical gradients, facilitates larger updates for infrequent parameters and smaller updates for frequent ones. This adaptability proves beneficial for handling sparse data and managing sparse gradients. The approach aims to expedite convergence in the presence of sparse data by adjusting the learning rate according to the loss function's geometry. This allows for swift convergence in steep gradient directions and more cautious updates in flatter gradient directions.

However, a notable drawback of learning rate adaption is the its potential reduction to infinitesimally small values over time, limiting the ability to acquire additional knowledge. To counteract this issue, a decaying average of squared gradients was proposed to adjust the step size for each parameter. This adaptive strategy stabilizes the learning process, preventing oscillations in the optimization trajectory and proving effective for non-convex optimization problems commonly encountered in machine learning.

3.5.3.1   Adaptive moment estimation: Adam

Adaptive Moment Estimation GD, commonly known as Adam [81], has become a powerful optimization technique standard in machine learning. Adam is an adaptive learning algorithm that refines SGD approach by incorporating both first-order moment estimates (m_t) and second-order moment estimates (v_t) of the gradients; that is

$$
\begin{aligned}
m_k &= \beta_1 m_{k-1} + \left(1 - \beta_1\right) \nabla_{\boldsymbol{x}_k} J(\boldsymbol{x}_k; \mathcal{D}) \\
v_k &= \beta_2 v_{k-1} + \left(1 - \beta_2\right) \left(\nabla_{\boldsymbol{x}_k} J(\boldsymbol{x}_k; \mathcal{D})\right)^2,
\end{aligned}
\tag{3.2}
$$

where the moving averages of the momentum term and the squared gradients is adjusted via the discount factors $\beta_1 \in [0, 1]$ and $\beta_2 \in [0, 1]$. The adaptive nature of Adam stems from dynamically adjusting learning rates $\{\eta_i\}_{i=1}^{d}$ for each parameter based on the historical gradient information. This feature addresses a longstanding challenge in machine learning—manual tuning of learning rate η. The ability of Adam to autonomously adjust rates during training alleviates the need for meticulous parameter tuning, a characteristic that sets it apart from its predecessors.

A fundamental strength of Adam lies in its incorporation of momentum, which enables Adam to persist in the correct direction, surmounting obstacles such as flat regions or saddle points in the cost function J. Furthermore, Adam introduces a bias correction mechanism, particularly beneficial in the initial stages of training when moment estimates might be im-

precise[*]. This correction, computed as

$$\hat{m}_k = \frac{m_k}{1 - \beta_1}$$
$$\hat{v}_k = \frac{v_k}{1 - \beta_2},$$

(3.3)

enhances the accuracy of the mean and uncentered variance estimates in (3.2), contributing to the algorithm's robustness. The update rule for Adam, based on Eqs. (3.3) and (3.2), yields

$$x_{k+1} = x_k - \frac{\eta}{\sqrt{\hat{v}_t} + \epsilon} \hat{m}_t.$$

(3.4)

Note that according to [81], the default values of $\beta_1 = 0.9$, $\beta_2 = 0.999$, and $\epsilon = 10^{-8}$ seem to work well in practice.

Adam's computational efficiency is a standout feature, showcasing its prowess in a diverse array of applications. It has demonstrated a tendency to converge faster and demand less hyperparameter tuning than traditional SGD. Notably, Adam's adaptability to sparse or high-dimensional data renders it well-suited for complex models where different parameters may necessitate varying learning rates. This adaptability is particularly advantageous in real-world scenarios where data is inherently noisy.

The algorithm's ability to handle noisy gradients effectively speaks to its stability during training, contributing to the robustness that makes Adam a preferred choice in modern machine learning applications.

3.5.3.2 AMS gradient descent

AMSGrad is a gradient descent method that aims at convergence issues faced by Adam and its variations [82]; which have been observed to exhibit issues related to the learning rate, where it may become too aggressive and lead to poor convergence behavior in certain scenarios. AMSGrad tries to address this issue by modifying the way it updates the parameters.

The key improvement introduced by AMSGrad lies in its handling of the adaptive learning rates for each parameter. In the original Adam algorithm, the learning rates are adapted based on the exponential moving averages of the first-order moment (mean) and the second-order moment (uncentered variance) of the gradients. However, Adam does not incorporate a mechanism to correct the bias in the estimate of the second-order moment, which can lead to an overly aggressive decrease in the learning rates.

AMSGrad addresses this limitation by modifying the update rule for the moving average of the second-order moment. Unlike Adam, AMSGrad maintains a running average of the past squared gradients for each parameter without bias correction. Particularly, instead of

---

[*]The terms v_k and m_k are biased towards their zero initial value.

using $\hat{v}_t$ as in (3.3), its previous value $\hat{v}_{k-1}$ is used as long as it is greater than the current value computed according to (3.2); i.e.,

$$\hat{v}_t = \max(\hat{v}_{t-1}, v_t). \tag{3.5}$$

The update rule for AMSGrad

$$x_{k+1} = x_k - \frac{\eta}{\sqrt{\hat{v}_t} + \epsilon} m_t \tag{3.6}$$

excludes also the regularization of the first momentum term. The alteration to the second moment ensures that the denominator in the learning rate calculation does not become excessively small over time, preventing the learning rate from growing uncontrollably small.

In essence, AMSGrad aims to provide a more stable and consistent learning rate, preventing scenarios where the learning rate diminishes rapidly and adversely affects convergence. By addressing this issue, AMSGrad aims to offer improved convergence behavior and generalization, particularly in cases where Adam might exhibit erratic behavior due to the uncorrected bias in the second-order moment estimate.

3.6   Fundamentals of graph theory

3.6.1   Definition of a graph

A graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is a mathematical construct the represents the interconnections between the elements of a system. These elements are represented as a set $\mathcal{V} = \{v_i\}_{i=1}^n$ of *n nodes* (also called *vertices*) and their connections are depicted as a set of *edges* $\mathcal{E} = \{e_i\}_{i=1}^m \subseteq \mathcal{V} \times \mathcal{V}$. Note that it is possible that an edge connects a node to itself, in which case the edge defines a self loop.
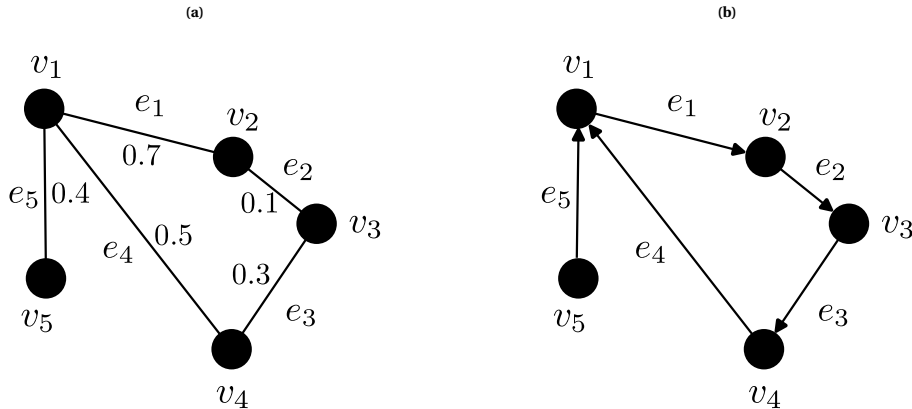

Figure 3.4: Two basic types of graphs. (a) An undirected and (b) a directed graph.

Two nodes *i* and *j* are said to be *adjacent* if there is an edge *e* connecting them. Similarly,

the edge *e* connecting the nodes is called *incident* to *i* and *j*. For example, the nodes v_1 and v_4 in Fig. 3.4a are adjacent to each other and the edge e_4 is incident to both nodes. The number of edges that are incident to a node is the *degree* of the vertex.

A *path* represents a potential sequence of edges connecting any two given nodes. The number of edges determines the *length* of the path. A graph is said to be *connected* if there is at least one path between every pair of nodes; otherwise, it is labeled *disconnected*. A *cycle* constitutes a path comprising a minimum of three edges, where the initial and final vertices are identical, and no vertices are repeated in between. Lastly, a graph is *complete* if there exists a path connecting every pair of nodes. Notice that the maximum number of edges in an undirected graph with *n* nodes is $n(n-1)/2$

An *undirected* graph lacks any specified direction assigned to its edges, meaning the connections between nodes are bidirectional. On the other hand, a *directed* graph, commonly referred to as a *digraph*, introduces directionality to its edges. In a directed graph, each edge has an explicit direction, indicating a one-way flow from one node to another. This directional information adds an extra layer of complexity to the relationships within the graph, as opposed to the bidirectional nature of edges in an undirected graph.

A *weighted* graph $\mathcal{G}(\mathcal{V}, \mathcal{E}, \boldsymbol{W})$ associates numerical value, a weight, to each edge. The weights with weights $\boldsymbol{W} : \mathcal{V} \times \mathcal{V} \to \mathbb{R}_+$ express some quantitative measure such as distance, cost, time, or any other relevant metric depending on the context of the graph. Unlike an unweighted graph, where edges simply represent connections between nodes, a weighted graph provides additional information about the relationships between nodes. For a weighted graph, the *strength* of a vertex corresponds to the sum of the edge weights associated with it.

3.6.2   Algebraic representation of a graph

A graph with *n* vertices can be represented as a square $n \times n$ matrix $\boldsymbol{A}$. This algebraic representation is called the *adjacency matrix* and depicts the graph's connectivity pattern; that is, the elements of the matrix indicate whether pairs of vertices (i, j) are adjacent or not in the graph. Formally, the elements of a *binary* adjacency matrix $\boldsymbol{A}$ are determined as follows

$$(\boldsymbol{A})_{i,j} = \begin{cases} 1 & \text{if and edge connects nodes } i \text{ and } j \\ 0 & \text{otherwise.} \end{cases} \tag{3.7}$$

By extension, a *weighted* adjacency matrix $\boldsymbol{W}$ denotes a weighted connection for some of the (i, j) entries, i.e $(\boldsymbol{W})_{i,j} \in \mathbb{R}_+$. If the graph is undirected, the adjacency matrix is symmetric, which implies that $\boldsymbol{A} = \boldsymbol{A}^\top$ and $\boldsymbol{W} = \boldsymbol{W}^\top$ respectively. Note, however, that for the case of digraphs, the adjacency matrix is not necessarily symmetric. To illustrate this, refer to the

graph in Fig. 3.4a, the corresponding binary and weighted adjacency matrices are

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

and

$$\boldsymbol{W} = \begin{bmatrix} 0 & 0.7 & 0 & 0.5 & 0.4 \\ 0.7 & 0 & 0.1 & 0 & 0 \\ 0 & 0.1 & 0 & 0.3 & 0 \\ 0.5 & 0 & 0.3 & 0 & 0 \\ 0.4 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Similarly, for the digraph in Fig. 3.4b, the adjacency matrix is

$$\boldsymbol{A}_D = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Notice that unless there are self loops in the graph, the main diagonal of an adjacency matrix contains only zero entries. Two other important matrices related to a graph $\mathcal{G}$ are the degree matrix $\boldsymbol{D} : (\boldsymbol{D}_{ii}) = \sum_{j=1}^{m} w_{ij}$, a diagonal matrix whose entries are the sum of the rows of $\boldsymbol{W}$, and the combinatorial graph Laplacian (CGL), defined as $\boldsymbol{L}_{\mathrm{CGL}} = \boldsymbol{D} - \boldsymbol{W}$ [83].

3.6.2.1   The normalized adjacency matrix

The normalized adjacency matrix, defined as

$$\mathcal{W} = \boldsymbol{D}^{-\frac{1}{2}} \boldsymbol{W} \boldsymbol{D}^{-\frac{1}{2}}, \tag{3.8}$$

is particularly employed in spectral graph theory [84] and related analyses. Normalization helps account for variations in node degrees and provides a more balanced representation of the graph structure. One common application is in spectral clustering algorithms [85], where the normalized adjacency matrix is used to compute eigenvectors and eigenvalues, aiding in the identification of clusters or communities within a graph.
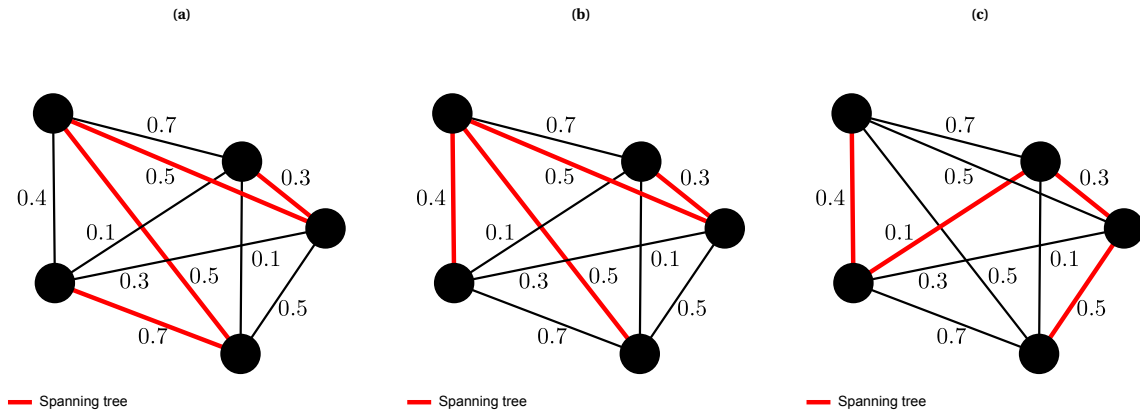
Figure 3.5: **Different spanning trees for the same graph**.

3.6.3 Subgraphs and spanning trees

A *subgraph* $\mathcal{G}_1$ is obtained by selecting a subset of the nodes of another graph $\mathcal{G}$ and a corresponding subset of the edges connecting those nodes. Formally, a graph $\mathcal{G}_1(\mathcal{V}_1, \mathcal{E}_1)$ is a subgraph of $\mathcal{G}$ if and only if $\mathcal{V}_1 \subseteq \mathcal{V}$ and $\mathcal{E}_1 \subseteq \mathcal{E}$. This is written $\mathcal{G}_1 \subseteq \mathcal{G}$. One particular subgraph of interest in this work are spanning trees. First, a *tree graph* is a graph that does not contain cycles. Such structure usually depicts a hierarchical arrangement. Consequently, a *spanning tree* $\mathcal{G}_1$ of a graph $\mathcal{G}$ is a subgraph containing all the nodes in $\mathcal{G}$ and whose edges form a tree that defines paths that ensure that all the nodes remain connected. The edges of $\mathcal{G}_1$ are denoted as the branches of the tree. In general, a graph $\mathcal{G}_1$ is said to be a spanning subgraph of $\mathcal{G}$ if and only if $\mathcal{V}_1 = \mathcal{V}$ and $\mathcal{E}_1 \subseteq \mathcal{E}$. It is worth noting that every connected graph has a spanning tree and that there are n^{n-2} distinct spanning trees with $n-1$ edges on a connected graph with n vertices [86]. Examples of spanning trees are shown in Fig. 3.5.

3.6.3.1 Minimum spanning tree and Kruskal's algorithm

The minimum spanning tree of an undirected, connected, and weighted graph is the spanning tree whose edge weight sum is less than or equal to that of all other spanning trees [87]. Kruskal's algorithm [88] is the standard method to find th minimum spanning tree of a graph. In this work we use an equivalent concept, the *maximum spanning tree* (MST), which corresponds to the spanning tree with maximum edge weight sum.

3.6.4 Metrics for graph comparison

==TODO==

Several works cover a number of metrics and measures to asses the similarity of any two given graphs. The most named metrics include: eigenvalue distribution analysis [89, 90], the graph adjacency spectral and matrix distances [91]...

3.7   Network topology inference

The process of identifying and visually representing relationships among various elements within a system, based on the available measurements, is termed *network topology inference* (NTI) [92]. Unveiling the structure of a network or graph through topology learning facilitates the analysis of interactions among entities.

3.7.1   Types connectivity

Structural connectivity   Functional connectivity   Effective connectivity
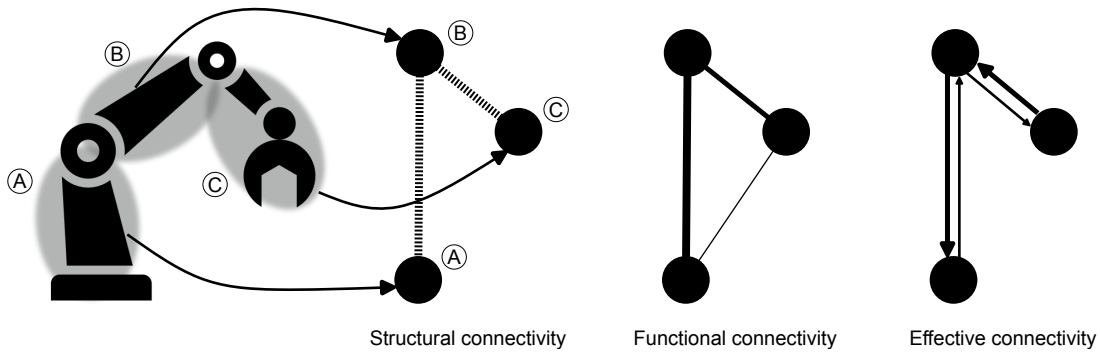
Figure 3.6: Types of connectivity. Every rigid body in the robot represents a node in the graphs.

Often used in the field of neuroscience [93], three different types of connectivity are distinguished [94, 95]:

Structural Connectivity (SC).   Refers to the physical connections between different elements in a network. In the context of neuroscience, it typically involves the anatomical connections between brain regions. It is measured with techniques such as diffusion-weighted imaging (DWI) or diffusion tensor imaging (DTI) are commonly used to infer structural connectivity in the brain.

Functional Connectivity (FC).   Refers to the statistical dependencies or temporal correlations between the activity of different elements in a network. As explained in [96], FC is an information-theoretic metric that characterizes dependencies using probability distributions of observed signals. It can be categorized into non-directed and directed forms, with the latter being associated with statistical causation derived from the data [97].

Effective Connectivity (EC).   Specifically addresses the dynamic, state-dependent influence that one network element has on another within a particular causal dynamics model. Essentially, effective connectivity denotes coupling or directed causal influence [94]. Exemplary applications of these concepts can be observed in the fields of biology [98] and neuroscience [93, 99].

To understand the intricate relationships among the three types of connectivity refer to Fig. 3.6. The figure depicts a simple robot arm as an example. The nodes represent the three links that compose the robot. Regarding SC and FC, a nuanced connection exists, albeit not strictly one-to-one. For example, the SC reflect the physical composition of the body, three bodies connected by two joints (the edges in the SC graph). Sensory signals coming from these bodies might lead to potential functional interactions of varied degree (the edges in the FC graph) that have a foundation on the bodily structure of the robot; yet, it does not ensure their occurrence. Actually, functional relationships can appear even in the absence of direct structural connections, not the light edge between nodes *A* and *C* in the FC graph. Moving to EC and FC, the former extends the latter by seeking to model the direction and strength of influence between different elements. While FC identifies statistical associations, EC strives to unveil the underlying causal relationships, the directed edges in the EC graph. In summary, these three connectivity types are interrelated components in the complex landscape of NTI. Structural connections provide the anatomical substrate, functional connections depict statistical dependencies, and effective connections aspire to model causal relationships within the network.

3.7.2 Inferring the connectivity

In NTI the elements $\mathcal{V}$ of a graph $\mathcal{G}$ are known but its connectivity (i.e. how these elements relate to each other) is unknown. Then, the graph topology inference problem consists in finding the edges $\mathcal{E}$ that best explain the relationships among the nodes $\mathcal{V}$ given some prior knowledge, such as data distribution, the location of the sensors, the physical relationships between the signals, or the data similarity [92, 100].

With **mutual information** [101]

With **GSP** [92]. We consider a method [102] which learns a relational matrix $\boldsymbol{W}_{GSP}$ without prior structural information considering the signals in $\boldsymbol{x}(t)$ as graph signals [92]. $\boldsymbol{W}_{GSP}$ is derived under the assumption that the signals on the graph change smoothly between connected nodes. Likewise, the Graph Signal Processing Toolbox (GSPBOX) from [103] was used to compute $\boldsymbol{W}_{GSP}$ using $\alpha = 0.6$, $\beta = 1$ (parameters that control the edge weight magnitude and the sparsity of $\boldsymbol{W}_{GSP}$, respectively) and normalizing the required pairwise distance matrix $\boldsymbol{Z}$ between $[0, 1]$.

With **correlation**. The first technique [104] upgrades standard correlation-based NTI by searching for an inverse covariance matrix with Laplacian (to find valid adjacency matrices $\boldsymbol{W}_{cor}$) and structural constraints (requiring a sparse matrix to reduce the graph edge density). We used the Graph Laplacian Learning (GLL) package [105] to calculate $\boldsymbol{W}_{cor}$, with regularization parameter $\gamma = 0.07$ and using a matrix $\boldsymbol{W}_0$ as connectivity prior with zero diagonal elements and ones elsewhere (denoting lack of structural knowledge).

3.7.2.1   Challenges in NTI

NTI brings with it challenges such as noisy measurements, lack of ground truth, large parameter spaces, and varying model complexity [106]. Moreover, inferring graph topology only from data is an ill-posed problem, having statistical models (e.g. correlation, entropy, mutual information) and physically motivated models (e.g. network diffusion) as general approaches [92]. Finally, the recently introduced paradigm of Graph Signal Processing (GSP) [100] considers samples from the signals at a given time as *graph signals*, whose properties are a consequence of the underlying graph.

3.7.3   Detecting linear dependencies with covariance

3.7.4   Graph signal processing

The fairly recent field of Graph Signal Processing (GSP) [83]

Under the assumption that the signals are related to the topology of the graph where they are supported, the goal of GSP is to develop algorithms that fruitfully leverage this relational structure and can make inferences about these relationships even when they are only partially observed.

A network may represent a conceptual model of pairwise relationships

A fundamental question in GSP is how to use the graph signals to infer the underlying structure of the network.

3.7.5   Based on statistic measures

3.8   Information theory

As discussed in [107] information is…

3.8.1   The meaning information

3.8.2   Entropy of random variables

3.8.3   Mutual Information: The correlation of the 21st century

Mutual information

The mutual information between two random variables is a symmetric measure of information computed as:

$$I(X;Y) = I(Y;X) = H(X) + H(Y) - H(X,Y) \tag{3.9}$$

with the Shannon's entropy of a variable X defined by

$$H(X) = -\sum_{i=1}^{n} p(x_i)\log_2\left(p(x_i)\right) \tag{3.10}$$

and the joint entropy between X and Y expressed as

$$H(X,Y) = -\sum_{i=1}^{n}\sum_{j=1}^{n} p(x_i, y_j)\log_2\left(p(x_i, y_j)\right). \tag{3.11}$$

3.8.3.1 Unexplored alternatives

The transfer entropy [108] is

3.8.4 Applications

Previous works have discussed metrics based on information entropy in the context of sensorimotor coordination [109]

3.9 Differential geometry

3.9.1 Fundamentals of differential geometry

3.9.2 Manifolds and the tangent space

3.9.3 Riemannian geometry and the metric

3.9.4 Applications in robotics

"The Symmetric Positive Definite (SPD) manifold is one specific type of Riemannian manifold. It is a smooth manifold where the tangent space is endowed with a Riemannian metric [2]. The Riemannian metric allows us to define various geometric notions such as the geodesic distance."

4
Discussion

The results and approach presented in this work bear resemblance to the work by Sturm [42], Hersch [37], Bongard [54], and Lipson [41]

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