Learning The-Self: Leveraging Proprioception to Guide the Autonomous Discovery of the Robotic Body Schema

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I am not Ultron, I am not Jarvis, I am… I am.

Vision, Avengers: Age of Ultron

Abstract

As robots become increasingly integral to human life, the imperative emerges for them to autonomously explore and construct models of their bodies. Robots should take cues from human capabilities, aspiring to build and utilize their body schema for advanced locomotion, finer manipulation, and adaptive interactions. Thus, a crucial foundation lies in seamlessly integrating the body schema to elevate learning, motor control, coordination, and spatial awareness. Furthermore, future robots should become self-sufficient entities that conduct monitoring, calibration, and adaptation exclusively through onboard sensing modalities. Standardizing constant self-monitoring nurtures spatial awareness and facilitates rapid error detection and correction. A profound understanding of their body structure will undoubtedly lead to enhanced, safe, and energy-aware interactions. However, current robot learning approaches encounter limitations, such as suboptimal generalization and sample efficiency, exhibiting a need for more structural knowledge. Versatile methods, like neural networks, confront challenges related to data and topology, confining learning to specific regions. On the other hand, learning robot physical attributes still rely on a presumed knowledge of the mechanical topology, often involving calibration and offline identification in controlled environments with a persistent reliance on external measurements, such as vision and motion-capturing systems. The research landscape generally reveals the lack of a unified framework that enables robots to build representations of their body schema to achieve improved body awareness and interaction capabilities. This study addresses these challenges by consolidating necessary and sufficient proprioceptive signal quantities, enabling robots to autonomously acquire knowledge about their body structure without relying on exteroceptive off-body sensors. It introduces an approach that reformulates robot kinematic calibration and system identification as a modular computational graph amenable to machine learning. This abstracted architecture, applied in online learning phases, seamlessly merges proprioceptive signals with first-order principles, extracting fundamental features of the robot body schema. Characterizing morphological properties of tree-like structures, the study infers mechanical topology through information-theoretic measures, validating and applying it independently of off-robot calibration. The research extends its scope

by complementing the robot body schema by instantiating inertial properties, ensuring online learning and physical feasibility. Ultimately, this work challenges the uncritical application of end-to-end learning in physical systems, urging a reevaluation of its limitations when excluding principled knowledge. It underscores opportunities for machine learning frameworks in embodied systems, emphasizing the untapped potential of synergizing structural knowledge with data-driven methods. This study catalyzes future research in an incipient field that underscores building and maintaining a body schema by demonstrating that fundamental properties of a robot's morphology can be deduced from proprioceptive signals. Its implications are far-reaching, addressing the needs of conventional and dynamic robotic structures with diverse sensory modalities that require a more profound sense of self.

Contents

1

Introduction

Imagine your body; do not look at it. Close your eyes and tell me what you see. What is the pose of your body right now? Are you standing, seated, lying? Where are your arms and hands? Do not open your eyes just yet. Now, touch your left knee with your right hand. Did you struggle to find your knee? Chances are you did not. How can we manage to know where our body parts are in space at all times without even looking at them? It is safe to assume that we humans have what is called an internal model of our physical self. That is, we have a representation of our body. Such a representation allows us to accomplish remarkable feats. Have you ever caught an object in mid-air? Certainly, succeeding in that involved not only estimating the trajectory of the object but also the motion that your arm and hand needed to accomplish to reach the catching point. Had your arm dimensions been different from the ones you *have in mind*, or had the accuracy of your knowledge about your arm's pose been insufficient, you would not have caught the object. This internal model of yourself is called the body schema in psychology and neuroscience. It plays a definite role in knowing about the state of our body in space and also in calculating how we move. Consider a different scenario. You are in a pool and have been floating around for quite some time now. By the time you leave the pool, you feel heavier, like you need to put more effort into moving your body. A similar example happens during a workout at the gym. You lift a heavy dumbbell for a number of repetitions, and then, when you are done, it feels like bending your arm takes no effort at all. Your body schema adapted its representation to the situation. The accuracy of this internal model and, certainly, its plasticity play an important role in the kinematic and dynamic control of our bodies. Not only that, but the schema is very much related to our ability to use tools, say a pencil, a hammer, or a golf club. Our body schema is plastic and can temporarily integrate external objects, giving us the capability to manipulate those tools with such dexterity as if they were parts of our own bodies.

Robots have been driving mass production in industries for a good number of decades. They are an essential part of assembly lines in factories. Teams of them take care of tasks demanding high precision. Yet, even as they have been undoubtedly the workforce of automation, they are not as autonomous as we would like them to be. They are designed, constructed, and programmed to be accurate. To really ensure accuracy in their tasks, it is not only the robots that need to move with high precision, but also their environments need to be controlled. Every possibility of interaction needs to be predefined, and disturbances are to be minimized if not completely eliminated. This is precisely the reason why robots in industries normally execute their tasks in confined spaces, having little or no interaction at all with the external environments, let alone with humans. However, in recent decades, there has been a shift. Technology has evolved enough so that robots are now leaving these enclosed and protected environments and are aiming at increased interaction with their surroundings, other robots, and people. This advanced interaction comes at a price: uncertainty. The world cannot be modeled; every single potential interaction scenario cannot be anticipated and controlled. Robots, if they are to succeed in the human world, need to improvise strategies, adapt to the situation, and overcome constant, ever-changing challenges. But robots are not (at least until now) adaptable; the ways in which they are modeled are fixed and rarely subject to change. Their control paradigms also do not account for changes. Naturally, their very own bodies are not supposed to change. But change is there, a constant factor. Wear and tear is a typical phenomenon. A localized malfunction is an event that, although undesired, could occur, and a robot should be able to handle it. Perhaps the task that it needs to execute has parameters slightly different from what was anticipated; tools could be different, and the environment could be different. To put it bluntly, modern robots need to have plasticity in their models, just as humans do.

Let us go back to our imagination. Say you are left-handed. Unfortunately, you had a light accident and broke your left arm. Nothing serious, but you will not be using that arm for a couple of weeks. You can adapt your internal model to account for this situation and will probably become dexterous with that good right arm. If a humanoid robot is programmed to accomplish tasks with both arms and one ceases to operate for some reason, it is unlikely that I will carry on with the task. At this point, the engineers responsible for this robot may determine that the robot can be repurposed while the arm is repaired. They adapt the model of the robot, tune some parameters, and off the robot goes to work on its new tasks. But this is not the level of autonomy we imagine when we see a future in which robots coexist with us and assist us in a myriad of things. Once again, robots need to adapt. And the very first aspect that needs to be adapted is their own body. For that, robots need to have the capability to understand and construct a model of their bodies. This brings back the discussion to the idea that a robot can certainly benefit from the understanding of its physical self, and for that, it would need to leverage information coming from its on-board sensors and fuse them with ground truths to make sense of its body and the world around it. But how?

1.1 Motivation and vision

Empowering Robots As the integration of robots into various facets of human life becomes more pronounced, there arises a crucial imperative for these machines to proactively participate in the exploration and development of models for their own bodies. At the core of this autonomous self-discovery is the development of an awareness of the physical self by developing and maintaining a body schema. This self-awareness represents a pivotal step towards endowing robots with a profound understanding of their own embodiment and becomes the foundation for the integration of sensory information and motor control. The robot body schema, akin to the internal representation of the human body in the brain, becomes a dynamic and evolving map that serves as a reference point for the robot's interactions with its surroundings. The establishment of a body schema contributes significantly to the enhancement of robot motor control. Robots, equipped with a coherent internal model of their bodies, can execute movements with a heightened level of precision and coordination. The fusion of sensory information and motor control within the body schema lays the scaffolding for efficient learning. Additionally, autonomous self-discovery is not a static process. It involves ongoing refinement and adaptation. As robots encounter diverse environments and engage in various tasks, their body schema adapts and expands, allowing for an extended understanding of their physical capabilities. This adaptability is crucial for robots to navigate the intricacies of real-world scenarios, adjusting their responses based on the context in which they operate. The significance of this autonomous self-discovery transcends the technical aspects of robotics; it resonates with the broader narrative of robots becoming integrated and adaptive participants in human-centric environments. By actively engaging in the exploration of their own bodies, robots pave the way for a future where they seamlessly navigate, interact, and adapt in tandem with human activities. This not only enhances their functional capabilities but also fosters a harmonious integration of robots into diverse and dynamic human-centric spaces.

Learning and the body schema The seamless integration of the body schema is indispensable for the evolution of robots, forming the foundational underpinning upon which multifaceted capabilities are constructed. This integration spans across various domains, encompassing learning for physical awareness, motor control, coordination, and interaction. Learning, a hallmark of intelligent systems, coexists in a dual relationship with the body schema: learning not only contributes to the development of the body schema but, reciprocally, the body schema enhances learning. Primarily, it plays a pivotal role in detecting the structure within the myriad afferent and efferent signals of the robot's sensorimotor system, facilitating the construction of the body schema. Simultaneously, the incorporation of the body schema into learning frameworks allows robots to explore their sensorimotor maps and develop models of their morphology. This understanding becomes a scaffold for acquiring new skills, refining existing ones, and assimilating knowledge gained from interactions with the environment. The integration of the body schema thus catalyzes a learning process that is not only adaptive but also inherently tied to the robot's physical embodiment. Moving to motor control, another pivotal aspect of intelligent systems, there exists an intricate link to learning the body schema. Internal body representations that can adapt through learning mechanisms contribute to the development of superior forward and inverse models, ultimately refining control precision. Lastly, a seamlessly integrated body schema empowers robots to learn diverse tasks, surpassing the limitations of rigid, pre-programmed functionalities. Ultimately, a plastic body representation provides versatility crucial in dynamic and unpredictable settings, where the cohesive ability to learn, control movements, coordinate actions, and perceive space allows robots to thrive in a myriad of scenarios.

Enhancing locomotion, manipulation, and adaptability. Taking inspiration from the nuanced abilities of humans, the vision for future robots is set to take advantage of the enhanced bodily awareness that the integration of a body schema will bring about, thereby improving adaptability and interaction in diverse situations. The development and maintenance of the robot body schema will unlock a spectrum of capabilities that enables precise and coordinated movements, fostering in particular advanced locomotion, motion planning, and intricate manipulation. Bodily awareness is expected to revolutionize the locomotive prowess of robots. Future machines, informed by this internal map of their physical structure, will navigate environments with an unprecedented level of sophistication. This extends beyond basic movement, enabling robots to traverse complex terrains, negotiate obstacles, and adapt seamlessly to changes in their surroundings. Motion planning, a core element of robotics, will harness the robot body schema as a dynamic blueprint, supporting not only precise and efficient movements but also the ability to determine near-optimal trajectories in real-time. The result is a more adaptive and resourceful approach to navigating intricate spaces and executing tasks with heightened precision. The mastery of a body schema extends to manipulation with profound implications. Future robots, leveraging this internal map, will exhibit a level of dexterity and precision in manipulating objects that mirrors the intricacies of human hand-eye coordination. This enhanced capability will represent a breakthrough in applications requiring delicate and precise interactions, from handling diverse items to executing complex manufacturing tasks. Moreover, coordination and collaboration with other robots or humans becomes more refined through a well-integrated body schema, as it allows robots to anticipate and adapt their interactions with other agents. This anticipatory and adaptive capability is fundamental for fostering safe, effective, and harmonious interactions.

Constant self-monitoring for autonomy Continuous self-monitoring signifies a fundamental imperative for future robotic systems. It is achieved through the amalgamation of internal models and the uninterrupted stream of sensorimotor signals. This seamless fusion enables a dynamic and real-time understanding of the robot's own state, creating a powerful feedback loop that is integral to the system's autonomy. Perpetual monitoring sets in motion successive phases of error detection and correction. This iterative process ensures that the robot is not only aware of its physical state but also capable of recognizing and rectifying discrepancies between intended actions and actual outcomes. The ability to identify errors in real-time positions future robots on a trajectory towards enhanced reliability and precision in its interactions. The profound impact of this ongoing adaptation is most evident in the rapid formulation and execution of contingency motion strategies. Armed with an enriched spatial awareness and a continuously evolving set of internal models, the robot becomes adept at anticipating and responding to unforeseen challenges. In dynamic and unpredictable environments, the ability to swiftly devise and implement contingency plans allows the robot to navigate complex scenarios with agility and efficiency. This advanced interaction capability with the environment is a hallmark of the paradigm of continuous self-monitoring. The robot not only perceives its surroundings in real-time but also possesses the foresight to proactively engage with its environment. This goes beyond mere reactionary responses; it encapsulates a proactive and intelligent engagement that significantly elevates the robot's efficacy in accomplishing tasks and navigating diverse scenarios.

Onboard sensing for self-sufficiency Achieving true autonomy requires robots to evolve into self-sufficient entities capable of independent learning, calibration, monitoring, and adaptation of their body representation. This transformation is predicated on the exclusive reliance on onboard sensing modalities, a fundamental transition that empowers robots with heightened versatility and adaptability. At the core of this self-sufficiency lie two fundamental sensory modalities: somatosensation, encompassing proprioception and touch, and vision. These modalities collectively provide the robot understanding of its own body and the surrounding environment. Proprioception provides the robot awareness of its own body in space. The sense of touch complements the understanding of the body and allows the robot to distinguish itself from its immediate environment. Vision, another cornerstone modality, extends the robot's perception beyond immediate physical contact. By abstaining from off-board sensing devices, robots liberate themselves from external dependencies and enhance their self-sufficiency. Leveraging onboard sensing modalities empowers robots to dynamically respond to changes in their surroundings in real-time. Whether it's navigating through a cluttered environment, adjusting movements for safety, or adapting to unforeseen obstacles, the reliance on somatosensation and vision will enable future robot to operate with a level of autonomy and adaptability previously unseen.

Safety- and energy-awareness Comprehending their own body structure empowers robots to engage in more effective and nuanced interactions with both their robotic and human counterparts. The body schema serves as a predictive tool, endowing robots with the foresight to anticipate potential challenges. This heightened understanding facilitates dynamic adjustments in their movements, prioritizing safety and fostering efficiency in diverse collaborative scenarios. When interacting with other robots, this capability enables seamless coordination, averting collisions or disruptions during joint tasks. Likewise, in human-robot interactions, the ability to adapt movements ensures a safer environment, mitigating the risk of accidental impacts or collisions. Moreover, the comprehension of their own body structure provides robots with a unique advantage in optimizing energy consumption. By adapting their motions based on the inherent physical properties of their structures, robots can execute tasks with greater efficiency. This optimization is particularly crucial in mobile robotics, where energy conservation is paramount given the limited resources. The dual capability of enhancing safety in interactions and contributing to energy-aware robotics underscores the significance of robots understanding their own body structure in fostering a more efficient, collaborative, and environmentally conscious robotic landscape.

1.2 Problem Statement

Navigating the complexities of learning a robot's physical attributes in the field of robotics reveals intricate challenges that demand a closer look. Traditional calibration routines, tailored for known kinematic structures and offline system identification methods, face limitations concerning generalization capabilities, sample efficiency, and the reliance on a predetermined mechanical topology. This is particularly evident in the context of global and local machine learning frameworks for physical systems, where the exclusion of structural knowledge often results in operational inefficiencies.

Compounding these challenges is the substantial reliance on off-robot measurement devices, such as vision and motion-capturing systems, during calibration and identification processes. Despite the wealth of sensor signals from modern robots, determining the minimum set required for constructing a body model based solely on robot sensing remains a persistent and unresolved challenge. This dependence on external measurements not only introduces limitations but also raises questions about the practicality and applicability of learned models in real-world scenarios.

Venturing into alternative learning methods, such as neural networks, introduces a unique set of challenges. These approaches frequently lack crucial information about the robot's body structure, necessitating vast amounts of data for effective learning. The design of neural networks further compounds the issue, as it becomes an expert-driven task requiring meticulous determination of topology. Often, these approaches suffer from generalization limitations, confining their learning capabilities to specific input-output regions

Examining the research landscape reveals substantial gaps, including an unclear understanding of how object handling extends the robotic body schema. Furthermore, limited exploration into the mechanical arrangement of joints and links, known as the mechanical topology, underscores untapped potential in understanding the intricate details of robotic physical structure. These research gaps collectively contribute to the overarching challenge of lacking a unifying scheme that seamlessly integrates all learning stages, hindering the development of a fully characterized robotic body schema based solely on knowledge about sensorimotor signals.

To address these multifaceted challenges effectively, there is a compelling need to bridge the existing gaps and explore innovative learning approaches. This includes the development of methods capable of incorporating structural knowledge, reducing reliance on external measurements, and enhancing the generalization capabilities of machine learning frameworks in the realm of robotics. Moreover, the establishment of a comprehensive unifying scheme stands as a pivotal step toward advancing the field and achieving a fully characterized robotic body schema.

1.3 Research Questions and Contribution

1.4 State of the Art

1.5 Thesis Structure

1.6 Curriculum Vitae

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Literature Review

One fundamental characteristic of humans is the awareness of the body, which allows its control with dexterity and plasticity. We know where our body is in space and we know how to take it from its present situation at time t, its current *state* $\boldsymbol{s}_t$, to a predicted or expected future state $\boldsymbol{s}_{t+1}$ at time $t+1$ given a carefully chosen *action* $\boldsymbol{a}_t$. The ability to anticipate the future and to determine the actions that can take us there is connected to the notions of forward and inverse models in cognitive science and robotics [1–3]. An alternative and complementary interpretation is that of sensorimotor contingencies [4–6], which denote the structured relations between the actions of an agent and the ensuing sensory inputs resulting from interaction. The potential sensorimotor interactions are connected to the concept of *embodiment*, which, as expressed by Pfeifer et al. [7], deals with "how the body shapes the way we think," putting a premium on the morphology and capacities of an agent. Last, a few strongly related concepts are pertinent to the discussion: the *ecological self* REFERENCE, the *physical self* REFERENCE, and the *body schema* REFERENCE. They are different interpretations of *body models* that describe its physical structure and capabilities, and integrate sensory information for prediction and action. Research in cognitive science tells us that we construct and maintain representations involving the previous concepts from the moment that we are in womb and during all our conscious existence. Maintaining those models is not only a matter of refining and tuning them but also may involve adapting them to accommodate unexpected drastic changes. The level of bodily awareness encoded in those representations allows to achieve remarkable feats, like reach for things without even looking, catching an object in midair, or contorting our bodies to maneuver or bodies in tight spaces.

As discussed in the Introduction the vision for future robots involves giving them bodily awareness capabilities inspired in humans. To contextualize what is the state of the art towards realizing that vision, this chapter provides a review and discussion of relevant works that address the field of modeling in robotics; in particular, model learning. The discussion will include a brief review of the fundamental methods to model robots and will gradually move to the paradigm of learning local and global models with and without prior knowledge. The discussion will connect with the above mentioned concepts pertaining body models and present pertinent works that have presented methods to learn the body schema and sensorimotor representations in robotics

2.1 Model learning in robotics

2.1.1 Classical and recent works in system identification

2.1.2 Local and global models linear models

2.1.3 End-to-end learning (black box models)

2.2 Data-driven learning with structure information

2.3 Model learning and the body schema

2.3.1 Internal representations

2.3.2 Sensorimotor maps

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Theoretical Framework: Fundamental concepts and tools

4

Methods: How to learn the robotic body schema

5 Results

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Discussion

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Conclusion

8

Test Chapter

8.1 Introduction

8.1.1 Contributions

In this paper we cast robot system identification into an incremental learning problem integrating machine learning methods and first-order principles from mechanics and differential geometry. Moreover, we enforce physical conformity of the inertial properties at all times by seamlessly moving on the Riemannian manifold of symmetric positive definite (SPD) matrices, where the inertial parameters of a rigid body are known to reside. We implement a Riemannian gradient descent algorithm with adaptive learning rate and extend it with an experience replay buffer to accelerate convergence and cope with sensor noise. Unlike other approaches, we do not depend on informed initial values. Additionally, we analyze different force/torque measurement setups and evaluate their influence on the learning process. Finally, we implement the method on a real robot and investigate the quality of the inverse dynamics torques generated by the learned parameters and assess the modeling error via a momentum observer.

8.2 Literature review

8.2.1 Related works

8.2.1.1 Classical neural networks model learning

Model-learning problems using typical neural networks (NN) involves, mainly, the following steps:

1. Input/output data collection: assumed to be available
2. Architecture design: usually found by trial and error
3. Parameter optimization/learning: via well understood schemes, e.g., backpropagation and variants

Overall, NN design involves many meta parameters, i.e. number of nodes, numbers of layers, connectivity, activation functions; requiring experts to determine the best topology for a particular problem [8]. Furthermore, it is difficult to achieve an acceptable level of generalization [9] [10] [8] [11] [12] [13] as the architecture needs to be balanced to obtain accurate results without overfitting, which may lead to poor generalization, [10] [14] [13]. Therefore, if a non-parametric model, such as a NN, is used without any model information, large amounts of data are required to generalize to unknown data [15].

8.2.1.2 Topology learning related works

Finding NN topologies is an important and challenging step [16] [9] [17]. Function approximation using NN uses subjective or empirical topologies that are, however, not suitable for interpretation and rely on numerous parameters. As a result, such models are not able to give insight into the actual relation between system variables.

Recent works have aimed to find optimal topologies automatically. Evolutionary methods are commonly utilized to optimize the topology and the weights of NN by adding or deleting connections and weights, as shown in [9] and [8] for Feed Forward Neural Networks (FFNN) or [16] for deep NN. Results tend to show satisfactory generalization capabilities, comparable to human designs. Another method used for FFNN represents the network as a graph and reduces its degrees-of-freedom (DoF), as shown in [10]. Constructive methods are also utilized for FFNN, shown in [11], and pruning methods as applied in [18]. Furthermore, reinforcement learning (through Q-learning) and topology learning (using variance analysis) have also been applied to generate the architectures, as shown in [17] [19]. Noticeably, for learning complex dynamical systems, such as articulated robot structures, results are still promising, however, limited in accuracy and generalization capabilities [3] [20] [21].

	Conventional NN	FOP Network	Functiona
Topology	Trial & error	FOP & system knowledge	System kn
Units	Homogeneous neurons	Parameterized operators	Functions
Activation function	Sigmoid, tanh, ReLU	Functions	Functions
Learned parameters	Connection weights	Operators parameters & topology	Neuron fu
Training	Backpropagation / optimization	Optimization algorithms	Standard g

Table 8.1: Comparison between traditional neural networks, first-order principles networks and functional networks.

8.2.1.3 Robot inverse dynamics estimation via classical NN

NN have been applied in numerous variants to model robot inverse dynamics. In [22], Hopfield NN were applied to identify the inertial parameters. Moreover, in [23] a FFNN that used the regressor matrix as training samples was applied. Extreme Learning Machines were utilized in [24] with the same purpose. More recently a two-hidden-layers network with rectified linear activation units (ReLU) was used in [25]. Similarly, recurrent NN have been used to account for the sequential nature of the data. In [26], a recurrent NN in the hidden layer of an otherwise conventional three-layer FFNN was proposed. Additionally, self-organizing-networks, in conjunction with echo state networks, were used in [27] via a real-time deep learning algorithm.

8.2.1.4 Inertial parameters

Determining the inertial parameters of a robot has been typically achieved using system identification [28]. Just recently, their physical feasibility has been brought to attention. Works such as [29] study the feasibility conditions on the inertial parameters as convex sets and propose solutions using linear matrix inequalities (LMI). Similarly, authors in [30] use LMIs to find feasible inertial parameters paying particular attention to the mass distribution. In [31] *full physical consistency* is introduced and linked to the triangle inequality and the principal moments of inertia of a rigid body. An approach that represents the feasible parameters on the manifold of symmetric positive definite (SPD) matrices is presented in [32]. Extensions to this work within the context of adaptive control are given in [33]. In [34], robot links are represented as a finite number of point masses and use LMIs to introduce physical feasibility constraints. Likewise, [35] uses an Extended Kalman Filter with sigmoidal constraint functions to estimate online feasible inertial parameters of a robot manipulator. Recently, in [36], system identification of a 7 degrees-of-freedom (DOF) robot was conducted while considering for the first time full physical consistency. None of the methods discussed in these works have considered the problem as an incremental learning problem and only a few have contemplated coupling online learning capability with physical feasibility as a desired feature. As such, their applicability in developmental robotics contexts is hindered. Table **??** contains relevant works with direct focus on full physical feasibility (PF) that are comparable to our work, whether the solution is computed online (OL) or not.

8.3 Theoretical framework

A *body schema* is an internal representation of the body, including the arrangement and geometry of its parts, and is built mainly from proprioceptive information [37, 38]. Adaptive and self-acquired, it is part of an agent's internal forward and inverse models and used to plan and predict sensorimotor interactions. In our view, a robot's body schema encompasses a description of its sensing and actuation capabilities together with its topological, morphological and dynamical characterization. Furthermore, from a developmental perspective, a robot is an embodied agent that autonomously learns and refines incrementally its body schema. Robotics research in this area has focused on discovering the kinematic structure of the robot from exploratory motions and sensorimotor information [39–43]. Yet, the inertial properties of the agent's body as part of the schema have been neglected. We argue that, in developing a body schema, the robot must incrementally acquire knowledge of these properties to cope with and adapt to alterations in its body. Typically, the classical system identification paradigm has been used to find sets of inertial parameters [44]; however, it does not contemplate a robot as an embodied agent capable of learning. Alternatively, the machine learning paradigm makes possible the definition of data-driven robot models. Yet, it suffers from a lack of interpretability [45, 46] and generalization capability limited by the size and variability of the training data. Inspired by both paradigms and starting from knowledge of the robot's kinematic structure, we present a method to incrementally learn the inertial parameters of the robot's constituent links guaranteeing physical feasibility at all times.

8.3.1 Learning the inertial parameters in the SPD manifold

8.3.1.1 The space of symmetric positive definite matrices

A *differentiable manifold* $\mathcal{M}$ is a topological space that is locally similar to Euclidean space and has a globally defined differential structure [47]. $\mathcal{T}_{\boldsymbol{P}}\mathcal{M}$ is the *tangent space* at a point $\boldsymbol{P} \in \mathcal{M}$ and represents the vector space of all the possible tangent vectors to the manifold that pass through $\boldsymbol{P}$. The pair $(\mathcal{M}, \rho)$ defines a *Riemannian manifold* if $\mathcal{M}$ is differentiable and is equipped with a positive definite metric tensor ρ at each point [48].

Let $\mathcal{S}^n \triangleq \{\boldsymbol{S} \in \mathbb{R}^{n\times n} : \boldsymbol{S} = \boldsymbol{S}^T\}$ be the space of real square symmetric matrices of dimension $n \times n$. Then, the space of $n \times n$ SPD matrices $\mathcal{S}^n_{++} \triangleq \{\boldsymbol{P} \in \mathcal{S}^n : \boldsymbol{P} \succ 0\}$ defines a smooth submanifold $\mathcal{M}$ of $\mathcal{S}^n$. By definition, its tangent space $\mathcal{T}\mathcal{M} \in \mathcal{S}^n$ is equipped with an *affine invariant Riemannian metric* ρ [32]. Consequently, $\mathcal{S}^n_{++}$ defines a Riemannian manifold. Fi-

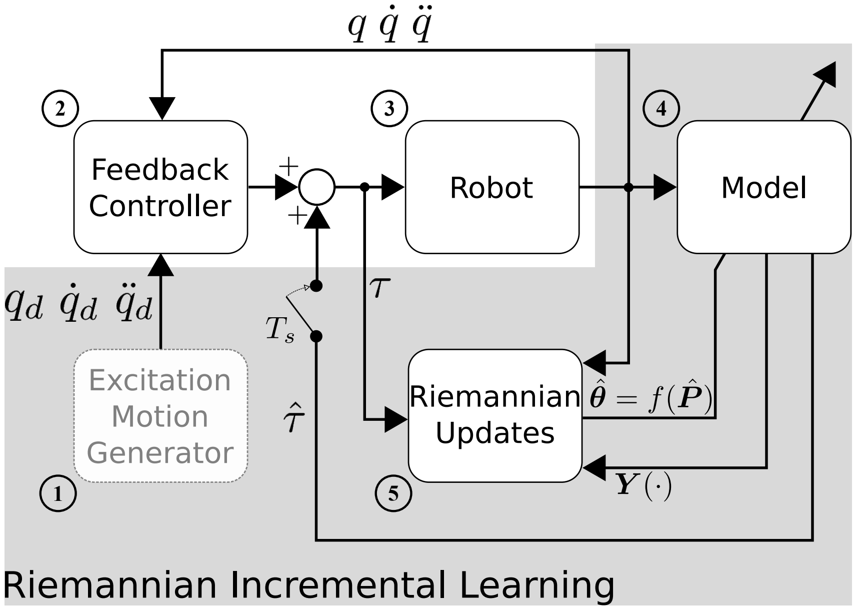

Figure 8.1: Overview of the proposed learning scheme.

nally, the product manifold $\mathcal{M}^N$ of SPD manifolds is the Cartesian product $\mathcal{M}^N = \mathcal{M}_1 \times \mathcal{M}_2 \times \ldots \times \mathcal{M}_N$. It is the set of matrices $\{(\boldsymbol{P}_1, \ldots, \boldsymbol{P}_N) : \boldsymbol{P}_i \in \mathcal{M}_i, \quad i = 1, \ldots, N\}$ and is also a Riemannian manifold with the metric $\boldsymbol{\rho} = \mathrm{diag}\left(\rho_1, \ldots, \rho_N\right)$. Similarly, the generalizations of the operators mentioned above to $\mathcal{M}^N$ are the concatenations of the individual operators for each $\mathcal{M}_i$.

8.3.2 The inverse dynamics problem

Let $\boldsymbol{w} = \boldsymbol{W}\left(\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}}\right)\boldsymbol{\theta}$ denote the inverse dynamics equation of a serial robot with N links, where the inputs are the vector of joint angles $\boldsymbol{q}$ and its first and second derivatives. We use here the formulation $\boldsymbol{w} = \boldsymbol{W}\left(\boldsymbol{\omega}, \dot{\boldsymbol{\omega}}, \dot{\boldsymbol{v}}\right)\boldsymbol{\theta}$ as it allows the decoupling of the kinematics and dynamics parts of the robot model [49]. The vector $\boldsymbol{w} = [\boldsymbol{w}_1^T, \cdots, \boldsymbol{w}_N^T]^T$ contains the wrenches of all the bodies in the kinematic chain expressed in the corresponding body frame. Each wrench is composed of the forces $\boldsymbol{f}_i$ and moments $\boldsymbol{n}_i$ acting on the i-th body, e.g. $\boldsymbol{w}_i = \left[\boldsymbol{f}_i^T, \boldsymbol{n}_i^T\right]^T$. The regressor matrix $\boldsymbol{W}(\cdot)$ depends on the robot kinematics and the Cartesian angular velocities $\boldsymbol{\omega}_i$, as well as on the angular $\dot{\boldsymbol{\omega}}$ and linear accelerations $\dot{\boldsymbol{v}}$*. The vector $\boldsymbol{\theta} = \begin{bmatrix}\boldsymbol{\theta}_1^\intercal & \ldots & \boldsymbol{\theta}_i^\intercal & \ldots & \boldsymbol{\theta}_N^\intercal\end{bmatrix}^\intercal$ contains the inertial parameters of the robot, with $\boldsymbol{\theta}_i = [m_i\ \boldsymbol{h}_i^\intercal\ XX_i\ XY_i\ XZ_i\ YY_i\ YZ_i\ ZZ$ $\mathbb{R}^{10}$ and $\boldsymbol{h}_i = \begin{bmatrix}mX_i & mY_i & mZ_i\end{bmatrix}^T$. The first element of $\boldsymbol{\theta}_i$ is the mass of link i, the vector $\boldsymbol{h}_i$ contains the first moments of mass, and the last six entries are the elements of the inertia matrix of link i expressed in joint frame i.

*It is worth mentioning that the vector $\boldsymbol{w}$ and, correspondingly, the matrix $\boldsymbol{W}$ can be adjusted according to the available sensors.

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