Dynamic sensorimotor graphs

Some Guy, Random Stranger and Anonymous Dude

***Abstract*— Regularities present in the somatosensory signals of a robotic agent can reflect its embodiment and the associations resulting from the active control policy. In this work, we analyze the dynamic functional connectivity of the somatosensory signals based on the instantaneous pairwise mutual information. As the robot performs exploratory motions based on motor babbling, we capture and study the time-varying changes in the signal relationships. We analyze the instantaneous and average information sharing and associate them with different information states. Results from a simulated planar system validate using the instantaneous mutual information as a tool for extracting and leveraging the relationships between the somatosensory signals and defining exploratory motions related to self-tocuh events.**

I. Introduction

The study of the structural and functional connectivity in the brain has furthered the understanding of its organizations and information processing. An analogy with the sensorimotor signals of an embodied agent is yet to be realized. While looking at the structural connectivity of the sensorimotor signals is difficult, studying their functional connectivity to extract information about the body structure and the emergence of behaviors based on information acquisition is undoubtedly realizable. The underlying premise is that regularities among the robot's sensorimotor signals connect tightly to the robot's embodiment. These regularities, known as *sensorimotor contingencies* (SMC) [1], can be identified and studied using information-theoretic methods. This work explores an embodied agent's time-varying sensorimotor functional structure leveraging concepts from information and graph theory.

A. Related works

As established in [1], SMCs play a role in acquiring body knowledge, generalization, and goal-directedness. Yet, although the literature about sensorimotor representations is extensive [2], the connections between sensorimotor regularities and body knowledge are poorly understood. Several works [3]–[7] have used information-theoretic metrics to study these relationships in the sensorimotor system. Additionally, when exploring SMCs, touch is an essential sensory modality. For example, the work by [12] discussed using intrinsic motivation and goal-babbling for learning self-touch on a simulated humanoid robot with artificially sensitive skin. Self-touch for calibration was used in [13], letting the robot close the kinematic chain by touching its own body. In [14], a multimodal variational autoencoder in a denoising framework is used to learn the latent representation of self-touch that allows the agent to reconstruct self-reaching configurations internally.

The study of functional connectivity has shown that the statistical properties of the considered signals and their relationships may vary over time. This *dynamical* functional connectivity (DFC) has mainly been used to study functional networks in the brain to, for example, show states of lower functional connectivity during onsets of epilepsy[9] or identify potentially abnormal connectivity patterns in The work in brains affected by illness[10]. Recently it was used to identify evolving relationships in the sensorimotor signals of a simulated infant based on age [11].

B. Contributions

This work considers that representing the functional connections between somatosensory signals (touch and proprioception) of an embodied agent as *dynamic* graphs and their corresponding analysis can help understand the formation and evolution of SMCs. Given that, based on the motion policy driving the robot or the task it executes, sensorimotor connectivity will also change, capturing and studying these varying connectivity patterns is essential. We achieve this by looking at the instantaneous mutual pairwise mutual information showing how the state of information sharing in the system is related to its motion and potential interactions with itself. Using non-negative matrix factorization, we identify different information-sharing states and classify them according to their information content. In a case study, we demonstrate how, after a motor babbling phase, by only focusing on the mutual information and without knowing the robot's morphology, an excitation trajectory for both the robot arms can be devised that avoids self-contact. A comparison of this trajectory against conventional trajectory design methods is presented.

II. The embodied agent

A. The planar dual arm model

We use as reference system the robot model presented in [14], [15], consisting of a simple six degrees-of-freedom planar dual arm system with three links per arm and a fixed torso, see Fig. 2. The robot is equipped with a set of tactile sensors distributed along the robot's body. The dynamics of the model were instantiated by assigning inertial properties to the robot's composing bodies. Its actuation mechanism is based on the biology-inspired model presented in [16]–[18], where each joint is driven by antagonistic muscles (modeled as spring-damper systems) whose pulling force is linearly controlled by the signal of a corresponding motor neuron σ. The generated joint torque, expressed as

$$\tau = \alpha \left(\sigma_{fx} - \sigma_{ex}\right) + \beta \left(\sigma_{fx} + \sigma_{ex} + \gamma\right) q + \delta \dot{q}, \qquad (1)$$

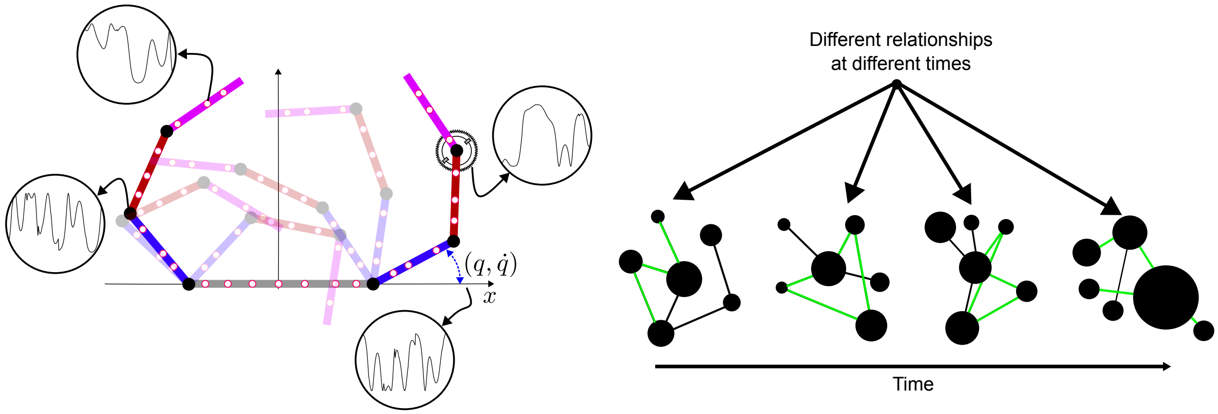

Fig. 1: General overview. While moving the sensorimotor functional connectivity changes in time

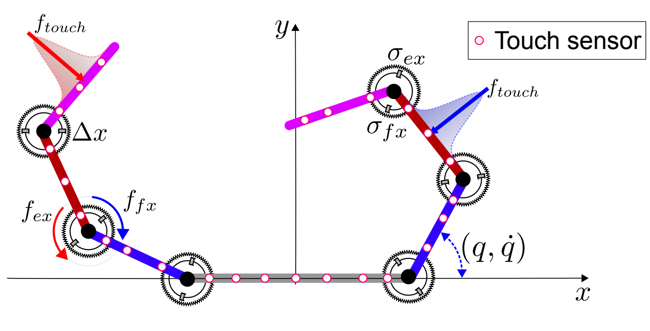

Fig. 2: The planar dual arm robot with antagonistic actuation.

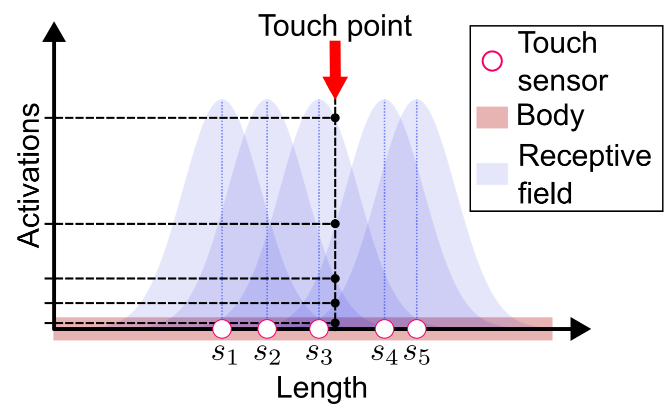

Fig. 3: The receptive fields used in population coding.

results from the difference between the flexion σ_{fx} and extension σ_{ex} activation signals which create the flexion and extension pulling forces f_{fx} and f_{ex}. The parameters are:

- α: muscle force gain
- β: stiffness gain
- γ: tonic stiffness
- δ: damping coefficient

B. The set of somatosensory signals

The tactile sensors on the robot's body are modeled based on population coding [19] represented as distance-dependent Gaussian receptive fields (see Fig. 3), with the location of the sensors (randomly located along the robot's one-dimensional body) being the means of the receptive fields. We modified the tactile sensors to account for the strength of touch. Essentially, the previously distance-based activation of the Gaussian receptive fields is now scaled by the contact force. Similar to the tactile sensors, the robot's proprioceptive measurements are also encoded using receptive fields. Therefore, the vector of somatosensory signals $\boldsymbol{s}$ in the extended model consists of proprioception that encompasses joint position $\boldsymbol{p}$, velocity $\boldsymbol{v}$, and effort $\boldsymbol{e}$, as well as touch signals $\boldsymbol{r}$ (scaled by the touch force); i.e.:

$$\boldsymbol{s} = \begin{bmatrix} \boldsymbol{p}^T & \boldsymbol{v}^T & \boldsymbol{e}^T & \boldsymbol{r}^T \end{bmatrix}^T \in \mathbb{R}_{\geq}^{N_s} \tag{2}$$

III. The Sensorimotor Dynamic Functional Connectivity

A. Functional connectivity

Functional connectivity (FC) is a method for network topology inference that characterizes the dependencies of the observed signals from a system based on their probability distributions[20]. It can be subdivided into undirected and directed; the latter being related to the analysis of statistical causation from the data [21]. By studying the FC it is possible to reveal a structure that aids in the analysis of the interaction among the entities.

Based on the connection between embodiment and information structure [22], our hypothesis is that properties of the sensorimotor interactions can be made apparent by studying the FC among the somatosensory signals $\boldsymbol{s}(t)$. From the various metrics that have been proposed to evaluate such relationships, we in particular leverage those based on information theory [23], [24], as their *model-free* nature can capture linear and nonlinear relationships between signals. Particularly, we use the *mutual information* (MI), a quantity that has been applied in different contexts to quantify the relationships between variables [25]. The MI between two signals $I(X, Y)$ can be interpreted as the amount by which a random signal Y reduces the uncertainty about a random signal X [26]. It is a symmetric measure of the information sharing by both signals and is computed as:

$$I(X;Y) = I(Y;X) = H(X) + H(Y) - H(X,Y) \tag{3}$$

with the Shannon's entropy of a variable X defined by

$$H(X) = -\sum_{i=1}^{n} p(x_i)\log_2\left(p\left(x_i\right)\right) \tag{4}$$

and the joint entropy between X and Y expressed as

$$H(X,Y) = -\sum_{i=1}^{n}\sum_{j=1}^{n} p(x_i, y_j)\log_2\left(p\left(x_i, y_j\right)\right). \tag{5}$$

By extension, the mutual information matrix $\boldsymbol{\mathcal{I}} \in \mathbb{R}^{m\times m}$ can be constructed by computing the pairwise MI between the $\left\{s_i\right\}_{i=1}^{N_s}$ somatosensory signals. Its (i,j) entries are given by

$$(\boldsymbol{\mathcal{I}})_{i,j} = I(s_i, s_j). \tag{6}$$

In practice, computing (6) for a pair $\left(s_i(t), s_j(t)\right)$ involves centering their samples (to zero mean and unit standard deviation) and using either binning, kernel, or nearest neighbor methods [27] to compute their mutual information. In this work, for the computation of $\boldsymbol{\mathcal{I}}$ we use the the open-source MATLAB package *Mutual information computation*[28].

B. Dynamic functional connectivity

When analyzing FC it might be interesting to look not only at the aggregated effect of a complete dataset of recordings but also at the instantaneous changes that occur in the relationships. Indeed, the functional relationships between sensorimotor signals can change rapidly depending on the motion policy and the interaction of the agent with the environment. To capture this time-varying, i.e. *dynamic*, functional connectivity with mutual information, it is common to use a sliding time window [29] with forward step Δt from which the MI is computed only for a small number of samples.

In particular, for a time window of length T. The mutual information $I_t(s_x(t); s_y(t))$ for a distinct pair of signals $s_x(t)$ and $s_y(t)$ at time t is computed for the set of signal samples spanning the interval $[t-T, t]$, see Fig. 4. We called this term the *instantaneous mutual information* (IMI). It follows that the mutual information matrix $\boldsymbol{\mathcal{I}}(t)$ at time t is constructed by computing the IMI for all the pairwise signals in the same time interval. The time series the time-varying mutual information matrix $\boldsymbol{\mathcal{I}}(t)$ shows the dynamical functional connectivity between the somatosensory signals.

C. Analyzing patterns

Given a set of m samples from the somatosensory signals, generated, for example, via motor babbling, a dataset $\boldsymbol{S} \in \mathbb{R}_{\geq}^{n\times m}$ can be used to search for repeating patterns of FC that encode certain modes of operation. It is also important to determine the strength and frequency of expression of these patterns to determine their relevance. Typical methods to detect repeating patterns in dynamic FC include the

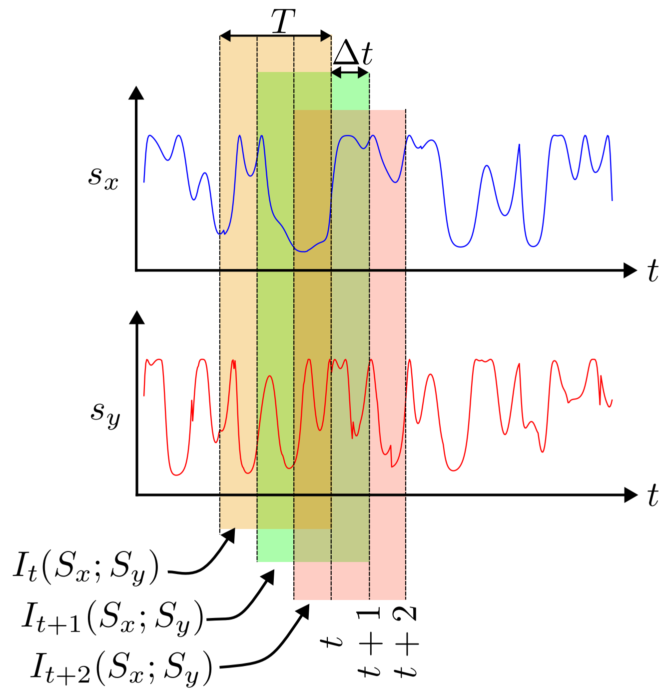

Fig. 4: Sliding window strategy to compute the instantaneous mutual information.

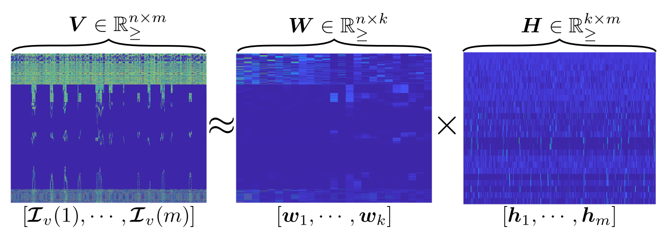

Fig. 5: Decomposition of the mutual information data matrix using non negative matrix factorization.

cosine similarity [30] and non-negative matrix factorization (NNMF) [31]. We chose the latter, as it proven useful in several studies analyzing brain dynamic functional networks and has also being used for community detection[32], [33].

NNMF is an unsupervised machine learning algorithm that can be used to split an input matrix $\boldsymbol{D}_{\boldsymbol{\mathcal{I}}} \in \mathbb{R}_{\geq}^{n\times m}$ into two parts: a matrix $\boldsymbol{W} \in \mathbb{R}_{\geq}^{n\times k}$ of bases and a matrix $\boldsymbol{H} \in \mathbb{R}_{\geq}^{k\times m}$ of their corresponding contributions to the input matrix $\boldsymbol{D}_{\boldsymbol{\mathcal{I}}}$; i.e.,

$$\boldsymbol{D}_{\boldsymbol{\mathcal{I}}} \approx \boldsymbol{W}\boldsymbol{H}. \tag{7}$$

In our case, the matrix $\boldsymbol{D}_{\boldsymbol{\mathcal{I}}}$ is constructed by vectorizing and concatenating each of the matrices $\boldsymbol{\mathcal{I}}_v(t) = \mathrm{vec}\left(\boldsymbol{\mathcal{I}}(t)\right)$; that is:

$$\boldsymbol{D}_{\boldsymbol{\mathcal{I}}} = [\boldsymbol{\mathcal{I}}_v(1), \cdots, \boldsymbol{\mathcal{I}}_v(m)] \tag{8}$$

Note that $n = N_s(N_s-1)/2$ is the total number of mutual information pairs. Since the matrix $\boldsymbol{D}_{\boldsymbol{\mathcal{I}}}$ is strictly non-negative, it is straight away suitable for its decomposition and analysis using NNMF. One crucial question is the number of factors k used to approximate the original dataset. From the various methods to select an adequate number [34], we chose k following the elbow method as in [35] by performing NNMF for ascending values of k and selecting the value where the residual error is not reduced any further.

Each of the factors $\{\boldsymbol{w}_i\}_i^k = 1$ can be interpreted as a base FC graph capturing a state of information sharing of multiple sensorimotor states. When the factors are aggregated using via the columns of $\boldsymbol{H}$ the particular mutual information matrix at time t is closely approximated.

To facilitate the analysis of the relevance of each of the factor, after factorization the factors are normalized to according to the L_2-norm.

IV. Simulation results

A. Exploratory phase

For the computation of the instantaneous mutual information we used sensors signals sampled at 100 Hz and a sliding window of $T = 0.1$ seconds; i.e., the previously seen 10 samples. With this short memory which is stored in a buffer, we can capture (fast) touch events.

B. Dimensionality reduction

To show graphically how the different factors cluster naturally depending on the touched regions we used the PaCMAP dimensionality reduction method [36] given its properties to preserve aspects of the global and local structure when reducing into the latent space [37].

V. Case study: robot excitation trajectories

TODOIn this section we use the instantaneous mutual information to generate trajectories for the left and right arms avoiding potential collisions. This is done agnostic to the actual morphology of the robot. In contrast we use a standard method for the design of excitation trajectories and compare the results.

VI. Beyond robotics

TODOValentin + Matej

VII. Conclusions

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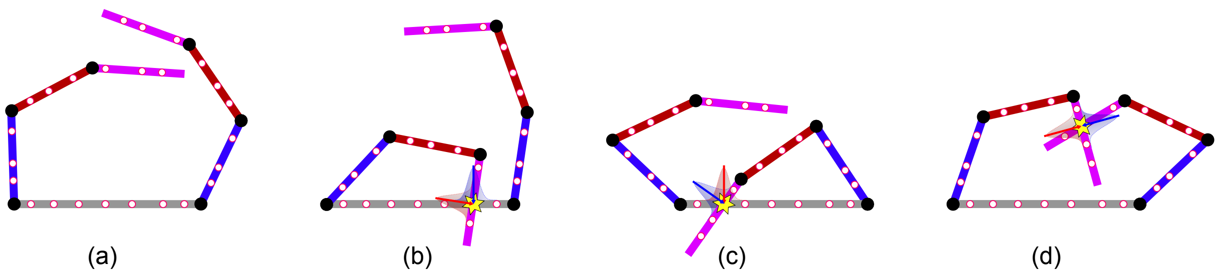

Fig. 6: Different events during exploration. (a) Pure proprioception (no touch), (b) contact with left arm, (c) contact with right arm, and (d) contact with both arms.

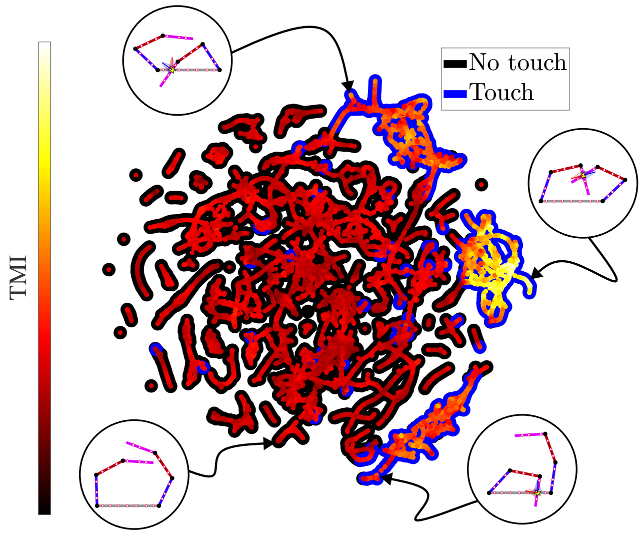

Fig. 7: A two dimensional projection using PaCMAP of the coefficients $\boldsymbol{H}$. The color of each point is scaled by its total mutual information value.

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