

# Dynamic sensorimotor graphs

Some Guy, Random Stranger and Anonymous Dude

**Abstract**—Regularities present in the sensorimotor signals of a robotic agent can reflect its embodiment, as well as the associations resulting from the active control policy. In this work, we analyze the functional connectivity of the sensorimotor signals based on pairwise mutual information. As the robot performs exploratory motions based on motor babbling we capture and study the time-varying changes in the relationships. We provide analysis of the instantaneous and average sharing of information and extrapolate the meaning in relation to the physical properties of the robot’s body. Results from a simulated planar system validate the use of mutual information as a tool not only for the analysis of the relationships between the sensorimotor signals but also to drive exploratory motions.

TODO

## I. INTRODUCTION

TODO

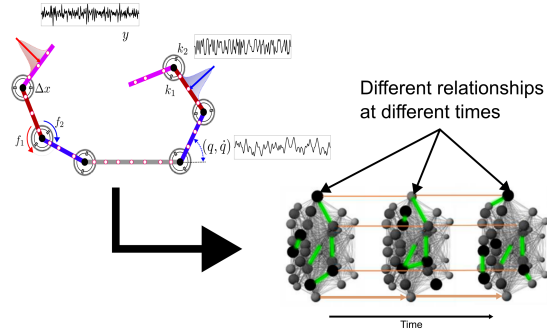


Fig. 1: General overview.

### A. Background

This work explores the sensorimotor structure of an embodied agent. The underlying premise is that regularities among the robot’s sensorimotor signals are a product of the robot’s embodiment. These regularities, known as *sensorimotor contingencies* (SMC) [1], can be identified by studying the relationships between the signals from its sensory apparatus (for both perception and actuation). Information-theoretic metrics are considered to study the relationships in the sensorimotor system. Some examples of previous works that have used information theory in this context include [2]–[6]. Although the literature about sensorimotor representations is extensive [7], this proposal considers that the representation of SMCs as *dynamic* graphs and their corresponding analysis aided by concepts from graph theory could provide further understanding about their formation and evolution. To achieve such a representation, methods from *network topology inference* (NTI)[8] are regarded to

study the relationships among the different constituent elements of the graph.

Dynamical functional connectivity (DFC) has been mainly used to study functional networks in the brain. For instance, in [9], it was used to show states of lower functional connectivity during onsets of epilepsy.

The work in [10] used NNMF on EEG signals to identify potentially abnormal connectivity patterns in brains affected by illness.

#### 1) Tactile exploration:

- The work by [11] discussed the usage of intrinsic motivation and goal-babbling for learning self-touch on a simulated humanoid robot with artificial sensitive skin.
- Self-touch for calibration was used in [12] letting the robot close the kinematic chain by touching its own body.
- [13] uses a multimodal variational autoencoder in a denoising framework to learn latent representation of self-touch that allows the agent to internally reconstruct self-reaching configurations. **The paper contributes to the sensorimotor contingency theory by providing a computational model that supports the hypothesis that the representation of self-touch can be learned simply through self-exploration.**

#### 2) Somatosensory relationships as graphs: **How can information and graph theory be leveraged to devise a graphical representation of the SMCs in the somatosensory signals of a robot?**

As mentioned in [1], the connections between sensorimotor regularities and body knowledge are not well understood. To contribute to this understanding, this proposal parts from the knowledge of the number and modality of the robot’s somatosensory signals (touch and proprioception) to

- use information-theoretic metrics, such as mutual information, predictive information, and/or transfer entropy, to explore the functional connectivity among sensorimotor signals and
- leverage graph theoretical concepts to identify and study the most important sensorimotor connections.

As the statistical properties of the signals and their relationships may vary depending on the motion policy driving the robot or the task being executed by it, it is essential that the estimated graph shows plasticity to reflect these effects (see Fig. ??). Therefore, unlike conventional NTI, methods that enable the estimated graph to change and adapt dynamically will be explored.

### B. Related works

Refer to Mannella, Gama, Marcel, Kanazawa, Husbands, Hoffmann (puppy).

### C. Contributions

With the goal of finding as much information as possible about body properties, we present an analysis based on the dynamic relationships existing among the sensorimotor signals of an embodied agent. In the analysis we show how the state of information sharing in the system is related to the motion of the robot and the tactile events that stem from its movement. By using non-negative matrix factorization we identified different information sharing states and classify them according to their information content. In a case study we demonstrate how after a motor babbling phase, by only focusing on the mutual information and without having knowledge about the morphology of the robot, an excitation trajectory for both the robot arms can be devised that avoids self contact. A comparison of this trajectory against conventional trajectory design methods is presented.

## II. THE EMBODIED AGENT

### A. Robot and measurements models

This work takes as reference the robot model used in [13], [14], consisting of a simple six degrees-of-freedom planar dual arm system with three links per arm and a fixed torso, see Fig. 2. The robot is equipped with a set of tactile sensors distributed along the robot's body. They are modeled based on population coding [15] represented as Gaussian receptive fields (see Fig. 3), with the means of the receptive fields randomly located along the robot's one-dimensional body. We modified the tactile sensors were modified to account for the strength of touch. Essentially, the previously distance-based activation of the Gaussian receptive fields is now scaled by the contact force. Furthermore, the dynamics of the model were instantiated by assigning inertial properties to the robot's composing bodies. Finally, we implemented the biology-inspired actuation model presented in [16]–[18], where each joint is driven by antagonistic muscles (modeled as springs) whose pulling force is linearly controlled by the signal of a corresponding motor neuron  $\sigma$ . The generated joint torque, expressed as

$$\tau = \alpha(\sigma_{fx} - \sigma_{ex}) + \beta(\sigma_{fx} + \sigma_{ex} + \gamma)q + \delta\dot{q}, \quad (1)$$

results from the difference between the flexion  $\sigma_{fx}$  and extension  $\sigma_{ex}$  activation signals which create the pulling forces  $f_{fx}$  and  $f_{ex}$ . The parameters are:

- $\alpha$ : muscle force gain
- $\beta$ : stiffness gain
- $\gamma$ : tonic stiffness
- $\delta$ : damping coefficient

Similar to the tactile sensors, the robot's proprioceptive measurements are encoded using receptive fields. Originally, this model was meant to capture static relationships between joint configurations and touch/no-touch events. Therefore, the somatosensory measurements in the extended model

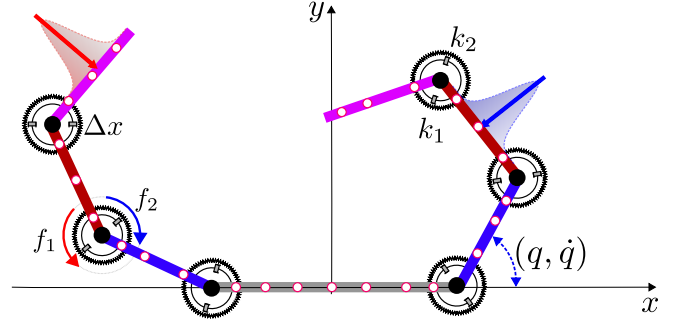


Fig. 2: The planar dual arm robot.

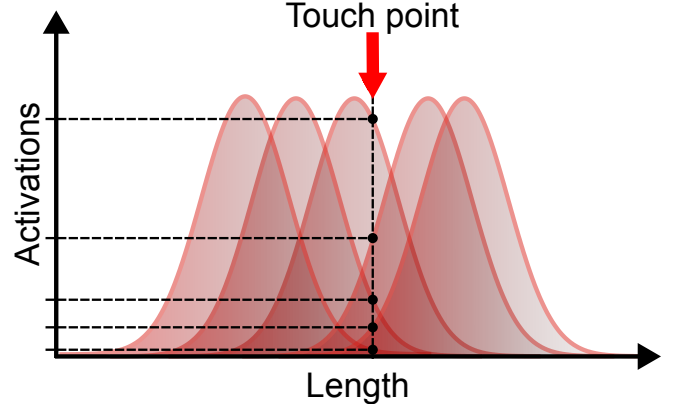


Fig. 3: The receptive fields used in population coding.

consist of touch signals (scaled by the touch force) and proprioception that encompasses joint position, velocity and effort.

## III. THE SENSORIMOTOR DYNAMIC FUNCTIONAL CONNECTIVITY

### A. Functional connectivity

*Functional connectivity* (FC) is a method for network topology inference that characterizes the dependencies of the observed signals from a system based on their probability distributions[19]. It can be subdivided into undirected and directed; the latter being related to the analysis of statistical causation from the data [20]. By studying the FC it is possible to reveal a structure that aids in the analysis of the interaction among the entities.

Based on the connection between embodiment and information structure [21], our hypothesis is that properties of the sensorimotor interactions can be made apparent by studying the FC among the somatosensory signals  $x(t)$ . From the various metrics that have been proposed to evaluate such relationships, we in particular leverage those based on information theory [22], [23], as their *model-free* nature can capture linear and nonlinear relationships between signals. Particularly, we use the *mutual information* (MI), a quantity that has been applied in different contexts to quantify the

relationships between variables [24]. The MI between two signals  $I(X, Y)$  can be interpreted as the amount by which a random signal  $Y$  reduces the uncertainty about a random signal  $X$  [25]. It is a symmetric measure of the information sharing by both signals and is computed as:

$$I(X; Y) = I(Y; X) = H(X) + H(Y) - H(X, Y) \quad (2)$$

with the Shannon's entropy of a variable  $X$  defined by

$$H(X) = - \sum_{i=1}^n p(x_i) \log_2(p(x_i)) \quad (3)$$

and the joint entropy between  $X$  and  $Y$  expressed as

$$H(X, Y) = - \sum_{i=1}^n \sum_{j=1}^n p(x_i, y_j) \log_2(p(x_i, y_j)). \quad (4)$$

By extension, the mutual information matrix  $\hat{\mathbf{W}}_{MI} \in \mathbb{R}^{m \times m}$  can be constructed by computing the pairwise MI between the  $\{x_i\}_{i=1}^m$  somatosensory signals. Its  $(i, j)$  entries are given by

$$(\hat{\mathbf{W}}_{MI})_{i,j} = I(x_i, x_j). \quad (5)$$

In practice, computing (5) for a pair  $(x_i(t), x_j(t))$  of time series from the data matrix  $\mathbf{X}$ , centering their samples (to zero mean and unit standard deviation) and using either binning, kernel, or nearest neighbor methods [26] to compute their mutual information.

In this work, for the computation of  $\hat{\mathbf{W}}_{MI}$  we use the open-source MATLAB package *Mutual information computation* [27].

### B. Dynamic functional connectivity

When analyzing FC it might be interesting to look not only at the aggregated effect of a complete dataset of recordings but also at the instantaneous changes that occur in the relationships. Indeed, the relationships between sensorimotor signals might change rapidly depending on the motion policy. To capture these time-varying functional connectivity with mutual information, the most common approach involves the usage of a sliding time window [28]. In particular, tactile events might have a short time scale, the mutual information was computed for a time window of length  $T$ , that is to say  $x_i(t, t-1, \dots, t-T)$ , we called this term the *instantaneous mutual information*.

**TODO**

### C. Analyzing patterns

Other methods to detect repeating patterns in DFC include the cosine similarity [29]

Once the data set  $\mathbf{D}_{MI}$  is available, it can be used to extract repeating patterns that encode certain modes of operation captured from the motor babbling. It is also important to determine the expression of these patterns during the recorded motion brain networks that encode resting states associated with altered patterns of connectivity—and track

<sup>1</sup>We use the *hat* notation to emphasize that this matrix is an estimate and there is no actual ground truth.

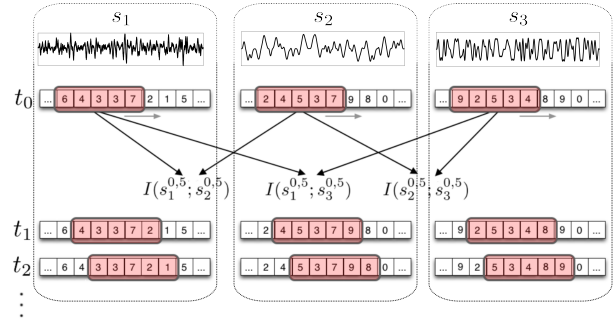


Fig. 4: Sliding window strategy to compute the instantaneous mutual information.

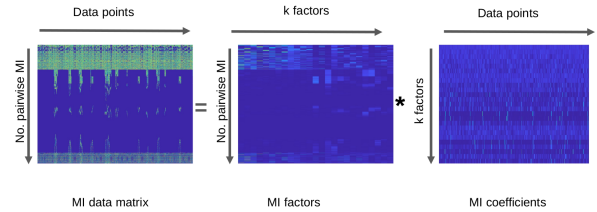


Fig. 5: Decomposition of the mutual information data matrix using non negative matrix factorization.

their expression over time—is required. Such a capability would improve our understanding of those patterns of brain networks that are significant for different groups of ASDs and how these patterns change over time

Intuitively, NMF decomposes functional brain networks into the following: (1) a set of subnetworks (patterns) overlapping in space and time and (2) corresponding coefficient time series that quantify the contribution of each subnetwork (pattern) at each time point (Chai et al., 2017; Khambhati et al., 2017, 2018a,b).

As compared to hard-partitioning schemes, the advantage of this method is that it provides information about brain-network dynamics in a continuous, overlapping manner in space and time rather than in discrete partitions. Furthermore, owing to the parts-based nature of the technique, we obtained subnetworks that resembled the localized features of largescale brain networks rather than generalized patterns of the overall network

Non-negative matrix factorization (NNMF) [30] was employed to extract data from the connectivity graphs defines by the instantaneous mutual information. NNMF is an unsupervised machine learning algorithm that can be used to split an input matrix  $\mathbf{V}$  into two parts: a matrix  $\mathbf{W}$  of bases, or subgraphs, and a matrix  $\mathbf{H}$  of their corresponding contributions to the input matrix  $\mathbf{V}$ . The input matrix ( $\mathbf{A}$ ) for the NMF, which produced the subgraphs ( $\mathbf{W}$ ) and associated time series of contributions ( $\mathbf{H}$ ), which were assumed to represent the relative temporal activation of each of these networks, was created by combining connectivity graphs.

NNMF has also being used to detect communities in

networks [31], [32]

There are various methods to select an adequate number of factors [33]. To choose the number of factors we followed as in [34] and "which was chosen by performing NMF iteratively for increasing values of  $k$  and selecting the inflection point, or elbow, in the reconstruction residual sum of squares error curve"

In [35] it is shown how the basis graph change their expressing during learning of a task

To show graphically how the different factors cluster naturally depending on the touched regions we used the PaCMAP dimensionality reduction method [36] given its properties to preserve aspects of the global and local structure when reducing into the latent space [37].

#### IV. SIMULATION RESULTS

**TODO** For the computation of the instantaneous mutual information we used sensors signals sampled at 100 Hz and a sliding window of  $T = 0.1$  seconds; i.e., the previously seen 10 samples. With this short memory which is stored in a buffer, we can capture (fast) touch events.

#### V. CASE STUDY: ROBOT EXCITATION TRAJECTORIES

**TODO** In this section we use the instantaneous mutual information to generate trajectories for the left and right arms avoiding potential collisions. This is done agnostic to the actual morphology of the robot. In contrast we use a standard method for the design of excitation trajectories and compare the results.

#### VI. BEYOND ROBOTICS

**TODO**

#### VII. CONCLUSIONS

##### REFERENCES

- [1] L. Jacquey, G. Baldassarre, V. G. Santucci, and J. K. O'Regan, "Sensorimotor contingencies as a key drive of development: From babies to robots," *Frontiers in neurorobotics*, vol. 13, p. 98, 2019.
- [2] N. M. Schmidt, M. Hoffmann, K. Nakajima, and R. Pfeifer, "Bootstrapping perception using information theory: Case studies in a quadruped robot running on different grounds," *Advances in Complex Systems*, vol. 16, no. 02n03, p. 1 250 078, 2013.
- [3] M. Lungarella and O. Sporns, "Mapping information flow in sensorimotor networks," *PLoS Comput Biol*, vol. 2, no. 10, e144, 2006.
- [4] D. Polani and M. Möller, "Models of information processing in the sensorimotor loop," in *Information Theory and Statistical Learning*, Springer, 2009, pp. 289–308.
- [5] T. Bossomaier, L. Barnett, M. Harré, and J. T. Lizier, "An introduction to transfer entropy," *Cham: Springer International Publishing*, vol. 65, 2016.
- [6] L. A. Olsson, C. L. Nehaniv, and D. Polani, "From unknown sensors and actuators to actions grounded in sensorimotor perceptions," *Connection Science*, vol. 18, no. 2, pp. 121–144, 2006.
- [7] P. D. Nguyen, Y. K. Georgie, E. Kayhan, M. Eppe, V. V. Hafner, and S. Wermter, "Sensorimotor representation learning for an "active self" in robots: A model survey," *KI-Künstliche Intelligenz*, vol. 35, no. 1, pp. 9–35, 2021.
- [8] X. Dong, D. Thanou, M. Rabbat, and P. Frossard, "Learning graphs from data: A signal representation perspective," *IEEE Signal Processing Magazine*, vol. 36, no. 3, pp. 44–63, 2019.
- [9] E. Christiaen, M.-G. Goossens, B. Descamps, L. E. Larsen, P. Boon, R. Raedt, and C. Vanhove, "Dynamic functional connectivity and graph theory metrics in a rat model of temporal lobe epilepsy reveal a preference for brain states with a lower functional connectivity, segregation and integration," *Neurobiology of disease*, vol. 139, p. 104 808, 2020.
- [10] T. Zhou, J. Kang, F. Cong, and X. Li, "Early childhood developmental functional connectivity of autistic brains with non-negative matrix factorization," *NeuroImage: Clinical*, vol. 26, p. 102 251, 2020. [Online]. Available: <https://www.sciencedirect.com/science/article/pii/S2213158220300887?via=ihub>.
- [11] F. Gama, M. Shcherban, M. Rolf, and M. Hoffmann, "Goal-directed tactile exploration for body model learning through self-touch on a humanoid robot," *IEEE Transactions on Cognitive and Developmental Systems*, 2021.
- [12] A. Roncone, M. Hoffmann, U. Pattacini, and G. Metta, "Automatic kinematic chain calibration using artificial skin: Self-touch in the icub humanoid robot," in *2014 IEEE International Conference on Robotics and Automation (ICRA)*, IEEE, 2014, pp. 2305–2312.
- [13] V. Marcel, J. K. O'Regan, and M. Hoffmann, "Learning to reach to own body from spontaneous self-touch using a generative model," in *2022 IEEE International Conference on Development and Learning (ICDL)*, IEEE, 2022, pp. 328–335.
- [14] F. Mannella, V. G. Santucci, E. Somogyi, L. Jacquey, K. J. O'Regan, and G. Baldassarre, "Know your body through intrinsic goals," *Frontiers in neurorobotics*, p. 30, 2018.
- [15] S. Panzeri, F. Montani, G. Notaro, C. Magri, and R. S. Peterson, "Population coding," in *Analysis of Parallel Spike Trains*, S. Grün and S. Rotter, Eds. Boston, MA: Springer US, 2010, pp. 303–319, ISBN: 978-1-4419-5675-0. DOI: 10.1007/978-1-4419-5675-0\_14. [Online]. Available: [https://doi.org/10.1007/978-1-4419-5675-0\\_14](https://doi.org/10.1007/978-1-4419-5675-0_14).
- [16] Ö. Ekeberg, "A combined neuronal and mechanical model of fish swimming," *Biological cybernetics*, vol. 69, no. 5-6, pp. 363–374, 1993.
- [17] T. Wadden and Ö. Ekeberg, "A neuro-mechanical model of legged locomotion: Single leg control," *Biological cybernetics*, vol. 79, no. 2, pp. 161–173, 1998.

- [18] Y. Shim and P. Husbands, "Chaotic exploration and learning of locomotion behaviors," *Neural computation*, vol. 24, no. 8, pp. 2185–2222, 2012.
- [19] K. J. Friston, "Functional and effective connectivity: A review," *Brain connectivity*, vol. 1, no. 1, pp. 13–36, 2011.
- [20] A. M. Bastos and J.-M. Schoffelen, "A tutorial review of functional connectivity analysis methods and their interpretational pitfalls," *Frontiers in systems neuroscience*, vol. 9, p. 175, 2016.
- [21] R. Pfeifer, M. Lungarella, and F. Iida, "Self-organization, embodiment, and biologically inspired robotics," *science*, vol. 318, no. 5853, pp. 1088–1093, 2007.
- [22] F. Bonsignorio, "Entropy based metrics of sensory motor coordination: A short survey," *Metrics of Sensory Motor Coordination and Integration in Robots and Animals: How to Measure the Success of Bioinspired Solutions with Respect to their Natural Models, and Against More 'Artificial' Solutions?*, pp. 89–110, 2020.
- [23] —, "Quantifying the evolutionary self-structuring of embodied cognitive networks," *Artificial life*, vol. 19, no. 2, pp. 267–289, 2013.
- [24] R. Steuer, J. Kurths, C. O. Daub, J. Weise, and J. Selbig, "The mutual information: Detecting and evaluating dependencies between variables," *Bioinformatics*, vol. 18, no. suppl\_2, S231–S240, 2002.
- [25] T. M. Cover, *Elements of information theory*. John Wiley & Sons, 1999.
- [26] J. Walters-Williams and Y. Li, "Estimation of mutual information: A survey," in *International Conference on Rough Sets and Knowledge Technology*, Springer, 2009, pp. 389–396.
- [27] H. Peng, *Mutual information computation*, <https://www.mathworks.com/matlabcentral/fileexchange/14888-mutual-information-computation>, Accessed: 2023-04-20.
- [28] M. G. Preti, T. A. Bolton, and D. Van De Ville, "The dynamic functional connectome: State-of-the-art and perspectives," *Neuroimage*, vol. 160, pp. 41–54, 2017.
- [29] S. S. Menon and K. Krishnamurthy, "A comparison of static and dynamic functional connectivities for identifying subjects and biological sex using intrinsic individual brain connectivity," *Scientific reports*, vol. 9, no. 1, p. 5729, 2019.
- [30] X. Fu, K. Huang, N. D. Sidiropoulos, and W.-K. Ma, "Nonnegative matrix factorization for signal and data analytics: Identifiability, algorithms, and applications," *IEEE Signal Process. Mag.*, vol. 36, no. 2, pp. 59–80, 2019.
- [31] F. Wang, T. Li, X. Wang, S. Zhu, and C. Ding, "Community discovery using nonnegative matrix factorization," *Data Mining and Knowledge Discovery*, vol. 22, pp. 493–521, 2011.
- [32] X. Luo, Z. Liu, L. Jin, Y. Zhou, and M. Zhou, "Symmetric nonnegative matrix factorization-based community detection models and their convergence analysis," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 33, no. 3, pp. 1203–1215, 2021.
- [33] L. Muzzarelli, S. Weis, S. B. Eickhoff, and K. R. Patil, "Rank selection in non-negative matrix factorization: Systematic comparison and a new mad metric," in *2019 International Joint Conference on Neural Networks (IJCNN)*, IEEE, 2019, pp. 1–8.
- [34] H. Phalen, B. A. Coffman, A. Ghuman, E. Sejdić, and D. F. Salisbury, "Non-negative matrix factorization reveals resting-state cortical alpha network abnormalities in the first-episode schizophrenia spectrum," *Biological Psychiatry: Cognitive Neuroscience and Neuroimaging*, vol. 5, no. 10, pp. 961–970, 2020.
- [35] J. Stiso, M.-C. Corsi, J. M. Vettel, J. Garcia, F. Pasqualetti, F. D. V. Fallani, T. H. Lucas, and D. S. Bassett, "Learning in brain-computer interface control evidenced by joint decomposition of brain and behavior," *Journal of neural engineering*, vol. 17, no. 4, p. 046018, 2020.
- [36] Y. Wang, H. Huang, C. Rudin, and Y. Shaposhnik, "Understanding how dimension reduction tools work: An empirical approach to deciphering t-sne, umap, trimap, and pacmap for data visualization," *The Journal of Machine Learning Research*, vol. 22, no. 1, pp. 9129–9201, 2021.
- [37] H. Huang, Y. Wang, C. Rudin, and E. P. Browne, "Towards a comprehensive evaluation of dimension reduction methods for transcriptomic data visualization," *Communications biology*, vol. 5, no. 1, p. 719, 2022.