Dynamic sensorimotor graphs

Some Guy, Random Stranger and Anonymous Dude

Abstract— **Regularities present in the sensorimotor signals of a robotic agent can reflect its embodiment, as well as the associations resulting from the active control policy. In this work, we analyze the functional connectivity of the sensorimotor signals based on pairwise mutual information. As the robot performs exploratory motions based on motor babbling we capture and study the time-varying changes in the relationships. We provide analysis of the instantaneous and average sharing of information and extrapolate the meaning in relation to the physical properties of the robot's body. Results from a simulated planar system validate the use of mutual information as a tool not only for the analysis of the relationships between the sensorimotor signals but also to drive exploratory motions.**

I. Introduction

The study of the structural and functional connectivity in the brain has furthered the understanding of its organizations and information processing. An analogy with the sensorimotor signals of an embodied agent is yet to be realized. The distributed nature of the sensorimotor signals excludes the exploration of its structural connectivity, yet it readily lends itself to inquire about its functional connectivity and the information about body structure and the emergence of behaviors that can be extracted or even caused by it. A natural question to ask is how can information and graph theory be leveraged to devise a graphical representation of the sensorimotor contingencies in the somatosensory signals of agent. This work explores the sensorimotor structure of an embodied agent. The underlying premise is that regularities among the robot's sensorimotor signals are a product of the robot's embodiment. These regularities, known as *sensorimotor contingencies* (SMC) [1], can be identified by studying the relationships between the signals from its sensory apparatus (for both perception and actuation). Information-theoretic metrics are considered to study the relationships in the sensorimotor system. Some examples of previous works that have used information theory in this context include [2]–[6]. Although the literature about sensorimotor representations is extensive [7], this proposal considers that the representation of SMCs as *dynamic* graphs and their corresponding analysis aided by concepts from graph theory could provide further understanding about their formation and evolution. To achieve such a representation, methods from *network topology inference* (NTI)[8] are regarded to study the relationships among the different constituent elements of the graph.

As mentioned in [1], the connections between sensorimotor regularities and body knowledge are not well understood. To contribute to this understanding, this proposal parts from the knowledge of the number and modality of the robot's somatosensory signals (touch and proprioception) to use mutual information to explore the functional connectivity among sensorimotor signals and leverage graph theoretical concepts to identify and study the most important sensorimotor connections.

As the statistical properties of the signals and their relationships may vary depending on the motion policy driving the robot or the task being executed by it, it is essential that the estimated graph shows plasticity to reflect these effects (see Fig. **??**). Therefore, unlike conventional NTI, methods that enable the estimated graph to change and adapt dynamically will be explored.

A. Background

This work explores the sensorimotor structure of an embodied agent. The underlying premise is that regularities among the robot's sensorimotor signals are a product of the robot's embodiment. These regularities, known as *sensorimotor contingencies* (SMC) [1], can be identified by studying the relationships between the signals from its sensory apparatus (for both perception and actuation). Information-theoretic metrics are considered to study the relationships in the sensorimotor system. Some examples of previous works that have used information theory in this context include [2]–[6]. Although the literature about sensorimotor representations is extensive [7], this proposal considers that the representation of SMCs as *dynamic* graphs and their corresponding analysis aided by concepts from graph theory could provide further understanding about their formation and evolution. To achieve such a representation, methods from *network topology inference* (NTI)[8] are regarded to study the relationships among the different constituent elements of the graph.

Dynamical functional connectivity (DFC) has has mainly being used to study functional networks in the brain. For instance, in [9], it was used to show states of lower functional connectivity during onsets of epilepsy.

The work in [10] used NNMF on EEG signals to to identify potentially abnormal connectivity patterns in brains affected by illness.

1) Tactile exploration:

- The work by [11] discussed the usage of intrinsic motivation and goal-babbling for learning self-touch on a simulated humanoid robot with artificial sensitive skin.
- Self-touch for calibration was used in [12] letting the robot close the kinematic chain by touching its own body.
- [13] uses a multimodal variational autoencoder in a denoising framework to learn latent representation of self-touch that allows the agent to internally reconstruct

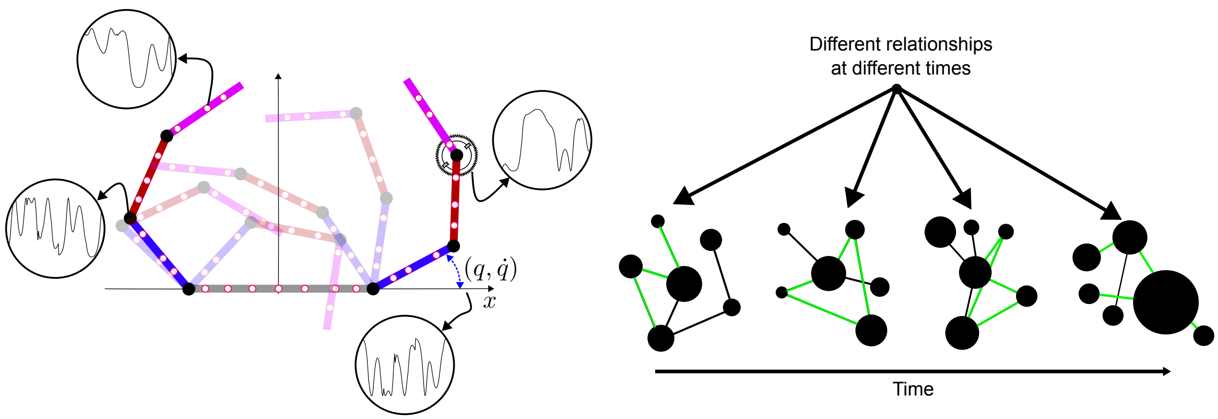

Fig. 1: General overview.

self-reaching configurations. The paper contributes to the sensorimotor contingency theory by providing a computational model that supports the hypothesis that the representation of self-touch can be learned simply through self-exploration.

B. Related works

Refer to Mannella, Gama, Marcel, Kanazawa, Husbands, Hoffmann (puppy).

C. Contributions

With the goal of finding as much information as possible about body properties, we present an analysis based on the dynamic relationships existing among the sensorimotor signals of a embodied agent. In the analysis we show how the state of information sharing in the system is related to the motion of the robot and the tactile events that stem from its movement. By using non-negative matrix factorization we identified different information sharing states and classify them according to their information content. In a case study we demonstrate how after a motor babbling phase, by only focusing on the mutual information and without having knowledge about the morphology of the robot, an excitation trajectory for both the robot arms can be devised that avoids self contact. A comparison of this trajectory against conventional trajectory design methods is presented.

II. The Embodied Agent

A. Robot and measurements models

This work takes as reference the robot model used in [13], [14], consisting of a simple six degrees-of-freedom planar dual arm system with three links per arm and a fixed torso, see Fig. 2. The robot is equipped with a set of tactile sensors distributed along the robot's body. They are modeled based on population coding [15] represented as Gaussian receptive fields (see Fig. 3), with the means of the receptive fields randomly located along the robot's one-dimensional body. We modified the tactile sensors were modified to account for the strength of touch. Essentially, the previously distance-based activation of the Gaussian receptive fields in now scaled by the contact force. Furthermore, the dynamics of

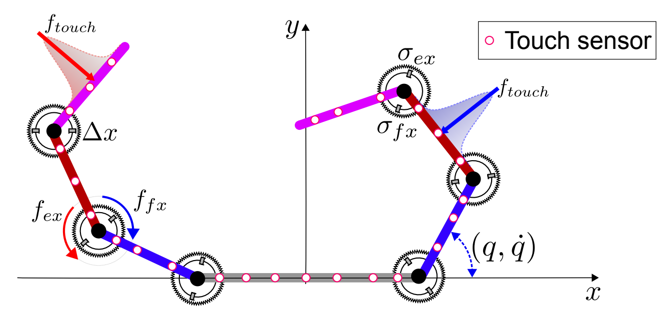

Fig. 2: The planar dual arm robot.

the model were instantiated by assigning inertial properties to the robot's composing bodies. Finally, we implemented the biology-inspired actuation model presented in [16]–[18], where each joint is driven by antagonistic muscles (modeled as springs) whose pulling force is linearly controlled by the signal of a corresponding motor neuron σ. The generated joint torque, expressed as

$$\tau = \alpha \left(\sigma_{fx} - \sigma_{ex}\right) + \beta \left(\sigma_{fx} + \sigma_{ex} + \gamma\right) q + \delta \dot{q}, \qquad (1)$$

results from the difference between the flexion σ_{fx} and extension σ_{ex} activation signals which create the pulling forces f_{fx} and f_{ex}. The parameters are:

- α: muscle force gain
- β: stiffness gain
- γ: tonic stiffness
- δ: damping coefficient

Similar to the tactile sensors, the robot's proprioceptive measurements are encoded using receptive fields. Therefore, the somatosensory measurements in the extended model consist of touch signals $\boldsymbol{r}$ (scaled by the touch force) and proprioception that that encompasses joint position $\boldsymbol{p}$, velocity $\boldsymbol{v}$, and effort $\boldsymbol{e}$.

In summary, the somatosensory signals vector $\boldsymbol{s}$ is composed as

$$\boldsymbol{s} = \begin{bmatrix} \boldsymbol{p}^T & \boldsymbol{v}^T & \boldsymbol{e}^T & \boldsymbol{r}^T \end{bmatrix}^T \qquad (2)$$

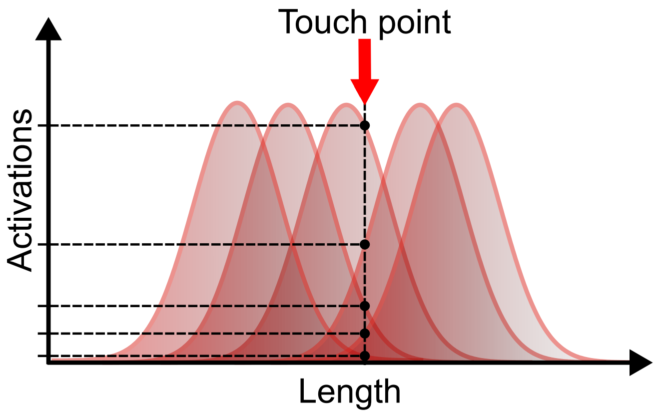

Fig. 3: The receptive fields used in population coding.

III. The sensorimotor dynamic functional connectivity

A. Functional connectivity

Functional connectivity (FC) is a method for network topology inference that characterizes the dependencies of the observed signals from a system based on their probability distributions[19]. It can be subdivided into undirected and directed; the latter being related to the analysis of statistical causation from the data [20]. By studying the FC it is possible to reveal a structure that aids in the analysis of the interaction among the entities.

Based on the connection between embodiment and information structure [21], our hypothesis is that properties of the sensorimotor interactions can be made apparent by studying the FC among the somatosensory signals $\boldsymbol{x}(t)$. From the various metrics that have been proposed to evaluate such relationships, we in particular leverage those based on information theory [22], [23], as their *model-free* nature can capture linear and nonlinear relationships between signals. Particularly, we use the *mutual information* (MI), a quantity that has been applied in different contexts to quantify the relationships between variables [24]. The MI between two signals $I(X,Y)$ can be interpreted as the amount by which a random signal Y reduces the uncertainty about a random signal X [25]. It is a symmetric measure of the information sharing by both signals and is computed as:

$$I(X;Y) = I(Y;X) = H(X) + H(Y) - H(X,Y) \tag{3}$$

with the Shannon's entropy of a variable X defined by

$$H(X) = -\sum_{i=1}^{n} p(x_i)\log_2\left(p\left(x_i\right)\right) \tag{4}$$

and the joint entropy between X and Y expressed as

$$H(X,Y) = -\sum_{i=1}^{n}\sum_{j=1}^{n} p(x_i,y_j)\log_2\left(p\left(x_i,y_j\right)\right). \tag{5}$$

By extension, the mutual information matrix $\boldsymbol{\mathcal{I}} \in \mathbb{R}^{m\times m}$ can be constructed by computing the pairwise MI between the $\left\{x_i\right\}_{i=1}^{m}$ somatosensory signals. Its (i,j) entries are given by

$$(\boldsymbol{\mathcal{I}})_{i,j} = I(x_i,x_j). \tag{6}$$

In practice, computing (6) for a pair $\left(x_i(t), x_j(t)\right)$ of time series from the data matrix $\boldsymbol{X}$, centering their samples (to zero mean and unit standard deviation) and using either binning, kernel, or nearest neighbor methods [26] to compute their mutual information.

In this work, for the computation of $\boldsymbol{\mathcal{I}}$ we use the the open-source MATLAB package *Mutual information computation*[27].

B. Dynamic functional connectivity

When analizing FC it might be interesting to look not only at the aggregated effect of a complete dataset of recordings but also at the instantaneous changes that occur in the relationships. Indeed, the relationships between sensorimotor signals might change rapidly depending on the motion policy. To capture these time-varying functional connectivity with mutual information, the most common approach involves the usage of a sliding time window [28].

In particular, as tactile events can occur in short time scale, the mutual information was computed for a time window of length T. For any two distinct pair of signals $s_x(t)$ and $s_y(t)$, the mutual information $I_t(s_x(t);s_y(t))$ at time t is computed in the interval $[t-T,t]$, see Fig. 4. We called this term the *instantaneous mutual information* (IMI). The forward step for the sliding window is Δt. The mutual information matrix $\boldsymbol{\mathcal{I}}(t)$ at time t is constructed by computing the IMI for all the pairwise signals. Given a data matrix of somatosensory signals a time series the time-varying mutual information matrix $\boldsymbol{\mathcal{I}}(t)$ defines a expresses the dynamical functional connectivity between the signals.

C. Analyzing patterns

Once the data set $\boldsymbol{D}_{MI}$ is available, it can be used to extract repeating patterns that encode certain modes of operation captured during motor babbling. It is also important to determine the expression of these patters during the recorded motion. Typical methods to detect repeating patterns in dynamic FC include the cosine similarity [29] and non-negative matrix factorization (NNMF) [30]. We chose the latter, as it proven useful in several studies analyzing brain dynamic functional networks and has also being used for community detection[31], [32] .

NNMF is an unsupervised machine learning algorithm that can be used to split an input matrix $\boldsymbol{V} \in \mathbb{R}_{\geq}^{n\times m}$ into two parts: a matrix $\boldsymbol{W} \in \mathbb{R}_{\geq}^{n\times k}$ of bases, or subgraphs, and a matrix $\boldsymbol{H} \in \mathbb{R}_{\geq}^{k\times m}$ of their corresponding contributions to the input matrix $\boldsymbol{V}$. As the matrix $\boldsymbol{\mathcal{I}}(t)$ is strictly non-negative, it is suitable for the analysis using NNMF. To achieve this, the matrix $\boldsymbol{V}$ is constructed by vectorizing and concatenating each of the matrices $\boldsymbol{\mathcal{I}}_v(t) = \mathrm{vec}\left(\boldsymbol{\mathcal{I}}(t)\right)$.

There are various methods to select an adequate number of factors [33]. To choose the number of factors we followed as in [34] and "which was chosen by performing NMF iteratively for increasing values of k and selecting the inflection

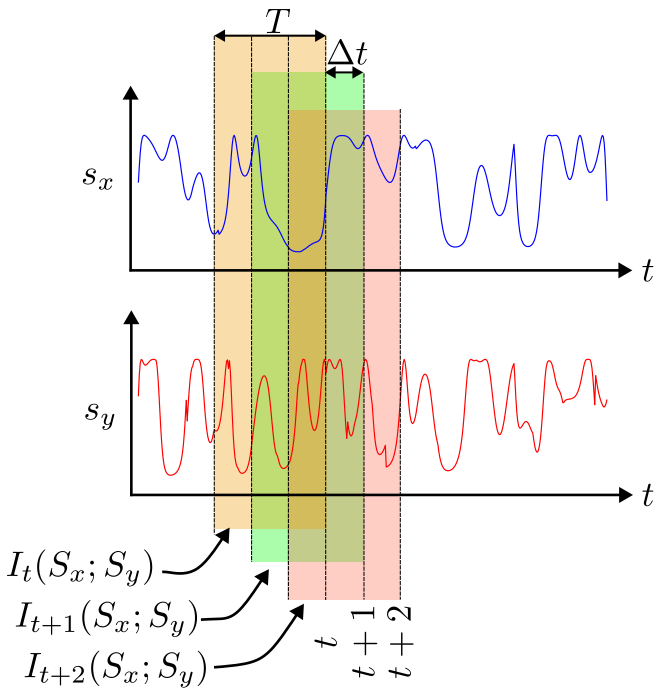

Fig. 4: Sliding window strategy to compute the instantaneous mutual information.

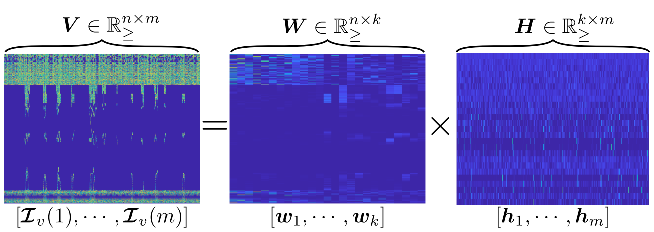

Fig. 5: Decomposition of the mutual information data matrix using non negative matrix factorization.

point, or elbow, in the reconstruction residual sum of squares error curve"

In [35] it is shown how the basis graph change their expressing during learning of a ask

To show graphically how the different factors cluster naturally depending on the touched regions we used the PaCMAP dimensionality reduction method [36] given its properties to preserve aspects of the global and local structure when reducing into the latent space [37].

IV. Simulation Results

TODOFor the computation of the instantaneous mutual information we used sensors signals sampled at 100 Hz and a sliding window of $T = 0.1$ seconds; i.e., the previously seen 10 samples. With this short memory which is stored in a buffer, we can capture (fast) touch events.

V. Case study: robot excitation trajectories

TODOIn this section we use the instantaneous mutual information to generate trajectories for the left and right arms avoiding potential collisions. This is done agnostic to the actual morphology of the robot. In contrast we use a standard method for the design of excitation trajectories and compare the results.

VI. Beyond robotics

TODO

VII. Conclusions

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