An analytical framework for the discovery of body features and behaviors from sensorimotor dynamic functional connectivity

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***Abstract*— Regularities present in the somatosensory signals of a robotic agent can reflect its embodiment and the associations resulting from an active control policy. In this work, we analyze the dynamic functional connectivity of the somatosensory signals based on the instantaneous pairwise mutual information. As the robot performs exploratory motions based on motor babbling, we capture and study the time-varying changes in the signal relationships. We analyze the instantaneous and average information sharing and associate them with different information states. A simulated planar system study shows that using instantaneous mutual information to extract and leverage the relationships between the agent's somatosensory signals exhibited during exploratory motions can yield information related to self-touch events.★ PENDING Needs to be re-worked**

I. Introduction

Understanding the brain's structural and functional connectivity (FC) has significantly advanced our knowledge of its organization and information-processing capabilities. A similar approach can be applied to studying the sensorimotor signals of an embodied agent, offering insights into how the agent processes information and adapts its behavior. While analyzing the structural connectivity of these signals may not always be feasible, investigating their functional connectivity provides a powerful way to understand an agent's body structure and the emergence of behavior through information acquisition. A key concept in this context is that of sensorimotor contingencies (SMC) [1], the regularities and dependencies between an organism's actions (motor actions) and the resulting sensory feedback linking an agent's sensorimotor signals to its physical embodiment and the world. Identifying and analyzing these regularities using information-theoretic methods can help uncover fundamental principles of sensorimotor learning.

A. Related works

Research suggests that SMCs play a fundamental role in acquiring body knowledge, generalization, and goal-directed behavior [1]. As such, they can be viewed as a form of sensorimotor representation—a framework that enables an embodied agent to learn, adapt, and interact with its environment. Despite extensive research on sensorimotor representations [2], the precise relationship between sensorimotor regularities, body knowledge, and behavior remains an open question.

Several studies have used information-theoretic metrics to examine these relationships in sensorimotor systems [3]–[7]. Among different sensory modalities, touch has been identified as particularly crucial in understanding SMCs. For example, Gama et al. [8] demonstrated how intrinsic motivation and goal-babbling can facilitate self-touch learning in a simulated humanoid robot with artificial tactile skin. Similarly, Roncone et al. [9] showed that self-touch could be used for kinematic calibration, allowing a robot to close its kinematic chain autonomously by touching its own body. Marcel et al. [10] further explored self-touch representation using a denoising framework with a multimodal variational autoencoder, enabling a robot to reconstruct its self-reaching configurations internally.

Another relevant area of study is dynamic functional connectivity (DFC), which explores how the statistical properties of sensorimotor signals evolve over time. Originally applied in neuroscience, DFC has been used to detect states of reduced functional connectivity during epilepsy onsets [11] and to identify abnormal connectivity patterns in brains affected by illness [12]. More recently, DFC has been used to investigate how sensorimotor connectivity evolves in simulated infants as they develop [13].

In robotics, functional connectivity is expected to change based on the robot's motion policy or the task it performs. Capturing and analyzing these evolving patterns using DFC could provide a deeper understanding of how an agent's sensory and motor systems interact over time.

B. Overview

This work introduces an analytical framework, illustrated in Fig. 1, to investigate the formation of relationships between the sensorimotor signals of a simulated two-dimensional embodied agent. Specifically, the framework examines the time-varying functional connectivity between the agent's proprioceptive, tactile, and visual inputs. During an initial exploratory phase using motor babbling, the agent gathers information about its evolving sensorimotor relationships. The framework models these changing relationships as time-varying graphs, serving as a proxy for understanding the formation and evolution of SMCs. To achieve this, we compute the instantaneous pairwise mutual information (MI), capturing the dynamic functional connectivity that correlates the agent's information-sharing state with its motion and potential interactions. A key step in the framework involves applying a Bayesian approach to uncover hidden structures within the mutual information-based connectivity. This analysis identifies clusters of highly related signals and their evolving interactions over time. Finally, the framework decomposes these dynamic relationships to extract meaningful patterns (i.e., subgraphs) in the graphs, which represent distinct information-sharing states. The insights derived from

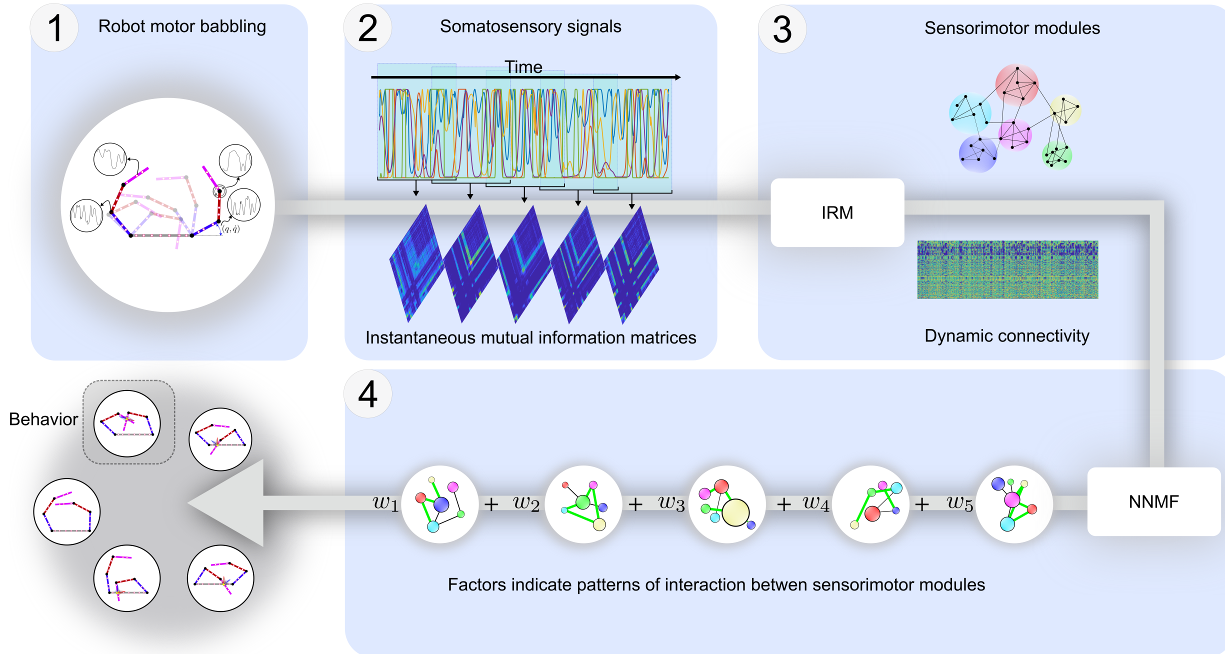

Fig. 1: **Analysis of the sensorimotor dynamic functional connectivity.** Motion excites the sensorimotor system of a robotic agent. While moving, the sensorimotor functional connectivity changes over time.

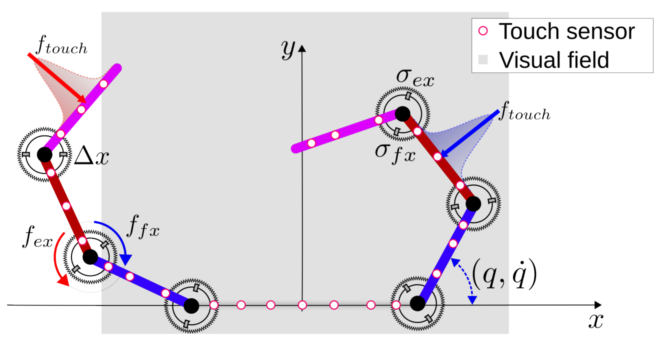

Fig. 2: **The embodied agent.** The planar dual arm robot with antagonistic actuation, tactile modules and vision.

these states and their transitions reveal fundamental aspects of the agent's body structure and behavior.

II. The Embodied Agent

A. The planar dual arm model

We employ the robot model described in [10], [14] as our reference system. This model consists of a simple planar dual-arm system with six degrees of freedom, featuring three links per arm and a fixed torso, see Fig. 2. The robot is equipped with tactile sensors distributed across its body. To instantiate the dynamics of the model, inertial properties are assigned to the robot's composing bodies. Its actuation mechanism is based on a biologically-inspired model presented in [15]–[17], where the position q and velocity $\dot{q}$ of each joint is driven by antagonistic muscles (modeled as spring-damper systems). The pulling force these muscles exert is linearly controlled by the signal generated by a corresponding motor neuron σ. The joint torque,

$$\tau = \alpha\left(\sigma_{\mathrm{fx}} - \sigma_{\mathrm{ex}}\right) + \beta\left(\sigma_{\mathrm{fx}} + \sigma_{\mathrm{ex}} + \gamma\right) q + \delta \dot{q}, \tag{1}$$

results from the difference between activation signals for flexion σ_{fx} and extension σ_{ex}. These activation signals contribute to the flexion and extension pulling forces, f_{fx} and f_{ex}, respectively. The remaining parameters in the model account for the muscle force gain (α), stiffness gain (β), tonic stiffness (γ), and damping coefficient (δ).

B. The sensory signals

Tactile sensors are randomly distributed along the robot's one-dimensional body and modeled using population coding [18]. Each sensor is represented by a Gaussian receptive field, whose mean is determined by the sensor's location—see Fig. 3. To incorporate touch strength, we modulate the activation of each receptive field based on the contact force, adjusting the distance-dependent response accordingly.

Similarly, the robot's proprioceptive measurements are also encoded using Gaussian receptive fields. In addition to somatosensation, the robot is equipped with visual inputs. The visual sensors consist of a fixed pixel receptive field with dimensions $(n_{\mathrm{x}}, n_{\mathrm{y}})$. When a limb segment intersects with a pixel in the visual field, the pixel value is set to one—i.e., pixels are sensitive to the positions of the agent's limbs. To account for spatial overlap in some visual sensors [19], we apply a convolution operation using a 3-by-3 kernel, to obtain the final pixel receptive field activations.

Ultimately, in our extended model, the visuo-somatosensory signal vector consists of N_{s} signals,

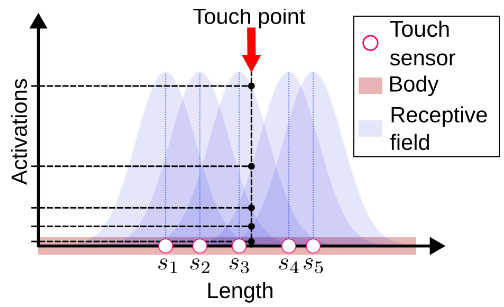

Fig. 3: **Population coding.** The receptive fields encode a signal into several distance-based activation functions.

including position-based proprioception ($\boldsymbol{p}$), touch-strength-modulated tactile sensation ($\boldsymbol{r}$), and visual inputs ($\boldsymbol{v}$):

$$\boldsymbol{s} = \left[\boldsymbol{p}^\top \quad \boldsymbol{r}^\top \quad \boldsymbol{v}^\top\right]^\top \in \mathbb{R}_{\geq}^{N_s}. \tag{2}$$

In stage 1 of our proposed analytical framework, the perceptual system of the embodied agent is stimulated to collect the signals $\boldsymbol{s}$ through a motor babbling strategy.

III. The Sensorimotor Dynamic Functional Connectivity

A. Functional connectivity

FC is a method for inferring network topology by characterizing the dependencies between observed signals based on their probability distributions [20]. Analyzing FC helps uncover underlying structures describing interactions between network entities. Building on the connection between embodiment and information structure [21], we hypothesize that an embodied agent's bodily structural properties and motor behaviors can be revealed by studying the FC among its sensorimotor signals $\boldsymbol{s}(t)$. To quantify these relationships, we leverage information-theoretic measures [22], [23], as their model-free nature captures both linear and nonlinear dependencies between signals. In particular, we focus on MI, a widely used measure for quantifying relationships between variables across different domains [24].

Given the Shannon's entropy of a variable X

$$H(X) = -\sum_{i=1}^{n} p(x_i)\log_2\left(p\left(x_i\right)\right) \tag{3}$$

and the joint entropy between two variables X and Y

$$H(X,Y) = -\sum_{i=1}^{n}\sum_{j=1}^{n} p(x_i,y_j)\log_2\left(p\left(x_i,y_j\right)\right), \tag{4}$$

the MI between two signals

$$I\left(X;Y\right) = I\left(Y;X\right) = H(X) + H(Y) - H(X,Y) \tag{5}$$

can be interpreted as the amount by which a random signal Y reduces the uncertainty about a random signal X [25]. It is a symmetric measure of the information sharing by both signals that depends on their marginal $p(\cdot)$ and joint $p(\cdot,\cdot)$ probability distributions.

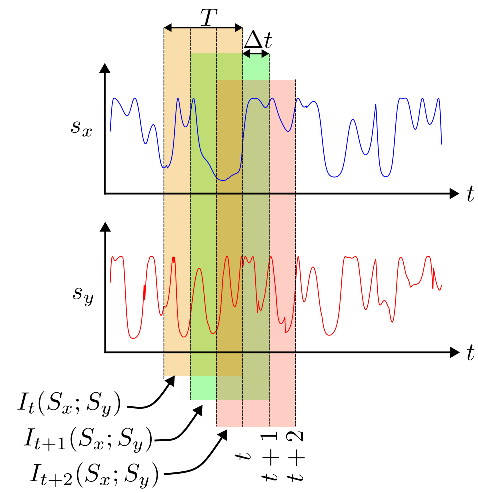

Fig. 4: **Instantaneous mutual information.** A sliding window strategy is used to compute the MI in an interval $[t-T,t]$.

By extension, the MI matrix $\boldsymbol{\mathcal{I}} \in \mathbb{R}^{N_s \times N_s}$ can be constructed by computing the pairwise MI between the $\left\{s_i\right\}_{i=1}^{N_s}$ sensorimotor signals. In practice, computing an entry

$$(\boldsymbol{\mathcal{I}})_{i,j} = I(s_i;s_j) \tag{6}$$

for a pair $\left(s_i(t),s_j(t)\right)$ involves centering their samples—to zero mean and unit standard deviation—and using either binning, kernel, or nearest neighbor methods to compute their MI [26]. In this work, we used a binning strategy to compute $\boldsymbol{\mathcal{I}}$ [1].

B. Dynamic functional connectivity

When analyzing FC, it might be interesting to look not only at the aggregated effect of a complete dataset of recordings but also at the instantaneous changes that occur in the relationships. Indeed, the functional relationships between sensorimotor signals can change rapidly depending on the motion policy and the agent's interaction with the environment. To capture this time-varying, i.e., dynamic, functional connectivity, it is common to use a sliding time window [28] with forward step Δt from which the MI is computed only for a small number of samples.

For a time window of length T, the MI $I_t(s_x(t);s_y(t))$ between a distinct pair of signals $s_x(t)$ and $s_y(t)$ at time t is computed using the set of signal samples spanning the $[t-T,t]$ interval[2], as illustrated in Fig. 4. We refer to this quantity as the instantaneous mutual information (IMI).

By extension, in stage 2 in Fig. 1, the MI matrix $\boldsymbol{\mathcal{I}}(t)$ at time t is constructed by calculating the IMI for all pairwise signals within the same time interval. The temporal evolution of this time-varying MI matrix $\boldsymbol{\mathcal{I}}(t)$ captures the DFC between the sensorimotor signals.

[1] We use the open-source MATLAB package *Mutual information computation* [27].

[2] In our particular case, the selection of a relatively short time window is motivated by the fact that tactile events often occur within a short timescale.

C. Finding structure in the functional relationships

Analyzing the time-varying relationships in $\mathcal{I}(t)$ at a local level may not be as informative as examining the global structure defined by interactions between groups of signals. To address this, we first identify clusters of closely related signals based on their MI values. However, conventional clustering techniques typically require a predetermined number of clusters. To overcome this limitation, in stage 3 of our framework, we employ a Bayesian approach—the infinite realtional model (IRM) [29]—to uncover hidden structures within the MI-based connectivity.

This probabilistic framework groups entities (e.g., signals) into N_c clusters, while simultaneously learning the relationships between them. Its primary objective is to group entities into clusters based on their relationships and predict new relationships based on these assignments. The IRM utilizes a Chinese Restaurant Process or a Dirichlet Process, allowing the number of clusters to dynamically expand as needed. By leveraging probabilistic methods, the IRM estimates the likelihood of entities belonging to the same cluster and models inter-cluster relationships using probability distributions.

For our purposes, the IRM taskes as an input the time series of IMI-matrices—representing N samples $\{\mathcal{I}(k)\}_{k=1}^{N}$—and assigns to the i-th signal in $\boldsymbol{s}$ a cluster z_i. To identify these latent cluster relationships, IRM assumes that the probability of a relation between two entities depends only on their cluster memberships. Specifically, if two entities belong to clusters z_i and z_j, the probability of a connection is determined by a parameter θ_{z_i,z_j}. These inter-cluster probabilities form a new bynary relational matrix $\boldsymbol{Z} \in \mathbb{R}^{N_\mathrm{c} \times N_\mathrm{s}}$, which the model infers from data. Additionally, the IRM produces a matrix $\boldsymbol{H} \in \mathbb{R}_{\geq}^{N_\mathrm{c} \times N_\mathrm{c} \times N}$, which captures the probability of relations between clusters.

D. Analyzing recurring patterns

The matrices $(\boldsymbol{Z}, \boldsymbol{H})$ in Sec. III-C capture the global changing relationships between the sensorimotor signals. In stage 4, the goal is to utilze the m relationships captured in $\boldsymbol{H}$ to identify recurrent patterns in the FC that corresnpond to distinct motor behaviors of the agent. Additionally, assessing the strenght and frequency of these patterns is crucial to determine their significance.

Common approaches to detect repeating patterns in DFC include the cosine similarity [30], k-means clustering [31], and non-negative matrix factorization (NNMF) [32]. We selected the latter, due to its demonstrated effectiveness in analyzing brain dynamic functional networks and its application in community detection [33], [34].

To use NNMF, a matrix $\bar{\boldsymbol{H}} \in \mathbb{R}^{N_\mathrm{r} \times N}$ is constructed by vectorizing and concatenating each of the matrices in $\bar{\boldsymbol{H}}$; where $N_\mathrm{r} = N_\mathrm{c}(N_\mathrm{c} - 1)/2$ is the total number of pairwise relationships. Since this matrix is strictly non-negative, it is amenable to be decomposed and analyzed using NNMF. Then, the NNMF algorithm can be used to split the non-negative matrix $\bar{\boldsymbol{H}}$ into two parts: a matrix $\boldsymbol{F} \in \mathbb{R}_{\geq}^{N_\mathrm{f} \times N_\mathrm{r}}$ of N_f baseis (or factors) and a matrix $\boldsymbol{W} \in \mathbb{R}_{\geq}^{N \times N_\mathrm{f}}$ of their corresponding contributions such that

$$\bar{\boldsymbol{H}}^\intercal \approx \boldsymbol{W}\boldsymbol{F}. \tag{7}$$

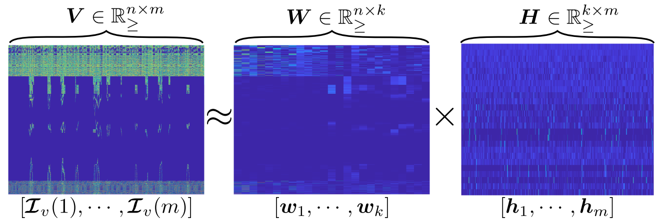

Fig. 5: **Non-negative matrix factorization.** Decomposition of the IMI data matrix $\boldsymbol{S}$ using NNMF helps identifying recurring patterns.★ **PENDING Replace the matrices with actual data**

One crucial question is the number of factors k used to approximate the original dataset. From the various methods to select an adequate number [35], we chose k following the elbow method as in [36] by performing NNMF for ascending values of k and selecting the value where the residual error is not reduced any further.

Each of the N_f rows of $\boldsymbol{F}$ can be interpreted as a basis FC graph capturing a state of information sharing between the N_c sensorimotor clusters. The factors can also be interpreted as components of body interaction or as proxis that encode SMCs. When the factors are aggregated using via the entries of $\boldsymbol{W}$, the particular realtional matrix at time t is closely approximated.

IV. Simulation Results

A. Exploratory phase

To compute the instantaneous mutual information, we used sensor signals sampled at 100 Hz and a sliding window of $T = 0.1$ seconds, i.e., the previously seen 10 samples. This short memory stored in a buffer can capture (fast) touch events. Excitatory signals: talk about them.

B. Sensorimotor modules

The output of the IRM are modules, corresponding to a group of sensors that have been regrouped by the IRM.

C. Factors NMMF

D. Tactile representation of Scores

V. Beyond Robotics

TODO★ **PENDING Valentin + Matej**

VI. Conclusions

VII. Discussion and Future Work

A. Limitations

One aspect not addressed is improving the motion policy used to collect the sensorimotor signals. The current study used motor babbling. However, better, perhaps goal-directed, or active exploration alternatives might lead to better results that are able to capture and reproduce particular sensorimotor behaviors.

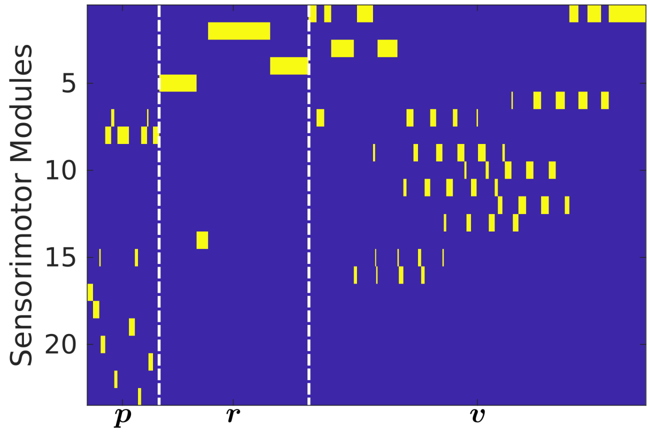

Fig. 6: . **Extracted modules from IRM.** Yellow-colored entries represent the sensors that corresponds to each extracted module (rows). From left to right, the sensory inputs are proprioceptive, tactile and visual.

The current implementation of our framework requires collecting and processing information offline. A method capable of doing online learning with the potential to adapt, for example, the clusters or the factors, is still an open question.

It is possible to add factors.

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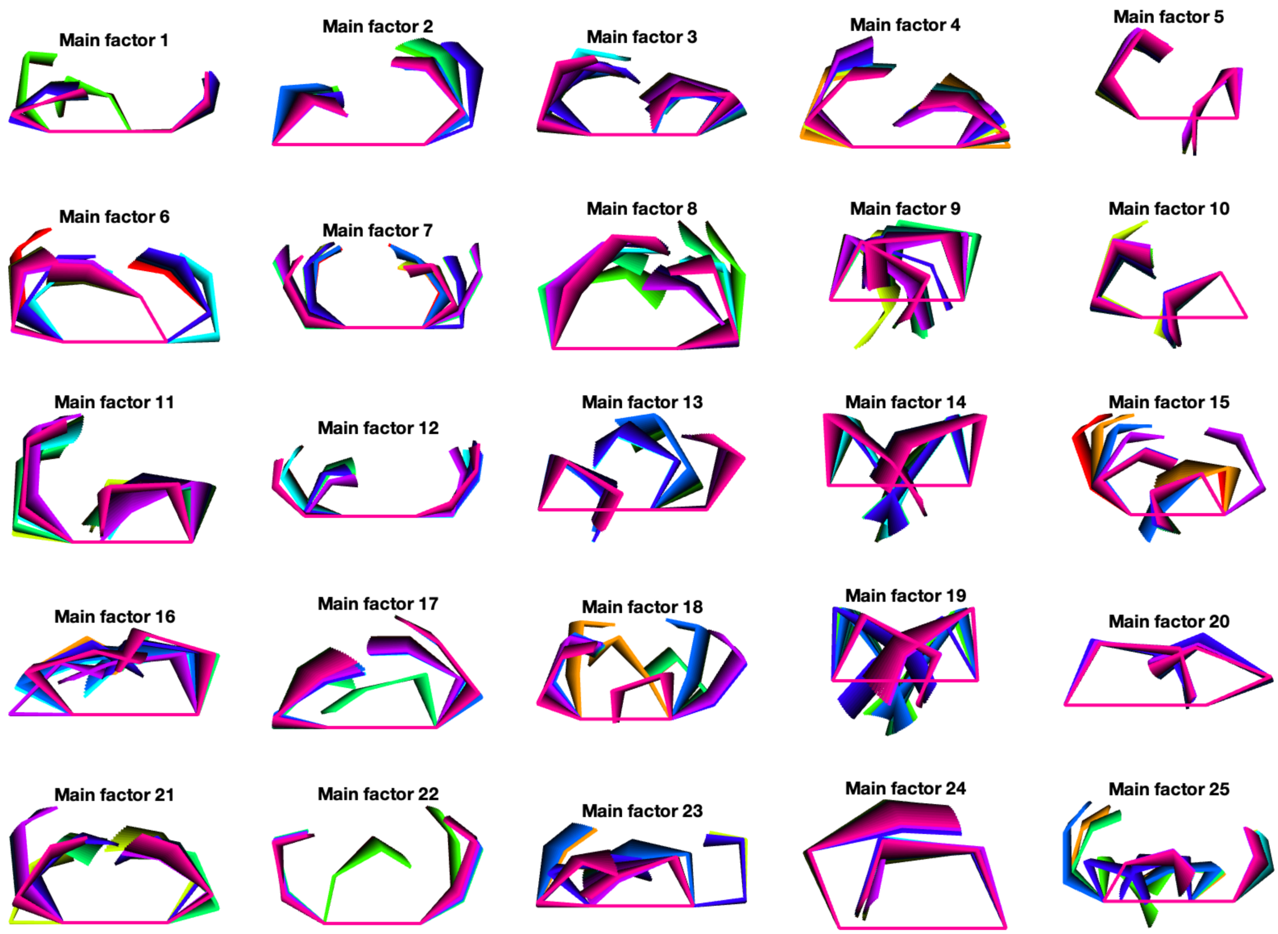

Fig. 7: **The factors and associated behaviors.** Different events during exploration link to a given factor; for example, pure proprioception (no touch), contact with left arm, right arm, and both arms.

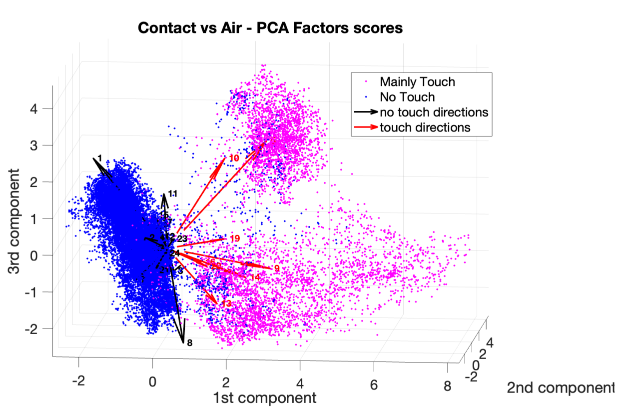

Fig. 8: .

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