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Comparing policies for the stochastic multi-period dual sourcing problem from a supply chain perspective

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Abstract

We study a supply chain that consists of a buyer and two suppliers. The buyer faces stochastic demand and has two different supply sources for the same product: a slower regular supplier and a more expensive expedited supplier. Such dual sourcing inventory systems have been widely studied in the literature, evaluating what is best for the buyer. As the decisions of the buyer may adversely affect the costs of the suppliers, and also, potentially, the supply chain profit, we adopt the perspective of the entire supply chain. We compare the performance of two different policies in a multi-period and two-echelon setting: the Dynamic Order Policy (DOP) and the Standing Order Policy (SOP). In the long run, the DOP policy converges to the Dual-Index Policy (DIP), which is optimal for the buyer in the case of a lead time difference of one period. On the other hand, the SOP policy is appealing from a practical perspective as a fixed (standing) quantity is ordered from the regular supplier in each period. We show that from our supply chain perspective, the DOP policy is not necessarily the better performing policy, and we define the conditions for the preferred policy. We evaluate the policies numerically under various conditions to obtain relevant managerial insights. We find that the preferred policy from a supply chain perspective is mainly the result of a trade-off between responsiveness, flexibility, and cost. Flexibility appears to be valuable if there is a substantial cost difference between the suppliers.

Keywords: *inventory management; dual sourcing; supply chain; policy evaluation; policy comparison*

1   Introduction

Many inventory models assume that there is only one source or transportation mode to procure, produce, or ship products. However, in practice, inventories can be replenished from more than one source or via different transportation modes. The case of two different sources or transportation modes is referred to as a dual sourcing system, and has become a popular strategy in many supply chains due to offshore production, outsourcing, and different transportation modes. The benefits of offshore sourcing include lower costs and better availability of specific workforce. However, it reduces flexibility and responsiveness in the sense that the replenishment lead times become longer. By contrast, onshore (or domestic) sourcing is faster but incurs higher costs. The resulting dual sourcing setting, that is, the trade-off between utilizing offshore production and an onshore source is the motivation for this study. In the remainder of this paper, we refer to offshore production as the *regular supplier* and to the onshore source as the *expedited supplier*.

Several studies have reported how dual sourcing strategies are beneficial, especially compared to a single sourcing strategy. For example, Tagaras and Vlachos (2001) analyze a dual sourcing inventory system and demonstrate that including a second source in the system leads to substantial cost savings. Du et al. (2006) indicate that dual sourcing can improve a firm's bargaining position. Burke et al. (2007) find that single sourcing strategies are only dominant in very exceptional cases, such as when the supplier capacities are ample and the buyer does not obtain diversification benefits. Otherwise, in all other studied cases, a dual sourcing strategy has been proven to be dominant. Hence, firms have several good reasons to adopt a strategy in which two (or more) different suppliers are used to supply products instead of relying on a single source or transportation mode (Inderst, 2008).

When adopting a dual sourcing strategy, the majority of the products would typically be supplied by the regular supplier, which usually has a longer lead time, and if needed, a portion would be supplied by the expedited supplier. Such dual sourcing practices are widespread in many industries. Rao et al. (2000) reported on Caterpillar's dual sourcing strategy for compact work tools, Beyer and Ward (2002) on Hewlett-Packard's dual sourcing strategy for its manufacturing plants, and Allon and Van Mieghem (2010) on a US-based \$10 billion manufacturer of wireless transmission components with two assembly plants: one in China and another in Mexico. Many companies have come to realize that a sourcing strategy with two (or more simultaneous) suppliers is more effective than one that relies on a single supplier. However, in contrast to most single sourcing inventory models, in which the optimal control policies are known (Zipkin (2000)), optimal policies for dual sourcing inventory systems are unknown or have a very complex structure (Sheopuri et al. (2010)). Therefore, several simpler heuristics have been proposed in the literature, such as the single-index (Zhou and Yang (2016)), the dual-index (Veeraraghavan and Scheller-Wolf (2008)), and the standing-order policies (Janssen and De Kok (1999)).

We study a single-product, multi-period, two-echelon stochastic periodic-review inventory system with one buyer and two suppliers. We assume a lead time difference of one time period. The buyer faces stochastic demand and can source the same product from two different suppliers: a regular supplier and an expedited one. The regular supplier is slower but offers the product at a lower price, whereas the expedited supplier is faster but offers the product at a higher price. The objective of our study is to compare the performance of two different policies: the Dynamic Order Policy (DOP) and the Standing Order Policy (SOP). The DOP policy has two basestock levels (control parameters), one for each of the two suppliers. In the long-run, the two control parameters become time-independent, and the DOP is referred to as the Dual-Index Policy (DIP). In a dual sourcing setting, considering the sourcing and inventory costs of the buyer, the DIP policy is optimal if the lead time difference between the two suppliers is exactly one period (Barankin (1961); Fukuda (1964)). The same policy becomes close to optimal if the lead time difference is larger than one period (Veeraraghavan and Scheller-Wolf, 2008). By contrast, the SOP policy is appealing from a practical perspective, as a constant quantity is ordered from the regular supplier in each time period, while the expedited supplier is utilized for emergency orders. Note that this policy is asymptotically optimal for infinite lead time differences.

We compare the performance of these two policies from *a supply chain perspective*. Although it is known that the DIP policy outperforms the SOP policy from the buyer's perspective (in case of a lead time difference of one period), we prove that this does not necessarily hold for the entire supply chain in a dual sourcing setting. Since the DIP policy employs two basestock levels, DIP can vary the order quantities to both suppliers, which SOP cannot. Therefore, one would expect that this lack of flexibility puts the SOP policy at a disadvantage and would lead to higher cost and lower profit for the supply chain. We prove that this is not always the case and that the SOP policy can outperform the DIP policy from a supply chain perspective, dependent on the cost structure and the model parameters, which can result in different levels of responsiveness, flexibility and cost. For example, when the wholesale price difference between the two suppliers is limited, the SOP policy typically outperforms DIP due to the fact that under SOP, a larger portion of the volume goes to the expedited supplier. That results in an increased responsiveness, that is, lower (expensive) penalty cost at a smaller additional wholesale price. The results of our study have strong implications and are relevant for practitioners that manage dual sourcing supply chains.

The contribution of this study is threefold. First, in contrast to the vast majority of the dual sourcing literature, we study the dual sourcing problem from a supply chain perspective by explicitly modeling the impact of the buyer's decisions on the suppliers' and the supply chain performance. The literature review of Svoboda et al. (2017) concludes that multi-echelon studies in the dual sourcing literature are still largely an open field. They state that 'the supplier's costs and decision making have hardly been considered in operational inventory control thus far'. Second, for the three-period version of our model, we present exact expressions for the optimal control parameters for both policies. Third, we reveal under which conditions each of the two policies is preferred from a supply chain perspective. Moreover, our numerical study reveals several interesting

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managerial insights.

The remainder of this paper is organized as follows. In section 2, we discuss the literature review. Then, in section 3, we present the model assumptions, model formulation and the mathematical analysis of both policies for the three-period version of the model. In section 4, we analytically compare the two policies. We present the results of our numerical study (for the multi-period version of our model) in section 5. Finally, in section 6, we draw the main conclusions and discuss ideas for future research.

2   Literature review

The dual sourcing inventory problem has been widely studied in the literature. Generally, the literature on dual sourcing inventory management can be divided into periodic versus continuous-review models. We find that for continuous and periodic-review systems with nonconsecutive lead times, most studies focus on evaluating and optimizing heuristic policies, as the general optimal policy is unknown. Regarding heuristic policies, the literature distinguishes between single- and dual-index policies, depending on whether one or two inventory positions are tracked, and between single- and dual basestock policies, depending on the number of basestock levels used by the proposed policy. Moreover, most dual sourcing models that assume deterministic lead times consider an inventory system with two suppliers with different unit costs and different lead times. The supplier with the shortest lead time has the highest unit cost. Due to the complexity of the dual sourcing problem, most models are restricted to zero setup costs, a lead time difference of one period, deterministic lead times, and are mostly further reduced to a single-period problem. We refer the reader to Minner (2003) and Svoboda et al. (2017) who provide excellent reviews of the literature on inventory models with multiple supply options and their contribution to supply chain management.

To the best of our knowledge, Barankin (1961) was the first contribution to the literature on dual sourcing inventory management. The author examines a single-period inventory model with emergency orders and finds that the optimal policy consists of two basestock levels when the lead times of the two sources are zero and one. Fukuda (1964) extends this work and studies a periodic-review inventory model with stochastic demand. The study derives optimal policies for the setting in which an order can be replenished from a supplier at a unit price c_k with lead time k or at a unit price c_{k+1} with lead time $k + 1$. For a lead time difference of one period, this study proves that the single-index, dual-basestock policy is optimal. The dual-basestock policy specifies two target basestock levels, one for each supplier. This implies that the regular and expedited supplier each have their own optimal target basestock level.

For larger differences in the lead times between the two suppliers, Whittemore and Saunders (1977) find that the optimal policy has a complex structure. If the lead time difference is more than one period, the optimal policy is no longer a function of one or two inventory positions. In this case, the optimal order decisions are based on all outstanding orders during the lead time difference, requiring a multi-dimensional modeling approach to determine the optimal parameters. This makes the curse of dimensionality becoming a significant

hurdle and makes it difficult to find the optimal policy. The authors also derive conditions under which only one supplier is used in the infinite horizon case. Li and Yu (2014) characterize the monotonicity of the optimal order quantities over the outstanding orders during the lead time difference.

For the case of larger lead time differences, Veeraraghavan and Scheller-Wolf (2008) introduce the Dual-Index Policy (DIP) for capacitated dual sourcing models. This policy implies that there are two basestock levels for each of the two suppliers. If the inventory position is below the lowest basestock level, then the difference is ordered from the fast supplier, and the inventory position is then raised to the higher basestock level by ordering from the slow supplier. The optimal policy parameters are computed using a simulation-based optimization procedure. The study reveals that DIP performs close to optimal compared to all the state-dependent policies found via dynamic programming. Sheopuri et al. (2010) extend the work of Veeraraghavan and Scheller-Wolf (2008) by developing two classes of policies: policies that have a basestock level structure for the expedited supplier and those that have an basestock level structure for the regular supplier. The proposed policies (vector base-stock, weighted dual index, and demand allocation policies) sometimes outperform DIP. Song and Zipkin (2009) and Arts et al. (2011) extend the DIP by considering stochastic lead times. Feng et al. (2006) find that when the number of sources is more than two, only the fastest two sources have optimal basestock levels. Exact and optimal solutions are usually not achieved for the DIP in the general case. Recently, Sun and Van Mieghem (2019) extend DIP by proposing a cap (upper limit) on the order to the regular supplier and prove the robust optimality of this policy.

Another relevant stream in the literature are the papers that study the Standing Order Policy (SOP). The SOP policy suggests that in each period, a constant and fixed quantity is ordered from the regular supplier, while a variable quantity is ordered from the expedited supplier. This is an example of a single-index policy and it combines a push system (fixed quantity) with a pull system (variable quantity). From a practical perspective, it is an attractive policy as it allows the regular supplier to focus on cost efficiency, whereas the expedited supplier is utilized to achieve a high service level. To the best of our knowledge, Rosenshine and Obee (1976) were the first to study SOP. While the regular supplier supplies a constant quantity in each period, the inventory position is tracked. Once the inventory drops below a critical level, an emergency order with zero lead time is released, which increases the inventory level to the base-stock level. Janssen and De Kok (1999) analyze a similar model. The replenishment decisions for the expedited supplier are made based on an (R, S) policy and they present an algorithm to determine the optimal policy parameters such that costs are minimized given a certain service level constraint. The study also reveals that the coefficient of variation of the demand is the determining factor for the optimal standing order quantity. Chiang (2007) derives the optimal parameters for such a system, given a predetermined standing order quantity. Allon and Van Mieghem (2010) propose the Tailored Base-Surge (TBS) policy for the dual sourcing problem. The policy suggests being replenished at a constant rate from the slow supplier and only ordering from the fast supplier if the inventory level is below a critical level. They present a simple square root formula to predict the near-optimal allocation and cost.

Janakiraman et al. (2014) extend this work and demonstrate that the cost difference between the TBS policy and the optimal policy decreases as the lead time of the regular supplier increases. Xin and Goldberg (2018) prove that the TBS policy is asymptotically optimal in unconstrained dual sourcing systems. Klosterhalfen et al. (2011) compare the SOP with the DIP policy. Based on a numerical study, they find that SOP delivers satisfactory results, and sometimes even better ones than DIP, especially when the lead time difference is large and the level of demand uncertainty is high, which has been confirmed by the results of Janakiraman et al. (2014) and Xin and Goldberg (2018).

There are several interesting extensions to the "standard" dual sourcing model. Yazlali and Erhun (2009) consider the dual sourcing case in which both suppliers place per period minimum and maximum capacity limits on the order quantities. Sting and Huchzermeier (2012) consider a dual sourcing system in which one source is considered unreliable, but has a high-margin, while the other source is reliable, but having a low-margin. Zhang et al. (2012) consider a dual sourcing system in which one supplier has a high variable cost but low fixed cost, whereas the other supplier has a low variable cost but high fixed cost as well as an order constraint. Janakiraman and Seshadri (2017) consider a dual sourcing model with the option of costless return to the cheaper, longer lead time supplier. Tan et al. (2016) consider the case in which both suppliers have uncertain capacities and different levels of reliability. Jakšič and Fransoo (2018) model the trade-off between uncertain supply availability of one source and the lead time. Zhu (2015) and Silbermayr and Minner (2016) consider a dual sourcing model with disruptions. Several other studies consider demand forecast updates in dual sourcing systems (Sethi et al. (2001),Sethi et al. (2003),Cheaitou et al. (2014)).

Another relevant extension is the order expediting in multi-echelon systems in which orders can be expedited (after they have been placed) or new emergency orders can be placed if current orders are delayed. Aggarwal and Moinzadeh (1994) examine expediting orders for retailers in a multi-echelon distribution system. Lawson and Porteus (2000) study a serial multi-echelon inventory system with the option to expedite orders between stages in the supply chain. The lead times between the different stages have been assumed to be one time period, but the decision maker has the option to expedite orders and obtain them immediately from the upstream stage or stop outstanding orders. Song et al. (2016) study a three-echelon, serial system with stochastic lead times. The middle echelon can be skipped at extra cost. The optimal policies have been derived by constructing an equivalent queueing system. Sapra (2017) also examines a serial, multi-echelon system in which the regular supplier supplies the most upstream stage, while the expedited supplier ships directly to the downstream stage. Höller et al. (2019) determines the optimal parameters for expediting policies under service level constraints. Although this stream of research is related to our paper, the main difference is that the suppliers' costs have not been taken into consideration.

Our main contribution to the dual sourcing inventory management literature is that, in opposite to the vast majority of the literature, we study the stochastic dual sourcing inventory management problem from a supply chain perspective by explicitly taking into consideration the impact of the buyer's decisions on both suppliers

and on the supply chain as a whole. By doing so, we find that the DIP policy is not necessarily optimal anymore. Also, for both policies that we analyze, we present an exact analysis and closed form expressions for the optimal policy parameters for the three-period model. In addition, we show under which conditions each of the two analyzed policies is the preferred policy from a supply chain perspective. Lastly, we conduct a numerical study with the multi-period version of our model that allows us to obtain several interesting managerial insights. One of the main insights is that the preferred policy from a supply chain perspective is the result of a different trade-off, namely the trade-off between the value of flexibility and cost; contrary to what would be the case if only the profit of the buyer is concerned. Flexibility appears to be only valuable if there is a substantial cost difference between the suppliers.

3 Model formulation and analysis

In this section, we first present our main modeling assumptions in section 3.1. Then, the mathematical formulation of the buyer's and the two suppliers', and the total supply chain model are presented for both policies in sections 3.2 and 3.3.

3.1 Model assumptions

The notations used in this study are defined below:

n	Number of remaining periods in the planning horizon ($1 \leq n \leq N$)
t	Period index in the planning horizon ($t = N - n + 1$)
Q_n^e	Order quantity from the expedited supplier under the DOP policy in n
Q_n^r	Order quantity from the regular supplier under DOP policy in n
$\widetilde{Q}_n^e$	Order quantity from the expedited supplier under the SOP policy in n
$\widetilde{Q}^r$	Standing order quantity from the regular supplier under the SOP policy
Y_n^e	Basestock level from the expedited supplier under the DOP policy in n
Y_n^r	Basestock level from the regular supplier under the DOP policy in n
$\widetilde{Y}_n^e$	Basestock level under the SOP policy in n
D_t	Stochastic demand faced by the buyer in period t
f	Probability density function of the demand
F	Cumulative distribution function of the demand
μ	Mean customer demand
$CV(D_t)$	Coefficient of variation of the demand (faced by the buyer)
x_n	On-hand inventory level of the buyer in n
h	Unit inventory holding cost
b	Unit backorder (penalty) cost
p	Unit selling price
w_e, w_r	Unit wholesale cost of the buyer from the expedited e and regular r suppliers
Δw	The wholesale price difference between the two suppliers ($\Delta w = w_e - w_r$)
c_e, c_r	Unit manufacturing (or procurement) cost of the expedited e and regular r suppliers
$\pi_n^B, \pi_n^e, \pi_n^r$	Expected profit of the buyer, expedited and regular supplier under the DOP policy
$\widetilde{\pi}_n^B, \widetilde{\pi}_n^e, \widetilde{\pi}_n^r$	Expected profit of the buyer, expedited and regular supplier under the SOP policy

We consider a single-product, three-period ($N = 3$), stochastic dual sourcing inventory management problem with one buyer and two suppliers: the regular (r) supplier and the expedited (e) supplier. Modeling this problem

as a three-period model allows us to conduct an exact analysis and to present closed-form expressions for the optimal policy parameters.

In each period t, the buyer faces stochastic demand D_t with the demands of different periods being stationary, independent and identically distributed (i.i.d.), and non-negative random variables. The buyer gets the same product replenished from two different suppliers: the expedited supplier e and the regular supplier r. Supplier e is considered to be the fast and expensive supplier, whereas supplier r is the slow and cheap supplier. Supplier e is a fast supplier with a replenishment lead time assumed to be equal to 0, while supplier r is located remote from the buyer and its replenishment lead time equals to 1. A unit inventory holding cost h is incurred for each unit of inventory at the end of a period and a backorder (penalty) cost of b is incurred for each unit of demand that is not satisfied. Except for the last period, unsatisfied demand is backordered. We also assume that the salvage value of the product at the end of the horizon is zero. The buyer sells the product at a unit price of p and buys them from the expedited and regular suppliers at wholesale prices of w_e and w_r respectively. Suppliers e and r produce (or procure) the product at a unit cost price of c_e and c_r respectively. We assume that $p > w_e > w_r$, $h < b$, and that decisions are made under uncertainty, that is, Q_n^e, Q_n^r, $\widetilde{Q}_n^e$ and $\widetilde{Q}^r$ are decided upon before demand is revealed (make-to-stock policy). Note that all decisions are made solely by the buyer, but we evaluate the impact of these decisions on the whole supply chain. The exact order of events is discussed in the next section. Furthermore, the two suppliers are assumed to decide on their production quantities after Q_n^e, Q_n^r, $\widetilde{Q}_n^e$ and $\widetilde{Q}^r$ are revealed (make-to-order policy). We use profit as performance measure in this paper. Without loss of generality, we assume that the initial stock $x_N = 0$. There is no structural change if we set this parameter differently.

The buyer's expected profit function in each period is equal to the expected revenue minus the expected holding and shortage cost minus the purchase cost plus the expected future profit. The suppliers' expected profit is the unit profit margin multiplied by the (expected) order quantities from the buyer. Consequently, the total supply chain profit is the sum of the buyer's profit and the two suppliers' profits. While the model optimizes only the decisions of the buyer, in this paper, we evaluate those decisions' impact on the total supply chain profit. We explicitly do not optimize for the supply chain profit in any of the decisions.

3.2 Dynamic Order Policy

In this subsection, we present the order of events and overall expected profit function for the buyer (section 3.2.1), the suppliers (section 3.2.2), and the total supply chain (section 3.2.3) under the DOP policy.

3.2.1 The buyer's problem

The buyer aims at maximizing his expected profit and needs to take decisions at the beginning of each period t. These decisions are made under uncertainty and based on the actual state of the system, which is the starting inventory level. Let $L(x)$ be the expected holding and shortage cost of a period given a starting on-hand

inventory x:

$$L(x) = h \int_0^x (x - t)f(t)dt + b \int_x^\infty (t - x)f(t)dt.$$

It follows that $L'(x) = (b + h)F(x) - b$.

For $n = 1$, the buyer needs to decide on the quantity to order from the expedited supplier before D_3 is revealed. Since this is the last period of the horizon, there is no order from the regular supplier at the beginning of the last period. Consequently, the expected profit of the buyer is given by:

$$\pi_1^B(x_1) = \max_{Y_1^e \geq x_1} p\mu - w_e(Y_1^e - x_1) - L_1(Y_1^e)$$

where $L_1(x) = h \int_0^x (x - t)f(t)dt + (p + b) \int_x^\infty (t - x)f(t)dt$. It easily follows that:

$$\frac{\partial \pi_1^B}{\partial Y_1^e} = -w_e - L_1'(Y_1^e) = -(p + b + h)F(Y_1^e) + p + b - w_e.$$

Solving $\frac{\partial \pi_1^B}{\partial Y_1^e} = 0$ yields $Y_1^{e^*} = F^{-1}\left(\frac{p + b - w_e}{p + b + h}\right)$.

For $n = 2$, following a dynamic programming approach, the buyer decides on Y_2^e and Y_2^r (before D_2 is revealed) such that the expected profit is maximized and subsequent decisions are made optimally. That is, the expected profit of the buyer is given by:

$$\pi_2^B(x_2) = \max_{Y_2^r \geq Y_2^e \geq x_2} p\mu - \Delta w Y_2^e + w_e x_2 - w_r Y_2^r - L(Y_2^e) + \int_0^\infty \pi_1^B(Y_2^r - t)f(t)dt.$$

Note that $\int_0^\infty \pi_1^B(y - t)f(t)dt = p\mu - \int_0^{y - Y_1^{e^*}} L_1(y - t)f(t)dt - w_e \int_{y - Y_1^{e^*}}^\infty (Y_1^{e^*} - y + t)f(t)dt - L_1(Y_1^{e^*})(1 - F(y - Y_1^{e^*}))$. Therefore, the expected profit $\pi_2^B(x_2)$ is expressed as:

$$\pi_2^B(x_2) = \max_{Y_2^r \geq Y_2^e \geq x_2} 2p\mu - \Delta w Y_2^e + w_e x_2 - w_r Y_2^r - L(Y_2^e) - \int_0^{Y_2^r - Y_1^{e^*}} L_1(Y_2^r - t)f(t)dt$$
$$-w_e \int_{Y_2^r - Y_1^{e^*}}^\infty (Y_1^{e^*} - Y_2^r + t)f(t)dt - L_1(Y_1^{e^*})(1 - F(Y_2^r - Y_1^{e^*})).$$

Taking the derivative with respect to Y_2^e and Y_2^r, it can be seen that:

$$\frac{\partial \pi_2^B}{\partial Y_2^e} = -(b + h)F(Y_2^e) + b - \Delta w,$$

$$\frac{\partial \pi_2^B}{\partial Y_2^r} = \Delta w - (p + b + h) \int_0^{Y_2^r - Y_1^{e^*}} F(Y_2^r - t)f(t)dt + (p + b - w_e)F(Y_2^r - Y_1^{e^*}).$$

When $b > \Delta w$, solving for $\frac{\partial \pi_2^B}{\partial Y_2^e} = 0$ and $\frac{\partial \pi_2^B}{\partial Y_2^r} = 0$ yields $Y_2^{e0} = F^{-1}(\gamma)$ and $Y_2^{r0} = \Phi^{-1}(\beta)$, where

$\Phi(y) = \int_0^{y-Y_1^{e^*}} F(y-t)f(t)dt - F(Y_1^{e^*})F(y-Y_1^{e^*})$, $\beta = \frac{\Delta w}{p+b+h}$, and $\gamma = \frac{b-\Delta w}{b+h}$. If $Y_2^{e0} < Y_2^{r0}$, we get $Y_2^{e^*} = Y_2^{e0}$ and $Y_2^{r^*} = Y_2^{r0}$. Otherwise, using Lagrange multipliers associated with the constraint $Y_2^e \leq Y_2^r$, we obtain $Y_2^{e^*} = Y_2^{r^*} = \Psi^{-1}(b)$ where $\Psi(y) = (b+h)F(y) + (p+b+h)\int_0^{y-Y_1^{e^*}} F(y-t)f(t)dt - (p+b-w_e)F(y-Y_1^{e^*})$. Note that in the case in which $b \leq \Delta w$, we have $Y_2^{e^*} = 0$ and $Y_2^{r^*} = \Phi^{-1}(\beta)$.

For $n = 3$, we assume $x_3 = 0$ and the buyer decides on Y_3^e and Y_3^r before D_1 is revealed in order to maximize his total expected profit:

$$\pi_3^B(0) = \max_{Y_3^r \geq Y_3^e \geq 0} p\mu - \Delta w Y_3^e - w_r Y_3^r - L(Y_3^e) + \int_0^\infty \pi_2^B(Y_3^r - u)f(u)du.$$

See the Appendix for the analysis of $\pi_3^B(0)$. The next Theorem, the proof of which is provided in the Appendix along with all other proofs, shows that the function π_3^B is jointly concave in Y_3^e and Y_3^r.

Theorem 1 *The total expected profit function π_3^B is jointly concave with respect to the variables Y_3^e and Y_3^r.*

It follows from the above that the optimality conditions could be expressed simultaneously as a variational inequality (see Hamdouch et al. (2017)) in which one seeks $(Y_3^{e^*}, Y_3^{r^*}) \in \mathbb{R}^2$ satisfying

$$\left[(b+h)F(Y_3^{e^*}) - b + \Delta w\right] \times \left[Y_3^e - Y_3^{e^*}\right] + \left[(b+h)\Theta(Y_3^{r^*}) - \Delta w\right] \times \left[Y_3^r - Y_3^{r^*}\right] \geq 0$$

subject to $Y_3^e \leq Y_3^r$, $\forall (Y_3^e, Y_3^r) \in \mathbb{R}^2$, where

$$\Theta(y) = \int_0^{y-Y_2^{e^*}} F(y-t)f(t)dt - \frac{b-\Delta w}{b+h}F(y-Y_2^{e^*})$$
$$+ \frac{p+b+h}{b+h}\int_0^{y-Y_2^{r^*}}\left[\int_0^{y-u-Y_1^{e^*}} F(y-u-t)f(t)dt - F(Y_1^{e^*})F(y-u-Y_1^{e^*}) - \beta\right]f(u)du.$$

After obtaining all optimal basestock levels, the optimal quantities ordered from both suppliers are given by:

$$Q_3^{e^*} = Y_3^{e^*}, Q_3^{r^*} = Y_3^{r^*} - Y_3^{e^*},$$
$$Q_2^{e^*} = \max\{Y_2^{e^*} - Y_3^{r^*} + D_1, 0\}, Q_2^{r^*} = Y_2^* - \max\{Y_2^{e^*}, Y_3^{r^*} - D_1\},$$
$$Q_1^{e^*} = \max\{Y_1^{e^*} - \max\{Y_2^*, Y_3^{r^*} - D_1\} + D_2, 0\}.$$

Note that $Q_3^{e^*}$ and $Q_3^{r^*}$ are deterministic variables while $Q_1^{e^*}$, $Q_2^{e^*}$ and $Q_2^{r^*}$ are random variables. The next Lemma provides an explicit expression for the optimal solution.

Lemma 1 *Let $Y_3^{r0} = \Theta^{-1}(\eta)$ with $\eta = \frac{\Delta w}{b+h}$, $\gamma = \frac{b-\Delta w}{b+h}$, and*

$$
\begin{aligned}
\Omega(y) = {} & (b+h)F(y) + (b+h)\int_0^{y-Y_2^{e^*}} F(t-u)f(u)du - (b-\Delta w)F(y - Y_2^{e^*}) \\
& + \int_0^{y-Y_2^{r^*}} \left[(p+b+h)\int_0^{y-u-Y_1^{e^*}} F(y-u-t)f(t)dt - (p+b-w_e)F(y-u-Y_1^{e^*}) - \Delta w \right] f(u)du.
\end{aligned}
$$

The optimal order quantities $Q_3^{e^*}$ and $Q_3^{r^*}$ are analytically expressed as:

- *For $\Delta w \geq b$, $Q_3^{e^*} = 0$ and $Q_3^{r^*} = \Theta^{-1}(\eta)$,*

- *For $\Delta w < b$ and $-(b+h)F(Y_3^{r0}) + b - \Delta w \geq 0$, $Q_3^{e^*} = \Omega^{-1}(b)$ and $Q_3^{r^*} = 0$,*

- *For $\Delta w < b$ and $-(b+h)F(Y_3^{r0}) + b - \Delta w < 0$, $Q_3^{e^*} = F^{-1}(\gamma)$ and $Q_3^{r^*} = \Theta^{-1}(\eta) - F^{-1}(\gamma)$.*

Note that the solution is a newsvendor-type solution and that the threshold values that determine the optimal order quantities depend on h and b. From Theorem 1 and Lemma 1, note that if the wholesale price difference ($=\Delta w$) between the two suppliers is larger than the unit backorder cost, it is optimal to order only from the regular supplier and nothing from the expedited supplier. Otherwise, there are two scenarios that are mainly dependent on the cost structure (h and b). In one scenario, it is optimal to order only from the expedited supplier and in the other scenario, it is optimal to order from both suppliers. The optimal order quantities are expressed in Lemma 1. Figure 1 shows an example of the optimal policy parameters which displays the several scenarios.
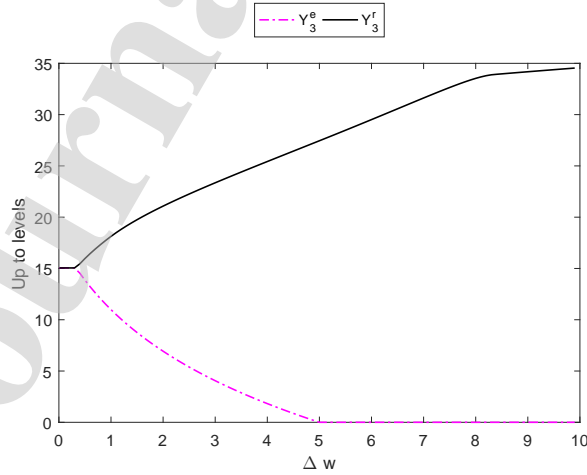

Figure 1: Optimal policy parameters for $n = 3$ as function of Δw ($p = 15$, $b = 5$, $h = 1$, $CV(D_t) = 1$)

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3.2.2 The suppliers' profits

For the expedited supplier e, his overall expected profit in the three periods is evaluated by:

$$
\begin{aligned}
\pi_3^e &= (w_e - c_e)\big(Q_3^{e^*} + E(Q_2^{e^*}) + E(Q_1^{e^*})\big) \\
&= (w_e - c_e)\bigg(Y_3^{e^*} + \int_{Y_3^{r^*}-Y_2^{e^*}}^{\infty}(t - Y_3^{r^*} + Y_2^{e^*})f(t)dt + \int_{Y_3^{r^*}-Y_2^{r^*}}^{\infty}\int_{Y_2^{r^*}-Y_1^{e^*}}^{\infty}(t - Y_2^{r^*} + Y_1^{e^*})f(t)dt f(u)du \\
&\qquad\qquad + \int_0^{Y_3^{r^*}-Y_2^{r^*}}\int_{Y_3^{r^*}-u-Y_1^{e^*}}^{\infty}(t - Y_3^{r^*} + u + Y_1^{e^*})f(t)dt f(u)du\bigg).
\end{aligned}
$$

The regular supplier r receives only two orders, as no orders will be received in the last period. Hence, the total expected profit is given by:

$$
\begin{aligned}
\pi_3^r &= (w_r - c_r)\big(Q_3^{r^*} + E(Q_2^{r^*})\big) \\
&= (w_r - c_r)\bigg(Y_3^{r^*} - Y_3^{e^*} + \int_{Y_3^{r^*}-Y_2^{r^*}}^{Y_3^{r^*}-Y_2^{e^*}}(t - Y_3^{r^*} + Y_2^{r^*})f(t)dt + \int_{Y_3^{r^*}-Y_2^{e^*}}^{\infty}(Y_2^{r^*} - Y_2^{e^*})f(t)dt\bigg).
\end{aligned}
$$

3.2.3 The total supply chain profit

The expected total supply chain profit under the DOP policy can be expressed as:

$$
\begin{aligned}
\pi_3^B + \pi_3^e + \pi_3^r &= 3p\mu - (c_e - c_r)Y_3^{e^*} - c_r Y_3^{r^*} - L(Y_3^{e^*}) \\
&\quad - \int_0^{Y_3^{r^*}-Y_2^{r^*}}\bigg[L(Y_3^{r^*} - u) + \int_0^{Y_3^{r^*}-u-Y_1^{e^*}}L_1(Y_3^{r^*} - u - t)f(t)dt \\
&\quad + c_e\int_{Y_3^{r^*}-u-Y_1^{e^*}}^{\infty}(Y_1^{e^*} - Y_3^{r^*} + u + t)f(t)dt + L_1(Y_1^{e^*})\big(1 - F(Y_3^{r^*} - u - Y_1^{e^*})\big)\bigg]f(u)du \\
&\quad - \int_{Y_3^{r^*}-Y_2^{r^*}}^{Y_3^{r^*}-Y_2^{e^*}}\bigg[c_r(Y_2^{r^*} - Y_3^{r^*} + u) + L(Y_3^{r^*} - u) + \int_0^{Y_2^{r^*}-Y_1^{e^*}}L_1(Y_2^{r^*} - t)f(t)dt \\
&\quad + c_e\int_{Y_2^{r^*}-Y_1^{e^*}}^{\infty}(Y_1^{e^*} - Y_2^{r^*} + t)f(t)dt + L_1(Y_1^{e^*})\big(1 - F(Y_2^{r^*} - Y_1^{e^*})\big)\bigg]f(u)du \\
&\quad - \int_{Y_3^{r^*}-Y_2^{e^*}}^{\infty}\bigg[c_e(Y_2^{e^*} - Y_3^{r^*} + u) + c_r(Y_2^{r^*} - Y_2^{e^*}) + L(Y_2^{e^*}) + \int_0^{Y_2^{r^*}-Y_1^{e^*}}L_1(Y_2^{r^*} - t)f(t)dt \\
&\quad + c_e\int_{Y_2^{r^*}-Y_1^{e^*}}^{\infty}(Y_1^{e^*} - Y_2^{r^*} + t)f(t)dt + L_1(Y_1^{e^*})\big(1 - F(Y_2^{r^*} - Y_1^{e^*})\big)\bigg]f(u)du.
\end{aligned}
$$

3.3 Standing Order Policy

This subsection analyzes the problem under the Standing Order Policy (SOP). The order of events, the overall expected profits, and the optimal solution for the buyer are presented in section 3.3.1. The suppliers' profits and the total supply chain profits are presented in section 3.3.2 and section 3.3.3.

3.3.1 The buyer's problem

Under this policy, the buyer decides on the quantities $\widetilde{Q}_n^e$ to order from the expedited supplier e $(1 \le n \le 3)$ and the standing quantity $\widetilde{Q}^r$ to order from the regular supplier r. The standing order $\widetilde{Q}^r$ is made at the beginning of period $t = 1$ and $t = 2$, but is only delivered in the subsequent periods. Similar to the case under DOP, we assume that the initial stock x_3 is zero. For $n = 1$, the buyer needs to decide on the quantity to order from the expedited supplier before D_3 is revealed. Therefore, the expected profit of the buyer is given by:

$$\widetilde{\pi}_1^B(x_1) = \max_{\widetilde{Y}_1^e \ge x_1} p\mu - w_e(\widetilde{Y}_1^e - x_1) - L_1(\widetilde{Y}_1^e).$$

It easily follows that:
$$\frac{\partial \widetilde{\pi}_1^B}{\partial \widetilde{Y}_1^e} = -w_e - L_1'(\widetilde{Y}_1^e) = -(p + b + h)F(\widetilde{Y}_1^e) + p + b - w_e.$$

Solving $\frac{\partial \widetilde{\pi}_1^B}{\partial \widetilde{Y}_1^e} = 0$ yields $\widetilde{Y}_1^{e*} = F^{-1}\left(\frac{p + b - w_e}{p + b + h}\right)$.

For $n = 2$, following a dynamic programming approach, the buyer decides on $\widetilde{Y}_2^e$ (before D_2 is revealed) to maximize his expected profit knowing that subsequent decisions are made optimally. That is, the expected profit of the buyer is given by:

$$\widetilde{\pi}_2^B(x_2) = \max_{\widetilde{Y}_2^e \ge x_2} p\mu - w_e\widetilde{Y}_2^e + w_e x_2 - w_r\widetilde{Q}^r - L(\widetilde{Y}_2^e) + \int_0^\infty \widetilde{\pi}_1^B(\widetilde{Y}_2^e + \widetilde{Q}^r - t)f(t)dt.$$

Note that $\int_0^\infty \widetilde{\pi}_1^B(y + \widetilde{Q}^r - t)f(t)dt = p\mu - \int_0^{y + \widetilde{Q}^r - \widetilde{Y}_1^{e*}} L_1(y + \widetilde{Q}^r - t)f(t)dt - w_e\int_{y + \widetilde{Q}^r - \widetilde{Y}_1^{e*}}^\infty (\widetilde{Y}_1^{e*} - y - \widetilde{Q}^r + t)f(t)dt - L_1(\widetilde{Y}_1^{e*})(1 - F(y + \widetilde{Q}^r - \widetilde{Y}_1^{e*}))$. Therefore, the expected profit $\widetilde{\pi}_2^B(x_2)$ is expressed as:

$$\widetilde{\pi}_2^B(x_2) = \max_{\widetilde{Y}_2^e \ge x_2} 2p\mu - w_e\widetilde{Y}_2^e + w_e x_2 - w_r\widetilde{Q}^r - L(\widetilde{Y}_2^e) - \int_0^{\widetilde{Y}_2^e + \widetilde{Q}^r - \widetilde{Y}_1^{e*}} L_1(\widetilde{Y}_2^e + \widetilde{Q}^r - t)f(t)dt$$
$$- w_e\int_{\widetilde{Y}_2^e + \widetilde{Q}^r - \widetilde{Y}_1^{e*}}^\infty (\widetilde{Y}_1^{e*} - \widetilde{Y}_2^e - \widetilde{Q}^r + t)f(t)dt - L_1(\widetilde{Y}_1^{e*})(1 - F(\widetilde{Y}_2^e + \widetilde{Q}^r - \widetilde{Y}_1^{e*})).$$

Taking the derivative with respect to $\widetilde{Y}_2^e$, one finds that:

$$\frac{\partial \widetilde{\pi}_2^B}{\partial \widetilde{Y}_2^e} = -w_e - L'(\widetilde{Y}_2^e) - \int_0^{\widetilde{Y}_2^e + \widetilde{Q}^r - \widetilde{Y}_1^{e*}} L_1'(\widetilde{Y}_2^e + \widetilde{Q}^r - t)f(t)dt + w_e(1 - F(\widetilde{Y}_2^e + \widetilde{Q}^r - \widetilde{Y}_1^{e*}))$$
$$= b - (b + h)F(\widetilde{Y}_2^e) - (p + b + h)\int_0^{\widetilde{Y}_2^e + \widetilde{Q}^r - \widetilde{Y}_1^{e*}} F(\widetilde{Y}_2^e + \widetilde{Q}^r - t)f(t)dt$$
$$+ (p + b - w_e)F(\widetilde{Y}_2^e + \widetilde{Q}^r - \widetilde{Y}_1^{e*}).$$

Solving $\frac{\partial \widetilde{\pi}_2^B}{\partial \widetilde{Y}_2^e} = 0$ yields $\widetilde{\Phi}(\widetilde{Y}_2^{e*}, \widetilde{Q}^r) = 0$, where $\widetilde{\Phi}(y, q) = b - (b + h)F(y) - (p + b + h)\int_0^{y + q - \widetilde{Y}_1^{e*}} F(y + q - $

$t)f(t)dt + (p + b - w_e)F(y + q - \widetilde{Y}_1^{e^*})$. Note that the optimal basestock level $\widetilde{Y}_2^{e^*}$ depends on the standing order quantity $\widetilde{Q}^r$.

For $n = 3$, the buyer decides on $\widetilde{Y}_3^e$ and $\widetilde{Q}^r$ to maximize his total expected profit:

$$\widetilde{\pi}_3^B(0) = \max_{\widetilde{Y}_3^e, \widetilde{Q}^r \geq 0} p\mu - w_e\widetilde{Y}_3^e - w_r\widetilde{Q}^r - L(\widetilde{Y}_3^e) + \int_0^\infty \widetilde{\pi}_2^B(\widetilde{Y}_3^e + \widetilde{Q}^r - u)f(u)du.$$

By setting $\widetilde{Z}_3^e = \widetilde{Y}_3^e + \widetilde{Q}^r$, the optimization problem can be written as:

$$\widetilde{\pi}_3^B(0) = \max_{\widetilde{Z}_3^e \geq \widetilde{Q}^r \geq 0} p\mu - w_e\widetilde{Z}_3^e + \Delta w\widetilde{Q}^r - L(\widetilde{Z}_3^e - \widetilde{Q}^r) + \int_0^\infty \widetilde{\pi}_2^B(\widetilde{Z}_3^e - u)f(u)du.$$

See the Appendix for a further analysis of $\widetilde{\pi}_3^B(0)$. The next Theorem establishes the joint concavity of $\widetilde{\pi}_3^B$ with respect to the variables $\widetilde{Z}_3^e$ and $\widetilde{Q}^r$.

Theorem 2 *The total expected profit function $\widetilde{\pi}_3^B$ is jointly concave with respect to the variables $\widetilde{Z}_3^e$ and $\widetilde{Q}^r$.*

It follows from the above that the optimal solution $(\widetilde{Z}_3^{e^*}, \widetilde{Y}_2^{e^*}, \widetilde{Q}^{r^*})$ is a solution of the system: $\frac{\partial \widetilde{\pi}_2^B}{\partial \widetilde{Y}_2^e} = 0$, $\frac{\partial \widetilde{\pi}_3^B}{\partial \widetilde{Z}_3^e} = 0$, $\frac{\partial \widetilde{\pi}_3^B}{\partial \widetilde{Q}^r} = 0$. By the joint concavity of $\widetilde{\pi}_3^B$, the optimality conditions could be expressed simultaneously as a variational inequality in which one seeks $(\widetilde{Z}_3^{e^*}, \widetilde{Y}_2^{e^*}, \widetilde{Q}^{r^*}) \in \mathbb{R}^3$ satisfying

$$\left[-\widetilde{\Theta}(\widetilde{Z}_3^{e^*}, \widetilde{Y}_2^{e^*}, \widetilde{Q}^{r^*})\right] \times \left[\widetilde{Z}_3^e - \widetilde{Z}_3^{e^*}\right] + \left[-\widetilde{\Phi}(\widetilde{Y}_2^{e^*}, \widetilde{Q}^{r^*})\right] \times \left[\widetilde{Y}_2^e - \widetilde{Y}_2^{e^*}\right] + \left[-\widetilde{\Omega}(\widetilde{Z}_3^{e^*}, \widetilde{Y}_2^{e^*}, \widetilde{Q}^{r^*})\right] \times \left[\widetilde{Q}^r - \widetilde{Q}^{r^*}\right] \geq 0,$$

subject to $\widetilde{Q}^r \leq \widetilde{Z}_3^e$, $\forall (\widetilde{Z}_3^e, \widetilde{Y}_2^e, \widetilde{Q}^r) \in \mathbb{R}^3$, where

$$\begin{aligned}
\widetilde{\Theta}(z, y, q) = {} & b - (b + h)F(z - q) - (b + h)\int_0^{z-y} F(z - u)f(u)du + bF(z - y) \\
& - (p + b + h)\int_0^{z-y}\int_0^{z+q-u-\widetilde{Y}_1^{e^*}} F(z + q - u - t)f(t)dtf(u)du \\
& + (p + b - w_e)\int_0^{z-y} F(z + q - u - \widetilde{Y}_1^{e^*})f(u)du.
\end{aligned}$$

and

$$\begin{aligned}
\widetilde{\Omega}(z, y, q) = {} & 2\Delta w + (b + h)F(z - q) - (p + b + h)\int_0^{z-y}\int_0^{z+q-u-\widetilde{Y}_1^{e^*}} F(z + q - u - t)f(t)dtf(u)du \\
& + (p + b - w_e)\int_0^{z-y} F(z + q - u - \widetilde{Y}_1^{e^*})f(u)du \\
& - (1 - F(z - y))\left[(p + b + h)\int_0^{y+q-\widetilde{Y}_1^{e^*}} F(y + q - t)f(t)dt - (p + b - w_e)F(y + q - \widetilde{Y}_1^{e^*})\right].
\end{aligned}$$

After obtaining all optimal basestock levels for the expedited supplier and the optimal standing quantity $\widetilde{Q}^{r^*}$,

the optimal quantities ordered from the expedited supplier e are given by:

$$\widetilde{Q}_3^{e^*} = \widetilde{Z}_3^{e^*} - \widetilde{Q}^{r^*},$$

$$\widetilde{Q}_2^{e^*} = \max\{\widetilde{Y}_2^{e^*} - \widetilde{Z}_3^{e^*} + D_1, 0\},$$

$$\widetilde{Q}_1^{e^*} = \max\{\widetilde{Y}_1^{e^*} - \max\{\widetilde{Y}_2^{e^*}, \widetilde{Z}_3^{e^*} - D_1\} - \widetilde{Q}^{r^*} + D_2, 0\}.$$

Note that $\widetilde{Q}_3^{e^*}$ is a deterministic variable while $\widetilde{Q}_1^{e^*}$ and $\widetilde{Q}_2^{e^*}$ are random variables. Using the above theorem, the next Lemma determines the analytical expression for the optimal solution.

Lemma 2 *The optimal quantities $\widetilde{Q}_3^{e^*}$ and $\widetilde{Q}^{r^*}$ are analytically expressed as:*

- *For $\widetilde{\Omega}(\widetilde{Z}_3^{e^*}, \widetilde{Y}_2^{e^*}, \widetilde{Z}_3^{e^*}) \geq 0$, $\widetilde{Q}_3^{e^*} = 0$ and $(\widetilde{Y}_2^{e^*}, \widetilde{Q}^{r^*})$ are the solutions of $\widetilde{\Phi}(\widetilde{Y}_2^{e^*}, \widetilde{Q}^{r^*}) = 0$ and $\widetilde{\Theta}(\widetilde{Q}^{r^*}, \widetilde{Y}_2^{e^*}, \widetilde{Q}^{r^*}) = 0$,*

- *For $\widetilde{\Omega}(\widetilde{Z}_3^{e^*}, \widetilde{Y}_2^{e^*}, 0) \leq 0$, $\widetilde{Q}^{r^*} = 0$ and $(\widetilde{Y}_2^{e^*}, \widetilde{Q}_3^{e^*})$ are the solutions of $\widetilde{\Phi}(\widetilde{Y}_2^{e^*}, 0) = 0$ and $\widetilde{\Theta}(\widetilde{Q}_3^{e^*}, \widetilde{Y}_2^{e^*}, 0) = 0$,*

- *For $\widetilde{\Omega}(\widetilde{Z}_3^{e^*}, \widetilde{Y}_2^{e^*}, 0) > 0$ and $\widetilde{\Omega}(\widetilde{Z}_3^{e^*}, \widetilde{Y}_2^{e^*}, \widetilde{Z}_3^{e^*}) < 0$, $\widetilde{Z}_3^{e^*}$, $\widetilde{Y}_2^{e^*}$ and $\widetilde{Q}^{r^*}$ are solutions of $\widetilde{\Phi}(\widetilde{Y}_2^{e^*}, \widetilde{Q}^{r^*}) = 0$, $\widetilde{\Theta}(\widetilde{Z}_3^{e^*}, \widetilde{Y}_2^{e^*}, \widetilde{Q}^{r^*}) = 0$ and $\widetilde{\Omega}(\widetilde{Z}_3^{e^*}, \widetilde{Y}_2^{e^*}, \widetilde{Q}^{r^*}) = 0$, and $\widetilde{Q}_3^{e^*} = \widetilde{Z}_3^{e^*} - \widetilde{Q}^{r^*}$.*

Similar to Lemma 1 (under the DOP policy), the solution is a newsvendor-type solution and the threshold values that determine the optimal order quantities are dependent on both parameters h and b. From Theorem 2 and Lemma 2, there are three scenarios that are mainly dependent on the cost structure (h and b). In one scenario, it is optimal to order only from the regular supplier and nothing from the expedited supplier. In the second scenario, it is optimal to order only from the expedited supplier and in the last case, it is optimal to order from both suppliers. The order quantities are expressed in Lemma 2. Figure 2 shows an example of the optimal policy parameters which displays the different scenarios.
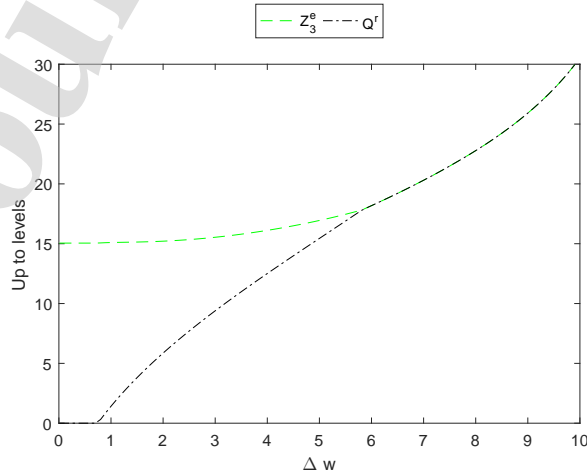

Figure 2: Optimal policy parameters for $n = 3$ as function of Δw ($p = 15$, $b = 5$, $h = 1$, $CV(D_t) = 1$)

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3.3.2 The suppliers' profits

For the expedited supplier e, the overall expected profit in the three periods is evaluated by:

$$
\begin{aligned}
\widetilde{\pi}_3^e &= (w_e - c_e)\big(\widetilde{Q}_3^{e^*} + E(\widetilde{Q}_2^{e^*}) + E(\widetilde{Q}_1^{e^*})\big) \\
&= (w_e - c_e)\bigg(\widetilde{Z}_3^{e^*} - \widetilde{Q}^{r^*} + \int_{\widetilde{Z}_3^{e^*} - \widetilde{Y}_2^{e^*}}^{\infty} (t - \widetilde{Z}_3^{e^*} + \widetilde{Y}_2^{e^*}) f(t) dt \\
&\quad + \int_{\widetilde{Z}_3^{e^*} - \widetilde{Y}_2^{e^*}}^{\infty} \int_{\widetilde{Y}_2^{e^*} + \widetilde{Q}^{r^*} - \widetilde{Y}_1^{e^*}}^{\infty} (t - \widetilde{Y}_2^{e^*} - \widetilde{Q}^{r^*} + \widetilde{Y}_1^{e^*}) f(t) dt f(u) du \\
&\quad + \int_0^{\widetilde{Z}_3^{e^*} - \widetilde{Y}_2^{e^*} (\widetilde{Q}^{r^*})} \int_{\widetilde{Z}_3^{e^*} + \widetilde{Q}^{r^*} - u - \widetilde{Y}_1^{e^*}}^{\infty} (\widetilde{Y}_1^{e^*} - \widetilde{Z}_3^{e^*} - \widetilde{Q}^{r^*} + u + t) f(t) dt f(u) du \bigg).
\end{aligned}
$$

The regular supplier r receives only two standing orders, as no orders will be received in the last period. Hence, the total profit is deterministic and given by

$$
\widetilde{\pi}_3^r = 2(w_r - c_r)\widetilde{Q}^{r^*}.
$$

3.3.3 The total supply chain profit

The expected total supply chain profit under the standing policy is given by:

$$
\begin{aligned}
\widetilde{\pi}_3^B + \widetilde{\pi}_3^e + \widetilde{\pi}_3^r &= 3p\mu - c_e \widetilde{Z}_3^{e^*} + (c_e - 2c_r)\widetilde{Q}^{r^*} - L(\widetilde{Z}_3^{e^*} - \widetilde{Q}^{r^*}) \\
&\quad - \int_0^{\widetilde{Z}_3^{e^*} - \widetilde{Y}_2^{e^*}} \bigg[L(\widetilde{Z}_3^{e^*} - u) + \int_0^{\widetilde{Z}_3^{e^*} + \widetilde{Q}^{r^*} - u - \widetilde{Y}_1^{e^*}} L_1(\widetilde{Z}_3^{e^*} + \widetilde{Q}^{r^*} - u - t) f(t) dt \\
&\quad + c_e \int_{\widetilde{Z}_3^{e^*} + \widetilde{Q}^{r^*} - u - \widetilde{Y}_1^{e^*}}^{\infty} (\widetilde{Y}_1^{e^*} - \widetilde{Z}_3^{e^*} - \widetilde{Q}^{r^*} + u + t) f(t) dt \\
&\quad + L_1(\widetilde{Y}_1^{e^*})(1 - F(\widetilde{Z}_3^{e^*} + \widetilde{Q}^{r^*} - u - \widetilde{Y}_1^{e^*})) \bigg] f(u) du \\
&\quad - \int_{\widetilde{Z}_3^{e^*} - \widetilde{Y}_2^{e^*}}^{\infty} \bigg[c_e(\widetilde{Y}_2^{e^*} - \widetilde{Z}_3^{e^*} + u) + L(\widetilde{Y}_2^{e^*}) + \int_0^{\widetilde{Y}_2^{e^*} + \widetilde{Q}^{r^*} - \widetilde{Y}_1^{e^*}} L_1(\widetilde{Y}_2^{e^*} + \widetilde{Q}^{r^*} - t) f(t) dt \\
&\quad + c_e \int_{\widetilde{Y}_2^{e^*} + \widetilde{Q}^{r^*} - \widetilde{Y}_1^{e^*}}^{\infty} (\widetilde{Y}_1^{e^*} - \widetilde{Y}_2^{e^*} - \widetilde{Q}^{r^*} + t) f(t) dt + L_1(\widetilde{Y}_1^{e^*})(1 - F(\widetilde{Y}_2^{e^*} + \widetilde{Q}^{r^*} - \widetilde{Y}_1^{e^*})) \bigg] f(u) du.
\end{aligned}
$$

4 Comparing the two policies

In the previous section, we analyzed the two policies separately. In this section, we will compare the two policies for the whole supply chain. Note that only the buyer makes decisions and that the suppliers react to the buyer's decisions. From the mathematical analysis, it is clear that Δw plays a crucial role in determining the preferred policies in the supply chain. However, it is mathematically impossible to get a closed form expression for Δw^*,

which is the critical value where $(\pi_3^B + \pi_3^e + \pi_3^r) - (\widetilde{\pi}_3^B + \widetilde{\pi}_3^e + \widetilde{\pi}_3^r) = 0$. In other words, Δw^* is the point of indifference where both policies generate the same supply chain profit. Therefore, in section 4.1 , we will present an analysis of the effect of this parameter (Δw) on the preferred policies. Moreover, the unit profit margins of the two suppliers ($\nu_e = w_e - c_e$ and $\nu_r = w_r - c_r$) are also affecting the preferred policy. Therefore, we present an analysis of the effect of the suppliers' unit profit margin on the preferred policy in section 4.2.

4.1 Effect of the wholesale price difference Δw

First, note that when $\Delta w = 0$, the system turns to a single sourcing system where both policies will have the same performance, see Figure 3. In this case, the product will be obviously only ordered from the expedited supplier. On the other hand, when $\Delta w \to \infty$, the system will also turn to a single sourcing system where the product will be only ordered from the regular supplier. In this case, the DOP policy will obviously outperform the SOP policy, because under SOP, the system will become completely rigid with no opportunity to adjust the order volumes from the regular supplier. The main question here is at what critical value of $\Delta w > 0$ (which we refer to as Δw^*) will the two policies perform equally. Basically, we derive the critical value Δw^* such that $\pi_3^B + \pi_3^e + \pi_3^r - \widetilde{\pi}_3^B + \widetilde{\pi}_3^e + \widetilde{\pi}_3^r = 0$, while fixing the other model parameters p, b, h, c_e and c_r. It follows that we have two scenarios:

- There is at least one feasible value Δw for which $(\pi_3^B + \pi_3^e + \pi_3^r) - (\widetilde{\pi}_3^B + \widetilde{\pi}_3^e + \widetilde{\pi}_3^r) = 0$. In this case, the preferred policy will switch from one policy to another one after each critical value Δw^*. For example, if the solution is unique, one policy will be preferred for $0 < \Delta w < \Delta w^*$ and the other policy will be preferred for $\Delta w \geq \Delta w^*$. An example of a case with one unique Δw^* is illustrated in Figure 3a and with two Δw^*'s in Figure 3b.

- There is no feasible value of Δw^* for which $(\pi_3^B + \pi_3^e + \pi_3^r) - (\widetilde{\pi}_3^B + \widetilde{\pi}_3^e + \widetilde{\pi}_3^r) = 0$, such as in Figure 3c. In this case, one policy will be preferred for all $\Delta w > 0$. In other words, either $\pi_3^B + \pi_3^e + \pi_3^r > \widetilde{\pi}_3^B + \widetilde{\pi}_3^e + \widetilde{\pi}_3^r$, $\forall \Delta w > 0$ or $\pi_3^B + \pi_3^e + \pi_3^r < \widetilde{\pi}_3^B + \widetilde{\pi}_3^e + \widetilde{\pi}_3^r$, $\forall \Delta w > 0$.

The mathematical analysis of this result is presented in the Appendix. In order to better understand these results, we plot in Figure 4 the following 4 functions for the same scenarios as in Figure 3:

- $\pi_3^B - \widetilde{\pi}_3^B$, which is the green colored function,

- $\pi_3^e - \widetilde{\pi}_3^e$, which is the red colored function,

- $\pi_3^r - \widetilde{\pi}_3^r$, which is the blue colored function, and

- $\pi_3^B + \pi_3^e + \pi_3^r - (\widetilde{\pi}_3^B + \widetilde{\pi}_3^e + \widetilde{\pi}_3^r)$, which is the black-colored function and the sum of the previous 3 functions.
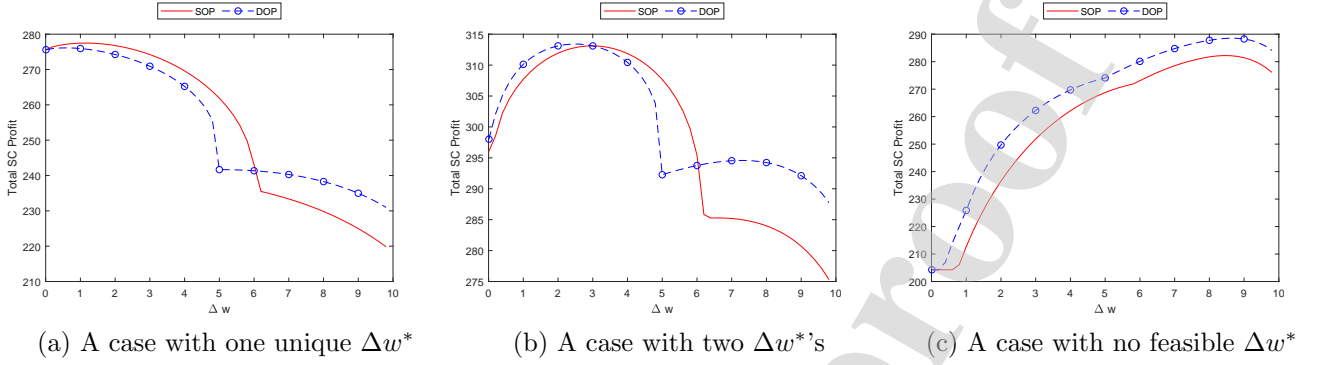
Figure 3: Total supply chain profit as function of Δw for three different cases

Obviously, the function $\pi_3^B - \widetilde{\pi}_3^B$ will be always positive, because the buyer prefers the DOP policy, independent of Δw. However, the impact of the buyer's decisions on the two suppliers and hence on the total supply chain performance is not straightforward. The plots in Figure 4 show that in these three cases, the expedited supplier prefers SOP, because the allocation of the expedite supplier is higher under SOP to compensate for the rigidity of the standing order.

Below, we explain the reasons for this in more detail. In Figure 4c, for example, the benefit of the expedited supplier from SOP is less than the benefit of the buyer and the regular supplier together from DOP. Therefore, in that case, the black colored function is positive in the whole domain of Δw, which means the benefit of the buyer and regular supplier together from DOP outweighs the benefit of the expedited supplier from SOP. Therefore, in this example, there is no feasible Δw^*. This does not hold for the other two cases (Figures 4a and 4b). In those cases, there is a domain of Δw where the benefit of the expedited supplier from SOP outweighs the benefit of the regular supplier and buyer together from DOP. Therefore, feasible Δw^*s exist in those cases, which is mainly due to the fact that at high values of Δw, i.e., when the expedited supplier becomes more
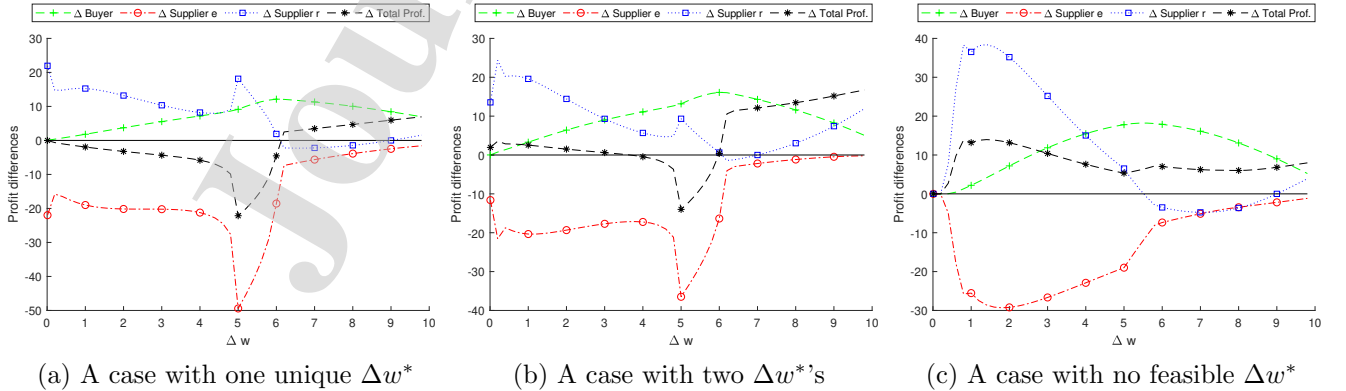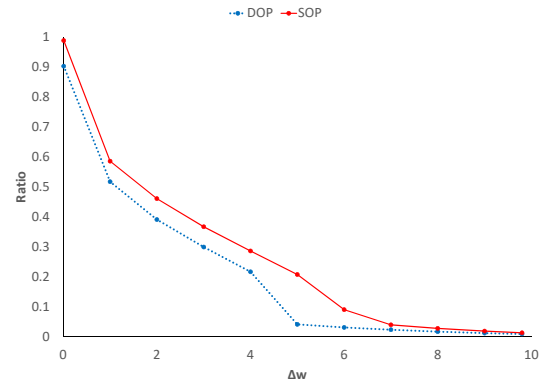

Figure 4: Difference in profit functions for each supply chain part as function of Δw for three different cases

expensive, its added value decreases.

In order to better understand the reason why the expedited supplier prefers SOP policy, we plot in Figure 5a the total order quantities to both suppliers under both policies (for our three-periods model), that is, the sum of the order quantities to the regular and expedited suppliers for the same setting as Figures 3a and 4a. We plot here only the results of one case, because the figures for the other cases look very similar. It becomes clear that under the SOP policy, the total order quantity is larger and the difference is growing in Δw. Note that this can only happen because we are studying here a three-periods model. In the long-run, the total order quantity will be equal to the expected demand, which means that Figure 5a will become flat. Figure 5b displays, for the same setting, the ratio of the total order volume that goes to the expedited supplier. We notice that under SOP, the expedited supplier gets always a larger part of the total order quantity. At high values of Δw, the difference and the ratio both converge to zero, because the expedited supplier is hardly utilized in that case. This behavior does not change when extending the horizon of our model. The volumes and allocation to both suppliers is different under both policies. Overall, the total order quantities are higher under SOP and the expedited supplier gets a larger share of the total order quantity. The increased reliance on the expedited supplier under the SOP policy is understandable, because it is the only source of flexibility for the buyer to be responsive to the final customer, that is, to avoid (expensive) penalty costs. This explains why the expedited supplier always prefers the SOP policy.
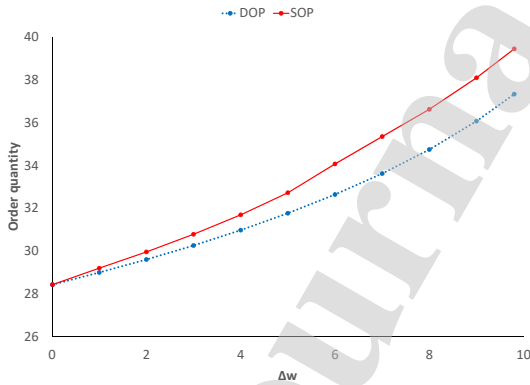
(a) Total order quantity to both suppliers   (b) Ratio of the order volume to the expedited supplier

Figure 5: Total order quantities and ratio of the total volume to the expedited supplier as function of Δw

To summarize, when explicitly incorporating the suppliers in the analysis, one can find that the additional profit of the buyer under DOP is less than the additional profit of (one of) the supplier(s) under SOP, in particular of the expedited supplier. In Section 5.7, we provide some more general managerial insights regarding what drives the preferred policy.

4.2   Effect of the suppliers' profit margin

The unit profit margins of the two suppliers (ν_e and ν_r) also play a key role for the preferred policies in the supply chain. In order to illustrate the effect of ν_e and ν_r on the preferred policy, we will fix Δw and c_r (in addition to fixing p, b and h). When doing so, we can derive the critical value of c_e^* such that $(\pi_3^B + \pi_3^e + \pi_3^r) - (\widetilde{\pi}_3^B + \widetilde{\pi}_3^e + \widetilde{\pi}_3^r) = 0$. See Appendix for the analysis. It follows that we have two different scenarios:

- If $0 < c_e^* < w_e$, then at $c_e = c_e^*$ we have $(\pi_3^B + \pi_3^e + \pi_3^r) - (\widetilde{\pi}_3^B + \widetilde{\pi}_3^e + \widetilde{\pi}_3^r) = 0$, like the case illustrated in Figure 6a. Consequently, either $\pi_3^B + \pi_3^e + \pi_3^r > \widetilde{\pi}_3^B + \widetilde{\pi}_3^e + \widetilde{\pi}_3^r, \forall 0 < c_e < c_e^*$ and $\pi_3^B + \pi_3^e + \pi_3^r < \widetilde{\pi}_3^B + \widetilde{\pi}_3^e + \widetilde{\pi}_3^r, \forall c_e^* < c_e < w_e$ or $\pi_3^B + \pi_3^e + \pi_3^r < \widetilde{\pi}_3^B + \widetilde{\pi}_3^e + \widetilde{\pi}_3^r, \forall 0 < c_e < c_e^*$ and $\pi_3^B + \pi_3^e + \pi_3^r > \widetilde{\pi}_3^B + \widetilde{\pi}_3^e + \widetilde{\pi}_3^r, \forall c_e^* < c_e < w_e$. This implies that from a supply chain perspective, one policy will be preferred for $0 < c_e < c_e^*$ and the other policy will be preferred for $c_e^* < c_e < w_e$.

- If $c_e^* \leq 0$ or $c_e^* > w_e$, then there is no feasible value of c_e for which $(\pi_3^B + \pi_3^e + \pi_3^r) - (\widetilde{\pi}_3^B + \widetilde{\pi}_3^e + \widetilde{\pi}_3^r) = 0$, like the case illustrated in Figure 6b. In this case, one of the policies will be preferred for all $0 < c_e < w_e$. In other words, either $\pi_3^B + \pi_3^e + \pi_3^r > \widetilde{\pi}_3^B + \widetilde{\pi}_3^e + \widetilde{\pi}_3^r, \forall 0 < c_e < w_e$ or $\pi_3^B + \pi_3^e + \pi_3^r < \widetilde{\pi}_3^B + \widetilde{\pi}_3^e + \widetilde{\pi}_3^r, \forall 0 < c_e < w_e$.

Note that when $N_e(p, h, b, \Delta w)$ converges to zero, the value of c_e^* will converge to $\pm\infty$ implying that there is no feasible value of c_e for which $(\pi_3^B + \pi_3^e + \pi_3^r) - (\widetilde{\pi}_3^B + \widetilde{\pi}_3^e + \widetilde{\pi}_3^r) = 0$ and therefore one policy will be preferred for all values of $0 < c_e \leq w_e$. See Appendix (section 'Analysis of Δw^* and c_e^*') for the definition of the function $N_e(p, h, b, \Delta w)$.

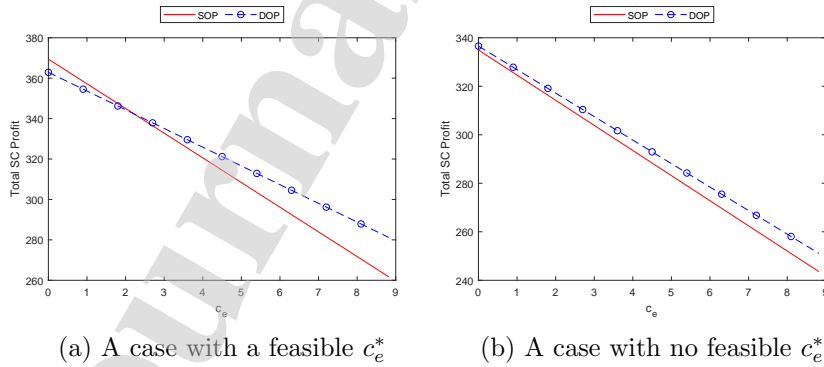|	
(a) A case with a feasible c_e^*	(b) A case with no feasible c_e^*

Figure 6: Total supply chain profit as function of c_e for two different cases

5   Numerical study and insights

In this section, we present the results of our numerical study and summarize the managerial insights. In section 5.1, we discuss the relationship between the three-period model that we analyzed in section 3 and the n-period model that we experiment with in this section. In section 5.2, we illustrate the base case and subsequently, a number of different experiments are presented in sections 5.3 till 5.6. In all numerical experiments, we assume that D_t follows the Gamma distribution with $\mu_t = 10$ and that the unit selling price is $p = 15$. We investigated a full test bed of all possible combinations of the parameters shown in the table below. Each parameter combination is called a *case*. The control parameters are optimized (from the buyer's perspective) via a simulation-based optimization procedure for which we use the interior point algorithm in MATLAB to search for the optimal policy parameters. Then, in section 5.7, we discuss some general managerial insights that we derive from our analysis and the numerical study.

$CV(D_t)$	Δw	b	h	c_r, c_e
0.25,0.5,1,2	[1,10]	[1,20]	1,2,5,10	[1,10]

5.1   From three-period to n-period model

In the previous section, we presented three-period mathematical models and the analysis of both policies. The models were limited to three-period models to allow for an exact mathematical analysis and to obtain closed-form expressions. In this section, we extend our model to an n-period model in this numerical study, which allows to obtain relevant insights. Below, we discuss how the policies behave in the long run.

For the **DOP policy**, in the long run, that is, when n goes to infinity, the DOP converges to DIP with $Y_n^{e^*} = \widehat{Y}^{e^*} = F^{-1}(\gamma), \forall n \geq 2$ and $\lim_{n \to \infty} Y_n^{r^*} = \widehat{\Theta}^{-1}(\eta)$ where $\eta = \frac{\Delta w}{b+h}$, $\widehat{\Theta}(y) = \int_0^{y - \widehat{Y}^{e^*}} F(y - t)f(t)dt - \frac{b - \Delta w}{b+h}F(y - \widehat{Y}^{e^*})$. For more details on this result, see Bulinskaya (1964).

For the **SOP policy**, in the long run, that is, when n goes to infinity, the optimal standing order quantity $\widetilde{Q}^{r^*}$ and the optimal quantity $\widetilde{Z}^{e^*}$ are solutions of the following functional equation:

$$\widetilde{\pi}^B(X) = \max_{\widetilde{Z}^e \geq \widetilde{Q}^r + X} p\mu - w_e(\widetilde{Z}^e - X) - w_r\widetilde{Q}^r - L(\widetilde{Z}^e - \widetilde{Q}^r) + \int_0^\infty \widetilde{\pi}^B(\widetilde{Z}^e - u)f(u)du,$$

where $\widetilde{\pi}^B$ is the average long run profit of the buyer. For more details on this result, see Chiang (2007). In the next sections, we present the results of several numerical tests of the n-period version of our model (with a horizon of 100 time periods). This allows us to better compare the performance of the two policies and to gain insights into the sensitivity of the policy parameters on the preferred policy as well as to obtain some managerial insights.

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5.2   Base case

In the base case, we assume that $CV(D_t) = 0.5$, $h = 1$, $b = 20$, and that $c_e = c_r = 3$. Figure 7 shows the total supply chain profit (Figure 7a), the buyer's profit (Figure 7b), the expedited supplier's profit (Figure 7c), and the regular supplier's profit (Figure 7d) as function of Δw. The results reveal that the buyer prefers the DIP policy, independent of Δw. This result is in line with the literature, namely that the DIP policy is optimal for the buyer in the case of a lead time difference of one period (Whittemore and Saunders, 1977). However, this is different for the two suppliers. The regular supplier prefers the DIP policy if w_r is sufficiently high, while the expedited supplier prefers the SOP policy for the entire domain of Δw. We had discussed in Section 4.1 that the expedited supplier prefers the SOP policy, because that supplier receives a higher order volume under SOP policy. Hence, the expedited supplier greatly benefits from the rigidity of the SOP policy. Under the SOP policy, the buyer becomes more dependent on the expedited supplier, because it is the buyer's only source of flexibility. Therefore, the order-up-to level of the expedited supplier is always higher under the SOP policy, which is beneficial for the expedited supplier.

(a) Total Supply Chain

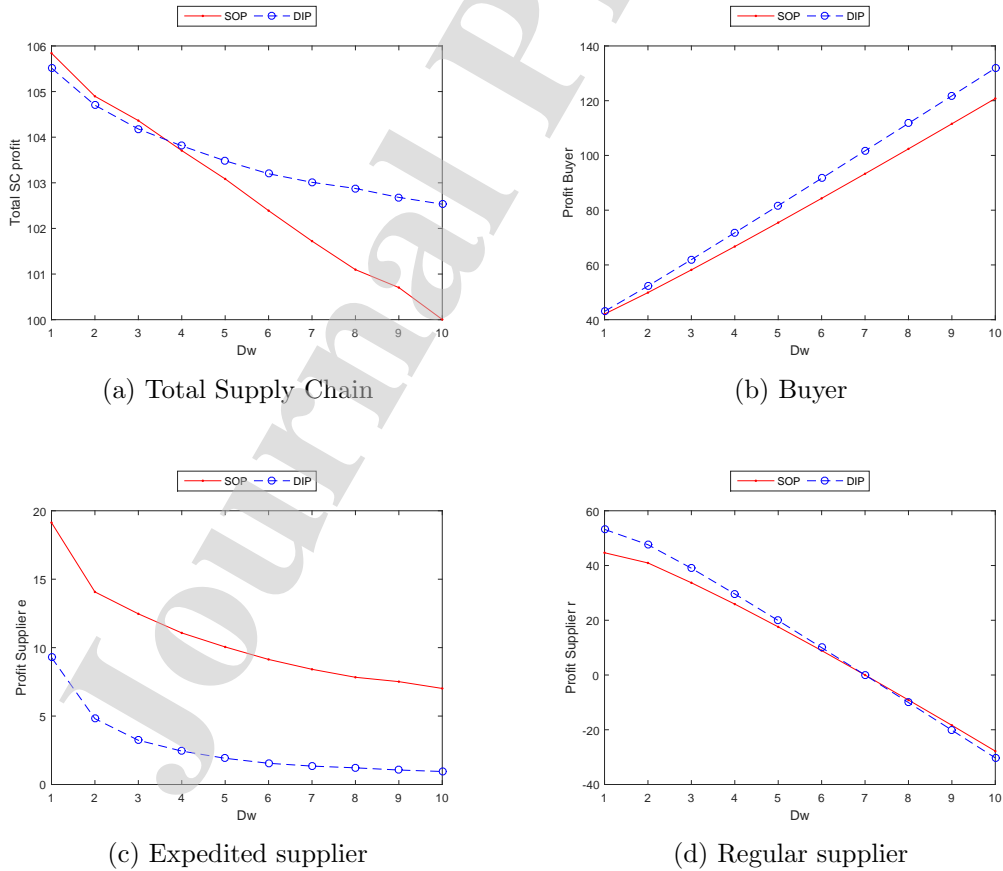(b) Buyer

(c) Expedited supplier

(d) Regular supplier

Figure 7: Profit functions for the supply chain, buyer, and the two suppliers as function of Δw

From the perspective of the entire supply chain, the results indicate that for small values of Δw, the SOP

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is the preferred policy and results in higher total supply chain profits with $\Delta w^* \simeq 3.5$. The explanation is as follows. When Δw increases, the standing order quantity increases as well as the basestock level for the regular supplier under the DIP policy. When Δw increases, the DIP policy benefits from its flexibility by increasing the order quantities towards the regular supplier, while the SOP suffers from its rigidness. In other words, when Δw increases, the benefit of the wholesale price decrease becomes smaller than the benefit of the flexibility of the regular supplier under the DIP policy.

For this base case, as well as for all cases presented in this section, we also compare the results of the n-periods model with the results of the equivalent three-periods model. The results, which we cannot display here due to limited space, show that there are no structural differences between the two models in terms of the preferred policy.

5.3 Varying the level of demand uncertainty

The base case was based on $CV(D_t) = 0.5$. When extending the number of experiments with different values of $CV(D_t)$, see Figure 8, we notice that as the level of demand uncertainty increases, the SOP policy becomes the preferred policy for a wider domain of Δw, which means that Δw^* increases. The two policies react differently to an increase in the level of demand uncertainty. Under the SOP policy, the higher the level of demand uncertainty, the lower the standing order quantity. On the other hand, the order-up-to levels from the expedited supplier increase in the level of demand uncertainty, that is, the order quantities placed at the expedited supplier increase. The buyer utilizes the expedited supplier more to handle the increased demand uncertainty. Under the DIP policy, the higher the level of demand uncertainty, the higher the order-up-to levels from the regular supplier, while the order-up-to levels from the expedited suppliers react differently to a change in the level of demand uncertainty, dependent on Δw.

This means that when the level of demand uncertainty increases, the SOP policy seeks for increased flexibility from the expedited supplier. When the expedited supplier is not much more expensive, that is, when Δw is low, the additional cost is limited, whereas the gain from the increased responsiveness from the expedited supplier outweighs the increased cost, especially at higher levels of demand uncertainty. On the other hand, DIP increases the flexibility by raising the order-up-to level of the regular supplier, which turns out to be only a more beneficial strategy when the expedited supplier is expensive, i.e., when Δw is high.

5.4 Varying the profit margins

When varying the profit margins of the two suppliers, that is, changing c_e and c_r, the effects are different. If the profit margins of both suppliers change, but set equally, then there is no structural change in terms of the preferred policy, see Figure 9a for an example. We run several experiments, which we do not display here due to space limitations, while varying c_e and c_r simultaneously. The results show that the preferred policy remains the same if the unit profit of the two suppliers is changed in an equal way. The reason is that the

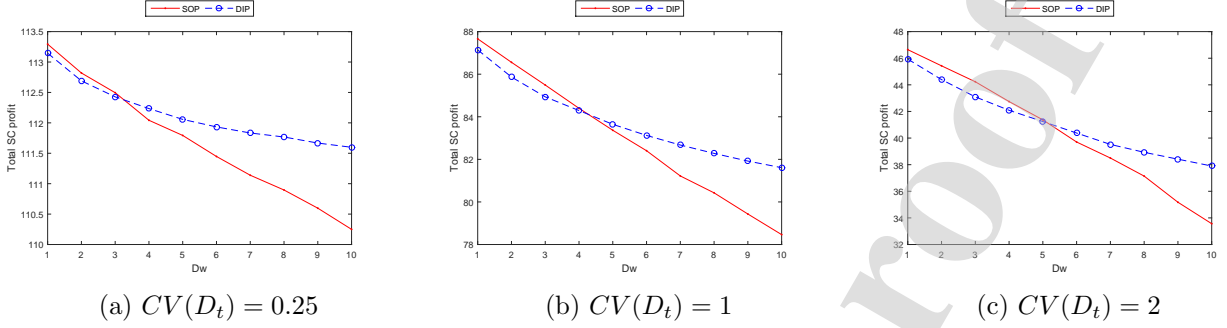(a) $CV(D_t) = 0.25$ (b) $CV(D_t) = 1$ (c) $CV(D_t) = 2$

Figure 8: Total supply chain profit as function of Δw for different levels of demand uncertainty $CV(D_t)$

policy parameters of both policies are independent of c_e and c_r. However, if we fix the profit margin of one supplier and vary the profit margin for the other, then the preferred policy does change. See Figures 9b and 9c for two cases in which c_r has been fixed equal to 1, while c_e has been varied. In other words, the regular supplier procures or produces at low cost, while the procurement or production cost for the expedited supplier increases. In this case, we observe that DIP is the preferred policy for the entire domain of Δw. Since the policy parameters are independent of c_e and c_r, it means that the contribution of the expedited supplier's profit to the total supply chain profit decreases. When the expedited supplier sources at higher cost, its unit profit decreases, and therefore, its contribution to the total supply chain profit decreases. Since the DIP policy sources more from the regular supplier, it is less affected by the profit decrease of the expedited supplier. Thus, the DIP policy becomes the preferred policy when the expedited supplier sources at higher cost. From other experiments (that we do not present here due to limited space), we know that the opposite holds in the case that the regular supplier procures at higher cost; SOP then becomes the preferred policy.
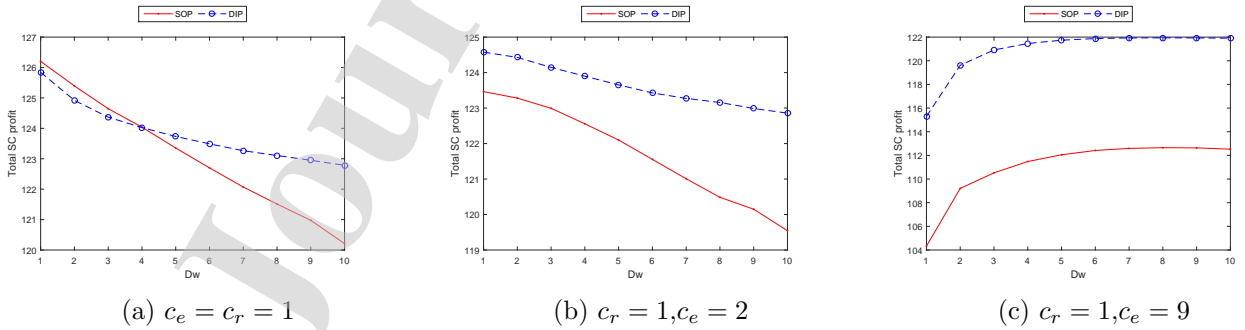

(a) $c_e = c_r = 1$ (b) $c_r = 1, c_e = 2$ (c) $c_r = 1, c_e = 9$

Figure 9: Total supply chain profit as function of Δw for different values of c_e and c_r

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5.5 Varying the backorder cost

The unit backorder cost b has a limited effect on the preferred policy. Figure 10 shows two (extreme) cases with $b = 2$ and $b = 100$. From the results, we notice that as b increases, SOP becomes the preferred policy over a larger domain of Δw. Under SOP, the standing order quantity is almost insensitive to b, but the order-up-to level from the expedited supplier increases significantly when b increases, which means that the expedited supplier is heavily utilized to avoid the expensive backorder costs. In other words, when the backorder cost increases, the rigidness of the SOP policy is compensated by a strong increase in responsiveness, that is, the expedited supplier is heavily utilized and a higher volume is ordered from the expedited supplier. In opposite to SOP, the order-up-to levels from the regular supplier is sensitive to b under DIP, but strongly dependent on Δw. Similar to SOP, the order-up-to level from the expedited supplier also increases when b increases, but the increase is more sensitive to Δw. This results in lower order quantities towards the expedited supplier, as compared to the SOP policy, whereas it is mostly the expedited supplier that is useful when b becomes larger. Hence, under DIP policy, when b increases, flexibility is partly exchanged by an increased responsiveness, but at a lower pace than under SOP policy.

(a) $b = 2$ (b) $b = 100$
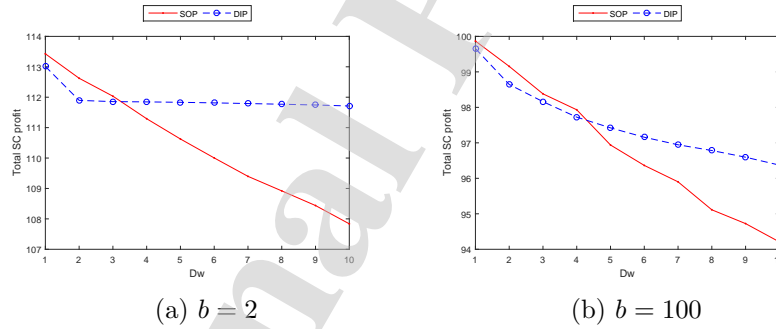
Figure 10: Total supply chain profit as function of Δw for different values of b

5.6 Varying the holding cost

Under the base scenario, we had set $h = 1$. Varying this parameter has a strong impact on the preferred policy. Figure 11 shows the total supply chain profit as function of Δw for $h = 2, 5, 10$. The results indicate that as h increases, SOP outperforms DIP and becomes the preferred policy.

When h increases, it is obvious that both policies become more conservative in their ordering: the standing order quantity and all order-up-levels decrease. In case of high inventories, not ordering is an option under both policies. However, the main difference is that the order-up-to level from the expedited supplier under the SOP policy remains relatively high, especially at high levels of Δw. As a result, under the SOP policy, the responsiveness remains higher, and therefore, the supply chain can better react in case of an imminent shortage.

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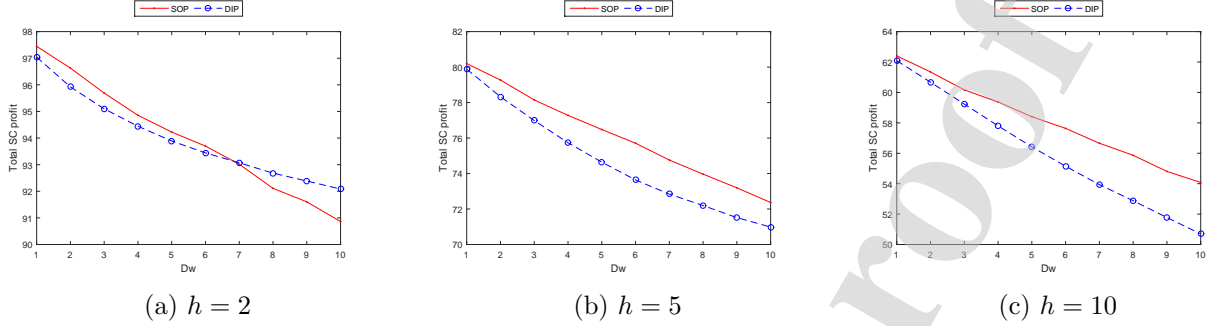
Figure 11: Total supply chain profit as function of Δw for different values of h

5.7 Managerial insights

In Section 4, we presented an analysis of the preferred policy that contains the mathematical conditions when each of the two policies becomes the preferred policy. In Table 1, we show (for all numerical tests presented in Section 5) the maximum relative difference in terms of the supply chain profit for both policies. Since the relative profit difference depends on Δw, we present the maximum relative differences. The column "SOP-DIP" ("DIP-SOP") presents the maximum relative difference in the supply chain profits in the domain where SOP (DIP) is the preferred policy. The results show that the difference in the supply chain profits is typically a few percent, which can be a substantial gain in some industries, especially those that produce products with a low profit margin. Furthermore, these gains come rather easily, as it only requires a change of the order policy. In addition, SOP also brings additional benefits to the supply chain which are not in our cost model as a result of the stable orders for the expedited supplier. The table shows also the Δw^*. A Δw^* of for example "3-4" means that Δw^* is between 3 and 4. Given that the buyer's selling price has been set equal to 15 in all our experiments and given the other model parameters, we are confident that the Δw^*s are in a very realistic range to make the preferred policy change.

Case	Figure	$CV(D_t)$	h	b	c_e	c_r	Δw^*	SOP-DIP	DIP-SOP
1	8a	0.25	1	20	3	3	3-4	0.13%	1.22%
2 (base case)	7a	0.5	1	20	3	3	3-4	0.26%	1.92%
3	8b	1	1	20	3	3	4-5	0.80%	4.01%
4	8c	2	1	20	3	3	5-6	3.16%	11.73%
5	9a	0.5	1	20	1	1	3-4	0.31%	1.86%
6	9b	0.5	1	20	2	1	N/A	N/A	5.07%
7	9c	0.5	1	20	9	1	N/A	N/A	10.32%
8	10a	0.5	1	2	3	3	4	0.65%	3.59%
9	10b	0.5	1	100	3	3	4-5	0.52%	2.30%
10	11a	0.5	2	20	3	3	6-7	0.92%	0.86%
11	11b	0.5	5	20	3	3	N/A	2.52%	N/A
12	11c	0.5	10	20	3	3	N/A	6.45%	N/A

Table 1: (Maximum) relative total supply chain profit difference between DIP and SOP

The buyer always prefer the DIP policy and the expedited supplier the SOP policy due to larger allocated order quantities. For the regular supplier, the policy preference varies, mostly dependent on Δw. Since we take the total supply chain perspective in this paper, we will present below, the main insights from this paper. Based on the analysis and numerical experiments, we find that what mainly drives the preferred policy is a trade-off between *responsiveness*, *flexibility*, and *cost*. When $\Delta w = 0$, the buyer will only source from the expedited supplier, because responsiveness is offered for free. However, when $\Delta w > 0$, then it becomes attractive to (slightly) give up on the costly responsiveness and start to source from the cheaper regular supplier, especially when the demand uncertainty is low.

When starting to source from the (cheaper) regular supplier, another trade-off arises between *flexibility* and *cost*. The DIP policy obviously provides more flexibility, because it allows to also vary the order quantities to the regular supplier in each period, which SOP does not. The results of this paper show that, dependent on the cost structure and level of demand uncertainty, the added value of this flexibility becomes usually only positive at high values of Δw, i.e., when the expedited supplier becomes substantially more expensive than the regular supplier. On the other hand, if the wholesale price difference (Δw) is limited, then the rigidness of the SOP policy typically outweighs the flexibility of the DIP policy. In that case, the increased order quantities allocated to the expedited supplier, which are meant to compensate for the rigidity of SOP, result in an increased responsiveness, which outweighs the profit difference for the buyer. Hence, our main managerial insight is that in a dual sourcing setting, flexibility is only valuable if there is a substantial cost difference between the suppliers. Otherwise, firms may consider implementing the SOP policy, because sufficient and beneficial responsiveness can be obtained due to an increased allocation of volume to the expedited supplier.

Our results and insights are all based on the assumption of a lead time difference of one time period. If the lead time difference is shortened to zero time periods, the buyer will obviously always order from the regular supplier and the system will become a single sourcing system. We also know from the literature that the Tailored Base Surge policy (a special case of the Standing Order Policy) is asymptotically optimal for the buyer (Janakiraman et al. (2014); Xin and Goldberg (2018)). In other words, if the lead time difference between the two suppliers grows large, the TBS (SOP) policy becomes even the preferred policy for the buyer instead of the optimal DIP policy when the lead time difference is only one time period. Therefore, we expect that the SOP policy will become even more dominant from a supply chain perspective if the lead time difference grows.

6   Conclusions and future research

The dual sourcing problem has been widely studied in the literature, but mostly from the buyer's perspective, that is, in a single-echelon setting. We examine the problem from a supply chain perspective by explicitly taking into consideration the effect of the control policies on both suppliers. More precisely, we compare the performance of two different policies: the dynamic order policy (DOP) and the standing order policy (SOP). The DOP specifies two basestock levels and has been proven to be optimal for a single-echelon system when the lead time difference between the two suppliers is one period. We compare this policy with SOP, which is a more appealing policy from a practical perspective. Under SOP, in each period, a fixed quantity is ordered from the regular supplier, while the expedited supplier is utilized to achieve a high(er) service level.

In a single-echelon system, that is, from the buyer's perspective, the DOP policy, which converges to DIP in the long-run, is known to be optimal and will obviously outperform SOP (in case of a lead time difference of one time period). However, from a supply chain perspective, we show that this is not necessarily the case. We define the conditions for the preferred policy and we demonstrate that in several cases, SOP outperforms DIP in a supply chain setting (with a lead time difference of one period), particularly when the wholesale price difference is small, the level of demand uncertainty is high, the unit inventory holding and backorder cost is high, and when the unit profit margin of the expedited supplier is large. This result is not only interesting from a theoretical perspective, but also has significant practical implications. In general, we find that the flexibility offered by the DOP policy is only valuable if there is a substantial cost difference between the suppliers. Otherwise, the SOP policy becomes a legitimate policy alternative, especially given its simple structure.

The insights from this study suggest a number of ideas to address in future research. First, our modeling approach was limited to one time period difference between the two suppliers. We expect that the SOP policy will become more dominant from a supply chain perspective if the lead time difference grows, but it would be interesting to investigate the performance of the two policies with larger lead time differences. Second, if SOP is implemented, interesting price negotiations can follow between the buyer and the suppliers regarding the price setting. The regular supplier can operate efficiently and the buyer will therefore require a lower rates. It would be interesting to investigate how a price equilibrium and supply chain coordination can be achieved by having the wholesale prices becoming decision variables in the model. When doing so, one can extend the rich literature of coordination in a single sourcing setting. Third, the optimization procedure is not convenient and easy to implement in a practical situation. It would be interesting to develop simpler heuristics to determine the control parameters.

Acknowledgement

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Appendix

The section outlines the proofs of results stated in this manuscript.

Analysis of π_3^B

Straightforward computations show that

$$
\begin{aligned}
\pi_3^B(0) = \max_{Y_3^r \geq Y_3^e \geq 0} & \; 3p\mu - \Delta w Y_3^e - w_r Y_3^r - L(Y_3^e) \\
& - \int_0^{Y_3^r - Y_2^{r^*}} \left[L(Y_3^r - u) + \int_0^{Y_3^r - u - Y_1^{e^*}} L_1(Y_3^r - u - t)f(t)dt \right. \\
& \left. + w_e \int_{Y_3^r - u - Y_1^{e^*}}^{\infty} (Y_1^{e^*} - Y_3^r + u + t)f(t)dt + L_1(Y_1^{e^*})\big(1 - F(Y_3^r - u - Y_1^{e^*})\big) \right] f(u)du \\
& - \int_{Y_3^r - Y_2^{r^*}}^{Y_3^r - Y_2^{e^*}} \left[w_r(Y_2^{r^*} - Y_3^r + u) + L(Y_3^r - u) + \int_0^{Y_2^{r^*} - Y_1^{e^*}} L_1(Y_2^{r^*} - t)f(t)dt \right. \\
& \left. + w_e \int_{Y_2^{r^*} - Y_1^{e^*}}^{\infty} (Y_1^{e^*} - Y_2^{r^*} + t)f(t)dt + L_1(Y_1^{e^*})\big(1 - F(Y_2^{r^*} - Y_1^{e^*})\big) \right] f(u)du \\
& - \int_{Y_3^r - Y_2^{e^*}}^{\infty} \left[w_e(Y_2^{e^*} - Y_3^r + u) + w_r(Y_2^{r^*} - Y_2^{e^*}) + L(Y_2^{e^*}) + \int_0^{Y_2^{r^*} - Y_1^{e^*}} L_1(Y_2^{r^*} - t)f(t)dt \right. \\
& \left. + w_e \int_{Y_2^{r^*} - Y_1^{e^*}}^{\infty} (Y_1^{e^*} - Y_2^{r^*} + t)f(t)dt + L_1(Y_1^{e^*})\big(1 - F(Y_2^{r^*} - Y_1^{e^*})\big) \right] f(u)du.
\end{aligned}
$$

Taking the derivative with respect to Y_3^e and Y_3^r, one gets

$$
\begin{aligned}
\frac{\partial \pi_3^B}{\partial Y_3^e} = & \; -(b+h)F(Y_3^e) + b - \Delta w, \\
\frac{\partial \pi_3^B}{\partial Y_3^r} = & \; \Delta w - (b+h)\int_0^{Y_3^r - Y_2^{e^*}} F(Y_3^r - u)f(u)du + (b - \Delta w)F(Y_3^r - Y_2^{e^*}) \\
& - \int_0^{Y_3^r - Y_2^{r^*}} \left[(p+b+h)\int_0^{Y_3^r - u - Y_1^{e^*}} F(Y_3^r - u - t)f(t)dt - (p+b-w_e)F(Y_3^r - u - Y_1^{e^*}) - \Delta w \right] f(u)du \\
= & \; \Delta w - (b+h)\Theta(Y_3^r),
\end{aligned}
$$

Proof of Theorem 1

One easily verifies that the Hessian H of the objective function π_3^B with respect to Y_3^e and Y_3^r is given by

$$
H = -(b+h)f(Y_3^e)\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} - (b+h)W_1(Y_3^r)\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} - (p+b+h)W_2(Y_3^r)\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix},
$$

where $W_1(x) = \int_0^{x-Y_2^{e^*}} f(x-t)f(t)dt$ when $b > \Delta w$, $W_1(x) = \int_0^x f(x-t)f(t)dt - \frac{b-\Delta w}{b+h}f(x)$ when $b \le \Delta w$ and $W_2(x) = \int_0^{x-Y_2^{r^*}} \int_0^{x-u-Y_1^{e^*}} f(x-u-t)f(t)dtf(u)du + (\Phi(Y_2^{r^*}) - \beta)f(x-Y_2^{r^*})$. Note that $W_1(x) > 0$ and the the first term of $W_2(x)$ is positive. The last term $(\Phi(Y_2^{r^*}) - \beta)f(x-Y_2^{r^*})$ is also positive since $\Phi(y)$ is an increasing function $(\Phi'(y) = \int_0^{y-Y_1^{r^*}} f(y-t)f(t)dt)$ and $\Phi(Y_2^{r^*}) > \beta$.

One easily sees that for any vector $v = (v_1, v_2) \in \mathbb{R}^2$ $v^T H v = -(b+h)f(Y_3^e)v_1^2 - (b+h)W_1(Y_3^r)v_2^2 - (p+b+h)W_2(Y_3^r)v_2^2 \le 0$, therefore, H is negative semi-definite implying that the objective function π_3^B is jointly concave with respect to Y_3^e and Y_3^r. ∎

Proof of Lemma 1

Recall that

$$\frac{\partial \pi_3^B}{\partial Y_3^e} = -(b+h)F(Y_3^e) + b - \Delta w,$$

$$\frac{\partial \pi_3^B}{\partial Y_3^r} = -(b+h)\Theta(Y_3^r) + \Delta w.$$

From the definition of Θ, one notices that $\Theta(Y_2^{e^*}) = 0$ and that $\lim_{x\to\infty} \Theta(x) = 1 - \frac{b-\Delta w}{b+h} + \frac{p+b+h}{b+h}[1 - F(Y_1^{e^*}) - \beta] = \frac{2h+w_e}{b+h}$, hence $\partial \pi_3^B/\partial Y_3^r = \Delta w > 0$ when $Y_3^r = Y_2^{e^*}$ and that $\partial \pi_3^B/\partial Y_3^r = -2h - w_r < 0$ when Y_3^r goes to infinity. Consequently, the equation $\partial \pi_3^B/\partial Y_3^r = 0$ has always a solution $Y_3^{r0} = \Theta^{-1}(\eta)$.

Next, notice that if $\Delta w \ge b$ then $\partial \pi_3^B/\partial Y_3^e \le 0$ for all Y_3^e and the optimal choice is given by $Y_3^{e^*} = 0$. Therefore, $Q_3^{e^*} = Y_3^{e^*} = 0$ and $Q_3^{r^*} = Y_3^{r0} = \Theta^{-1}(\eta)$.

Now, for $\Delta w < b$, we have two cases. If $-(b+h)F(Y_3^{r0}) + b - \Delta w < 0$, then $Y_3^{e^*} = F^{-1}(\gamma)$ and $Y_3^{r^*} = \Theta^{-1}(\eta)$ which implies that $Q_3^{e^*} = Y_3^{e^*}$ and $Q_3^{r^*} = Y_3^{r^*} - Y_3^{e^*}$. Otherwise, using Lagrange multipliers associated to the constraint $Y_3^e \le Y_3^r$, we obtain $Y_3^{e^*} = Y_3^{r^*} = \Omega^{-1}(b)$ and we have $Q_3^{e^*} = Y_3^{e^*} = Y_3^{r^*}$ and $Q_3^{r^*} = 0$. ∎

Analysis of $\widetilde{\pi}_3^B$

Straightforward computations show that

$$\widetilde{\pi}_3^B(0) = \max_{\widetilde{Z}_3^e \ge \widetilde{Q}^r \ge 0} 3p\mu - w_e \widetilde{Z}_3^e + (\Delta w - w_r)\widetilde{Q}^r - L(\widetilde{Z}_3^e - \widetilde{Q}^r)$$

$$- \int_0^{\widetilde{Z}_3^e - \widetilde{Y}_2^{e^*}} \left[L(\widetilde{Z}_3^e - u) + \int_0^{\widetilde{Z}_3^e + \widetilde{Q}^r - u - \widetilde{Y}_1^{e^*}} L_1(\widetilde{Z}_3^e + \widetilde{Q}^r - u - t)f(t)dt \right.$$

$$\left. + w_e \int_{\widetilde{Z}_3^e + \widetilde{Q}^r - u - \widetilde{Y}_1^{e^*}}^{\infty} (\widetilde{Y}_1^{e^*} - \widetilde{Z}_3^e - \widetilde{Q}^r + u + t)f(t)dt + L_1(\widetilde{Y}_1^{e^*})(1 - F(\widetilde{Z}_3^e + \widetilde{Q}^r - u - \widetilde{Y}_1^{e^*})) \right] f(u)du$$

$$- \int_{\widetilde{Z}_3^e - \widetilde{Y}_2^{e^*}}^{\infty} \left[w_e(\widetilde{Y}_2^{e^*} - \widetilde{Z}_3^e + u) + L(\widetilde{Y}_2^{e^*}) + \int_0^{\widetilde{Y}_2^{e^*} + \widetilde{Q}^r - \widetilde{Y}_1^{e^*}} L_1(\widetilde{Y}_2^{e^*} + \widetilde{Q}^r - t)f(t)dt \right.$$

$$\left. + w_e \int_{\widetilde{Y}_2^{e^*} + \widetilde{Q}^r - \widetilde{Y}_1^{e^*}}^{\infty} (\widetilde{Y}_1^{e^*} - \widetilde{Y}_2^{e^*} - \widetilde{Q}^r + t)f(t)dt + L_1(\widetilde{Y}_1^{e^*})(1 - F(\widetilde{Y}_2^{e^*} + \widetilde{Q}^r - \widetilde{Y}_1^{e^*})) \right] f(u)du.$$

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Taking the derivative with respect to $\widetilde{Z}_3^e$ and $\widetilde{Q}^r$, one gets:

$$
\begin{aligned}
\frac{\partial \widetilde{\pi}_3^B}{\partial \widetilde{Z}_3^e} &= -w_e - L'(\widetilde{Z}_3^e - \widetilde{Q}^r) - \int_0^{\widetilde{Z}_3^e - \widetilde{Y}_2^{e^*}} \left[L'(\widetilde{Z}_3^e - u) + \int_0^{\widetilde{Z}_3^e + \widetilde{Q}^r - u - \widetilde{Y}_1^{e^*}} L_1'(\widetilde{Z}_3^e + \widetilde{Q}^r - u - t) f(t) dt \right] f(u) du \\
&\quad + w_e \left[\int_0^{\widetilde{Z}_3^e - \widetilde{Y}_2^{e^*}} (1 - F(\widetilde{Z}_3^e + \widetilde{Q}^r - u - \widetilde{Y}_1^{e^*})) f(u) du + 1 - F(\widetilde{Z}_3^e - \widetilde{Y}_2^{e^*}) \right], \\
&= b - (b+h) F(\widetilde{Z}_3^e - \widetilde{Q}^r) - (b+h) \int_0^{\widetilde{Z}_3^e - \widetilde{Y}_2^{e^*}} F(\widetilde{Z}_3^e - u) f(u) du + b F(\widetilde{Z}_3^e - \widetilde{Y}_2^{e^*}) \\
&\quad - (p+b+h) \int_0^{\widetilde{Z}_3^e - \widetilde{Y}_2^{e^*}} \int_0^{\widetilde{Z}_3^e + \widetilde{Q}^r - u - \widetilde{Y}_1^{e^*}} F(\widetilde{Z}_3^e + \widetilde{Q}^r - u - t) f(t) dt f(u) du \\
&\quad + (p+b-w_e) \int_0^{\widetilde{Z}_3^e - \widetilde{Y}_2^{e^*}} F(\widetilde{Z}_3^e + \widetilde{Q}^r - u - \widetilde{Y}_1^{e^*}) f(u) du \\
&= \widetilde{\Theta}(\widetilde{Z}_3^e, \widetilde{Y}_2^{e^*}, \widetilde{Q}^r)
\end{aligned}
$$

Next,

$$
\begin{aligned}
\frac{\partial \widetilde{\pi}_3^B}{\partial \widetilde{Q}^r} &= \Delta w - w_r + L'(\widetilde{Z}_3^e - \widetilde{Q}^r) - \int_0^{\widetilde{Z}_3^e - \widetilde{Y}_2^{e^*}} \int_0^{\widetilde{Z}_3^e + \widetilde{Q}^r - u - \widetilde{Y}_1^{e^*}} L_1'(\widetilde{Z}_3^e + \widetilde{Q}^r - u - t) f(t) dt f(u) du \\
&\quad + w_e \int_0^{\widetilde{Z}_3^e - \widetilde{Y}_2^{e^*}} (1 - F(\widetilde{Z}_3^e + \widetilde{Q}^r - u - \widetilde{Y}_1^{e^*})) f(u) du \\
&\quad - (1 - F(\widetilde{Z}_3^e - \widetilde{Y}_2^{e^*})) \left[\int_0^{\widetilde{Y}_2^{e^*} - \widetilde{Y}_1^{e^*}} L_1'(\widetilde{Y}_2^{e^*} + \widetilde{Q}^r - t) f(t) dt - w_e (1 - F(\widetilde{Y}_2^{e^*} + \widetilde{Q}^r - \widetilde{Y}_1^{e^*})) \right] \\
&= 2\Delta w + (b+h) F(\widetilde{Z}_3^e - \widetilde{Q}^r)) - (p+b+h) \int_0^{\widetilde{Z}_3^e - \widetilde{Y}_2^{e^*}} \int_0^{\widetilde{Z}_3^e + \widetilde{Q}^r - u - \widetilde{Y}_1^{e^*}} F(\widetilde{Z}_3^e + \widetilde{Q}^r - u - t) f(t) dt f(u) du \\
&\quad + (p+b-w_e) \int_0^{\widetilde{Z}_3^e - \widetilde{Y}_2^{e^*}} F(\widetilde{Z}_3^e + \widetilde{Q}^r - u - \widetilde{Y}_1^{e^*}) f(u) du - (1 - F(\widetilde{Z}_3^e - \widetilde{Y}_2^{e^*})) \Big[(p+b+h) \\
&\quad \int_0^{\widetilde{Y}_2^{e^*} + \widetilde{Q}^r - \widetilde{Y}_1^{e^*}} F(\widetilde{Y}_2^{e^*} + \widetilde{Q}^r - t) f(t) dt - (p+b-w_e) F(\widetilde{Y}_2^{e^*} + \widetilde{Q}^r - \widetilde{Y}_1^{e^*}) \Big] \\
&= \widetilde{\Omega}(\widetilde{Z}_3^e, \widetilde{Y}_2^{e^*}, \widetilde{Q}^r)]
\end{aligned}
$$

Proof of Theorem 2

One easily checks that the Hessian $\widetilde{H}$ of the objective function $\widetilde{\pi}_3^B$ with respect to $\widetilde{Z}_3^e$ and $\widetilde{Q}^r$ is given by

$$
\widetilde{H} = \begin{bmatrix} \widetilde{H}_{11} & \widetilde{H}_{12} \\ \widetilde{H}_{12} & \widetilde{H}_{22} \end{bmatrix},
$$

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where

$$
\begin{aligned}
\widetilde{H}_{11} &= -(b+h)\big[f(\widetilde{Z}_3^e - \widetilde{Q}^r) + \widetilde{U}(\widetilde{Z}_3^e, \widetilde{Y}_2^e)\big] - (p+b+h)\widetilde{W}(\widetilde{Z}_3^e, \widetilde{Y}_2^e, \widetilde{Q}^r), \\
\widetilde{H}_{12} &= (b+h)f(\widetilde{Z}_3^e - \widetilde{Q}^r) - (p+b+h)\widetilde{W}(\widetilde{Z}_3^e, \widetilde{Y}_2^e, \widetilde{Q}^r), \\
\widetilde{H}_{22} &= -(b+h)f(\widetilde{Z}_3^e - \widetilde{Q}^r) - (p+b+h)\widetilde{W}(\widetilde{Z}_3^e, \widetilde{Y}_2^e, \widetilde{Q}^r).
\end{aligned}
$$

with

$$
\begin{aligned}
\widetilde{U}(z,y) &= \int_0^{z-y} f(z-u)f(u)du, \\
\widetilde{W}(z,y,q) &= \int_0^{z-y} \int_0^{z+q-u-\widetilde{Y}_1^{e^*}} \big[f(z+q-u-t)f(t)dt\big]f(u)du.
\end{aligned}
$$

It is easy to see that $\widetilde{H}_{11} < 0$ and $\widetilde{H}_{22} < 0$ and

$$
det(-\widetilde{H}) = -\bigg[(b+h)\widetilde{U}(\widetilde{Z}_3^e, \widetilde{Q}^r)\widetilde{H}_{22} - 4(b+h)(p+b+h)f(\widetilde{Z}_3^e - \widetilde{Q}^r)\widetilde{W}(\widetilde{Z}_3^e, \widetilde{Q}^r)\bigg].
$$

$det(-\widetilde{H}) > 0$, consequently, $\widetilde{H}$ is negative semi-definite implying that the objective function $\widetilde{\pi}_3^B$ is jointly concave with respect to $\widetilde{Z}_3^e$ and $\widetilde{Q}^r$. ∎

Proof of Lemma 2

Recall that

$$
\begin{aligned}
\frac{\partial \widetilde{\pi}_2^B}{\partial \widetilde{Y}_2^e} &= \widetilde{\Phi}(\widetilde{Y}_2^e, \widetilde{Q}^r), \\
\frac{\partial \widetilde{\pi}_3^B}{\partial \widetilde{Z}_3^e} &= \widetilde{\Theta}(\widetilde{Z}_3^e, \widetilde{Y}_2^e, \widetilde{Q}^r), \\
\frac{\partial \widetilde{\pi}_3^B}{\partial \widetilde{Q}^r} &= \widetilde{\Omega}(\widetilde{Z}_3^e, \widetilde{Y}_2^e, \widetilde{Q}^r)].
\end{aligned}
$$

From the definition of $\widetilde{\Phi}$ and $\widetilde{\Theta}$, one notices that when $\widetilde{Y}_2^e = \widetilde{Z}_3^e = 0$, $\widetilde{\Phi}(0, \widetilde{Q}^r) = \widetilde{\Theta}(0, 0, \widetilde{Q}^r) = b > 0$ and that $\lim_{\widetilde{Y}_2^e \to \infty} \widetilde{\Phi}(\widetilde{Y}_2^e, \widetilde{Q}^r) = -2h - w_e < 0$ and $\lim_{\substack{\widetilde{Y}_2^e \to \infty \\ \widetilde{Z}_3^e \to \infty}} \widetilde{\Theta}(\widetilde{Z}_3^e, \widetilde{Y}_2^e, \widetilde{Q}^r) = -3h - w_e < 0$. Consequently, the equations $\partial \widetilde{\pi}_2^B / \partial \widetilde{Y}_2^e = 0$ and $\partial \widetilde{\pi}_3^B / \partial \widetilde{Z}_3^e = 0$ have always solutions satisfying $\widetilde{\Phi}(\widetilde{Y}_2^{e^*}, \widetilde{Q}^{r^*}) = 0$ and $\widetilde{\Theta}(\widetilde{Z}_3^{e^*}, \widetilde{Y}_2^{e^*}, \widetilde{Q}^{r^*}) = 0$. For the standing order quantity $\widetilde{Q}^r$, we have three cases. First, notice that if $\widetilde{\Omega}(\widetilde{Z}_3^{e^*}, \widetilde{Y}_2^{e^*}, \widetilde{Z}_3^{e^*}) \geq 0$ then $\frac{\partial \widetilde{\pi}_3^B}{\partial \widetilde{Q}^r} \geq 0$ for all $\widetilde{Q}^r$ since $\widetilde{\Omega}(\widetilde{Z}_3^{e^*}, \widetilde{Y}_2^{e^*}, \widetilde{Q}^r) \geq \widetilde{\Omega}(\widetilde{Z}_3^{e^*}, \widetilde{Y}_2^{e^*}, \widetilde{Z}_3^{e^*}), \forall \widetilde{Q}^r$ and the optimal choice is given by $\widetilde{Q}^{r^*} = \widetilde{Z}_3^{e^*}$. Therefore, $\widetilde{Q}_3^{e^*} = \widetilde{Y}_3^{e^*} = 0$ and $(\widetilde{Y}_2^{e^*}, \widetilde{Q}^{r^*})$ are the solutions of $\widetilde{\Phi}(\widetilde{Y}_2^{e^*}, \widetilde{Q}^{r^*}) = 0$ and $\widetilde{\Theta}(\widetilde{Q}^{r^*}, \widetilde{Y}_2^{e^*}, \widetilde{Q}^{r^*}) = 0$. Now, if $\widetilde{\Omega}(\widetilde{Z}_3^{e^*}, \widetilde{Y}_2^{e^*}, 0) \leq 0$ then $\frac{\partial \widetilde{\pi}_3^B}{\partial \widetilde{Q}^r} \leq 0$ for all $\widetilde{Q}^r$ since $\widetilde{\Omega}(\widetilde{Z}_3^{e^*}, \widetilde{Y}_2^{e^*}, \widetilde{Q}^r) \leq \widetilde{\Omega}(\widetilde{Z}_3^{e^*}, \widetilde{Y}_2^{e^*}, 0), \forall \widetilde{Q}^r$. Therefore $\widetilde{Q}^{r^*} = 0$ and $(\widetilde{Y}_2^{e^*}, \widetilde{Q}_3^{e^*})$ are the solutions of $\widetilde{\Phi}(\widetilde{Y}_2^{e^*}, 0) = 0$ and $\widetilde{\Theta}(\widetilde{Q}_3^{e^*}, \widetilde{Y}_2^{e^*}, 0) = 0$. Finally, the third case corresponds to

the scenario when $\widetilde{\Omega}(\widetilde{Z}_3^{e^*}, \widetilde{Y}_2^{e^*}, 0) > 0$ and $\widetilde{\Omega}(\widetilde{Z}_3^{e^*}, \widetilde{Y}_2^{e^*}, \widetilde{Z}_3^{e^*}) < 0$. In that case, it exists $0 < \widetilde{Q}^{r^*} < \widetilde{Z}_3^{e^*}$ such that $\frac{\partial \widetilde{\pi}_3^B(\widetilde{Z}_3^{e^*}, \widetilde{Y}_2^{e^*}, \widetilde{Q}^{r^*})}{\partial \widetilde{Q}^r} = 0$. Therefore, $\widetilde{Z}_3^{e^*}$, $\widetilde{Y}_2^{e^*}$ and $\widetilde{Q}^{r^*}$ are solutions of $\widetilde{\Phi}(\widetilde{Y}_2^{e^*}, \widetilde{Q}^{r^*}) = 0$, $\widetilde{\Theta}(\widetilde{Z}_3^{e^*}, \widetilde{Y}_2^{e^*}, \widetilde{Q}^{r^*}) = 0$ and $\widetilde{\Omega}(\widetilde{Z}_3^{e^*}, \widetilde{Y}_2^{e^*}, \widetilde{Q}^{r^*}) = 0$, and $\widetilde{Q}_3^{e^*} = \widetilde{Z}_3^{e^*} - \widetilde{Q}^{r^*}$. ∎

Analysis of Δw^* and c_e^*

Defining $\Lambda(x) = \int_0^{x-Y_1^{e^*}} L_1(x-t)f(t)dt + L_1(Y_1^{e^*})\big(1 - F(x - Y_1^{e^*})\big)$ and $\Gamma(x) = \int_x^\infty (t-x)f(t)dt$ and after algebraic manipulation, Δw^* is the solution of:

$$M(p,h,b,\Delta w) - c_e N_e(p,h,b,\Delta w) - c_r N_r(p,h,b,\Delta w) = 0$$

where

$$
\begin{aligned}
M(p,h,b,\Delta w) = {}& L(\widetilde{Z}_3^{e^*} - \widetilde{Q}^{r^*}) + \int_0^{\widetilde{Z}_3^{e^*} - \widetilde{Y}_2^{e^*}} \big[L(\widetilde{Z}_3^{e^*} - u) + \Lambda(\widetilde{Z}_3^{e^*} + \widetilde{Q}^{r^*} - u)\big] f(u)du \\
& + \int_{\widetilde{Z}_3^{e^*} - \widetilde{Y}_2^{e^*}}^{\infty} \big[L(\widetilde{Y}_2^{e^*}) + \Lambda(\widetilde{Y}_2^{e^*} + \widetilde{Q}^{r^*} - u)\big] f(u)du \\
& - L(Y_3^{e^*}) - \int_0^{Y_3^{r^*} - Y_2^{r^*}} \big[L(Y_3^{r^*} - u) + \Lambda(Y_3^{r^*} - u)\big] f(u)du \\
& - \int_{Y_3^{r^*} - Y_2^{r^*}}^{Y_3^{r^*} - Y_2^{e^*}} \big[L(Y_3^{r^*} - u) + \Lambda(Y_2^{r^*})\big] f(u)du - \int_{Y_3^{r^*} - Y_2^{e^*}}^{\infty} \big[L(Y_2^{e^*}) + \Lambda(Y_2^{r^*})\big] f(u)du
\end{aligned}
$$

$$
\begin{aligned}
N_e(p,h,b,\Delta w) = {}& Y_3^{e^*} + \int_0^{Y_3^{r^*} - Y_2^{r^*}} \Gamma(Y_3^{r^*} - u - Y_1^{e^*}) f(u)du + \Gamma(Y_3^{r^*} - Y_2^{e^*}) \\
& + \int_{Y_3^{r^*} - Y_2^{r^*}}^{\infty} \Gamma(Y_2^{r^*} - Y_1^{e^*}) f(u)du - \int_0^{\widetilde{Z}_3^{e^*} - \widetilde{Y}_2^{e^*}} \Gamma(\widetilde{Z}_3^{e^*} + \widetilde{Q}^{r^*} - u - \widetilde{Y}_1^{e^*}) f(u)du \\
& - (\widetilde{Z}_3^{e^*} - \widetilde{Q}^{r^*}) - \Gamma(\widetilde{Z}_3^{e^*} - \widetilde{Y}_2^{e^*}) - \int_{\widetilde{Z}_3^{e^*} - \widetilde{Y}_2^{e^*}}^{\infty} \Gamma(\widetilde{Y}_2^{e^*} + \widetilde{Q}^{r^*} - \widetilde{Y}_1^{e^*}) f(u)du
\end{aligned}
$$

$$
N_r(p,h,b,\Delta w) = Y_3^{r^*} - Y_3^{e^*} + \int_{Y_3^{r^*} - Y_2^{r^*}}^{Y_3^{r^*} - Y_2^{e^*}} \Gamma(Y_3^{r^*} - Y_2^{r^*}) f(u)du + (Y_2^{r^*} - Y_2^{e^*})(1 - F(Y_3^{r^*} - Y_2^{r^*}) - 2\widetilde{Q}^{r^*}
$$

From the previous, it follows that

$$c_e^* = \frac{M(p,h,b,\Delta w) - c_r N_r(p,h,b,\Delta w)}{N_e(p,h,b,\Delta w)}$$

∎

Highlights

- We study a stochastic dual sourcing inventory system from a supply chain perspective
- When explicitly considering the suppliers, the DIP policy may not remain optimal
- We define conditions for the preferred policy, which depends on several parameters
- The preferred policy is mainly based on a trade-off between flexibility and cost
- Flexibility is valuable if there is a large cost difference between the suppliers