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A comparison of the constant-order and dual-index policy for dual sourcing

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Abstract

We analyze a periodic-review, single-stage stochastic demand inventory system with back-orders and two supply options, a regular and a more expensive expedited one. Since the optimal policy for such a problem is generally unknown, we restrict our analysis to two simpler policies: the constant-order and the dual-index policy. We numerically explore the relative cost performance of both policies in a detailed simulation study and find that for long regular lead times the much simpler and from a practical point of view more attractive constant-order policy delivers very good or even better results than the dual-index one, which in previous research has been found to perform closely to the optimal policy.

1 Introduction

In practice, it can be frequently encountered that companies rely on multiple suppliers (or supply modes/channels) for their material procurement. Such sourcing strategies enable them to serve demand at low costs without compromising service. (Although we only refer to “suppliers” in the rest of the paper, different modes of delivery from a single supplier can also be thought of.) Having two suppliers available, the majority of materials can be replenished from the cheaper one, which usually has a longer procurement lead time (*regular supplier*). In case of peaks in demand, replenishment orders can be placed with the more expensive, but faster supplier (*expedited supplier*) in order to avoid

future stockouts. Examples for such dual-sourcing practices can be found at Caterpillar or Hewlett Packard, amongst others. (See e.g. Beyer and Ward [1] or Rao et al. [7] for details.) Although companies already employ dual sourcing, they are still asking for simple, yet effective policies to support their replenishment decisions.

In contrast to many single-sourcing models, where optimal inventory control policies are readily available, results for dual-sourcing models are limited to very special cases. So far, the optimal policy has only been derived for dual-sourcing models with a lead-time difference of one period between both suppliers (see Daniel [3] and Fukuda [4]). For more general lead-time settings, Whittemore and Saunders [11] find that the optimal policy has a highly complex structure. Ordering decisions need to be based not only on a single inventory position, but the system needs to keep track of as many different inventory positions as the lead-time difference is between the two suppliers. Due to this complexity, several simpler policies have been proposed in the literature for such cases (see e.g. Sheopuri et al. [9] for several dual-sourcing heuristics). For an overview of multiple-supplier models we refer the reader to Minner [6].

We focus on two prominent dual-sourcing policies, the constant-order (COP) and the dual-index policy (DIP). Under the COP, the order quantity with the regular supplier is fixed in each period and the expedited order is determined according to a simple order-up-to logic. Such a policy is first introduced by Rosenshine and Obee [8]. They call it a standing-order system and incorporate possible sell-offs if a maximum inventory is exceeded. Optimal policy parameters are found by modeling the system as a Markov Chain. Chiang [2] uses a dynamic programming approach to derive the optimal parameters for the setting considered in Rosenshine and Obee [8]. Later works by Zhang and Hausman [12] and Janssen and de Kok [5] address this policy without a sell-off option. Both use approximations for the determination of the policy parameters.

The DIP, on the other hand, specifies two order-up-to levels, one for regular and one for expedited ordering. For the execution of replenishment orders, it keeps track of two inventory positions. Such a policy represents a simple alternative to the optimal policy and has been shown to perform very closely to the optimal solution for many cases (see Veeraraghavan and Scheller-Wolf [10]). Veeraraghavan and Scheller-Wolf [10] further provide a simulation-based optimization procedure for the computation of the optimal policy

parameters.

Both above-mentioned policies result in very different order processes with the two suppliers. Due to the employment of two order-up-to levels the DIP can vary both order quantities. In periods with high demand, large replenishment orders can be placed. In case of low demand, a small order can be made. The COP, on the other hand, does not have such flexibility options. The minimum quantity that is delivered each period corresponds to the fixed order quantity from the regular supplier. Under this policy, only an increase of the total order quantity through an expedited order is possible. Thus, one would conjecture that this lack of flexibility puts the COP at a major disadvantage, i.e. causing higher costs and rendering the COP less favorable compared to the DIP.

We address this question in a periodic-review setting with stochastic demand and linear holding, backorder and procurement costs. We present a detailed comparison between the two policies for deterministic lead times. Insights are drawn from an extensive simulation study. In terms of costs, we identify major drivers for the performance gap between the two policies. We find that with an increase in the lead time of the regular supplier, the cost difference between the two policies diminishes and the much simpler and from a practical point of view more attractive COP matches or even outperforms the DIP.

The remainder of the paper is organized as follows. In Section 2, we describe the inventory model. The COP and DIP are introduced in Sections 3 and 4, respectively. Section 5 presents theoretical and numerical results of the policy comparison. The major findings are summarized in Section 6, where also a short outlook on future research is provided.

2 Model

We consider an inventory model with a single stockpoint, which faces stochastic customer demand per period, D , with the demands of different periods being i.i.d. non-negative discrete random variables. The stockpoint can replenish its materials each period by placing orders with two suppliers. Via the regular supplier, r , it takes L_r periods for the order to be delivered. The expedited order arrives after L_e periods with $L_e < L_r$. Both lead times are assumed to be deterministic and an integer multiple of the base (review) period, e.g. one day or one week. The shorter lead time of the expedited order comes at

a higher procurement cost per unit, which is expressed by $c_e > c_r$. (Note that otherwise one would only use the expedited supplier.) For each unit of on-hand stock at the end of a period, inventory holding costs h are incurred. In case customer demand cannot be completely satisfied, the missing amount is backordered resulting in costs of b per unit and period. Thus, the total expected average cost function for the system consists of (1) inventory holding costs for the on-hand stock, I^+ , (2) backorder costs for the backordered amount, I^- , and (3) procurement costs for the quantities ordered with both suppliers, Q^r and Q^e . It is given as

$$C = h \cdot \mathbb{E}[I^+] + b \cdot \mathbb{E}[I^-] + c_r \cdot \mathbb{E}[Q^r] + c_e \cdot \mathbb{E}[Q^e] \quad (1)$$

Note that we do not account for any pipeline costs. We assume that all materials are paid for after reception. On average, the sum of both order quantities has to equal the demand during a period, i.e. $\mathbb{E}[D] = \mathbb{E}[Q^r] + \mathbb{E}[Q^e]$. Exploiting this relation and the fact that the term $c_r \cdot \mathbb{E}[D]$ cannot be influenced by the inventory control parameters, the total relevant cost function reduces to

$$C = h \cdot \mathbb{E}[I^+] + b \cdot \mathbb{E}[I^-] + (c_e - c_r) \cdot \mathbb{E}[Q^e] \quad (2)$$

All terms in equation (2) depend on the inventory control policy the stockpoint pursues, because this influences not only the on-hand stock and backorder amount, but also the orders placed with each supplier and therefore the procurement costs. We consider two different policies: a constant-order policy (COP) and a dual-index policy (DIP). Furthermore, we assume that in each period the sequence of events is as follows: arrival of orders (at the beginning of a period), placement of orders and satisfaction of backorders (at the beginning of a period), occurrence and satisfaction of demand (sometime during the period), assessment of costs (at the end of a period).

3 Constant-order policy

Under the COP, each period a fixed quantity, $Q^r = Q$, is ordered from the regular supplier, which arrives after L_r periods. The expedited order, Q^e , is determined according to an order-up-to logic and arrives after L_e periods. We define the inventory position at the

beginning of period t before ordering, IP_t , as the net stock at the end of the previous period, I_{t-1} , plus all outstanding orders with both suppliers that will arrive prior to the expedited order:

$$IP_t = I_{t-1} + \sum_{i=L_r-L_e+1}^{L_r} Q_{t-i}^r + \sum_{j=1}^{L_e} Q_{t-j}^e = I_{t-1} + L_e \cdot Q + \sum_{j=1}^{L_e} Q_{t-j}^e \quad . \quad (3)$$

In order to determine the expedited order quantity, the inventory position is first raised by Q units. If this is not sufficient to bring the inventory position up to or above the order-up-to level, S_e , an expedited order is placed for $(S_e - (IP_t + Q))$ units. Otherwise, no expedited order is placed. Consequently, the order quantities are given as

$$Q_t^r = Q \quad (4)$$

$$Q_t^e = \begin{cases} 0 & IP_t + Q \geq S_e \\ S_e - (IP_t + Q) & IP_t + Q < S_e \end{cases} \quad (5)$$

As can be seen from equation (3), the inventory position and therefore order quantities with the two suppliers are independent of the regular lead time. Only the expedited lead time is relevant. The decision variables in the COP are Q and S_e . In order to evaluate a parameter combination we need to find expressions for the expected on-hand stock, backorders, and expedited order quantity. For a given Q , the latter one immediately results from $\mathbb{E}[Q^e] = \mathbb{E}[D] - Q$. The other two quantities are more difficult to determine. Janssen and de Kok [5] use an approximation which is based on similarities of this problem to a $GI|D|1$ queue. We pursue an exact approach, which is in line with Rosenshine and Obee [8], and model the system as a Markov Chain. Therefore, we define with X_t the inventory position after ordering at both suppliers in period t . Note that the state space of this Markov Chain is infinite and given as $X_t \in SS := \{S_e, S_e + 1, \dots\}$. The inventory balance equation is given as

$$X_{t+1} = X_t - D_t + Q + Q_{t+1}^e \quad (6)$$

where D_t denotes the demand in period t . Combining equations (4) and (5) with (6) we get for the inventory position after ordering:

$$X_{t+1} = \max\{S_e, X_t - D_t + Q\} \quad (7)$$

In order to be able to compute the average inventory holding and backorder costs we have to determine the stationary distribution of X_t

$$\lim_{t \rightarrow \infty} P\{X_t = i\} = \nu_i, \quad i \in SS \quad (8)$$

which can be obtained as the eigenvector of the transition matrix $\mathcal{P} = (p_{ij})$ for the eigenvalue 1

$$v = \mathcal{P}v \quad (9)$$

where $v = (S_e, S_e + 1, \dots)$. The transition probabilities p_{ij} are given as follows:

$$p_{iS_e} = P\{X_{t+1} = S_e \mid X_t = i\} = P\{D \geq i + Q - S_e\} \quad (10)$$

and for all $j > S_e$

$$p_{ij} = P\{X_{t+1} = j \mid X_t = i\} = P\{D = i + Q - j\} \quad (11)$$

Given the stationary distribution of the inventory position, we can calculate the average on-hand stock

$$\mathbb{E}[I^+(Q, S_e)] = \sum_{i=S_e}^{\infty} \sum_{j=0}^i (i - j) \cdot P\{D(L_e + 1) = j\} \cdot \nu_i \quad (12)$$

and the average backorders

$$\mathbb{E}[I^-(Q, S_e)] = \sum_{i=S_e}^{\infty} \sum_{j=i}^{\infty} (j - i) \cdot P\{D(L_e + 1) = j\} \cdot \nu_i \quad (13)$$

with $D(i)$ denoting the cumulative demand over i periods. Due to computational reasons, we determine a maximum inventory position state i_{max} such that the probability for larger states is negligible. We validate the appropriate choice of i_{max} by simulation. In order to get accurate results, a quite large state space has to be chosen, resulting in long computation times. Similar to the number of possible inventory position states, also the demand D is restricted to a maximum value D_{max} .

With the above-described approach, we can evaluate for every set of policy parameters (Q, S_e) the cost performance of the COP. In order to find the optimal policy parameters, we compute the optimal $S_e^*(Q)$ for a fixed value of Q . Note that the steady state

probabilities only have to be calculated once for a given value of Q . Moreover, it is straightforward to show that the cost function is convex in S_e for a fixed value of Q , such that we can easily find the optimum by using a bisection algorithm. Since in each period Q is ordered from the regular supplier and the expected expedited order quantity, $\mathbb{E}[Q^e]$, is also non-negative, Q cannot exceed the average demand in a period. Otherwise, the inventory in the system would grow unboundedly. Hence, in order to find the optimal (Q, S_e) -combination we can start with a value for Q which is slightly below $\mathbb{E}[D]$ and then decrease Q gradually until the total costs increase for the first time. Although it could not be shown so far that the total cost function is unimodal with respect to Q (see Janssen and de Kok [5]), no counterexample to this assumption has been found, either.

As an extreme case, the COP can mimic single sourcing from the expedited supplier in form of an order-up-to policy. If $Q = 0$, all materials are ordered from the expedited supplier. Single sourcing from the regular supplier would mean that $Q = \mathbb{E}[D]$. This is not a reasonable strategy, however, because there is no flexibility in adjusting the order quantities due to changes in demand. Such a strategy would lead to long replenishment cycles and cause high inventory holding or backorder costs. Consequently, it is presumed to be far worse than expedited single sourcing.

4 Dual-index policy

The DIP specifies two order-up-to levels, one for the regular (S_r) and one for the expedited supplier (S_e). The difference between the two order-up-to levels is denoted as Δ ($= S_r - S_e$). For the execution of replenishment orders, such an inventory control system keeps track of two inventory positions. The inventory position at the beginning of a period t is given by the net stock at the end of the previous period, I_{t-1} , plus all outstanding orders with any of the two suppliers that will arrive no later than the order which is to be determined. In each period, the inventory position of the expedited supplier, IP_t^e , is checked first in order to find out whether a replenishment order has to be placed with the faster supply option. This is the case if $IP_t^e < S_e$. The inventory position of the regular supplier, IP_t^r , takes this potential expedited order into account when deciding on

the order quantity with the regular supplier, which is given by $(S_r - IP_t^r)$. Consequently, the inventory positions are defined as

$$IP_t^e = I_{t-1} + \sum_{i=L_r-L_e}^{L_r} Q_{t-i}^r + \sum_{j=1}^{L_e} Q_{t-j}^e \quad (14)$$

$$IP_t^r = I_{t-1} + \sum_{i=1}^{L_r} Q_{t-i}^r + \sum_{j=0}^{L_e} Q_{t-j}^e \quad (15)$$

IP_t^r is always equal to or larger than S_e . Thus, the maximum regular order quantity is Δ and both order quantities are given as

$$Q_t^e = \begin{cases} 0 & IP_t^e \geq S_e \\ S_e - IP_t^e & IP_t^e < S_e \end{cases} \quad (16)$$

$$Q_t^r = S_r - IP_t^r = d_{t-1} - Q_t^e \quad (17)$$

Due to the order-up-to logic in the DIP, in each period t the sum of both order quantities, $Q_t^r + Q_t^e$, equals the demand realization of the previous period, d_{t-1} . For the evaluation of the cost function we follow the approach given in Veeraraghavan and Scheller-Wolf [10]. Since on-hand stock, I^+ , minus backorders, I^- , equals net stock, I , it is in a first step sufficient to come up with an expression for calculating the net stock. As in the COP case, it is possible that the expedited inventory position lies above the respective order-up-to level, S_e . We denote this amount as the overshoot, O_t , i.e.

$$O_t = (IP_t^e - S_e)^+ \quad (18)$$

Referring to the expedited order-up-to level, the net stock at the end of period t is then given as

$$I_t = S_e + O_{t-L_e} - D(t-L_e, t) = S_e - (D(t-L_e, t) - O_{t-L_e}) \quad (19)$$

where $D(t-k, t) = d_{t-k} + d_{t-k+1} + \dots + d_t$. Equation (19) states that $t-L_e$ periods ago the expedited inventory position was equal to S_e plus some overshoot (if any), i.e. $S_e + O_{t-L_e}$. Since then, it has been depleted by all demands including the one in period t , $D(t-L_e, t)$, resulting in the net stock at the end of period t . Denote $\tilde{D}(t-L_e, t) = D(t-L_e, t) - O_{t-L_e}$ as the net demand, i.e. the convolution of the demand and overshoot distribution. Assuming that we can determine the steady-state distribution of $\tilde{D}(t-L_e, t)$, equation (19) resembles

the simple newsvendor net stock calculation. The optimal expedited order-up-to level can therefore be derived from the newsvendor equation

$$P\{\tilde{D}(t - L_e, t) \leq S_e\} = \frac{b}{b + h} \quad . \quad (20)$$

Determining the net demand distribution is not easy since the overshoot distribution is unknown. One possibility to derive this distribution is to use a Markov Chain approach as in the COP case. This time, the system gets more complex, however, due to multiple state variables that are required, one for each period of lead-time difference. We therefore employ the simulation-based optimization procedure proposed by Veeraraghavan and Scheller-Wolf [10] to calculate the steady-state net demand distribution. Given this distribution, the average on-hand stock and backorders can be calculated using equation (19). Another advantage of this approach lies in the simple determination of the average expedited order quantity, $\mathbb{E}[Q^e]$.

A computational simplification results from the fact that the overshoot distribution does not depend on the specific values of the order-up-to levels, S_r and S_e , but only on their difference Δ . An analytical proof is given in Veeraraghavan and Scheller-Wolf [10]. Intuitively, this can be explained as follows. For a fixed value of Δ , the order processes with both suppliers are identical and independent of the specific value of S_e because the probability that an order is placed with the expedited supplier is the same for any value of S_e . In the simplest case of a lead-time difference of one, the probability that an expedited order is placed in period t is $P\{d_{t-1} > \Delta\}$. Only the relation between the on-hand stock and backorders changes with S_e . This property means that in order to find the optimal order-up-to levels, it is sufficient to perform a one-dimensional search over Δ . For each Δ the corresponding expedited order-up-to level, S_e , follows from equation (20) and the regular order-up-to level from $S_r = S_e + \Delta$. In contrast to the COP, where Q is bounded from below by 0 and from above by $\mathbb{E}[D]$, only a (finite) lower bound, 0, exists for Δ in the DIP. $\Delta = 0$ corresponds to single sourcing from the expedited supplier. The upper bound is $\Delta = \infty$, which represents regular single sourcing. Thus, the DIP can mimic both single-sourcing strategies in form of order-up-to policies. The optimal Δ can be found by a numerical search method like “golden section search”. Although no proof for the total

cost function being unimodal is available yet, this seems to be the case (see Veeraraghavan and Scheller-Wolf [10]).

5 Policy comparison

5.1 Theoretical considerations

In this section, we analyze the cost performance of the two policies and compare them. From the extreme strategies that both policies can prescribe, i.e. regular and expedited single sourcing, we can obtain a first insight into the costs. Since the DIP can mimic both strategies in form of order-up-to policies, the optimal DIP is at least as good as the better of the two single-sourcing strategies. Whether regular or expedited single sourcing is advantageous depends on the trade-off between the premium that has to be paid on the procurement costs when sourcing from the expedited supplier and the higher inventory holding costs in case of regular single sourcing due to the longer regular lead time and therefore larger amount of safety stock. Thus, as the regular lead time increases single sourcing from the regular supplier becomes less attractive and, at some point, is dominated by expedited single sourcing. With an increase in the regular lead time (or, more generally, the lead-time difference), the DIP approaches expedited single sourcing and therefore these costs. The costs of the COP, on the other hand, are independent of the regular lead time because it does not influence the setting of the control parameters (see Section 3). Since the maximum possible COP costs also correspond to the expedited single-sourcing costs, there ought to be some intersection (or at least cost equality) of the two policies. If this is true, the COP would result in lower (or equal) costs than the DIP for situations with long regular lead times. Figure 1 illustrates these cost relationships. Due to the complexity of the dual-sourcing policies an analytical derivation of a possible intersection is not possible. We therefore conduct an extensive numerical study to gain further insights.

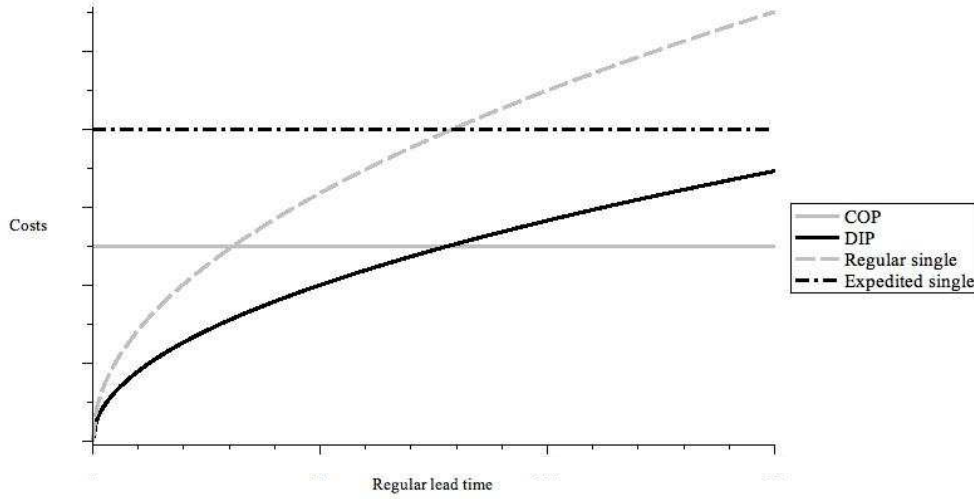


Figure 1: Relationship between single- and dual-sourcing policies

5.2 Numerical study

In order to be able to model a wide range of demand variability we assume a discrete demand distribution based on a Gamma distribution as follows:

$$P\{D = 0\} := F(0.5) \quad (21)$$

$$P\{D = i\} := F(i + 0.5) - F(i - 0.5), \quad i = 1, 2, \dots, D_{max} \quad (22)$$

$$P\{D = D_{max}\} := 1 - F(D_{max} - 0.5) \quad (23)$$

where $F(x)$ denotes the cumulative distribution function of a Gamma distributed random variable. As lead-time difference we choose, on the one hand, the smallest lead-time difference for which the optimal result is unknown, i.e. $L_r - L_e = 2$. On the other hand, we consider a rather large difference of 12 periods. Furthermore, we assume in our parameter setting that one period equals one week. For each holding cost parameter the corresponding backorder cost per unit and period is determined according to a $\frac{b}{b+h}$ ratio of 95% and 99%. All parameters are summarized in Table 1.

First, we analyze the advantage of dual sourcing over single sourcing. We find that in instances with low holding costs ($h = 0.1$), the COP performs worse than the best single-sourcing strategy in almost all of the cases. Also, in instances with a lead-time difference of only 2 periods, the COP delivers poorer results in more than half of the cases. The cost

	Parameter	Value
Demand	Mean per period	10
	Coefficient of variation	0.4, 1.0, 1.6
Lead times	Replenishment lead time of expedited supplier, L_e	1, 3
	Difference in replenishment lead times, $L_r - L_e$	2, 12
Costs	Unit procurement cost of regular supplier, c_r	100
	Unit procurement cost of expedited supplier, c_e	102, 106, 110
	Holding cost per unit and period, h	0.1, 0.5, 1.0
	<i>(i.e. yearly interest rate of 5%, 25%, 50% on a product value of $c_r = 100$ with 1 period = 1 week)</i>	

Table 1: Parameter settings

inferiority ranges between 18% and 164%. The reason is that in most of these instances regular single sourcing is optimal and therefore an average Q close to 10. The largest feasible value for Q is 9, however. That means that still 10% of the expected demand per period is replenished from the expedited supplier which causes higher procurement costs. If the mean demand per period was higher, say 100, the largest feasible Q would be 99, i.e. on average, only 1% of the materials would have to be sourced from the expedited supplier. Therefore, the gap between the COP and regular single sourcing should be closing. Since the premium for expedited ordering is the dominant component in the total costs compared to the holding and backorder costs, such large cost differences can occur. This observation basically shows that in instances where inventory holding is inexpensive and the lead-time difference is small, regular single sourcing is a very reasonable strategy. It can also be observed, however, that in those instances where the COP delivers rather poor results, the overall benefit that can be gained from dual sourcing (using the DIP) is not very large, either. It ranges between 0% and 10%. Larger benefits from dual sourcing of 20% to 26% can be realized in instances with a large lead-time difference and more expensive inventory holding. So let us focus on those instances where dual sourcing is most valuable and conduct a more detailed comparison between the two dual-sourcing policies.

Table 2 presents the COP and DIP costs of the relevant instances. It can be seen that overall the COP performs only about 2.5% worse than the DIP. It is particularly inter-

esting that, although the COP seems to be less flexible with its order quantities, it gets better for increasing demand variability. The cost gap to the DIP narrows from 3.82% to 2.28%. Furthermore, the more expensive inventory holding gets, the better the COP seems to perform. For $h = 0.5$, the COP results in about 6% higher costs than the DIP. For $h = 1.0$, the two policies perform nearly equally well. The most striking result, however, is that if the premium for expedited ordering is small ($c_e = 102$), the COP even causes about 1% lower costs than the DIP, on average. The maximum COP advantage amounts to 3.61% (see Table 3). Surely, this is not significant, but the major finding here is that the COP can outperform the DIP at all. Table 3 indicates the instances with COP advantage for $(L_e = 1; L_r = 13)$. Furthermore, the results in this table reveal another finding with regard to demand variability. From the aggregate results of Table 2, we concluded that demand variability favors the COP. The detailed results of Table 3, however, show that the effect, which demand variability has on the two policies, seems to depend on which policy is superior. In instances with $c_e = 102$, the COP results in lower costs for all holding cost and demand variability values. Here, an increase in the demand variability reduces the cost advantage of the COP. In instances with $c_e = 106$ and $h = 0.5$, the DIP performs better. If demand becomes more variable now, the cost gap between the two policies narrows, i.e. the COP performs better in relation to the DIP.

The superiority of the COP in some of the instances can be explained as follows. The major COP advantage results from a reduction in the average on-hand stock. (See Table 4 for details on some sample instances.) Demand fluctuations have no influence on the regular (constant) order. For short regular lead times, this is a disadvantage because a quick reaction to demand peaks (or drops) is not possible. Consequently, the effects of these disruptions cannot be alleviated in a timely manner unless the expedited supplier is used at a higher procurement cost. In case of long regular lead times, a quick rectifying action via the regular supplier is no longer possible. Nevertheless, the DIP would sometimes still place very large orders, which, at the time they arrive, are not required to such an extent and therefore are put on stock. The COP avoids these extreme cases and thus reduces the extent of the supply-demand mismatch which results in left-over stock at the end of a period. Although backorders might increase slightly, this effect seems less severe.

$(L_e; L_r)$	COP			DIP			Regular single			Expedited single			$\frac{\text{Best single} - \text{Dual}}{\text{Best single}}$		$\frac{\text{COP} - \text{DIP}}{\text{DIP}}$
	Avg.	Min	Max	Avg.	Min	Max	Avg.	Min	Max	Avg.	Min	Max	DIP (Avg.)	COP (Avg.)	Avg.
(1; 13)	55.06	10.22	150.07	53.81	10.60	146.97	80.84	16.34	211.87	98.46	26.70	209.20	24.16%	22.40%	2.31%
(3; 15)	65.69	12.47	176.78	63.93	12.83	173.07	85.46	17.40	222.97	109.42	29.13	236.04	18.52%	16.27%	2.75%
CV	Avg.	Min	Max	Avg.	Min	Max	Avg.	Min	Max	Avg.	Min	Max	Avg.	Avg.	Avg.
0.4	22.15	10.22	37.48	21.34	10.60	36.38	29.37	16.34	45.97	74.01	26.70	124.62	26.70%	23.90%	3.82%
1.0	58.39	27.11	100.05	56.94	27.69	97.16	80.45	44.04	127.22	101.67	39.59	173.65	23.14%	21.19%	2.55%
1.6	100.58	44.96	176.78	98.34	45.60	173.07	139.64	75.31	222.97	136.14	54.88	236.04	18.67%	16.82%	2.28%
h	Avg.	Min	Max	Avg.	Min	Max	Avg.	Min	Max	Avg.	Min	Max	Avg.	Avg.	Avg.
0.5	44.10	10.22	100.69	41.58	10.60	92.78	55.43	16.34	111.48	89.29	26.70	168.02	20.50%	15.68%	6.06%
1.0	76.64	18.09	176.78	76.16	18.68	173.07	110.87	32.67	222.97	118.59	33.41	236.04	21.57%	21.07%	0.64%
c_e	Avg.	Min	Max	Avg.	Min	Max	Avg.	Min	Max	Avg.	Min	Max	Avg.	Avg.	Avg.
102	51.84	10.22	149.46	52.45	10.60	149.52	83.15	16.34	222.97	63.94	26.70	156.04	15.59%	16.58%	-1.17%
106	60.99	14.22	164.78	59.94	13.07	163.81	83.15	16.34	222.97	103.94	66.70	196.04	24.06%	22.73%	1.75%
110	68.29	18.23	176.78	64.22	14.21	173.07	83.15	16.34	222.97	143.94	106.70	236.04	22.66%	17.76%	6.34%
Overall	60.37	10.22	176.78	58.87	10.60	173.07	83.15	16.34	222.97	103.94	26.70	236.04	21.19%	19.18%	2.55%

Table 2: Dual- and single-sourcing costs

h	CV	$\frac{b}{b+h}$	$c_e = 102$			$c_e = 106$		
			COP	DIP	$\frac{\text{COP} - \text{DIP}}{\text{DIP}}$	COP	DIP	$\frac{\text{COP} - \text{DIP}}{\text{DIP}}$
0.5	0.4	95%	10.2203	10.6028	-3.61%	14.2228	13.0697	8.82%
		99%	12.7529	13.2156	-3.50%	16.7554	15.8305	5.84%
	1.0	95%	27.1054	27.6891	-2.11%	35.4707	33.6102	5.54%
		99%	36.3225	36.7894	-1.27%	44.7866	42.8571	4.50%
	1.6	95%	44.9624	45.5990	-1.40%	57.5803	54.8737	4.93%
		99%	65.3437	65.7482	-0.62%	78.0350	75.2247	3.74%
1.0	0.4	95%	18.0945	18.6833	-3.15%	22.4419	22.9291	-2.13%
		99%	23.1169	23.7879	-2.82%	27.5071	28.2269	-2.55%
	1.0	95%	48.1946	49.5839	-2.80%	58.8930	59.4285	-0.90%
		99%	66.6288	68.0159	-2.04%	77.5248	77.6970	-0.22%
	1.6	95%	81.7733	83.0220	-1.50%	97.1747	97.3280	-0.16%
		99%	122.2641	123.1814	-0.74%	138.0841	137.8430	0.17%

Table 3: Dual-sourcing costs for ($L_e = 1; L_r = 13$)

CV	On-hand stock		Backorders		Regular order		Expedited order	
	COP	DIP	COP	DIP	COP	DIP	COP	DIP
0.4	12.6535	13.7447	0.1992	0.1720	9.0000	8.9522	1.0006	1.0484
1.0	30.8888	34.9148	0.5942	0.5779	7.0000	7.6373	3.0081	2.3709
1.6	50.2577	54.0394	1.2461	1.2227	6.0000	6.5160	3.9977	3.4821

Table 4: Quantity details for ($L_e = 1; L_r = 13$), $c_e = 102$, $h = 0.5$, and $\frac{b}{b+h} = 95\%$

6 Conclusions and outlook

In this paper, we analyzed two dual-sourcing policies, the constant-order (COP) and the dual-index policy (DIP). We explored the relative cost performance of both policies in an extensive numerical study.

Our results indicate that in settings where dual-sourcing is most valuable, the simple COP performs only about 2.5% worse than the much more complex DIP, on average. Moreover, we find that for larger lead-time differences the cost gap not only closes, but the COP can even outperform the DIP.

This finding is particularly of practical relevance for two reasons. First, the COP is the more easily implementable and controllable policy in practice. Secondly, such a policy is of particular benefit when negotiating with suppliers, in this case the regular supplier. Being able to guarantee the supplier a constant order each period increases a company's bargaining power. The regular supplier will be more willing to make concessions because, from his point of view, a constant order each period facilitates his production planning significantly.

Based on this work, several issues seem worthwhile to be addressed in future research. So far, we have analyzed the cost performance of the DIP and COP in a single-stage setting. The constant-order property might be even more advantageous in a multi-stage context. The regular supplier does not face any uncertainty and therefore does not need to hold any safety stock. The expedited supplier, on the other hand, is confronted with very sporadic demand that requires larger amounts of safety stock. Under a DIP both suppliers have to cope with stochastic demand. It is not easy to say which of these policies is superior. The presented optimization procedures for both policies are quite cumbersome. In case of the DIP, a simulation model needs to be developed. The COP requires the solution of a Markov Chain with a large state space. Therefore, the development of simple, yet effective heuristics for setting the inventory control parameters for both policies is of interest.

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