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1 Example

1.1 Problem Description

The Bin Packing Problem (BPP) is a classic combinatorial optimization problem where a set of items, each with a given size, must be packed into a minimum number of bins of fixed capacity. The goal is to assign items to bins such that the total size of items in each bin does not exceed its capacity while using the fewest possible bins.

1.2 Model

1.2.1 Parameters

- n : Number of items.
- I : set of items, $I = \{1, \dots, n\}$.
- m : Maximum number of bins. (This must be an upper bound on the optimal solution. A simple upper bound is the number of items).
- J : set of bins, $J = \{1, \dots, m\}$.
- k : Capacity of each bin (bins have the same capacity).
- w_i : Weight of item i, for $i \in I$.

1.2.2 Variables and Model

- $x_{ij} \in \{0, 1\}$: 1 if item $i \in I$ is assigned to bin $j \in J$, 0 otherwise
- $y_j \in \{0, 1\}$: 1 if bin j is used, 0 otherwise

$$
\begin{aligned}
\min \quad & \sum_{j \in J} y_j && \text{(Minimize number of bins used)} && (1)\\
\text{s.t.} \quad & \sum_{j \in J} x_{ij} = 1, && i \in I, \quad \text{(Each item in one bin)}, && (2)\\
& \sum_{i \in I} w_i x_{ij} \leq k y_j, && j \in J, \quad \text{(Bin capacity constraint)}, && (3)\\
& y_j \in \{0, 1\} && j \in J, \quad \text{(Escope of variables)}, && (4)\\
& x_{ij} \in \{0, 1\} && i \in I, j \in J, \quad \text{(Escope of variables)}. && (5)\\
& && && (6)
\end{aligned}
$$

The objective function minimizes the total number of bins used. The first constraint ensures that each item is assigned to exactly one bin. The second constraint guarantees that the total weight of the items in any given bin does not exceed the bin's capacity.

1.3 Numerical answers

The optimal objective function for instance 1 is:
The optimal objective function for instance 2 is: