MECHANISM DESIGN

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Lecture #9 – 02/21/2022

CMSC498T
Mondays & Wednesdays
2:00pm – 3:15pm

COMPUTER SCIENCE
UNIVERSITY OF MARYLAND

QUIZ REVIEW!

LAST WEEK

Stable marriage problem

- Bipartite, one vertex to one vertex
- Gale-Shapley can always find this in poly-time by having jockeys propose to horses, but this favors jockeys
- There are lots of variants of the problem that break theory

Stable roommates problem

- Not bipartite, one vertex to one vertex
- Irving's algorithm finds a stable matching if it exists, otherwise reports failure

Hospitals/Residents problem

- Bipartite, one vertex to many vertices
- Actually used in practice (NRMP, lawyers, sororities)
- Lots of finicky details for handling complementaries

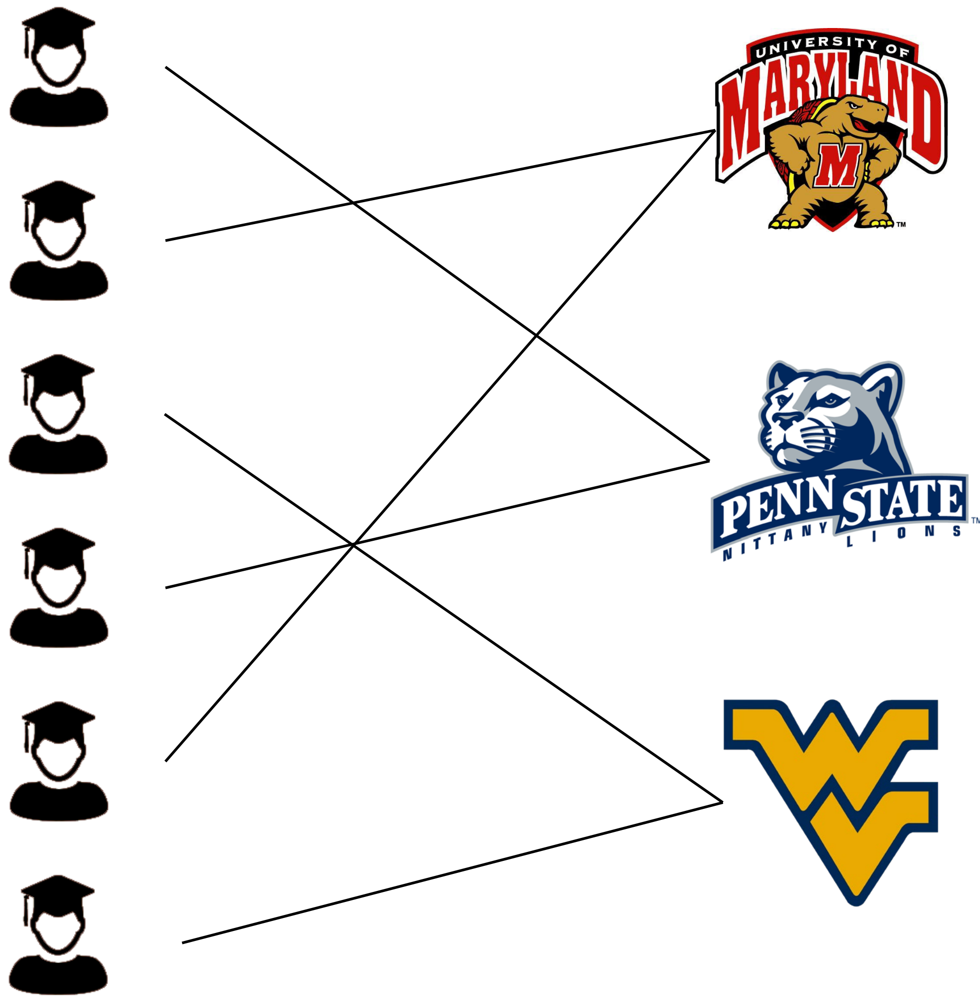
THIS CLASS:
THE AFFILIATE MATCHING PROBLEM (DOOLEY & DICKERSON '20)

ACTIVITY TIME!

Take this survey:

https://tinyurl.com/affmatch

THE BASIS: ONE-TO-MANY



Capacity 2

Capacity 2

Capacity 2

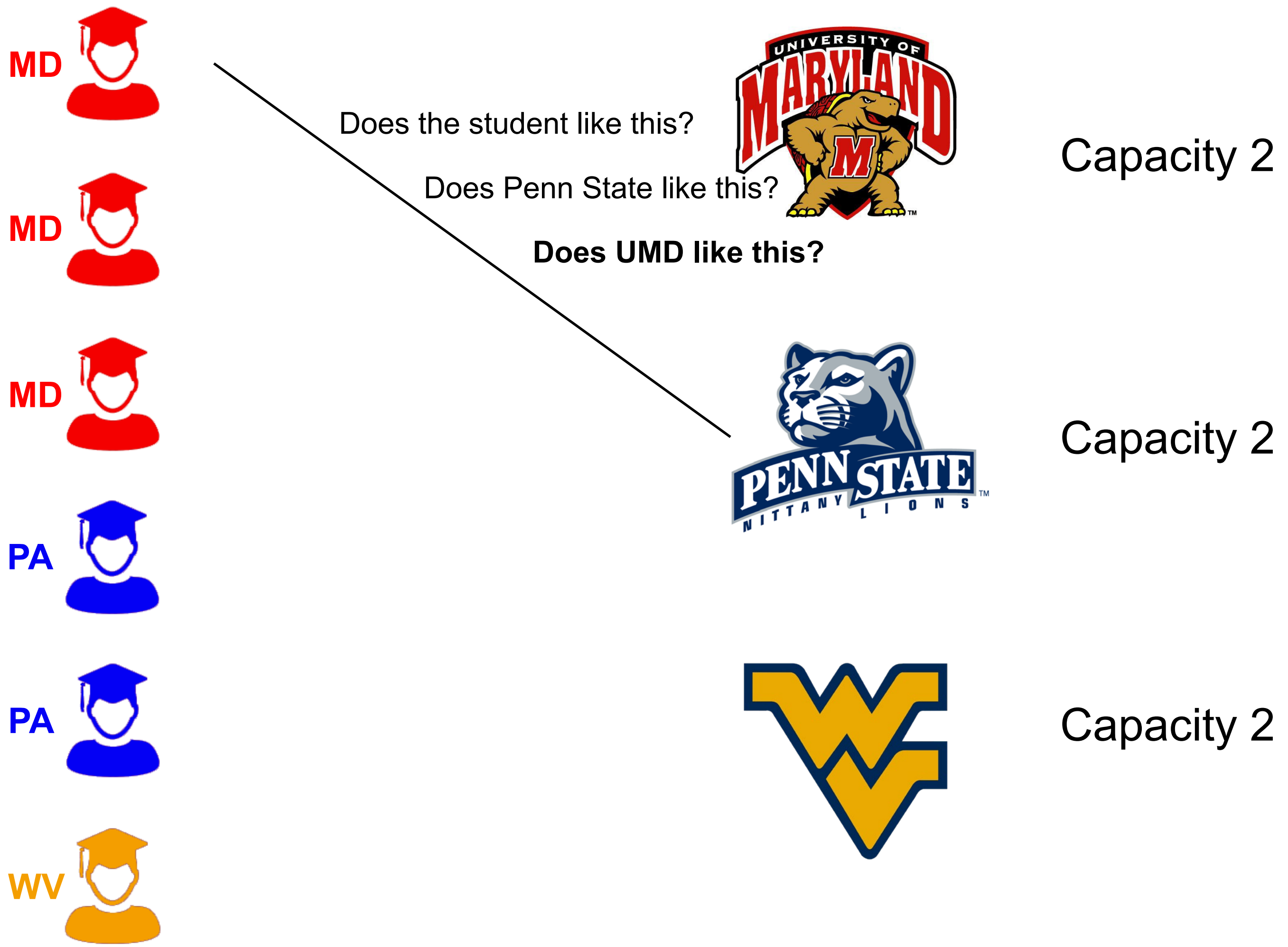
AFFILIATIONS

Capacity 2

Capacity 2

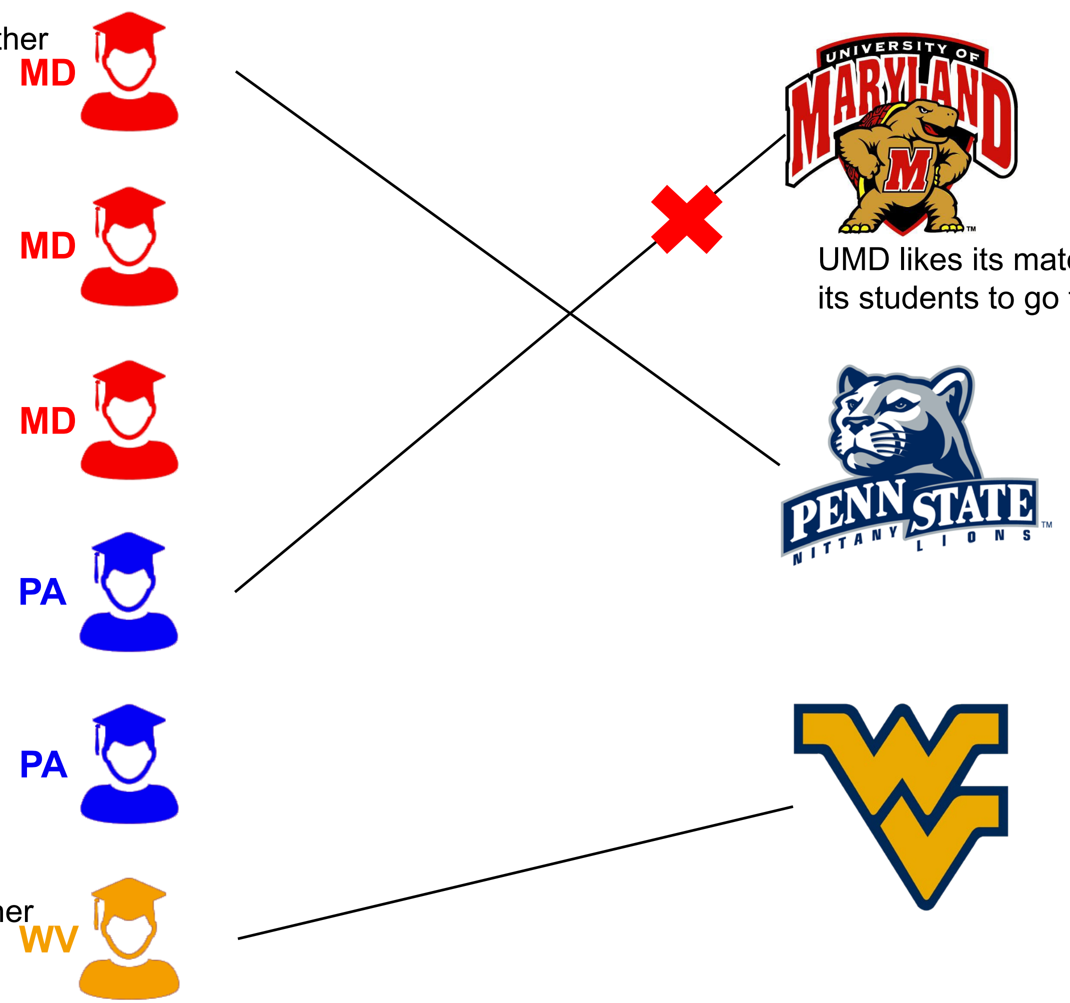
Capacity 2

AFFILIATIONS



AFFILIATIONS

Student would rather match to WVU



Capacity 1

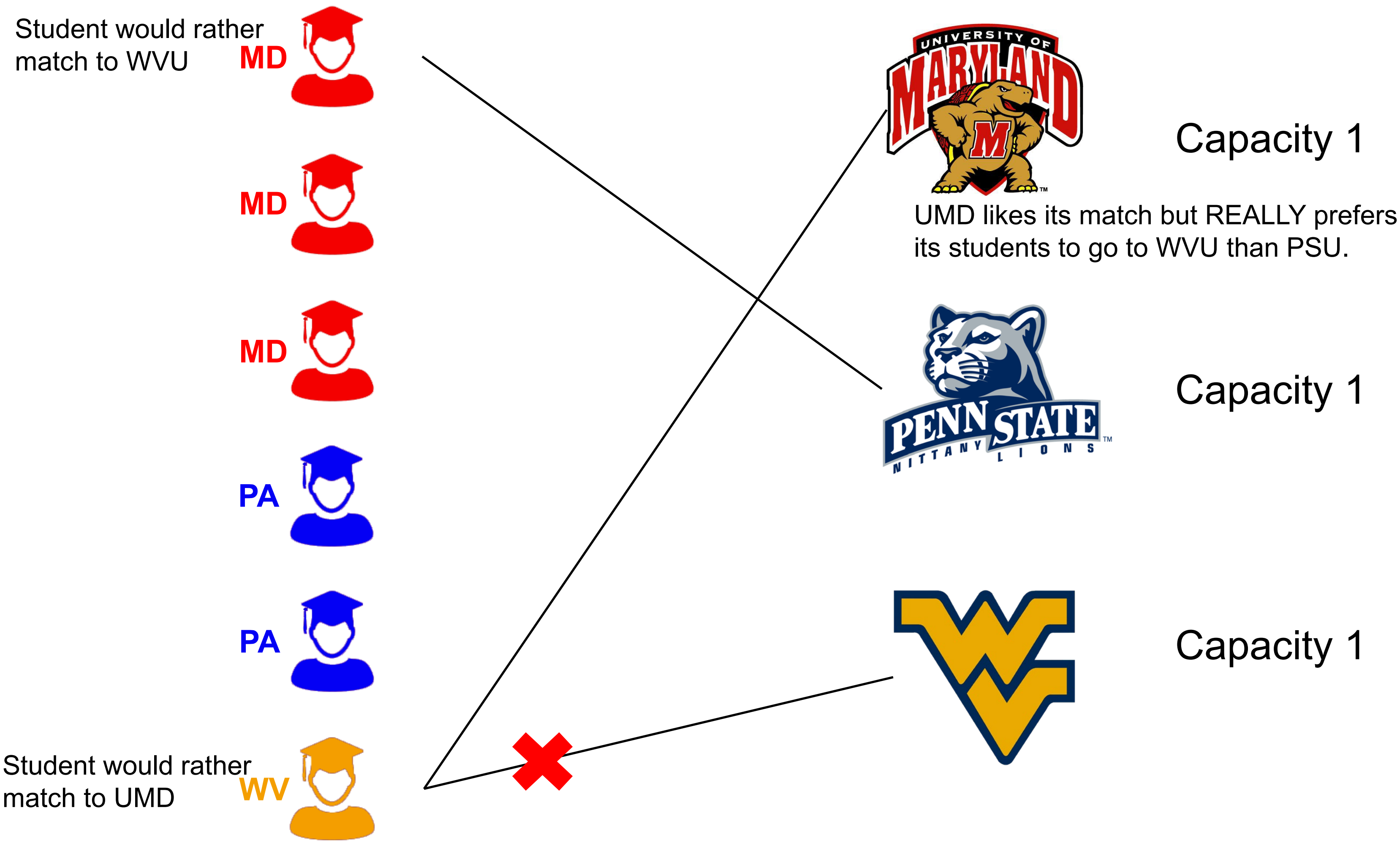
UMD likes its match but REALLY prefers its students to go to WVU than PSU.

Capacity 1

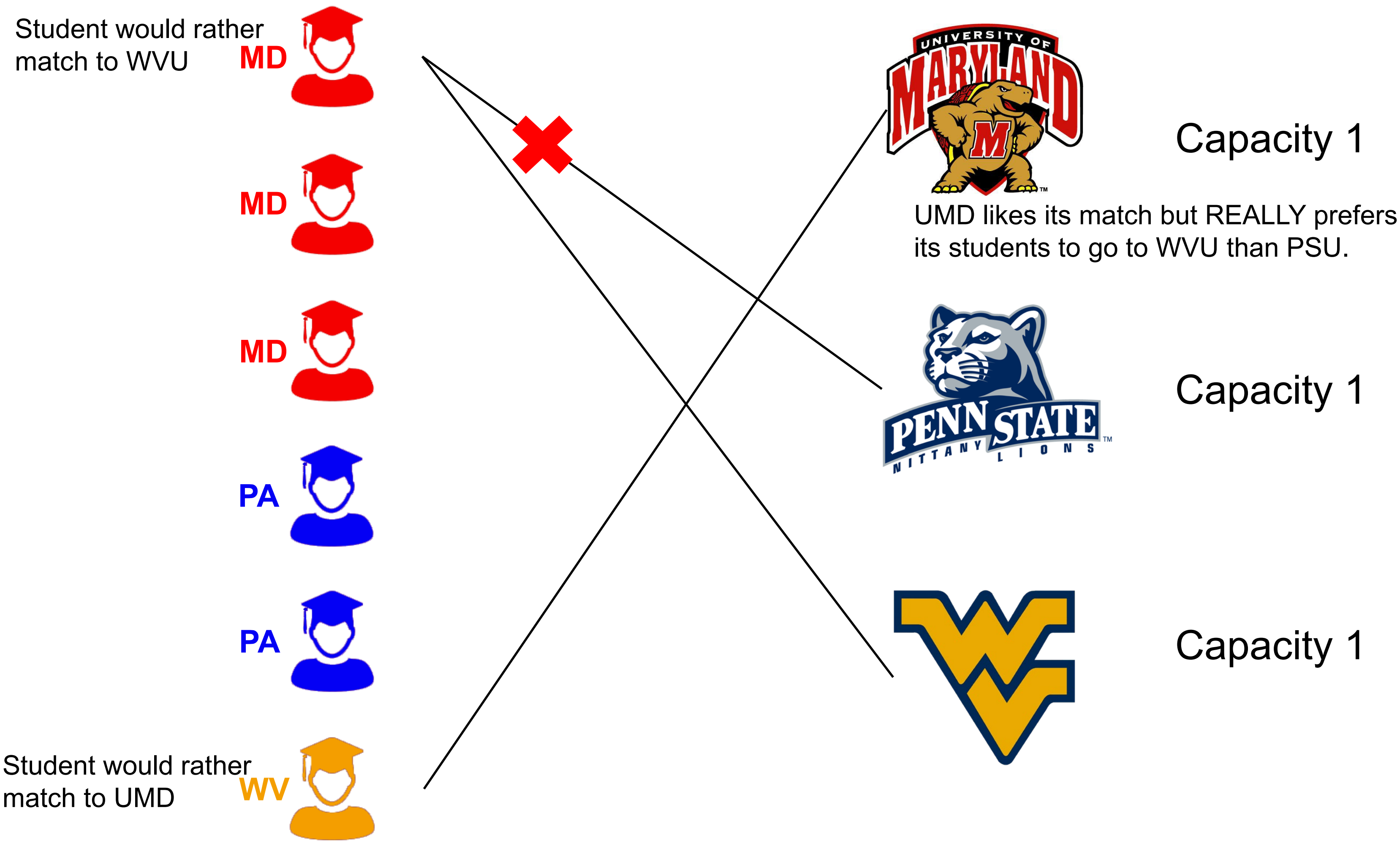
Capacity 1

Student would rather match to UMD

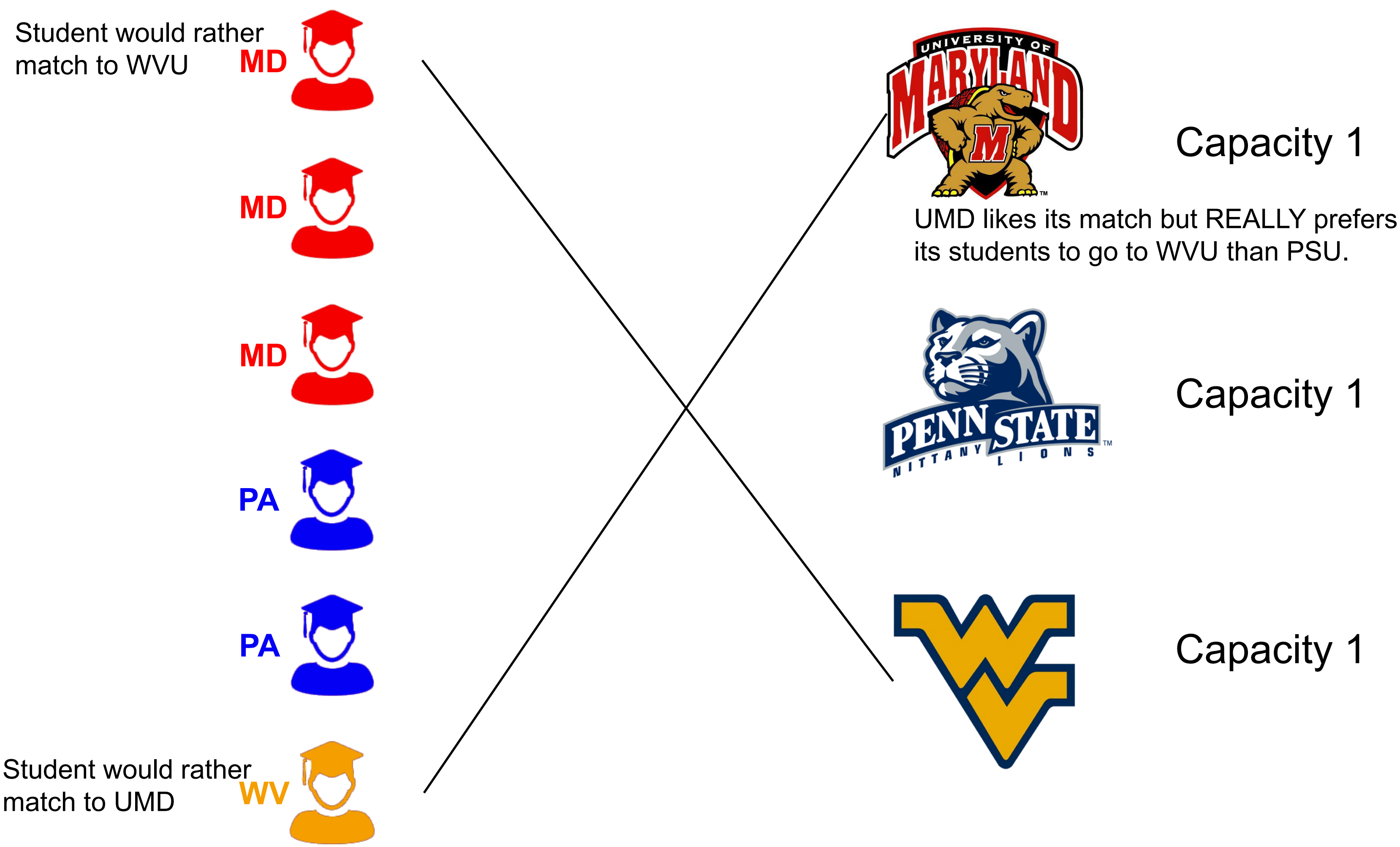
AFFILIATIONS



AFFILIATIONS



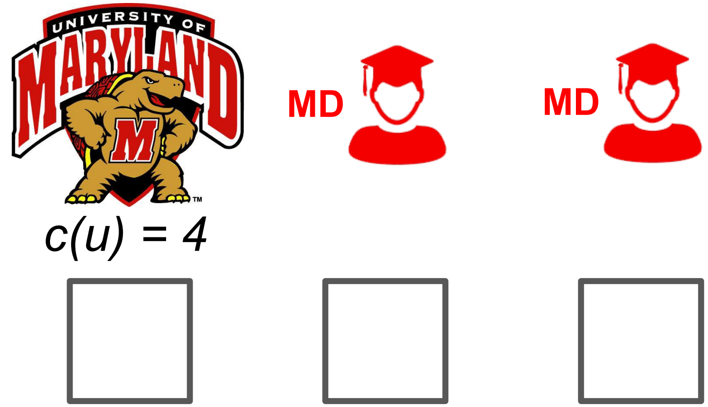
AFFILIATIONS



AFFILIATIONS: A MATHEMATICAL FORMULATION

Let U be the universities and S be the students.

Student preferences are just a ranked list over universities.



What about university preferences?

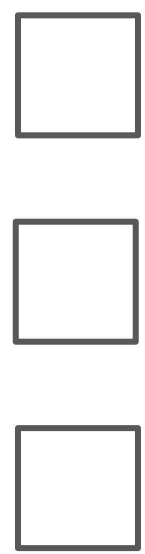
Let the capacity of a university be $c(u)$

For a university u, let its affiliates be $aff(u)$ where $n(u)=|aff(u)|$

- $n(u)$ is the number of affiliates

u has a ranked list over $S^{c(u)} \times U^{n(u)}$

- Its and its affiliates matchings



UMD cares about:

- Its 4 matches
- Affiliate 1 match
- Affiliate 2 match

This is a 6-tuple:
$(s, s', s'', s''', u', u'')$

*Note: this notation **differs** from the associated paper*

UNDERLYING PREFERENCES

MD Alex

PA Ryan

WV Taylor

Capacity 1

Capacity 1

Capacity 1

Setting

- 3 schools: UMD, PSU, WVU
- 3 students: Alex, Ryan, Taylor

Say UMD has a general preference over applicants it wants and schools it likes for Alex.

- *Alex* $>_{UMD}$ *Ryan* $>_{UMD}$ *Taylor*
- *Penn State* $>_{UMD}$ *Maryland* $>_{UMD}$ *West Virginia*

How will UMD rank overall matchings? It only cares about it's and Alex's match.

Represent a matching as: *(UMD's match, Alex's match)*

UNDERLYING PREFERENCES

MD Alex

PA Ryan

WV Taylor

Capacity 1

Capacity 1

Capacity 1

Alex $>_{UMD}$ *Ryan* $>_{UMD}$ *Taylor*

Penn State $>_{UMD}$ *Maryland* $>_{UMD}$ *West Virginia*

Matchings: (UMD's match, Alex's match)

Consider the following *partial* ranks UMD might have over matchings:

1. *(Alex, PSU)* $>_{UMD}$ *(Alex, WVU)* $>_{UMD}$ *(Ryan, UMD)*
2. *(Alex, WVU)* $>_{UMD}$ *(Alex, PSU)* $>_{UMD}$ *(Ryan, UMD)*
3. *(Alex, UMD)* $>_{UMD}$ *(Taylor, PSU)* $>_{UMD}$ *(Ryan, PSU)*
4. *(Alex, UMD)* $>_{UMD}$ *(Ryan, PSU)* $>_{UMD}$ *(Taylor, PSU)*

Which of these matches are impossible?

Disregarding impossible matches, which of these seem "rational" given UMD's underlying preferences?

UNDERLYING PREFERENCES

Alex $>_{UMD}$ Ryan $>_{UMD}$ Taylor

Penn State $>_{UMD}$ Maryland $>_{UMD}$ West Virginia

Matchings: (UMD's match, Alex's match)

MD Alex

PA Ryan

WV Taylor

Capacity 1

Capacity 1

Capacity 1

Consider the following *partial* ranks UMD might have over matchings:

1. ***(Alex, PSU) $>_{UMD}$ (Alex, WVU) $>_{UMD}$ (Ryan, UMD)***
2. ***(Alex, WVU) $>_{UMD}$ (Alex, PSU) $>_{UMD}$ (Ryan, UMD)***
3. *(Alex, UMD) $>_{UMD}$ (Taylor, PSU) $>_{UMD}$ (Ryan, PSU)*
4. *(Alex, UMD) $>_{UMD}$ (Ryan, PSU) $>_{UMD}$ (Taylor, PSU)*

Which of these matches are impossible?

(Alex, PSU): how can UMD be matched to Alex, but Alex is matched to PSU?

Disregarding impossible matches, which of these seem "rational" given UMD's underlying preferences?

UNDERLYING PREFERENCES

MD Alex

PA Ryan

WV Taylor

Capacity 1

Capacity 1

Capacity 1

Alex $>_{UMD}$ Ryan $>_{UMD}$ Taylor

Penn State $>_{UMD}$ Maryland $>_{UMD}$ West Virginia

Matchings: (UMD's match, Alex's match)

Consider the following *partial* ranks UMD might have over matchings:

1. *(Alex, PSU) $>_{UMD}$ (Alex, WVU) $>_{UMD}$ (Ryan, UMD)*
2. *(Alex, WVU) $>_{UMD}$ (Alex, PSU) $>_{UMD}$ (Ryan, UMD)*
3. *(Alex, UMD) $>_{UMD}$ (Taylor, PSU) $>_{UMD}$ (Ryan, PSU)*
4. *(Alex, UMD) $>_{UMD}$ (Ryan, PSU) $>_{UMD}$ (Taylor, PSU)*

Which of these matches are impossible?

(Alex, PSU): how can UMD be matched to Alex, but Alex is matched to PSU?

Disregarding impossible matches, which of these seem "rational" given UMD's underlying preferences?

UNDERLYING PREFERENCES

MD Alex

PA Ryan

WV Taylor

Capacity 1

Capacity 1

Capacity 1

Alex $>_{UMD}$ Ryan $>_{UMD}$ Taylor

Penn State $>_{UMD}$ Maryland $>_{UMD}$ West Virginia

Matchings: (UMD's match, Alex's match)

Consider the following *partial* ranks UMD might have over matchings:

1. *(Alex, **PSU**) $>_{UMD}$ (Alex, **WVU**) $>_{UMD}$ (Ryan, UMD)*
2. *(Alex, **WVU**) $>_{UMD}$ (Alex, **PSU**) $>_{UMD}$ (Ryan, UMD)*
3. *(Alex, UMD) $>_{UMD}$ (Taylor, PSU) $>_{UMD}$ (Ryan, PSU)*
4. *(Alex, UMD) $>_{UMD}$ (Ryan, PSU) $>_{UMD}$ (Taylor, PSU)*

Which of these matches are impossible?

(Alex, PSU): how can UMD be matched to Alex, but Alex is matched to PSU?

Disregarding impossible matches, which of these seem "rational" given UMD's underlying preferences?

1 seems rational, 2 does not.

UNDERLYING PREFERENCES

MD Alex

PA Ryan

WV Taylor

Capacity 1

Capacity 1

Capacity 1

Alex $>_{UMD}$ Ryan $>_{UMD}$ Taylor

Penn State $>_{UMD}$ Maryland $>_{UMD}$ West Virginia

Matchings: (UMD's match, Alex's match)

Consider the following *partial* ranks UMD might have over matchings:

1. *(Alex, PSU) $>_{UMD}$ (Alex, WVU) $>_{UMD}$ (Ryan, UMD)*
2. *(Alex, WVU) $>_{UMD}$ (Alex, PSU) $>_{UMD}$ (Ryan, UMD)*
3. *(Alex, UMD) $>_{UMD}$ (**Taylor**, PSU) $>_{UMD}$ (**Ryan**, PSU)*
4. *(Alex, UMD) $>_{UMD}$ (**Ryan**, PSU) $>_{UMD}$ (**Taylor**, PSU)*

Which of these matches are impossible?

(Alex, PSU): how can UMD be matched to Alex, but Alex is matched to PSU?

Disregarding impossible matches, which of these seem "rational" given UMD's underlying preferences?

1 seems rational, 2 does not.

4 seems rational, 3 does not.

How do we capture "rationality"?

CONSISTENT PREFERENCES

In this model, we call "rational" preferences ***consistent***. Say $>_u$ is *u*'s preference over complete matchings, $>'_u$ is its preference over students, and $>''_u$ is its preference over universities.

Formally:

> An employer's preference profile $>_u$ *consistent* with $>'_u$ (resp. $>''_u$) if the ordering of the first element of each tuple preserves $>'_u$ *(resp.* $>''_u$ *).*

Alex $>'_{UMD}$ *Ryan* $>'_{UMD}$ *Taylor*

Penn State $>''_{UMD}$ *Maryland* $>''_{UMD}$ *West Virginia*

Then which is consistent with $>'_{UMD}$ and $>''_{UMD}$?

(Ryan, PSU) $>_{UMD}$ *(Taylor, PSU)* $>_{UMD}$ *(Ryan, WVU)*

(Ryan, PSU) $>_{UMD}$ *(Ryan, WVU)* $>_{UMD}$ *(Taylor, PSU)*

CONSISTENT PREFERENCES

In this model, we call "rational" preferences ***consistent***. Say $>_u$ is *u*'s preference over complete matchings, $>'_u$ is its preference over students, and $>''_u$ is its preference over universities.

Formally:

> An employer's preference profile $>_u$ *consistent* with $>'_u$ (resp. $>''_u$) if the ordering of the first element of each tuple preserves $>'_u$ *(resp.* $>''_u$ *).*

Alex $>'_{UMD}$ *Ryan* $>'_{UMD}$ *Taylor*

Penn State $>''_{UMD}$ *Maryland* $>''_{UMD}$ *West Virginia*

Then which is consistent with $>'_{UMD}$ and $>''_{UMD}$?

(Ryan, PSU) $>_{UMD}$ *(Taylor, PSU)* $>_{UMD}$ *(Ryan, WVU)* $>''_{UMD}$

(Ryan, PSU) $>_{UMD}$ *(Ryan, WVU)* $>_{UMD}$ *(Taylor, PSU)* $>'_{UMD}$

We call this "affiliate-agnostic"

SURVEY: DO REAL PREFERENCES VARY?

You are BMU, your affiliate is Ryan.

BMU and Ryan top tier ← → *BMU and Ryan bottom tier*

	Scenario								
	1	2	3	4	5	6	7	8	9
Top 1	80%	49%	40%	47%	50%	42%	38%	42%	38%
Top 2	48%	26%	24%	27%	25%	18%	20%	22%	18%
Top 3	19%	20%	13%	23%	18%	12%	11%	10%	11%
Top 4	18%	15%	12%	14%	13%	10%	9%	9%	9%
Top 5	18%	15%	12%	14%	13%	10%	9%	9%	9%

Table 1: Percentage agreement for *most common* Top k elements of the of the preference profile across the nine scenarios. The 'Top 1' condition reports the percentage of respondents who agreed with the most common outcome which they *most prefer*. The 'Top 2' condition reports the percentage of respondents who agreed with the most common outcome for the *most and second most preferred*. 'Top 5' reports the percentage of respondents who agreed on the most common full preference profile $\succ$.

Takeaways:

- 1/10 – 1/5 respondents agreed on complete profiles each question
- Agreement is much higher for Top 1 and 2 (anecdotal strategies)
- Agreement is higher when BMU and Ryan are top-tier

ARE PEOPLE CONSISTENT?

In other words, did people agree with any consistent profile (there are four possible options)?

- This varied from 1/100 to 1/4 respondents
- Generally higher when BMU and Ryan are higher tier
- 1st, 4th, 7th: Ryan is top-tier -> consistency is higher

BMU and Ryan top tier ← → *BMU and Ryan bottom tier*

	Scenario								
	1	2	3	4	5	6	7	8	9
Consistency Frequency	26%	7%	4%	14%	4%	4%	9%	2%	1%

Table 2: Percentage of full preference profiles which were consistent for each scenario.

SURVEY LIMITATIONS

What are some survey limitations (either ones you know where true or suspect are true)?

- Respondents were non-experts in faculty hiring
  - Social desirability: you respond in ways you think would be viewed as favorable, if someone were to see your responses
  - More noisy
- Only 154 “successful” respondents
- Priming: randomly assign some participants to believe prioritizing affiliate’s matches would be good
  - No effect was found – either priming was done poorly or there is simply no effect in this setting

GREEDY STABILITY

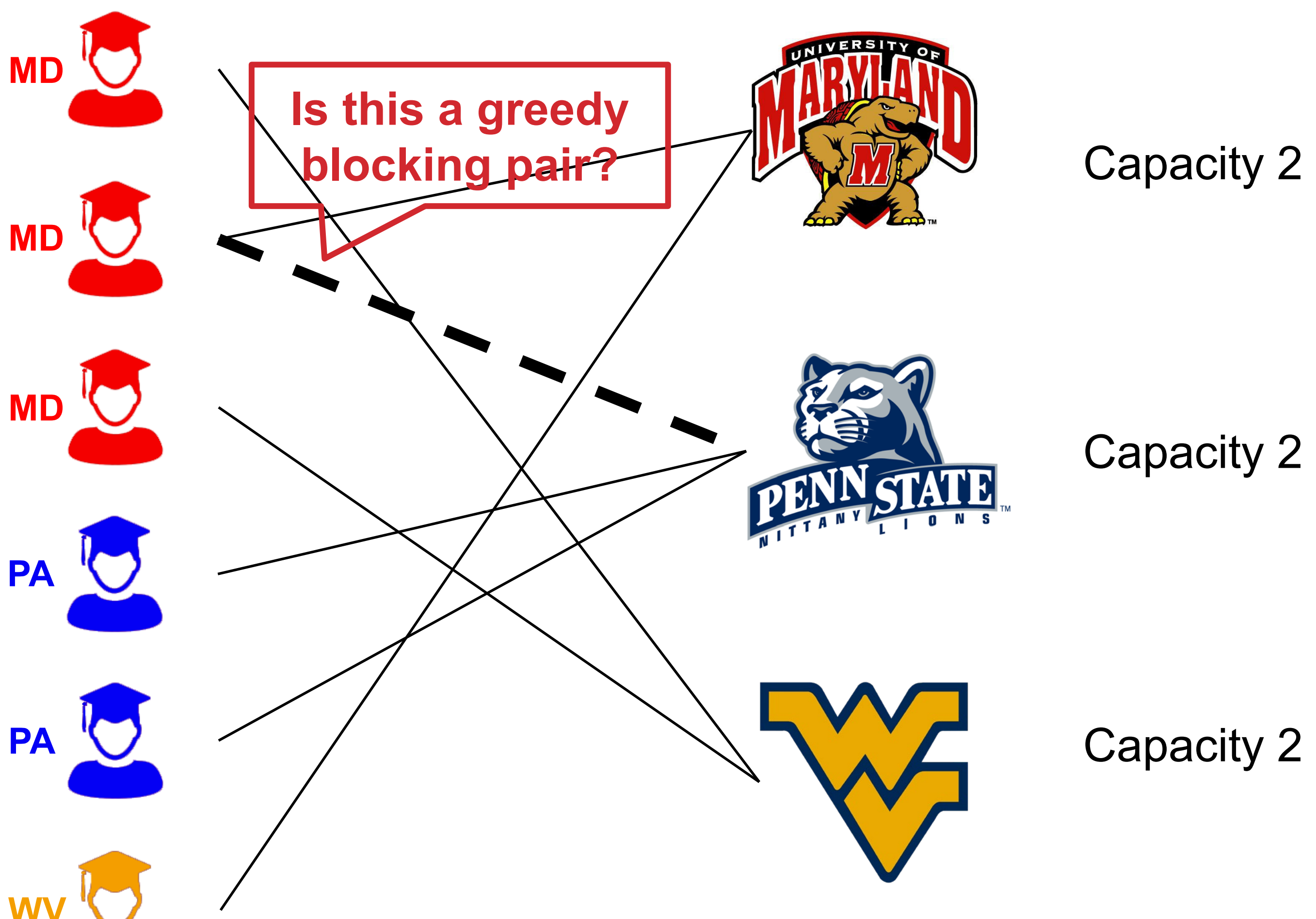
Consider a matching *M*, and say the match of any agent *a* under *M* is *M(a).*

(Alex, UMD) is a ***greedy blocking pair*** iff $UMD >_{Alex} M(Alex)$ and there exists some other matching *M'* where $M'(Alex) = UMD$ and $(M'(UMD), M'(aff(UMD))) >_{UMD} (M(UMD), M(aff(UMD)))$.

- AKA, *(Alex, UMD)* is a greedy blocking pair exactly when they are not matched and there is another matching that matches them which both prefer

A matching is ***greedy stable*** if there are no greedy blocking pairs.

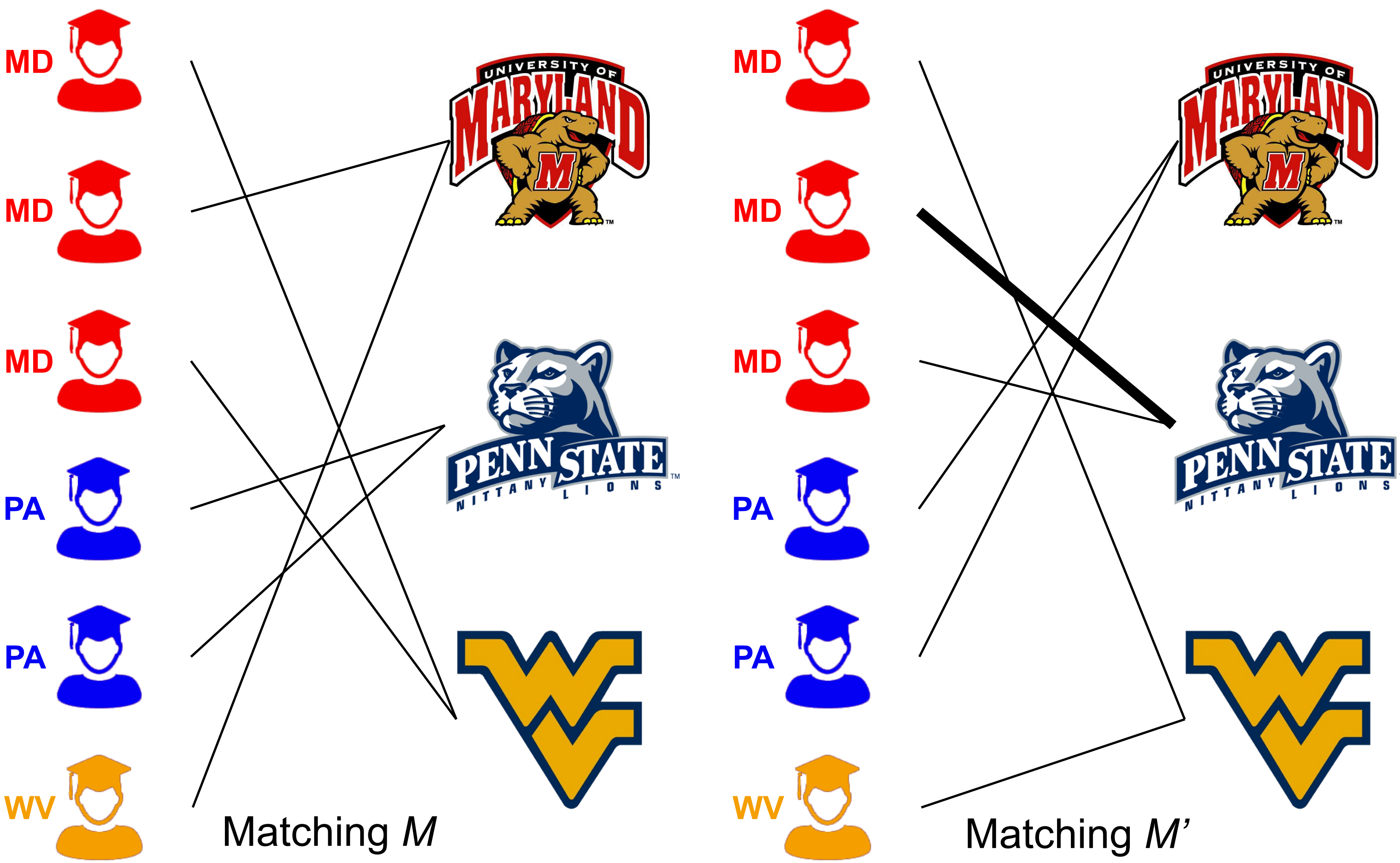
GREEDY STABILITY



Matching *M*

GREEDY STABILITY

If the newly matched pair is happier in *M'*, then *M* is unstable



PROPERTIES OF GREEDY STABILITY

Recall: A marketplace is *affiliate-agnostic* if all university preferences are consistent with their preferences over students.

- AKA: Universities care about their own matches first and foremost, and then their students' matches

Proposition

In an affiliate-agnostic marketplace, the problem reduces to stable marriage.

PROPERTIES OF GREEDY STABILITY

We no longer assume the marketplace is affiliate-agnostic.

Then, there may be no stable matchings.

WV > PA > MD	MD A	MD	*(R, WV) > (A, MD) > (T, PA) > (R, PA) > (T, WV)*
MD > WV > PA	PA R	PA	*(A, WV) > (T, WV) > (A, MD) > (T, MD) > (R, PA)*
WV > MD > PA	WV T	WV	*(A, MD) > (R, MD) > (T, WV) > (R, PA) > (A, PA)*

INT. LINEAR PROGRAM FOR GREEDY STABILITY

We do not have any combinatorial strategies so far, so an ILP is the best we have. Additionally, it allows us to encode any optimization function we'd like. Say all capacities are zero.

Let x_{su} for each student $s \in \{1, \dots, n\}$ and university $u \in \{1, \dots, m\}$ indicate if s and u are matched.

This looks gross! But if you ignore this part it looks the same as our stable marriage ILP

Optimize: ***whatever***

Subject to:

$\sum_{u' >_s u} x_{su'} + \sum_{s', M' : M'(s') = u, (M'(u), M'(aff(u))) >_u (M(u), M(aff(u)))} x_{s'u} + x_{su} \geq 1$ **for all** $s \in \{1, \dots, n\}$ **and** $u \in \{1, \dots, m\}$

$x_{su} \in \{0, 1\}$ **for all** $s \in \{1, \dots, n\}$ **and** $u \in \{1, \dots, m\}$

DESIGNING AFFILIATE MATCHING MECHANISMS

Does an affiliate matching mechanism reify notions of prestige in a way that produces harm?

- If you go to a prestigious school, you likely access to better resources that give you a better outcome.

How do current affiliate marketplaces operate?

- Can this model be used in some way? Is preference elicitation reasonable? Can we make it strategy-proof?

How much do employers care about their affiliates? How much ***should*** *they care?*

- Qualitative results in the survey yielded many different philosophies.

What is the right definition of stability?

- Greedy stability is not the only notion of stability. Is it too weak? To strong? Helpful? Unhelpful?

EXTENSION: DICHOTOMOUS PREFERENCES

Similar existing marketplaces are simplified: each applicant submits their top 2 preferences, schools do initial acceptances based off of this, then it is decentralized.

Why is this good?

- It's easy for people to submit their top 2

Affiliate matching: should students give entire rankings over schools? Can they? Can schools give entire rankings over tuples?

Simplification: Everyone has a binary approval/disapproval indicator for each university/student

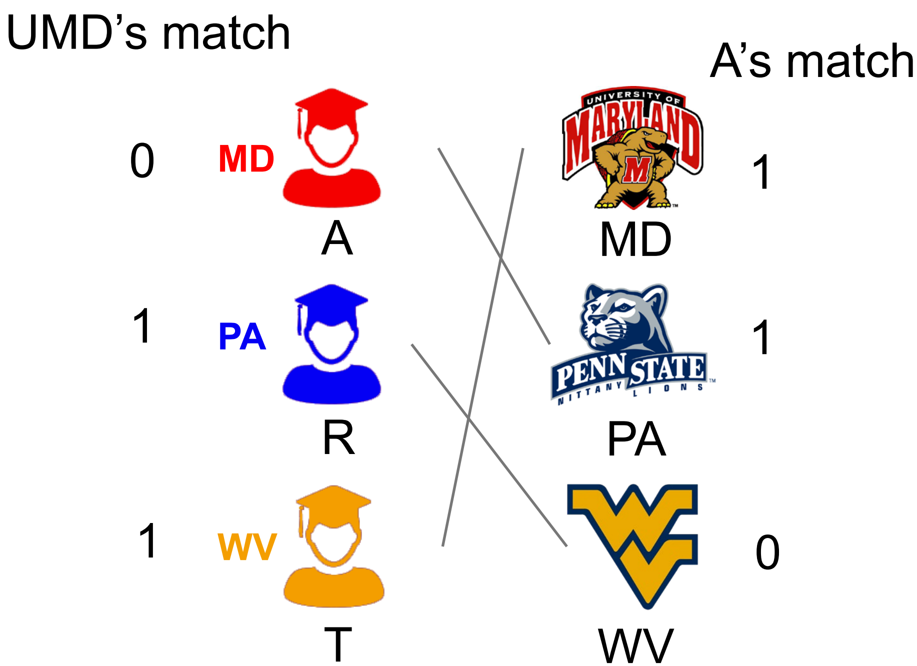
EXTENSION: DICHOTOMOUS PREFERENCES

UMD's Preferences:

- Approve or disapprove of each student for itself
- Approve or disapprove of each university for each affiliate

Value of a match:

- Start at 0
- Add 1 for every match it receives that it likes
- Add λ for every match an affiliate receives that it likes



Value of match: 1+ λ

THE VALUE OF DICHOTOMOUS PREFS

Preference elicitation – simple, easy to get, will never contradict itself

Problem solvability – there is always a stable solution and it can be found efficiently (even in many-to-many matches)

Problems? – is it over simplified? Is our notion of valuation "right"? Does this really model real world problems

Proposed problems – faculty hiring *interviews*, playdates!, study abroad, student projects, dog breeding

HERE IS A DOGGO