

Coefficients of conservative second order derivative

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Calculation of fourth-order accurate second derivative

- If $i = j$ (Equation 15 (Algorithm-Eng document))

$$\begin{aligned} \frac{\partial}{\partial x_j} \left(\frac{\partial u}{\partial x_i} \right) = & \\ u_{i+3} & \left(\frac{1}{24\Delta x} \frac{1}{24\Delta x} \right) + \\ u_{i+2} & \left(-\frac{9}{8\Delta x} \frac{1}{24\Delta x} - \frac{1}{24\Delta x} \frac{9}{8\Delta x} \right) + \\ u_{i+1} & \left(\frac{9}{8\Delta x} \frac{9}{8\Delta x} + \frac{9}{8\Delta x} \frac{1}{24\Delta x} + \frac{1}{24\Delta x} \frac{9}{8\Delta x} \right) + \\ u_i & \left(-\frac{9}{8\Delta x} \frac{9}{8\Delta x} - \frac{9}{8\Delta x} \frac{9}{8\Delta x} - \frac{1}{24\Delta x} \frac{1}{24\Delta x} - \frac{1}{24\Delta x} \frac{1}{24\Delta x} \right) + \\ u_{i-1} & \left(\frac{9}{8\Delta x} \frac{9}{8\Delta x} + \frac{9}{8\Delta x} \frac{1}{24\Delta x} + \frac{1}{24\Delta x} \frac{9}{8\Delta x} \right) + \\ u_{i-2} & \left(-\frac{9}{8\Delta x} \frac{1}{24\Delta x} - \frac{1}{24\Delta x} \frac{9}{8\Delta x} \right) + \\ u_{i-3} & \left(\frac{1}{24\Delta x} \frac{1}{24\Delta x} \right) \end{aligned}$$

- if $i \neq j$ ((Equation 17 (Algorithm-Eng document)))

$$\begin{aligned}
& \frac{\partial}{\partial x_j} \left(\frac{\partial u}{\partial x_i} \right) = \\
& \frac{4}{6\Delta x_i} \left(\frac{4}{3} \left(\frac{u_{i+1,j+1} - u_{i+1,j-1}}{2\Delta x_j} \right) - \frac{1}{3} \left(\frac{u_{i+1,j+2} - u_{i+1,j-2}}{4\Delta x_j} \right) \right. \\
& \quad \left. - \left(\frac{4}{3} \left(\frac{u_{i-1,j+1} - u_{i-1,j-1}}{2\Delta x_j} \right) - \frac{1}{3} \left(\frac{u_{i-1,j+2} - u_{i-1,j-2}}{4\Delta x_j} \right) \right) \right) \\
& \quad - \frac{1}{12\Delta x_i} \left(\frac{4}{3} \left(\frac{u_{i+2,j+1} - u_{i+2,j-1}}{2\Delta x_j} \right) - \frac{1}{3} \left(\frac{u_{i+2,j+2} - u_{i+2,j-2}}{4\Delta x_j} \right) \right. \\
& \quad \left. - \left(\frac{4}{3} \left(\frac{u_{i-2,j+1} - u_{i-2,j-1}}{2\Delta x_j} \right) - \frac{1}{3} \left(\frac{u_{i-2,j+2} - u_{i-2,j-2}}{4\Delta x_j} \right) \right) \right)
\end{aligned}$$

- Second Order Accurate derivative in non-uniform scheme

$$\begin{aligned}
& u_{i+1} \left(\frac{y_i - y_{i-1}}{(y_{i+0.5} - y_{i-0.5})(y_{i+1} - y_i)(y_i - y_{i-1}))} \right) + \\
& u_i \left(\frac{(y_{i+1} - y_i)(y_i - y_{i-1})}{(y_{i+0.5} - y_{i-0.5})(y_{i+1} - y_i)(y_i - y_{i-1}))} \right) + \\
& u_{i-1} \left(\frac{y_{i+1} - y_i}{(y_{i+0.5} - y_{i-0.5})(y_{i+1} - y_i)(y_i - y_{i-1}))} \right)
\end{aligned}$$