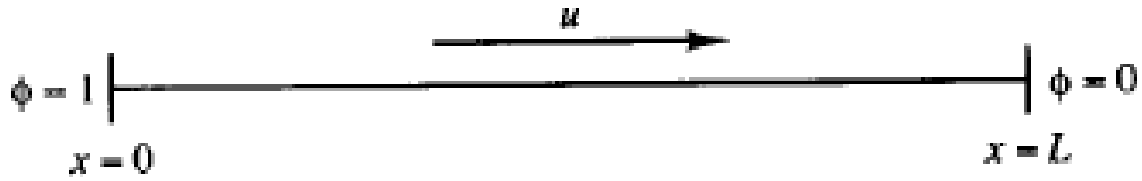
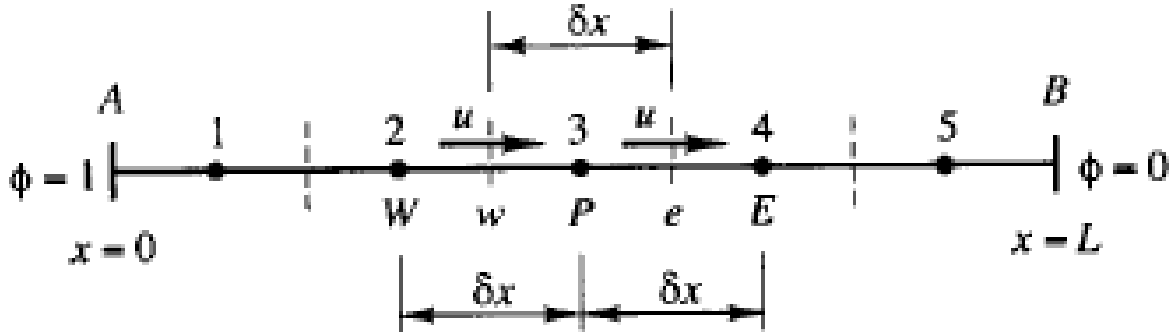
5.3 The central differencing scheme

Example 5.1 1-D convection-diffusion problem with $\phi_o = 1$, $\phi_L = 0$



$$\frac{\phi - \phi_0}{\phi_L - \phi_0} = \frac{\exp(\rho u x / \Gamma) - 1}{\exp(\rho u L / \Gamma) - 1} \qquad (5.15)$$ Exact solution



5.6 The upwind differencing scheme

Example 5.2 Upwind scheme for (i) u=0.1 m/s, (ii) u=2.5 m/s with 5-point grid

$F = F_e = F_w = \rho u$ and $D = D_e = D_w = \Gamma / \delta x$ everywhere.

$$F_e\phi_P - F_A\phi_A = D_e(\phi_E - \phi_P) - D_A(\phi_P - \phi_A) \qquad (5.32)$$

$$F_B\phi_P - F_w\phi_W = D_B(\phi_B - \phi_P) - D_w(\phi_P - \phi_W) \qquad (5.33)$$

$D_A = D_B = 2\Gamma / \delta x = 2D$ and $F_A = F_B$

$$a_P\phi_P = a_W\phi_W + a_E\phi_E + S_u \qquad (5.34)$$

$$a_P = a_W + a_E + (F_e - F_w) - S_P$$

5.7 The hybrid differencing scheme

Example 5.3; Hybid scheme for u = 2.5 m/s, 5-point and 25-point grid

$$F_e\phi_P - F_A\phi_A = 0 - D_A(\phi_P - \phi_A) \qquad (5.38)$$

$$F_B\phi_P - F_w\phi_W = D_B(\phi_B - \phi_P) - 0 \qquad (5.39)$$

$$a_P\phi_P = a_W\phi_W + a_E\phi_E + S_u \qquad (5.40)$$

$$a_P = a_W + a_E + (F_e - F_w) - S_P$$

Node	a_W	a_E	S_p	S_u
1	0	0	$-(2D+F)$	$(2D+F)\phi_A$
2, 3, 4	F	0	0	0
5	F	0	$-2D$	$2D\phi_B$

5.9 Higher order differencing schemes

Example 5.4; QUICK for u = 0.2 m/s 5-point grid

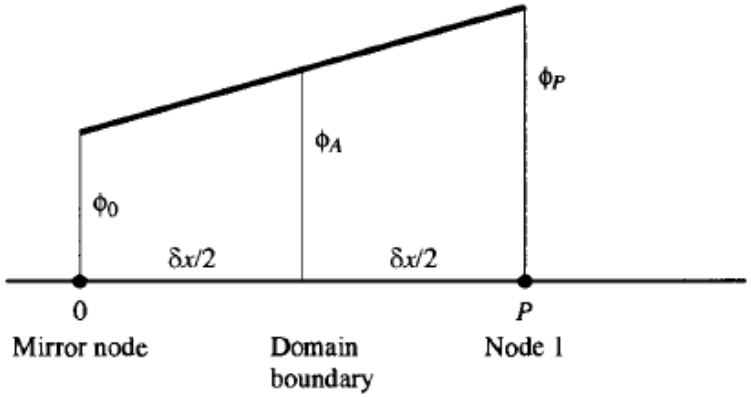


Fig 5.18

$$\phi_0 = 2\phi_A - \phi_P \qquad (5.52)$$

$$\phi_e = \frac{6}{8}\phi_P + \frac{3}{8}\phi_E - \frac{1}{8}(2\phi_A - \phi_P) = \frac{7}{8}\phi_P + \frac{3}{8}\phi_E - \frac{2}{8}\phi_A \qquad (5.53)$$

$$\Gamma \left.\frac{\partial \phi}{\partial x}\right|_A = \frac{D_A}{3}(9\phi_P - 8\phi_A - \phi_E) \qquad (5.54)$$