FE 5217: Seminar in Risk Management and Alternative Investment: Algorithmic Trading and Quantitative Strategies–Assignment 1, Solutions

1. (a) Table 1 shows the sample mean $\hat{\mu}$, standard deviation $\hat{\sigma}$ and first-order autocorrelation coefficient $\hat{\rho}$ for daily simple returns over the entire sample period for the 15 stocks and two indexes. Figure 1, 2 and 3 present the means, standard deviations and first-order autocorrelation coefficients for 15 stocks for four equal subperiods respectively. These three figures show that AIG is not stable.

Similarly, Figure 4, 5 and 6 show the means, standard deviations and first-order autocorrelation coefficients of the two indexes (VWRETD, EWRETD) for four subperiods. From them, we can see that these two indexes are stable.

1. (b) The left panel in Figure 7 (8) presents the histogram of returns of VWRETD (EWRETD) for sample period and the right panel presents the histogram of normal distribution with mean and variance equal to the sample mean and variance of the returns of VWRETD (EWRETD). The red solid line represents the density of normal distribution with same sample mean and variance. We can see the returns of the two indexes distribute symmetrically and have a heavier tail than normal distribution with same sample mean and variance. Comparing the estimated density of returns to density of normal, we can conclude that EWRETD is closer to normal distribution.

1.(c) Table 2 presents 99% confidence intervals for 15 stocks and two indexes and Figure 9 shows means and confidence intervals for 15 stocks. From Figure 9 we can see AIG has larger confidence interval than the other stocks.

Figure 10 shows the 99% confidence intervals of 15 stocks for four equal subperiods. We can see from it that AIG shifted a lot. It has much wider confidence interval in the third period, which is due to large standard deviation for the third period.

Figure 11 (12) shows the 99% confidence intervals of VWRETD (EWRETD) for four equal subperiods. It can be seen that both of them had wider confidence intervals

Names	mean	SD	First-order AR
MSFT	0.00009904	0.02083959	-0.04780523
T	0.00028915	0.01812412	-0.00882292
DD	0.00021848	0.01907428	-0.03507677
IBM	0.00038236	0.01771069	-0.04953134
BA	0.00049291	0.02042177	-0.01194315
MRK	0.00019233	0.01880311	0.00771627
DIS	0.00040628	0.02097049	-0.04068132
HPQ	0.00018854	0.02547714	-0.01509675
MCD	0.00049318	0.01610964	-0.02148319
JPM	0.00051539	0.02836653	-0.08481484
WMT	0.00022346	0.01627357	-0.03534592
AXP	0.00049417	0.02546958	-0.06858817
VZ	0.00030373	0.01723925	-0.02941973
HD	0.00034667	0.02197002	0.01609763
AIG	-0.00003672	0.04379550	0.13459958
VWRETD	0.00018740	0.01336952	-0.08466268
EWRETD	0.00043655	0.01471896	-0.04082585

Table 1: The sample mean, standard deviation and first-order autocorrelation coefficients for daily simple returns over the entire sample period for 15 stocks and 2 indexes

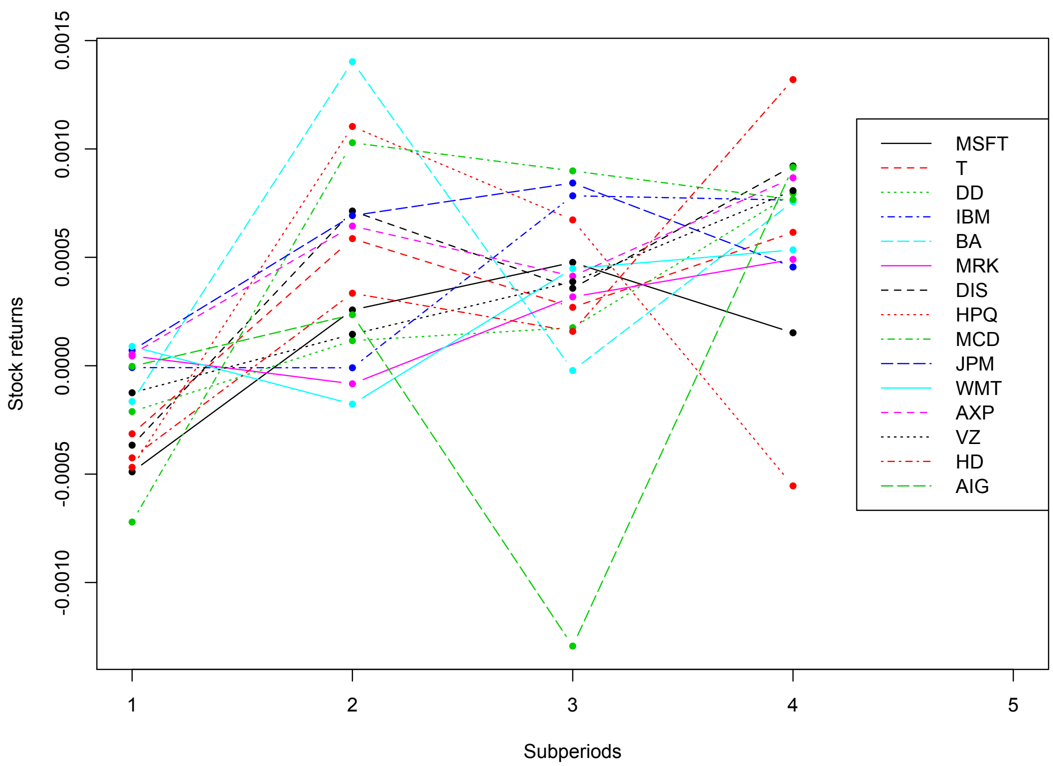

Figure 1: Plot of means of daily returns for 15 stocks for four equal subperiods

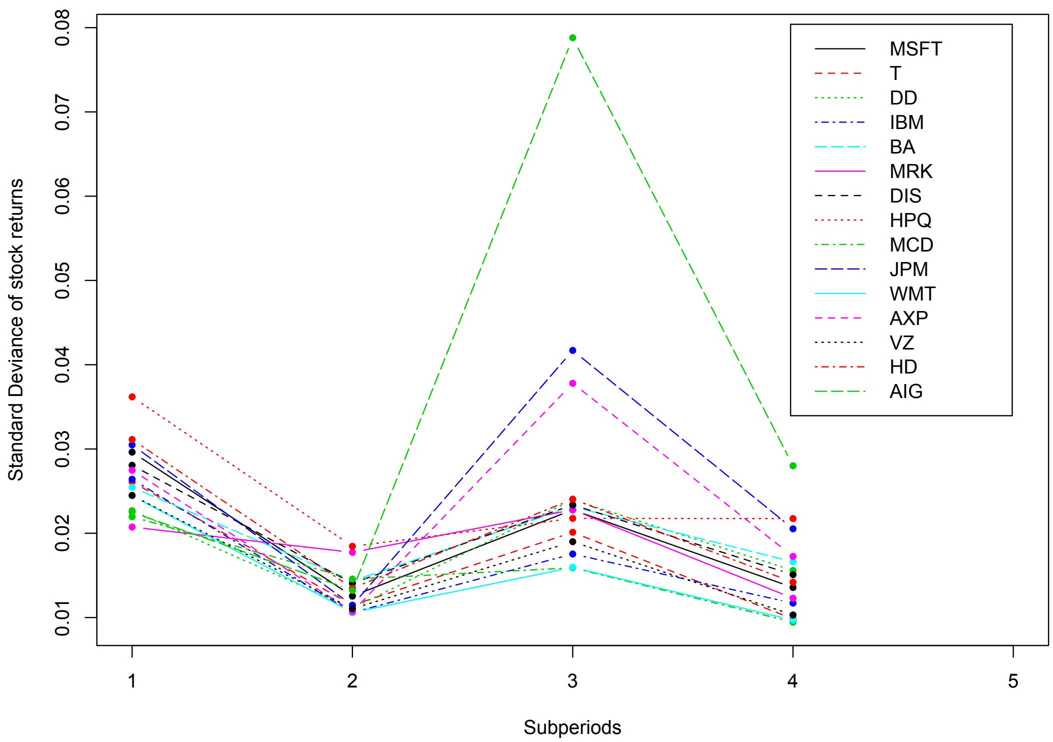

Figure 2: Plot of standard deviations of daily returns for 15 stocks for four equal subperiods

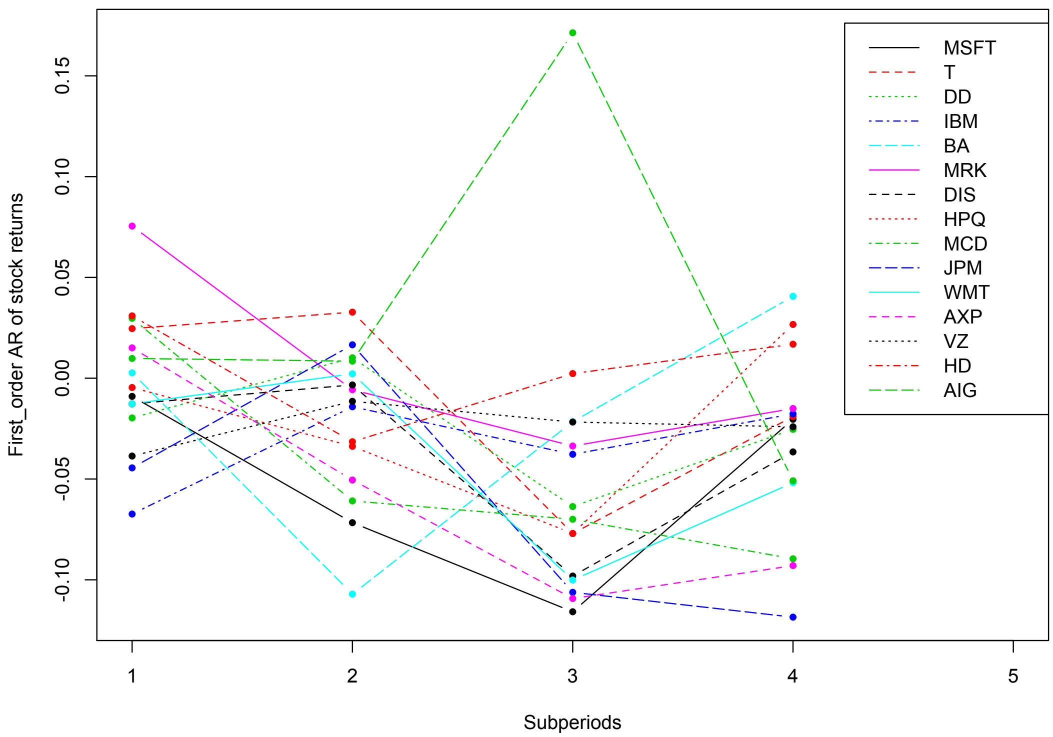

Figure 3: Plot of first-order autocorrelation coefficients of daily returns for 15 stocks for four equal subperiods

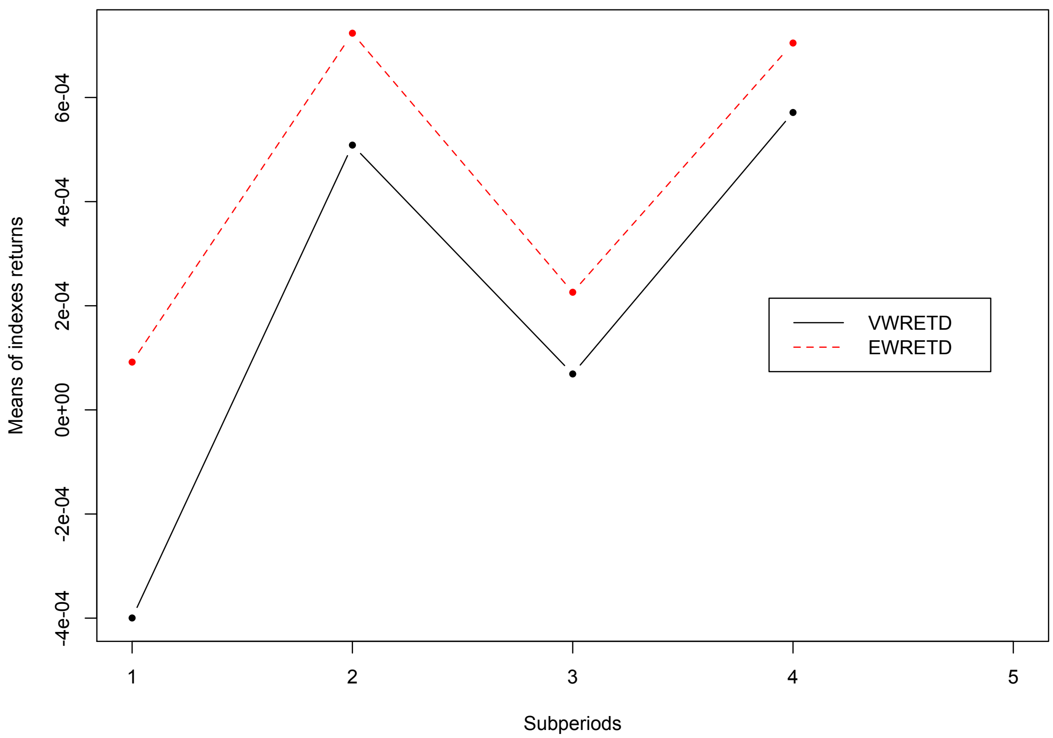

Figure 4: Plot of means of daily returns for two indexes for four equal subperiods

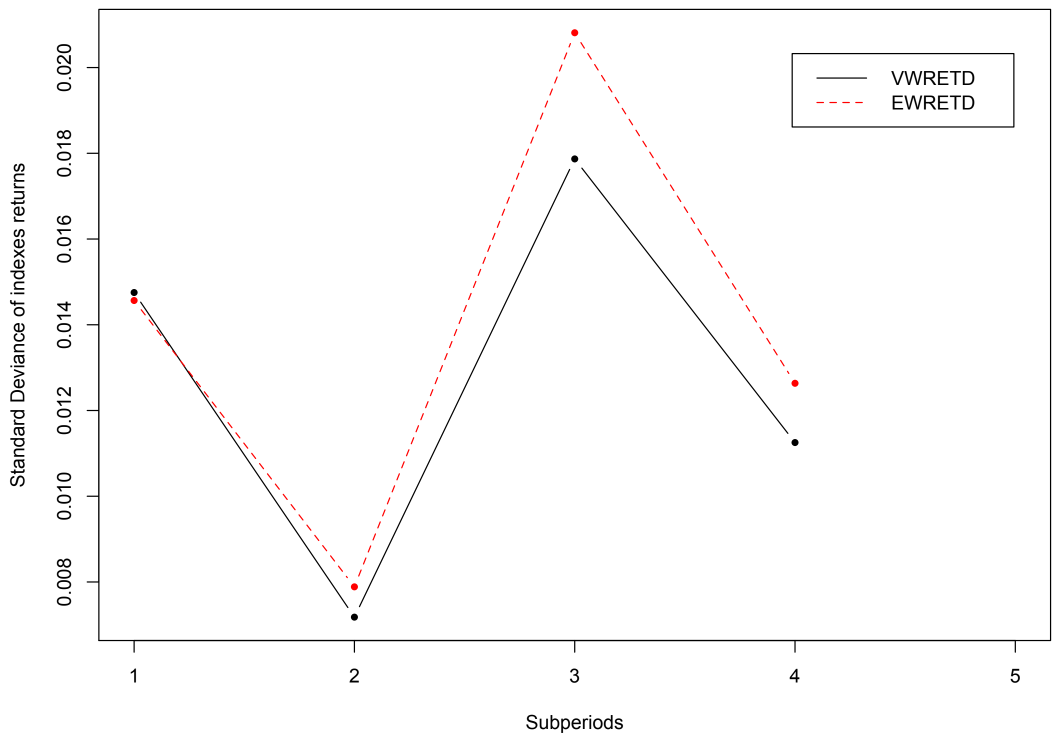

Figure 5: Plot of standard deviations of daily returns for two indexes for four equal subperiods

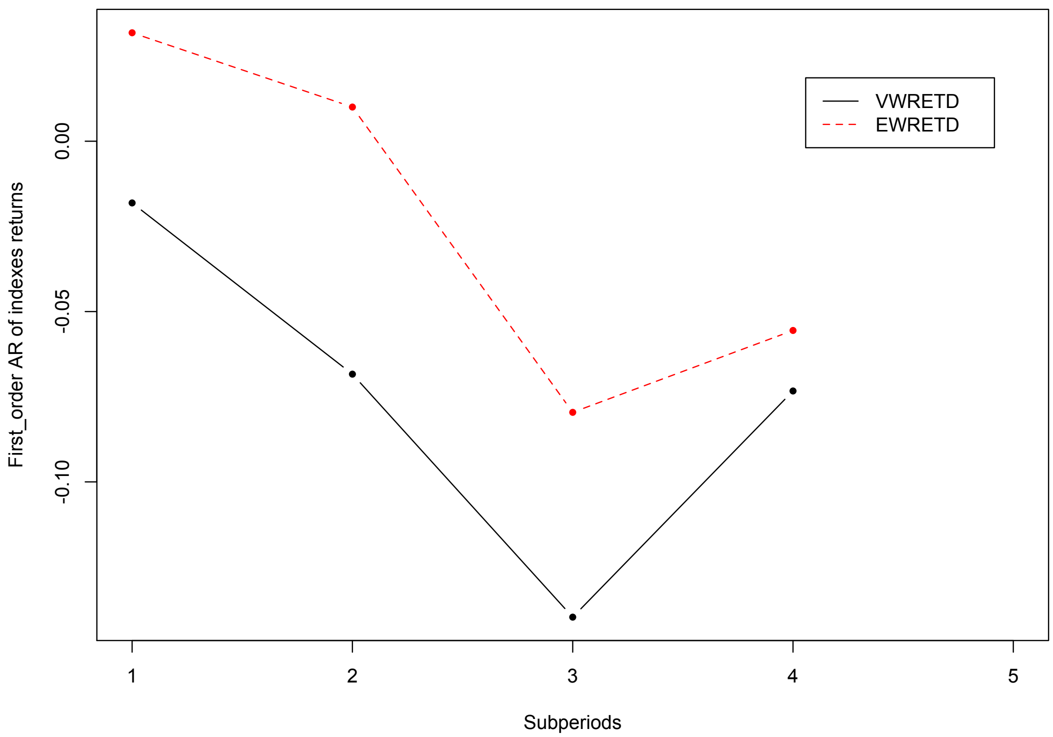

Figure 6: Plot of first-order autocorrelation coefficients of daily returns for two indexes for four equal subperiods

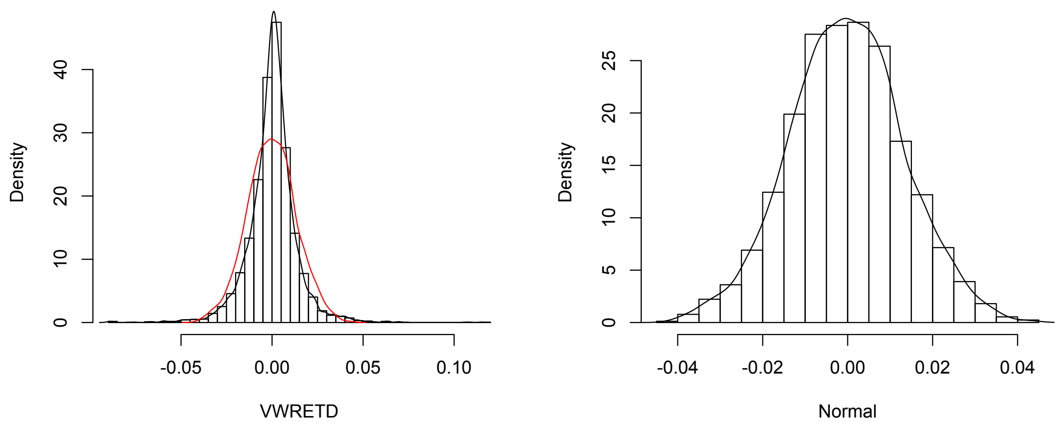

Figure 7: Histogram of returns of VWRETD for sample period (Left panel) and histogram of normal distribution with mean and variance equal to the sample mean and variance of the returns of VWRETD

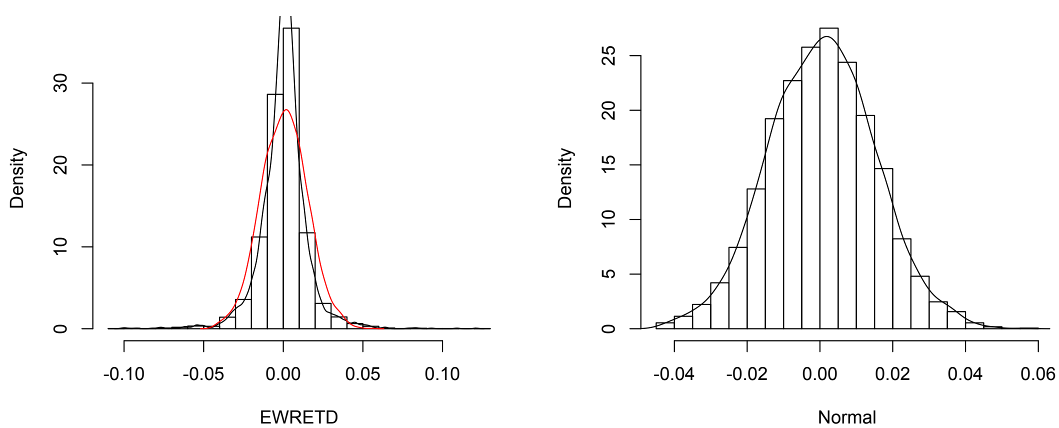

Figure 8: Histogram of returns of EWRETD for sample period (Left panel) and histogram of normal distribution with mean and variance equal to the sample mean and variance of the returns of EWRETD

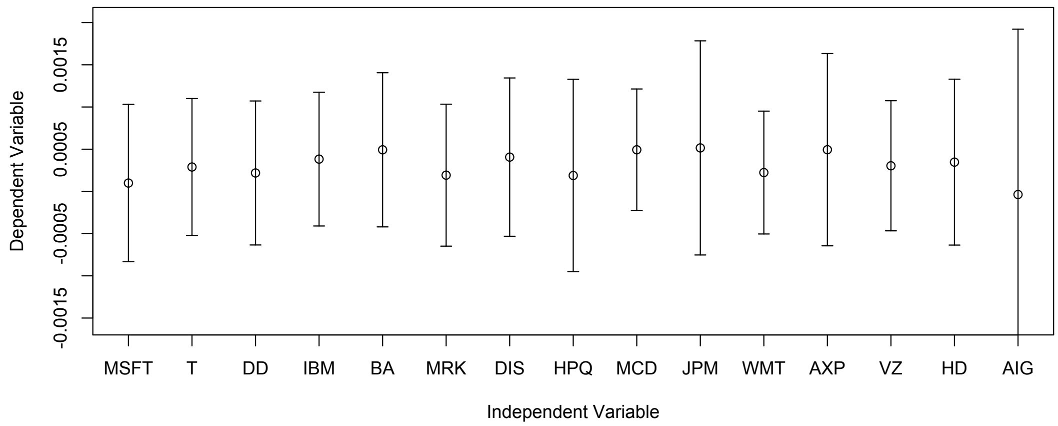

Figure 9: Means and confidence intervals of 15 stocks for sample periods

in the first and third subperiods.

1. (d) Table 3 presents skewness, kurtosis and studentized range of daily returns for 15 stocks and two indexes for sample period. Figure 13, 14 and 15 show skewness, kurtosis and studentized range of daily returns for 15 stocks for four subperiods. We can see from them that MRK fluctuates greatly. It has large skewness, kurtosis and studentized range for the second period.

Table 4 presents the results of D'Agostino skewness test for 15 stocks and two indexes for sample period. From it, we can see at 5% level, except DD, BA, HPQ, MCD, all the other stocks have skewness. The two indexes are believed not to have skewness at 5% level. Also, we apply same test for monthly data, we have that, except DD, BA, HPQ and MCD, all the other stocks have skewness. Both of the two indexes have no skewness.

Table 5 presents the results of Anscombe-Glynn kurtosis test for 15 stocks and two indexes for sample period. It can be seen that the kurtosis of all stocks and indexes are not equal to 3. We repeat the same test for monthly data of stocks and indexes at 5% level. We have that except for HD and EWRETD, the kurtosis of all the other stocks

Names	Confidence Interval	
	Lower bound	Upper bound
MSFT	-0.00083269	0.00103077
T	-0.00052117	0.00109947
DD	-0.00063432	0.00107129
IBM	-0.00040948	0.00117420
BA	-0.00042015	0.00140596
MRK	-0.00064835	0.00103301
DIS	-0.00053131	0.00134386
HPQ	-0.00095054	0.00132761
MCD	-0.00022708	0.00121343
JPM	-0.00075287	0.00178365
WMT	-0.00050412	0.00095105
AXP	-0.00064457	0.00163290
VZ	-0.00046703	0.00107449
HD	-0.00063560	0.00132894
AIG	-0.00199480	0.00192136
VWRETD	-0.00040981	0.00078460
EWRETD	-0.00022094	0.00109403

Table 2: 99% confidence intervals for 15 stocks and two indexes

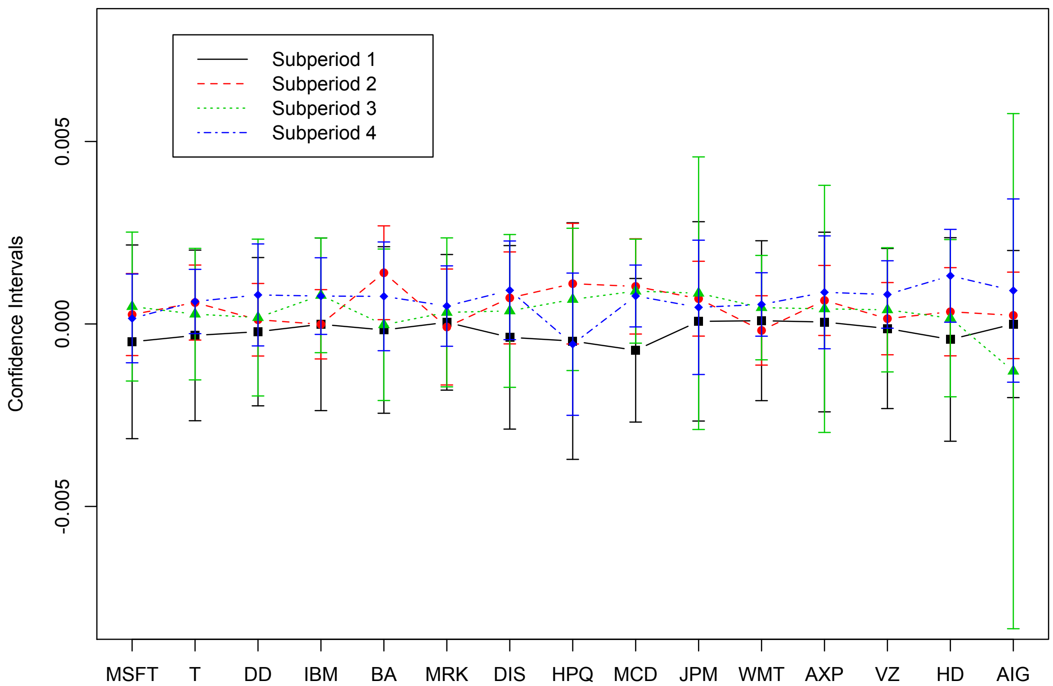

Figure 10: Means and confidence intervals of 15 stocks for four subperiods

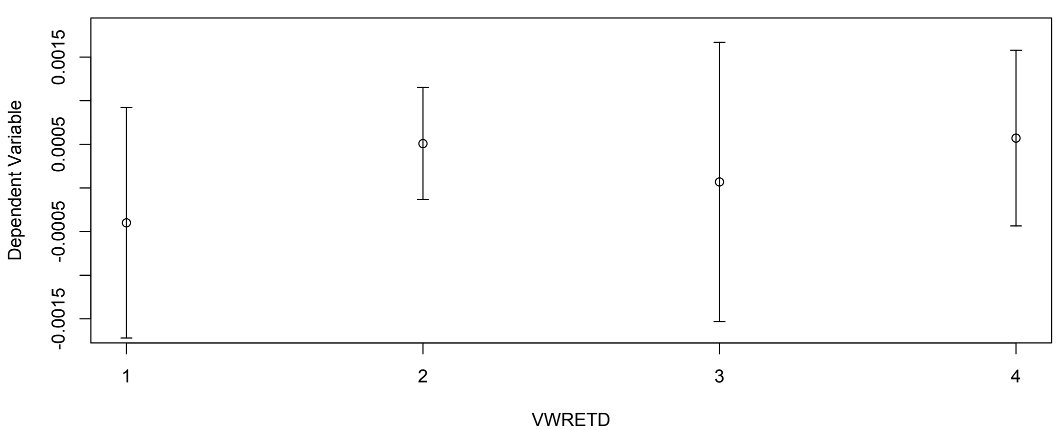

Figure 11: Means and confidence intervals of VWRETD for for subperiods

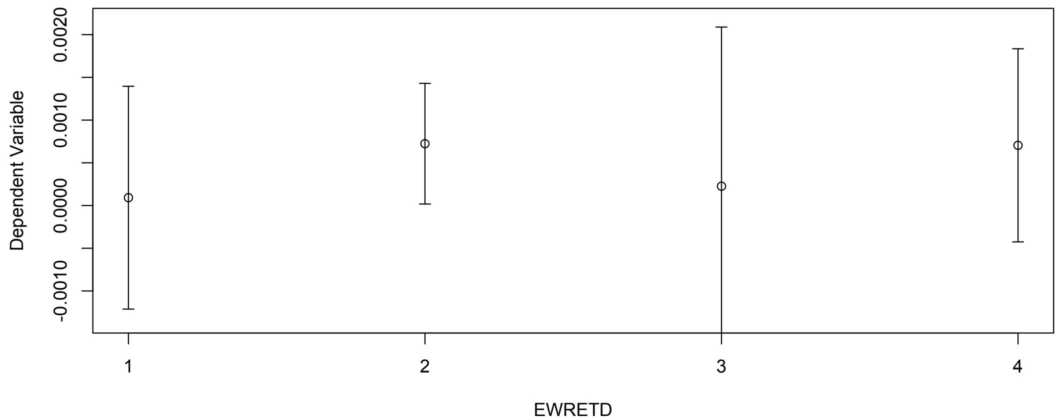

Figure 12: Means and confidence intervals of EWRETD for for subperiods

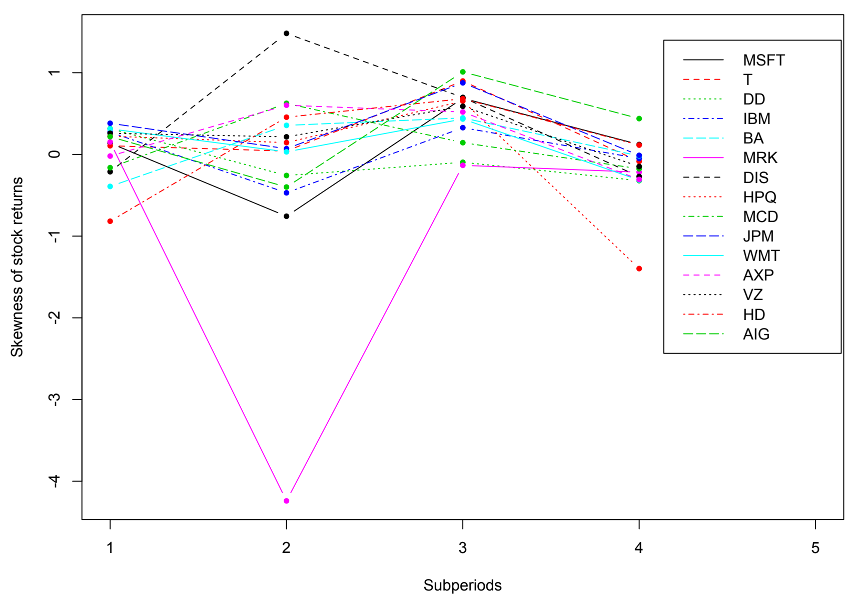

Figure 13: Plot of skewness of daily returns for 15 stocks for four equal subperiods

	Skewness	Kurtosis	Studentized range
MSFT	0.26712151	9.35517826	16.87317454
T	0.34166508	6.45620774	15.96839088
DD	0.01571746	4.74412380	11.95321824
IBM	0.25832177	7.68563395	16.14025011
BA	-0.03741358	4.78422967	16.20236642
MRK	-0.91763385	18.67945884	21.17388833
DIS	0.16376832	7.48862819	16.37310465
HPQ	0.02804137	6.76836048	14.64661999
MCD	-0.02312892	5.28923082	13.78460211
JPM	0.81922208	12.07439271	16.15428649
WMT	0.34671438	5.33974485	12.23413515
AXP	0.39103655	8.63989084	15.01532583
VZ	0.36090480	6.57895429	15.35948283
HD	-0.37088818	12.58005752	19.48209743
AIG	1.44224536	57.64378532	28.95064328
VWRETD	0.03201377	7.47400349	15.34012036
EWRETD	0.00736678	7.82015212	15.06539849

Table 3: Skewness, kurtosis and studentized range of daily returns for 15 stocks and two indexes for sample period

Name	D'Agostino skewness test
MSFT	skew = 0.2672, z = 4.0836, p-value = 4.434e-05 alternative hypothesis: data have a skewness
T	skew = 0.3418, z = 5.1700, p-value = 2.342e-07 alternative hypothesis: data have a skewness
DD	skew = 0.0157, z = 0.2444, p-value = 0.8069 alternative hypothesis: data have a skewness
IBM	skew = 0.2584, z = 3.9533, p-value = 7.708e-05 alternative hypothesis: data have a skewness
BA	skew = -0.0374, z = -0.5816, p-value = 0.5608 alternative hypothesis: data have a skewness
MRK	skew = -0.9180, z = -12.2775, p-value $<$ 2.2e-16 alternative hypothesis: data have a skewness
DIS	skew = 0.1638, z = 2.5300, p-value = 0.0114 alternative hypothesis: data have a skewness
HPQ	skew = 0.0281, z = 0.4360, p-value = 0.6629 alternative hypothesis: data have a skewness
MCD	skew = -0.0231, z = -0.3596, p-value = 0.7191 alternative hypothesis: data have a skewness
JPM	skew = 0.8196, z = 11.2274, p-value $<$ 2.2e-16 alternative hypothesis: data have a skewness
WMT	skew = 0.3469, z = 5.2423, p-value = 1.586e-07 alternative hypothesis: data have a skewness
AXP	skew = 0.3912, z = 5.8707, p-value = 4.339e-09 alternative hypothesis: data have a skewness
VZ	skew = 0.3611, z = 5.4449, p-value = 5.183e-08 alternative hypothesis: data have a skewness
HD	skew = -0.3711, z = -5.5867, p-value = 2.315e-08 alternative hypothesis: data have a skewness
AIG	skew = 1.4429, z = 16.9552, p-value $<$ 2.2e-16 alternative hypothesis: data have a skewness
VWRETD	skew = 0.0320, z = 0.4982, p-value = 0.6184 alternative hypothesis: data have a skewness
EWRETD	skew = 0.0074, z = 0.1147, p-value = 0.9087 alternative hypothesis: data have a skewness

Table 4: D'Agostino skewness test for 15 stocks and two indexes for sample period

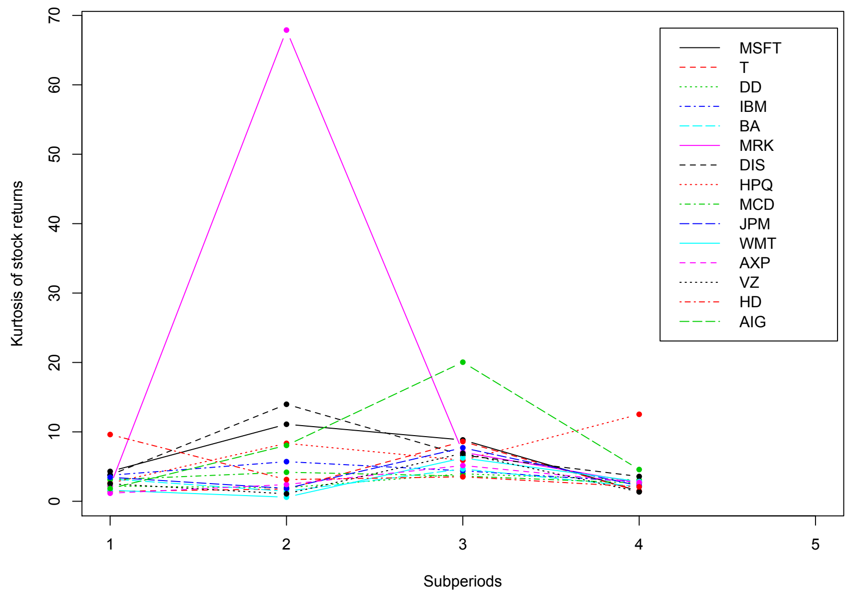

Figure 14: Plot of kurtosis of daily returns for 15 stocks for four equal subperiods

and indexes are not equal to 3.

Therefore, we conclude that these 17 series stocks and 2 indexes do not normally distributed since they do not have similar skewness and kurtosis to normal distribution. (We can have the same conclusion if we use Jarque-Bera test).

2. (a)

Table 6 presents Ljung-Box test for log return series of AAPL stock. The Ljung statistics is 19.507 and p-value is 0.0343. Since the p-value is less than 5% level, we reject the null hypothesis and believe there exists serial correlation in the daily log returns.

(b) The *ar* function with the maximum likelihood method specifies an AR(6) model for the log return series.

Figure 16 shows the results of model fitting where $x_i (i = 1, \ldots, 6)$ represent variable

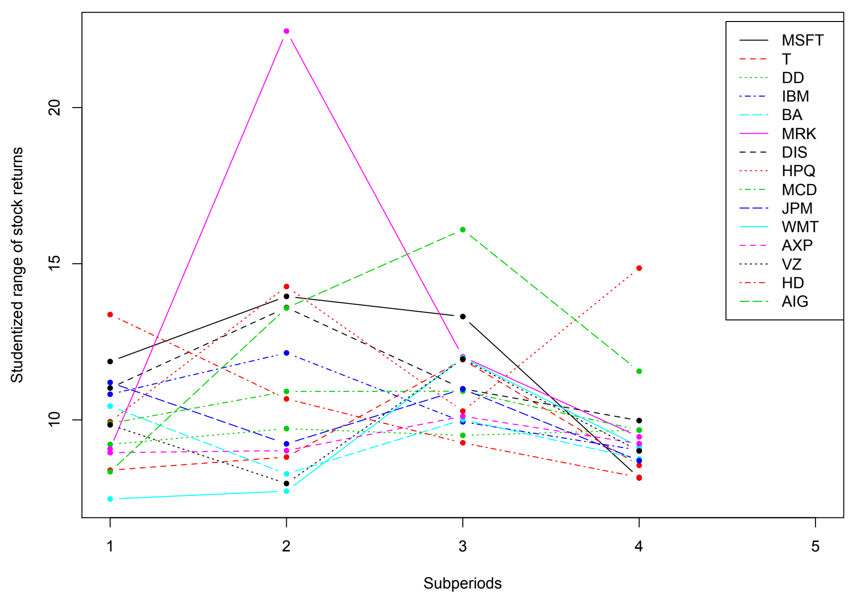

Figure 15: Plot of studentized range of daily returns for 15 stocks for four equal subperiods

Names	Anscombe-Glynn kurtosis test
MSFT	kurt = 12.3626, z = 23.8171, p-value < 2.2e-16 alternative hypothesis: kurtosis is not equal to 3
T	kurt = 9.4619, z = 21.1718, p-value < 2.2e-16 alternative hypothesis: kurtosis is not equal to 3
DD	kurt = 7.7488, z = 18.8935, p-value < 2.2e-16 alternative hypothesis: kurtosis is not equal to 3
IBM	kurt = 10.6921, z = 22.4317, p-value < 2.2e-16 alternative hypothesis: kurtosis is not equal to 3
BA	kurt = 7.7889, z = 18.9564, p-value < 2.2e-16 alternative hypothesis: kurtosis is not equal to 3
MRK	kurt = 21.6925, z = 28.3067, p-value < 2.2e-16 alternative hypothesis: kurtosis is not equal to 3
DIS	kurt = 10.4949, z = 22.2457, p-value < 2.2e-16 alternative hypothesis: kurtosis is not equal to 3
HPQ	kurt = 9.7742, z = 21.5156, p-value < 2.2e-16 alternative hypothesis: kurtosis is not equal to 3
MCD	kurt = 8.2942, z = 19.7033, p-value < 2.2e-16 alternative hypothesis: kurtosis is not equal to 3
JPM	kurt = 15.0835, z = 25.5485, p-value < 2.2e-16 alternative hypothesis: kurtosis is not equal to 3
WMT	kurt = 8.3448, z = 19.7738, p-value < 2.2e-16 alternative hypothesis: kurtosis is not equal to 3
AXP	kurt = 11.6469, z = 23.2615, p-value < 2.2e-16 alternative hypothesis: kurtosis is not equal to 3
VZ	kurt = 9.5847, z = 21.3092, p-value < 2.2e-16 alternative hypothesis: kurtosis is not equal to 3
HD	kurt = 15.5894, z = 25.8191, p-value < 2.2e-16 alternative hypothesis: kurtosis is not equal to 3
AIG	kurt = 60.6803, z = 34.1507, p-value < 2.2e-16 alternative hypothesis: kurtosis is not equal to 3
VWRETD	kurt = 10.4803, z = 22.2513, p-value < 2.2e-16 alternative hypothesis: kurtosis is not equal to 3
EWRETD	kurt = 10.8267, z = 22.5755, p-value < 2.2e-16 alternative hypothesis: kurtosis is not equal to 3

Table 5: Anscombe-Glynn kurtosis test for 15 stocks and two indexes for sample period

Ljung-Box test
Ljung-Box statistic = 19.50658, df = 10 p.value = 0.034281
Null hypothesis: rho(1) = rho(2) = ... = rho(10) = 0

Table 6: Kjung-Box test for log return series of AAPL stock

r_{t-i}. THe fitted model is

$$\begin{aligned} r_t &= 0.001467935 + 0.001303642 r_{t-1} - 0.026192404 r_{t-2} + 0.010542232 r_{t-3} \\ &\quad + 0.059773308 r_{t-4} + 0.024155045 r_{t-5} - 0.038964763 r_{t-6} \end{aligned}$$

The AR parameters are significant only for intercept as well as lag4 and lag6. A possible reason for not all coefficients being significant is lag4 and lag6 contain information brought by other lags.

(c) The fitted mode is

$$r_t = -0.0014677 + 0.0597905 r_{t-4} - 0.0389441 r_{t-6} + \epsilon_t \tag{1}$$

(d) In order to check if the log price series is unit-root stationary, we chose to use Augmented Dickey-Fuller Test. From Figure 17, we can see the p-value for lag 4 is less than 5% level, which means we reject null hypothesis for lag 4 and believe unit root is present.

3 (a) We use the fitted model (1) to remove the serial correlations. Figure 18 shows residuals, autocorrelations coefficients (adf) of residuals and partial autocorrelations coefficients (pacf) of residuals. ACF shows that we have removed serial correlations in the log returns but the PACF shows that after lag 6, the pacf of residuals are not close

```
Call:
lm(formula = log_return ~ x1 + x2 + x3 + x4 + x5 + x6)

Residuals:
      Min        1Q    Median        3Q       Max 
-0.126420 -0.012552  0.000266  0.012401  0.201298 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)   
(Intercept) -0.0014677  0.0004646  -3.159  0.00160 **
x1           0.0012950  0.0194773   0.066  0.94699   
x2          -0.0261946  0.0194717  -1.345  0.17865   
x3           0.0105296  0.0194468   0.541  0.58824   
x4           0.0597905  0.0194472   3.075  0.00213 **
x5           0.0241860  0.0194754   1.242  0.21439   
x6          -0.0389441  0.0194815  -1.999  0.04571 * 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.02358 on 2632 degrees of freedom
Multiple R-squared:  0.006804,	Adjusted R-squared:  0.00454 
F-statistic: 3.005 on 6 and 2632 DF,  p-value: 0.006274
```

Figure 16: Regression analysis of log return series of AAPL stock

```
Title:
 Augmented Dickey-Fuller Unit Root Test

Test Results:

  Test regression none

  Call:
  lm(formula = z.diff ~ z.lag.1 - 1 + z.diff.lag)

  Residuals:
        Min        1Q    Median        3Q       Max
  -0.192352 -0.011924  0.000222  0.013246  0.132945

  Coefficients:
               Estimate Std. Error t value Pr(>|t|)
  z.lag.1      2.418e-04  9.704e-05   2.492  0.01278 *
  z.diff.lag1  2.537e-03  1.951e-02   0.130  0.89652
  z.diff.lag2 -2.494e-02  1.950e-02  -1.279  0.20110
  z.diff.lag3  1.173e-02  1.947e-02   0.602  0.54697
  z.diff.lag4  6.104e-02  1.947e-02   3.135  0.00174 **
  z.diff.lag5  2.530e-02  1.950e-02   1.297  0.19462
  z.diff.lag6 -3.785e-02  1.951e-02  -1.940  0.05249 .
  ---
  Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

  Residual standard error: 0.02362 on 2626 degrees of freedom
  Multiple R-squared:  0.009451,	Adjusted R-squared:  0.006811
  F-statistic: 3.579 on 7 and 2626 DF,  p-value: 0.0007712


  Value of test-statistic is: 2.4915

  Critical values for test statistics:
        1pct  5pct 10pct
  tau1 -2.58 -1.95 -1.62
```

Figure 17: Results of Augmented Dickey-Fuller Test

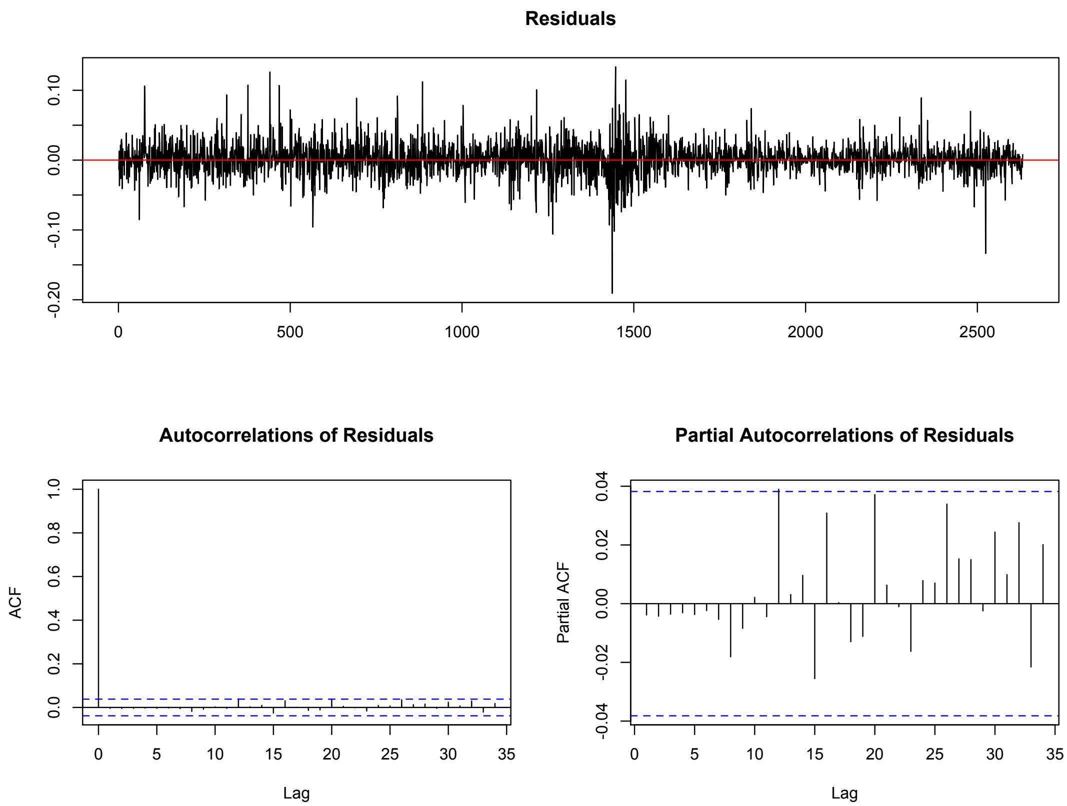

Figure 18: Residuals, autocorrelations (adf) of residuals and partial autocorrelations (pacf) of residuals

to zero. Therefore, we believe there exist some ARCH effects in the daily log returns after removing serial correlations.

(b) Figure 19 shows the results of fitted GARCH(1,1) model with Gaussian innovations. We can see from it that only intercept as well as the coefficients of lag4 and lag6 are significant. The fitted model is

$$\begin{aligned} r_t &= 2.091^{-3} + 4.076 \times 10^{-2} r_{t-4} - 4.5 \times 10^{-2} r_{t-6} + u_t \\ u_t &= \sigma_t \epsilon_t \\ \sigma_t^2 &= 7.437 \times 10^{-6} + 5.243 \times 10^{-2} u_{t-1}^2 + 9.352 \times 10^{-1} \sigma_{t-1}^2 \end{aligned}$$

(c) Figure 20 shows the results of fitted GARCH(1,1) model with studentized t-

```
Error Analysis:
         Estimate  Std. Error  t value Pr(>|t|)    
mu      2.091e-03   4.162e-04    5.023 5.09e-07 ***
ar1     1.849e-02   2.045e-02    0.904  0.36603    
ar2    -7.267e-03   2.051e-02   -0.354  0.72312    
ar3    -1.033e-02   2.042e-02   -0.506  0.61303    
ar4     4.076e-02   2.048e-02    1.991  0.04653 *  
ar5     1.653e-02   2.044e-02    0.809  0.41873    
ar6    -4.500e-02   2.038e-02   -2.208  0.02722 *  
omega   7.437e-06   2.459e-06    3.024  0.00249 ** 
alpha1  5.243e-02   8.443e-03    6.210 5.30e-10 ***
beta1   9.352e-01   1.087e-02   86.072  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Figure 19: Fit GARCH(1,1) model with Gaussian innovations for log return series of AAPL stock

distribution innovation We can see from it that only intercept as well as the coefficients of lag4 are significant. The fitted model is

$$
\begin{aligned}
r_t &= 1.683^{-3} + 4.388 \times 10^{-2} r_{t-4} + u_t \\
u_t &= \sigma_t \epsilon_t \\
\sigma_t^2 &= 5.624 \times 10^{-6} + 4.64 \times 10^{-2} u_{t-1}^2 + 9.446 \times 10^{-1} \sigma_{t-1}^2 \\
t &\sim t_{5.343}
\end{aligned}
$$

(d) Figure 21 shows the results of fitted integrated GARCH(1,1) model with Gaussian innovation The fitted model is

$$
\begin{aligned}
r_t &= 2.787^{-3} + 5.634 \times 10^{-2} r_{t-4} - 6.4286 \times 10^{-2} r_{t-6} + u_t \\
u_t &= \sigma_t \epsilon_t \\
\sigma_t^2 &= 3.0 \times 10^{-6} + 0.0527 u_{t-1}^2 + 0.9473 \sigma_{t-1}^2
\end{aligned}
$$

(e) Since the sum of α_1 and β_1 is close to 1, IGARCH(1,1) is the best to be used.

```
Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)    
mu     1.683e-03   3.862e-04    4.357 1.32e-05 ***
ar1    1.488e-03   1.902e-02    0.078   0.9376    
ar2   -3.315e-03   1.911e-02   -0.173   0.8623    
ar3   -1.521e-03   1.890e-02   -0.080   0.9359    
ar4    4.388e-02   1.879e-02    2.335   0.0195 *  
ar5    1.597e-02   1.848e-02    0.864   0.3874    
ar6   -2.726e-02   1.871e-02   -1.457   0.1450    
omega  5.624e-06   2.348e-06    2.395   0.0166 *  
alpha1 4.640e-02   1.006e-02    4.611 4.01e-06 ***
beta1  9.446e-01   1.197e-02   78.890  < 2e-16 ***
shape  5.343e+00   5.480e-01    9.751  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Figure 20: Fit GARCH(1,1) model with studentized t-distribution innovations for log return series of AAPL stock

```
Conditional Variance Dynamics
-----------------------------------
GARCH Model    : iGARCH(1,1)
Mean Model : ARFIMA(6,0,0)
Distribution   : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.002787    0.000594  4.69463 0.000003
ar1     0.010177    0.024495  0.41546 0.677802
ar2    -0.030028    0.024511 -1.22506 0.220554
ar3     0.002594    0.024418  0.10624 0.915394
ar4     0.056340    0.024478  2.30162 0.021357
ar5     0.034453    0.024574  1.40203 0.160906
ar6    -0.064286    0.024462 -2.62804 0.008588
omega   0.000003    0.000007  0.46111 0.644722
alpha1  0.052702    0.031403  1.67827 0.093295
beta1   0.947298          NA       NA       NA

Robust Standard Errors:
        Estimate  Std. Error   t value Pr(>|t|)
mu      0.002787    0.002068  1.347837 0.177711
ar1     0.010177    0.024936  0.408122 0.683184
ar2    -0.030028    0.029393 -1.021590 0.306975
ar3     0.002594    0.023588  0.109971 0.912432
ar4     0.056340    0.025471  2.211927 0.026972
ar5     0.034453    0.039443  0.873488 0.382397
ar6    -0.064286    0.044176 -1.455239 0.145603
omega   0.000003    0.000064  0.053154 0.957610
alpha1  0.052702    0.266448  0.197795 0.843206
beta1   0.947298          NA        NA       NA
```

Figure 21: Fit IGARCH(1,1) model for log return series of AAPL stock