Урок №9 Д/З.

① Найти $\frac{dz}{dt}$, если $z(x;y)$, $x=x(t)$, $y=y(t)$

1) $z=x^2+y^2+xy$, $x=a\cdot\sin t$, $y=a\cdot\cos t$

$$\dot{z}=\frac{dz}{dt}=\frac{\partial z}{\partial x}\cdot\frac{dx}{dt}+\frac{\partial z}{\partial y}\cdot\frac{dy}{dt}$$

$\frac{\partial z}{\partial x}=2x+1$ $\qquad$ $\frac{\partial z}{\partial y}=2y+1$

$\frac{dx}{dt}=a\cdot\cos t$ $\qquad$ $\frac{dy}{dt}=-a\cdot\sin t$

$$\frac{dz}{dt}=(2x+1)\cdot a\cdot\cos t+(2y+1)\cdot(-a)\cdot\sin t=$$

$$=a\left((2x+1)\cdot\cos t-(2y+1)\cdot\sin t\right)$$

2) $z=x^2y^3\cdot u$, $x=t$, $y=t^2$, $u=\sin t$

$\frac{\partial z}{\partial x}=2xy^3\cdot u$ $\qquad$ $\frac{\partial z}{\partial y}=3y^2\cdot x^2\cdot u$ $\qquad$ $\frac{\partial z}{\partial u}=x^2y^3$

$\frac{dx}{dt}=1$ $\qquad$ $\frac{dy}{dt}=2t$ $\qquad$ $\frac{du}{dt}=\cos t$

$$\frac{dz}{dt}=2xy^3\cdot u\cdot 1+3y^2x^2\cdot u\cdot 2t+x^2y^3\cdot\cos t=xy^2(2y\cdot u+6xt+xy\cdot\cos t)$$

② Найти $\frac{\partial z}{\partial u}$, $\frac{\partial z}{\partial v}$, dz

1) $z=x^3+y^3$, где $x=uv$, $y=\frac{u}{v}$

$\frac{\partial z}{\partial x}=3x$ $\qquad$ $\frac{\partial z}{\partial y}=3y$

$\frac{\partial x}{\partial u}=v$ $\qquad$ $\frac{\partial y}{\partial u}=\frac{1}{v}$

$\frac{\partial x}{\partial v}=u$ $\qquad$ $\frac{\partial y}{\partial v}=-\frac{1}{v^2}$

$\frac{\partial z}{\partial u}=3x\cdot v+3y\cdot\frac{1}{v}$; $\qquad$ $\frac{\partial z}{\partial v}=3x\cdot u-3y\frac{1}{v^2}$

$$dz=\left(3x\cdot v+3y\cdot\frac{1}{v}\right)\cdot du+\left(3xu-3y\frac{1}{v^2}\right)\cdot dv$$

2) $z=\cos xy$, $x=ue^v$ $\quad$ $y=v\cdot\ln u$

$\frac{\partial z}{\partial x}=-\sin xy\cdot y$ $\qquad$ $\frac{\partial z}{\partial y}=-\sin xy\cdot x$

$$\frac{\partial x}{\partial u} = e^v \qquad \frac{\partial y}{\partial u} = v \cdot \frac{1}{u}$$

$$\frac{\partial x}{\partial v} = u \cdot e^v \qquad \frac{\partial y}{\partial v} = \ln u$$

$$\frac{\partial z}{\partial u} = -y \cdot \sin yx \cdot e^v - x \cdot \sin xy \cdot v \cdot \frac{1}{u} = -\sin xy \cdot \left(y \cdot e^v + x \cdot \frac{v}{u}\right)$$

$$\frac{\partial z}{\partial v} = -y \cdot \sin xy \cdot u \cdot e^v - x \cdot \sin xy \cdot \ln u = -\sin xy \cdot (y \cdot u \cdot e^v + x \cdot \ln u)$$

$$dz = -\sin xy \cdot \left(y \cdot e^v + x \cdot \frac{v}{u}\right) \cdot du - \sin xy \cdot (y \cdot u e^v + x \cdot \ln u)\, dv$$

③ Найти $y'(x)$

1) $x e^{2y} - y \cdot \ln x = 8$

$$y': \quad 1 \cdot e^{2y} + x \cdot 2y' \cdot e^{2y} - \left(y' \cdot \ln(x) + y \cdot \frac{1}{x}\right) = 0$$

$$x \cdot 2y' \cdot e^{2y} - y' \cdot \ln x = y \cdot \frac{1}{x} - e^{2y}$$

$$y' = \frac{\frac{y}{x} - e^{2y}}{2x \cdot e^{2y} - \ln x}$$

④ Составить ур-е касат и нормали

$x^3 \cdot y - y^3 \cdot x = 6 \qquad M_0(2;1)$

$F(2;1) = x^3 \cdot y - y^3 \cdot x - 6 = 0$

$F'_y = x^3 - 3y^2 x \qquad F'_y(2;1) = 8 - 3 \cdot 1 \cdot 2 = 2$

$F'_x = 3x^2 y - y^3 \qquad F'_x(2;1) = 3 \cdot 4 \cdot 1 - 1 = 11$

$$k = y'(x_0) = -\frac{F'_x}{F'_y} = -\frac{11}{2}$$

$$(t): \quad y - 1 = -\frac{11}{2}(x-2) \quad ; \quad (n): \quad y - 1 = \frac{2}{11} \cdot (x-2)$$

⑤ Найти частную произв. или дифф.

1) $z = \sin x \cdot \sin y \qquad d^2 z = ?$

$$dz = \frac{\partial z}{\partial x} \cdot dx + \frac{\partial z}{\partial y} \cdot dy = \cos x \cdot \sin y \cdot dx + \sin x \cdot \cos y \cdot dy$$

$$d^2 z = d(dz) = \frac{\partial^2 z}{\partial x^2} \cdot dx^2 + 2 \frac{\partial^2 z}{\partial x \partial y} dx dy + \frac{\partial^2 z}{\partial y^2} \cdot dy^2$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x} \cdot \cos x \cdot \sin y = -\sin x \cdot \sin y$$

$$\frac{\partial^2 z}{\partial y^2} = \frac{\partial}{\partial y} \cdot \cos y \cdot \sin x = -\sin y \cdot \sin x$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial y} \cdot (\cos x \cdot \sin y) = \cos x \cdot \cos y$$

$$d^2 z = -\sin x \cdot \sin y \cdot dx^2 + 2 \cdot \cos x \cdot \cos y \cdot dx \cdot dy - \sin y \cdot \sin x \cdot dy^2 =$$

$$= -\sin x \cdot \sin y \, (dx^2 + dy^2) + 2 \cdot \cos x \cdot \cos y \cdot dx \cdot dy$$

2) $z = xy + \sin(x+y)$, $\frac{\partial^2 z}{\partial x^2} = ?$

$$\frac{\partial z}{\partial x} = y + \cos(x+y)$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x} \cdot (y \cdot \cos(x+y)) = -\sin(x+y)$$

3) $z = \operatorname{arctg} \frac{x+y}{1-xy}$, $\frac{\partial^2 z}{\partial x \partial y} = ?$

$$\frac{\partial z}{\partial x} = \frac{\left(\frac{x+y}{1-xy}\right)'}{1 - \left(\frac{x+y}{1-xy}\right)^2} = \frac{(x+y)' \cdot (1-xy) - (x+y)(1-xy)'}{\cancel{(1-xy)^2} \cdot \frac{1}{\cancel{(1-xy)^2}} \cdot ((1-xy)^2 - (x+y)^2)} = \frac{1+y^2}{(1-xy)^2 - (x+y)^2}$$

$$\frac{\partial z}{\partial y} = \frac{1+x^2}{(1-xy)^2 - (x+y)^2}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial y} \cdot \left(\frac{1+y^2}{(1-xy)^2 - (x+y)^2}\right) = \frac{(1+y^2)' \cdot (x^2y^2 - x^2 - 4xy - y^2 + 1) - (1+y^2) \cdot (x^2y^2 - x^2 - 4xy - y^2 + 1)'}{(x^2y^2 - x^2 - 4xy - y^2 + 1)^2} =$$

$$= \frac{-4x^2y - 4xy^2 + 4y - 2y^3 + 2y^4 + x^2y^2 + x^2 + 4x}{(x^2y^2 - x^2 - 4xy - y^2 + 1)^2}$$

6) Найти y', y'', y'''.

$x^2 - xy + 2y^2 + x - y = 1$, при $x = 0$, $y(0) = 1$

y': $2x - y - xy' + 4y \cdot y' + 1 - y' = 0$

y'': $2 - y' - y' - x \cdot y'' + 4y'y' + 4y \cdot y'' - y'' = 0$

$2 - 2y' - xy'' + 4(y')^2 + 4y \cdot y'' - y'' = 0$

$y''':\quad -2y'' - (y'')^2 - x\cdot y''' + 4\cdot 2y'\cdot y'' + 4y'' + 4y\cdot y''' - y''' = 0$

$2y'' - (y'')^2 - xy''' + 8\cdot y'\cdot y'' + 4y\cdot y''' - y''' = 0$

$y'(0) = 0 \quad ; \quad y''(0) = 2 \quad ; \quad y'''(0) = 0$

(7) Построить градиент и линии уровня

$z = \operatorname{arctg}\frac{y}{x}$, в т. $(1;1)$, $(1;-1)$

$z'_y = \frac{(y/x)'}{1+\left(\frac{y}{x}\right)^2} = \frac{x-y}{x^2+y^2}$ $\qquad$ $z'_x = \frac{y-x}{x^2-y^2}$

$z'_x(1;1) = 0$ $\qquad$ $z'_x(1;-1) = -1$

$z'_y(1;1) = 0$ $\qquad$ $z'_y(1;-1) = 1$

$\overrightarrow{grad}\, z(1;1) = (0;0) = 0$

$\overrightarrow{grad}\, z(1;-1) = \sqrt{(-1)^2+1^2} = \sqrt{2}$