Д/З к уроку №6

① Найти производную

1) $y = x^3 \cdot \log_2 x$

$y' = (x^3)' \cdot \log_2 x + x^3 \cdot (\log_2 x)' = 3x^2 \cdot \log_2 x + \cancel{x^3}^{\,2} \cdot \frac{1}{\cancel{x} \cdot \ln 2} =$

$= x^2 \cdot \left(3 \cdot \log_2 x + \frac{1}{\ln 2}\right)$

2) $y = -10 \operatorname{arctg} x + 7e^x$

$y' = -10 \cdot \frac{1}{1+x^2} + 7e^x = 7e^x - \frac{10}{1+x^2}$

3) $y = \frac{1}{\sqrt[3]{x^2}} - \frac{2}{x^3} + \sqrt{7} \cdot x$

$y' = (x^{-\frac{2}{3}})' - 2 \cdot (x^{-3})' + \sqrt{7} \cdot x' = -\frac{2}{3} \cdot x^{-\frac{5}{3}} - 2 \cdot (-3) \cdot x^{-4} + \sqrt{7} \cdot 1 =$

$= \sqrt{7} - \frac{2}{3\sqrt[3]{x^5}} + \frac{6}{x^4}$

4) $y = \cos\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)$

$y' = \left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)' \cdot \left(-\sin\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)\right) = \frac{(1-\sqrt{x})' \cdot (1+\sqrt{x}) - (1-\sqrt{x}) \cdot (1+\sqrt{x})'}{(1+\sqrt{x})^2} \cdot$

$\cdot \left(-\sin\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)\right) = \frac{-\frac{1}{2}x^{-\frac{1}{2}}(1+\sqrt{x}) - (1-\sqrt{x}) \cdot \frac{1}{2}x^{-\frac{1}{2}}}{(1+\sqrt{x})^2} \cdot \left(-\sin\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)\right) =$

$= \frac{-2}{2\sqrt{x}\,(1+\sqrt{x})^2} \cdot \sin\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)$

5) $y = e^{sh^2 5x}$

$y' = e^{sh^2 5x} \cdot (sh^2 5x)' = e^{sh^2 5x} \cdot 2 \cdot sh(5x) \cdot 5 \cdot ch(5x) =$

$= 10 e^{sh^2 5x} \cdot sh(5x) \cdot ch(5x)$

6) $y = \ln\left(\frac{(x+1)(x+3)^3}{(x+2)^3(x+4)}\right) = \ln(x+1) + \ln(x+3)^3 - \ln(x+2)^3 - \ln(x+4) =$

$= \ln(x+1) + 3\ln(x+3) - 3\ln(x+2) - \ln(x+4)$

$y' = \frac{1}{x+1} + \frac{3}{x+3} - \frac{3}{x+2} - \frac{1}{x+4}$

7). $y = \underbrace{\frac{\sin^2 x}{ctg\, x + 1}}_{y_1} + \underbrace{\frac{\cos^2 x}{tg\, x + 1}}_{y_2}$

$$y_1' = \frac{(\sin^2 x)' \cdot (ctg\, x + 1) - \sin^2 x \cdot (ctg\, x + 1)'}{(ctg\, x + 1)^2} = \frac{2\sin x \cdot \cos x \cdot (ctg\, x + 1) - \cancel{\sin^2 x} \cdot \left(-\frac{1}{\cancel{\sin^2 x}}\right)}{(ctg\, x + 1)^2} =$$

$$= \frac{2\sin x \cdot \cos x}{ctg\, x + 1} + \frac{1}{(ctg\, x + 1)^2}$$

$$y_2' = \frac{(\cos^2 x)' \cdot (tg\, x + 1) - \cos^2 x \cdot (tg\, x + 1)'}{(tg\, x + 1)^2} = \frac{-2\cos x \cdot \sin x \cdot (tg + 1) - \cancel{\cos^2 x} \cdot \frac{1}{\cancel{\cos^2 x}}}{(tg\, x + 1)^2}$$

$$= -\frac{2\cos x \cdot \sin x}{tg\, x + 1} - \frac{1}{(tg\, x + 1)^2}$$

$$y' = (y_1 + y_2)' = y_1' + y_2' = \frac{2\sin x \cdot \cos x}{ctg\, x + 1} + \frac{1}{(ctg\, x + 1)^2} - \frac{2\cos x \cdot \sin x}{tg\, x + 1} - \frac{1}{(tg\, x + 1)^2}$$

(2) Найти производную в т.

1). $y = \frac{\ln x}{x}$, $x_0 = e$

$$y' = \frac{(\ln x)' \cdot x - \ln x \cdot x'}{x^2} = \frac{\frac{1}{x} \cdot x - \ln x}{x^2} = \frac{1 - \ln x}{x^2}$$

$$y'(x_0) = \frac{1 - \overset{=1}{\ln e}}{e^2} = 0$$

2). $y = \frac{\sqrt{x}}{\sqrt{x} + 1}$, $x_0 = 9$

$$y' = \frac{(\sqrt{x})' \cdot (\sqrt{x} + 1) - \sqrt{x} \cdot (\sqrt{x} + 1)'}{(\sqrt{x} + 1)^2} = \frac{\frac{1}{2}(x^{-\frac{1}{2}})}{(x + 1)} - \frac{\sqrt{x} \cdot \left(\frac{1}{2\sqrt{x}} + 0\right)}{(\sqrt{x} + 1)^2} =$$

$$= \frac{1}{2\sqrt{x}(\sqrt{x} + 1)} - \frac{1}{2(\sqrt{x} + 1)^2}$$

$$y'(x_0) = \frac{1}{2 \cdot 3 \cdot (3 + 1)} - \frac{1}{2(3 + 1)^2} = \frac{1}{96}$$

③ Используя лог. производную найти произв. ф-и

1) $y = x^{\ln x}$

$\ln y = \ln(x^{\ln x}) \Rightarrow \ln y = \ln x \cdot \ln x = \ln^2 x$

$(\ln y)' = (\ln^2 x)'$

$\frac{y'}{y} = 2\,\frac{\ln x}{x} \Rightarrow y' = \frac{2\ln x \cdot x^{\ln x}}{x}$

2) $y = \frac{(x^3-2)\cdot\sqrt[3]{(x-1)}}{(x+5)^4}$

$\ln y = \ln\left(\frac{(x^3-2)\cdot\sqrt[3]{(x-1)}}{(x+5)^4}\right) = \ln(x^3-2) + \frac{1}{3}\ln(x-1) - 4\ln(x+5)$

$\frac{y'}{y} = \frac{3x^2}{x^3-2} + \frac{1}{3}\cdot\frac{1}{x-1} - \frac{4}{x+5}$

$y' = \frac{(x^3-2)\cdot\sqrt[3]{(x-1)}}{(x+5)^4}\cdot\left(\frac{3x^2}{x^3-2} + \frac{1}{3(x-1)} - \frac{4}{x+5}\right)$

3) $y = (tg\,x)^{\cos x}$

$\ln y = \ln((tg\,x)^{\cos x}) = \cos x\cdot\ln(tg\,x)$

$\frac{y'}{y} = (\cos x)'\cdot\ln(tg\,x) + \cos x\cdot(\ln(tg\,x))' = -\sin x\cdot\ln tg\,x + \cos x\cdot\frac{\frac{1}{\cos^2 x}}{tg\,x} =$

$= \frac{1}{\cos x\cdot tg\,x} - \sin x\cdot\ln(tg\,x)$

$y' = (tg\,x)^{\cos x}\cdot\left(\frac{1}{\cos x\cdot tg\,x} - \sin x\cdot\ln(tg\,x)\right)$

④ Найти производную неявно задан. ф-и

1) $e^{xy} - \cos(x^2+y^2) = 0$

$(x\cdot y)'\cdot e^{xy} - (x^2+y^2)'\cdot(-\sin(x^2+y^2)) = 0$

$(y + y\cdot y')\cdot e^{xy} - 2y\cdot y'\cdot(-\sin(x^2+y^2)) = 0$

$y\cdot e^{xy} + y'\cdot y\cdot e^{xy} - 2y\cdot y'\cdot(-\sin(x^2+y^2)) = 0$

$y'\cdot(y\cdot e^{xy} - 2y(-\sin(x^2+y^2))) = -y\cdot e^{xy}$

$y' = \frac{-y\cdot e^{xy}}{y\cdot e^{xy} + 2y\cdot\sin(x^2+y^2)}$

2) $x \cdot \sin y + y \cdot \sin x = 0$

$1 \cdot \sin y + x \cdot y' \cdot \cos y + y' \cdot \sin x - y \cdot \cos x = 0$

$y' \cdot (x \cdot \cos y + \sin x) = y \cdot \cos x - \sin y$

$y' = \dfrac{y \cdot \cos x - \sin y}{x \cdot \cos y + \sin x}$

5. Найти произв. для фий, задан. параметрически

1) $x = t^3 + t$, $y = t^2 + t + 1$

$y'(x) = \dfrac{y'(t)}{x'(t)} \Rightarrow y'(x) = \dfrac{(t^2+t+1)'}{(t^3+t)'} = \dfrac{2t+1}{3t^2+1}$

2) $x = e^t \cdot \sin t$, $y = e^t \cdot \cos t$

$y'(x) = \dfrac{(e^t \cdot \cos t)'}{(e^t \cdot \sin t)'} = \dfrac{\cancel{e^t} \cdot \cos t + \cancel{e^t} \cdot (-\sin t)}{\cancel{e^t} \cdot \sin t + \cancel{e^t} \cdot \cos t} = \dfrac{\cos t - \sin t}{\cos t + \sin t}$

6. Найти ур-е касат. и нормали в данной точке

1) $y = e^x$, $x_0 = 0$

$y' = e^x$

$y'(x_0) = e^0 = 1$

$y - y_0 = f'(x_0) \cdot (x - x_0)$

$y - 1 = 1 \cdot (x - 0) \Rightarrow y = x + 1$ – ур-е касательной

$y - y_0 = -\dfrac{1}{f'(x_0)} \cdot (x - x_0)$

$y - 1 = -1 \cdot (x - 0) \Rightarrow y = 1 - x$ – ур-е нормали

7. Найти производные указ. порядка

1) $y = -x \cdot \cos x$, $y'' = ?$

$y' = -(x' \cdot \cos x + x \cdot (\cos x)') = -1 \cdot \cos x + x \cdot \sin x = x \cdot \sin x - \cos x$

$y'' = (y')' = x' \cdot \sin x + x \cdot \cos x + \sin x = 2\sin x + x \cdot \cos x$

2) $y = e^{2x}$, $y^{(V)} = ?$

$y' = 2 \cdot e^{2x}$; $y'' = 4 \cdot e^{2x}$; $y''' = 8e^{2x}$, $y^{IV} = 16e^{2x}$, $\underline{y^{V} = 32e^{2x}}$

$y^{n} = 2^{n} \cdot e^{2x}$

3). $y = \ln(1+x)$, $y^n = ?$

$y' = \frac{(1+x)'}{1+x} = \frac{1}{1+x}$; $y'' = \left(\frac{1}{1+x}\right)' = -\frac{1}{(1+x)^2}$;

$y''' = \left(-\frac{1}{(1+x)^2}\right)' = \frac{2}{(1+x)^3}$; $y^{IV} = \left(\frac{2}{(1+x)^3}\right)' = -\frac{6}{(1+x)^4}$

$y^{V} = \left(-\frac{6}{(1+x)^4}\right)' = \frac{24}{(1+x)^5}$ $\{ n! = 1, 2, 4, 6, 24 \dots n \}$

$\Rightarrow$ $y^n = \frac{(-1)^{n-1} \cdot (n-1)!}{(1+x)^n}$