Урок 54. Д/З.

(1) Найти область опред. ф-ии.

а) $f(x) = \ln(x+2)$

$D(f) = (-2; +\infty)$

(2) Найти множ-ва значений ф-ии

а) $f(x) = 2^{x^2}$

$E(f) = (0; +\infty)$

б) $f(x) = 3 - 5\cos x$

Если $-1 \le \cos x \le 1$, то $-5 \le -5\cos x \le 5$ и $-2 \le 3 - 5\cos x \le 8$

$E(f) = [-2; 8]$

(3) Построить график ф-ии.

а) $y = x^2 + 4x + 3$

x	-5	-3	-2	-1	2
y	8	0	-1	0	15

$D = 16 - 4 \cdot 1 \cdot 3 = 4$

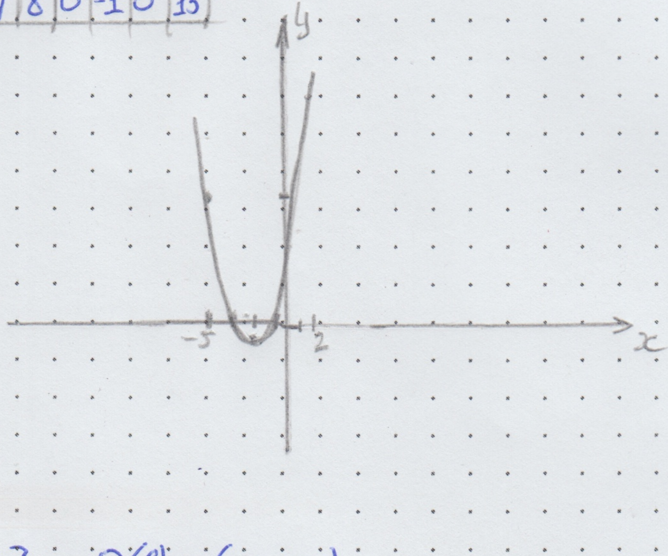
$x_1 = -1 \qquad x_2 = -3$

$f(-5) = 25 - 20 + 3 = 8$

$f(-2) = 4 - 8 + 3 = -1$

$f(2) = 4 + 8 + 3 = 15$

$f'(x) = 2x + 4 \Rightarrow x = -2$

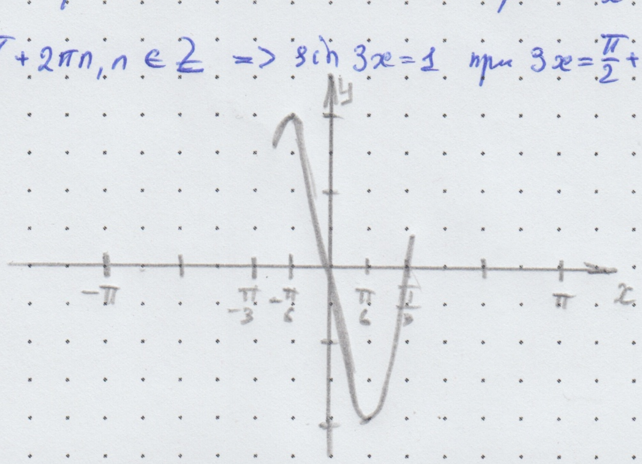


б) $y = -2\sin 3x \qquad E(f) = [-2; 2] \qquad D(f) = (-\infty; \infty)$

$\sin x = 0$ при $x = \pi n$, $n \in Z \Rightarrow \sin 3x = 0$ при $3x = \pi n$; $x_0 = \frac{\pi n}{3}, n \in Z$

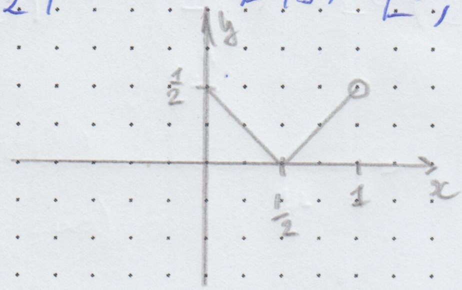
$\sin x = 1$ при $x = \frac{\pi}{2} + 2\pi n, n \in Z \Rightarrow \sin 3x = 1$ при $3x = \frac{\pi}{2} + 2\pi n$; $x_1 = \frac{\pi}{6} + 6\pi n, n \in Z$

x	$-\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{6}$	0
y	+2	0	-2	0



в) $y = |\{x\} - \frac{1}{2}|$ $\qquad E(f) = [0; \frac{1}{2})$

x	0	(1)	$1/2$
y	$1/2$	$(1/2)$	0



④ Найти обратную ф-ю

а) $y = x - 1$ $\qquad D(f) = (-\infty; \infty) \Rightarrow$ имеет $f(x)^{-1}$

$x = y + 1 \quad \Rightarrow \quad f(x)^{-1} = y = x + 1$

б) $y = \sqrt{x}$ $\qquad D(f) = (-\infty; \infty) \Rightarrow$ имеет $f(x)^{-1}$

$x = y^2 \quad \Rightarrow \quad f(x)^{-1} = y = x^2$

⑤ Найти предел

1) $\lim\limits_{x \to -2} (5x^2 + 2x - 1) = 20 - 4 - 1 = 15$

2) $\lim\limits_{x \to 0} \frac{x}{x^2 - x} = \left[\frac{0}{0}\right] = \lim\limits_{x \to 0} \frac{\cancel{x}}{\cancel{x}(x-1)} = \frac{1}{-1} = -1$

3) $\lim\limits_{x \to 5} \frac{x^2 - 6x + 5}{x^2 - 25} = \left[\frac{0}{0}\right] = \left| \begin{matrix} x^2 - 6x + 5 = 0 \\ D = 16, \; x_1 = 1, \; x_2 = 5 \end{matrix} \right| =$

$= \lim\limits_{x \to 5} \frac{(x-1)\cancel{(x-5)}}{(x+5)\cancel{(x-5)}} = \frac{4}{10} = \frac{2}{5}$

4) $\lim\limits_{x \to -1} \frac{x^3 + x + 2}{x^3 + 1} = \left[\frac{0}{0}\right] = \left| \begin{matrix} x^3 + x + 2 = x^3 - x^2 + 2x + x^2 - x + 2 = \\ = x(x^2 - x + 2) + (x^2 - x + 2) = \\ = (x+1)(x^2 - x + 2) \end{matrix} \right| =$

$= \lim\limits_{x \to -1} \frac{\cancel{(x+1)}(x^2 - x + 2)}{\cancel{(x+1)}(x^2 - x + 1)} = \frac{4}{3}$

5) $\lim\limits_{x \to 3} \frac{\sqrt{2x+3} - 3}{\sqrt{x-2} - 1} = \frac{\sqrt{6+3} - 3}{\sqrt{3-2} - 1} = \left[\frac{0}{0}\right] = \lim\limits_{x \to 3} \frac{(\sqrt{2x+3} - 3) \cdot (\sqrt{2x+3} + 3)}{(\sqrt{x-2} - 1)(\sqrt{2x+3} + 3)} =$

$= \lim\limits_{x \to 3} \frac{2 \cdot (x-3) \cdot (\sqrt{x-2} + 1)}{(\sqrt{x-2} - 1) \cdot (\sqrt{2x+3} + 3)(\sqrt{x-2} + 1)} = \lim\limits_{x \to 3} \frac{2\cancel{(x-3)}(\sqrt{x-2} + 1)}{\cancel{(x-3)} \cdot (\sqrt{2x+3} + 3)} = \frac{2}{3}$

6) $\lim\limits_{x\to+\infty}\left(\sqrt{x^2+4}-x\right)=[\infty-\infty]=\lim\limits_{x\to+\infty}\frac{(\sqrt{x^2+4}-x)(\sqrt{x^2+4}+x)}{(\sqrt{x^2+4}+x)}=$

$=\lim\limits_{x\to+\infty}\frac{\cancel{x^2}+4-\cancel{x^2}}{\underbrace{\sqrt{x^2+4}+x}_{\to\infty}}=0$

7) $\lim\limits_{x\to0}\frac{1-\cos x}{x^2}=\left[\frac{0}{0}\right]=\lim\limits_{x\to0}\left(\frac{1-\cos x}{x^2}\cdot\frac{(1+\cos x)}{(1+\cos x)}\right)=$

$=\lim\limits_{x\to0}\frac{1-\cos^2x}{x^2\cdot(1+\cos x)}=\underbrace{\lim\limits_{x\to0}\frac{\sin^2x}{x^2}}_{1}\cdot\lim\limits_{x\to0}\frac{1}{1+\cos x}=\frac{1}{2}$

8) $\lim\limits_{x\to0}x\cdot ctg\,x=\lim\limits_{x\to0}x\cdot\frac{\cos x}{\sin x}=[0\cdot0]=\lim\limits_{x\to0}\cos x\cdot\lim\limits_{x\to0}\frac{x}{\sin x}=$

$=\lim\limits_{x\to0}\underbrace{\left(\frac{1}{\cos x}\right)^{-1}}_{1}\cdot\lim\limits_{x\to0}\underbrace{\left(\frac{\sin}{x}\right)^{-1}}_{1}=1$

9) $\lim\limits_{x\to0}\frac{\cos5x-\cos3x}{x^2}=\left[\frac{0}{0}\right]=\left|\begin{array}{l}\cos\alpha-\cos\beta=\\=2\sin\frac{\alpha+\beta}{2}\cdot\sin\frac{\beta-\alpha}{2}\end{array}\right|=$

$=\lim\limits_{x\to0}\frac{2\cdot\sin\frac{8x}{2}\cdot\sin\frac{-2x}{2}}{x^2}=\lim\limits_{x\to0}\frac{2\cdot\sin4x\cdot\sin(-x)}{x^2}=$

$=2\cdot\lim\limits_{x\to0}\frac{\sin4x}{x}\cdot\lim\limits_{x\to0}\frac{\sin(-x)}{x}=\left|\cdot\frac{4}{4};\cdot\frac{-1}{-1}\right|=$

$=2\cdot\lim\limits_{x\to0}4\underbrace{\frac{\sin4x}{4x}}_{=1}\cdot\lim\limits_{x\to0}-1\underbrace{\frac{\sin(-x)}{-x}}_{=1}=2\cdot4\cdot(-1)=-8$

10) $\lim\limits_{x\to0}\sqrt[2x]{1+3x}=[1^\infty]=\lim\limits_{x\to0}(1+3x)^{\frac{1}{1,5\cdot2x}\cdot1,5}=$

$=\left[\lim\limits_{x\to0}(1+3x)^{\frac{1}{3x}}\right]^{\frac{3}{2}}=\sqrt{e^3}$

12) $\lim\limits_{x\to0}\left(\frac{3+5x}{3+2x}\right)^{\frac{1}{x}}=\frac{\lim\limits_{x\to0}(3+5x)^{\frac{1}{x}}}{\lim\limits_{x\to0}(3+2x)^{\frac{1}{x}}}=\frac{\lim\limits_{x\to0}\left(\frac{3}{3}+\frac{5x}{3}\right)^{\frac{1}{x}}}{\lim\limits_{x\to0}\left(\frac{3}{3}+\frac{2x}{3}\right)^{\frac{1}{x}}}=$

$=\frac{\left(\lim\limits_{x\to0}\left(1+\frac{5x}{3}\right)^{\frac{3}{5x}}\right)^{\frac{5}{3}}}{\left(\lim\limits_{x\to0}\left(1+\frac{2x}{3}\right)^{\frac{3}{2x}}\right)^{\frac{2}{3}}}=\frac{\sqrt[3]{e^5}}{\sqrt[3]{e^2}}=e$