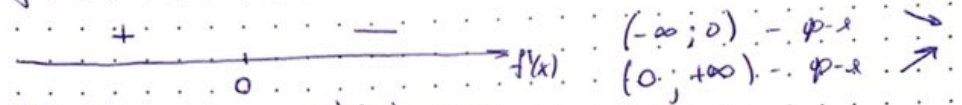
Д/З к уроку №7

① Найти интервалы возр. и убыв. ф-и

1) $f(x) = x + e^{-x}$

$f'(x) = 1 - e^{-x}$

$f'(x) = 0$ в т. $x = 0$ – стац. точки



$(-\infty; 0)$ – ф-я ↘

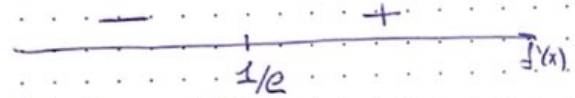
$(0; +\infty)$ – ф-я ↗

$f'(-1) > 0$; $f'(1) < 0$

2) $f(x) = x \cdot \ln x$ $\quad D(f) = (0; \infty)$

$f'(x) = x' \cdot \ln x + x(\ln x)' = 1 \cdot \ln x + x \cdot \frac{1}{x} = \ln x + 1$

$f'(x) = 0$ в т. $x = 1/e$

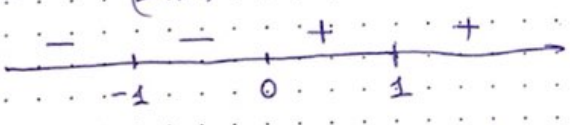


$f'(0{,}1) \approx -1{,}302 < 0$

$f'(2) \approx 0{,}69 > 0$

3) $y = \frac{1}{1-x^2}$

$y' = +\frac{2x}{(1-x^2)^2}$; $y' = 0$ при $x = 0$, т. $x = \pm 1$ – асимптоты



$(-\infty; -1) \cup (-1; 0)$ – ф-я ↘

$(0; 1) \cup (1; +\infty)$ – ф-я ↗

$y'(-2) = -\frac{4}{9} < 0$, $y'(-0{,}5) \approx -0{,}64 < 0$

$y'(0{,}5) \approx 1{,}7 > 0$, $y'(2) = \frac{4}{9} > 0$

② Найти экстремумы функции

1) $f(x) = x^3 - 3x + 1$

$f'(x) = 3x^2 - 3$, $f'(x) = 0$ при $x = \pm 1$

$f''(x) = 6x$

$f''(\pm 1) = \pm 6$ $\Rightarrow$ $f'(x) = 0$, $f''(-1) < 0 \Rightarrow x = -1$ – т. лок. максимума

$f'(x) = 0$, $f''(1) > 0 \Rightarrow x = 1$ – т. лок. минимума

2) $y = e^{x^2-4x+5}$

$y' = e^{x^2-4x+5} \cdot (x^2-4x+5)' = e^{x^2-4x+5}(2x-4)$

$y'(x) = 0$ в т. $x = 2$

$y''(x) = (e^{x^2-4x+5})' (2x-4) + e^{x^2-4x+5} \cdot (2x-4)' = e^{x^2-4x+5} \cdot ((2x-4)^2 + 2)$

$y''(2) = e \cdot 2 = 2e$

$y'(2) = 0$, $y''(2) > 0$ $\Rightarrow$ $x = 2$ – т. локального минимума

3) $y = x - \operatorname{arctg} x$

$y' = 1 - \frac{1}{1+x^2}$, $y'(0)$ – не существует $\Rightarrow$ нет лок. мин. и макс.

$x = 0$ – является критической точкой.

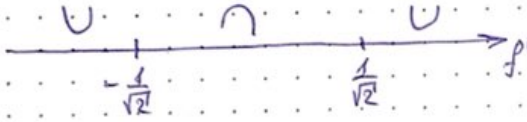
(3) Найти интервалы выпуклости и т. перегиба.

1) $f(x) = e^{-x^2}$

$f'(x) = (-x^2)' \cdot e^{-x^2} = -2x \cdot e^{-x^2}$

$f''(x) = (-2x)' \cdot e^{-x^2} + (-2x) \cdot (e^{-x^2})' = -2 \cdot e^{-x^2} + 4x^2 \cdot e^{-x^2} = 2e^{-x^2}(2x^2 - 1)$

$f''(x) = 0$ в т. $x = \pm \frac{1}{\sqrt{2}}$



$f''(-2) = 7 \geq 0$

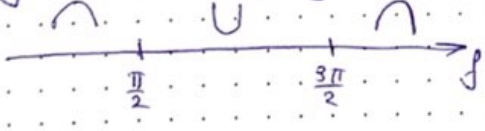
$f''(0) = -1 \leq 0$ $\Rightarrow$ $(-\infty; -\frac{1}{\sqrt{2}}) \cup (\frac{1}{\sqrt{2}}; +\infty)$ – ф-я ∪

$f''(2) = 7 \geq 0$ $(-\frac{1}{\sqrt{2}}; \frac{1}{\sqrt{2}})$ – ф-я ∩

2) $y = \cos x$

$y' = -\sin x$

$y'' = -\cos x$, $y''(x) = 0$ в т. $x_1 = \frac{\pi}{2} + 2\pi n$, $x_2 = \frac{3\pi}{2} + 2\pi n$



$y''(0) = -1 \leq 0$, $y''(\pi) = 1 \geq 0$, $y''(2\pi) = -1 \leq 0$

$(-\frac{\pi}{2} + 2\pi n; \frac{\pi}{2} + 2\pi n)$ – ф-я ∩

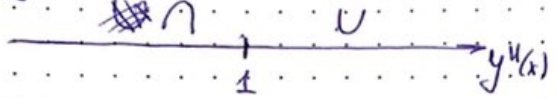
$(\frac{\pi}{2} + 2\pi n; \frac{3\pi}{2} + 2\pi n)$ – ф-я ∪

3) $y = x^5 - 10x^2 + 7x$

$y' = 5x^4 - 20x + 7$

$y'' = 20x^3 - 20$

$y''(x) = 0$ в т. $x = 1$



$y''(0) = -20 \leq 0$ $\Rightarrow$ $(-\infty; 1)$ – вып. $\cap$

$y''(2) = 140 \geqslant 0$ $\Rightarrow$ $(1; +\infty)$ – вып. $\cup$