Д/З к уроку № 8

(1) 1) $z = \sqrt{y \cdot \sin x}$

$\sqrt{y - \sin x} \geq 0 \Rightarrow |y| \geq |\sin x| \Rightarrow |y| \geq 1$

$D(f): \begin{cases} x: (-\infty; \infty) \\ |y|: [1; +\infty) \end{cases}$

2) $z = x + \arccos y$

$D(f): \begin{cases} x: (-\infty; \infty) \\ y: [-1; 1] \end{cases}$

(2) 1) $\lim\limits_{\substack{x \to \infty \\ y \to \infty}} (x+y) \underbrace{\sin \frac{1}{x}}_{\text{не опред.}} \cdot \underbrace{\cos \frac{1}{y}}_{\text{не опред.}} = 0$

2) $\lim\limits_{\substack{x \to 1 \\ y \to 2}} \frac{2 \cdot (x-1)(y-2)}{(x-1)^2 + (y-2)^2} = \left| \begin{matrix} x-1 = t, & y-2 = z \\ \begin{matrix} x \to 1 \\ y \to 2 \end{matrix} & \Rightarrow \begin{matrix} t \to 0 \\ z \to 0 \end{matrix} \end{matrix} \right| = \lim\limits_{\substack{t \to 0 \\ z \to 0}} \frac{2tz}{t^2+z^2} =$

Вар. 1

$= \left| \begin{matrix} t^2 + z^2 \geq 2tz \\ \frac{2tz}{t^2+z^2} \leq \frac{t^2+z^2}{t^2+z^2} \leq 1 \end{matrix} \right| = 1 \quad ?$

Вар. 2

$= \left| \begin{matrix} (x-1) = r \cdot \sin\varphi \\ (y-2) = r \cdot \cos\varphi \\ r \to 0 \end{matrix} \right| = \lim\limits_{r \to 0} \frac{2 \cancel{r^2} \cdot \sin\varphi \cdot \cos\varphi}{\cancel{r^2} \cdot (\sin^2\varphi + \cos^2\varphi)} =$

$= \lim\limits_{r \to 0} \frac{\sin 2\varphi}{1} = \sin 2\varphi \quad ?$

3) $\lim\limits_{\substack{x \to \infty \\ y \to \infty}} \left(\frac{xy}{x^2+y^2}\right)^{x^2} = e^{\ln\left(\lim\limits_{\substack{x \to \infty \\ y \to \infty}} \left(\frac{xy}{x^2+y^2}\right)^{x^2}\right)} =$

Вар 1

$[\infty \cdot \infty]$

$\lim\limits_{\substack{x \to \infty \\ y \to \infty}} \left(x^2 \cdot \ln\left(\frac{xy}{x^2+y^2}\right)\right) \overset{|:x^2}{=} \ln\left(\lim\limits_{\substack{x \to \infty \\ y \to \infty}} \frac{xy}{x^2+y^2}\right) = \left| \begin{matrix} x = r \cdot \cos\varphi \\ y = r \cdot \sin\varphi \\ r \to \infty \end{matrix} \right| =$

$= \ln\left(\lim\limits_{\substack{x \to \infty \\ y \to \infty}} \frac{\cancel{r^2} \cdot \sin\varphi \cdot \cos\varphi}{\cancel{r^2}(\sin^2\varphi + \cos^2\varphi)}\right) = \ln(\sin\varphi \cdot \cos\varphi)$

Вар. 2

$$= \left| \begin{array}{l} x\to\infty \\ y\to\infty \end{array} \right\} \Rightarrow \begin{array}{l} z\to 0 \\ t\to 0 \end{array} \quad x=\frac{1}{z} \quad y=\frac{1}{t} \Bigg| = \lim_{\substack{z\to 0 \\ t\to 0}} \left(\frac{zt}{z^2+t^2}\right)^{\frac{1}{z^2}} =$$

$$= \left| \begin{array}{l} z^2+t^2 \geq 2zt \\ \frac{zt}{z^2+t^2} \leq \frac{1}{2} \end{array} \right\} 0 \leq \left(\frac{zt}{z^2+t^2}\right)^{\frac{1}{z^2}} < \frac{1}{2}^{\frac{1}{z^2}} \Bigg| = \lim_{\substack{z\to 0 \\ t\to 0}} \frac{1}{2}^{\overset{\to\infty}{\frac{1}{z^2}}} = 0$$

4) $$\lim_{\substack{x\to\infty \\ y\to\infty}} (x^2+y^2)^{\to 0} \cdot \underbrace{\sin^3 \frac{1}{xy}}_{\text{не опред}} = 0$$

5) $$\lim_{\substack{x\to 0 \\ y\to 0}} \frac{x^2 \cdot y^3}{x^2+y^2} = \left| \begin{array}{l} \lim\limits_{x\to 0} \lim\limits_{y\to 0} \frac{x^2 \cdot \overset{\to 0}{y^3}}{x^2+y^2} = 0 \\ \lim\limits_{y\to 0} \lim\limits_{x\to 0} \frac{\overset{\to 0}{x^2} \cdot y^3}{x^2+y^2} = 0 \end{array} \right| = 0$$

(3) Найти частные и полное приращение ф-и в т.

$z = x^2 \cdot y$; $M_0(1;2)$, $\Delta x = 0{,}1$, $\Delta y = -0{,}2$

$x_0 = 1$, $y_0 = 2$

$x_0 + \Delta x = x = 1{,}1$ $\quad y_0 + \Delta y = y = 1{,}8$ $\Rightarrow M_1(1{,}1; 1{,}8)$

$z(M_0) = z(1,2) = 2$

$z(x_0+\Delta x); y_0) = z(1{,}1; 2) = 2{,}42$

$z(x_0; y_0+\Delta y) = z(1; 1{,}8) = 1{,}8$

$z(M_1) = z(1{,}1; 1{,}8) = 2{,}178$

$\Delta_x z = z(x_0+\Delta x; y_0) - z(x_0; y_0) = 0{,}42$

$\Delta_y z = z(x_0; y_0+\Delta y) - z(x_0; y_0) = -0{,}2$

$\Delta z = z(x_0+\Delta x; y_0+\Delta y) - z(x_0; y_0) = 0{,}178$

$\Delta z = 0{,}178 \neq \Delta_x z + \Delta_y z = 0{,}4$

(4) Найти частные произв-ые ф-ий.

1) $z = e^{x^2+y^2}$

$z'_x = e^{x^2+y^2} \cdot (2x+0) = 2x \cdot e^{x^2+y^2}$

$z'_y = e^{x^2+y^2} \cdot (0+2y) = 2y \cdot e^{x^2+y^2}$

2) $u = x^y + (xy)^z + z^{xy}$

$u'_x = y \cdot x^{y-1} + y^z \cdot z \cdot x^{z-1} + z^{x\cdot y} \cdot y \cdot \ln(z)$

$u'_y = x^y \cdot \ln(x) + x^z \cdot z \cdot y^{z-1} + z^{xy} \cdot x \cdot \ln(z)$

$u'_z = 0 + (xy)^z \cdot \ln(xy) + x \cdot y \cdot z^{xy-1}$

5 Вычислить приближённо

1) $1{,}04^{2.03}$

$f(x;y) = x^y$, $x = 1{,}04$, $y = 2{,}03$

$f(x_0, y_0) = 1^2$, $x_0 = 1$, $y_0 = 2$

$\Delta x = x - x_0 = 1{,}04 - 1 = 0{,}04$

$\Delta y = y - y_0 = 2{,}03 - 2 = 0{,}03$

$f(x + \Delta x; y + \Delta y) \approx f(x_0; y_0) + df(x_0; y_0)$

$f'_x = y \cdot x^{y-1}$, $f'_y = x^y \cdot \ln x$

$f'_x(1;2) = 2 \cdot 1^1 = 2$ $f'_y(1;2) = 1^2 \cdot \ln 1 = 0$

$df(1;2) = 2 \cdot 0{,}04 + 0 \cdot (0{,}03) = 0{,}08 \Rightarrow 1{,}04^{2{,}03} \approx 1 + 0{,}08 = 1{,}08$

2) $\sin 28° \cdot \cos 61°$

$f(x,y) = \sin y \cdot \cos y$, $x = 28°$, $y = 61°$

$f(x_0, y_0) = \sin 30° \cdot \cos 60° = \frac{1}{4}$, $x_0 = 30°$, $y_0 = 60°$

$\Delta x = x - x_0 = 28 - 30 = -2$, $\Delta y = y - y_0 = 61 - 60 = 1$

$f(x + \Delta x; y + \Delta y) \approx f(x_0; y_0) + df(x_0, y_0)$

$f'_x = \cos x \cdot \cos y$

$f'_y = -\sin x \cdot \sin y$

$f'_x(30;60) = \cos 30° \cdot \cos 60° = \frac{\sqrt{3}}{4}$

$f'_y(30;60) = -\sin 30° \cdot \sin 60° = -\frac{\sqrt{3}}{4}$

$df(30;60) = f'_x \cdot \Delta x + f'_y \cdot \Delta y = \frac{\sqrt{3}}{4} \cdot \frac{-2\pi}{180} + \frac{-\sqrt{3}}{4} \cdot \frac{\pi}{180} = -\frac{\pi}{180} \cdot \left(\sqrt{3} + \frac{\sqrt{3}}{4}\right)$

$\sin 28° \cdot \cos 61° \approx \frac{1}{4} - \frac{\pi}{180}\left(\sqrt{3} + \frac{\sqrt{3}}{4}\right) \approx 0{,}2122$

3) $\sqrt{(\sin^2 1{,}55 + 8e^{0{,}015})^5}$

$f(x;y) = \sqrt{(\sin^2 x + 8e^y)^5}$, $x = 1{,}55$, $y = 0{,}015$

$x_0 = 1{,}57$, $y_0 = 0$

$f(x_0; y_0) = \sqrt{(\sin^2 1{,}57 + 8e^0)^5} = \sqrt{9^5} = 243$

$\Delta x = x - x_0 = -0{,}02$ $\quad \Delta y = y - y_0 = 0 - 0{,}015 = -0{,}015$

$f'_x = \frac{5}{2} \cdot (\sin^2 x + 8e^y)^{\frac{3}{2}} \cdot 2 \cdot \cos x \cdot \sin x$

$f'_y = \frac{5}{2} \cdot (\sin^2 x + 8e^y)^{\frac{3}{2}} \cdot 8e^y$

$f'_x(1{,}57; 0) = \frac{5}{2} \cdot (1+8)^{\frac{3}{2}} \cdot 0 = 0 \quad f'_y = \frac{5}{2} \cdot 9^{\frac{3}{2}} \cdot 8 = 540$

$df(1{,}57; 0) = f'_x \cdot \Delta x + f'_y \cdot \Delta y \approx 0 + 540 \cdot (-0{,}015) = -8{,}1$

$\sqrt{(\sin^2 1{,}55 + 8e^{0{,}015})^5} \approx 243 - 8{,}1 = 234{,}9$