Урок № 10 Д/З

1. Найти интегралы:

1) $\int \frac{x^4+x^2-6x}{x^3}dx = \int \frac{\cancel{x}(x^3+x-6)}{x^{\cancel{3}2}}dx = \int \frac{x^3}{x^2}dx + \int \frac{x}{x^2}dx - \int \frac{6}{x^2}dx =$

$= \int x\cdot dx + \int \frac{1}{x}dx + \int \frac{6}{x^2}dx = \frac{x^2}{2} + \ln|x| + \frac{6}{x} + C$

2) $\int \cos(2x)dx = \left| \begin{matrix} t=2x \\ dt=2dx \end{matrix} \right\} \Rightarrow dx = \frac{1}{2}dt \bigg| = \frac{1}{2}\int \cos t\, dt = \frac{1}{2}\sin(2x) + C$

3) $\int \frac{dx}{(3x+2)^4} = \left| \begin{matrix} t=(3x+2) \\ dt=3dx \Rightarrow dx=\frac{1}{3}dt \end{matrix} \right| = \frac{1}{3}\int t^{-4}dt = -\frac{1}{9\cdot t^3} + C = -\frac{1}{9\cdot(3x+2)^3} + C$

4) $\int \frac{5x-1}{\sqrt{4-x^2}}dx = \int \frac{5x}{\sqrt{4-x^2}}dx - \int \frac{1}{\sqrt{4+x^2}}dx \ (=)$

1. $\int \frac{5x}{\sqrt{4-x^2}}dx = 5\cdot\int \frac{x}{\sqrt{4-x^2}}dx = \left| \begin{matrix} t=4-x^2 \\ dt=d(4-x^2) \\ dt=-2x\,dx \\ xdx=-\frac{1}{2}dt \end{matrix} \right| = 5\left(-\frac{1}{2}\right)\int \frac{1}{\sqrt{t}}dt =$

$= 5\cdot\cancel{\left(-\frac{1}{2}\right)}\cdot\frac{t^{\frac{1}{2}}}{\cancel{\frac{1}{2}}} + C = -5\cdot\sqrt{t} + C = -5\cdot\sqrt{4-x^2} + C$

2. $\int \frac{1}{\sqrt{4-x^2}}dx = \arcsin\left(\frac{x}{2}\right) + C$

$(=) \ -5\cdot\sqrt{4+x^2} - \arcsin\left(\frac{x}{2}\right) + C$

5) $\int \frac{dx}{\sqrt{x}\cdot(1+\sqrt{x})} = \left| \begin{matrix} x=t^2 \\ dx=2t\,dt \end{matrix} \right| = \int \frac{2\cancel{t}dt}{\cancel{t}\cdot(1+t)} = 2\int \frac{1}{1+t}dt = 2\,\mathrm{arctg}\left(\sqrt[4]{x}\right) + C$

6) $\int x^2\cdot\cos x\cdot dx = \left| \begin{matrix} u=x^2 \Rightarrow du=2x\,dx \\ dv=\cos x \\ \Rightarrow v=\sin x + C \\ \int u\,dv = uv - \int v\,du \end{matrix} \right| = x^2\sin x - \int \sin x\cdot 2x\cdot dx =$

$= x^2\sin x - 2\int x\cdot\sin x\,dx = \left| \begin{matrix} dv=\sin x \Rightarrow v=-\cos x + C \\ u=x \Rightarrow du=dx \end{matrix} \right| = x^2\sin x - 2\cdot(x\cdot(-\cos x) -$

$- \int(-\cos x)\cdot dx) = x^2\cdot\sin x + 2x\cdot\cos x - 2\sin x + C$

7) $\int \operatorname{arctan} x \cdot dx = \left| \begin{matrix} u = \operatorname{arctg} x \Rightarrow du = \frac{1}{1+x^2} dx \\ dv = 1 \Rightarrow v = x + c \end{matrix} \right| = \operatorname{arctg} x \cdot (x) - \int \frac{x}{1+x^2} dx =$

$= \left| \begin{matrix} t = 1 + x^2 \\ dt = 2x\,dx \end{matrix} \right\} \Rightarrow x\,dx = \frac{1}{2} dt \Big| = \operatorname{arctg} x \cdot x - \frac{1}{2} \int \frac{1}{t} dt = x \cdot \operatorname{arctg} x - \frac{1}{2} \ln|1 + x^2| + C$

8) $\int \frac{dx}{(x-1)^5} = \left| \begin{matrix} t = (x-1) \\ dt = \cancel{4(x-1)}\,dx \end{matrix} \right| = \int \frac{1}{t^5} dt = \int t^{-5} dt = \frac{t^{-4}}{-4} + C = \frac{1}{-4 \cdot (x-1)^4} + C$

9) $\int \frac{(x+6)dx}{x^2 - 2x + 17} = \frac{1}{2} \cdot \ln(x^2 - 2x + 17) + \frac{12 + 2}{\sqrt{4 \cdot 17 - 17^2}} \cdot \operatorname{arctg} \frac{2x - 2}{\sqrt{4 \cdot 17 - 17^2}} + C =$

$= \frac{1}{2} \ln(x^2 - 2x + 17) + \frac{14}{\sqrt{357}} \cdot \operatorname{arctg} \frac{2x - 2}{\sqrt{357}} + C$

10) $\int \frac{x\,dx}{(x^2-1)(x^2+1)} = \int \frac{x\,dx}{x^4 - 1} = \left| \begin{matrix} t = x^2 \\ dt = 2x\,dx \\ x\,dx = \frac{1}{2} dt \end{matrix} \right| = \frac{1}{2} \int \frac{1}{t^2 - 1} dt = -\frac{1}{2} \int \frac{1}{1 - t^2} dt =$

$= -\frac{1}{2} \cdot \left(\frac{1}{2} \cdot \ln\left|\frac{1+t}{1-t}\right| + C \right) = -\frac{1}{4} \cdot \ln\left|\frac{1+x^2}{1-x^2}\right| + C$

11) $\int \frac{\sqrt[3]{x}\,dx}{\sqrt[3]{x^2} - \sqrt{x}} = \left| \begin{matrix} x = t^6 \\ dx = 6t^5 dt \end{matrix} \right| = \int \frac{t^2 \cdot 6 \cdot t^5\,dt}{t^4 - t^3} = \int \frac{6t^{4}\,dt}{\cancel{t^3}(t-1)} =$

$= \int \frac{6t^4}{t-1} dt = \left| \begin{matrix} u = t - 1 \\ dv = t^4 \end{matrix} \right\} \Rightarrow \begin{matrix} du = 1 \\ v = \int t^4 dt = \frac{1}{5} t^5 + c \end{matrix} \Big| = 6 \cdot \left(t - 1 - \int \frac{1}{5} \cdot t^5 dt \right) =$

$= 6 \cdot \left(t - 1 - \frac{1}{5} \cdot \frac{1}{6} \cdot t^6 + C \right)$

12) $\int \sqrt[3]{x} \cdot \sqrt[3]{1 + 3 \cdot \sqrt[3]{x^2}}\,dx = \left| \begin{matrix} x^m (a + bx^n)^p\,dx \\ m = \frac{1}{3},\ n = \frac{2}{3} \\ p = \frac{1}{3} \end{matrix} \right\} \Rightarrow \begin{matrix} \frac{m+1}{n} = 2 \text{ - целое число} \\ a + bx^n = t^r \\ r = 3 \end{matrix} \Big| =$

$= \left| \begin{matrix} 1 + 3x^{\frac{2}{3}} = t^3 ; & d(1 + 3x^{\frac{2}{3}}) = d(t^3) \\ x^{\frac{2}{3}} = \frac{t^3 - 1}{3} ; & 2 \cdot \frac{1}{\sqrt[3]{x}} dx = 3t^2 \cdot dt \\ & \sqrt[3]{x}\,dx = \frac{3}{2} t^2 \cdot \sqrt[3]{x^2}\,dt \end{matrix} \right\} \Rightarrow \begin{matrix} \sqrt[3]{x}\,dx = \frac{\cancel{3}}{2} \cdot t^2 \cdot \frac{t^3 - 1}{\cancel{3}} dt \\ \sqrt[3]{x}\,dx = \frac{t^2 (t^3 - 1)}{2} dt \end{matrix} \Big| =$

$= \frac{1}{2} \int t \cdot t^2 \cdot (t^3 - 1)\,dt = \frac{1}{2} \left(\int t^6 dt - \int t^3 dt \right) = \frac{1}{2} \cdot \left(\frac{t^7}{7} - \frac{t^4}{4} \right) + C =$

$= \frac{t^4 \cdot (4 \cdot t^3 + 7)}{56} + C = \frac{(1 + 3\sqrt[3]{x^2}) \cdot \sqrt[3]{1 + 3\sqrt[3]{x^2}} \cdot (4 \cdot (1 + 3\sqrt[3]{x^2}) + 7)}{56} + C$

13) $\int \frac{dx}{\sin x \cdot \sin 2x} = \frac{1}{2}\int \frac{dx}{(1-\cos^2 x)\cos x} = \frac{1}{2}\int \frac{dx}{\cos - \cos^3 x} = \left| \begin{matrix} u = \cos x - \cos^3 x \\ dv = 1 \end{matrix} \right\} \Rightarrow$

$\Rightarrow \left. \begin{matrix} du = -\sin x - 3\cdot\cos^2 x \cdot \sin x \\ v = x + c \end{matrix} \right| = \frac{1}{2}\cdot\left(x\cdot(\cos x - \cos^3 x) - \int((-\sin x) - 3\cos^2 x\cdot\sin x)\,dx\right) =$

$= \frac{1}{2}\left(x\cdot(\cos x - \cos^3 x) - \cos x - 3\int \cos^2 x\cdot \sin x\,dx\right) \ominus$

$\int \cos^2 x\cdot\sin x\,dx = \left| \begin{matrix} \cos x = t \\ dt = -\sin x \end{matrix} \right| = -\int t^2\,dt = -\frac{t^3}{3} + c = -\frac{\cos^3 x}{3} + c$

$\ominus \frac{1}{2}\left(x(\cos x - \cos^3 x) - \cos x + \cancel{3}\cdot\frac{\cos^3 x}{\cancel{3}} + c\right) = \frac{1}{2}\cdot(x-1)(\cos x - \cos^3 x) + C$

14) $\int \cos 5x\cdot\cos 3x\,dx = \frac{1}{2}\int(\cos 2x + \cos 8x)\,dx = \frac{1}{2}\left(\underset{①}{\int \cos 2x\,dx} + \underset{②}{\int \cos 8x\,dx}\right) \ominus$

① $\int \cos 2x\,dx = \left| \begin{matrix} 2x = t \\ dx = \frac{dt}{2} \end{matrix} \right| = \frac{1}{2}\int \cos t\,dt = \frac{1}{2}\sin t + c = \frac{1}{2}\sin 2x + c$

② $\int \cos 8x\,dx = \left| \begin{matrix} 8x = t \\ dx = \frac{dt}{8} \end{matrix} \right| = \frac{1}{8}\sin 8x + C$

$\ominus \frac{\sin 2x}{4} + \frac{\sin 8x}{16} + C$

15) $\int_0^{\pi}(2x + \sin 2x)\,dx = 2\underset{①}{\int_0^{\pi} x\,dx} + \underset{②}{\int_0^{\pi}\sin 2x\,dx} \ominus$

① $= 2\frac{x^2}{\cancel{2}}\Big|_0^{\pi} = x^2\Big|_0^{\pi} = \pi^2$

② $= \left| \begin{matrix} 2x = t \\ dx = \frac{dt}{2} \end{matrix} \right| = \frac{1}{2}\int_0^{\pi}\sin t\,dt = -\frac{1}{2}\cdot\cos 2x\Big|_0^{\pi} = -\frac{1}{2}\cdot(1-1) = 0$

$\ominus \pi^2$

16) $\int\sqrt{4x-2} = \begin{matrix} t = 4x-2 \\ dt = 4\,dx \\ dx = \frac{dt}{4} \end{matrix} = \int\sqrt{t}\cdot\frac{1}{4}dt = \frac{1}{4}\,\frac{t^{\frac{3}{2}}}{\frac{3}{2}}dt = \frac{1}{\cancel{4}_2}\cdot\frac{\cancel{2}}{3}\cdot t^{3/2} = \frac{1}{6}\cdot t^{3/2}$

$= \frac{1}{6}\cdot(4x-2)^{3/2} + C$

$\left[\frac{1}{6}\cdot\left(\cancel{4}^2\cdot\frac{1}{\cancel{2}} - 2\right)^{3/2} = 0 \qquad \frac{1}{6}\cdot(4-2)^{3/2} = \frac{1}{6}\sqrt{2^3} = \frac{\sqrt{8}}{6}\right]$

$\int_{\frac{1}{2}}^{1}\sqrt{4x-2}\,dx = \frac{1}{6}(4x-2)^{3/2}\Big|_{\frac{1}{2}}^{1} = 0 - \frac{\sqrt{8}}{6} = -\frac{\sqrt{8}}{6}$

17) $\int_0^{+\infty} e^{-4x} dx = \left| \begin{matrix} t=-4x \\ dt=-4dx \\ dx=-\frac{dt}{4} \end{matrix} \right| = \int_0^{+\infty} e^t \cdot \left(-\frac{dt}{4}\right) = -\frac{1}{4} e^t \Big|_0^{+\infty} = -\frac{1}{4} e^{-4x} \Big|_0^{+\infty}$

$= -\frac{1}{4} \cdot (0-1) = \frac{1}{4}$

18) $\int_0^1 \ln x \, dx = \left| \begin{matrix} u=\ln x \\ dv=1 \end{matrix} \right\} \Rightarrow \begin{matrix} du=\frac{1}{x}dx \\ v=x \end{matrix} \right| = \ln(x) \cdot x - \int_0^1 \cancel{x} \cdot \frac{1}{\cancel{x}} dx =$

$= \ln(x) \cdot x - x \Big|_0^1 = -1 - 0 = -1$