SAN FRANCISCO STATE UNIVERSITY
COMPUTER SCIENCE DEPARTMENT

SORTING REVIEW (CSC220)
SELECTION SORT, INSERTION SORT, SHELL SORT

DUC TA

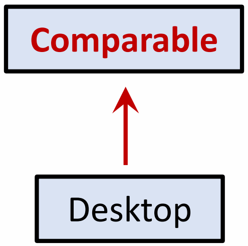
Generic Types?

```
// Selection Sort
public static <T extends Comparable<? super T>> void selectionSort(T[] a, int n){
```

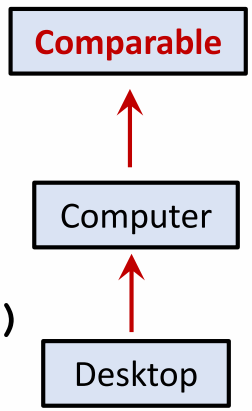
Bounded Type Parameters

```
// Selection Sort, too restrictive

public static <T extends Comparable<T>> T selectionSort(T[] a, int n){
```

- Objects in the passed array be strings, Integer Objects, or any **comparable** objects.
- The class of these objects must implement the interface **Comparable**.
- T is bounded to represent class type that provides the method **compareTo()**.



- The green (1st) **T** is the type parameter's name
- The blue (2nd) **T** is the upper bound of the green (1st) T
- The brown (3rd) **T** is the return type (if used or if not void)
- The black (4th) **T** is the parameter type



- The `extends` keyword used in general sense to mean either extends (as in classes) or implements (as in interfaces). "implements" in this case.

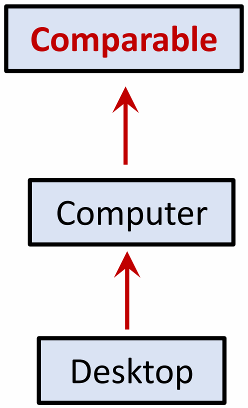
- Problems: it is **more restrictive** than necessary
  - `compareTo` only works for T, but not subclasses or super class of T.
  - `Desktop smallestDesktop = MyClass.arrayMinimum(myArray)`
  - Insisting comparing a Desktop vs. a Desktop, not even a Computer

Bounded WildCard

```
    // CORRECT

    public static <T extends Comparable<? super T>> T arrayMinimum(T[] anArray) {
```

- Problems: it is **more restrictive** than necessary
  - `Desktop smallestDesktop = MyClass.arrayMinimum(myArray)`

- `<? super T>` allows comparing Desktop vs. Computer. That is Desktop vs. an object of Desktop's superclass.
- `<? super T>` means **any superclass** of T
- `<? super T>` T is the **lower bound** of the wild card "`?`"
- `? extends T` T is the **upper bound** of the wild card "`?`"




```
// Selection Sort

public static <T extends Comparable<? super T>> void selectionSort(T[] a, int n){
```

- T[] a: an array of Comparable objects
- int n: the first n objects in the array

Selection Sort

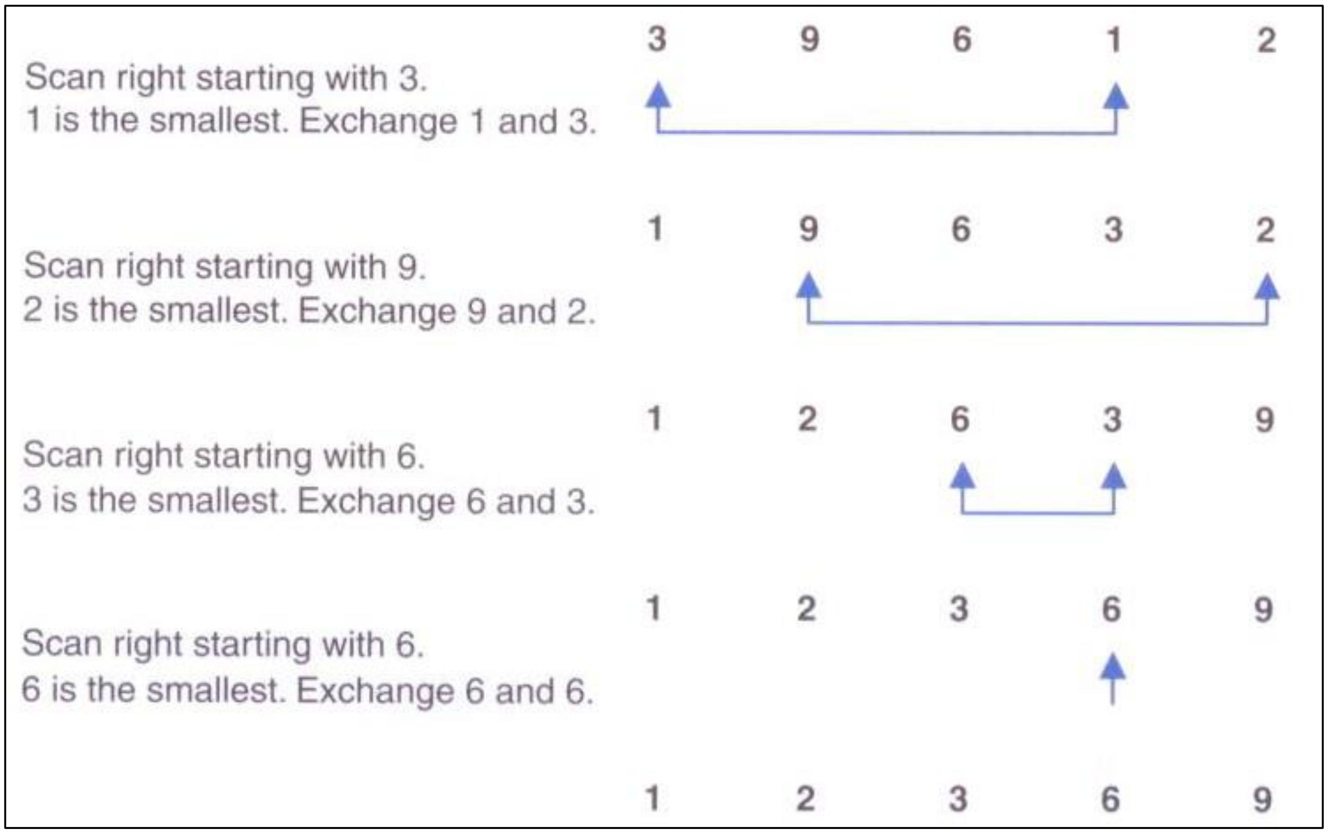

Selection Sort

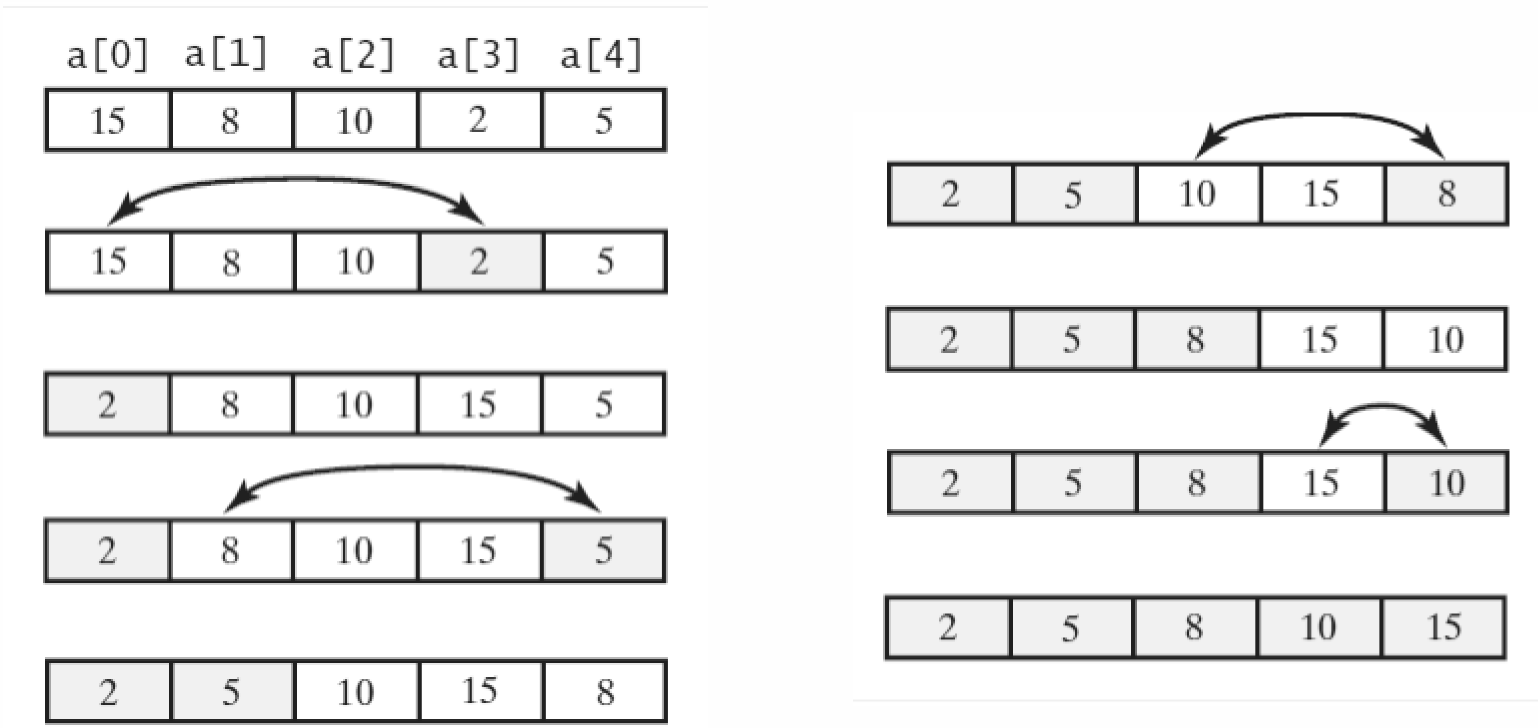
- **Selection sort** orders a list of values by repetitively **putting a particular value into its final position**
- More specifically:
    1. **Find** the **smallest** value in **the list**
    2. **Switch** it with the value in **the first position**
    3. **Find** the next **smallest** value in **the list**
    4. **Switch** it with the value in **the second position**
    5. Repeat **until all values are in their proper places**

Smallest → Final Position

Selection Sort

1. **Find** the **smallest** value in **the list**
2. **Switch** it with the value in **the first position**
3. **Find** the next **smallest** value in **the list**
4. **Switch** it with the value in **the second position**
5. Repeat **until all values are in their proper places**




a[0] a[1] a[2] a[3] a[4]
15 8 10 2 5
15 8 10 2 5
2 8 10 15 5
2 8 10 15 5
2 5 10 15 8
2 5 10 15 8
2 5 8 15 10
2 5 8 15 10
2 5 8 10 15

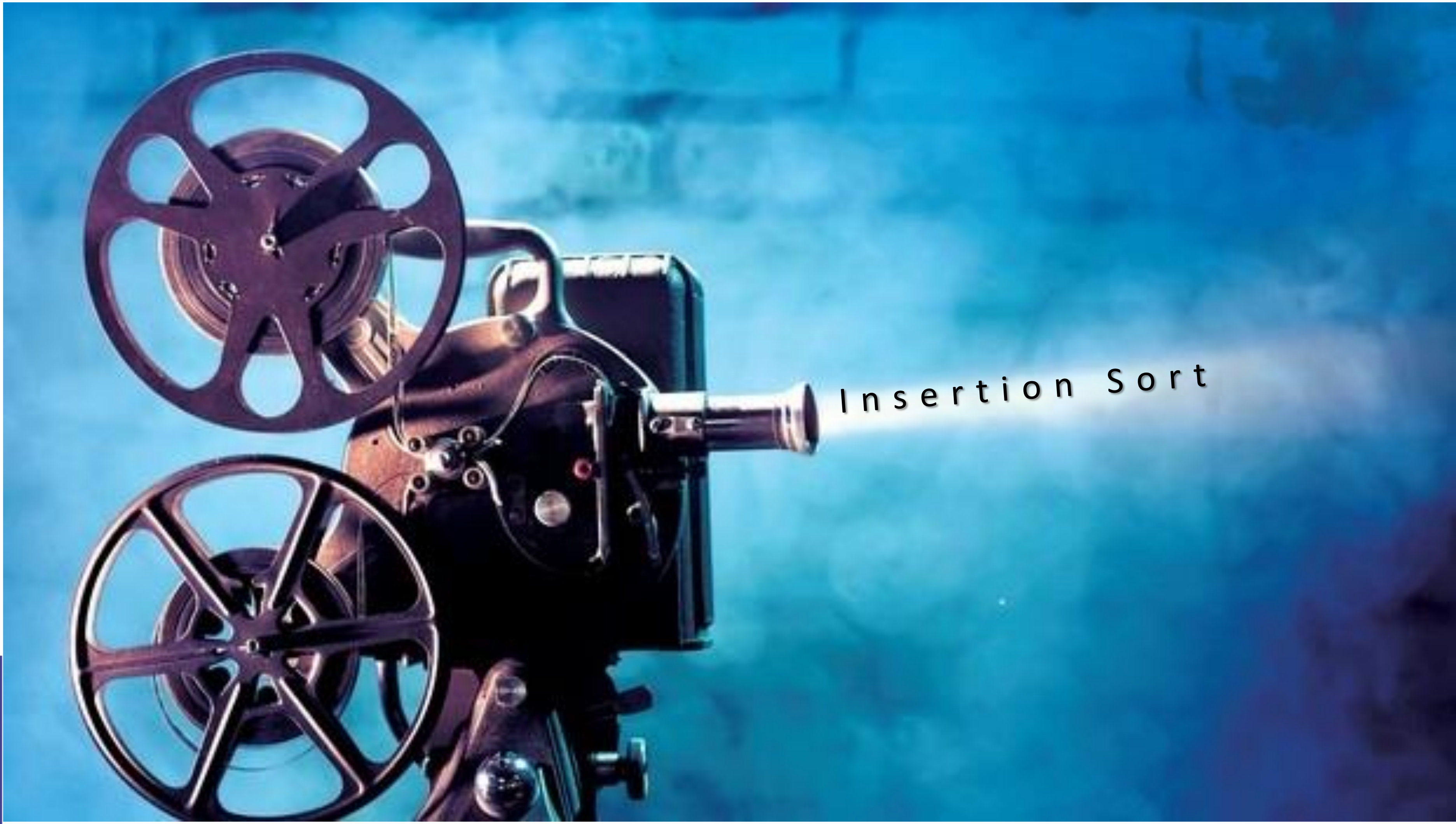
Selection Sort

```java
// Finds the index of the smallest value in a portion of an array a.
// Returns the index of the smallest value among

private static <T extends Comparable<? super T>>
        int getIndexOfSmallest(T[] a, int first, int last) {

    T min = a[first];
    int indexOfMin = first;

    for (int index = first + 1; index <= last; index++) {

        if (a[index].compareTo(min) < 0) {
            min = a[index];
            indexOfMin = index;
        }
    }

    return indexOfMin;
}
```

```java

public static <T extends Comparable<? super T>> void selectionSort(T[] a, int n) {

        for (int index = 0; index < n - 1; index++) {

            int indexOfNextSmallest = getIndexOfSmallest(a, index, n - 1);

            swap(a, index, indexOfNextSmallest);
        }
}

```

```java

    // Swaps the array entries a[i] and a[j].
    private static void swap(Object[] a, int i, int j) {
        Object temp = a[i];
        a[i] = a[j];
        a[j] = temp;
    }

```

Insertion Sort

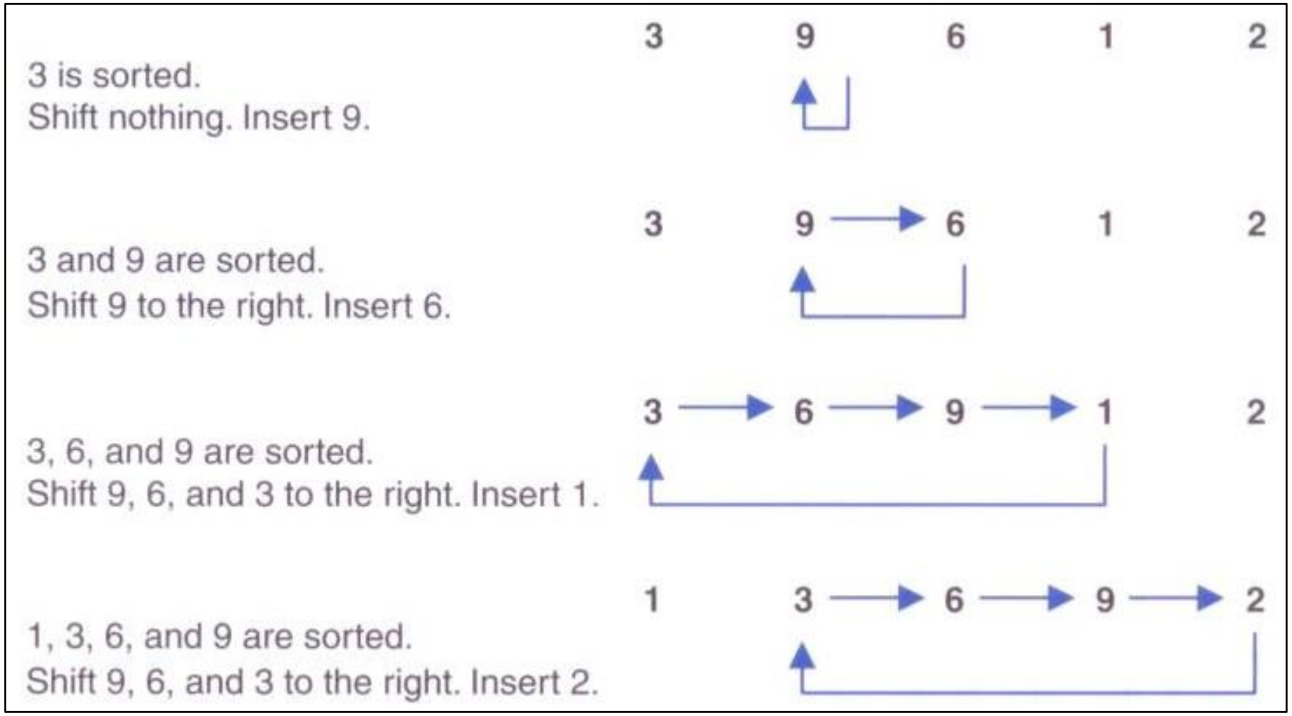

Insertion Sort

- **Insertion sort** orders values by repetitively **inserting a particular value** into a **sorted subset** of the list
- More specifically:
    1. Consider the **first** item to be a **sorted sublist** of length 1
    2. Insert the **second** item into the **sorted sublist**, shifting the first item if needed
    3. Insert the **third** item into the **sorted sublist**, shifting the other items as needed
    4. Repeat **until all values have been inserted into their proper positions**

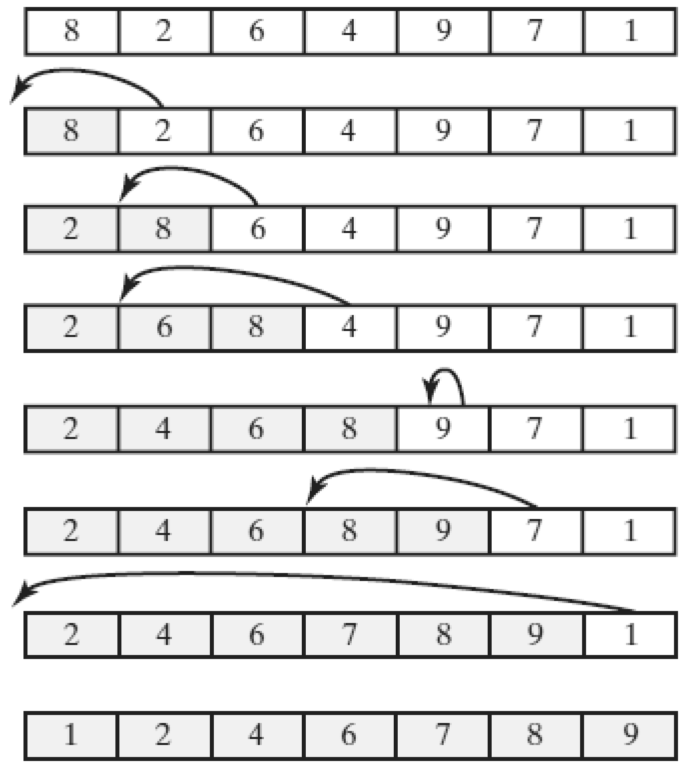
Sorted Sublist

INSERTION SORT

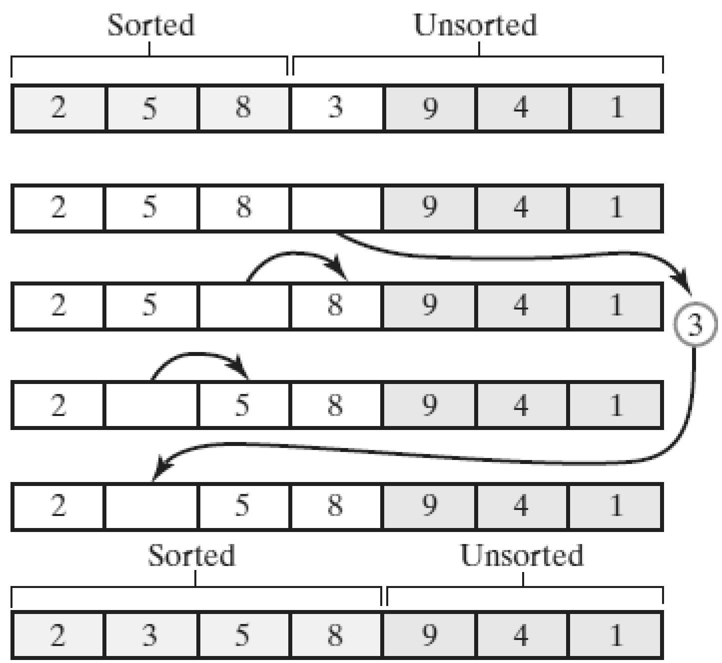
1. Consider the **first** item to be a **sorted sublist** of length 1
2. Insert the **second** item into the **sorted sublist**, shifting the first item if needed
3. Insert the **third** item into the **sorted sublist**, shifting the other items as needed
4. Repeat **until all values have been inserted into their proper positions**



Insertion Sort



INSERTION SORT



```java
    // Insert anEntry, the next unsorted entry, in the sorted sublist
    //
    private static <T extends Comparable<? super T>>
            void insertInOrder(T anEntry, T[] a, int begin, int end) {

        int index = end; // End of the sublist

        // Keep scanning the sublist from end to beginning
        // Compare anEntry vs. last entry (a[index])
        // Shift a[index] towards the end to make room for anEntry
        //
        while ((index >= begin) && (anEntry.compareTo(a[index]) < 0)) {
            a[index + 1] = a[index];                    // Make room
            index--;                                    // Next left entry
        } // end for

        a[index + 1] = anEntry;  // Insert
    }

```

Insertion Sort

```java

public static <T extends Comparable<? super T>>
        void insertionSort(T[] a, int n) {

    insertionSort(a, 0, n - 1);
}

public static <T extends Comparable<? super T>>
        void insertionSort(T[] a, int first, int last) {

    // Start at position 1, not 0
    //
    for (int unsorted = first + 1; unsorted <= last; unsorted++) {

        T firstUnsorted = a[unsorted]; // first of Unsorted

        insertInOrder(firstUnsorted, a, first, unsorted - 1);
    }
}

```

SHELL SORT


Shell Sort

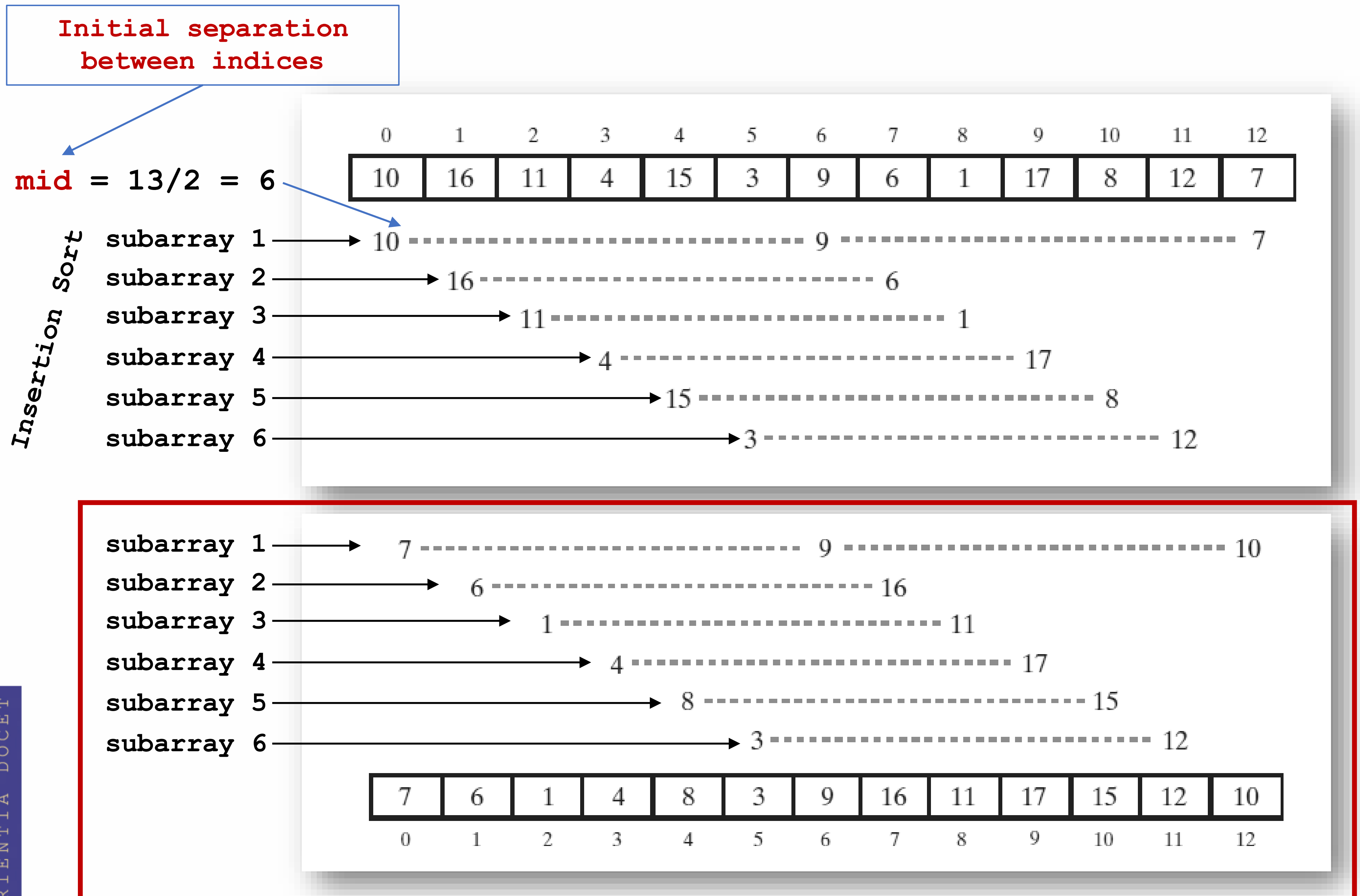
Shell Sort

- **Shell Sort** is a variation of the Insertion Sort which is faster than $O(n^2)$
- Observation during Insertion Sort:
  - When an entry is far from its correct sorted position → **many moves**
  - When an array is completely scrambled → Insertion Sort: **extremely inefficient**
  - When an array is almost sorted → Insertion Sort = Shell Sort: **more efficient**
  - Donald Shell in 1959:
    - **(Insertion) sort subarrays of entries** at **equally spaced indices**.
      - **Initial separation between indices be n/2**
      - **Sort each subarrays separately using insertion sort**
      - **Halve this value each pass**
      - **Sort each subarrays separately using insertion sort**
      - **Repeat** until the **separation is 1**
      - **Final step: Insertion sort the entire array.**

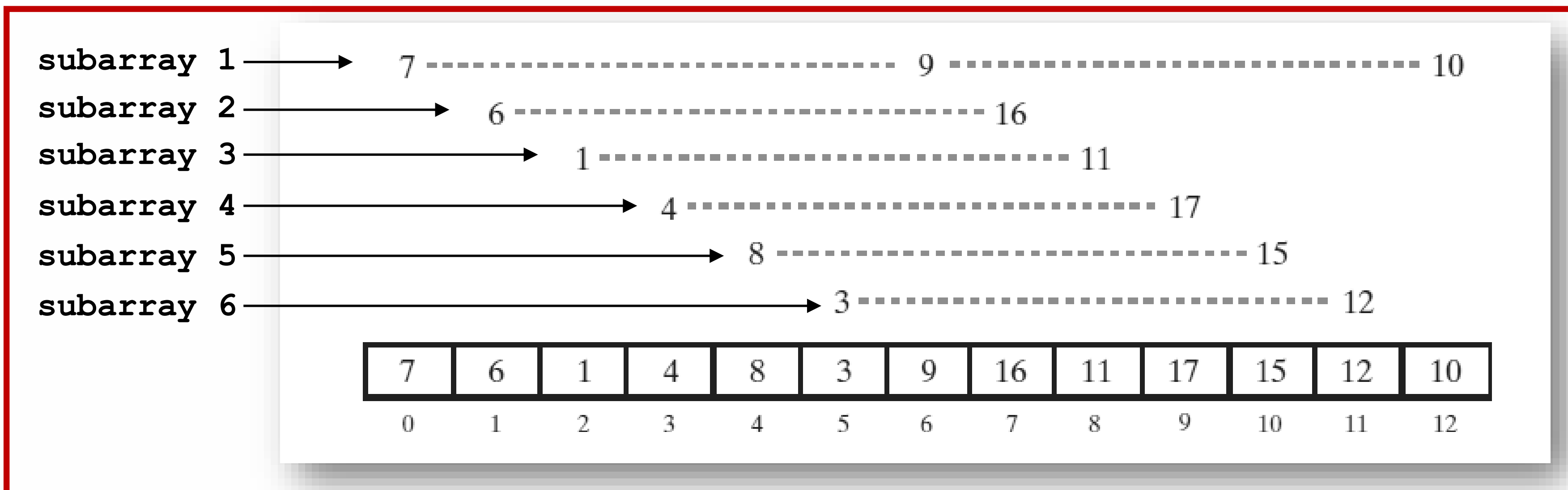
Almost Sorted n/2

EXPERIENTIA DOCET

Shell Sort


Initial separation between indices
mid = 13/2 = 6
Insertion Sort
0 1 2 3 4 5 6 7 8 9 10 11 12
10 16 11 4 15 3 9 6 1 17 8 12 7
subarray 1 → 10 9 7
subarray 2 → 16 6
subarray 3 → 11 1
subarray 4 → 4 17
subarray 5 → 15 8
subarray 6 → 3 12
subarray 1 → 7 9 10
subarray 2 → 6 16
subarray 3 → 1 11
subarray 4 → 4 17
subarray 5 → 8 15
subarray 6 → 3 12
7 6 1 4 8 3 9 16 11 17 15 12 10
0 1 2 3 4 5 6 7 8 9 10 11 12

SHELL SORT



subarray 1
subarray 2
subarray 3
subarray 4
subarray 5
subarray 6
7 9 10
6 16
1 11
4 17
8 15
3 12
7 6 1 4 8 3 9 16 11 17 15 12 10
0 1 2 3 4 5 6 7 8 9 10 11 12

Shell Sort

2rd round, separation between indices

mid = 6/2 = 3

0	1	2	3	4	5	6	7	8	9	10	11	12
7	6	1	4	8	3	9	16	11	17	15	12	10

subarray 7 → 7 - - - 4 - - - 9 - - - 17 - - - 10
subarray 8 → 6 - - - 8 - - - 16 - - - 15
subarray 9 → 1 - - - 3 - - - 11 - - - 12

subarray 1 → 7 - - - 9 - - - 10
subarray 2 → 6 - - - 16
subarray 3 → 1 - - - 11
subarray 4 → 4 - - - 17
subarray 5 → 8 - - - 15
subarray 6 → 3 - - - 12

0	1	2	3	4	5	6	7	8	9	10	11	12
7	6	1	4	8	3	9	16	11	17	15	12	10

SHELL SORT

2rd round, separation between indices

mid = 6/2 = 3

0	1	2	3	4	5	6	7	8	9	10	11	12
7	6	1	4	8	3	9	16	11	17	15	12	10

Insertion Sort

subarray 7 → 7 - - - 4 - - - 9 - - - 17 - - - 10

subarray 8 → 6 - - - 8 - - - 16 - - - 15

subarray 9 → 1 - - - 3 - - - 11 - - - 12

subarray 7 → 4 - - - 7 - - - 9 - - - 10 - - - 17

subarray 8 → 6 - - - 8 - - - 15 - - - 16

subarray 9 → 1 - - - 3 - - - 11 - - - 12

4	6	1	7	8	3	9	15	11	10	16	12	17
0	1	2	3	4	5	6	7	8	9	10	11	12

3rd round, separation between indices

mid = 3/2 = 1

Insertion Sort

4	6	1	7	8	3	9	15	11	10	16	12	17
0	1	2	3	4	5	6	7	8	9	10	11	12

3rd round, separation between indices

mid = 3/2 = 1

Insertion Sort

4	6	1	7	8	3	9	15	11	10	16	12	17
0	1	2	3	4	5	6	7	8	9	10	11	12

Sorted Sublist

1	3	4	6	7	8	9	10	11	12	15	16	17

```java
// Sorts equally spaced elements of an array into ascending order.
// Parameters:
//     a      An array of Comparable objects.
//     first  The integer index of the first array entry to
//            consider; first >= 0 and < a.length.
//     last   The integer index of the last array entry to
//            consider; last >= first and < a.length.
//     space  The difference between the indices of the
//            entries to sort.
private static <T extends Comparable<? super T>>
        void incrementalInsertionSort(T[] a, int first, int last, int space) {

    int unsorted, index;

    for (unsorted = first + space; unsorted <= last;
            unsorted = unsorted + space) {

        T nextToInsert = a[unsorted];
        index = unsorted - space;

        while ((index >= first) && (nextToInsert.compareTo(a[index]) < 0)) {
            a[index + space] = a[index];
            index = index - space;
        }

        a[index + space] = nextToInsert;
    }
}
```

```java

    public static <T extends Comparable<? super T>>
            void shellSort(T[] a, int n) {

        shellSort(a, 0, n - 1);

    } // end shellSort

    /**
     * Sorts equally spaced elements of an array into ascending order.
     *
     * @param a An array of Comparable objects.
     * @param first An integer >= 0 that is the index of the first array element
     * to consider.
     * @param last An integer >= first and < a.length that is the index of the
     * last array element to consider. @param space The difference between the
     * indices of the eleme nts to sort.
     */
    public static <T extends Comparable<? super T>>
            void shellSort(T[] a, int first, int last) {

        int n = last - first + 1; // Number of array entries
        int space = n / 2;

        while (space > 0) {
            for (int begin = first; begin < first + space; begin++) {
                incrementalInsertionSort(a, begin, last, space);
            }
            space = space / 2;
        }
    }

```

MORE EXAMPLES

Selection Sort

7 5 2 9 1 1 4 3 6

Show the contents of the array above **each time** an **Selection Sort** changes it while sorting the array into **ascending order**.

Smallest → Final Position

	7.	5	2	9	1.	1	4	3	6
	1	5.	2	9	7	1.	4	3	6
	✓	1	2✓	9	7	5	4	3	6
		✓	2	9.	7	5	4	3.	6
			✓	3	7.	5	4.	9	6
				✓	4	5✓	7	9	6
					✓	5	7.	9	6.
						✓	6	9.	7.
							✓	7	9
								✓	✓
✓	1	1	2	3	4	5	6	7	9

7 5 2 9 1 1 4 3 6

Show the contents of the array above **each time** an **Insertion Sort** changes it while sorting the array into **ascending order**.

Sorted [Sublist]

[7]	5	2	9	1	1	4	3	6
[7.	5.]							
[5.	7	2.]						
[2	5	7	9]					
[2.	5	7	9	1.]				
[1	2.	5	7	9	1.]			
[1	1	2	5.	7	9	4.]		
[1	1	2	4.	5	7	9	3.]	
[1	1	2	3	4	5	7.	9	6.]
1	1	2	3	4	5	6	7	9

8 5 3 2 2 9 3 7 4 6 Almost Sorted n/2

Show the contents of the array above **each time a Shell Sort** changes it while sorting the array into **ascending order**.

											mid
8	5	3	2	2	9	3	7	4	6		10/2=5
8.					9.						
	3.					5.					
		3.					7.				
			2.					4.			
				2.					6.		
8	3	3	2	2	9	5	7	4	6		5/2=2
2.		3.		4.		5.		8.			
	2.		3.		6.		7.		9.		
[2]	2	3	3	4	6	5	7	8	9		2/2=1 *
[2	2]										
[2	2	3]									
[2	2	3	3]								
[2	2	3	3	4]							
[2	2	3	3	4	6]						
[2	2	3	3	4	6.	5.]					
[2	2	3	3	4	5	6	7]				
[2	2	3	3	4	5	6	7	8]			
[2	2	3	3	4	5	6	7	8	9]		
2	2	3	3	4	5	6	7	8	9		

8 5 3 2 2 9 3 7 4 Almost Sorted n/2

Show the contents of the array above **each time a Shell Sort** changes it while sorting the array into **ascending order**.

nud

	8	5	3	2	2	9	3	7	4		9/2 = 4
	2.				4.				8.		
		5.				9.					
			3.				3.				
				2.				7.			
	2	5	3	2	4	9	3	7	8		4/2 = 2
	2.		3.		3.		4.		8.		
		2.		5.		7.		9.			
*	[2]	2	3	5	3	7	4	9	8		2/2 = 1
	[2	2]									
	[2	2	3]								
	[2	2	3	5]							
	[2	2	3	5.	3.]						
	[2	2	3	3	5	7]					
	[2	2	3	3	5.	7	4.]				
	[2	2	3	3	4	5	7	9]			
	[2	2	3	3	4	5	7	9.	8.]		
✓	2	2	3	3	4	5	7	8	9		

9 5 6 4 3 9 3 8 2 Almost Sorted n/2

Show the contents of the array above **each time a Shell Sort** changes it while sorting the array into **ascending order**.

mid

9	5	6	4	3	9	3	8	2		9/2 = 4	
2.				3.				9.			
	5.				9.						
		3.				6.					
			4.				8.				
2	5	3	4	3	9	6	8	9		4/2 = 2	
2.		3.		3.		6.		9.			
	4.		5.		8.		9.				
[2]	4	3	5	3	8	6	9	9		2/2 = 1	*
[2	4]										
[2	4.	3.]									
[2	3	4	5]								
[2	3	4.	5	3.]							
[2	3	3	4	5	8]						
[2	3	3	4	5	8.	6.]					
[2	3	3	4	5	6	8	9]				
[2	3	3	4	5	6	8	9	9]			
2	3	3	4	5	6	8	9	9			

COMPARE THE SORTING ALGORITHMS

Selection Sort

```
public static <T extends Comparable<? super T>> void selectionSort(T[] a, int n) {
        for (int index = 0; index < n - 1; index++) {
            int indexOfNextSmallest = getIndexOfSmallest(a, index, n - 1);
            swap(a, index, indexOfNextSmallest);
        }
}
```

n – 1, for-loop

Worst case, **O(n)**

Always 3 ops, **O(1)**

$(n - 1) * (O(n) + O(1)) = (n - 1) * O(n) = O(n^2)$

simplify -> **simplify ->**

swap():

Always 3 steps → Number of Operations is independent of number of entries in an array → **O(1)**

Data movement, **O(n)** swap, little movement.

```
private static void swap(Object[] a, int i, int j) {
    Object temp = a[i];
    a[i] = a[j];
    a[j] = temp;
}
```

```
private static <T extends Comparable<? super T>>
        int getIndexOfSmallest(T[] a, int first, int last) {

    T min = a[first];
    int indexOfMin = first;

    for (int index = first + 1; index <= last; index++) {

        if (a[index].compareTo(min) < 0) {
            min = a[index];
            indexOfMin = index;
        }
    }

    return indexOfMin;
}
```

getIndexOfSmallest():

last = n – 1

first ranges from 0 to n-2

Each time the 2 loops execute last – first. Worst (n - 1) – 0 = n - 1. Best (n - 1) – (n - 2) = 1.

Then total: (n - 1) + (n - 2) + … + 1 = n (n – 1) / 2 = ½ $\mathbf{n^2}$ – ½ **n**

The selection sort then is $\mathbf{O(n^2)}$

or

The worst: **O (n)** for n comparisons, 1 basic operation compareTo() each time.

Outer for-loop runs n -1 so the selection sort then is (n – 1) * $\mathbf{O(n) = O(n^2)}$

Insertion Sort

```
public static <T extends Comparable<? super T>>
        void insertionSort(T[] a, int n) {

    insertionSort(a, 0, n - 1);
}

public static <T extends Comparable<? super T>>
        void insertionSort(T[] a, int first, int last) {

    // Start at position 1, not 0
    //
    for (int unsorted = first + 1; unsorted <= last; unsorted++) {

        T firstUnsorted = a[unsorted]; // first of Unsorted

        insertInOrder(firstUnsorted, a, first, unsorted - 1);
    }
}
```

n – 1, for loop

Worst case, **O(n)**

Worst Case: (n – 1) * O(n) = O(n^2)
Best Case: (n – 1) * O(1) = O(n)

Insertion Sort

```java
private static <T extends Comparable<? super T>>
        void insertInOrder(T anEntry, T[] a, int begin, int end) {

    int index = end; // End of the sublist

    // Keep scanning the sublist from end to beginning
    // Compare anEntry vs. last entry (a[index])
    // Shift a[index] towards the end to make room for anEntry
    //
    while ((index >= begin) && (anEntry.compareTo(a[index]) < 0)) {
        a[index + 1] = a[index];            // Make room
        index--;                            // Next left entry
    } // end for

    a[index + 1] = anEntry;  // Insert
}
```

insertInOrder():

begin is 0

end ranges from 0 to n – 2

1 basic operation compareTo() each time.

Worst case O(n). Best case O(1).

Total: 1 + 2 + … + (n-1) = n(n-1)/2

Thus, insertion sort is **O(n²)** ← worst case

When array already sorted, **O(n)** ← best case

```
public static <T extends Comparable<? super T>>
        void shellSort(T[] a, int first, int last) {

    int n = last - first + 1; // Number of array entries
    int space = n / 2;

    while (space > 0) {
        for (int begin = first; begin < first + space; begin++) {
  1         incrementalInsertionSort(a, begin, last, space);
        }
        space = space / 2;
    }
}
```

```
private static <T extends Comparable<? super T>>
        void incrementalInsertionSort(T[] a, int first, int last, int space) {

    int unsorted, index;

    for (unsorted = first + space; unsorted <= last;
2            unsorted = unsorted + space) {

        T nextToInsert = a[unsorted];
        index = unsorted - space;

        while ((index >= first) && (nextToInsert.compareTo(a[index]) < 0)) {
     3      a[index + space] = a[index];
            index = index - space;
        }

        a[index + space] = nextToInsert;
    }
}
```

O(n^3), 3 nested loops
but space n/2 each time
O($n^{1.5}$) is the worst-case of Shell
O(n) is the best, an ordinary insertion sort

Time efficiencies in Big Oh notation			
	Best	Average	Worst
Selection Sort	**$O(n^2)$**	**$O(n^2)$**	**$O(n^2)$**
Insertion Sort	**$O(n)$**	**$O(n^2)$**	**$O(n^2)$**
Shell Sort	**$O(n)$**	**$O(n^{1.5})$**	**$O(n^{1.5})$**

Array Sorting Algorithms

Algorithm	Time Complexity			Space Complexity
	Best	Average	Worst	Worst
Quicksort	Ω(n log(n))	Θ(n log(n))	O(n^2)	O(log(n))
Mergesort	Ω(n log(n))	Θ(n log(n))	O(n log(n))	O(n)
Timsort	Ω(n)	Θ(n log(n))	O(n log(n))	O(n)
Heapsort	Ω(n log(n))	Θ(n log(n))	O(n log(n))	O(1)
Bubble Sort	Ω(n)	Θ(n^2)	O(n^2)	O(1)
Insertion Sort	Ω(n)	Θ(n^2)	O(n^2)	O(1)
Selection Sort	Ω(n^2)	Θ(n^2)	O(n^2)	O(1)
Tree Sort	Ω(n log(n))	Θ(n log(n))	O(n^2)	O(n)
Shell Sort	Ω(n log(n))	Θ(n(log(n))^2)	O(n(log(n))^2)	O(1)
Bucket Sort	Ω(n+k)	Θ(n+k)	O(n^2)	O(n)

	Best Case	Average Case	Worst Case
Selection sort	$O(n^2)$	$O(n^2)$	$O(n^2)$
Insertion sort	$O(n)$	$O(n^2)$	$O(n^2)$
Shell sort	$O(n)$	$O(n^{1.5})$	$O(n^2)$ or $O(n^{1.5})$

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Comparing the Algorithms

Array Sorting Algorithms

Algorithm	Time Complexity			Space Complexity
	Best	Average	Worst	Worst
Quicksort	Ω(n log(n))	Θ(n log(n))	O(n^2)	O(log(n))
Mergesort	Ω(n log(n))	Θ(n log(n))	O(n log(n))	O(n)
Timsort	Ω(n)	Θ(n log(n))	O(n log(n))	O(n)
Heapsort	Ω(n log(n))	Θ(n log(n))	O(n log(n))	O(1)
Bubble Sort	Ω(n)	Θ(n^2)	O(n^2)	O(1)
Insertion Sort	Ω(n)	Θ(n^2)	O(n^2)	O(1)
Selection Sort	Ω(n^2)	Θ(n^2)	O(n^2)	O(1)
Tree Sort	Ω(n log(n))	Θ(n log(n))	O(n^2)	O(n)
Shell Sort	Ω(n log(n))	Θ(n(log(n))^2)	O(n(log(n))^2)	O(1)
Bucket Sort	Ω(n+k)	Θ(n+k)	O(n^2)	O(n)
Radix Sort	Ω(nk)	Θ(nk)	O(nk)	O(n+k)
Counting Sort	Ω(n+k)	Θ(n+k)	O(n+k)	O(k)
Cubesort	Ω(n)	Θ(n log(n))	O(n log(n))	O(n)

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See you next class!

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